Efficient Prices under Uncertainty and Non-Convexity

Brent Eldridge[1], Bernard Knueven[2], and Jacob Mays*[3]

[1]Environmental Health and Engineering Department, Johns Hopkins University
[2]Computational Science Center, National Renewable Energy Laboratory†
[3]School of Civil and Environmental Engineering, Cornell University

January 25, 2022

Abstract

Operators of organized wholesale electricity markets attempt to form prices in such a way that the private incentives of market participants are consistent with a socially optimal commitment and dispatch schedule. In the U.S. context, several competing price formation schemes have been proposed to address the non-convex production cost functions characteristic of most generation technologies. This paper considers how the design and analysis of price formation policies for non-convex markets is affected by the uncertainty inherent in electricity demand and supply. We argue that by excluding uncertainty, the analytical framework underlying existing policies mischaracterizes the incentives of market participants, leading to inefficient price formation and poor incentives for flexibility. We establish favorable theoretical properties of a new construct, *ex ante convex hull pricing*, demonstrate the difference between this policy and existing methods on a large-scale test system, and discuss the implications for price formation in organized wholesale markets.

Keywords: Electricity market design, price formation, uplift

1 Introduction

In recent years, many organized wholesale electricity markets in the U.S. have implemented enhanced pricing schemes intending to address the non-convex production cost functions characteristic of most generation technologies. Since it prevents the formation of uniform clearing prices, non-convexity can lead to a difference between the profit market participants would gain by maximizing their individual benefit and that they obtain by following the socially optimal schedule. The non-convex price formation problem has proven challenging, motivating a large number of

*jacobmays@cornell.edu

†This work was authored in part by the National Renewable Energy Laboratory, operated by Alliance for Sustainable Energy, LLC, for the U.S. Department of Energy (DOE) under Contract No. DE-AC36-08GO28308. The views expressed in the article do not necessarily represent the views of the DOE or the U.S. Government.

competing proposals [33], modeling advances to enable computation of prices under these proposals [20, 29], and large-scale tests aimed at understanding the economic consequences of their adoption [8, 16, 21, 37]. Efforts to address non-convexity have led to the introduction of Extended LMP in the Midcontinent Independent System Operator (MISO) [48] and Fast-Start Pricing in ISO New England (ISO-NE) [22], the New York Independent System Operator (NYISO) [9], PJM Interconnection (PJM) [10], and the Southwest Power Pool (SPP) [11].

This paper argues that non-convex price formation efforts to date have been aimed at an incorrect target, with negative consequences for the remuneration of flexible resources. While difficult in its own right, the version of the non-convex price formation problem generally addressed in the literature understates the real-world challenge. The main focus in the literature is the binary commitment variables included in deterministic day-ahead market models covering at least 24 hours of operations. Deterministic models that include only physical resources, however, do not fully capture the price formation process in U.S. day-ahead markets. As a forward market that includes financial participants attempting to profit through arbitrage, the primary determinant of prices in the day-ahead market is expectations for real-time prices on the subsequent day [23, 32]. In U.S. markets, these real-time prices are calculated every five minutes using a simpler economic dispatch model that excludes binary variables, covers a much smaller time frame than the day-ahead market, and updates as new information is available throughout the day. In this setting, the challenge is not to identify a complete set of prices that accurately conveys the cost to serve load over a 24-hour operating period, but instead to identify a policy for setting spot prices in each individual five-minute period such that the generated sequence of prices preserves any needed information about non-convexity and intertemporal constraints contributing to overall cost. Due to this complication, the relationship between price formation proposals with favorable properties described in the literature and those implemented in real-world markets is unclear. Along these lines, this paper proposes a new target for the non-convex price formation problem, the definition of which requires a clear distinction between spot and forward prices within the price formation framework.

Establishing this new target entails a consideration of uncertainty in addition to non-convexity. Price formation often depends on past decisions that were made without perfect information, so accounting for uncertainty has important implications for the interpretation of competing policies.

Among uniform pricing schemes, convex hull pricing was proposed as a "best compromise" between a convex market equilibrium and what is obtainable in a market with non-convexities [15, 19, 44]. It has the property of minimizing a particular definition of lost opportunity costs, i.e., the difference between the profit market participants could have gained by maximizing their individual benefit and what they obtain by following a socially optimal schedule. Informally, these prices are often described as "ideal" because they are thought to align the incentives of market participants and the system operator to the extent possible given underlying non-convexity [4]. This description of aligned incentives, however, results from a deterministic analysis where generators respond to a known market price. As shown in [36], convex hull pricing as traditionally defined can give poor incentives in the presence of uncertainty. In addition to non-convex production costs, thermal generators have intertemporal constraints linking decisions in present intervals with the future. An appropriate characterization of generator incentives therefore requires consideration of commitment decisions that are made under uncertainty of real-time conditions. The common definition of lost opportunity costs overestimates the incentives for generators to deviate from the system operator's commitment and dispatch instructions because it assumes that these past commitment decisions could have been made with perfect knowledge of real-time prices.

To achieve better alignment between social and private incentives given both non-convexity and uncertainty, this paper defines a new pricing construct, *ex ante convex hull pricing*, and shows that generating prices with this policy minimizes expected lost opportunity costs at the time commitment decisions must be made. We complement this theoretical result with a computational demonstration on an ISO-scale system with a large scenario set leveraging state-of-the-art stochastic programming methods to compare market outcomes of different pricing schemes, including the traditional ex post locational marginal pricing (LMP), the ex post convex hull pricing (EP-CHP) sometimes described as an ideal in the literature, two variants of the fast-start pricing (FSP) implemented in several real-world markets, and the proposed ex ante convex hull pricing (EA-CHP). We discuss the results in the context of ongoing debates about price formation in U.S. markets.

The numerical tests support two recommendations relevant to these policy debates. First, with the growth of wind and solar, many systems have become particularly concerned with ensuring sufficient flexible resources to manage variability and uncertainty. Consistent with [35], our tests suggest that current pricing schemes could suppress the volatility in prices necessary to attract investment in flexible resources. Second, currently accepted definitions of uplift include losses due to uncertainty as well as non-convexity. By misdiagnosing losses from uncertainty as instead arising due to non-convexity, uplift payments often described as "necessary" to providing good incentives can instead inappropriately subsidize inflexibility [36].

Given the potential for greater price volatility and a higher probability of incurring losses under efficient prices, the paper also assesses the need for instruments to reduce financial risk and align the incentives of risk-averse market participants in short-term markets. Particularly important in the context of U.S. systems are day-ahead markets. In addition to helping manage financial risk, trading in the day-ahead market can improve the physical performance of power systems by pushing the solution of the deterministic market clearing model toward that of the true underlying stochastic problem [23, 24, 25, 34]. In practice, many complications can interfere with this salubrious property [41]. In an extreme example, the root cause analysis of the August 2020 outages in California argues that virtual bidding contributed to the need for rolling blackouts [3]. This paper highlights that the effect of forward contracts on physical system performance depends on the efficiency of the spot prices on which they are based. In this context, while suppressing volatility of spot prices relative to the ideal may have risk reduction benefits, a better remedy for losses due to uncertainty may be ensuring that market participants have greater ability to trade risk [7, 12]. To manage increasing variability and uncertainty due to the growth of wind and solar, some have suggested the introduction of intraday markets [18, 47]. Our results suggest that intraday markets could help reduce the perceived need for uplift payments to address misaligned incentives. However, along the lines of [40], fixed-quantity swaps alone do not give an efficient way to manage risk associated with the positive correlation between price and dispatch quantity for most near-marginal generators, suggesting that a more effective approach may be to supplement day-ahead markets with option-like instruments.

We describe the price formation problem, define ex ante convex hull pricing in a general way, and describe its theoretical properties in Section 2. Section 3 develops a small example system to motivate the discussion, illustrate the key economic phenomena, and demonstrate the potential failure modes of seemingly plausible price formation policies that neglect the effect of uncertainty. Section 4 builds a large-scale example to provide a more complete demonstration of the properties of competing pricing schemes. We conclude in Section 5.

2 Commitment, Dispatch, and Pricing

Operators of organized wholesale markets seek to identify commitment and dispatch schedules that maximize market surplus, as well as prices that support those optimal operational schedules. Our first task is therefore to identify a surplus-maximizing solution to the operational problem, for which we define a stochastic program. It bears mentioning that the stochastic program we define is a simplification of the true operational problem, which would be better characterized in the framework of sequential decision making under uncertainty. Given our focus on pricing rather than operations, our strategy is to describe a problem that is complex enough to reveal the economic properties of competing pricing policies, but not so complex as to obscure the economic analysis.

At the same time, we do not assume that the market operator explicitly employs stochastic programming for either operations or market clearing (cf. [6, 42, 50, 51]). At present, system operators typically use a series of deterministic models to calculate commitment and dispatch schedules and market prices. While the formulations are deterministic, the presence of reserve products, virtual bidders, and operator actions can push toward a higher-quality solution of the underlying stochastic problem [23, 24, 25, 31, 34, 36]. What is important for our purposes is not the means by which a commitment and dispatch is identified, but the ability to describe a complete solution for all scenarios.

The section then turns to the question of what price formation policy would best support that optimal commitment and dispatch. Focusing on the joint impact of uncertainty and non-convexity, we define a set of pricing models associated with the commitment and dispatch decisions. Here our representation of the system as a stochastic program becomes important, as it determines market schedules in each possible real-time scenario. Three ex post pricing models are defined that consider each real-time scenario independently. An ex ante pricing model is then defined that considers linkages between real-time scenarios that arise due to uncertainty and intertemporal constraints. We consider real-time scenarios from a discrete probability distribution that accurately represents all possible states of the world and the market operator and participants' beliefs. As will become clear, this assumption poses a serious challenge for implementation of the idealized ex ante convex hull pricing. In pursuit of practical implementations, future in-depth studies of competing approaches would benefit from more realistic simulations of rolling horizon decision models, the dynamic revelation of uncertainties, agents with asymmetric information and risk preferences, and decentralized methods to coordinate market-based pricing and scheduling decisions.

2.1 Models

2.1.1 Stochastic Unit Commitment.

The stochastic unit commitment problem (SUC) below minimizes the total production cost z^{SUC} considering dispatch decisions $x_{ns} \in \mathbb{R}$ and commitment $y_{ns} \in \mathbb{Z}$ for a set of generators $n \in \mathcal{N}$ and scenarios $s \in \mathcal{S}$. Here we consider a slight modification of the classic two-stage setup [5, 46], adding an additional intermediate stage to capture fast-start commitments, for a total of three stages. The generators are split into a subset of fast-start generators $\mathcal{F} \subset \mathcal{N}$ and slow-start generators $\hat{\mathcal{F}} = \mathcal{N} \setminus \mathcal{F}$. In the first stage, commitment decisions for all slow-start generators must be determined. Between the first and second stages, we gain knowledge about the uncertainties, allowing us to narrow the possible scenarios to a subset $\mathcal{S}_r \subset \mathcal{S}$. In the second stage, commitment decisions for all fast-start generators are made. Lastly, in the third stage, uncertainties are revealed and dispatch decisions for all generators are made subject to the commitment decisions from prior epochs. The optimization problem is formulated below.

$$\min \quad z^{SUC} = \sum_s \rho_s \left(c^\top x_s + d^\top y_s \right) \tag{1a}$$

$$\text{s.t.:} \quad \rho_s \left(A_0 x_s - b_{0s} \right) \geq 0 \qquad \forall s \in \mathcal{S} \tag{1b}$$

$$A_{ns} x_{ns} + B_{ns} y_{ns} \geq b_{ns} \qquad \forall n \in \mathcal{N}, \forall s \in \mathcal{S} \tag{1c}$$

$$y_{ns} \in \{0, 1\} \qquad \forall n \in \mathcal{N}, \forall s \in \mathcal{S} \tag{1d}$$

$$y_{ns} = y'_{n0} \qquad \forall n \in \hat{\mathcal{F}}, \forall s \in \mathcal{S} \tag{1e}$$

$$y_{ns} = y'_{nr} \qquad \forall n \in \mathcal{F}, \forall r \in \mathcal{R}, \forall s \in \mathcal{S}_r. \tag{1f}$$

Note that the generator dispatch and commitment decisions are written more succinctly as the column vectors $x_s = [x_{1s}, \ldots, x_{Ns}]$ and $y_s = [y_{1s}, \ldots, y_{Ns}]$ where appropriate. Additional decision variables y'_{n0} and y'_{nr} define the first stage commitment decisions of slow-start resources and the second stage commitment decisions of fast-start resources, respectively. Constraint (1b) includes the system-wide constraints that define each scenario s. Constraints (1c) and (1d) include generator-specific feasibility and binary constraints, and constraints (1e) and (1f) define the nonanticipativity constraints of slow- and fast-start resources, respectively. The problem's parameters include scenario probability ρ_s, dispatch costs c, fixed start-up and no-load costs d, system-wide constraint matrix A_0, scenario s's system requirements b_{0s}, generator n's constraint matrices A_{ns} and B_{ns}, and generator n's constraint limits b_{ns}. Scenario-dependent generator constraints can be used to model economic offers by wind, solar, or other renewable resources that have uncertain maximum output, as well as possible contingency scenarios for traditional thermal generators. Solutions to (1) provide optimal commitment y_s^* and dispatch x_s^* decisions for each scenario $s \in \mathcal{S}$. When the s is dropped from the notation, it will be understood that x^* and y^* refer to the full solution to model (1). For convenience, z_s^* will be used to refer to the optimal system cost in each scenario.

We define the following notation for the nonanticipativity constraints (1e) and (1f):

$$\mathcal{Y}_{ns} := \left\{ y_{ns} : \left\{ \begin{array}{ll} y_{ns} = y'_{n0}, & \forall n \in \hat{\mathcal{F}}, \forall s \in \mathcal{S}, \text{ or} \\ y_{ns} = y'_{nr}, & \forall n \in \mathcal{F}, \forall s \in \mathcal{S}_r, \forall r \in \mathcal{R} \end{array} \right\} \right\}$$

To simplify notation of the generator-level constraints, we define feasibility sets utilizing these representations of the nonanticipativity constraints:

$$\begin{aligned} \mathcal{X}_{ns} &:= \{(x_{ns}, y_{ns}) : A_{ns}x_{ns} + B_{ns}y_{ns} \geq b_{ns}, y_{ns} \in \{0,1\}\} && \forall n \in \mathcal{N}, \forall s \in \mathcal{S}; \\ \mathcal{X}_n &:= \{(x_n, y_n) : (x_{ns}, y_{ns}) \in \mathcal{X}_{ns},\ y_{ns} \in \mathcal{Y}_{ns},\ \forall s \in \mathcal{S}\} && \forall n \in \mathcal{N}, \end{aligned}$$

where $x_n = [x_{n1}, \ldots, x_{nS}]$ and $y_n = [y_{n1}, \ldots, y_{nS}]$ will refer to generator n's dispatch and commitment decisions, respectively, across all scenarios $\mathcal{S} = \{1, \ldots, S\}$.

In addition, we define the sets $\mathcal{X}_{ns}^C$ and $\mathcal{X}_n^C$ to be the convex hull relaxation of the associated set, i.e.:

$$\mathcal{X}_{ns}^C := \text{conv}(\mathcal{X}_{ns}); \qquad \mathcal{X}_n^C := \text{conv}(\mathcal{X}_n).$$

In general, a formulation for $\mathcal{X}_{ns}^C$ may be insufficient to ensure that a compact formulation for $\mathcal{X}_n^C$ is easily obtainable [38].

By formulating (1) generically, the proceeding analysis can be applied to many unit commitment-based (or more generally integer programming-based) market designs. In addition to system power balance, the constraint matrix A_0 may include constraints defining power flows in the transmission system and ancillary service definitions including spinning reserves, operating reserves and/or ramping products. Similarly, system requirements b_{0s} would typically define each scenario based on different levels of hourly inelastic energy demand and behind-the-meter solar output. It may also reflect scenario-specific transmission line limits in cases where dynamic line ratings are used. Also note that, while demand information would typically be given as fixed information in the b_{0s} parameters, the dispatch decisions x_{ns} have no explicit sign restriction and could therefore be used to model controllable or price-responsive loads in addition to economic offers from renewable resources and traditional thermal generators.

2.1.2 Ex Post Pricing.

Prices are calculated after dispatch decisions are decided. Therefore, the following pricing models consider the scenario s to be fixed and do not consider the probabilities ρ_s. This section will first formulate a few of the standard pricing models that are found in the electricity pricing literature. The dual variables of each pricing model constraint are shown in brackets to the right of each model.

The traditional method to calculate LMPs is formally presented in [39] and consists of fixing all binary variables to their optimal value in the solution to (1). It can be written as follows.

$$\min_{x_s, y_s} \quad z_s^{LMP} = c^\top x_s + d^\top y_s \tag{2a}$$

$$\text{s.t.:} \quad A_0 x_s \geq b_{0s} \qquad [\lambda_s^{LMP}] \tag{2b}$$

$$A_{ns} x_{ns} + B_{ns} y_{ns} \geq b_{ns} \qquad [\sigma_{ns}^{LMP}] \quad \forall n \in \mathcal{N} \tag{2c}$$

$$y_{ns} = y_{ns}^* \qquad [\delta_{ns}^{LMP}] \quad \forall n \in \mathcal{N}. \tag{2d}$$

The pricing model (2) has two advantageous properties since it is a linear program and obtains the same optimal objective function value as model (1) when weighted across all scenarios. Similarly to the optimal dispatch and commitment variables x^* and y^*, for LMP and subsequent pricing schemes, the notation λ^{LMP} indicates the vector of prices in all scenarios, i.e., $\lambda^{LMP} = [\lambda_s^{LMP}]_{s \in \mathcal{S}}$.

The model's prices λ^{LMP} provide the correct signal for each generator to produce the socially optimal production quantities x_s^* in each scenario s, given that the generators follow the socially optimal commitment solution y_s^*. Since generators may be otherwise unwilling to follow the commitment schedule y_s^*, revenues based on this traditional method of calculating LMPs are typically supplemented by make-whole payments equal to the difference between the generator's as-offered costs and market revenues, if positive. It is argued that large make-whole payments tend to dilute the pricing signals from model (2), which has motivated attempts to find new pricing policies.

The ex post convex hull pricing (EP-CHP) model, proposed by [15], is based on minimizing a broad category of side-payments called uplift payments. The pricing model can be formulated as below.

$$\min_{x_s,y_s} \quad z_s^{EP} = c^\top x_s + d^\top y_s \tag{3a}$$

$$\text{s.t.:} \quad A_0 x_s \geq b_{0s}, \qquad [\lambda_s^{EP}] \tag{3b}$$

$$(x_{ns}, y_{ns}) \in \mathcal{X}_{ns}^C \qquad [\sigma_{ns}^{EP}] \quad \forall n \in \mathcal{N}. \tag{3c}$$

It will be useful to compare the objective function values of each pricing model. Since constraint (3c) relaxes the constraints (2c) and (2d), it is clear that the EP-CHP model is a relaxation of the LMP model and therefore $z_s^{LMP} \geq z_s^{EP}$. The subsequent pricing schemes will also refer to the convex hull constraint set $\mathcal{X}_{ns}^C$.

Arguments in favor of the EP-CHP model are often based on improved generator dispatch incentives, since the perceived need for uplift payments is minimized. In contrast to make-whole payments, uplift is defined by the difference between the maximum operating profit that a generator can earn by changing its production schedule and the actual operating profit it earns by following the production schedule provided by the ISO. If it is assumed that generators can produce a zero quantity and therefore earn at least zero operating profit, then uplift payments are always greater than make-whole payments, and make-whole payments can be considered a component of uplift.

Despite the purported benefits of ex post CHP, no organized electricity market has implemented it to date. Instead, the MISO, PJM, ISO-NE, and NYISO markets have implemented Extended LMP and fast-start pricing (FSP) models that have some similarities to the original convex hull pricing proposal by [15] but only allow certain types of generators under certain conditions to be relaxed. Although some specifics differ between each market's implementation pricing models (see [9, 10, 22, 48]), the main aspects are captured in the model formulation below.

$$\min_{x_s,y_s} \quad z_s^{F1} = c^\top x_s + d^\top y_s \tag{4a}$$

$$\text{s.t.:} \quad A_0 x_s \geq b_{0s}, \qquad [\lambda_s^{F1}] \tag{4b}$$

$$(x_{ns}, y_{ns}) \in \mathcal{X}_{ns}^C \qquad [\sigma_{ns}^{F1}] \quad \forall n \in \mathcal{N} \tag{4c}$$

$$y_{ns} = y_{ns}^*, \qquad [\delta_{ns}^{\hat{F}1}] \quad \forall n \in \hat{\mathcal{F}} \tag{4d}$$

$$y_{ns} \leq y_{ns}^*, \qquad [\delta_{ns}^{F1}] \quad \forall n \in \mathcal{F}. \tag{4e}$$

Rather than relaxing the commitment constraints of all generators like the EP-CHP pricing model, the fast-start pricing model above (FSP-I) only relaxes the commitment variables of fast-start resources and only if those resources are committed in the schedule calculated by the ISO. Arguments in favor of adopting FSP models often make similar, though vague, appeals to improved price signals and lower uplift or make-whole payments as previously discussed for the EP-CHP model. However, the model (4) differs from (3) since far fewer commitments will typically be relaxed, comparing (2d) with (4d) and (4e). Therefore model (4) may be seen as only a small adjustment to the traditional LMP model (2).

An additional FSP model called FSP-II modifies FSP-I to relax all fast-start resources instead of only those that are committed by the ISO. This pricing model is formulated below.

$$\min_{x_s, y_s} \quad z_s^{F2} = c^\top x_s + d^\top y_s \tag{5a}$$

$$\text{s.t.:} \quad A_0 x_s \geq b_{0s}, \qquad [\lambda_s^{F2}] \tag{5b}$$

$$(x_{ns}, y_{ns}) \in \mathcal{X}_{ns}^C \qquad [\sigma_{ns}^{F2}] \quad \forall n \in \mathcal{N} \tag{5c}$$

$$y_{ns} = y_{ns}^*, \qquad [\delta_{ns}^{\hat{F}2}] \quad \forall n \in \hat{\mathcal{F}} \tag{5d}$$

$$y_{ns} \leq 1 \qquad [\delta_{ns}^{F2}] \quad \forall n \in \mathcal{F}. \tag{5e}$$

Due to the binary variable restrictions in each formulation, it can be seen that (3) is a relaxation of (5), (5) is a relaxation of (4), and (4) is a relaxation of (2). Therefore, the pricing model objective functions can be arranged in the order $z_s^{LMP} \geq z_s^{F1} \geq z_s^{F2} \geq z_s^{EP}$ for each scenario s.

Like EP-CHP and in contrast to FSP-I, prices from FSP-II may reflect the production costs of fast-start resources that are not dispatched by the ISO. However, this aspect of FSP-II and EP-CHP may plausibly be justified by the same appeals to improved price signals that motivate other modifications to the traditional LMP model (2).

2.1.3 Ex Ante Pricing.

A potential shortcoming of ex post pricing methods is that they may not provide incentives for generators to make efficient commitment decisions ex ante. For example, inefficient generators may have incentives to self-commit ex ante if there is an expectation of high ex post prices. Conversely, efficient generators may fail to make necessary ex ante arrangements, such as purchasing fuel contracts, if the expectation of ex post prices is too low. This paper therefore proposes a new pricing model called ex ante convex hull pricing (EA-CHP) which attempts to provide the best possible ex ante incentives.

In contrast to the ex post pricing policies, under which the spot price can be calculated from a deterministic model using data from only the realized scenario, EA-CHP computes a vector of prices covering all possible future states of the world and allows the prices in any state to be partially dependent upon conditions that might occur in other states. The appropriate price can then be selected from this vector after uncertainty is realized. This construction raises an important practical question regarding how to choose prices when the scenario realized in real time is not included in the lookahead stochastic model. Since we are primarily interested in comparing the economic properties of EA-CHP against the ex post policies, for this analysis we assume that the stochastic unit commitment problem includes all possible future scenarios.

The EA-CHP model is formulated analogously to the EP-CHP model for a deterministic unit commitment model, but is based on the stochastic unit commitment model (1), as shown below.

$$\min_{x,y} \quad z^{EA} = \sum_{s} \rho_s \left(c^\top x_s + d^\top y_s \right) \tag{6a}$$

$$\text{s.t.:} \quad \rho_s \left(A_0 x_s - b_{0s} \right) \geq 0, \qquad [\lambda_s^{EA}] \quad \forall s \in \mathcal{S} \tag{6b}$$

$$(x_n, y_n) \in \mathcal{X}_n^C \qquad \forall n \in \mathcal{N}. \tag{6c}$$

By construction, model (6) is a convex relaxation of (1). While the convex hull representation of an individual generator's schedule, $\mathcal{X}_{ns}^C$, may have a known compact formulation [13, 26], it does not necessarily generate a compact description of $\mathcal{X}_n^C$. Accordingly, in the computational experiments we use a relaxation of (6) which considers the convex hull of every generator per scenario:

$$\min \quad \hat{z}^{EA} = \sum_{s} \rho_s \left(c^\top x_s + d^\top y_s \right) \tag{7a}$$

$$\text{s.t.:} \quad \rho_s \left(A_0 x_s - b_{0s} \right) \geq 0 \qquad [\hat{\lambda}_s^{EA}] \quad \forall s \in \mathcal{S} \tag{7b}$$

$$(x_{ns}, y_{ns}) \in \mathcal{X}_{ns}^C \qquad \forall n \in \mathcal{N}, \forall s \in \mathcal{S} \tag{7c}$$

$$y_{ns} = y'_{n0} \qquad \forall n \in \hat{\mathcal{F}}, \forall s \in \mathcal{S} \tag{7d}$$

$$y_{ns} = y'_{nr} \qquad \forall n \in \mathcal{F}, \forall s \in \mathcal{S}_r, \forall r \in \mathcal{R}. \tag{7e}$$

To compare the objective values z^{EA} and $\hat{z}^{EA}$ with the other price models, first note that $z^{SUC} \geq z^{EA} \geq \hat{z}^{EA}$ since the EA-CHP model (6) is a relaxation of (1), and (7) is in turn a relaxation of (6). Since the objective function in (2) is equal to (1) in each scenario, it is also clear that

$z^{SUC} = \mathbb{E}_s[z_s^{LMP}]$. Additionally, the commitment and dispatch decisions of (3) are unrestricted by any nonanticipativity constraints such as (7d) and (7e), so the objective's expectation is $\mathbb{E}_s[z_s^{EP}] \leq \hat{z}^{EA}$. As a result, objective values are ordered by $z^{SUC} = \mathbb{E}_s[z_s^{LMP}] \geq z^{EA} \geq \hat{z}^{EA} \geq \mathbb{E}_s[z_s^{EP}]$.

The above descriptions of the stochastic unit commitment problem and pricing models are kept brief for easy reference. Although the models include simplifying assumptions such as the discrete probability distribution ($\rho_s, s \in \mathcal{S}$), the formulations are sufficient to illustrate shortcomings of standard pricing methods and to describe how the proposed EA-CHP pricing policy's properties overcome these shortcomings. Descriptions and proofs of the pricing model's properties are described in more detail in the following two sections.

2.2 Properties

The following subsections introduce the concepts of expected value of perfect information (EVPI), Lagrangian duality, and lost opportunity cost (LOC). First, EVPI is introduced for the SUC problem (1), which can be viewed as either the expected reduction in production costs with perfect forecast accuracy, or as the expected regret of ex ante decisions after the ex post scenario is realized. Next, a Lagrangian dual formulation of (1) is presented. Then, ex ante and ex post definitions of lost opportunity costs apply the LOC definition provided by [15] for the SUC problem. Using these definitions, it is proven that the EA-CHP pricing model minimizes ex ante LOC. Lastly, this section provides a dual interpretation of the EVPI called the expected nonanticipativity opportunity cost (ENOC) that is equal to the difference between ex post and ex ante LOC. The analysis in this section largely follows standard Lagrangian relaxation procedures for integer programs (e.g., see Section 11.4 in [1] and Chapter 10 in [49]).

2.2.1 Expected Value of Perfect Information.

As described above, EVPI is the expected reduction in production costs if the future scenarios could be forecasted with perfect accuracy. To calculate EVPI, the nonanticipativity constraints in (1) are dropped so that an ex post optimal solution can be determined for each scenario. The ex post optimal unit commitment problem is formulated as

$$\begin{aligned}
\min_{x,y} \quad & z^{EPO} = \sum_s \rho_s \left(c^\top x_s + d^\top y_s \right) && && \text{(8a)} \\
\text{s.t.:} \quad & \rho_s \left(A_0 x_s - b_{0s} \right) \geq 0 && \forall s \in \mathcal{S} && \text{(8b)} \\
& A_{ns} x_{ns} + B_{ns} y_{ns} \geq b_{ns} && \forall n \in \mathcal{N}, \forall s \in \mathcal{S} && \text{(8c)} \\
& y_{ns} \in \{0, 1\} && \forall n \in \mathcal{N}, \forall s \in \mathcal{S}. && \text{(8d)}
\end{aligned}$$

Since (8) is a relaxation of (1), the total expected production cost is $z^{EPO} \leq z^{SUC}$, and the inequality is strict if a perfectly accurate forecast would necessarily change decisions that are made ex ante. EVPI is simply $EVPI = z^{SUC} - z^{EPO}$, or more explicitly,

$$\begin{aligned}
EVPI := & \min_{x,y} \left\{ \sum_s \rho_s \left(c^\top x_s + d^\top y_s \right) : (x_n, y_n) \in \mathcal{X}_n, \forall n \in \mathcal{N} \right\} \\
& - \min_{x,y} \left\{ \sum_s \rho_s \left(c^\top x_s + d^\top y_s \right) : (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N}, \forall s \in \mathcal{S} \right\}.
\end{aligned} \tag{9}$$

Recall that $\mathcal{X}_n$ includes the set of nonanticipativity constraints (1e) and (1f). Because the two optimizations only differ due to the presence of nonanticipativity constraints, any difference in objective values can be solely attributed to the need to make decisions in advance.

2.2.2 Lagrangian Duality.

To present the UC problem's economic properties, the Lagrangian relaxation of (1) is defined as

$$L(\lambda) = \min_{x,y} \left\{ \sum_s \rho_s \left(c^\top x_s + d^\top y_s - \lambda^\top (A_0 x_s - b_{0s}) \right) : (x_n, y_n) \in \mathcal{X}_n, \forall n \in \mathcal{N} \right\}. \tag{10}$$

Note that for $\lambda \geq 0$, the problem (10) is a relaxation of (1) for the following reasons: all feasible solutions to (1) are also feasible in (10), any feasible solution to (1) will have as objective function value in (10) that is no more than the objective value in (1), and there may be feasible solutions to (10) that are not feasible in (1).

Next, let the Lagrangian dual of (1) be defined as

$$L^* = \max_{\lambda \geq 0} L(\lambda). \tag{11}$$

Similarly, a Lagrangian relaxation and Lagrangian dual can be defined for each scenario in the ex post optimal UC problem (8), as follows:

$$\tilde{L}_s(\lambda_s) = \min_{x_s, y_s} \left\{ c^\top x_s + d^\top y_s - \lambda^\top (A_0 x_s - b_{0s}) : (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N} \right\} \tag{12}$$

$$\tilde{L}_s^* = \max_{\lambda_s \geq 0} \tilde{L}_s(\lambda_s). \tag{13}$$

2.2.3 Lost Opportunity Costs.

LOCs quantify the foregone profits of a generator due to following the market operator's commitment and dispatch instructions instead of a commitment and dispatch solution that would have been profit-maximizing at the given market prices. The standard LOC is calculated ex post as follows:

$$U_s^P(\lambda_s) = \max_{x_s, y_s} \left\{ \left(A_0^\top \lambda_s - c \right)^\top x_s - d^\top y_s : (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N} \right\} - \left(\lambda_s^\top b_{0s} - c^\top x_s^* - d^\top y_s^* \right). \tag{14}$$

We note that, in contrast to the deterministic analysis applied in [15], Eq. (14) uses the optimal solution (x^*, y^*) to the stochastic problem (1). To reduce the potential for poor incentives, [15] suggests that market operators set prices to minimize LOCs and pay uplift payments equal to the LOC of each market participant. Such prices can be defined as the vector $\lambda_s \geq 0$ that minimizes $U_s^P(\lambda_s)$ and can be determined by solving the Lagrangian dual (13):

$$\begin{aligned}
&\min_{\lambda_s \geq 0} U_s^P(\lambda_s) \\
&\quad = \min_{\lambda_s \geq 0} \left\{ \max_{x_s, y_s} \left\{ \left(A_0^\top \lambda_s - c \right)^\top x_s - d^\top y_s : (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N} \right\} - \left(\lambda_s^\top b_{0s} - c^\top x_s^* - d^\top y_s^* \right) \right\} \\
&\quad = \min_{\lambda_s \geq 0} \left\{ \max_{x_s, y_s} \left\{ \left(A_0^\top \lambda_s - c \right)^\top x_s - d^\top y_s : (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N} \right\} - \lambda_s^\top b_{0s} + z_s^* \right\} \\
&\quad = \min_{\lambda_s \geq 0} \max_{x_s, y_s} \left\{ - \left(c^\top x_s + d^\top y_s - \lambda_s^\top \left(A_0 x_s - b_{0s} \right) \right) : (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N} \right\} + z_s^* \\
&\quad = z_s^* - \max_{\lambda_s \geq 0} \min_{x_s, y_s} \left\{ c^\top x_s + d^\top y_s - \lambda_s^\top \left(A_0 x_s - b_{0s} \right) : (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N} \right\} \\
&\quad = z_s^* - \tilde{L}_s^*.
\end{aligned}$$

That is, the minimum ex post LOC is exactly equal to the gap between the schedule cost z_s^* and the optimal Lagrangian cost $\tilde{L}_s^*$ in each scenario s.

An alternative approach would be to apply nonanticipativity constraints on the commitment and dispatch decisions in each potential scenario, resulting in the following ex ante LOC definition:

$$U^A(\lambda) = \max_{x,y} \left\{ \sum_s \rho_s \left(\left(A_0^\top \lambda_s - c \right)^\top x_s - d^\top y_s \right) : (x_n, y_n) \in \mathcal{X}_n, \forall n \in \mathcal{N} \right\} \\ - \sum_s \rho_s \left(\lambda_s^\top b_{0s} - c^\top x_s^* - d^\top y_s^* \right). \tag{15}$$

In contrast to the ex post LOC, the ex ante LOC defined above requires that the profit maximizing schedules obey the same nonanticipativity constraints that are satisfied in the set of socially optimal schedules, $\{(x_s^*, y_s^*), \forall s \in \mathcal{S}\}$, from the solution to (1).

Following the same steps as before, minimizing ex ante LOC is equivalent to solving the SUC problem's Lagrangian dual (11).

$$\min_{\lambda \geq 0} U^A(\lambda) = z^{SUC} - L^*$$

Assuming that the ex ante optimal commitment solution is chosen, then $z^{SUC} = \sum_s \rho_s z_s^*$ and definitions (14) and (15) imply that the expected ex post LOC must be greater than or equal to the ex ante LOC, i.e., $\mathbb{E}[U_s^P(\lambda_s)] \geq U^A(\lambda)$. Accordingly, the ex post LOC definition may include some LOC that is not included in the ex ante definition. This difference will be called the expected nonanticipativity opportunity cost (ENOC), $U^N(\lambda)$, calculated as follows:

$$\begin{aligned} U^N(\lambda) &= \sum_s \rho_s U_s^P(\lambda_s) - U^A(\lambda) \\ &= \sum_s \rho_s \left(\max_{x,y} \left\{ \left(A_0^\top \lambda_s - c \right)^\top x_s - d^\top y_s : (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N} \right\} \right) \\ &\quad - \max_{x,y} \left\{ \sum_s \rho_s \left(\left(A_0^\top \lambda_s - c \right)^\top x_s - d^\top y_s \right) : (x_n, y_n) \in \mathcal{X}_n, \forall n \in \mathcal{N} \right\}. \end{aligned} \tag{16}$$

Notice that (16) can be rewritten as the difference of two minimization problems:

$$
\begin{aligned}
U^N(\lambda) = \min_{x,y} & \left\{ \sum_s \rho_s \left(c^\top x_s + d^\top y_s - \lambda_s^\top A_0 x_s \right) : (x_n, y_n) \in \mathcal{X}_n, \forall n \in \mathcal{N} \right\} \\
& - \min_{x,y} \left\{ \sum_s \rho_s \left(c^\top x_s + d^\top y_s - \lambda_s^\top A_0 x_s \right) : (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N} \right\} + \sum_s (\lambda_s b_{0s} - \lambda_s b_{0s}) \\
= \min_{x,y} & \left\{ \sum_s \rho_s \left(c^\top x_s + d^\top y_s - \lambda_s^\top (A_0 x_s - b_{0s}) \right) : (x_n, y_n) \in \mathcal{X}_n, \forall n \in \mathcal{N} \right\} \\
& - \min_{x,y} \left\{ \sum_s \rho_s \left(c^\top x_s + d^\top y_s - \lambda_s^\top (A_0 x_s - b_{0s}) \right) : (x_{ns}, y_{ns}) \in \mathcal{X}_{ns}, \forall n \in \mathcal{N} \right\} \\
= L(\lambda) & - \sum_s \tilde{L}_s(\lambda_s).
\end{aligned}
$$

In other words, the ENOC is a dual formulation of EVPI that can be written as the difference of the Lagrangian relaxations of the SUC (10) and EPO (12) problems given the same price vector λ.

Remark 1. The derived expressions for ex ante LOC (15) and ENOC (16) are defined in the context of the three-stage stochastic unit commitment model (1), which is a simplification of the real-world operational problem. We note, however, that the existence of a gap between ex ante and ex post LOC only requires that nonanticipativity constraints will limit the ability of market participants to maximize their operating profits in all possible scenarios. While more sophisticated stochastic models of real-world operations would be able to generate better estimates of the magnitude of the gap, our focus is instead on a clear exposition of the directional consequences.

2.2.4 Primal Convex Hull Equivalence.

The ex post CHP model (3) and ex ante CHP model (6) were previously described in terms of minimizing a specific set of side-payments called uplift. Those statements can be made more precise now that the ex post and ex ante LOC definitions, (14) and (15), have been provided. Uplift payments are intended to remove incentives for participants to deviate from the socially optimal schedule calculated by the market operator since they would ensure that all participants receive their maximum profit, and they would be paid on condition that the participant follows the market operator's dispatch schedule to a reasonable degree of accuracy.

Supposing that the ex ante CHP model (6) produces a set of prices λ^{EA} that solve the Lagrangian dual problem (11), then it would be clear that λ^{EA} minimizes the expected ex ante LOC in equation (15). Accordingly, the following proofs show that the prices calculated by solving (6) in fact also solve the Lagrangian dual problem (11). Let the ex ante CHP model's Lagrangian relaxation be defined as follows:

$$L^{EA}(\lambda) = \min_{x,y} \left\{ \sum_s \rho_s \left(c^\top x_s + d^\top y_s - \lambda_s^\top (A_0 x_s - b_{0s}) \right) : (x_n, y_n) \in \mathcal{X}_n^C, \forall n \in \mathcal{N} \right\}.$$

Lemma 1. Suppose $L^{EA*} = \max_{\lambda \geq 0} L^{EA}(\lambda)$ is the Lagrangian dual solution for problem (6). Then $L^{EA*} = L^*$.

Proof. Proof of Lemma 1: For given λ, the feasible region of $L(\lambda)$ only differs from the feasible region of $L^{EA}(\lambda)$ in that the variables (x, y) are optimized over the possibly non-convex set $\mathcal{X}_n$ for each $n \in \mathcal{N}$ rather than the convex hull relaxation $\mathcal{X}_n^C$. Clearly, $L(\lambda) \geq L^{EA}(\lambda)$. Since the optimal points $(x_n^*, y_n^*) \in \mathcal{X}_n^C$ for L^{EA} will be extreme, the relaxed solution will be such that $(x_n^*, y_n^*) \in \mathcal{X}_n$ for every $n \in \mathcal{N}$. Therefore $L^{EA}(\lambda) = L(\lambda)$ for every λ, so $L^{EA*} = L^*$. □ □

Theorem 1. Suppose λ^{EA} is the optimal dual variable of constraint (6b). Then $z^{EA} = L^{EA*} = L^*$, and λ^{EA} solves $L^* = L(\lambda^{EA})$.

Proof. Proof of Theorem 1: For the first part of the theorem, note that (6) is a convex linear program. Therefore, strong duality implies that $z^{EA} = L^{EA*}$. The second equality $L^{EA*} = L^*$ is implied by Lemma 1.

The second part of the proof will show, first, that $L^{EA}(\lambda^{EA}) \leq z^{EA}$, and second, that $L^{EA}(\lambda^{EA}) \geq z^{EA}$. As a result, $L^{EA}(\lambda^{EA}) = L^*$, so we can conclude that λ^{EA} solves the Lagrangian dual problem.

For the upper bound, let (x^{EA}, y^{EA}) be the optimal primal solution to (6) and λ^{EA} the optimal dual variable to constraint (6b). Then we have the following upper bound based on the minimization in $L(\lambda^{EA})$ and complementary slackness in the primal and dual solutions of (6):

$$\begin{aligned} L(\lambda^{EA}) &\leq \sum_s \rho_s \left(c^\top x_s^{EA} + d^\top y_s^{EA} - (\lambda^{EA})^\top \left(A_0^\top x_s^{EA} - b_{0s} \right) \right) \\ &= \sum_s \rho_s \left(c^\top x_s^{EA} + d^\top y_s^{EA} \right) \\ &= z^{EA}. \end{aligned}$$

For the lower bound, it must be true that $L(\lambda^{EA}) \geq L^{EA}(\lambda^{EA})$ since $L^{EA}(\lambda)$ is a relaxation of minimization problem $L(\lambda)$. Further, since $L^{EA}(\lambda)$ is a convex optimization problem, strong duality implies that $L^{EA}(\lambda^{EA}) = z^{EA}$. Applying the first part of the proof, we have shown that $L(\lambda^{EA}) = L^*$. Therefore, λ^{EA} is a solution to the Lagrangian dual problem. □ □

Corollary 1. Suppose $\hat{L}^{EA}(\lambda)$ is the Lagrangian relaxation of (7) and $\hat{\lambda}^{EA}$ is the optimal dual variable of constraint (7b). Then $\hat{z}^{EA} = \hat{L}^{EA}(\hat{\lambda}^{EA}) \leq L^*$.

Proof. Proof: The proof is immediate from the fact that (7) is a relaxation of (6). □

Remark 2. There may be considerable difficulty in solving either the exact EA-CHP model (6) or the Lagrangian problem Ł^{EA*}. If the relaxed EA-CHP model (7) is solved instead, there may be uncertainty whether the resulting prices provide similar economic properties as the theoretically ideal prices. Even if the approximation is unable to reproduce the theoretically ideal price vectors, it is unlikely that the approximation introduces significantly different incentives so long as ex ante LOC is nearly minimized. From Corollary 1, the increase in ex ante LOC due to approximation is $z^{EA} - \hat{z}^{EA} \geq 0$, which will be close to zero if (7) is a close approximation of (6). In addition, recalling the result of Corollary 1 in [8], there is an upper bound on the redistribution of market surplus that results from sub-optimal scheduling decisions, provided that the pricing model is a convex relaxation of a mixed-integer scheduling problem. Those conditions are satisfied by the approximated ex ante pricing model (6) or any other reasonably tight relaxation of the stochastic scheduling problem, which implies a limited amount of economic distortions in the approximated model.

3 Example System

The analysis in Section 2 highlights that lost opportunity costs for market participants can arise from both uncertainty and non-convexity. Accordingly, addressing lost opportunity costs in an efficient way requires attention to both. Since prices formed in spot markets serve as the basis for trades used to manage financial risk in forward markets, strategies to handle uncertainty and non-convexity are necessarily interlinked.

To elaborate on the incentives of market participants, the potential failure of pricing policies that neglect uncertainty, and the role of forward markets, here we adapt an example from [36] and construct a system with a single uncertain demand, single hour-long time period, and single node. The set of scenarios $\mathcal{S} = \{1, 2, \ldots, 100\}$ contains 100 possible outcomes for demand b, with

$b_s = (99.5 + s)$ MW $\forall s \in \mathcal{S}$. The system is served by 101 thermal generators. Generator 0 has capacity 100 MW, has a marginal production cost of \$50/MWh, and has no minimum operating level, start-up cost, or no-load cost. Generators $n = 1, 2, \ldots, 100$ are each block loaded units of size 1 MW with zero energy cost and a start-up cost of $n + 50$, such that Generator 1 costs \$51 to commit and operate for the hour while Generator 100 costs \$150. All generators with even indices are slow-start generators, while those with odd indices have fast-start capability. Lastly, the system can also engage a demand-side resource at a cost of \$500/MWh in the event that the committed generators are insufficient to meet demand. In stochastic programming terms, the presence of this resource guarantees complete recourse.

At the time of first stage decisions, the probability $\rho_s = 0.01$ for each scenario $s \in \mathcal{S}$. The scenarios are partitioned into 20 sets of cardinality 5; i.e., with $\mathcal{R} = \{1, \ldots, 20\}$, $\mathcal{S}_r = \{5(r-1)+1, 5(r-1)+2, \ldots, 5(r-1)+5\}$ $\forall r \in \mathcal{R}$. The conditional probability of scenario $s \in \mathcal{S}$ is 0.20 if $s \in \mathcal{S}_r$ and zero otherwise.

3.1 Optimal solution

If the demand scenario s were known in advance, it can be seen that the optimal solution would be to commit exactly s of the block-loaded units. For instance, $s = 40$ would give a total demand of 139.5, optimally served by 99.5 MW from generator 0 and 40 MW from generators 1 through 40. Instead, the stochastic problem requires the system operator to weigh the commitment cost of the block-loaded generators against the probability that a more expensive recourse action will be required in the next epoch. Table 1 shows the optimal commitment solution for the example system, which depends on the subset of demand scenarios $\mathcal{S}_r$ identified before the commitment of fast-start units. The system commits 38 slow-start units in stage one. In the event of low demand, i.e, $r \in \{1, \ldots, 7\}$, no fast-start units are committed. For $r \in \{8, \ldots, 12\}$, the operator commits enough fast-start units to cover demand in all scenarios that remain possible. For $r \in \{13, \ldots, 17\}$, it is optimal to engage the \$500/MWh demand-side resource 20 percent of the time rather than commit an additional fast-start unit. Lastly, in the highest demand scenarios with $r \in \{18, 19, 20\}$, the system commits all available fast-start units but the demand-side resource is frequently needed since many slow-start units have been held offline.

Table 1: Commitment solution for example system for each demand range

Range	Max Demand	Block-loaded Units Committed	Probability of Shortfall
1	104.5	38	0.0
2	109.5	38	0.0
3	114.5	38	0.0
4	119.5	38	0.0
5	124.5	38	0.0
6	129.5	38	0.0
7	134.5	38	0.0
8	139.5	40	0.0
9	144.5	45	0.0
10	149.5	50	0.0
11	154.5	55	0.0
12	159.5	60	0.0
13	164.5	64	0.2
14	169.5	69	0.2
15	174.5	74	0.2
16	179.5	79	0.2
17	184.5	84	0.2
18	189.5	88	0.4
19	194.5	88	1.0
20	199.5	88	1.0

3.2 Pricing

The pricing challenge is to establish a policy that supports this optimal solution. Building on the discussion in Section 2, we make several aspects of this challenge more concrete here. Table 2 shows the average price across all 100 scenarios under the five pricing schemes defined in Section 2. The system is constructed such that the marginal unit in every scenario is either generator 0, with

Table 2: Expected spot price under different pricing schemes ($/MWh)

LMP	EP-CHP	FSP-I	FSP-II	EA-CHP
126.50	100.50	147.85	129.00	127.90

a marginal cost of $50/MWh, or the demand-side resource, with a marginal cost of $500/MWh. Examination of Table 1 shows that the demand-side resource is engaged in 17 of the 100 scenarios (one scenario each when $r \in \{13, \ldots, 17\}$, two scenarios when $r = 18$, and five scenarios each when $r \in \{19, 20\}$). Under the traditional LMP pricing policy, this means that the price set after the dispatch is determined will be $500/MWh in 17 percent of scenarios and $50/MWh in the remaining 83 percent, for an average of $126.50/MWh.

The price under EP-CHP is substantially lower. Suppose we knew in advance that $s = 40$ and were able to optimally commit 40 block-loaded units. In this instance, generator 0 would still be the price-setting resource under traditional LMP, leading to a price of \$50/MWh and losses for generators 1 through 40. The EP-CHP scheme attempts to minimize conflicts between the system operator and individual market participants by instead setting a price of \$90/MWh, i.e., the total cost of the most expensive unit that would have been committed had demand been known in advance. Given this price, generators 0 through 40 would be content with their non-negative profit, while generators 41 through 100 would still prefer not to operate. Applying this logic across all scenarios, an EP-CHP policy would lead to generator s setting the price in each scenario $s \in \mathcal{S}$, giving an average price of \$100.50.

3.2.1 Forward Contracting and Ex Post Lost Opportunity Costs.

In the optimal solution, 38 slow-start units are committed in the first stage, the most expensive of which is generator 76 at a total cost of \$126. With the LMP pricing policy, generator 76 incurs a loss of \$76 in the 83 scenarios with a price of \$50/MWh. In the other 17 scenarios, the uncommitted slow-start generator 78 does not operate even though it could hypothetically earn a profit of \$500 − \$128 = \$372. In both cases, owners of the generator may complain that the commitment and dispatch schedule directed by the system operator led to lower profits than would have been obtained under a different schedule.

Suppose we identify the first stage of the model as a day-ahead market that includes virtual bidders driving the day-ahead price to the expected real-time price. Under LMP, this expected price is \$126.50/MWh, while EP-CHP gives \$100.50/MWh. Under LMP or EA-CHP, forward contracts awarded in the day-ahead market alleviate the lost opportunity costs problem noted above: generator 76 locks in a profit of \$0.50, allowing it to avoid losses in scenarios with a price of \$50/MWh. Under EP-CHP, generator 76 would not clear in the day-ahead market, being supplanted by virtual suppliers bidding closer to \$100.50. In order to restore an efficient commitment solution, system operators may nevertheless commit the unit as part of a residual unit commitment process. In this case, generator 76 would be fully exposed to real-time prices and would expect to incur an operating loss of \$25.50 under EP-CHP.

The FSP-I scheme exhibits the opposite issue. With an expected price of \$147.85/MWh, many slow-start units will be awarded a contract in the day-ahead market despite not being in the socially efficient solution to the stochastic unit commitment problem. While not a precise match, the FSP-II policy produces an expected price closer to that of EA-CHP in this example.

3.2.2 Ex Ante Lost Opportunity Costs.

The discussion so far suggests that attempts to address incentive issues through EP-CHP may in fact be counterproductive: for generator 76, instituting EP-CHP results in poor incentives at the time a commitment decision must be made. In effect, the pricing policy misdiagnoses losses arising due to uncertainty as instead resulting from non-convexity. This observation, however, does not imply that retaining LMP removes the potential for misaligned incentives.

Table 3: Total cost of most expensive committed fast-start unit compared to conditional mean of price in each demand range

Range (r)	Highest FS Cost ($)	LMP ($/MWh)	EP-CHP ($/MWh)	FSP-I ($/MWh)	FSP-II ($/MWh)	EA-CHP ($/MWh)
1	0	50.00	53.00	50.00	50.00	50.00
2	0	50.00	58.00	50.00	50.00	50.00
3	0	50.00	63.00	50.00	50.00	50.00
4	0	50.00	68.00	50.00	50.00	50.00
5	0	50.00	73.00	50.00	50.00	50.00
6	0	50.00	78.00	50.00	50.00	50.00
7	0	50.00	83.00	50.00	50.00	50.00
8	53	50.00	88.00	50.80	50.80	53.00
9	63	50.00	93.00	59.00	59.00	63.00
10	73	50.00	98.00	69.00	69.00	73.00
11	83	50.00	103.00	79.00	79.00	83.00
12	93	50.00	108.00	89.00	89.00	93.00
13	101	140.00	113.00	178.40	99.00	103.00
14	111	140.00	118.00	186.40	109.00	113.00
15	121	140.00	123.00	194.40	119.00	123.00
16	131	140.00	128.00	202.40	129.00	133.00
17	141	140.00	133.00	210.40	139.00	141.00
18	149	230.00	138.00	288.20	288.20	230.00
19	149	500.00	143.00	500.00	500.00	500.00
20	149	500.00	148.00	500.00	500.00	500.00

Table 3 shows the cost of the most expensive fast-start unit committed for each $r \in \mathcal{R}$, as well as the conditional mean of the price under each pricing policy. Consider the case of $r = 10$, in which 12 fast-start units are committed. Since the demand-side resource is never engaged, under LMP the price will be \$50/MWh. Since the 12 fast-start units all have a start-up cost above \$50, they are guaranteed to lose money despite being included in the optimal commitment. In other words, the fast-start generators would prefer not to be committed in the second stage given the conditional distribution of prices in the third stage. Under EA-CHP, the conditional mean is determined by the most expensive committed fast-start unit, which has a total cost of $73.

3.2.3 Uplift and Incomplete Markets.

The presence of a day-ahead market substantially changes the lost opportunity costs calculation for slow-start units. In U.S. markets, however, market operators do not provide an opportunity to update financial positions between the day-ahead and real-time markets. In the second stage of the example problem, consider the case of $r = 16$, which leads to the commitment of 41 fast-start units and a 20 percent chance of engaging the demand-side resource. In this case, the most expensive fast-start unit is generator 81, with a cost of $131. The conditional mean of the price under LMP is $140/MWh, i.e., high enough to make commitment of the generator profitable in expectation. Without a mechanism to lock in the new expected price of $140/MWh, however, generator 81 will incur losses 80 percent of the time.

To alleviate problems with potentially misaligned incentives, market operators in the U.S. use discriminatory side payments to supplement compensation from the uniform prices seen by all market participants. These side payments can be seen as part of the overall pricing policy. While one possible approach would be to pay all lost opportunity costs, such a policy would lead to indefensibly high compensation in practice [37]. Instead, most systems authorize smaller categories of uplift payments. The most important of these are make-whole payments, which guarantee non-negative profit for generators that are included in the efficient commitment solution.

In the case of $r = 16$ discussed above, generator 81 could be entitled to a make-whole payment of $81 to cover its losses in the 4 out of 5 scenarios that result in an LMP of $50/MWh. While the justification for these side payments relies on the need to satisfy individual rationality constraints, the example highlights the potential issue with assessing profitability ex post. Given $r = 16$, at the time of commitment generator 81 is profitable in expectation, earning $9 on average. In this case, providing make-whole payments socializes the losses that occur 80 percent of the time and privatizes the gains that occur in the remaining 20 percent. A more efficient route to resolving the incentive issues associated with these potential losses could be to attach a financial position to commitments occurring in the second stage, moving toward more complete markets in risk.

3.3 Forward Market Models

The example invites a solution in the form of an intraday market. In place of a discriminatory side payment, a forward market executed at the time of the second stage would allow generators to update their financial positions based on the conditionally expected price. In the case of $r = 16$, generator 81 could sell its power in this intraday market under either LMP or EA-CHP, enabling it to avoid losses in the scenarios with a real-time price below \$131/MWh. Here we construct models for forward markets corresponding to the first stage as well as each node of the second stage. In each forward market, we assume that virtual bidders participate in a perfectly competitive, risk-neutral manner, such that prices converge to the expected spot price under the chosen pricing policy. We note the contrast between this assumption and previous examinations of stochastic market clearing, where the absence of virtual bidders can lead to inconsistencies between day-ahead and expected real-time prices [51]. Quantities awarded in the day-ahead market assuming pricing scheme $PS \in \{LMP, EP, F1, F2, EA\}$ are calculated by solving

$$
\begin{aligned}
\min_{x^{DAM}, y^{DAM}} \quad & z^{DAM} = c^\top x^{DAM} + d^\top y^{DAM} - \overline{\lambda}^{PS} \left(A_0 x^{DAM} - \overline{b}_0 \right) && && \text{(17a)} \\
\text{s.t.:} \quad & A_n x_n^{DAM} + B_n y_n^{DAM} \geq \overline{b}_n && \forall n \in \mathcal{N} && \text{(17b)} \\
& y_n^{DAM} \in \{0, 1\} && \forall n \in \mathcal{N}. && \text{(17c)}
\end{aligned}
$$

Here $\overline{\lambda}^{PS} = \mathbb{E}[\lambda_s^{PS}]$, $\overline{b}_0 = \mathbb{E}[b_{0s}]$, and $\overline{b}_n = \mathbb{E}[b_{ns}]$. As a result, the forward market is a deterministic unit commitment problem with the power balance constraint relaxed at a penalty corresponding to the expected real-time price. The constraints in Eq. (17b) ensure that generators are awarded a feasible schedule in the day-ahead market.

Similarly, positions in the intraday market in scenario r using pricing scheme PS are calculated by solving

$$
\begin{aligned}
\min_{x_r^{IDM}, y_r^{IDM}} \quad & z_r^{IDM} = c^\top x_r^{IDM} + d^\top y_r^{IDM} - \overline{\lambda}_r^{PS} \left(A_0 x_r^{IDM} - \overline{b}_0^r \right) && && \text{(18a)} \\
\text{s.t.:} \quad & A_n x_{nr}^{IDM} + B_n y_{nr}^{IDM} \geq \overline{b}_n^r && \forall n \in \mathcal{N} && \text{(18b)} \\
& y_{nr}^{IDM} = y_{n0}^{'*} && \forall n \in \hat{\mathcal{F}} && \text{(18c)} \\
& y_{nr}^{IDM} \in \{0, 1\} && \forall n \in \mathcal{F}. && \text{(18d)}
\end{aligned}
$$

Here $\overline{\lambda}_r^{PS} = \mathbb{E}[\lambda_s^{PS} | s \in \mathcal{S}_r]$, $\overline{b}_0 = \mathbb{E}[b_{0s} | s \in \mathcal{S}_r]$, and $\overline{b}_n = \mathbb{E}[b_{ns} | s \in \mathcal{S}_r]$. We note that slow start generators in the intraday market are fixed to their optimal commitment in the stochastic unit commitment, rather than the position awarded in the day-ahead market. In terms of real-world processes, this choice corresponds to an assumption that if the day-ahead market yields suboptimal first-stage decisions, it will be overruled by a reliability unit commitment process performed by system operators that restores the optimal commitment.

3.4 Scenario Profits

With prices and quantities in forward markets defined, we can calculate the profit earned by generators in each scenario under each pricing policy and trading regime. With the superscript 1 indicating a single settlement, i.e., no forward trades, profit for generator n under policy PS is entirely dependent on the spot price in the given scenario and the production according to the efficient schedule:

$$\pi^1_{ns} = (\lambda^{PS}_s)^\top x^*_{ns} - \left(c^\top x^*_{ns} + d^\top y^*_{ns}\right).$$

With superscript 2 indicating a two-settlement system with a day-ahead market in addition to the spot market, profit is calculated as

$$\pi^2_{ns} = (\overline{\lambda}^{PS})^\top x^{DAM}_{ns} + (\lambda^{PS}_s)^\top (x^*_{ns} - x^{DAM}_n) - \left(c^\top x^*_{ns} + d^\top y^*_{ns}\right).$$

With the two-settlement system, sales in the real-time market are calculated with reference to the forward position awarded in the day-ahead market. With superscript 3 indicating an additional settlement in an intraday market, profit is calculated as

$$\pi^3_{ns} = (\overline{\lambda}^{PS})^\top x^{DAM}_{ns} + (\overline{\lambda}^{PS}_r)^\top (x^{IDM}_{ns} - x^{DAM}_n) + (\lambda^{PS}_s)^\top (x^*_{ns} - x^{IDM}_n) - \left(c^\top x^*_{ns} + d^\top y^*_{ns}\right).$$

The expected prices in Table 2 are an indication of the superior ex ante incentives offered by EA-CHP, and to a lesser extent LMP and FSP-II, in the example system. Here we turn the focus to ex post results, in particular the potential for losses in individual scenarios. Table 4 shows the number of generator scenarios which result in a negative profit under each pricing policy and trading regime. Given 100 scenarios and 101 generators, the total count of generator scenarios is 10,100. With a price of \$50/MWh frequently set by generator 0, LMP and EA-CHP lead to frequent occurrence of negative profit scenarios. However, the introduction of a day-ahead market substantially reduces

Table 4: Generator scenarios with negative profit

Settlements	LMP	EP-CHP	FSP-I	FSP-II	EA-CHP
One	4,324	1,523	2,250	2,271	4,106
Two	35	953	247	35	159
Three	50	984	250	29	0

this number, and in the case of EA-CHP the addition of an intraday market brings the number of negative profit generator scenarios to zero. Forward trading brings limited benefit in the case of EP-CHP, because the underlying prices are too low to result in a profit for many committed generators.

Table 5 shows the make-whole payments that would be authorized on average if generators were guaranteed non-negative profit in each scenario. Table 5 exhibits the degree to which make-whole payments may be driven more by the variance of prices created by a pricing policy, rather than the expected value. Despite a lower expected price, EP-CHP generates fewer make-whole payments in the single settlement regime by lifting prices above \$50/MWh in low-demand scenarios. We return to the topic of price volatility, and corresponding incentives for flexibility, in the larger test system.

Table 5: Average make-whole payments to all generators, with losses computed by scenario inclusive of financial trades

Settlements	LMP	EP-CHP	FSP-I	FSP-II	EA-CHP
One	\$1,623	\$377	\$754	\$756	\$1,545
Two	\$146	\$272	\$828	\$46	\$164
Three	\$172	\$272	\$843	\$46	\$0

4 Large-Scale Test System

4.1 Case Study

To demonstrate the difference between pricing policies on a more realistic example, we selected a known-to-be-challenging day (2013-05-11) from [43], which considers 100 hypothetical wind scenarios drawn from real-world data from the Bonneville Power Administration over the WECC-240 system. To stress the system further, load was increased 10% from its given value, resulting in a mean wind penetration rate of 26% for this day over 100 wind scenarios, with a maximum hourly variability (at hour 20) of 79% of load, and a maximum net-load at hour 15 in scenario 38. Load is modeled deterministically, and wind is considered a zero marginal cost resource which is fully curtailable. Finally, to create a slightly less flexible system, we down-selected from 50 fast-start resources to 27 fast-start resources out of a total of 85 thermal units.

As is common in power systems operations, it may be difficult to distinguish between different policies on "typical" days. In our selected case study, FSP-I and FSP-II resulted in identical prices across all scenarios and time periods. It should be noted that these identical prices were obtained from the optimal solution of a modestly sized test system. Suboptimal commitments are common in real-world systems, in which case the FSP-II policy's results would be relatively stable while those of FSP-I could change substantially [8]. In this section, we merge the results into a single policy labeled FSP. Further convergence in policies can occur. In tests that used the original set of 50 fast-start resources instead of our selected 27, for example, FSP yielded the same prices as EP-CHP. Here we present an instance with some separation so as to distinguish between the different pricing policies.

4.2 Computational Setup

All computations were done on a MacBook Pro (16-inch, 2019) with an 8-core 2.4GHz Intel Core i9 processor with 64 GB of DDR4 memory. All optimization problems were solved using Gurobi 9.0.2 [17]. Stochastic unit commitment problems were formulated using `EGRET` [2, 30] and `mpi-sppy` [27, 28]. All stochastic unit commitment problems were solved using the "extensive form" with 0% optimality gap to ensure an optimal commitment schedule is obtained (within numerical tolerances). Suboptimal commitments are a practical reality in large markets and can have a large effect on pricing results [8]. Accordingly, the LMP and FSP-I results could change substantially if the first or second stage commitments are not optimal, whereas results would be relatively stable due to suboptimality in either stage for EA-CHP or the second stage for FSP-II. Pricing problems were solved using the "extensive form" for EA-CHP, FSP, and LMP. To ensure convex hull prices (or variants thereof) were obtained, the row-generation procedure introduced in [29] was adapted to iteratively refine the convex hull relaxations of individual generators within stochastic pricing problems. The deterministic-equivalent formulation of the problem, i.e., the load-balance (1c) and generator (1b) constraints, was also taken from [29].

The 100-scenario stochastic unit commitment used in this case study, with 85 thermal generators and a power-balance constraint over 24 time periods, was readily solvable by Gurobi within

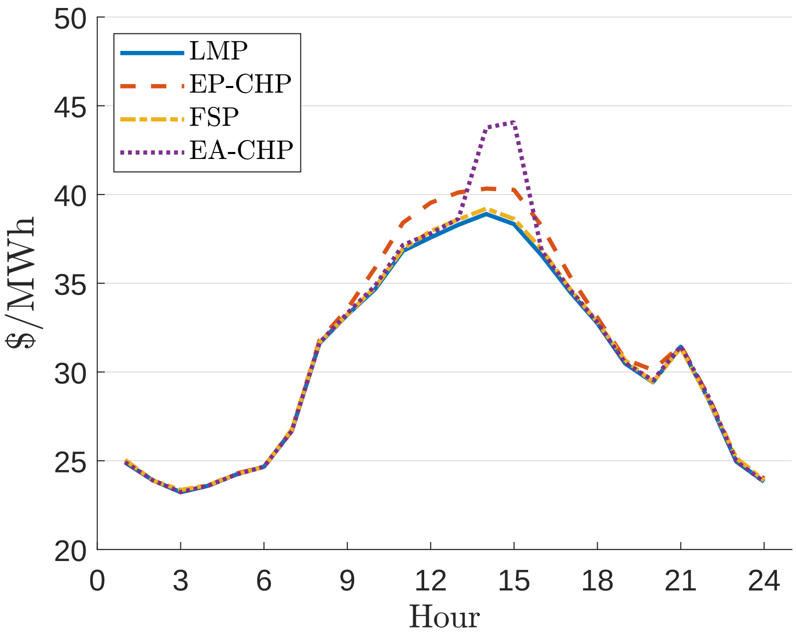


Figure 1: Average Price by Hour

reasonable wall-clock times of less than 1 hour. With a larger test system or a transmission network further decomposition schemes would need to be considered; however, approaches such as lazy-constraint generation for transmission constraints and scenario decomposition techniques such as progressive hedging [45] are readily applicable to the pricing problems demonstrated here.

4.3 Average Prices

Figure 1 shows the average price in each hour under the four pricing schemes. In the low-demand hours of the early morning and late evening, all the pricing schemes result in similar prices, with EP-CHP tracking slightly above the other three. Relative to LMP and FSP, EP-CHP results in slightly elevated prices on average through the middle of the day. EA-CHP also result in higher prices than LMP overall, but price spikes are concentrated in hours 14 and 15. Because the set of fast-start resources is relatively small, LMP and FSP return similar results. In other instances with a larger number of fast-start resources, FSP could instead mirror EP-CHP.

Figure 2 shows the distribution of load-weighted average prices arising in the 100 scenarios, corresponding to the total revenue earned by generators over the course of the day. It can be seen that, while average compensation is similar under the four pricing schemes, EA-CHP leads to a wider spread between scenarios.

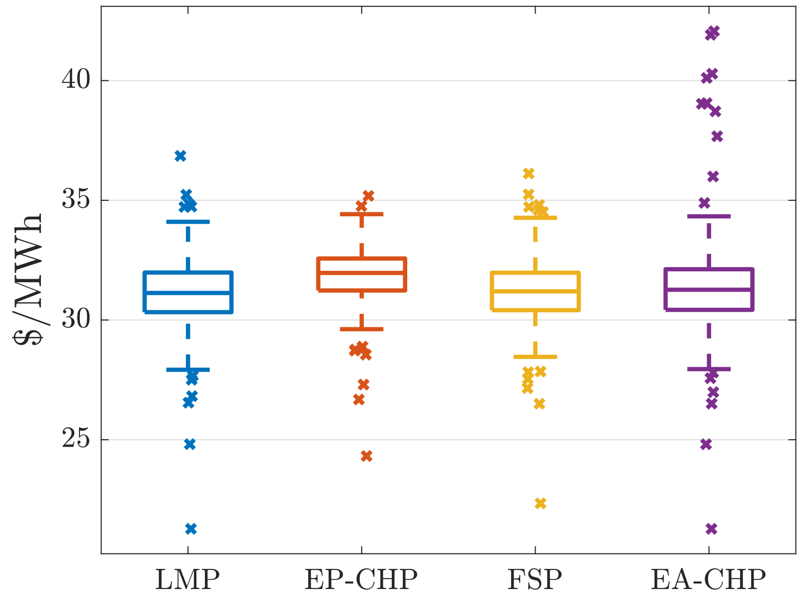


Figure 2: Distribution of Weighted Average Prices

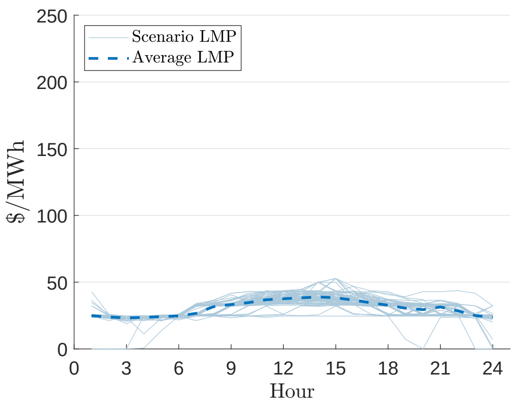
4.4 Scenario Prices and Incentives for Flexibility

The spread of daily compensation observed in Figure 2 suggests that under EA-CHP, there may be greater value to being able to defer a commitment decision until closer to real time. Differing incentives for flexibility also arise in real-time operations. Figure 3 shows the paths that prices take in each scenario under each pricing scheme. As reflected by the averages in Figure 1, it can be seen that EA-CHP leads to more significant price spikes in the afternoon of some scenarios, corresponding to the hours and scenarios that drive the need for commitment of the highest-priced generators.

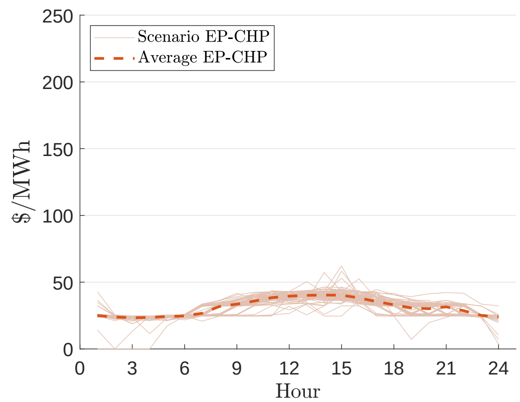
Since price volatility is an important signal for flexibility in operations, these pricing patterns suggest that EA-CHP may provide stronger incentives for long-run investment than LMP, EP-CHP, and FSP [35]. As a metric for price volatility, we compute the value

$$\frac{1}{100} \sum_{s \in \mathcal{S}} \sum_{t \in 2 \ldots 24} |\lambda_{st}^{PS} - \lambda_{s(t-1)}^{PS}|,$$

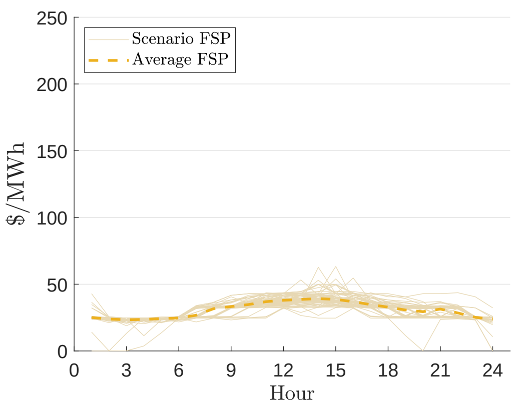
where λ_{st}^{PS} indicates the price in scenario s and hour t under pricing scheme PS. Table 6 reports the value of this price volatility metric under each pricing scheme.



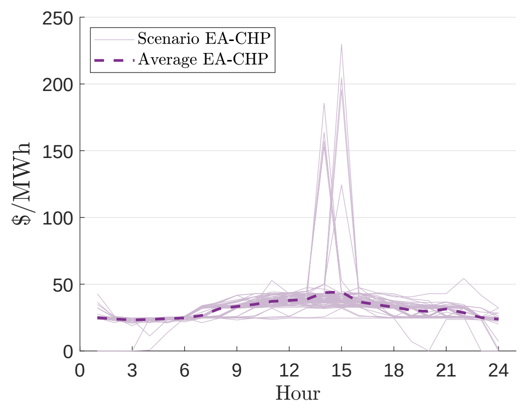
(a) LMP



(b) EP-CHP



(c) FSP



(d) EA-CHP

Figure 3: Hourly Price in Each Scenario under Each Pricing Policy

Table 6: Average of Absolute Hourly Price Differences

LMP ($/MWh)	EP-CHP ($/MWh)	FSP ($/MWh)	EA-CHP ($/MWh)
44.68	49.34	46.07	65.38

4.5 Scenario Profits and Forward Markets

The distribution of profits faced by generators at the time of commitment informs their willingness to follow a socially optimal schedule. The results in this subsection consider the ability of the pricing policies to support the socially optimal schedule, as well as the effect of risk trading in ensuring that incentives are aligned. Table 7 shows the number of generators with negative expected profit under each pricing scheme before accounting for any side payments. We note that these values would be zero in a convex setting. Consistent with Theorem 1, EA-CHP exhibits superior performance on this metric, with expected losses limited to a single generator incurring a loss amounting to 0.0002% of total expected operating cost. Because the EA-CHP prices are approximated by solving model (7) instead of model (6), the near-zero expected losses provide considerable confidence that the approximation is very close for our test problems (see Remark 2).

Table 7: Number of generators (out of 85) with negative expected profit and total negative expected profit as a percentage of expected operating cost.

	LMP	EP-CHP	FSP	EA-CHP
Number of generators	6	2	4	1
Relative expected losses	0.1877%	0.0433%	0.1517%	0.0002%

Suppose the market satisfies the assumptions of complete risk trading [7, 12], with the added assumption of at least one risk-neutral market participant. In this setting, the risk-adjusted probability attached by each market participant to each scenario $s \in \mathcal{S}$ would be 1%, and an Arrow–Debreu security for scenario would be priced at \$0.01, returning \$1.00 in scenario s and \$0.00 otherwise [14]. Accordingly, by selling securities for each scenario in a quantity equal to its profit in that scenario, a generator could lock in its expected profit. Given this ability to trade, the expected losses in Table 7 correspond to the make-whole payments that would be required under each scheme. As such, the need for make-whole payments would almost be eliminated under EA-CHP, while the other policies leave greater need for side payments.

Now suppose that markets in risk are incomplete. In this case, market participants may not use the same probability measure, due either to differing underlying assessments or to risk aversion. The current two-settlement system used in U.S. electricity markets, for example, does not meet the idealized conditions of complete markets. In the example of Section 3, introduction of day-ahead and intraday markets was enough to eliminate scenarios with losses under EA-CHP (Table 4). The forward markets defined in Section 3.3, however, are not sufficient to complete the market in the larger scale test system. Table 8 shows the expected make whole payments as a percentage of operating costs under each pricing scheme when trading is limited to these two contracts. In

Table 8: Total expected make whole payments as a percentage of expected operating cost.

Settlements	LMP	EP-CHP	FSP	EA-CHP
One	0.4658%	0.1560%	0.4037%	0.4298%
Two	0.3943%	0.0842%	0.3645%	0.1708%
Three	0.3141%	0.0757%	0.2989%	0.1157%

contrast to the results in Section 3, we see even that three settlements is not enough to drive the expected make-whole payments to the idealized values presented in Table 7. The inability of EA-CHP to outperform EP-CHP under conditions of incomplete trading is an indication of the underlying price volatility combined with incomplete forward markets. In Section 3, because all generators except for the always-profitable generator 0 were block loaded, the quantity sold by committed generators was decoupled from the price. In the large-scale example, generators produce more when prices are higher, leading to a risk that cannot be hedged through fixed-volume swaps alone. The larger variability in prices in EA-CHP demonstrated in Figure 1 and quantified in Table 6 means that market participants are unable to hedge completely in the modeled forward markets, driving relatively larger losses in a few scenarios. Comparatively, under EP-CHP such market participants experience smaller losses in several scenarios (but also smaller gains), driving expected make-whole payments lower.

The results in Tables 6 and 8 suggest an important trade-off in the choice of a price formation policy. With greater price volatility, market participants may be exposed to a higher chance of losses. This potential for losses could in turn affect the offer behavior of risk-averse market participants, degrading efficiency in operations [7]. At the same time, price volatility is an important incentive to invest in resources that are flexible enough to take advantage of that volatility. Accordingly, suppressed volatility relative to the ideal could degrade efficiency on longer timescales [35].

Resolving this trade-off would entail both producing efficient underlying spot prices and ensuring the availability of a broader range of hedging instruments with low transaction costs.

5 Conclusion

This article investigates the combined effect of uncertainty and non-convexity when evaluating policies for price formation in wholesale electricity markets. Our results emphasize the importance of correctly diagnosing the source of misaligned incentives for market participants. Policies developed on the basis of deterministic models or ex post analysis may be counterproductive, leading to poor incentives at the time generator commitment decisions must be made. In particular, uplift payments that appear "necessary" in a deterministic analysis may be revealed as inefficient subsidies in a stochastic analysis, while enhanced pricing schemes that neglect the effect of uncertainty may have the negative consequence of suppressing volatility in prices and hampering efforts to attract an efficient level of investment in flexible resources.

To help elucidate the economic phenomena the paper defines a new construct, ex ante convex hull pricing, that minimizes expected lost opportunity costs for market participants. Generators are nevertheless exposed to the possibility of realizing losses due to underlying non-convexity as well as the uncertainty inherent in electricity systems. Results from the case study indicate that intraday markets may help reduce the potential for losses, but more complete risk management would require introduction of option-like instruments enabling market participants to manage the positive correlation between price and quantity. In current markets, uplift payments and enhanced pricing schemes may have the effect of partially managing risk on behalf of participants. In doing so, however, they may introduce distortions and subsidies that reduce overall efficiency.

Despite its appealing theoretical properties, EA-CHP faces an important epistemic challenge to implementation in practice. While ex ante prices in the paper are determined through a stochastic program that includes all possible future states of the world, a real-world system would encounter scenarios in real-time that were not included in the model. With this complication, it is not clear how to produce EA-CHP prices in real time. Just as an inability to produce exact EP-CHP prices has not prevented market operators from implementing approximations, however, our results suggest that market designers should seek workable policies able to approximate the properties of EA-CHP. Conceptually simple approximations will be the most compelling candidates for use in practical settings, possibly being formulated as an ex post pricing policy with similar form as currently implemented fast-start pricing models. In addition to variants of schemes proposed in the deterministic context, an alternate route to forming prices that provide similar incentives to EA-CHP may be through the design of operating reserve demand curves [36]. Further tests on multiperiod stochastic models are needed to assess the empirical performance of competing proposals.

References

[1] Dimitris Bertsimas and J Tsitsiklis. *Introduction to linear programming.* Athena Scientific, Belmont, MA, fifth edition, 1997.

[2] Michael Lee Bynum, Andrea R Castillo, Bernard Knueven, Carl Damon Laird, and Jean-Paul Watson. Egret: Electrical grid research and engineering tools. Available at `https://github.com/grid-parity-exchange/Egret`, 2021.

[3] California Independent System Operator. Final root cause analysis, mid-august 2020 extreme heat wave. Available at `http://www.caiso.com/Documents/Final-Root-Cause-Analysis-Mid-August-2020-Extreme-Heat-Wave.pdf`, January 2021.

[4] H. Chao. Incentives for efficient pricing mechanism in markets with non-convexities. *Journal of Regulatory Economics*, 56(1):33–58, Aug 2019.

[5] K. Cheung, D. Gade, C. Silva-Monroy, S.M. Ryan, J.P. Watson, R.J.B. Wets, and D.L. Woodruff. Toward scalable stochastic unit commitment. *Energy Systems*, 6(3):417–438, 2015.

[6] R. Cory-Wright, A. Philpott, and G. Zakeri. Payment mechanisms for electricity markets with uncertain supply. *Operations Research Letters*, 46(1):116–121, 2018.

[7] Ryan Cory-Wright and Golbon Zakeri. On stochastic auctions in risk-averse electricity markets with uncertain supply. *Operations Research Letters*, 48(3):376–384, 2020.

[8] B. Eldridge, R. O'Neill, and B.F. Hobbs. Near-optimal scheduling in day-ahead markets: Pricing models and payment redistribution bounds. *IEEE Transactions on Power Systems*, 35(3):1684–1694, 2020.

[9] Federal Energy Regulatory Commission. Docket no. EL18-33-000. Available at `https://`

cms.ferc.gov/sites/default/files/whats-new/comm-meet/2019/041819/E-2.pdf, April 2019.

[10] Federal Energy Regulatory Commission. Docket no. EL18-34-000. Available at https://cms.ferc.gov/sites/default/files/whats-new/comm-meet/2019/041819/E-3.pdf, April 2019.

[11] Federal Energy Regulatory Commission. Docket no. EL18-35-000. Available at https://www.ferc.gov/sites/default/files/2020-06/20190612145037-EL18-35-000.pdf, June 2019.

[12] M. Ferris and A. Philpott. Dynamic risked equilibria. Available at http://www.optimization-online.org/DB_HTML/2018/04/6577.html, April 2018.

[13] Claudio Gentile, Germán Morales-España, and Andres Ramos. A tight mip formulation of the unit commitment problem with start-up and shut-down constraints. *EURO Journal on Computational Optimization*, 5(1-2):177–201, 2017.

[14] H. Gérard, V. Leclère, and A. Philpott. On risk averse competitive equilibrium. *Operations Research Letters*, 46(1):19–26, 2018.

[15] P. Gribik, W.W. Hogan, and S.L. Pope. Market-clearing electricity prices and energy uplift. December 2007.

[16] C. Guo, M. Bodur, and D.J. Papageorgiou. Generation expansion planning with revenue adequacy constraints. Available at http://www.optimization-online.org/DB_HTML/2020/04/7725.html, April 2020.

[17] LLC Gurobi Optimization. Gurobi optimizer reference manual, 2021.

[18] I. Herrero, P. Rodilla, and C. Batlle. Enhancing intraday price signals in U.S. ISO markets for a better integration of variable energy resources. *The Energy Journal*, 39(3):141–165, 2018.

[19] William W Hogan and Brendan J Ring. On minimum-uplift pricing for electricity markets. *Electricity Policy Group*, pages 1–30, 2003.

[20] B. Hua and R. Baldick. A convex primal formulation for convex hull pricing. *IEEE Transactions on Power Systems*, 32(5):3814–3823, 2017.

[21] R.B. Hytowitz, B. Frew, G. Stephen, E. Ela, N. Singhal, A. Bloom, and J. Lau. Revenue

sufficiency and reliability in a zero marginal cost future. Technical report, National Renewable Energy Laboratory, Golden, Colorado, USA, May 2020.

[22] ISO New England. Real-time fast-start pricing project. Available at https://www.iso-ne.com/participate/support/customer-readiness-outlook/real-time-fast-start-pricing-project, March 2017.

[23] Akshaya Jha and Frank A. Wolak. Can forward commodity markets improve spot market performance? evidence from wholesale electricity. Available at SSRN: https://ssrn.com/abstract=3568372, 2020.

[24] J. Kazempour and B.F. Hobbs. Value of flexible resources, virtual bidding, and self-scheduling in two-settlement electricity markets with wind generation—part i: Principles and competitive model. *IEEE Transactions on Power Systems*, 33(1):749–759, 2018.

[25] J. Kazempour and B.F. Hobbs. Value of flexible resources, virtual bidding, and self-scheduling in two-settlement electricity markets with wind generation—part ii: Iso models and application. *IEEE Transactions on Power Systems*, 33(1):760–770, 2018.

[26] Ben Knueven, Jim Ostrowski, and Jianhui Wang. The ramping polytope and cut generation for the unit commitment problem. *INFORMS Journal on Computing*, 30(4):739–749, 2018.

[27] Bernard Knueven, David Mildebrath, Christopher Muir, John D Siirola, Jean-Paul Watson, and David L Woodruff. A parallel hub-and-spoke system for large-scale scenario-based optimization under uncertainty. Available at http://www.optimization-online.org/DB_FILE/2020/11/8088.pdf, 2020.

[28] Bernard Knueven, David Mildebrath, Christopher Muir, John D Siirola, Jean-Paul Watson, and David L Woodruff. mpi-sppy: Optimization under uncertainty for pyomo models. Available at https://github.com/Pyomo/mpi-sppy, 2021.

[29] Bernard Knueven, James Ostrowski, Anya Castillo, and Jean-Paul Watson. A computationally efficient algorithm for computing convex hull prices. *Computers & Industrial Engineering*, page 107806, 2021.

[30] Bernard Knueven, James Ostrowski, and Jean-Paul Watson. On mixed-integer programming

formulations for the unit commitment problem. *INFORMS Journal on Computing*, 32(4):857–876, 2020.

[31] Dheepak Krishnamurthy, Wanning Li, and Leigh Tesfatsion. An 8-zone test system based on iso new england data: Development and application. *IEEE Transactions on Power Systems*, 31(1):234–246, 2015.

[32] Ruoyang Li, Alva J. Svoboda, and Shmuel S. Oren. Efficiency impact of convergence bidding in the California electricity market. *Journal of Regulatory Economics*, 48(3):245–284, Dec 2015.

[33] G. Liberopoulos and P. Andrianesis. Critical review of pricing schemes in markets with non-convex costs. *Operations Research*, 64(1):17–31, 2016.

[34] J. Mather, E. Bitar, and K. Poolla. Virtual bidding: Equilibrium, learning, and the wisdom of crowds. *IFAC-PapersOnLine*, 50(1):225–232, 2017. 20th IFAC World Congress.

[35] J. Mays. Missing incentives for flexibility in wholesale electricity markets. *Energy Policy*, 149:112010, 2021.

[36] J. Mays. Quasi-stochastic electricity markets. *INFORMS Journal on Optimization*, 3(4):350–372, 2021.

[37] J. Mays, D.P. Morton, and R.P. O'Neill. Investment effects of pricing schemes for non-convex markets. *European Journal of Operational Research*, 289(2):712–726, 2021.

[38] David Mildebrath, Victor Gonzalez, Mehdi Hemmati, and Andrew J Schaefer. Relating single-scenario facets to the convex hull of the extensive form of a stochastic single-node flow polytope. *Operations Research Letters*, 48(3):342–349, 2020.

[39] R.P. O'Neill, P.M. Sotkiewicz, B.F. Hobbs, M.H. Rothkopf, and W.R. Stewart. Efficient market-clearing prices in markets with nonconvexities. *European Journal of Operational Research*, 164(1):269–285, 2005.

[40] Yumi Oum, Shmuel Oren, and Shijie Deng. Hedging quantity risks with standard power options in a competitive wholesale electricity market. *Naval Research Logistics (NRL)*, 53(7):697–712, 2006.

[41] J.E. Parsons, C. Colbert, J. Larrieu, T. Martin, and E. Mastrangelo. Financial arbitrage and efficient dispatch in wholesale electricity markets. February 2015.

[42] G. Pritchard, G. Zakeri, and A. Philpott. A single-settlement, energy-only electric power market for unpredictable and intermittent participants. *Operations Research*, 58(4-part-2):1210–1219, 2010.

[43] Benjamin Rachunok, Andrea Staid, Jean-Paul Watson, David L Woodruff, and Dominic Yang. Stochastic unit commitment performance considering monte carlo wind power scenarios. In *2018 IEEE international conference on probabilistic methods applied to power systems (PMAPS)*, pages 1–6. IEEE, 2018.

[44] Brendan J Ring. *Dispatch Based Pricing in Decentralised Power Systems.* PhD thesis, University of Canterbury, Christchurch, NZ, 1995.

[45] R Tyrrell Rockafellar and Roger J-B Wets. Scenarios and policy aggregation in optimization under uncertainty. *Mathematics of operations research*, 16(1):119–147, 1991.

[46] Samer Takriti, John R Birge, and Erik Long. A stochastic model for the unit commitment problem. *IEEE Transactions on Power Systems*, 11(3):1497–1508, 1996.

[47] L. Tesfatsion. *A New Swing-Contract Design for Wholesale Power Markets.* Wiley, New York, NY, USA, 2020.

[48] C. Wang, P.B. Luh, P. Gribik, T. Peng, and L. Zhang. Commitment cost allocation of fast-start units for approximate extended locational marginal prices. *IEEE Transactions on Power Systems*, 31(6):4176–4184, 2016.

[49] Laurence A Wolsey. *Integer programming.* John Wiley & Sons, 2020.

[50] G. Zakeri, G. Pritchard, M. Bjorndal, and E. Bjorndal. Pricing wind: A revenue adequate, cost recovering uniform price auction for electricity markets with intermittent generation. *INFORMS Journal on Optimization*, 1(1):35–48, 2019.

[51] V.M. Zavala, K. Kim, M. Anitescu, and J. Birge. A stochastic electricity market clearing formulation with consistent pricing properties. *Operations Research*, 65(3):557–576, 2017.