High quality timetables for Italian schools

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Abstract

This work introduces a complex variant of the timetabling problem, which is motivated by the case of Italian schools. The new requirements enforce to (i) provide teachers with the same idle times, (ii) avoid consecutive days with heavy workload, (iii) limit multiple daily lessons for each class, (iv) introduce shorter time units to differentiate entry and exit times. We present an integer programming model for this problem, which is denoted by Italian High School Timetabling Problem (IHSTP). However, requirements (i), (ii), (iii) and (iv) cannot be expressed according to the current XHSTT standard. Since the IHSTP model is very hard to solve by an off-the-shelf solver, we present a two-step optimization method: the first step optimally assigns teachers to lesson times and the second step assigns classes to teachers. The computational experimentation shows that the method is effective in solving both this new problem and the simplified problem without the new requirements.

Keywords: *Integer Programming, Timetabling, High School Timetabling.*

1 Introduction

The High School Timetabling problem (HST) is a relevant research area, which aims to schedule lectures to time slots. Its characteristics are country-dependent [12] and several solution approaches were proposed [13]. The introduction of the XHSTT (XML High School Time-Table) format for the HST problem has provided a uniform way to support a variety of possible constraints. However, new requirements have emerged in the case of Italian High School and some of them do not fit with this format. This research is motivated by such a case.

The recent reforms in 2008 and 2015 deeply changed the educational structure of Italian high schools to make a service more oriented to students and decrease system costs [1]. The reduction in the weekly extension of lessons has led to an irregular distribution of lectures. Moreover, when two classes of the same year (or level) have few students, schools are requested to merge them into the so-called *articulated class*, even if students have a different curriculum. Therefore, the students of articulated classes may have few subjects in common and must be split when the different characterizing subjects are taught. In addition, full-time teachers must give lessons for 18 hours a week and, if this workload is not complete, they must be enrolled in other schools. Yet, some teachers may have additional days-off to account for possible additional duties.

The former reforms increased the number of idle times for teachers. They claim that this number must be the same for all of them for the sake of equity. The new rules may also result in the planning of timetables with heavy workloads in consecutive days and lead to the burn-out of teachers. Moreover, it is recommended to schedule school days of same duration for a class to plan the transportation of students smoothly, even if this situation leads to the growth in the number of lessons. In addition, it is important to diversify the entrance and exit times of classes to limit crowds, as emphasized by the recent pandemic event.

Although relevant research exists in the HST problem for Italian schools [21, 3], it dates back in time and the former requirements were not taken into account. In this paper they are investigated and added to well-recognized requirements for the HST problem (assign teachers to classes, full-time and part-time teachers with one or more days off, surveillance in each class at any time slot, etc.). The complete list of requirements is provided in Section 2. All in all, the new problem is denoted by the *Italian High-School Timetabling Problem* [IHSTP].

This paper presents an Integer Programming formulation for the [IHSTP]. Since large-scale instances cannot be solved efficiently by an off-the-shelf optimization solver, we present a two-step algorithm. In the first step, we assign teachers to time slots by solving the so-called *Teacher Profile Problem* [TPP]. In the second step, we solve a restricted version of the former formulation of [IHSTP] using the solution of the [TPP]. Since some of the new requirements are not supported by the XHSTT format, all models are implemented by a general-purpose modeling language and solved by a MIP (Mixed-Integer Programming) solver.

The two-step method is extensively tested in several instances, in order to asses to what extent it can be adopted. More precisely, in the first part of experimentation we consider all requirements and compare the solutions of the MIP solver for the [IHSTP] model and those provided by the two-step method, in which each sub-problem is solved by the same MIP solver. In the second part we focus on a simpler problem without the new features of [IHSTP] and compare the solutions of the MIP solver running our formulation and that by [11]. The program KHE was also adopted to enrich the comparison. The former approaches are tested with and without the [TPP] step.

The experiments show the effectiveness of the two-step method, because it determines high-quality solutions for the problem at hand in terms of times and gaps. Moreover, it also effective for a simplified problem devoid of the new requirements: the method can be successfully applied both to the XHSTT model and the IHSTP formulation, but the results are far better in the latter case.

This paper is organized as follows. In section 2 the specific requirements for Italian High School are presented. In section 3 the related work is critically discussed, comparing the our problem with the case of other countries. In section 4 a complete Integer Programming formulation for Italian High School is defined. In section 5 the two-step method is presented and the [TPP] is described and formulated. In section 6 experimental results are presented. Finally, in section 7 the conclusion is reported.

2 Italian High School Timetabling Problem

In Italian schools each student belongs to a class (or group of students) sharing the same lessons according to a *curriculum*). All students in a class must follow the same set of subjects for a fixed number of weekly hours. Lessons are daily organized in time slots (e.g. 1 hour, but fractional lesson units are also possible) and must be placed in a time horizon, which normally spans over a week and is repeated periodically for the entire school year. Lessons may span over multiple consecutive time slots, to accommodate special needs as in-class works or lab activities. These lessons are called multiple lessons (e.g. double and triple lessons).

Each subject is taught by a teacher or, more rarely, by a teacher and a co-presence teacher (or co-teacher), who has to teach always together with another colleague supervising the activity. From now on, for the sake of simplicity, teachers and co-teachers will be denoted as teachers, unless one refers explicitly to co-teachers and non-co-teachers.

Teachers may give lessons on more than one subject in one or more classes. Schools open from Monday to Saturday (very seldom until Friday) and teachers must have a day off for rest. They are classified in full-time and part-time teachers. Full-time teachers have to teach for a fixed number of hours a week (typically 18 hours, but some reductions are possible to do some management tasks) and must work in others schools to complete their workload. Part-time teachers have a shorter workload according to their annual contract and may work for several schools. As a result, some teachers must receive more than one day-off from each school. For the same reasons, some teachers may not be available to teach in some specific times.

Clearly, a teacher should not be employed for very few time slots a day. Conversely, a workload spanning all time slots in a day in not recommended, to prevent burn-out. Time-slot breaks are possible between lessons, even if are they not always required or appreciated.

The objective is to build a timetable, i.e. assign each lesson to a specific time slot of each day, such that a number of requirements is satisfied. They are divided into mandatory (or hard), desirable (or soft) or both. For the sake of clarity, in the following we enumerate all requirements and denote if they are hard, soft or both.

- R1 (hard) - Each class has to attend lessons for a given set of weekly days and a consecutive set of hours a day, as established by the school. For example, in a school all classes of the fifth year have to attend 32 hours a week and 5 hours a day, except on Tuesday and on Thursday, in which lessons are given for six hours. Every class of the second year has to attend lessons for 33 hours a week, in which the additional hour w.r.t. fifth year classes is given on Saturday.
- R2 (hard) - Every teacher has to teach for a fixed number of hours as established by national laws or school rules.
- R3 (hard) - Every teacher must have at least one day off a week. It can be determined according to two school-dependent policies: the day off can be *a priori* selected by the school or its decision is left *a posteriori* during timetable planning. Therefore, any methodological proposal must be able to deal with both policies.
- R4 (hard/soft) - A subset of teachers must/may receive additional days off according to specific conditions (e.g. employment in several schools, special contracts, additional administrative tasks, etc.). Unlike R5, these conditions affect whole days instead of specific daily parts.
- R5 (hard) - Since classes spend different time periods at school (on a daily and weekly basis), a lesson must be scheduled for a class only when the class is at school. For example, a fifth-year class cannot attend any lesson in the sixth hour on Saturday, if only five hours of lessons are scheduled for that class.
- R6 (hard) - A lesson must not be scheduled for a teacher in the case of specific commitments in specific periods of a day (e.g. employment in another school, special contracts, additional administrative tasks, etc.).
- R7 (hard) - Each class has to be taught by a given teacher for a fixed number of weekly hours. This number is called *week requirement* and is established by laws or school rules.
- R8 (hard) - A teacher-clash must be avoided: a teacher cannot teach simultaneously in two classes, unless they form an articulated class.
- R9 (hard) - A class-clash must be avoided: two teachers cannot teach the same class at the same time; the only exception is represented by the so-called co-teaching lessons (e.g. in some lab lessons).
- R10 (hard/soft) - The multiple lessons of a teacher in a class should be consecutive. It is important for multiple lessons of the same teacher in a class to be consecutive in a day. Clearly, a hard requirement for not splitting lessons could prevent the determination of a feasible timetable. As a result, both hard and soft options are possible. Moreover, consecutive lessons are welcome to have in-class works or written exams.
- R11 (hard) - For a limited number of hours, an articulated class must be divided into two or more groups attending different lessons with dedicated teachers. For example, a class could attend the lessons on the second foreign language with two different teachers at the same time: one for French and one for Spanish. The problem doubles in the case of co-teachers in articulated class: for example, if this class has two groups of students and the split groups must attend a lab lesson in co-teaching, four teachers must be involved with the class at the same time.
- R12 (hard) - Block lessons must be scheduled. These lessons take place at the same time for two or more classes, in order to share possible resources (e.g., gym or specialized language teachers). Blocks could also support the ordering of lessons by an optional offset, to enforce one lesson to precede another one in a class by a given number of periods.

- R13 (hard) - Preassigned lessons must be scheduled. In these lessons a teacher is already assigned to a class in a given period of a given day. They are often adopted when a teacher gives lessons for a short number of hours in a school.
- R14 (soft) - This requirement enforces a balanced distribution of the lessons among the workdays for a teacher in a class. This requirement can be denoted by horizontal distribution. For example, it holds for a teacher working for one hour on Monday, Tuesday, Thursday and Friday and for two hours on Wednesday (on Saturday no lessons are possible because the teacher must have a day-off).
- R15 (soft) - This requirement guarantees a balanced distribution among daily periods for a teacher in a class. This requirement can be referred to as vertical distribution. For example, it holds when a teacher gives lessons in a class no more than once in the first daily period, no more than once in the second daily period and so on.
- R16 (hard/soft) - Every teacher must/may give lessons in between a minimum and maximum number of daily periods over all classes taught in a day. These numbers can be conveniently set to zero, if appropriate.
- R17 (soft) - Every teacher is willing to have a weekly timetable with no idle times between consecutive lessons. However, this requirement is often difficult to achieve in practice for every teacher.
- R18 (hard/soft) - The multiple lessons of specific pairs of teachers and classes must/should be limited in a week between a minimum and a maximum number of periods.
- R19 (hard/soft) - Each teacher must/should not to reach the maximum workload in two consecutive days.
- R20 (hard/soft) - A minimum and maximum number of double lessons must/should be scheduled in the week for some pairs of classes and teachers.
- R21 (hard/soft) - A minimum and maximum number of triple lessons must/should be scheduled in the week for some pairs of classes and teachers.
- R22 (hard/soft) - The timetable must/should avoid the occurrence of too many multiple lessons for a class in a particular day.
- R23 (hard/soft) - The daily periods of a teacher in a class must/should be in between a minimum and a maximum value.
- R24 (hard) - Fractional periods must be introduced to differentiate the times to start and end lessons for groups of classes. As a result, the duration of all lessons must be multiple quantities of this fractional time unit. This requirement could be hard and enforced for all classes, but it could also be ignored at all for the sake of equity.
- R25 (hard) - All teachers must have the same number of idle times. This requirement is set to be hard, because it must be enforced for all teachers or ignored at all for the sake of equity.

3 Related work

Several studies investigated the HST problem by Integer Programming formulations. The problem characteristics are country-dependent and depend on the organizational model, which could be class-teacher (e.g. Australia, Bosnia, Brazil, Greece, Italy and South-Africa), course-based (e.g. USA) or a mix of them (e.g. Denmark, England, Finland and Netherlands). In the class-teacher model lessons are given to all students of a class, whereas in the course-based variant students attend lessons according to their individual plan. In the first case compact timetables are built from classes, which do not have idle times, whereas teachers typically have. In the second case, the timetable of teachers has no idle times, which can take place for students. This paper is in the area of class-teacher models and, to our knowledge, this is the first study investigating requirements R19, R22, R24 and R25.

Several constraints were analyzed in [16] on Bosnia and Herzegovina, but no computational experiments are not provided. Moreover, it also neglects requirements on days off, obligation to take lessons (R5), balance of lessons spread in the week (R14, R15), teachers' idle times (R17) and limits on multiple daily lessons for classes (R18).

A lot of research was carried in Brazil on the so-called Class-Teacher Timetabling Problem with Compactness Constraints (CTTPC) ([17], [6], [7], [8], [18], [19], [20]). Owing to the specific characteristics of Brazilian schools, the former papers do not consider requirements on irregular weekly class layout (R5), articulated class (R11), block lessons (R12), preassigned lessons (R13), balance of lessons spread in the week (R14, R15), multiple daily lessons limit for classes (R18) and restrictions on triple lessons (R21).

The Danish HST problem was described in ([23], [22]). However, they do not take into account requirements on weekly workload of teachers (R2), additional days off (R4), split lessons (R10), articulated classes (R11), balance of lessons spread in the week (R14, R15), multiple daily lessons limit for classes (R18), limits for number of double (R20) and triple lessons (R21), and restrictions on daily class-teacher workload (R23).

The French school is investigated only in [14]. However, it ignores the requirements on articulated classes (R11), block lessons (R12), preassigned lessons (R13), balance of lessons spread in the week (R14, R15), limited idle times (R17), multiple daily lessons limit for classes (R18), limits on number of double (R20) and triple lessons (R21), and class-teacher workload (R23). Experimental results were provided only for one instance and presented very synthetically.

The case of the Greece school is investigated in [4], [15], [5], [25] and [24]. Unlike in the Italian case, there are no requirements on lessons spread in the week with respect to daily periods (R15), limits for number of double (R20) and triple lessons (R21).

The HST problem was investigated in Italy by [21], [3], who did not take into account the recent scholastic reforms. As a result, they could not consider the requirements on lessons spread in the week with respect to daily periods (R15), limits on the number of double (R20) and triple lessons (R21) and restrictions on the class-teacher workload (R23).

The HST problem was also generalized by ([11], [9]) to support the XHSTT format and adopt Integer Programming formulations. Although the set of requirements is wide, it is not exhaustive for the Italian case.

Table 1 reports which problem requirements are faced in the most recent literature on HST problem. Therefore, one can notice that requirements R19, R22, R24 and R25 have not been investigated so far. This paper covers this gap.

Year	Ref	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20	R21	R22	R23	R24	R25
1997	[4]	X	X	X	X		X	X							X		X	X								
1999	[21]		X	X		X	X	X	X			X	X	X			X		X							
2003	[15]	X	X	X	X			X	X	X		X			X		X	X								
2007	[3]	X	X	X	X	X	X	X	X	X	X			X	X		X	X								
2009	[5]	X	X	X	X		X	X	X	X	X		X	X				X	X							
2012	[6]			X	X		X	X	X	X	X						X	X			X			X		
2012	[17]				X				X	X	X							X			X			X		
2012	[25]	X		X	X	X	X	X	X	X			X		X		X	X								
2014	[7]			X	X		X	X	X	X	X						X	X			X			X		
2014	[22]	X		X		X	X	X	X	X			X	X			X									
2015	[11]	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X		X	X		X		
2015	[16]	X	X				X	X	X	X	X	X	X	X			X				X	X		X		
2016	[8]			X	X		X	X	X	X	X						X	X			X	X		X		
2016	[14]	X	X	X		X	X	X	X	X	X	X					X									
2017	[9]	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X		X	X		X		
2017	[18]	X			X		X	X	X	X	X							X			X			X		
2018	[19]	X			X		X	X	X	X	X							X			X			X		
2020	[24]	X		X	X	X	X		X	X							X	X								
2020	[20]	X			X		X	X	X	X	X							X			X			X		
2021	**this**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**

Table 1: Comparison between High School Timetable works relatively requirements used in this document

Moreover, the current version of XHSTT (XHSTT-2014 [2]) does not support the new requirements R22, R24 and R25. The implementation of requirement R19 is possible, but it requires much effort and has some limitations. More precisely, at the moment this requirement must be implemented with a different value for any pair of consecutive days and any teacher. As a result, it would be simpler and more effective to *a priori* enforce the maximum workload between consecutive days, in order to simplify the implementation and decrease the memory issues.

Finally, the timetabling XHSTT logic is based on *a priori* enumeration of variable length sub-events covering an event (week requirement). However, another logic is possible: the division of an event into unitary time-slots. Since the second logic is expected to be less memory-consuming, it is of interest to develop a time-slot-based model and make a comparison to an event-based model. Clearly this comparison is possible only in simplified problem without the new requirements. This paper investigates this comparison.

4 Mathematical model for the [IHSTP]

In this section we present the mathematical formulation for the [IHSTP] for a single school. The formulation is based on the following sets: let C be the set of classes (or groups of students), T the set of teachers, $F \subseteq T$ the set of co-teachers, D the set of days, H the set of daily periods, N set of possible periods in multiple lessons. Note that the duration of a single period is shorter than the length of a lesson in the case of multiple lessons and/or fractional time units.

The following parameters are defined. Let χ_{ct} be the number of weekly lessons for class $c \in C$ with teacher $t \in T$ (this number is typically called *timetable requirement*). Preassigned lessons are denoted by parameter π_{ctdh}, which takes value 1 if a lesson has to be scheduled in day $d \in D$ at period $h \in H$ for class $c \in C$ with teacher $t \in T$, 0 otherwise.

In order to handle block lessons, consider any two classes $c', c'' \in C$ and two any teachers $t', t'' \in T$, and define the quantity $\phi_{c't'c''t''}$, which denotes the number of lessons that must be located in the same time slot for teacher $t' \in T$ in class $c' \in C$ and teacher $t'' \in T$ and class $c'' \in C$. Let μ_{ctf} be the number of weekly lessons of both teacher $t \in T \backslash F$ and co-teacher $f \in F$ in class $c \in C$.

Let $\underline{\epsilon}_{ct}$ and $\overline{\epsilon}_{ct}$ the minimum and the maximum duration of a multiple lesson for class $c \in C$ with teacher $t \in T$, whereas $\underline{\zeta}_{ct}$ and $\overline{\zeta}_{ct}$ are its minimum and maximum occurrence of multiple lessons of class $c \in C$ with teacher $t \in T$ in the week. We also denote by θ_{ct} the penalty for the violation of multiple lessons of class $c \in C$ with teacher $t \in T$.

The following parameters are defined to link classes, days and periods. Let δ_{cdh} be a coefficient which takes value 1 if class $c \in C$ has to have a lesson in day $d \in D$ at period $h \in H$, 0 otherwise (note that, if $\delta_{cdh} = 0$, it is also possible for class $c \in C$ to have a lesson in day $d \in D$ at period $h \in H$). If class $c \in C$ is recommended to have a lesson in day $d \in D$ at period $h \in H$, consider the boolean parameter β_{cdh} instead. If a class c must not have a lesson in day d at period h, the parameter β_{cdh} has value 0, and 1 otherwise.

The following parameters are defined to link teachers, days and periods. Let γ_{tdh} a boolean parameter with value 1 if teacher $t \in T$ is available to give a lesson in day $d \in D$ at period $h \in H$, 0 otherwise. According to the values of γ_{tdh}, one can easily detect the last duty period ν_{td} for teacher $t \in T$ in day $d \in D$. Moreover, teacher $t \in T$ must or could have in between $\underline{\alpha}_{td}$ and $\overline{\alpha}_{td}$ lessons on day $d \in D$.

The days-off of any teacher $t \in T$ are controlled by parameter τ_{ti}, in which index i takes integer values from 0 to 3. If $i = 0$, teacher $t \in T$ must have a day off in day $\tau_{t0} \in D \cup \{0\}$ (where 0 indicates a day off selected in the model solution); if $i = 1$, τ_{ti} represents the minimum number of additional days off of teacher $t \in T$ (since one day off must be guaranteed, the number of days off a week is at least $\tau_{t1} + 1$); if $i = 2$, τ_{ti} is the maximum number of additional days off for teacher $t \in T$ (hence, the number of days off a week is at most $\tau_{t2} + 1$); if $i = 3$, index τ_{ti} represents the (high) cost of violation of days off. Let $\tilde{D}_t$ be the singleton of the day off for teacher $t \in T$: $\tilde{D}_t = \{\tau_{t0}\}$.

The following parameters are defined to link teachers and classes. Teacher $t \in T$ must or could have in between $\underline{\rho}_{ct}$ and $\overline{\rho}_{ct}$ lessons with class $c \in C$.

In order to introduce a possible fractional time duration for all classes of a school, consider an integer positive parameter η, which represents the number of daily periods in a single lesson. For example, if the lesson takes 1 hour and the daily periods of set H represent 30 minutes, η takes value 2. Since some lessons cannot have a duration multiple of η, they need to be removed from the planning of fractional time units. As a result, define the set of incompatible periods $\tilde{N}_\eta = \{n \in N | (n \bmod \eta) \neq 0\}$.

The first decision variable is denoted by x_{ctdh}. It takes value 1 if class $c \in C$ is assigned to teacher $t \in T$ on day $d \in D$ at period $h \in H$, 0 otherwise. Note that $x_{ctdh} = 0$ if $d \in D \cap \tilde{D}_t$, $t \in T$, $c \in C$,

$h \in H$. Clearly, this is the main decision variables, because its entries with value 1 define the timetable. The following auxiliary variables are also defined:

- a'_{td} is equal to 1 if at least one lesson of teacher $t \in T$ is scheduled on day $d \in D$, 0 otherwise;
- a''_{ctd} is equal to 1 if at least one lesson of teacher $t \in T$ and class $c \in C$ is scheduled on day $d \in D$, 0 otherwise;
- $b_{c't'c''t''dh}$ takes value 1 if teacher $t' \in T$ has a lesson on class $c' \in C$ and teacher $t'' \in T$ has a lesson on class $c'' \in C$ in the same period $h \in H$ of the same day $d \in D$, 0 otherwise;
- e_{ctfdh} is equal to 1 if teachers $t \in T$ and $f \in F$ have a lesson in class c on day $d \in D$ at period $h \in H$, 0 otherwise;
- m_{nctdh} is equal to 1 if a multiple lesson with duration $n \in N$ of teacher $t \in T$ starts at period $h \in H$ of day $d \in D$ in class $c \in C$, 0 otherwise;
- s^{min} is the minimum idle times for all teachers;
- s^{max} is the maximum idle times for all teachers;
- u'_{td} is the ordinal number of the first activity period of teacher $t \in T$ on day $d \in D$;
- v'_{td} is the ordinal number of the last activity period of teacher $t \in T$ on day $d \in D$;
- u''_{ctd} is the ordinal number of the first activity period of teacher $t \in T$ in class $c \in C$ on day $d \in D$;
- v''_{ctd} is the ordinal number of the last activity period of teacher $t \in T$ in class $c \in C$ on day $d \in D$.

For the sake of clarity, constraints are clustered in types depending on the requirements presented in Section 2. The link between constraints are requirements is reported in Table 2, which also reports a brief description of the types of constraints.

Requirements	**Constraint**	**Description**
-	C_0	Service constraints (required for the implementation of each requirement)
R1,R2,R7	C_1	Weekly requirement
R1	C_2	Class presence
R5,R9	C_3	Class unavailability
R6,R8	C_4	Teacher unavailability
R10	C_5	Split lessons
R3,R4	C_6	Days off
R9	C_7	Co-teaching
R11,R12	C_8	Block
R13	C_9	Pre-assigned lessons
R25	C_{10}	Equity in idle times
R17	C_{11}	Idle times
R18,R20,R21	C_{12}	Multiple lessons
R14	C_{13}	Horizontal distribution
R15	C_{14}	Vertical distribution
R16	C_{15}	Teacher workload restrictions
R23	C_{16}	Class/teacher workload restrictions
R22	C_{17}	Excessive multiple lessons
R19	C_{18}	Maximum workload
R24	C_{19}	Fractional time unit

Table 2: Grouping of requirements in types of constraints

All constraints are described hereafter.

C_0 **- Service constraints.**

(1) $a'_{td} \geq x_{ctdh}$ $\qquad \forall c \in C, \forall t \in T, \forall d \in D, \forall h \in H$

$$(2) \quad \nu_{td} a''_{ctd} \geq \sum_{h \in H} x_{ctdh} \qquad \forall c \in C, \forall t \in T, \forall d \in D$$

According to (1), any teacher $t \in T$ cannot be assigned to any class $c \in C$ in any period $h \in H$ of day $d \in D$ if he/she is not scheduled on this day. Constraint (2) enforces that any teacher $t \in T$ cannot be assigned to any period in class $c \in C$ on day $d \in D$ if he/she is not scheduled in this class on this day.
C_1 **- Weekly requirement (R1, R2, R7, hard).** The sum of all lessons of teacher $t \in T$ in class $c \in C$ cannot differ from those the required value (χ_{ct}). Since the satisfaction of this hard constraint could not be guaranteed, a non-negative integer variable $s^{C_1}_{ct}$ is introduced. More formally,

$$(3) \quad \sum_{d \in D} \sum_{h \in H} x_{ctdh} - s^{C_1}_{ct} \leq \chi_{ct} \qquad \forall c \in C, \forall t \in T$$

$$(4) \quad \sum_{d \in D} \sum_{h \in H} x_{ctdh} + s^{C_1}_{ct} \geq \chi_{ct} \qquad \forall c \in C, \forall t \in T$$

(3) and (4) is similar to the analogous constraint introduced in [21].
Before introducing constraint types C_2 and C_3, it is worth noting that in each class there is at most a teacher $t \in T \setminus F$ and, if there is no teacher, the class cannot attend a lesson. These requirements can be directly enforced by the boolean parameters δ_{cdh} on class presence and β_{cdh} on class availability, in fact
$\delta_{cdh} \leq \sum_{t \in T \setminus F} x_{ctdh} \leq \beta_{cdh} \qquad \forall c \in C, \forall d \in D, \forall h \in H$
However, these constraints are not implemented as reported above, because we need to penalize their violation. Therefore, in what follows, we consider inequalities separately and introduce suitable auxiliary variables.
C_2 **- Class presence (R1, hard).** The following constraint enforces that each class must attend lessons in some periods and days of the weekly timetable:

$$(5) \quad \sum_{t \in T \setminus F} x_{ctdh} + s^{C_2}_{cdh} \geq \delta_{cdh} \qquad \forall c \in C, \forall d \in D, \forall h \in H$$

Note that the inequality constraint holds despite the non-negative integer variable, because a teacher could be assigned to a class in a daily period, even if the class does not have to attend a lesson in that period. Clearly, this situation must not be penalized unlike in the converse case.
C_3 - **Class unavailability (R5, R9, hard).** The following constraint enforces that a class could attend lessons in some periods and days only if it is available in these periods and days of the weekly timetable:

$$(6) \quad \sum_{t \in T \setminus F} x_{ctdh} - s^{C_3}_{cdh} \leq \beta_{cdh} \qquad \forall c \in C, \forall d \in D, \forall h \in H$$

Note that the inequality constraint holds despite the non-negative integer variable, because it is possible to have the availability of a class in a period of a day, but no teacher is assigned to the class. Clearly, this situation must not be penalized unlike in the converse case.
C_4 **- Teacher unavailability (R6, R8, hard).** Excluding the case of articulated classes, teacher $t \in T$ cannot be assigned to more than one class in each period of each day, i.e. $\sum_{c \in C} x_{ctdh} <= 1$. The (un)availability of teachers is controlled by the boolean parameter γ_{tdh} and we must penalize the assignment of teachers when they are not available. Therefore, we introduce a boolean variable $s^{C_4}_{tdh}$, which takes value 1 of this critical situation occurs, 0 otherwise. Therefore, this constraint can be formulated as follows:

$$(7) \quad \sum_{c \in C} x_{ctdh} - s^{C_4}_{tdh} \leq \gamma_{tdh} a'_{td} \qquad \forall t \in T, \forall d \in D, \forall h \in H$$

C_5 **- Split lessons (R10, hard/soft).** Multiple lessons of any teacher $t \in T$ in class $c \in C$ must be consecutive on any day $d \in D$ (or without splits). This constraint can be enforced in period $h \in H$ by an upper bound of value h on the period of the first lesson and a lower bound of value h on the period of the last lesson for teacher $t \in T$ in class $c \in C$ on day $d \in D$, if this teacher is on duty in this class on this day. If $x_{ctdh} = 0$, these bounds must not be effective. More formally,

$$(8) \quad u''_{ctd} \leq (|H|+1) - (|H|+1-h) x_{ctdh} \qquad \forall c \in C, \forall t \in T, \forall d \in D, \forall h \in H$$

$$(9) \quad v''_{ctd} \geq h x_{ctdh} \qquad \forall c \in C, \forall t \in T, \forall d \in D, \forall h \in H$$

However, one must still link the time interval between the first and last teaching period to the number of lessons of a teacher in a day. The boolean variable $s^{C_5}_{ctd}$ is introduced to detect the split lessons of teacher $t \in T$ in class $c \in C$ on day $d \in D$ when it takes value 1, 0 otherwise. More formally,

$$(10)\quad a''_{ctd}+v''_{ctd}-u''_{ctd} \leq \sum_{h\in H} x_{ctdh}+s^{C_5}_{ctd}(|H|-2) \qquad \forall c\in C, \forall t\in T, \forall d\in D$$

A minor change in these constraints will be reported later to handle idle times.

C_6 **- Days off (R3, R4, hard/soft).** The overall number of days off must be in between the minimum and the maximum values, which are $1 + \tau_{t1}$ and $1 + \tau_{t2}$ for teacher $t \in T$, respectively. A non-negative integer variable $s^{C_6}_t$ is introduced to report how many times these constraints are not satisfied for teacher $t \in T$. Therefore,

$$(11)\quad 1+\tau_{t1}-s^{C_6}_t \leq |D|-\sum_{d\in D} a'_{td} \qquad \forall t \in T$$

$$(12)\quad |D|-\sum_{d\in D} a'_{td} \leq 1+\tau_{t2}+s^{C_6}_t \qquad \forall t \in T$$

C_7 **- Co-teaching (R9, hard).** Co-teaching cannot be performed either when the class or the teacher or the co-teacher are not available in a daily period.

$$(13)\quad e_{ctfdh} \leq \beta_{cdh}\cdot\gamma_{tdh}\cdot\gamma_{fdh}\cdot x_{ctdh} \qquad \forall c\in C, \forall t\in T\backslash F, \forall f\in F, \forall d\in D, \forall h\in H$$

$$(14)\quad e_{ctfdh} \leq \beta_{cdh}\cdot\gamma_{tdh}\cdot\gamma_{fdh}\cdot x_{cfdh} \qquad \forall c\in C, \forall t\in T\backslash F, \forall f\in F, \forall d\in D, \forall h\in H$$

Moreover, one must guarantee exactly μ_{ctf} co-teaching lessons in a week and a possible violation must be taken into account. Therefore, we introduce a non-negative integer variable $s^{C_7}_{ctf}$, which is an excess or lack of lessons for class $c \in C$ with teacher $t \in T$ and co-teacher $f \in F$:

$$(15)\quad \sum_{d\in D}\sum_{h\in H} e_{ctfdh}+s^{C_7}_{ctf} \geq \mu_{ctf} \qquad \forall c\in C, \forall t\in T\backslash F, \forall f\in F$$

$$(16)\quad \sum_{d\in D}\sum_{h\in H} e_{ctfdh}-s^{C_7}_{ctf} \leq \mu_{ctf} \qquad \forall c\in C, \forall t\in T\backslash F, \forall f\in F$$

C_8 **- Block lessons (R11, R12, hard).** Block lessons cannot be performed either when the first class or the second class or their teachers are not available in a daily period:

$$(17)\quad b_{c't'c''t''dh} \leq \beta_{c'dh}\cdot\beta_{c''dh}\cdot\gamma_{t'dh}\cdot\gamma_{t''dh}\cdot x_{c't'dh} \qquad \forall c', c''\in C, \forall t', t''\in T, \forall d\in D, \forall h\in H$$

$$(18)\quad b_{c't'c''t''dh} \leq \beta_{c'dh}\cdot\beta_{c''dh}\cdot\gamma_{t'dh}\cdot\gamma_{t''dh}\cdot x_{c''t''dh} \qquad \forall c', c''\in C, \forall t', t''\in T, \forall d\in D, \forall h\in H$$

Moreover, one must guarantee exactly $\phi_{c't'c''t''}$ co-teaching lessons in a week and a possible violation must be taken into account. Therefore, we introduce a non-negative integer variable $s^{C_8}_{c't'c''t''}$, which is an excess or lack of block lessons for classes $c', c'' \in C$ with teachers $t', t'' \in T$:

$$(19)\quad \sum_{d\in D}\sum_{h\in H} b_{c't'c''t''dh}+s^{C_8}_{c't'c''t''} \geq \phi_{c't'c''t''} \qquad \forall c', c''\in C, \forall t', t''\in T$$

$$(20)\quad \sum_{d\in D}\sum_{h\in H} b_{c't'c''t''dh}-s^{C_8}_{c't'c''t''} \leq \phi_{c't'c''t''} \qquad \forall c', c''\in C, \forall t', t''\in T$$

In the case of articulated classes, one could represent a teacher by an alias (i.e. the pair of teachers t' and t'' represent the same person).

C_9 **- Preassigned lessons (R13, hard).** The lessons of teacher $t \in T$ in class $c \in C$ have to be scheduled in period $h \in H$ of day $d \in D$ when the boolean parameter π_{ctdh} takes value 1. Since lessons could also be scheduled when π_{ctdh} is 0, the satisfaction of preassigned lessons can be enforced by

$$(21)\quad x_{ctdh} \geq \pi_{ctdh} \qquad \forall c\in C, \forall t\in T, \forall d\in D, \forall h\in H$$

However, the the sake of consistency with the other constraints, the former expression is presented by a boolean variable $s^{C_9}_{ctdh}$, which takes value 1 only if the compulsory lesson of class $c \in C$ with teacher $t \in T$ in period $h \in H$ of day $d \in D$ is not scheduled:

(22) $x_{ctdh}+s^{C_9}_{ctdh} \geq \pi_{ctdh}$ $\quad \forall c\in C, \forall t\in T, \forall d\in D, \forall h\in H$

C_{10} **- Equity in idle times (R25, hard).** The same number of idle times among all teachers can be pursued by the minimization of the difference between the maximum and the minimum idle times among all teachers. Therefore, one needs to introduce a new non-negative integer variable $s^{C_{10}} = s^{max} - s^{min}$ and minimize its value. Clearly, the number of idle times of each teacher must be in between s^{min} and s^{max} in the weekly planning horizon. More formally,

(23) $s^{min} + s^{C_{10}} = s^{max}$

(24) $\sum_{d\in D}(v'_{td}+a'_{td}-u'_{td}-\sum_{c\in C}\sum_{h\in H} x_{ctdh}) \leq s^{max}$ $\quad \forall t\in T$

(25) $\sum_{d\in D}(v'_{td}+a'_{td}-u'_{td}-\sum_{c\in C}\sum_{h\in H} x_{ctdh}) \geq s^{min}$ $\quad \forall t\in T$

C_{11} **- Idle times (R17, soft).** The idle times of teacher $t \in T$ in day $d \in D$ can be derived from the first and the last activity daily period in a way similar to constraints C_5 on split lessons. More precisely, we compute for each period daily and teacher a lower bound on the last activity period and an upper bound on the first activity period. Their difference must be limited above by the number of lessons given by teacher $t \in T$ over all classes in a day. In order to guarantee the satisfaction of this constraint, an additional non-negative integer variable $s^{C_{11}}_{td}$ is introduced to report how many times a idle time occurs for teacher $t \in T$ over all periods $h \in H$ of day $d \in D$. Clearly, this variable will be minimized in this formulation. Therefore:

(26) $u'_{td} \leq (|H|+1)-(|H|+1-h)\sum_{c\in C} x_{ctdh}$ $\quad \forall t\in T, \forall d\in D, \forall h\in H$

(27) $v'_{td} \geq h\sum_{c\in C} x_{ctdh}$ $\quad \forall t\in T, \forall d\in D, \forall h\in H$

(28) $a'_{td} + v'_{td} - u'_{td} \leq \sum_{c\in C}\sum_{h\in H} x_{ctdh} + s^{C_{11}}_{td}$ $\quad \forall t\in T, \forall d\in D$

C_{12} **- Multiple lessons (R18, R20, R21, hard/soft).** Consider a multiple lessons of length n starting in period 1 for teacher $t \in T$ in class $c \in C$ on day $d \in D$. As a consequence, in period $n+1$, $x_{ctd(n+1)}$ must take value 0. More formally:

(29) $\sum_{i=1}^{n} x_{ctdi}+1-x_{ctd(n+1)} \leq n+m_{nctd1}$ $\quad \forall n\in N\setminus\{|H|\}, \forall c\in C, \forall t\in T, \forall d\in D$

Hence, m_{nctd1} must take value 1 when n consecutive lessons are followed by a period with no lesson between teacher t and class c.

If the multiple lesson of length n is scheduled in the last periods of a day, the former constraint is modified as follows:

(30) $1-x_{ctd(\nu_{td}-n)}+\sum_{i=1}^{n} x_{ctd(\nu_{td}-n+i)} \leq n+m_{nctd(\nu_{td}-n+1)}$ $\quad \forall n\in N\setminus\{|H|\}, \forall c\in C, \forall t\in T, \forall d\in D$

The following constraint introduces for the special case in which multiple lessons spans over all periods in a day:

(31) $\sum_{h\in H} x_{ctdh} \leq n-1+m_{nctd1}$ $\quad \forall n \in \{|H|\}, \forall c\in C, \forall t\in T, \forall d\in D$

We still need to introduce the case of multiple lessons starting after the first period and ending before the last one:

$$(32)\quad 1-x_{ctd(h-1)}+\sum_{i=1}^{n}x_{ctd(h+i-1)}+1-x_{ctd(h+n)}\leq n+1+m_{nctdh}\ \forall n\in N\backslash\{|H|\}, \forall c\in C, \forall t\in T, \forall d\in D, \forall h\in\{2..(\nu_{td}-n)\}$$

Sometimes the minimum ($\underline{\zeta}_{ct}$) and the maximum ($\overline{\zeta}_{ct}$) number of multiple lessons of predefined length (ranging between $\underline{\epsilon}_{ct}$ and $\overline{\epsilon}_{ct}$) must be considered for some teachers in some classes. The weekly number of multiple lessons can be computed by variable m_{nctdh}, but an additional non-negative integer variable $s_{ct}^{C_{12}}$ must be adopted to compute the number of violations for teacher $t \in T$ in class $c \in C$:

$$(33)\quad \sum_{n=\underline{\epsilon}_{ct}}^{\overline{\epsilon}_{ct}}\sum_{d\in D}\sum_{h=1}^{|H|+1-n} m_{nctdh}+s_{ct}^{C_{12}}\geq\underline{\zeta}_{ct} \qquad \forall c\in C, \forall t\in T$$

$$(34)\quad \sum_{n=\underline{\epsilon}_{ct}}^{\overline{\epsilon}_{ct}}\sum_{d\in D}\sum_{h=1}^{|H|+1-n} m_{nctdh}-s_{ct}^{C_{12}}\leq\overline{\zeta}_{ct} \qquad \forall c\in C, \forall t\in T$$

A congruence check of m_{nctdh} is needed: the sum of all lessons (multiple or single) must be equal to the week requirement

$$(35)\quad \sum_{n\in N}\sum_{d\in D}\sum_{h=1}^{|H|+1-n}(n\cdot m_{nctdh})=\chi_{ct} \qquad \forall c\in C, \forall t\in T$$

C_{13} **- Horizontal distribution (R14, soft).** The lessons of a teacher in a class should not be clustered either in the first part or in the second part of a week. If the weekly number of lessons χ_{ct} of teacher $t \in T$ in class $c \in C$ is even, we enforce to have the same number of lessons in the two parts of the week; if χ_{ct} is odd, their difference is almost one. Since this ideal balance could not be guaranteed in both previous cases, a non-negative integer variable $s_{ct}^{C_{13}}$ is introduced to report the difference between the periods of teacher $t \in T$ in class $c \in C$ in the two parts of the week. More formally:

$$(36)\quad \sum_{d=1}^{\lfloor|D|/2\rfloor}\sum_{h\in H}x_{ctdh}-\sum_{d=\lceil|D|/2\rceil+1}^{|D|}\sum_{h\in H}x_{ctdh}-s_{ct}^{C_{13}}\leq\lceil\frac{\chi_{ct}}{2}\rceil-\lfloor\frac{\chi_{ct}}{2}\rfloor \qquad \forall c\in C, \forall t\in T$$

$$(37)\quad \sum_{d=\lceil|D|/2\rceil+1}^{|D|}\sum_{h\in H}x_{ctdh}-\sum_{d=1}^{\lfloor|D|/2\rfloor}\sum_{h\in H}x_{ctdh}-s_{ct}^{C_{13}}\leq\lceil\frac{\chi_{ct}}{2}\rceil-\lfloor\frac{\chi_{ct}}{2}\rfloor \qquad \forall c\in C, \forall t\in T$$

Note that both former constraints hold when $|D|$ is even or odd.

C_{14} **- Vertical distribution (R15, soft).** The number of lessons of teacher $t \in T$ in class $c \in C$ should not be clustered in a specific period $h \in H$ over all days of the weekly planning horizon. Therefore, we enforce an upper bound $\lceil\frac{\chi_{ct}}{|H|}\rceil$ and a lower bound $\lfloor\frac{\chi_{ct}}{|H|}\rfloor$ on the number of lessons scheduled for any teacher in any class in a given period. Since these bounds could be violated, a non-negative integer variable $s_{cth}^{C_{14}}$ is defined to report how many times they are not met for teacher $t \in T$ in class $c \in C$ in period $h \in H$. Therefore,

$$(38)\quad \sum_{d\in D}x_{ctdh}-s_{cth}^{C_{14}}\leq\lceil\frac{\chi_{ct}}{|H|}\rceil \qquad \forall c\in C, \forall t\in T, \forall h\in H$$

$$(39)\quad \sum_{d\in D}x_{ctdh}+s_{cth}^{C_{14}}\geq\lfloor\frac{\chi_{ct}}{|H|}\rfloor \qquad \forall c\in C, \forall t\in T, \forall h\in H$$

In (38) and in (39) the lessons must have a balanced distribution over all daily periods.

C_{15} **- Teacher workload restrictions (R16, hard/soft).** The number of activity periods of each teacher $t \in T$ in any day $d \in D$ must be in between the lower bound $\underline{\alpha}_{td}$ and the upper bound $\overline{\alpha}_{td}$, if at least a lesson is scheduled for teacher $t \in T$ on day $d \in D$ (this is checked by the values of variable

a'_{td}). Since this situation could not occur, the non-negative integer variable $s^{C_{15}}_{td}$ is defined to report how many times the violation occurs.

$$(40)\quad \sum_{c\in C}\sum_{h\in H} x_{ctdh} - \eta s^{C_{15}}_{td} \leq a'_{td}\overline{\alpha}_{td} \qquad \forall t\in T, d\in D$$

$$(41)\quad \sum_{c\in C}\sum_{h\in H} x_{ctdh} + \eta s^{C_{15}}_{td} \geq a'_{td}\underline{\alpha}_{td} \qquad \forall t\in T, \forall d\in D$$

Note that the parameter η accounts for the possible use of fractional periods.

C_{16} **- Class/teacher workload restrictions (R23, hard/soft).** The number of daily activity periods of each teacher $t \in T$ with class $c \in C$ must be in between the lower bound $\underline{\rho}_{ct}$ and the upper bound $\overline{\rho}_{ct}$, if at least a lesson is scheduled for teacher $t \in T$ with class $c \in C$ on day $d \in D$ (this is checked by the values of variable a''_{ctd}). Since this situation could not occur, the non-negative integer variable $s^{C_{16}}_{ctd}$ is defined to report how many times the violation occurs.

$$(42)\quad \sum_{h\in H} x_{ctdh} - \eta s^{C_{16}}_{ctd} \leq a''_{ctd}\overline{\rho}_{ct} \qquad \forall c\in C, \forall t\in T, \forall d\in D$$

$$(43)\quad \sum_{h\in H} x_{ctdh} + \eta s^{C_{16}}_{ctd} \geq a''_{ctd}\underline{\rho}_{ct} \qquad \forall c\in C, \forall t\in T, \forall d\in D$$

Note that the parameter η accounts for the possible use of fractional periods.

C_{17} **- Excessive multiple lessons (R22, hard/soft).** The number of periods with multiple daily lessons for class $c \in C$ in day $d \in D$ cannot be larger that a threshold value, which can be reasonably set to $\lceil \frac{|H|-1}{2} \rceil$ (e.g. half of the periods in a day, when $|H|$ is even). Since this situation could not occur, the non-negative integer variable $s^{C_{17}}_{cd}$ is defined to report how often it occurs.

$$(44)\quad \sum_{n\in N\setminus\{1\}}\sum_{t\in T\setminus F}\sum_{h=1}^{|H|-(n-1)} n\cdot m_{nctdh} - s^{C_{17}}_{cd} \leq \lceil \frac{|H|-1}{2} \rceil \qquad \forall c\in C, \forall d\in D$$

Note that n allows to consider duration of multiple lessons, which cannot take the trivial length of one period. Moreover, co-teachers are no included to consider this constraint only once for a class in a day. Yet, multiple lessons with duration n cannot start after period $|H| - (n - 1)$.

C_{18} **- Teacher maximum workload (R19, hard/soft).** The workload of teacher $t \in T$ in any two consecutive days $d \in D$ and $(d + 1) \in D$ must take value one period less than the sum of maximum workload in these days (i.e. $\overline{\alpha}_{td} + \overline{\alpha}_{t(d+1)}$). Since the former situation could not occur, the non-negative integer variable $s^{C_{18}}_{td}$ is introduced to quantify the former violation.

$$(45)\quad \sum_{c\in C}\sum_{h\in H}(x_{ctdh} + x_{ct(d+1)h}) - s^{C_{18}}_{td} \leq \overline{\alpha}_{td} + \overline{\alpha}_{t(d+1)} - 1 \qquad \forall t\in T, \forall d\in D\setminus\{|D|\}$$

Although the implementation of this constraint is possible in the standard XHSTT format, it is very complex in practice owing to the size of data in $\overline{\alpha}_{td}$ and $\overline{\alpha}_{t(d+1)}$.

C_{19} **- Fractional time unit (R24, hard).** When fractional time units are possible, the duration of lessons must be multiple of parameter η. Since this condition could not occur, a non-negative integer variable $s^{C_{19}}_{nct}$ is defined to report how often it is not satisfied.

$$(46)\quad \sum_{d\in D}\sum_{h=1}^{|H|+1-n} m_{nctdh} - s^{C_{19}}_{nct} = 0 \qquad \forall n\in \tilde{N}_{\eta}, \forall c\in C, \forall t\in T$$

The objective function is a linear combination of the violation of all types of constraints. Let o_i the overall violation of i-th constraint type and ω_i its weight. More formally, the violation of each constraint type is reported below:

$$o_1 = \sum_{c\in C}\sum_{t\in T} s^{C_1}_{ct} \quad o_2 = \sum_{c\in C}\sum_{d\in D}\sum_{h\in H} s^{C_2}_{cdh} \quad o_3 = \sum_{c\in C}\sum_{d\in D}\sum_{h\in H} s^{C_3}_{cdh} \quad o_4 = \sum_{t\in T}\sum_{d\in D}\sum_{h\in H} s^{C_4}_{tdh}$$

$$o_5 = \sum_{c\in C}\sum_{t\in T}\sum_{d\in D} s^{C_5}_{ctd} \quad o_6 = \sum_{t\in T} \tau_{t3} s^{C_6}_{t} \quad o_7 = \sum_{c\in C}\sum_{d\in D}\sum_{h\in H} s^{C_7}_{cdh}$$

$$o_8 = \sum_{c \in C} \sum_{d \in D} \sum_{h \in H} s^{C_8}_{cdh} \quad o_9 = \sum_{c \in C} \sum_{t \in T} \sum_{d \in D} \sum_{h \in H} s^{C_9}_{ctdh} \quad o_{10} = s^{C_{10}} \quad o_{11} = \sum_{t \in T} \sum_{d \in D} s^{C_{11}}_{td}$$

$$o_{12} = \sum_{c \in C} \sum_{t \in T} \theta_{ct} s^{C_{12}}_{ct} \quad o_{13} = \sum_{c \in C} \sum_{t \in T} s^{C_{13}}_{ct} \quad o_{14} = \sum_{c \in C} \sum_{t \in T} \sum_{h \in H} s^{C_{14}}_{cth} \quad o_{15} = \sum_{t \in T} \sum_{d \in D} s^{C_{15}}_{td}$$

$$o_{16} = \sum_{c \in C} \sum_{t \in T} \sum_{d \in D} s^{C_{16}}_{ctd} \quad o_{17} = \sum_{c \in C} \sum_{d \in D} s^{C_{17}}_{cd} \quad o_{18} = \sum_{t \in T} \sum_{d \in D \setminus \{|D|\}} s^{C_{18}}_{td} \quad o_{19} = \sum_{n \in N} \sum_{c \in C} \sum_{t \in T} s^{C_{19}}_{nct}$$

Hence, the objective function of [IHSTP] is:

$$(47) \quad f = \sum_{i=1}^{19} \omega_i o_i$$

The complete MIP model consists in minimizing f, subject to constraints (1)-(46).

5 A two-step method for the [IHSTP]

Since the model for [IHSTP] is expected to be very hard to solve, we present a two-step method to determine high-quality solutions within a reasonable time interval. The method is motivated by many possibilities for selecting the activity periods of each teacher, who gives lessons in a class for a limited number of periods w.r.t the the overall number of periods spent by students in the same class (e.g. a teachers must say in a class for 4 hours a week and the same class attends lessons for 32 hours a week). Therefore, the [IHSTP] would be simplified if one *a priori* knows the schedule of teachers without details on the classes taught in each period.

Therefore, the proposed method decomposes [IHSTP] into two problems:

- The first problem assigns teaching periods to teachers to determine the so-called teacher profile. This problem is called Teacher Profile Problem [TPP].
- The second problem assign classes to teachers according to the solution of the [TPP] and results in a simplified version of the [IHSTP], which is called restricted [IHSTP] and denoted by [RIHSTP].

The details about the mathematical formulations of these problems are provided in Section 5.1 and Section 5.2. The figure 1 shows the connection between [TPP] and [RIHSTP].

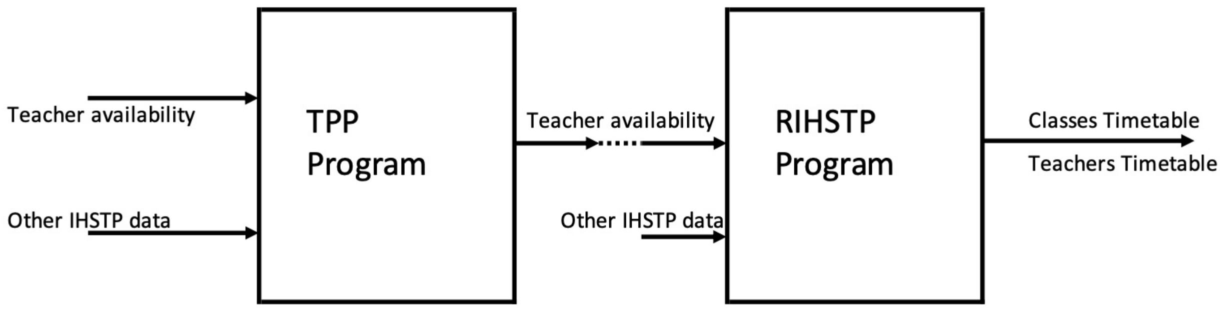

Figure 1: The connection between [TPP] and [RIHSTP]

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5.1 The teacher profile problem [TPP]

5.1.1 Problem description

Relevant data for the [TPP] are the periods in which teachers are available and lessons are given for each class in each day. Classes may not spend the same number of periods at school, because usually the number of school days in a week is not an exact divider of the overall week requirements (e.g. 33

hours over 6 days from Monday to Saturday). Schools have two choices to face this situation: fixing in advance which days have extra periods or letting this decision to the optimization phase. In the first case, parameters β_{cdh} and δ_{cdh} take the same value for all the weekly periods; in the second case they differ when the extra daily periods occur. Generally speaking, Italian schools prefer the first choice, because it results in a greater management control. Moreover, it would not be possible to determine the work shifts of the teachers if the attendance periods of classes at school are not known. Since the teacher profile is determined before the final timetable, in what follows the values of β_{cdh} and δ_{cdh} are supposed to be identical.

We aim to obtain a subset of the profiles for each teacher who is not a co-teacher (or teacher profile), while taking into account some requirements of the [IHSTP], but their determination must be computationally viable. Clearly, the periods in a (non-co-)teacher profile must be consecutive in a day, in order a priori minimize idle times. In the [TPP] we consider daily profiles starting for all teachers in the first period or ending in the last period. This assumption decreases the number of possible profiles and is also motivated by equity issues. In fact, teachers starting in the second period have a edge versus those starting in the first one, because they wake up later and come across less congested roads in their trips. Similarly, teachers ending in the last hour are more tired than those ending before and can go home later. Therefore, two possible shifts are considered: the first shift starts in the first period, the last shift ends in the last period. Note that the profiles of co-teachers are not determined in the [TPP], because the may end up working with teachers with different profiles and it may be impossible to satisfy all the requirements at the same time.

Figure 2 shows an example on the construction of a profile. The teacher has a day off on Wednesday and is available to teach from period 1 to period 6 in the other days. Assume to select in the first shift 3 periods on Monday, 4 periods on Tuesday and 3 periods on Saturday (a). In the last shift assume to select 2 periods on Monday, 3 periods on Thursday and 3 periods on Friday (b). The shifts can be merged and result in the final teacher profile (c). Although the profile of Monday has one idle period, it is acceptable owing to the relevant workload in this day. Note that this profile also satisfies the horizontal and vertical distribution, as defined in requirements R15 and R16.

In what follows, we enumerate all requirements of the profiles (or shifts).

1. **R26** (Shift selection). For non-co-teachers, the first and/or the second shift could be selected in a day.

2. **R27** (Duration of shifts). The length of shifts cannot be larger than the daily availability and teachers cannot be on duty in days which are not selected.

3. **R28** (Allocation of periods to shifts): Non-co-teachers must be on duty for all periods in a shift, if it is selected.

4. **R29** (Teacher profile definition). A period is part of a teacher profile if and only if the first shift or the second shift are selected.

5. **R30** (Profile consistency). Teachers cannot be assigned to profiles with periods in which they are not available. Moreover, the daily profiles cannot be selected in days off.

6. **R31** (Class surveillance). The profile of teachers must guarantee that each class is monitored by one of its non-co-teachers in each daily period.

7. **R32** (Alternated shifts). The profiles of teachers should encourage the alternation between the first and the last shift between any pair of consecutive days to incentive a good vertical distribution possibly.

8. **R33'** (Day off). Teachers must have one day off in a selected day.

Finally, we need to restate a number of requirements of the [IHSTP] in terms of teacher profiles. More precisely, these requirements concern additional days off (**R34**), soft), pre-assigned lessons (**R35**), hard), horizontal distribution (**R36**), hard), vertical distribution (**R37**), hard), block (**R38**), hard), fractional time unit (**R39**), hard), teacher workload restrictions (**R40**), soft), idle times (**R41**, soft), equity in idle times (**R42**), soft).

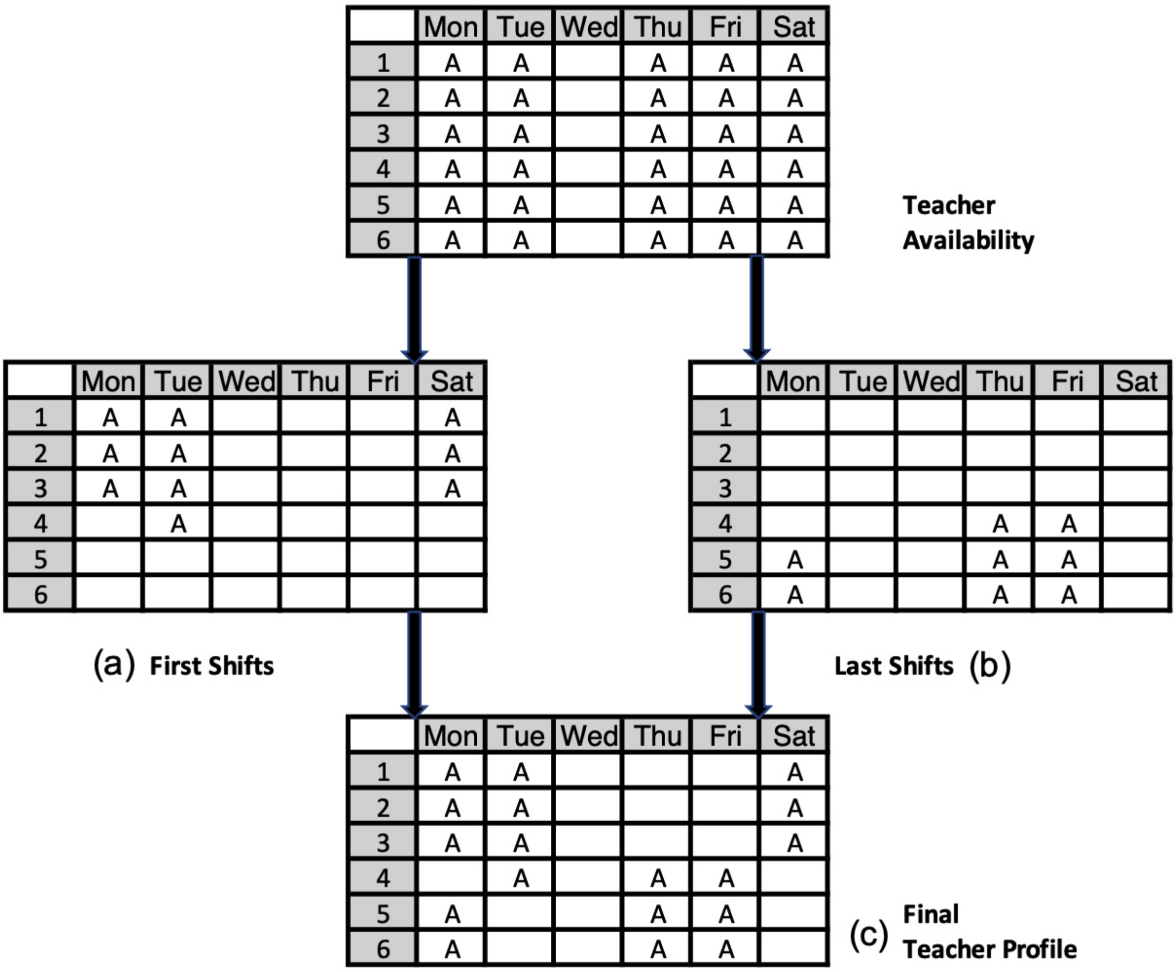

Figure 2: An example of optimized Teacher Profile

5.1.2 Optimization model

The [TPP] formulation is based on the notation already presented for the [IHSTP]. However, some additional notation needs to be introduced. Let ψ^c be a $|T\backslash F|$-column-vector, in which ψ^c_t takes value 1 if teacher $t \in T\backslash F$ teaches in class $c \in C$, and 0 otherwise. Let $\psi^{c\top}$ be the transpose of ψ^c. Moreover, let L_c be the set of classes with some teachers in common with class $c \in C$, including class c itself. It is possible to compute L_c from ψ^c as follows:
$L_c = \{c' \in C | \psi^{c\top}\psi^{c'} > 0\}$

Let y_{tdh} be a decision variable, which takes value 1 if the teacher $t \in T$ is on duty in day $d \in D$ at period $h \in H$, 0 otherwise. Clearly, y_{tdh} is the main decision variable of the [TPP], because all entries with value 1 represent the profile of teacher $t \in T$. The following variables of the [IHSTP] are also used with the same meaning in [TPP] model: $a'_{td}, b_{c't'c''t''dh}, u'_{td}, v'_{td}, s^{min}, s^{max}$. In addition, the following auxiliary variables are defined:
f_{tdh} 1 if $h \in H$ is the last period of the first shift of teacher $t \in T$ on day $d \in D$, 0 otherwise;
l_{tdh} 1 if $h \in H$ is the first period in last shift of teacher $t \in T$ on day $d \in D$, 0 otherwise;
n'_{td} length of the first shift of teacher $t \in T$ on day $d \in D$;
n''_{td} length of the last shift of teacher $t \in T$ on day $d \in D$;
$\tilde{m}_{ntdh}$ is equal to 1 if a block of duration $n \in N$ of teacher $t \in T$ starts at period $h \in H$ of day $d \in D$ in one of the shifts, 0 otherwise.
HC_1 - **Shift selection (R26, hard)**. The following constraint states that the first shift could be selected for any teacher in each day:

(48) $\sum_{h=1}^{\nu_{td}} \gamma_{tdh} f_{tdh} \leq 1 \qquad \forall t \in T \backslash F, \forall d \in D$

Note that the first shift includes all periods between the first one and time slot such that f_{tdh} has value 1. A similar constraint is formulated for the last shift:

(49) $\sum_{h=2}^{\nu_{td}} \gamma_{tdh} l_{tdh} \leq 1 \qquad \forall t \in T \backslash F, \forall d \in D$

Clearly, the last shift includes all periods between the time slot for which l_{tdh} has value 1 and the last one.

HC_2 - **Duration of shifts (R27, hard)**. The following constraints determine the duration of shifts for each teacher in each day from the values of variables f_{tdh} and l_{tdh} :

(50) $n'_{td} = \sum_{h \in H} h \gamma_{tdh} f_{tdh} \qquad \forall t \in T \backslash F, \forall d \in D$

(51) $n''_{td} = \sum_{h \in H} (\nu_{td} + 1 - h) \gamma_{tdh} l_{tdh} \qquad \forall t \in T \backslash F, \forall d \in D$

Each teacher cannot be on duty for a number of periods larger than the daily availability:

(52) $n'_{td} + n''_{td} \leq a'_{td} \sum_{h \in H} \gamma_{tdh} \qquad \forall t \in T \backslash F, \forall d \in D$

In a workday at least one lesson has to be given by a teacher:

(53) $n'_{td} + n''_{td} \geq a'_{td} \qquad \forall t \in T \backslash F, \forall d \in D \backslash \tilde{D}_t$

HC_3 - **Allocation of periods to shifts (R28, hard)**. The following constraints link variable y_{tdj} to f_{tdh} and l_{tdh}:

(54) $y_{tdj} \geq f_{tdh} \qquad \forall t \in T \backslash F, \forall d \in D, \forall h \in H, j \in \{1, .., h\}$

(55) $y_{tdj} \geq l_{tdh} \qquad \forall t \in T \backslash F, \forall d \in D, \forall h \in H, j \in \{h, .., \nu_{td}\}$

HC_4 - **Teacher profile definition (R29, hard)**.
The values of y_{tdh} are computed in the following constraint:

(56) $y_{tdh} = \sum_{i=h}^{\nu_{td}} f_{tdi} + \sum_{i=h}^{\nu_{td}} l_{td(\nu_{td}+1-i)} \qquad \forall t \in T \backslash F, \forall d \in D, \forall h \in H$

Note that three cases may occur: period $h \in H$ does not belong to any shift, or it is part of the first shift or part of the second shift.

HC_5 - **Profile consistency (R30, hard)**. Teachers cannot be assigned to profiles with periods in which they are not available. Moreover, profiles cannot be assigned to days which are not selected to give lessons:

(57) $y_{tdh} \leq a'_{td} \gamma_{tdh} \qquad \forall t \in T \backslash F, \forall d \in D, \forall h \in H$

HC_6 - **Class surveillance (R31, hard)**. Since no class should be left unattended, for each daily period the number of classes must be equal to the number of teachers:

(58) $\sum_{t \in T \backslash F} y_{tdh} = \sum_{c \in C} \delta_{cdh} \qquad \forall d \in D, \forall h \in H$

The previous constraint does not guarantee that each class is attended by one of its teachers (for the sake of simplicity, we do not consider co-teachers). This is possible only if the sets of classes and teachers represent one partition or can be decomposed in several partitions (i.e when the subset of teachers gives lessons only in a subset of classes in a partition and vice versa). Therefore, we need to recall the definition of L_c from the values of ψ_t^c, to report the partition associated with class $c \in C$. If $L_c \equiv C$, there is one partition, else there are at least two partitions. Therefore, for each period of any day and class, a balance must be guaranteed between the number of teachers of the class and the number of classes in the same partition, provided that the class is available:

$$\sum_{t\in T\setminus F:\psi_t^c=1} y_{tdh} = \delta_{cdh}|L_c|$$

However, some classes may not be available in the same daily periods. As a result, the former formula is modified as follows:

$$(59)\quad \sum_{t\in T\setminus F:\psi_t^c=1} y_{tdh} = \sum_{c'\in L_c} \delta_{c'dh} \qquad \forall c\in C, \forall d\in D, \forall h\in H$$

HC_7 - **Day off selection (R33, hard)**. Day off must be guaranteed for each teacher:

$$(60)\quad a'_{td} = 0 \qquad \forall t\in T\setminus F, \forall d \in D\cap \tilde{D}_t$$

HC_8 - **Preassigned lessons (R35, hard)**. Preassigned lessons must have to be scheduled

$$(61)\quad y_{tdh} \geq \pi_{ctdh} \qquad \forall c\in C, \forall t\in T\setminus F, \forall d\in D, \forall h\in H$$

(61) is very similar to (21) in constraint C_9 of the [IHSTP].

HC_9 - **Horizontal distribution (R36, hard)**. Unlike in the [IHSTP], in the [TPP] the horizontal distribution of lessons is enforced on the overall activity of each teacher without paying attention classes:

$$(62)\quad \sum_{d=1}^{|D|/2}\sum_{h\in H} y_{tdh} + \lceil \frac{\sum_{c\in C}\chi_{ct}}{2}\rceil - \lfloor \frac{\sum_{c\in C}\chi_{ct}}{2}\rfloor \geq \sum_{d=|D|/2+1}^{|D|}\sum_{h\in H} y_{tdh} \qquad \forall t\in T\setminus F$$

$$(63)\quad \sum_{d=1}^{|D|/2}\sum_{h\in H} y_{tdh} - \lceil \frac{\sum_{c\in C}\chi_{ct}}{2}\rceil + \lfloor \frac{\sum_{c\in C}\chi_{ct}}{2}\rfloor \leq \sum_{d=|D|/2+1}^{|D|}\sum_{h\in H} y_{tdh} \qquad \forall t\in T\setminus F$$

Note that (62)-(63) are similar to (36)-(37) in C_{13} of the [IHSTP].

HC_{10} - **Vertical distribution (R37, soft)**. The same logic holds for the vertical distribution:

$$(64)\quad \sum_{d\in D} y_{tdh} \leq \lceil \frac{\sum_{c\in C}\chi_{ct}}{|H|}\rceil \qquad \forall t\in T\setminus F, \forall h\in H$$

$$(65)\quad \sum_{d\in D} y_{tdh} \geq \lfloor \frac{\sum_{c\in C}\chi_{ct}}{|H|}\rfloor \qquad \forall t\in T\setminus F, \forall h\in H$$

Clearly, these (64)-(65) are similar to (38)-(39) in C_{13} of the [IHSTP].
HC_{11} - **Block (R38, hard)**. Constrains on block lessons are enforced.

$$(66)\quad b_{c't'c''t''dh} \leq \beta_{c'dh}\cdot\beta_{c''dh}\cdot\gamma_{t'dh}\cdot\gamma_{t''dh}\cdot y_{t'dh} \qquad \forall c', c''\in C, \forall t', t''\in T\setminus F, \forall d\in D, \forall h\in H$$

$$(67)\quad b_{c't'c''t''dh} \leq \beta_{c'dh}\cdot\beta_{c''dh}\cdot\gamma_{t'dh}\cdot\gamma_{t''dh}\cdot y_{t''dh} \qquad \forall c', c''\in C, \forall t', t''\in T\setminus F, \forall d\in D, \forall h\in H$$

$$(68)\quad \sum_{d\in D}\sum_{h\in H} b_{c't'c''t''dh} = \phi_{c't'c''t''dh} \qquad \forall c', c''\in C, \forall t', t''\in T\setminus F$$

Note that (66)-(68) exhibit minor changes w.r.t. (17)-(20) in constraint C_8 of the [IHSTP].

HC_{12} - **Fractional time unit (R39, hard)**. The duration of both shifts of every teacher must be a multiple quantity of the fractional time unit η.

$$(69)\quad \sum_{h\in H} y_{tdh} \leq n-1+\tilde{m}_{ntd1} \qquad \forall n \in \{|H|\}, \forall t\in T\setminus F, \forall d\in D$$

$$(70)\quad \sum_{i=1}^{n} y_{tdi} + (1-y_{td(n+1)}) \leq n+\tilde{m}_{ntd1} \qquad \forall n\in N, \forall t\in T\setminus F, \forall d\in D$$

$$(71)\quad 1-y_{td(h-1)} + \sum_{i=1}^{n} y_{td(h+i-1)} + 1 - y_{td(h+n)} \leq n+1+\tilde{m}_{ntdh} \qquad \forall n\in N, \forall t\in T\setminus F, \forall d\in D, \forall h\in\{2,..,(\nu_{td}-n)\}$$

$$(72)\quad 1-y_{td(\nu_{td}-n)} + \sum_{i=1}^{n} y_{td(\nu_{td}-n+i)} \leq n+\tilde{m}_{ntd(\nu_{td}-n+1)} \qquad \forall n\in N, \forall t\in T\setminus F, \forall d\in D$$

(73) $\sum_{d\in D}\sum_{h=1}^{\nu_{td}+1-n}\tilde{m}_{ntdh}=0$ $\quad\forall n\in\tilde{N}_\eta,\forall t\in T\setminus F$

Note that (69)-(73) are similar to (46) C_{19} and C_{12}. Clearly, these constraints can be skipped if $\eta=1$.

SC_1 - **Alternated shifts (R32, soft)**. The first shift and last shift are recommended to be alternate in consecutive days.

(74) $1-s_{tdh}^{SC_1}\le y_{tdh}+y_{t(d+1)h}$ $\quad\forall t\in T\setminus F,\forall d\in D\setminus\{|D|\},\forall h\in\{1,\nu_{td}\}$

(75) $1-s_{tdh}^{SC_1}\ge y_{tdh}-y_{t(d+1)h}$ $\quad\forall t\in T\setminus F,\forall d\in D\setminus\{|D|\},\forall h\in\{1,\nu_{td}\}$

(76) $1-s_{tdh}^{SC_1}\ge y_{t(d+1)h}-y_{tdh}$ $\quad\forall t\in T\setminus F,\forall d\in D\setminus\{|D|\},\forall h\in\{1,\nu_{td}\}$

(77) $1-s_{tdh}^{SC_1}\le 2-y_{tdh}-y_{t(d+1)h}$ $\quad\forall t\in T\setminus F,\forall d\in D\setminus\{|D|\},\forall h\in\{1,\nu_{td}\}$

SC_2 - **Days off placement (R34, soft)**. It is recommended to provide additional days off to each teacher:

(78) $\sum_{d\in D}a'_{td}+1+\tau_{t1}-s_t^{SC_2}\le|D|$ $\quad\forall t\in T\setminus F$

(79) $\sum_{d\in D}a'_{td}+1+\tau_{t2}+s_t^{SC_2}\ge|D|$ $\quad\forall t\in T\setminus F$

Note that these constraints enforce the assignment of days off, when they were not indicated by teachers.

SC_3 - **Teacher workload restrictions (R40, soft)**. The following constraints play the same role of those in C_{15}, where $\sum_{c\in C}x_{ctdh}$ is replaced by y_{tdh}:

(80) $\sum_{h\in H}y_{tdh}+\eta s_{td}^{SC_3}\ge a'_{td}\underline{\alpha}_{td}$ $\quad\forall t\in T\setminus F,\forall d\in D$

(81) $\sum_{h\in H}y_{tdh}\le a'_{td}\overline{\alpha}_{td}+\eta s_{td}^{SC_3}$ $\quad\forall t\in T\setminus F,d\in D$

SC_4 - **Idle times (R41, soft)**. The following constraints play the same role of those in C_{11}, where $\sum_{c\in C}x_{ctdh}$ is replaced by y_{tdh}:

(82) $u'_{td}\le(\nu_{td}+1)-(\nu_{td}+1-h)y_{tdh}$ $\quad\forall t\in T\setminus F,\forall d\in D,\forall h\in H$

(83) $v'_{td}\ge h\cdot y_{tdh}$ $\quad\forall t\in T\setminus F,\forall d\in D,\forall h\in H$

(84) $a'_{td}+v'_{td}-u'_{td}\le\sum_{h\in H}y_{tdh}+s_{td}^{SC_4}$ $\quad\forall t\in T\setminus F,\forall d\in D$

SC_5 - **Equity in idle times (R42, soft)**. It is recommended for the teachers to have the same minimum idle times:

(85) $\sum_{d\in D}s_{td}^{SC_4}\le s^{max}$ $\quad\forall t\in T\setminus F$

(86) $\sum_{d\in D}s_{td}^{SC_4}\ge s^{min}$ $\quad\forall t\in T\setminus F$

(87) $s^{min}+s^{SC_5}\ge s^{max}$

Teacher Profile Problem objective function

The objective function of the [TPP] is the sum of all constraint deviation multiplied by a proper weight:

(88) $$f=\omega_1\sum_{t\in T\setminus F}\sum_{d\in D}\sum_{h\in\{1,\nu_{td}\}}s_{tdh}^{SC_1}+\omega_3\sum_{t\in T\setminus F}s_t^{SC_3}+\omega_4\sum_{t\in T\setminus F}\sum_{d\in D}s_{td}^{SC_4}+\omega_4\sum_{t\in T\setminus F}\sum_{d\in D}s_{td}^{SC_4}+\omega_5 s^{SC_5}$$

The complete formulation of the [TPP] consists in minimizing f, subject to constraints (48)-(87)

5.2 The restricted [IHSTP]

This problem is obtained by replacing γ_{tdh} with y_{tdh} in [IHSTP], as determined in the solution of the [TPP]. This substitution occurs in constraints (7), (13)-(14), (17)-(18). All in all, the two-step method is supposed to be effective owing to the larger number of null entries of y_{tdh} as opposed to γ_{tdh}. The real effectiveness of the method will be evaluated in the following experimentation.

Figure 3 shows how the solution of the [TPP] can be adopted to obtain a possible solution of the restricted [IHSTP] for teacher A, who must give lessons in classes denoted by 3C, 4C and 5C. For example, according to the TPP, teacher A must give lessons on Monday from period 1 to period 3 and from period 5 to period 6. The restricted [IHSTP] assigns the selected work periods of teacher A to each class. In Figure 3, teacher A is assigned to class 3C from period 1 to period 2, class 5C in period 3, class 4C from period 5 to period 6.

	Mon	Tue	Wed	Thu	Fri	Sat
1	A	A				A
2	A	A				A
3	A	A				A
4		A		A	A	
5	A			A	A	
6	A			A	A	

↓

	Mon	Tue	Wed	Thu	Fri	Sat
1	3C	5C				4C
2	3C	5C				4C
3	5C	4C				3C
4		3C		5C	4C	
5	4C			5C	3C	
6	4C			3C	5C	

Figure 3: A Teacher Timetable obtained as the solution of [RIHSTP] program from the Teacher Profile

6 Experimentation

6.1 Experimental settings

The objectives of this section are threefold. First, we aim to show to what extent the [IHSTP] formulation can be solved to tackle realistic instances including also requirements R19, R22, R24 and R25. Second, we want to assess how much the two-step method is effective both in terms of time and objective function, and how long the computation is performed in each step. Third, we compare the formulation for the [IHSTP] and the two-step method to the [XHSTT] model ([11], [9]) and the KHE heuristic[1] in a simpler experimental setting, in which the former requirements are ignored. The KHE is a well-known freeware open-source C program, that implements an advanced heuristic described in [10] and supports the XHSTT format. Although KHE does not have any time limit for optimization, the option of multiple separate threads can be introduced to obtain better solutions. Clearly, the XHSTT model supports XHSTT format and can be solved by any MIP solver, but additional implementations were performed to remove redundancies on decision variables and scale to larger problems instances.

The experimentation is performed according to two experimental settings, which differ for which constraints are hard, soft or disabled. These settings are called *Setting1* and *Setting2*. For the sake of clarity we denote by *Setting1* the experimentation with requirements R19, R22, R24 and R25, whereas in *Setting2* they are ignored. Therefore, *Setting1* represents the current case of Italian schools and *Setting2* is a simplified problem. The types of constraints in these settings are reported in Table 3.

Constraints	**Requirements**	*Setting1* [IHSTP]	*Setting2* KHE/XHSTT/[IHSTP]
C1	R1,R2,R7	Hard	Hard
C2	R1	Hard	Hard
C3	R5,R9	Hard	Hard
C4	R6,R8	Hard	Hard
C5	R10	Hard/Disabled	Hard/Disabled
C6	R3,R4	Hard	Hard
C7	R9	Hard/Disabled	Hard/Disabled
C8	R11,R12	Hard/Disabled	Hard
C9	R13	Hard/Disabled	Hard/Disabled
C10	**R25**	**Hard**	**Disabled**
C11	R17	Soft	Soft
C12	R18,R20,R21	Soft/Hard	Soft/Hard
C13	R14	Soft	Disabled
C14	R15	Soft	Disabled
C15	R16	Hard	Hard
C16	R23	Hard	Hard
C17	**R22**	**Soft**	**Disabled**
C18	**R19**	**Soft**	**Disabled**
C19	**R24**	**Disabled/Hard**	**Disabled**

Table 3: Types of constraints in the two experimental settings.

The values of coefficients ω_i in the objective function of the [IHSTP] indicate whether the i-th constraint is hard, soft or disabled. They can be set by schools according to their policies. When two options are reported (e.g. hard and disabled) in Table 3, some instances consider one option and other instances the other option. In this experimentation, hard constraints have values of ω_i equal to $100,000$. Soft-constraints have values of ω_i much lower than $100,000$ and typically range between 1 and 100. If i-th constraint is not used (or disabled), the corresponding value of ω_i is 0. The details about the values are reported for each instance in tables 9 and 10 of Appendix C.

In *Setting1* we compare the solutions provided by a MIP solver for the [IHSTP] model and those of the same solver for the [RIHSTP] after the [TPP] step. The outcomes of *Setting1* are reported in Table

[1] *http://jeffreykingston.id.au/khe/*

5. In *Setting2* we compare the KHE program and the same MIP solver running the [XHSTT] model and the [IHSTP] model separately. The three approaches are run with and without the [TPP] step. These outcomes are reported in Table 6.

Twenty realistic instances are created to describe possible situations in Italian high schools. They are divided into two groups: in the first, instances are denoted from 1 to 9 and their size ranges from small to medium size; in the second, instances are denoted from 10 to 20 and their size ranges from medium to huge.

Instance 1 has the data of a very small-size timetable with only 4 daily periods. Instance 2 is more realistic than the first one, because it has 6 daily periods. Instance 3 is the Instance 2 plus co-teaching with additional 1 full-time teacher and 1 part-time teacher. Instance 4 is more complex than the previous ones, because it features 3 articulated class (one teacher must teach two class in the same time) and blocks. Instance 5 represents a school in which classes have 5 hours and half every day. Instance 6 and Instance 8 are similar for experimenting fractional time units (30 and 15 minutes respectively). Instance 7 is quite realistic, because every class has 12 teachers in a week. Instance 9 is more complex and larger than Instance 7: it features 18 classes and teachers with variable week requirements ranging from 2 to 4 time slots. The second group (instances between 10 and 20) is computer generated using a common block with 12 teachers and 6 classes with fixed week requirement (3 periods). They are useful for showing how well a MIP solver runs the [IHSTP] and [TPP] models.

The most important problem data of each instance are reported in table 4, where *Requir. (1)* indicates the number of timetable requirements, *CoTea (2)* lessons in co-teaching, *Artic. (3)* lessons in articulated classes, *Blocks (4)* block lessons. The data files of instances in *Setting2* are also available in XHSTT format[2], whereas this is not possible for *Setting1*, because some constraints of [IHSTP] are not supported in XHSTT standard. In this experimentation, we adopt the modeling language IBM OPL to call the MIP solver CPLEX 20.1 for implementing and solving all models([XHSTT], [IHSTP] and [TPP]). All the experiments are performed on a computer with an Intel I5-4460 3.20 GHz 4-core CPU equipped with 32 GBytes of DDR3 RAM and 1 TBytes SSD drive running Ubuntu 20.04 LTS. The time limit is 3 hours.

Instance	$\|C\|$	$\|T\|$	*Requir. (1)*	$\|D\|$	$\|H\|$	$\|D\| \cdot \|H\|$	$\|C\| \cdot \|D\| \cdot \|H\|$	η	$\underline{\rho}$	$\overline{\rho}$	$\underline{\alpha}$	$\overline{\alpha}$	*CoTea (2)*	*Artic (3)*	*Blocks (4)*
1	3	6	18	6	4	24	72	1	1	2	2	3	-	-	-
2	3	6	18	6	6	36	108	1	1	2	3	5	-	-	-
3	3	7	36	6	6	36	126	1	1	2	3	5	21	-	3
4	6	6	36	6	6	36	144	1	1	2	3	5	-	18	90
5	3	6	18	6	11	66	198	1	2	7	3	10	-	-	-
6	3	6	18	6	12	72	216	2	2	4	6	10	-	-	-
7	6	12	72	6	6	36	216	1	1	2	3	5	-	-	-
8	3	6	18	6	16	96	288	4	4	8	4	12	-	-	-
9	18	36	216	6	6	36	648	1	1	1	3	5	-	-	648
10	18	36	216	6	6	36	648	1	1	1	3	5	-	-	-
11	24	48	288	6	6	36	864	1	1	1	3	5	-	-	-
12	30	60	360	6	6	36	1080	1	1	1	3	5	-	-	-
13	36	72	432	6	6	36	1296	1	1	1	3	5	-	-	-
14	42	84	504	6	6	36	1512	1	1	1	3	5	-	-	-
15	42	85	504	6	6	36	1512	1	1	1	3	5	18	-	-
16	48	96	576	6	6	36	1728	1	1	1	3	5	-	-	-
17	54	108	648	6	6	36	1944	1	1	1	3	5	-	-	-
18	60	120	720	6	6	36	2160	1	1	1	3	5	-	-	-
19	78	156	936	6	6	36	2808	1	1	1	3	5	-	-	-
20	156	312	1872	6	6	36	5616	1	1	1	3	5	-	-	-

Table 4: Description of the instances

Table 5 pertains *Setting1* and is organized into three groups of columns. The first column lists the instances, the second group reports the outcomes of the [IHSTP] formulation, the third group shows the results obtained by the [TPP] and the [RIHSTP]. For example, in instance 5 the [TPP] is solved in 58.3 seconds and the overall two-step method in 875.2 seconds. Almost of instances cannot be solved by the [IHSTP] formulation within the time limit.

The columns denoted by *Idle times* reports the average value of idle times for all teachers. For example, according to the solution of the [IHSTP], in instance 16 this value is 6.2, but it is obtained at

[2] *https://github.com/ClaudioCrobu/IHSTP*

the time limit, when constraint C10 (*Equity Idle Times*) is not satisfied (note also that fractional values in this column indicate that teachers do not have the same idle times). The same instance is effectively solved by the pair [TPP] and [RIHSTP], which returns a much lower value of idle times for all teachers.

Two types of gaps are reported. The column *Gap* indicates the relative difference between the best known integer solution and a value that bounds the best possible solution. It is computed as:

$$Gap = \left[100 \frac{U_B - L_B}{L_B} \right]$$

where U_B means Upper Bound and L_B means Lower Bound. This gap is not reported in the group of columns *[TPP]* & *[RIHSTP]*, because it takes always value zero. The column *LB Gap* indicates the relative difference between the best integer solution at the end of the CPLEX's Branch & Bound and the lower bound at the root node after CPLEX cuts. The best outcomes are emphasized in bold.

Setting1	*[IHSTP]*				*[TPP] & [RIHSTP]*			
Instance	*Time*	*Gap*	*Idle times*	*LB Gap*	*Total time*	*[TPP] time*	*Idle times*	*LB Gap*
1	1705.1	**0**	**1.0**	100	**1.1**	0.3	**1.0**	0
2	TL	234	3.0	100	**20.5**	1.3	**1.0**	2
3	TL	398	3.8	100	**1026.4**	3.2	**1.0**	8
4	TL	546	5.1	100	**134.0**	32.1	**1.0**	2
5	TL	930	7.8	100	**875.2**	58.3	**1.0**	22
6	TL	236	4.7	100	**289.5**	1.3	**2.0**	8
7	TL	404	3.4	100	**352.3**	0.1	**1.0**	0
8	TL	6	**4.0**	100	**18.3**	9.7	**4.0**	6
9	TL	300	2.8	100	**174.3**	30.9	**1.0**	0
10	TL	600	5.0	100	**150.3**	24.7	**1.0**	0
11	TL	700	5.2	100	**1893.9**	59.8	**1.0**	0
12	TL	700	5.4	100	**602.5**	241.9	**1.0**	0
13	TL	800	5.6	100	**2891.4**	559.6	**1.0**	0
14	TL	800	5.4	100	**2981.7**	920.2	**1.0**	0
15	TL	800	5.3	100	**10606.9**	925.0	**1.0**	0
16	TL	1000	6.2	100	**1139.1**	725.4	**1.0**	0
17	TL	900	5.6	100	**8889.5**	1216.3	**1.0**	0
18	TL	1118	5.5	100	**3038.6**	2779.9	**1.0**	0
19	TL	1001	5.8	100	**2116.4**	1832.7	**1.0**	0
20	TL	1118	5.1	100	**9639.0**	8632.8	**1.0**	0

Table 5: Results of *Setting1* (all times are expressed in seconds; TL = Time Limit = 10800 seconds)

Table 6 pertains *Setting2* and is organized into the same three groups of columns. However, the second and the third differ from Table 5, because they report the outcomes of KHE, XHSTT and [IHSTP] with and without the [TPP] step. In order to make a fairer comparison on KHE, it was used with the option of parallel threads and the best solution was selected. For the first group of instances 1, 10, 100 and 1000 threads number were used; the option with 1000 threads was not used for the second group of instances. Since KHE does not compute a lower bound, in the computation of *Gap*, this is replaced by the best upper bound computed by the other methods. The string MEM means that the computer's available memory was insufficient for the computation.

Setting2	*without [TPP]*						*with [TPP]*					
	KHE		*XHSTT*		*[IHSTP]*		*KHE*		*XHSTT*		*[IHSTP]*	*[TPP]*
Instance	*Time*	*Gap*	*Time*	*Gap*	*Time*	*Gap*	*Total Time*	*Gap*	*Total Time*	*Gap*	*Total Time*	*Time*
1	**0.5**	0	25.2	0	389.6	0	2.3	0	3.2	0	**0.4**	0.3
2	366.0	I	TL	0	**1862.3**	0	136.8	I	12.5	0	**2.2**	1.9
3	313.0	I	1740.1	0	**468.5**	0	155.6	I	30.0	0	**3.3**	1.8
4	953.9	I	TL	9	TL	**0**	516.1	I	208.1	0	**45.3**	36.0
5	417.4	500	TL	N	TL	**17**	547.2	I	629.1	0	**79.0**	74.9
6	969.9	I	TL	0	TL	**0**	676.9	I	52.7	0	**2.3**	1.7
7	1025.8	42	TL	8	**258.8**	0	36.2	0	330.5	0	**13.5**	0.1
8	**4.6**	0	TL	0	1149.1	0	40.6	0	172.8	0	**12.6**	12.1
9	2453.2	**8**	TL	N	TL	8	106.9	0	155.9	0	**101.3**	98.4
10	300.7	11	TL	192	TL	**8**	95.1	0	159.2	0	**91.0**	87.8
11	357.5	13	TL	N	TL	**0**	270.2	0	346.5	0	**264.8**	259.2
12	435.1	12	TL	27	TL	**8**	171.2	0	261.3	0	**164.3**	157.5
13	586.4	15	TL	N	**535.9**	0	197.5	0	1186.5	0	**194.3**	179.2
14	725.7	14	TL	N	TL	**8**	602.5	0	1114.4	0	**592.8**	580.2
15	723.3	**12**	TL	N	TL	14	602.3	0	1723.3	0	**599.5**	579.4
16	924.9	14	TL	N	**752.8**	0	554.2	0	1823.9	0	**545.4**	527.8
17	1124.2	15	TL	N	TL	**8**	500.7	0	1470.6	0	**497.7**	469.7
18	1073.2	12	MEM		TL	**8**	697.1	0	MEM		**690.9**	662.8
19	1599.8	15	MEM		**596.4**	0	1469.5	0	MEM		**1456.2**	1408.2
20	3241.0	19	MEM		**10185.9**	0	**3704.9**	0	MEM		3772.3	3580.9

Table 6: Results of *Setting2* (All times are expressed in seconds; TL = Time Limit = 10800 seconds; I = Infeasible; N = No feasible within the time limit; MEM = memory exhausted)

6.2 Analysis of results

6.2.1 *Setting1* without and with [TPP] step

Consider the columns denoted by *[IHSTP]* in Table 5. They show that, when the [TPP] step is not run, only the first instance is optimally solved within the time limit. The other instances use the overall available time to determine low quality upper-bounds and the lower bounds at the root node after Cplex cuts are always zero. In addition, the average idle times are not acceptable.

Consider the columns denoted by *[TPP]* & *[RIHSTP]* in Table 5. When the [TPP] is run before [IHSTP], all instances are optimally solved within the time limit. The column "*[TPP] time*" shows that an acceptable time is spent for solving the [TPP]. The time spent in the [TPP] is on average 15% of the total running time for first group of instances and 42% for second group, if the default parameters are used for the configuration of CPLEX. The lower bounds at the root node after Cplex cuts are often equal to the final integer solution. Generally speaking, the two-step method returns lower values of the average idle times, i.e. higher-quality timetables from the viewpoint of teachers. Therefore, the two-step method looks a promising approach for solving [HST] problems and it is worth investigating its viability also in *Setting2*.

6.2.2 *Setting2* without [TPP]

Consider the columns denoted by *without [TPP]* in Table 6. In the first group of instances (1-9), [IHSTP] outperforms XHSTT: XHSTT obtains the optimum for 2 times out of 9, while [IHSTP] determines the optimal solutions for 5 instances out of 9. Furthermore XHSTT does not get the first feasible solution within the time limit for two times; such a situation never occurs to [IHSTP]. Although KHE software does not give guarantees of optimality, the comparison to [IHSTP] shows that it determines the optimal solution for two times very quickly. However, it does not return a feasible solution in four instances.

In the second group of instances [IHSTP] returns four optimal solution out of eleven and the other solutions have an optimality gap ranging from 0% to 14%. XHSTT gets a feasible solution two times out of eleven, in six instances out of nine it does not return the first feasible solution within the time limit and, in other cases, it runs out of memory. Therefore, [IHSTP] is always superior to XHSTT in all instances. KHE always obtains feasible solutions, even if the relative gap to the best known solution ranges from 11% to 19%. Therefore, KHE looks better than XHSTT and slightly worse than [IHSTP].

6.2.3 *Setting2* with [TPP]

Consider the columns denoted by *with [TPP]* in Table 6. In the first group of instances (1-9), [IHSTP] proves to be superior to KHE and XHSTT in 9 instances out of 9. KHE does not take advantage of [TPP] step and in two cases (instances 5 and 8) it worsens w.r.t the case without [TPP] step. The benefits of the [TPP] step are instead very clear for both XHSTT and [IHSTP], as they show significant improvements in gaps and optimization times.

In the second group of instances (10-20) KHE obtains the best solution in one case and improves the other 10 solutions owing to the [TPP] step. The comparison between XHSTT and [IHSTP] indicates a better effectiveness of [IHSTP] in terms of running times. Furthermore, XHSTT is more demanding from the point of view of memory use, as the 3 largest instances cannot be solved and compared to [IHSTP].

6.3 Final tests

According to the former results, it is of interest to solve the [IHSTP] formulation for a larger time limit, to possibly obtain optimal solutions for all instances. A final experimentation is carried out with a time limit of 24 hours and these optimal solutions are obtained for all instances. These solutions are equal to those of the two-step method, i.e. the proposed method returns the optimal solutions for all instances in this paper. Moreover, the method exhibits a considerable speed-up in running times.

Finally, since this problem is naturally affected by symmetry, we rerun all tests while disabling the symmetry control options in Cplex. However, these results are never better than those presented so far.

7 Conclusion

This paper has investigated the Italian High School Timetabling Problem. It has well-established characteristics like co-teachers, articulated classes, multiple lessons, additional days-off, as well as quality indicators, such as the horizontal and vertical distributions of lessons. However, it exhibits new features which have not been investigated so far: fractional time units, equity in idle times, avoidance of consecutive heavy days and excessive workload for classes. All in all, this problem is more complex than those in the literature on the class-teacher paradigm. Moreover, the generalized HST problem based on XHSTT [11] [9] does not incorporate all requirements in the [IHSTP].

A mixed integer programming model has been proposed for [IHSTP], in order to pursue the maximum compatibility with XHSTT. Since XHSTT timetabling is based on the decomposition into sub-events and the [IHSTP] is built on equally-sized sub-events, the larger cardinality of the sets of sub-events makes the model for [IHSTP] easier to solve also realistic size instances of a simplified problem, in which the new requirements are omitted.

In order to obtain fast solutions for both the complete and the simplified [IHSTP], a two-step method is proposed. In the first step, the [TPP] problem is solved to cleverly decrease the initial solution space of IHSTP and determine the profiles of teachers. Next, a restricted version of the [IHSTP] is solved very effectively in the second step. The two-step method results in good-shaped timetables and suitable computing times even for the most complex problem instances. Although the method does not guarantee the optimality, it returns the optimal solutions for all instances in this paper. In our opinion, the two-step method is quite general and could be applied to other class-teacher problems for countries with a similar problem setting.

Some possible research developments are reported below:

- investigating the applicability of the TPP also to the XHSTT standard instances;
- bringing a complete compatibility with XHSTT on-line database [2];
- treating the multiple school timetabling problem with some teachers among two or more schools;
- planning temporary timetables in which some teachers maybe not available or they have to be substituted;
- planning timetables according to teachers' preferences;
- completing teacher-class requirements assignment accounting for continuity over time.

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A Appendix A - Summary of [IHSTP]-[TPP] slack variables

Var.	Type	Description
$s_{ct}^{C_1}$	Integer non-negative	Not assigned weekly lessons for class c and teacher t
$s_{cdh}^{C_2}$	Boolean	Not assigned required lesson for class c on day d at period h
$s_{cdh}^{C_3}$	Boolean	Violated availability periods of class c on day d at period h
$s_{tdh}^{C_4}$	Boolean	Violated availability periods of teacher t on day d at period h
$s_{ctd}^{C_5}$	Boolean	Split lessons for class $c \in C$ and teacher t on day d
$s_{t}^{C_6}$	Integer non-negative	Lack/excess of days off for teacher t
$s_{ctf}^{C_7}$	Integer non-negative	Lab lessons for class c, teacher t and co-teacher f in excess or in lack
$s_{c't'c''t''}^{C_8}$	Integer non-negative	Block lessons for classes c', c'' with teachers t', t'' in excess or in lack
$s_{cdh}^{C_9}$	Boolean	Not assigned preassigned lesson for class c on day d at period h
$s^{C_{10}}$	Integer non-negative	Difference between maximum and minimum idle times for teachers
$s_{td}^{C_{11}}$	Integer non-negative	Idle times for teacher t on day d
$s_{l}^{C_{12}}$	Integer non-negative	Violation for multiple lessons limit $l \in L$
$s_{ct}^{C_{13}}$	Integer non-negative	Violation of ideal weekly lessons' distribution for class c and teacher t
$s_{cth}^{C_{14}}$	Integer non-negative	Violation of ideal daily lessons' distribution for class c and teacher t for period h
$s_{td}^{C_{15}}$	Integer non-negative	Violation of under-load/over-load limits for teacher t on day d
$s_{ctd}^{C_{16}}$	Integer non-negative	Violation of under-load/over-load limits for class c/teacher t on day d
$s_{cd}^{C_{17}}$	Integer non-negative	Presence of multiple lessons overload for class c on day d
$s_{td}^{C_{18}}$	Integer non-negative	Presence of two consecutive heavy days $d, d+1$ for teacher t
$s_{td}^{C_{19}}$	Integer non-negative	Violation of fractional time units for teacher t on day d

Table 7: [IHSTP] Slack variables summary

Var.	Type	Description
s_{tdh}^{SC1}	Boolean	1 if teacher t teaches in the same period h in two consecutive days $d, d+1$, 0 otherwise
s_{t}^{SC2}	Integer non-negative	Violation of minimum/maximum days off required
s_{td}^{SC3}	Integer non-negative	Violation of under-load/over-load limits for teacher t on day d
s_{td}^{SC4}	Integer non-negative	Idle times for teacher t on day d
s^{SC5}	Integer non-negative	0 if all teachers have the same minimum idle times, positive otherwise

Table 8: [TPP] Slack variables summary

B Appendix B - Glossary

Articulated class
A class made with the union of two or more *classes* with a small number of students.

Block
Two lessons for two pairs of *classes* and teachers who have to work together or separately in the same time slot.

Class (or group)
Group of students taking lessons from the same *curriculum* at the same time.

Co-presence teacher (co-teacher)
A teacher who always works together with another colleague.

Curriculum
It is the set of subjects in a class and the number of lessons for each subject.

Daily period (time slot)
A time interval with a constant duration equal to the minimum lesson unit.

Day off
A day when the teacher does not teach.

Double lesson
A lesson with length of two periods which must be consecutive for the same class and teacher.

Fractional time unit
A period with a duration of a fraction of an hour.

Full-time teacher
A teacher with a weekly workload equal to a fixed number of hours.

Idle time
A pause between two lessons non-consecutive of a teacher.

Lesson unit
The minimum interval of time of a lesson (normally it is equal to one hour).

Multiple lesson
A lesson with length of some periods which must be consecutive for the same class and teacher.

Part-time teacher
A teacher with a reduced weekly workload compared to a *full-time teacher*.

Split lesson
A lesson which is not given in consecutive periods by a teacher in a class.

Time slot (period)
A time interval in which a lesson can be allocated.

Triple lesson
A lesson with length of three periods which must be consecutive for the same class and teacher.

C Appendix C - Weights of instances used in experimentation

Instance	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19
1	100,000	100,000	100,000	100,000	100,000	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	0	0
2	100,000	100,000	100,000	100,000	100,000	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
3	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100	0	10	3	100,000	100,000	1	1,000	0
4	100,000	100,000	100,000	100,000	100,000	100,000	0	100,000	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
5	100,000	100,000	100,000	100,000	100,000	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
6	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100	1	10	3	100,000	100,000	1	1,000	1
7	100,000	100,000	100,000	100,000	100,000	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
8	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100	1	10	3	100,000	100,000	1	1,000	1
9	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
10	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
11	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
12	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
13	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
14	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
15	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
16	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
17	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
18	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
19	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0
20	100,000	100,000	100,000	100,000	0	100,000	0	0	0	100,000	100	0	10	3	100,000	100,000	1	1,000	0

Table 9: Weights of constraints C1-C19 used in *Setting1*

Instance	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19
1	100,000	100,000	100,000	100,000	100,000	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
2	100,000	100,000	100,000	100,000	100,000	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
3	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	0	100	0	0	0	100,000	100,000	0	0	0
4	100,000	100,000	100,000	100,000	100,000	100,000	0	100,000	0	0	100	0	0	0	100,000	100,000	0	0	0
5	100,000	100,000	100,000	100,000	100,000	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
6	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	0	100	1	0	0	100,000	100,000	0	0	1
7	100,000	100,000	100,000	100,000	100,000	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
8	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000	0	100	1	0	0	100,000	100,000	0	0	1
9	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
10	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
11	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
12	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
13	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
14	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
15	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
16	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
17	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
18	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
19	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0
20	100,000	100,000	100,000	100,000	0	100,000	0	0	0	0	100	0	0	0	100,000	100,000	0	0	0

Table 10: Weights of constraints C1-C19 used in *Setting2*