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Markov Chain Sampling of Hidden Relay States for Economic Dispatch with Cascading Failures

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Independent system operators (ISO) of electric power networks aim to dispatch electricity economically while maintaining system reliability. NERC (North America Electric Reliability Council) requires the transmission network to be $(N-1)$-secure, i.e., to have sufficient supply to satisfy demand in the event of the failure of any single resource in the network. Such a policy is at best an ad hoc rule that may be both overly conservative in considering all potential single-resource failures and excessively optimistic in inherently underestimating the economic consequences of correlated and cascading failures. More conservative approaches consider all possible combinations of $N-k$ resources for $k>1$ but including such combinations significantly increases the number of contingency scenarios even for low values of k and ignores the actual likelihood of these events. A significant challenge in determining event likelihoods is that the failure states of network elements are highly correlated through their interactions and common exposures, making direct determination of their joint distribution intractable. To address this issue, we develop a computational methodology to generate samples from the distribution of potential failure scenarios including correlated and cascading events using a Markov chain Monte Carlo (MCMC) algorithm. We demonstrate the method using the well-known IEEE 118-bus system and highlight the significant differences between the expected costs of dispatch using the MCMC model to generate failures and the costs that results from assuming only single unit failures and of assuming that failures are independent instead of being drawn from the correlated joint distribution.

Key words: DC optimal power flow, Security constrained economic dispatch, Markov chain approximation, Protective relays, Cascading failures, Optimal transmission switching.

History:

1. Introduction.

Independent system operators (ISO) of electric power transmission networks have an obligation to dispatch electricity economically while maintaining system reliability. Currently, the North American Electricity Reliability Corporation (NERC) requires that the transmission network should be $N-1$ secure, that is, the load should be fully met under a single component failure. The rationale for this policy is partly that a single component failure is much more likely than the failure of multiple components; however, multiple component failures can occur in the cases of cascading failures and resulting blackouts. The policy also ignores differences in the likelihood of the single-resource failures, and, hence, may also lead to overly conservative policies. Ensuring a given level of reliability or minimizing total expected costs, however, would require the ability to generate a distribution over potential failure scenarios and their potential remediation actions. While finding such a distribution is not directly tractable, this paper proposes a computational methodology to provide consistent estimates and demonstrates the significant differences from results of simplified models.

Our focus in this paper is on disruptions of the power distribution system, which have been responsible for over 90% of electric power interruptions, both in terms of the duration and frequency of outages, according to The Quadrennial Energy Review (QER) Task Force (see the second instalment of the QER report published in 2017 United States Department of Energy (2017)). According to that report, damage to the transmission system can result in widespread major power outages including cascading failures and affect numerous customers with significant economic consequences. The total number of these events include 24478 blackouts recorded in the US form 2008 to 2015 (Alhelou et al. (2019)). The fewest annual number of these power outages was 2169, recorded in 2008, when approximately 25.8 million people were affected. The highest annual number of outages recorded was 3634 in 2014, and up to 41.8 million were affected in 2011. While the probability of such events on a daily basis may be low, the consequences can be quite substantial.

Random outages may begin with single triggering events, largely related to weather conditions, but may then cause additional failures depending on the states of adjacent resources, potentially resulting in a cascade of failure. Alhelou et al. (2019) mentioned that from 2011 to 2019 the initial cause of a blackout or cascading failure was mostly a weather related condition, which led to transmission line overload and line or generator tripping. Recent major blackouts due to cascading failures in the US include the August 14, 2003, Northeast blackout which affected 55 million people (US-Canada Power System Outage Task Force (2004)), the September 8, 2011 San Diego blackout, which left 2.7

million customers without power, the March 1, 2017 New York blackout, the September 10, 2017 Southeast blackout, and the July 13, 2019 Manhattan blackout affecting 73,000 people. Major blackouts around the world due to cascading failures include the June 22, 1999 Southern Brazil blackout, which affected 75 to 97 million people, the July 30 and 31, 2012 Northern India blackout, which left 600 million people stranded, the September 28, 2016 South Australia blackout affecting 1.7 million people, and the March 21, 2018 Brazil blackout which left 10 million people without power. For details on these catastrophic blackouts due to cascading failures, we refer to Alhelou et al. (2019). Very recently, the entire Mumbai was without power for a few hours on October 12, 2020 due to power grid failure (Mirror (2020)).

The IEEE PES-CAMS Task Force on understanding, prediction, mitigation and restoration of cascading failures (Vaiman et al. (2012)) reports on a variety of simulation models developed to study cascading failures. Analyzing these phenomena is particularly challenging since exhaustive computation of potential conditions includes all possible combinations of failures (2^N combinations, for N number of transmission lines), which quickly becomes infeasible for moderate values of N (Chen and Mili (2013). To reduce this computational burden, Thorp et al. (1998) and Chen et al. (2005) implemented importance sampling to identify and focus on failure clusters. He et al. (2010) and Green et al. (2010) used partitioning and pruning techniques on the state space (consisting of all possible combinations) to reduce the computational effort. Steady-state based stochastic models to capture cascading failures were also proposed in Anghel et al. (2007), Wang et al. (2012), Jiang and Singh (2011) and Chen and Mili (2013), which includes hidden Markov models as well. A state duration sampling or state transition sampling approach are primarily used in these stochastic models.

All these models have certain sets of assumptions to approximate the physical power system, but a well-accepted model is still absent due to the complexity of the interconnected power grids and cascading failures themselves (Vaiman et al. (2012)). The IEEE PES-CAMS report also highlights that, apart from events caused by extreme weather conditions or shortage of supply, many, if not all, of the registered blackouts were aggravated by cascading failures. Assessing the potential impact of such failures and ensuring grid operations to mitigate these effects requires the computational capability to sample these events and their operational responses. This paper provides this capability.

Much of the difficulty in evaluating these is that hidden failures of protective relays and circuit breakers are particularly difficult to detect (Yang et al. (2006), Zhao et al. (2019)). While advanced control systems can allow for additional detection and mitigation capabilities (Biswal et al. (2016), Guo et al. (2018),Pal et al. (2020)), research continues to try to identify the probabilistic relationship between hidden failures and cascading events (Pal et al. (2019)). This issue is, moreover, a key feature of all biological, physical, financial, and social networks that are prone to interconnected failures (Valdez et al. (2020)). A critical feature for determining the effects of the cascades is the correlation among the hidden elements, which creates cosusceptibility of connected elements to failure (Yang et al. (2017)). This paper provides a mechanism to identify this cosusceptibility by allowing sampling from the states of systems with correlated hidden failures.

The general approach of using a hidden Markov model requires understanding of the underlying relay failure process. This paper provides a methodology to sample from these hidden states and, hence, to estimate the cost consequences of economic dispatch policies in the presence of such hidden risks. We present an algorithm to calculate the expected minimum dispatch cost under cascading failures following the commitment of generation units. Previous papers investigating the effects of relay failures have either assumed independent failures (e.g., Chen et al. (2005), Pal et al. (2019), Pal et al. (2020)) or have included additional dynamics, such as loaded variability, in addition to initial line failures (Guo et al. (2018)). In our approach, we treat the transmission network as static and simulate random cascading failure of the transmission lines using dynamics dependent on the hidden failure states.

To represent the cascade of failures, we follow the evolution of the system conditional on the hidden failure states resulting in relays' misoperation. In this process, if a transmission line trips, there is a small but significant probability that lines topologically connected to either end of the tripped line may incorrectly trip due to the relay misoperation. The Electric Reliability Organization (ERO), which is comprised of the North American Electric Reliability Corporation (NERC) and the seven Regional Entities (REs), reported in 2019 that the overall protection system misoperations rate in 2018 was 8%, which is slightly higher than in 2017 and that the regional misoperations rate ranged from 5.7% to 13.3% (Electric Reliability Organization Report (2019)). The report also mentioned that according to the data collected from 2013 to 2018, one of the top three reasons for misoperations is relay failure/mispprations. This cause code accounted for 19% of all misoperations. Hence, combining the above two findings of the report, we can place the overall relay misoperations rate at 1.52%. Each of these relay misoperations can cause cascading failure of the lines. In 2003, NERC (North American Electric Reliability Council) studies of major disturbances shows that over a long interval, more than 70% of major disturbances involved relay systems, not necessarily as the initiating event, but as contributing to the cascading nature of the event (Chen et al. (2005)). Moreover, most, if not all, of the above mentioned major blackouts in the last two decades due to cascading failures were caused by relay misoperaions, however, weather related conditions were the initiating event in most cases.

Among many of these incorrect relay operations, a common scenario exists: the relay has an undetected defect that remains dormant until abnormal operating conditions are reached, which we refer to as a *hidden failure* (Thorp et al. (1998)). We consider the status of these protective relays as the critical random element in the transmission system for our consideration of potential failure cascades. Their role in maintaining system reliability is to eliminate unnecessary trips, isolate faults, protect motors and breakers and provide system information to help operators to manage the network better. The misoperation of relays due to hidden failure generally remains unobserved unless an abnormal operational situation occurs and the relay fails. Hence, we consider the failure of relays due to undetected faults as the hidden factor to develop cascading failure.

Even in the hidden failure embedded models, the cascading failures are often formulated as a function of relay states of the network. For example, Chen et al. (2005) addresses this issue by assuming a load dependent failure (tripping) probability of the exposed line, which depends on the relay state of that line. Any line which shares a common bus with an open line is considered to be exposed. Therefore, failure of a line depends on the hidden failure of the relay and the adjacent lines which share a common bus. Jiang and Singh (2011) proposed a four-state protection model from the complete Markov model of power systems with protection system failures formulated based on protective relays. Wang et al. (2012) also assumed that the relay states of the individual lines are independent. In this paper, the state transition probability of the network topology is based on the time varying line tripping rate $\lambda(t)$ and the line restoration rate $\mu(t)$, which depends on the reclosing mechanism of the protective relays. Chen and Mili (2013) proposed a model based on the state duration sampling approach. They used the sequential Monte Carlo sampling technique to simulate the states and combined it with importance sampling to reduce the computational burden. The relays are assumed independent across the nodes, but this ignores the correlation of relay failures due to adjacency and common environmental conditions for shared locations. Moreover, most of the literature on state transition sampling approach are based on the steady-state Markov chain where the transition rates are usually assumed to be constant (Chen and Mili (2013)). The model in this paper directly addresses this shortcoming.

The motivation of this work is to capture the cascading failure resulting from a given initiator more rigorously than in the prior literature, a key step in addressing the issue identified in the IEEE PES-CAMS Task Force (Vaiman et al. (2012)) as the absence of a well-accepted model. As mentioned earlier, following the QER report's finding that 90% of the major outages are caused by the disruption on the transmission system, we focus on the transmission system state (power flow equations) to address the reliability issue. Following the 2019 ERO report's findings about relay misoperations (Electric Reliability Organization Report (2019)) as well as the 2003 NERC report's observation of 70% of the cascading failures caused by relay misoperations (Chen et al. (2005)), we in turn model cascading failures as a function of hidden relay states of the network system.

The hidden failures (undetected defects) of relays can be modeled as random variables. The major difficulty in their analysis is to capture their joint distribution since they are not directly observed. The state space consists of all possible combinations of the relay states of the network. Given the lack of an explicit distribution, independent and identically distributed (i.i.d.) samples cannot be directly generated since the states evolve in a complex interdependent manner as a function of adjacent states, local conditions, and idiosyncratic interventions. To generate from this complex distribution, we assume that conditional information on the failure and repair rates of the protective relays can be used to form a continuous time steady-state Markov chain (jump process) of the relay states evolution over a particular period of time, whose stationary distribution is the underlying distribution of the relay states. Each observation of the Markov chain represents a relay state of the network. The (probabilistically) realized relay states may then trigger failures of the lines in response to other events. That is, while they may not be the initiating event, they contribute to the production of cascading failures.

We follow the failure mechanism of lines as described in Chen et al. (2005), but, instead of assuming a load dependent tripping probability that is independent across nodes, we use samples from the steady state of the Markov chain to capture spatial correlation in the tripping probabilities across nodes (which, as we show below, can significantly increase the chances of cascades). The probability associated with each state (cascading outcome) is calculated using the occupation time of the semi-Markov process at that state. The probability of occurrence of an outcome is inversely proportional to the occupation time. Thus, the improbable outcomes are being ruled out. Therefore, instead of assuming exponential failure and repair rates of the transmission lines as Wang et al. (2012), we assume exponential failure and repair rates of the relays to simulate realizations of the relay state and the cascading failure of the lines are generated by using them. Moreover, we also focus on the resulting dispatch and load-loss cost in contrast to the reliability score used in Chen et al. (2005).

As transmission system operations have become more data driven, complex and interconnected, so has data analysis become an increasingly important aspect of today's grid management. Moreover, the time scales of power balancing have shifted from daily to hourly, minute, second-to-second to millisecond-to-millisecond at the distribution end of the supply chain. Therefore, we consider a continuous time framework in this paper to study the transmission system disruptions.

Given an observed state of the transmission network, the system operator aims to minimize the dispatch cost (i.e., the level of generation at each generating unit, the demand or load to be served, the resulting cost of that supply, and penalty for any loss of load) over the network topology. Moreover, in some cases, operators can, and do, change the network topology to improve voltage profiles or increase capacity of a flowgate (Fisher et al. (2008)). Instead of designing a new network, operators can alter the existing network by switching off lines to improve system dispatch and to make it economically efficient. These actions change the network topology temporarily and adjust generation to meet the load. Articles in the literature which explore the implications of automating this practice include O'Neill et al. (2005), Fisher et al. (2008), Hedman et al. (2009) and Hedman et al. (2008). In these articles, a mixed-integer linear programming model decides the lines to open to dispatch electricity cost efficiently with bounds on the maximum number of such flexible transmission lines. The authors studied this issue under the name of *optimal transmission* *switching* (OTS) as it explores the optimal use of the existing transmission network to dispatch power.

In this paper, a constrained optimization model is developed to minimize the operational cost under cascading failure scenarios, allowing for dispatch adjustments including load shedding due to failures. That is, the model aims to maintain reliability as much as possible while dispatching power cost-efficiently under cascading failures. A hidden failure embedded DC optimal power flow (DCOPF) model in association with mixed integer linear programming is proposed to dispatch power cost efficiently. To consider potential reactions to failure cascades, we also combine cascading failure outcomes with transmission switching as one form of mitigating the effects of the hidden failures. In particular, we combine the OTS model with our formulation of the hidden relay states. In this model, we allow additional lines to be open apart from the lines which are already open due to the cascading failures. The transmission switching process of opening additional lines under operator control uses the existing network more efficiently under the cascading failure regime. This will satisfy our goal to further minimize the operational cost (including penalty due to loss of load) under cascading failure while maintaining the reliability as much as possible. To reflect actual operations, we also consider the production limits of the generators.

To test our model, we assume an initial failure to be random (e.g., due to a weather event) across the network and that then the cascading failure propagates according to the generated jump process (samples) based on the unobserved relay states. We calculate the average dispatch cost over the samples of the relay states using the IEEE 118 bus model (Christie (1993)) for each line to represent the initial failure as well as the corresponding cascades. Assuming a common cost for lost load, the cost of the resulting dispatch is then compared with the cost assumed with the other mechanisms, such as maintaining generation under an $N-1$ contingency dispatch and following the mechanism described in Chen et al. (2005) (ref. Table 1). The numerical findings suggest that our model incorporating the correlation structure of hidden failures results in significantly different expected operational costs than produced by previous models. In particular, ignoring multiple failures and the positive correlation among relays generally underestimates operational cost and could lead to sub-optimal commitment of generators. To allow for mitigation responses, we also calculate the dispatch cost combining the stochastic model with transmission switching, i.e., including additional lines opened by the optimization model apart from the lines open due to cascading failures. The dispatch cost reduces as we allow more additional lines to be opened, but, as we show, the expected costs using the sample distribution of relay states are still much different from those assuming a single uniform failure across all lines and failures assuming independent relay states.

The paper is organized as follows. We first want to establish that optimizing generation with our sampling procedure is consistent. Section 2 presents results that ensure that solving a sample average approximation (SAA) model using samples from a consistent Markov chain produces a consistent estimate of expected costs under a true underlying distribution. Section 3 then presents the specification of these results for the expected cost of the economic dispatch of generation and optimal transmission switching in the presence of potential line failures and cascades resulting from faulty relays. Next, Section 4 presents computational results using the 118-bus data. Finally, Section 5 presents conclusions.

2. Stochastic Linear Program Markov Chain Results

Our main approach in this paper is to solve for optimal generation and transmission switches using samples of hidden relay states generated by a Markov chain. We show that sampling from the Markov chain allows us to estimate the expected cost of various dispatch and transmission switching policies. We then compare these results to results assuming only single and independent failures to demonstrate the bias in these approaches. In this section, we establish the consistency of the solutions of a stochastic linear programming problem involving expectations over the steady-state distribution of relay states. This result ensures then that, with a sufficient number of Markov chain samples, a system operator could determine generation capabilities that minimize the expected cost over the distribution of possible hidden failures.

The main difficulty in solving such an optimization model is to evaluate the expectations in a closed form, particularly since the steady-state distribution has no analytical characterization. Therefore, instead of evaluating the expectation directly, the expectations are replaced by their sample sums where the samples are chosen from a Markov chain. In cases with samples drawn from explicit distributions, this technique is known as *sample average approximation* (SAA). Large sample properties of this method are well studied in the literature when the sample is independent and identically distributed (i.i.d.). As we show below, the SAA approximation using Markov chain samples also produces a consistent estimate for expectations with respect to the stationary distribution of the chain under suitable assumptions. We provide such large sample properties of SAA when the sample is taken from a Harris recurrent Markov chain. First, we begin with a law of large numbers to establish that our sampled Markov chain provides the proper steady-state expectation.

2.1. Uniform Law of Large Numbers

In this section, we present results concerning the convergence of optimal solutions of an SAA model using Markov chain samples to optimal solutions for an underlying stochastic optimization model with expectation with respect to the stationary distribution of the Markov chain. We consider a function, $f:\mathbb{R}^n \times \Xi \to \mathbb{R}$, and random vector, $\xi:\mathbb{R}^n \times \Omega \to \Xi$, where $\Xi \subset \mathbb{R}^m$ is the support set of the random vector. To study the uniform convergence result, we construct a Markov chain with state space $(\Xi, \mathcal{B})$ and stationary distribution π. $E_\pi[\cdot]$ denotes the expected value with respect to π. A Markov chain is said to be *Harris recurrent* with respect to a reference measure ϕ if $A \in \mathcal{B}$, $\phi(A) > 0$ implies $Q_{\boldsymbol{\xi}}(T_A < \infty) = 1$ for all $\boldsymbol{\xi} \in \Xi$, where T_A is the first entrance time or hitting time to A, $Q(\cdot,\cdot)$ is the transition probability function on Ξ, and $Q_{\boldsymbol{\xi}}$ is the probability measure of the sequence of transitions under Q given initial state $\boldsymbol{\xi}$ (for example, see Athreya and Lahiri (2006)). The following law of large numbers is an extension of Theorem 7.48 in Shapiro et al. (2009) to the case of Markov chains. The proof of this result appears in Sur and Birge (2020)

THEOREM 1. *Let us assume C be a non-empty compact subset of $\mathbb{R}^n$ and suppose that:*
i) the function $f(x,\xi)$ is continuous on C, π-a.s.
ii) $f(x,\xi)$ is dominated by an integrable function on C,
iii) the random sample $\{\xi^k\}_{k\geq 1}$ is a Harris recurrent Markov chain with state space $(\Xi, \mathfrak{B}(\Xi))$ and transition function $P(\cdot,\cdot)$. Then the expected value function $E_\pi[f(x,\xi)]$ is finite valued and continuous on C and for all initial state $\boldsymbol{\xi_1} \in \Xi$, $\frac{1}{K}\sum_{k=1}^{K} f(x,\xi^k)$ converges uniformly on C to $E_\pi[f(x,\xi)]$, $P_{\boldsymbol{\xi_1}}$-a.s.

2.2. Consistency of Solutions of the Stochastic Linear Program

The model focus of this paper is a special case of the following stochastic linear programming problem:

$$\begin{aligned} \min_{x\in\mathbb{R}^n} \quad & E_\pi[f(x,\xi)] \\ \text{s. t.} \quad & Ax \leq b, \end{aligned} \tag{1}$$

where again $E_\pi[\cdot]$ denotes the expected value with respect to π, the probability distribution of ξ, $f:\mathbb{R}^n\times\Xi\to\mathbb{R}$, is continuous in x, π-a.s., $A\in\mathbb{R}^{l\times n}$, $b\in\mathbb{R}^l$, and $\xi:\Omega\to\Xi$, where $\Xi\subset\mathbb{R}^m$ is the support set of the random variable. In the formulation below of the economic dispatch model with random contingencies, $f(x,\xi)$ represents a second-stage (recourse) function value such that

$$f(x,\xi)=\min_{y\in Y(x,\xi)} q(y,\xi), \tag{2}$$

where y is a recourse decision within a feasible region $Y(x,\xi)$ that may depend on the random vector ξ (so that $f(x,\xi)=q(y(\xi),\xi)$ where $y(\xi)$ is a solution of the minimization problem in (2)).

The corresponding Markov chain SAA subproblem is the following:

$$\begin{aligned}\min_{x\in\mathbb{R}^n}\quad & \frac{1}{K}\sum_{k=1}^{K} f(x,\xi_k)\\ \text{s. t.}\quad & Ax\le b,\end{aligned} \tag{3}$$

where $\{\xi_k:k=1,\dots K\}$ is a Markov chain with stationary distribution π. The following theorem from Sur and Birge (2020) achieves the consistency of the Markov chain SAA estimators of (3) to solutions of (1). We apply this result on consistency of solutions in the following section to the stochastic economic dispatch problem.

THEOREM 2. *Consider the stochastic program (1) and its corresponding sample average approximation problem* (3)*. Let us assume that X and X_K are feasible sets of* (1) *and the corresponding* (3)*. Suppose that $\{\xi_k:k=1,\dots K\}$ forms a Harris recurrent Markov chain with stationary distribution π, for each $K\in\mathbb{N}$; then:*
i) if $\{\tilde{x}_K:K=1,2\dots\}$ is a sequence of global minimizers of sample average approximation problems (3) for given $\boldsymbol{\xi}_1\in\Xi$, i.e., $\tilde{x}_K$ is a global minimizer of the problem (3), for each $K\in\mathbb{N}$, and $\tilde{x}$ is an accumulation point of this sequence, $P_{\boldsymbol{\xi}_1}$-a.s. Further, assume that all in $\{\tilde{x}_K:K=1,2\dots\}$ belong to X except a finite members. Then $\tilde{x}$ is a global minimizer of problem (1) and $\frac{1}{K}\sum_{k=1}^{K} f(\tilde{x}_K,\xi_k)$ converges to $E_\pi[f(\tilde{x},\xi)]$, $P_{\boldsymbol{\xi}_1}$-a.s.;
ii) if $\{\tilde{x}_K:K=1,2.....\}$ is a sequence of local minimizers of the sample average approximation problems (3) for given $\boldsymbol{\xi}_1\in\Xi$ sharing a common radius of attraction $\rho>0$, $P_{\boldsymbol{\xi}_1}$-a.s. (i.e., for each $K\in\mathbb{N}$, $\frac{1}{K}\sum_{k=1}^{K} f(\tilde{x}_K,\xi_k)\le\frac{1}{K}\sum_{k=1}^{K} f(x,\xi_k)$ for all $x\in X$ such that $\|\tilde{x}_K-x\|<\rho$, $P_{\boldsymbol{\xi}_1}$-a.s.) and $\tilde{x}$ is an accumulation point of this sequence, $P_{\boldsymbol{\xi}_1}$-a.s. Further, assume that

all in $\{\tilde{x}_K : K = 1,2\dots\}$ *belong to* X *except a finite members. Then* $\tilde{x}$ *is a local minimizer of the problem (1) and* $\frac{1}{K}\sum_{k=1}^{K} f(\tilde{x}_K, \xi_k)$ *converges to* $E_\pi[f(\tilde{x},\xi)]$, $P_{\boldsymbol{\xi}_1}$*-a.s.*

3. Economic Dispatch Model with Transmission Switching under Cascading Failure

In this section we present comparative formulations of security-constrained economic dispatch (SCED) models. Before presenting these SCED models, we introduce the following notation.

Sets and indices:

G: set of all generators;

L: set of lines;

K: set of indices of the relay states;

$\mathcal{N}$: set of nodes;

G_n: set of generators that are located in bus n;

$LI_n = \{l \in L : l = (k,n), k \in \mathcal{N}\}$;

$LO_n = \{l \in L : l = (n,k), k \in \mathcal{N}\}$.

Decision variables:

p_g^i: production of generator g for failure at line i

(where, in this case and elsewhere, the absence of an additional relay state argument indicates a decision for a contingency without cascading failures);

$p_g^i(\boldsymbol{\xi_k})$: production of generator g for initial failure at line i and relay state $\boldsymbol{\xi_k}$

(where, in this case and elsewhere, the explicit relay state argument indicates a decision for a contingency that includes cascading failures resulting from the relay state and initial failure);

p_g^{ik}: production of generator g for initial failure at line i and sample k chosen assuming independent relay states as generated by Procedure 2 below (from Chen et al. (2005));

θ_n^i: phase angle at bus n for initial failure at line i;

p_l^i: power flow on line l for failure at line i;

$p_l^i(\boldsymbol{\xi_k})$: power flow on line l for initial failure at line i and relay state $\boldsymbol{\xi_k}$;

p_l^{ik}: power flow on line l for initial failure at line i and sample k sample from independent relay states;

z_l^i: on-off status of line l for initial failure of line i due to transmission switching as described in (5);

$z_l^i(\boldsymbol{\xi_k})$: on-off status of line l for initial failure of line i and relay state $\boldsymbol{\xi_k}$ due to transmission switching;

z_l^{ik}: on-off status of line l with initial failure at line i and sample k sample from independent relay states;

L_n^i: load shedding on bus n for failure at line i;

$L_n^i(\boldsymbol{\xi_k})$: load shedding on bus n for initial failure at line i and relay state $\boldsymbol{\xi_k}$;

L_n^{ik}: load shedding on bus n for initial failure at line i and sample k sample from independent relay states.

Parameters:

η_i: probability of initial failure occurring at line i with $\sum_{i\in L}\eta_i = 1$;

N_l^i: on-off status of the line l during i^{th} contingency where $N_l^i = 0$ for $l = i$ and $N_l^i = 1$ for $l \neq i$;

$N_l^i(\boldsymbol{\xi_k})$: on-off status of line l during cascading failures for initial failure at line i and relay state $\boldsymbol{\xi_k}$;

N_l^{ik}: on-off status of line l during cascading failures for initial failure at line i and sample k of independent relay states $\boldsymbol{\xi_k}$: the k^{th} sample of the relay state of the lines;

ϕ_k: probability of occurrence of k^{th} relay state $\boldsymbol{\xi_k}$, such that $\sum_{k\in K}\phi_k = 1$;

C_g: marginal cost of generator g;

ρ_n: penalty cost of load shedding at bus n;

D_n: demand in bus n;

p_l^{min}, p_l^{max}: minimum and maximum flow capacity of line l;

p_g^{min}, p_g^{max}: minimum and maximum capacity of generator g;

B_l: susceptance of line l;

M_l: a large non-negative scalar to enforce constraints for binary variables of each line l;

J: maximum number of lines that may be open due to transmission switching.

These variables and parameters are used in models for security constrained economic dispatch, a process of allocating generation and transmission resources to supply electricity to a particular geographical region with low cost and high reliability. As noted earlier, transmission line failures pose a considerable threat to the power supply affecting reliability issues. Two main feature of these systems, cost efficiency and reliability, are often conflicting. Cost efficiency demand capacities of the transmission lines should be fully used. On the contrary, then a certain amount of risk of failure of transmission lines is involved which may affect the reliability. A tradeoff between low cost and high reliability is inevitable. In practice, finding an optimal tradeoff is highly complex.

The security-constrained economic dispatch problem aims to provide distributed power that can supply the required load in a variety of potential failure conditions. This generally is accomplished by removing each (or several) of the N system components[1] (e.g., lines or generators) (leaving $N-1$ or fewer) and ensuring that the load can be met in all of these contingencies. In our examples, we only consider failures on lines (and, therefore, $N=|L|$ in those examples), but our approach generalizes for other component failures, such as those including generators.

To represent these contingencies, we use the binary variables N_l^i and z_l^i, in which, as given above, N_l^i denotes on-off status of line l during contingency i, i.e.,

$$N_l^i = \begin{cases} 0, & \text{if the line is down during contingency } i; \\ 1, & \text{if the line is working during contingency } i. \end{cases} \tag{4}$$

Initially, $N_l^i = 0$ only for $l = i$, but, as we consider the possibility of cascading failure (which in turn depend on relay states ξ), additional lines may subsequently go down.

To allow for potential mitigation of a cascade, we consider transmission switching in which additional lines may be opened to enable more efficient use of the generation and to isolate the damage from the initial event. Within contingency i, we use a binary variable z_l^i to denote the on-off status of line l under transmission switching, such that

$$z_l^i = \begin{cases} 1, & \text{if line } l \text{ is working or if } N_l^i = 0 \\ & \text{(indicating line } l \text{ is not available for transmission switching),} \\ 0, & \text{if line } l \text{ is available for transmission switching and is switched off under contingency } i. \end{cases} \tag{5}$$

[1] We use N here as a generic number of components following the common notation in the literature. We use $\mathcal{N}$ for the number of buses or nodes in the network.

As noted above, in contingency i (with no cascades), if we have $N_i^i = 0$, while $N_l^i = 1$ for all $l \neq i$. This indicates that only one line (i) has failed during the contingency and transmission switching of additional lines is available up to some maximum (the parameter J). With these definitions, a basic form of the $(N-1)$-secured SCED model (which we refer to below as Model $N-1$) with transmission switching can be written as follows:

$$\min_{p,z} \quad \sum_{i \in L} \eta_i \left(\sum_{g \in G} C_g p_g^i + \sum_{n \in \mathcal{N}} \rho_n L_n^i \right) \tag{6}$$

$$\text{s. t.} \sum_{l \in LI_n} p_l^i + \sum_{g \in G_n} p_g^i + L_n^i = D_n + \sum_{l \in LO_n} p_l^i, \qquad n \in \mathcal{N}, \tag{7}$$

$$\theta_n^{min} \leq \theta_n^i \leq \theta_n^{max}, \qquad n \in \mathcal{N}, \tag{8}$$

$$B_l(\theta_n - \theta_m) - p_l^i + (2 - N_l^i - z_l^i) M_l \geq 0, \tag{9}$$

$$B_l(\theta_n - \theta_m) - p_l^i - (2 - N_l^i - z_l^i) M_l \leq 0, \; \forall \, l \text{ with endpoints } n \text{ and } m, \tag{10}$$

$$\sum_l (1 - z_l^i) \; \leq \; J, \tag{11}$$

$$p_l^{min} z_l^i N_l^i \; \leq \; p_l^i \; \leq \; p_l^{max} z_l^i N_l^i, \tag{12}$$

$$p_g^{min} \; \leq \; p_g^i \; \leq \; p_g^{max}. \tag{13}$$

The above model minimizes the average dispatch cost across all the failures of single lines $i \in L$. In contrast to models that prohibit loss of load for each of the contingencies, this model allows for lost load with a penalty cost. Each scenario represents random failure of a single line which could, for example, follow a uniform distribution so that $\eta_i = \frac{1}{|L|}$. In that case, the objective then can be written as: $\sum_{i \in L} \frac{1}{|L|} \left(\sum_{g \in G} C_g p_g^i + \sum_{n \in N} \rho_n L_n^i \right)$. We note that we assume in (6) that the decisions p and z can depend on random outcomes, given as the line failure contingencies. In this way, they represent the y variables in (2). In this formulation, we have suppressed the x decisions that appear in (1), which would correspond to generation and commitment decisions that cannot respond to the contingencies (although such variables can be included without loss of generality in our results below).

The objective (6) in this economic dispatch problem is to minimize system costs, which consist of fuel and other marginal generation costs as well as the load shedding cost for not meeting demand. The market clearing constraint of Equation (7) requires that the amount of power that is injected and produced in each bus equals the amount of power consumed and exported from the bus. Equations (8)-(10) are required to incorporate the physical properties of the transmission lines, e.g., voltage profiles and Kirchoff's laws, and Equation (12) limits the current flow through each line. We suppose that the original network is represented by an $|L| \times |\mathcal{N}|$ adjacency matrix $A = [a_{ij}]$ where $a_{ij} = 1$ if line i exits bus j, $a_{ij} = -1$ if line i enters bus j, and $a_{ij} = 0$ otherwise. The susceptance of each line l is represented by B_l so that decisions on bus voltage angles θ and net power inflow p_l through each line l are given by the power-flow equations (9) and (10). Equation (11) is added to limit the number of open lines and is known as generalized upper bound (GUB). J represents the maximum number of lines that can be open due to transmission switching, so that $J \geq 0$. Equations (9)-(11) are included in our model following the literature on transmission switching, see Fisher et al. (2008). The binary parameter N_l^i denotes the on-off status of line l during the i^{th} contingency, i.e., such that the i^{th} line has failed, and z_l^i is the binary variable represents the state of the line l due to transmission switching during i^{th} contingency. The limit of the generation capacity of each generator is represented in Equation (13).

In this model, we assume that the system can re-dispatch the power in response to line outages in a manner that minimizes the dispatch cost (including penalty due to loss of load). The same set of generators are, however, committed across all the contingencies. In assuming these commitments, this formulation does not capture a full $(N-1)$ (or, indeed, $|L|-1$) contingency model in which load must always be met in the event of any single component failure (i.e., such that no load shedding is allowed). We relax this requirement in particular because it is not possible to meet the load in all such line contingencies for the example 118-bus network that we use for our experiments. In this formulation, z_l and N_l are not allowed to be zero for the same line because of Equation (5) which ensures that additional lines are allowed to be open apart from the contingency, i.e., the lines are being opened due to transmission switching after the contingency has occurred, which, as the results below show, can mitigate some of the costs of failures. This feature models a smart grid consisting of a self-sufficient system based on automation technology for monitoring, control, and analysis. We follow the authors in Hedman et al. (2009) who formulated a model coupled with transmission switching and contingency to minimize the expected dispatch cost across all possible $(N-1)$ contingencies (excluding those which would lead to loss of load) with a probability associated with each contingency scenario. If the probability of a single line failure (contingency) is uniformly distributed (and no other failures occur), the average of the minimum dispatch costs obtained by solving the Equations (6)–(13) for each single line failure provides the minimum expected dispatch cost.

The solution value from (6)-(13) does not necessarily capture the expected cost of these initial failures because of the potential for cascades or clusters of failures, e.g., because of a common cause. Our goal is to extend this system from an initial $N-1$ contingency to include additional events (e.g., multiple failures among the $N-2$ remaining lines) that result directly from the initial failure and hidden failures in the relay system intended to protect system elements. One possibility to help ensure operations under cascading failure is to consider all pairs or sets of k failures (i.e., $N-k$ reliability), but the size of the resulting models generally becomes prohibitive and these cases may also miss the impact of failure clusters and cascades that may include more than k failures.

We expand on the model in (6) to consider the distribution of hidden failures and possible cascading dynamics, as for example, in Wang and Kong (2010), to minimize the expected cost over the distribution of hidden failures ξ (over a given time interval). In this model, the additional argument of the variables in ξ indicates that these decisions may depend on observations of hidden failure states as revealed through additional failure cascades following the initial line failure. With the expectations taken over all potential hidden states, the model for a given distribution of failures can be expressed as the following:

$$\min_{p,z} \quad \sum_{i\in L} \eta_i \left(\sum_{g\in G} E[C_g p_g^i(\xi)] + \sum_{n\in N} E[\rho_n L_n^i(\xi)] \right) \tag{14}$$

$$\text{s. t.} \sum_{l\in LI_n} p_l^i(\xi) + \sum_{g\in G_n} p_g^i(\xi) + L_n^i(\xi) = D_n + \sum_{l\in LO_n} p_l^i(\xi)\,, \qquad n\in N, \tag{15}$$

$$\theta_n^{min} \le \theta_n^i \le \theta_n^{max}\,, \qquad n\in N, \tag{16}$$

$$B_l(\theta_n - \theta_m) - p_l^i(\xi) + (2 - N_l^i(\xi) - z_l^i(\xi))M_l \ge 0, \tag{17}$$

$$B_l(\theta_n - \theta_m) - p_l(\xi) - (2 - N_l^i(\xi) - z_l^i(\xi))M_l \le 0, \; \forall\, l \text{ with endpoints } n \text{ and } m, \tag{18}$$

$$\sum_l (1 - z_l^i(\xi)) \;\le\; j, \tag{19}$$

$$p_l^{min} z_l^i(\xi) N_l^i(\xi) \;\le\; p_l^i(\xi) \;\le\; p_l^{max} z_l^i(\xi) N_l^i(\xi), \tag{20}$$

$$p_g^{min} \;\le\; p_g^i(\xi) \;\le\; p_g^{max}. \tag{21}$$

In the above model, the initial failure occurs in line i with probability η_i and then the subsequent failures in lines (i.e., the cascade) depend on the hidden random factor ξ. The on-off status $N_l^i(\xi)$ (network topology) of the lines during each contingency depends on the initial failure and the distribution of hidden failures. Moreover, additional lines may be opened up due to transmission switching, represented by $z_l^i(\xi)$ as in (5).

A key difficulty in implementing (14) is that sampling from a steady-state distribution of hidden failures ξ is quite difficult since the failures are contingent on each other. These contingencies can, however, be represented by a Markov chain so that the results on the convergence of Markov chain samples can then apply.

3.1. Generation of Cascading Failures

The report of IEEE PES-CAMS Task Force (Vaiman et al. (2012)) highlighted that many, if not all, of the registered blackouts were aggravated by the cascading failures. As noted earlier, NERC studies of major disturbances in 2003 indicated that that over a long interval, more than 70% of major disturbances involved relay systems, not necessarily as the initiating event, but as contributing to the cascading nature of the event (Chen et al. (2005)). Therefore, in the literature, researchers have modelled cascading failure depending on the state of the protective relays of the network. Moreover, ERO report's findings in 2019 about relay misoperations (Electric Reliability Organization Report (2019)) also validate such a formulation. We follow the assumption by Thorp et al. (1998) that the protective relays of the network may have an undetected error which leads to relay misoperations and remains hidden unless an abnormal operation is observed. The potential for cascades depends on these hidden states of relays that are not observed (or are only observed infrequently).

We assume an initial failure following a known distribution and that subsequent failures depend on the hidden relay state. For this process, we assume that tripping of a given line l can lead to incorrect trips of any of the lines connected to the nodes of line l, corresponding to failures of the protection relays associated with those lines. Chen et al. (2005) assumed that such failures are load-dependent, but are otherwise independent, which as we noted can underestimate the operational cost during cascading failure when hidden relay failures are actually correlated due to earlier loading conditions on the lines. Moreover, realizing the distribution of ξ (hidden relay state) is quite troublesome due to the correlation among the relays. We use a Markov chain to capture this complex correlation structure of the hidden failures. This steady state Markov chain structure also appears in other models applied to power failure cascades, such as Roy et al. (2001), Anghel et al. (2007), Wang et al. (2012), Jiang and Singh (2011) and Chen and Mili (2013), which also assume independence of the relay states.

The Markov chain approach here is to generate a set of K relay-failure states from the Markov chain of relay states and then to generate associated K failure cascades of lines (one for each of the relay states) following an initial failure. Instead of drawing samples directly from the stationary distribution of ξ in model (14)-(21) (which is not directly representable), we generate a Markov chain of relay states whose steady-state distribution is the distribution of ξ and then approximate the objective (14) by the sample average subproblems associated with the Markov chain. To generate the Markov chain, we assume that the exponential failure and repair rates of the relay of each line (depending on the relay status of the connected lines) are known.

For the relay failure and repair process, we define $\boldsymbol{\xi_k} = (\boldsymbol{\xi_k}(i), i \in L) \in \{0,1\}^{|L|}$ as the relay state at stage k where $\boldsymbol{\xi_k}(i) = 0$ corresponds to a working (non-failed) state and $\boldsymbol{\xi_k}(i) = 1$ corresponds to a failed state. (We set these as binary initially but could allow for continuous states as well.) We denote the line adjacency in the network by an $|L| \times |L|$ matrix $M = [m_{ij}]$ where $m_{ij} = 1$ if line i and line j share a common node and $m_{ij} = 0$ otherwise. The indices are partitioned as

$$\mathcal{A}(\boldsymbol{\xi_k}) = \{i | \boldsymbol{\xi_k}(i) = 0, \sum_{j \neq i} m_{ij} \boldsymbol{\xi_k}(j) = 0\},$$

$$\mathcal{B}(\boldsymbol{\xi_k}) = \{i | \boldsymbol{\xi_k}(i) = 0, \sum_{j \neq i} m_{ij} \boldsymbol{\xi_k}(j) > 0\},$$

$$\mathcal{C}(\boldsymbol{\xi_k}) = \{i | \boldsymbol{\xi_k}(i) = 1, \sum_{j \neq i} m_{ij} \boldsymbol{\xi_k}(j) = 0\},$$

$$\mathcal{D}(\boldsymbol{\xi_k}) = \{i | \boldsymbol{\xi_k}(i) = 1, \sum_{j \neq i} m_{ij} \boldsymbol{\xi_k}(j) > 0\}.$$

For simplicity, we initially assume that each functioning relay has exponential failure rate λ_0 for all $i \in \mathcal{A}(\boldsymbol{\xi_k})$ and $\lambda_1 \geq \lambda_0$ for all $i \in \mathcal{B}(\boldsymbol{\xi_k})$ (where again these could be generalized to other distributions and to depend on power flow as well). Similarly, we assume that each functioning relay has exponential repair rate μ_0 for all $i \in \mathcal{C}(\boldsymbol{\xi_k})$ and $\mu_1 \geq \mu_0$ for all $i \in \mathcal{D}(\boldsymbol{\xi_k})$. We then let $\lambda(\boldsymbol{\xi_k}) = \lambda_0 |\mathcal{A}(\boldsymbol{\xi_k})| + \lambda_1 |\mathcal{B}(\boldsymbol{\xi_k})|$ and $\mu(\boldsymbol{\xi_k}) = \mu_0 |\mathcal{C}(\boldsymbol{\xi_k})| + \mu_1 |\mathcal{D}(\boldsymbol{\xi_k})|$.

With these definitions, we can define the transition probabilities Q from $\boldsymbol{\xi_k}$ to $\boldsymbol{\xi_{k+1}}$ as follows:

$$Q(\boldsymbol{\xi_k}, \boldsymbol{\xi_{k+1}}) = \begin{cases} \frac{\lambda_0}{\lambda(\boldsymbol{\xi_k})+\mu(\boldsymbol{\xi_k})}, & \text{if } \exists j \in \mathcal{A}(\boldsymbol{\xi_k}), \boldsymbol{\xi_{k+1}}(j) = 1, \boldsymbol{\xi_k}(j) = 0, \boldsymbol{\xi_{k+1}}(i) = \boldsymbol{\xi_k}(i), i \neq j; \\ \frac{\lambda_1}{\lambda(\boldsymbol{\xi_k})+\mu(\boldsymbol{\xi_k})}, & \text{if } \exists j \in \mathcal{B}(\boldsymbol{\xi_k}), \boldsymbol{\xi_{k+1}}(j) = 1, \boldsymbol{\xi_k}(j) = 0, \boldsymbol{\xi_{k+1}}(i) = \boldsymbol{\xi_k}(i), i \neq j; \\ \frac{\mu_0}{\lambda(\boldsymbol{\xi_k})+\mu(\boldsymbol{\xi_k})}, & \text{if } \exists j \in \mathcal{C}(\boldsymbol{\xi_k}), \boldsymbol{\xi_{k+1}}(j) = 0, \boldsymbol{\xi_k}(j) = 1, \boldsymbol{\xi_{k+1}}(i) = \boldsymbol{\xi_k}(i), i \neq j; \\ \frac{\mu_1}{\lambda(\boldsymbol{\xi_k})+\mu(\boldsymbol{\xi_k})}, & \text{if } \exists j \in \mathcal{D}(\boldsymbol{\xi_k}), \boldsymbol{\xi_{k+1}}(j) = 0, \boldsymbol{\xi_k}(j) = 1, \boldsymbol{\xi_{k+1}}(i) = \boldsymbol{\xi_k}(i), i \neq j; \\ 0, & \text{otherwise.} \end{cases} \tag{22}$$

By using a Markov chain simulation following Q, we can generate a sequence $\{\boldsymbol{\xi_k}\}$ that mixes to a stationary distribution. Note that this stationary distribution is not necessarily the long-run probability distribution of the semi-Markov process of relay states (since the time until the next transition depends on $\boldsymbol{\xi_k}$), but the long-run distribution of the relay states can be recovered by weighting the Markov chain stationary distribution with the average time in each state, $\frac{1}{\lambda(\boldsymbol{\xi_k})+\mu(\boldsymbol{\xi_k})}$. Moreover, it is noted that the transition probabilities are usually assumed to be constant in the literature of state transition sampling approach based on the steady-state Markov chain (Chen and Mili (2013)). We address that issue as the transition probabilities in our model are state dependent.

The set of K samples of hidden relay states is then taken by simulating the semi-Markov process. For each sample $k \in \{1, \ldots, K\}$, we then attach a probability ϕ_k (to correspond to the time weighting on each of the simulated states). The probability ϕ_k associated to each relay state is calculated using the occupation time of the semi-Markov process at that state. The probability of occurrence of an outcome (relay state) is inversely proportional to the occupation time. Moreover, for each of these states, we then generate an initial failure for some line i according to a failure probability η_i. This initial failure means that the network is now updated and a cascading failure is then generated according to each sample of the relay states.

Following the generation of an initial fault in line i, we generate the samples of the state of the network topology (cascading failures) by the following iterative procedure using the relay state $\boldsymbol{\xi_k}$.

Procedure 1: Initial failure and hidden relay state network topology generation

1. Set the initial failure (only tripped line) as i. Let the hidden relay state be $\boldsymbol{\xi_k}$.
2. For any tripped line j, consider all adjacent untripped lines l such that $\boldsymbol{\xi_k}(l) = 1$, trip line l.
3. Given a previous network topology and using the adjacency matrix A, update the values of $N_l^i(\boldsymbol{\xi_k})$ for all l, where the $N_l^i(\boldsymbol{\xi_k})$ are binary variables that represent the state of the lines, i.e., either open due to failure or closed, corresponding to the initial failure i and relay state $\boldsymbol{\xi_k}$.
4. Repeat the process in the first two steps until no further lines trip.
5. Solve for a feasible decision vector and find the updated cost for a single sample in a sample-average approximation of the objective in (14).

The above algorithm generates K samples of cascading failures $\{N^i(\boldsymbol{\xi_k})\}$ (states of the network topology) based on a random initial failure i and the sequence of the relay states $\{\boldsymbol{\xi_k}\}$ of the network , for each i. $N^i(\boldsymbol{\xi_k}) = (N_1^i(\boldsymbol{\xi_k}) \dots N_{(i-1)}^i(\boldsymbol{\xi_k}), 0, N_{(i+1)}^i(\boldsymbol{\xi_k}) \dots N_{|L|}^i(\boldsymbol{\xi_k})) \in \{0,1\}^{|L|}$ is a $|L|$-dimensional vector which denotes the state of the network topology under a cascading failure which starts at line i and propagates according to the relay state $\boldsymbol{\xi_k}$, i.e., $N_l^i(\boldsymbol{\xi_k})$ is a binary variable representing the on-off status of line l under the cascading failure. A random sample of cascading failures can be generated for the occurrence of initial failure at each line i. We attach the probability ϕ_k with each member of the random sample corresponding to the relay state $\boldsymbol{\xi_k}$, which is independent of the initial failure i. Similarly, the probability of occurrence of a cascading outcome is also inversely proportional to the occupation time of the semi-Markov process.Thus, instead of drawing samples directly from the distribution of the hidden failure ξ, we generate a Markov chain $\{\boldsymbol{\xi_k}\}$ of hidden relay states whose steady-state distribution coincides with the distribution of ξ. In our model, we replace the objective (14) by the sample averages corresponding to $\{\boldsymbol{\xi_k}\}$ and solve the constraints for each $\boldsymbol{\xi_k}$ as in the following section.

3.2. Model Description

The following stochastic mixed integer program (SMIP) based on DCOPF model is the resulting Markov chain sample-average approximation (MC-SAA) of the model (14)–(21) using samples $\boldsymbol{\xi_k}, k = 1, \dots, K$ and the resulting states of the network topology generated following Procedure 1.

$$\min_{p,z} \quad \sum_{i \in L} \eta_i \left(\sum_{k=1}^{K} \phi_k \left[\sum_{g \in G} C_g p_g^i(\boldsymbol{\xi_k}) + \sum_{n \in N} \rho_n L_n^i(\boldsymbol{\xi_k}) \right] \right) \tag{23}$$

$$\text{s. t.} \sum_{l \in LI_n} p_l^i(\boldsymbol{\xi_k}) + \sum_{g \in G_n} p_g^i(\boldsymbol{\xi_k}) + L_n^i(\boldsymbol{\xi_k}) = D_n + \sum_{l \in LO_n} p_l^i(\boldsymbol{\xi_k}), \qquad n \in N, i \in L, \tag{24}$$

$$\theta_n^{min} \le \theta_n^i \le \theta_n^{max}, \qquad n \in N, \tag{25}$$

$$B_l(\theta_n - \theta_m) - p_l^i(\boldsymbol{\xi_k}) + (2 - N_l^i(\boldsymbol{\xi_k}) - z_l^i(\boldsymbol{\xi_k}))M_l \ge 0, \tag{26}$$

$$B_l(\theta_n - \theta_m) - p_l^i(\boldsymbol{\xi_k}) - (2 - N_l^i(\boldsymbol{\xi_k}) - z_l^i(\boldsymbol{\xi_k}))M_l \le 0, \forall l \text{ with endpoints } n, m, \tag{27}$$

$$\sum_l (1 - z_l^i(\boldsymbol{\xi_k})) \le j, \tag{28}$$

$$p_l^{min} z_l^i(\boldsymbol{\xi_k}) N_l^i(\boldsymbol{\xi_k}) \le p_l^i(\boldsymbol{\xi_k}) \le p_l^{max} z_l^i(\boldsymbol{\xi_k}) N_l^i(\boldsymbol{\xi_k}), \tag{29}$$

$$p_g^{min} \le p_g^i(\boldsymbol{\xi_k}) \le p_g^{max}. \tag{30}$$

The Markov chain generation procedure used to simulate the samples $\boldsymbol{\xi_k}, k = 1, \ldots, K$ includes a new state transition for each sample including the possibility of recovery and multiple failures. Note that the network topology in this case depends on the initial failure, adjacency matrix and the hidden relay state of the network.

The simulation procedure to realize the states of the network topology (and potential cascading failure) is similar to the procedure used in Chen et al. (2005). The procedure relies on three different factors: a tripped line, the adjacency matrix and the hidden relay states. As noted earlier, the failure probability of the relay state of an exposed line is load dependent in Chen et al. (2005) but otherwise independent across lines, whereas the steady-state distribution resulting from Procedure 1 includes relay-failure dependencies due to line adjacency. Assuming independence of failure of the relays in adjacent lines as in the literature, for example in Anghel et al. (2007), Wang et al. (2012), Jiang and Singh (2011) and Chen and Mili (2013), may severely underestimate the possibility of cascading failures due to the interdependence of the relay state of the lines (e.g., with independent failure probability η, a sequence of 3 consecutive faults on one path of flow has only probability η^3 while high correlation could make this probability closer to η in reality). The Markov chain procedure given in Procedure 1 instead considers the correlations among the relay states of the adjacent lines. In addition to accounting for correlation among relay states, we consider transmission switching as a strategy to further minimize the dispatch cost and to explore the extent to which transmission switching reduces or increases the error produced in models assuming independent failures of the relay states.

The procedure used in Chen et al. (2005) differs from the solution resulting from Procedure 1 in the MC-SAA model ((23)–(30)) because that process asssumes that the hidden relay states are no longer taken from the Markov chain but are generated independently according to a failure probability distribution as a function of loads on the line. The result is that this alternative process effectively uses a different set of samples for each line failure.

To define the model following the independent relay-state assumption, for a given initial line failure i, we denote the kth sample ($k = 1, \ldots, K$) of the resulting network topology as N_l^{ik} for the status of each line l. The corresponding decisions for power generation p_g, line flow p_l, load losses L_n, and transmission switching decision z_l are given as $p_g^{ik}, p_l^{ik}, L_n^{ik}$ and z_l^{ik} for initial failure i, sample k, line l, generator g, and bus n. The resulting independent-relay-state sample average approximation model (IRS-SAA) is given as follows (with each sample weight again given as $\phi_k = \frac{1}{K}$ in our examples):

$$\min_{p,z} \quad \sum_{i \in L} \eta_i \left(\sum_{k=1}^{K} \phi_k \left[\sum_{g \in G} C_g p_g^{ik}) + \sum_{n \in N} \rho_n L_n^{ik}) \right] \right) \tag{31}$$

$$\text{s. t.} \sum_{l \in LI_n} p_l^{ik} + \sum_{g \in G_n} p_g^{ik} + L_n^{ik} = D_n + \sum_{l \in LO_n} p_l^{ik}, \qquad n \in N, i \in L, \tag{32}$$

$$\theta_n^{min} \le \theta_n^i \le \theta_n^{max}, \qquad n \in N, \tag{33}$$

$$B_l(\theta_n - \theta_m) - p_l^{ik}) + (2 - N_l^{ik}) - z_l^{ik}) M_l \ge 0, \tag{34}$$

$$B_l(\theta_n - \theta_m) - p_l^{ik}) - (2 - N_l^{ik}) - z_l^{ik}) M_l \le 0, \; \forall \, l \text{ with endpoints } n \text{ and } m, \tag{35}$$

$$\sum_l (1 - z_l^{ik}) \; \le \; j, \tag{36}$$

$$p_l^{min} z_l^{ik} N_l^{ik} \; \le \; p_l^{ik} \; \le \; p_l^{max} z_l^{ik} N_l^{ik}, \tag{37}$$

$$p_g^{min} \; \le \; p_g^{ik} \; \le \; p_g^{max}. \tag{38}$$

The process for generating the network topologies N_l^{ik} is given as follows.

Procedure 2: IRS-SAA relay state network topology generation

1. Set the initial failure as i. The k^{th} sample of the relay state of the network is generated independently according to the failure probability (load dependent) of the relays.

2. For any tripped line j, consider all adjacent untripped lines l such that if the k^{th} sample relay state of any of these lines is not working, then trip line l.

3. Given a previous network topology and using the adjacency matrix A, update the values of N_l^{ik} for all l, where the N_l^{ik} are binary variables that represent the state of the lines, i.e., either open due to failure or closed, corresponding to the initial failure i and k^{th} sample of the independently generated relay state.

4. Repeat the process in the first two steps until no further lines trip.
5. Solve for a feasible decision vector and find the updated cost using objective in (31).

In the following section, we compare solutions of Model $N-1$ (6)–(13), which assumes only a single line failure for each contingency, with our Markov chain hidden relay sample average approximation (MC-SAA) in (23)–(30) as well as the independent relay state sample average approximation (IRS-SAA) in (31)–(38). We calculate and compare the minimum expected dispatch costs for each different approach. Since mitigation efforts may reduce the effects of such losses, we also consider transmission switching with varying flexibility level (indexed by J) and compare the expected costs of the three models in these settings as well.

4. Computational Results

To see the impact of failure cascade on the dispatch cost, we compare the models using the IEEE 118-bus engineering test network (see, e.g., Christie (1993)). The system consists of 118 buses, 186 transmission lines, 19 committed generators with a total capacity of 5859 MW, and 99 load buses with a total consumption of 4519 MW.

For generating the Markov chain for model (MC-SAA), we fix the failure rate to be $\lambda = 0.001$ and repair rate $\mu = 0.1$ (as representative values that can be adjusted for varying conditions). Based on these values, we construct a Markov chain of relay states of size $K = 1000$ for the sample of relay states and then construct another set of Markov chains of cascading failures following Procedure 1 using the relay states and adjacency matrix of the network. For this network, we use an exhaustive sample for the initial failure in each of the 186 lines.

The average cost from the three models over the 186 initial line failure instances is shown in Figure 1. The figure shows that the average costs for MC-SAA are higher than those of IRS-SAA which in turn are higher than those of the $N-1$ model. Table 1 provides the data on average costs and standard errors over 1000 samples of the Markov chain and the resulting cascading failure as described in this paper. The resulting values of $N^i(\boldsymbol{\xi_k})$ are then used in Equations (23)–(30) to solve for the dispatch cost where any unmet load $L^i_n(\boldsymbol{\xi_k})$ has a penalty of USD 100/MWh. The table gives the originating line failure in the first column, the Markov Chain sample average approximation (MC-SAA) results in the second column, the results assuming independent relay failures in the third column, and the results using the solution from the $N-1$ model in the fourth column. The average dispatch cost across all the initial failures using MC-SAA is \$2642/hr. The average dispatch costs for IRS-SAA are \$2611/hr and for $N-1$ the average cost is \$2584/hr.

These differences in average cost are highly statistically different (i.e., t-statistic on the difference in means is approximately 87) between MC-SAA and IRS-SAA. The average costs decrease from MC-SAA to IRS-SAA because the IRS-SAA assumption misses the correlation in relay failures between adjacent lines, leading generally to under-estimates of the cost of longer cascades. This does not occur in all cases however. In 37 of the 186 initial line failures, the average MC-SAA cost is lower than the average IRS-SAA cost. In these cases, additional line failures in the MC-SAA case are actually fortuitous, opening an additional line that allows for more efficient transmission. Since such circumstances are rare, the overall trend, however, is that the IRS-SAA method produces significantly low-biased results relative to the expectation in MC-SAA, which considers correlation among relay failures. With higher loss-of-load penalties, the difference becomes even greater.

The difference in average cost between the MC-SAA solution and the cost of the $N-1$ solution is unsurprisingly more significant, given that the $N-1$ solution does not consider any additional failures beyond the initial line failure (and indeed generation has been committed so that load is not lost in these cases). In this comparison to the MC-SAA average cost, there are however still four instances in which MC-SAA actually produces lower cost. Again, these anomalies result from random failures along lines for which opening of the line leads to overall lower cost of generation.

Overall, these results indicate that ignoring the potential for line failure cascades or assuming that failures are caused by independent events can lead to significant errors in cost estimates. Higher-level decisions, such as commitment of generation units or overall network design, can then also be significantly altered and lead to substantial inefficiencies. Our use of samples from the Markov chain of the hidden states enables the correction of such errors with sufficient sampling and assuming the conditions given in Section 2. The results then apply to any network in which failures, losses, infections, misinformation, or other contagion can spread due to unseen states that propagate along the links of the network in the pattern of a Harris recurrent Markov chain.

Mitigating actions may reduce the effects of line failures and potentially avoid cascading failures. As an example of such actions, we consider the possibility of transmission control, which has been shown to offer potentially significant savings, even for a fully functioning network (e.g., Fisher et al. (2008)). The availability of these potential savings from transmission control (sometimes accidentally triggered as in the examples in which more failures lead to lower cost) motivates us to combine transmission switching with the MC-SAA model to study the impact of opening additional lines to those already open due to relay failures. A histogram of the average costs for the MC-SAA model in allowing 0 additional open lines (i.e., the same results as shown for MC-SAA in Figure 1), 1 additional open line, and 2 additional open lines appears in Figure 2. The figure shows how the availability of transmission switching results in substantial reductions in average cost from the base case with no additional open lines to one and two additional open lines. The overall mean of the costs is reduced from the original \$2643/hr for no additional lines to \$2368/hr when allowing one additional open line and \$2199/hr when allowing two additional open lines. These reductions represent a 10.4% reduction in average cost for allowing one additional open line and 16.8% reduction in average cost for allowing two additional open lines. These reductions should be compared to cost savings of 6.3% for allowing one additional open line and 12.4% for allowing two additional open lines for the deterministic optimal transmission switching solution with no additional failures (as reported in Fisher et al. (2008)). This finding suggests that transmission switching becomes more valuable in the presence of line failures and potential cascades. The values for the use of additional lines for each initial line failure appear in Table 2 in the appendix. As observed in Fisher et al. (2008), Hedman et al. (2008), and Hedman et al. (2009), additional transmission control (i.e., allowing three or more additional open lines) can lead to further reductions in total system cost (up to 25% for the basic deterministic system without line failures as reported in Fisher et al. (2008)).

To test whether the inclusion of transmission switching might also help to reduce the operational costs associated with line failures in the IRS-SAA and $N-1$ models, and the errors in costs estimated from these models, we include transmission switching in those models as well. Figure 3 provides a histogram of the average costs for the MC-SAA, IRS-SAA, and $N-1$ models for each initial line failure while allowing one additional transmission line to open (in addition to those that are failed) using the MC-SAA, IRS-SAA, and $N-1$ model solutions as before. The data for each initial line failure appears in Table 3 in the appendix. The overall average costs are reduced from the results with no transmission switching in each case: by 10.4% (\$2643/hr to \$2368/hr) for MC-SAA, by 10.3% for IRS-SAA (\$2611/hr to \$2342/hr.) and by 10.3% for the $N-1$ model (\$2584/hr to \$2318/hr). As the results show, transmission switching has approximately the same overall benefit in each case and does not substantially change the relative differences in the average costs among the different models. While this observation does not imply that differences might occur with other mitigation actions, it suggests that this particularly mitigation strategy is most beneficial in stressed situations such as those that result from drawing samples from the distribution of the underlying hidden states of the network.

5. Conclusion

Security constrained economic dispatch models often only consider the $(N-1)$ security standard, i.e., they aim to ensure that load is met in the event that any single component fails. In reality, however, component failures are not isolated single events and may occur in clusters. $(N-1)$ contingency analysis underestimates the consequences of such cascading failures. Although the probability of cascading failures is low, the economic consequences can be substantial and providing informed and accurate estimates of their costs can be critical for achieving efficiency in overall network operations. This motivates our consideration of cascading failures in estimating the operational cost of flow across a network.

We incorporate cascading failures of the transmission lines in our model to more accurately capture the potential costs. We model the cascading failures as a function of hidden relay state of the network, which is the primary mechanism for cascades, as in the NERC report (Chen et al. (2005)) that states that 70% of the major disturbances over a long interval of time are due to the relay misoperation and the recent ERO report's findings (Electric Reliability Organization Report (2019)) that emphasize modelling cascading failures from relay misoperations. To overcome the complexity in directly sampling from the distribution of these hidden relay failures, we develop a Markov chain Monte Carlo model enables a consistent estimate of expected costs.

We presented a case study of a well known engineering test system to demonstrate the use of the algorithm developed in this paper to calculate the expected dispatch cost under cascading failures and compared it with the results from only assuming single line failures as in $(N-1)$ contingency analysis and in assuming independence of relay failures among lines. The results demonstrate that the average dispatch cost for this network system under cascading failures is significantly higher than the dispatch cost assuming a single or independent failures. The result demonstrates the economic consequences of not including cascading failures. Moreover, our model combined with transmission switching as a mitigation mechanism further reduces the average dispatch cost and perhaps creates even greater differences relative to the results from antecedent models.

The model used here improves the process of estimation of the operational cost during cascading events. In addition, this model can be incorporated into higher level decisions, such as unit commitment, medium-term scheduling (e.g., hydro resources), and longer-term transmission network design. The approach also applies to other biological, financial, physical, and social networks in which contagion may spread due to underlying hidden states of network connections that are correlated through the network structure.

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Appendix: Figures and Tables of Numerical Results by Initial Line Failure

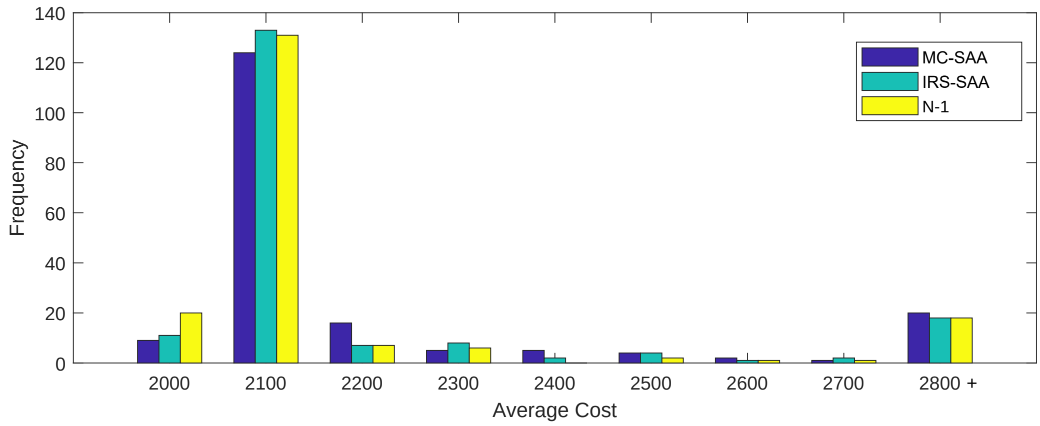

Figure 1 Histogram of the average costs (in $/hour) of the model in (3) using Markov chain Monte Carlo to generate scenarios (MC-SAA) with the independent relay-state (IRS-SAA, Chen et al. (2005)) and the cost using the $(N-1)$-contingency dispatch model $(N-1)$.

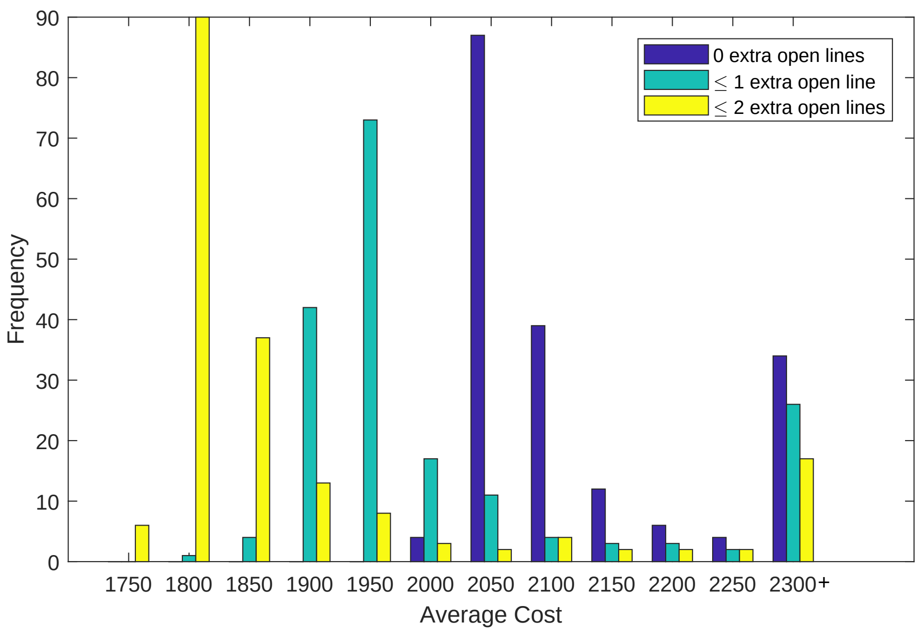

Figure 2 Histogram of average dispatch cost under the MC-SAA model for initial failure events on each transmission line with additional transmission switching allowing for 0, 1, and 2 additional open lines.

Table 1 **This tables gives the average cost and standard error for the solutions assuming an initial failure of each line for the optimal dispatch problem (with no transmission switching) using the (MC-SAA) model for 1000 scenarios, the IRS-SAA model in Chen et al. (2005), and the model assuming only a single line failure (i.e., the initial line failure).**

Model:	MC-SAA		IRS-SAA		$N-1$		MC-SAA		IRS-SAA		$N-1$
Initial Failure	**Avg.Cost**	**Std. Dev.**	**Avg.Cost**	**Std. Dev.**	**Cost**	**Initial Failure**	**Avg.Cost**	**Std. Dev.**	**Avg.Cost**	**Std. Dev.**	**Cost**
1	2116	17.53	2062	7.86	2054	94	2054	0.02	2054	0.01	2054
2	2179	26.19	2168	24.97	2055	95	2057	1.89	2054	0.003	2054
3	2105	10.5	2102	10.14	2054	96	2054	0.01	2054	0.01	2054
4	2057	0.36	2057	0.36	2055	97	2071	4.58	2068	4.17	2055
5	2057	2.21	2074	6.61	2054	98	2064	3.53	2065	3.74	2055
6	2118	16.03	2057	0.4	2056	99	2054	0.01	2054	0.01	2054
7	2068	7.46	2061	4.32	2055	100	2054	0.01	2054	0.01	2054
8	2092	14.07	2057	0.22	2056	101	2063	8.65	2054	0.01	2054
9	2057	0.36	2057	0.38	2056	102	2054	0.01	2054	0.02	2054
10	2084	12.83	2107	17.19	2055	103	2054	0.006	2054	0.005	2054
11	2070	5.59	2078	7.09	2055	104	2054	0.004	2054	0.005	2054
12	2146	0.37	2145	0.53	2145	105	2054	0.007	2054	0.007	2054
13	2261	0.17	2260	0.56	2262	106	2055	0.05	2055	0.02	2055
14	2076	0.76	2075	0.71	2072	107	2080	8.74	2055	0.004	2055
15	2262	0	2262	0	2262	108	2058	0.04	2058	0.04	2058
16	2058	2.2	2075	6.6	2055	109	2058	0.06	2058	0.06	2058
17	2108	14.02	2055	0.01	2055	110	2053	0.06	2053	0.05	2053
18	2064	3.8	2079	6.36	2055	111	2044	0.41	2042	0.13	2041
19	2070	6.55	2104	11.21	2054	112	2058	0.17	2058	0.2	2058
20	4046	0.0075	4046	0.01	4046	113	2408	66	2312	63.79	2106
21	2113	14.29	2123	15.05	2054	114	2099	11.53	2080	8.72	2055
22	3448	0.0083	3403	8.21	3448	115	2451	56.11	2403	55.1	2226
23	2075	6.89	2083	7.49	2054	116	2439	53.53	2600	77.39	2297
24	2167	26.48	2116	18.12	2054	117	20358	0.83	20357	0.85	20354
25	2072	5.93	2067	4.08	2056	118	2080	17.14	2045	3.62	2035
26	4543	0.0042	4434	17.76	4543	119	2167	10.84	2142	8.11	2114
27	2175	26.98	2091	12.17	2055	120	2517	26.48	2348	10.62	2321
28	2084	8.95	2103	11.37	2054	121	2062	2.29	2059	1.87	2054
29	2078	6.62	2077	7.82	2054	122	2111	21.24	2050	0.04	2050
30	8028	0.0042	7888	30.07	8028	123	2052	0.62	2055	3.32	2057
31	2195	27.41	2081	13.25	2055	124	2100	7.4	2075	4.35	2054
32	2162	26.33	2122	20.88	2055	125	2652	0.002	2652	0.002	2652
33	2099	8.41	2069	5.25	2055	126	2057	0.32	2112	20.12	2056
34	2100	7.7	2069	4.5	2055	127	2142	19.14	2083	14.62	2044
35	2080	5.14	2061	2.47	2056	128	2175	21.26	2039	0.62	2037
36	2052	0.0011	2052	0.02	2052	129	2404	46.16	2115	13.49	2058
37	2052	0.083	2052	0.02	2052	130	2140	22.22	2107	20.13	2043
38	2057	0.04	2060	2.65	2057	131	2191	17.66	2124	8.62	2102
39	2053	0.08	2054	0.04	2054	132	2036	17.49	1993	10.35	1935
40	2064	3.75	2055	1.44	2053	133	2114	16.93	2075	14.02	2014
41	2055	0.09	2055	0.09	2054	134	5406	84.31	5101	41.63	5008
42	2054	0.42	2052	0.13	2052	135	2036	4.27	2107	25.33	2032
43	2103	0.09	2102	0.26	2103	136	2005	2.25	2067	16.28	1994
44	2054	3.13	2057	2.66	2054	137	2579	15.63	2530	23.12	2536
45	2054	0.01	2054	0.01	2054	138	2794	34.31	2665	23.84	2605
46	2099	10.26	2054	2.11	2054	139	2268	13.66	2223	6.62	2194
47	2071	6.52	2073	7	2054	140	2244	13.42	2195	4.97	2176
48	2054	0.01	2054	0.01	2054	141	2315	3.25	2314	3.59	2297
49	4444	1.61	4417	8.35	4444	142	16909	0.16	16503	78.66	16908
50	2069	6.17	2072	6.75	2054	143	2823	116	2501	79.39	2179
51	2072	0.21	2072	0.18	2077	144	2521	68.84	2368	56.62	2154
52	2070	6.51	2086	9.14	2054	145	4105	72.79	3861	57.4	3602
53	2061	2.09	2058	1.62	2054	146	2467	60.05	2306	35.61	2135
54	2070	3.73	2054	0.01	2054	147	5872	26.68	5723	19.55	5667
55	2100	10.69	2075	7.17	2054	148	4744	51.78	4471	19.22	4418
56	2107	13.26	2054	0.004	2054	149	4184	76.94	3785	47.67	3596
57	2054	0.014	2054	0.02	2054	150	7224	0.98	7229	1.69	7222
58	2076	6.3	2055	0.01	2055	151	2364	46.94	2506	61.98	2167
59	2093	11.51	2106	13.17	2054	152	12852	124.07	12087	47.66	12338
60	2110	14.12	2083	10.28	2054	153	2046	28.23	2262	79.47	1945
61	2089	9.84	2056	0.02	2056	154	5715	23.26	5591	36.14	5585
62	2056	0.03	2056	0.03	2056	155	3069	32.08	3658	164.74	2927
63	2051	0.02	2051	0.03	2051	156	10849	59.82	10381	72.83	10686
64	2045	0.1	2045	0.41	2044	157	2057	13.44	2078	22.16	1985
65	2064	5.16	2085	9.39	2055	158	2067	13.61	2091	24.28	1978
66	2117	15.73	2055	0.01	2055	159	2118	12.47	2163	24.94	2052
67	2054	0.01	2054	0.01	2054	160	2119	12.21	2151	24.6	2052
68	2074	9.1	2107	14.65	2054	161	2019	2.54	2030	4.79	2010
69	2053	0.23	2053	0.19	2053	162	2067	17.48	1991	1.28	1987
70	2053	0.23	2053	0.04	2053	163	1983	5.04	1955	2.78	1938
71	2083	7.39	2068	5.25	2054	164	2359	5.61	2303	2.56	2313
72	2071	5.6	2058	3.06	2054	165	2018	14.97	1973	9.71	1936
73	2054	0.005	2054	0.005	2054	166	2599	28.56	2542	19.59	2464
74	2054	0.23	2053	0.04	2053	167	2203	14.44	2167	9.92	2141
75	2055	0.03	2055	0.02	2054	168	2158	13.19	2144	11.33	2117
76	2057	2.2	2054	0.002	2054	169	2171	25.67	2084	3.34	2071
77	2055	0.24	2054	0.04	2054	170	2061	2.01	2059	1.33	2054
78	2073	4.5	2066	1.27	2060	171	2061	2.01	2059	1.3	2054
79	2099	9.78	2080	7.61	2054	172	2061	2.01	2065	5.52	2054
80	2082	7.46	2056	0.07	2056	173	2110	11.4	2062	4.3	2052
81	2058	2	2056	0.04	2056	174	2054	1.36	2054	0	2054
82	2058	2.56	2055	0.03	2055	175	2054	1.28	2054	0	2054
83	2058	2.56	2055	0.03	2055	176	2173	29.36	2107	19.5	2054
84	2051	0.23	2050	0.05	2050	177	2054	1.32	2054	0	2054
85	2051	0.24	2053	3.1	2050	178	2054	1.28	2060	5.43	2054
86	2071	4.48	2063	1	2059	179	2158	23.47	2054	0	2054
87	2091	7.32	2070	5.29	2055	180	2060	1.12	2054	0	2054
88	2093	9.31	2055	0.01	2055	181	2054	1.49	2130	20.19	2054
89	2086	7.52	2055	0.005	2055	182	2060	1.08	2057	1.03	2054
90	2185	16.12	2078	6.85	2054	183	2197	29.6	2097	16.62	2054
91	2100	10.69	2075	7.16	2054	184	2283	28.87	2246	23.98	2166
92	2054	0.008	2054	0.006	2054	185	8710	0	8710	0	8710
93	2057	1.89	2056	1.33	2054	186	2082	5.62	2077	6.49	2054

Table 2 **This tables gives the average cost and standard error for the solutions using the (MC-SAA) model with transmission switching allowing at most one and two additional open lines.**

	One Extra Open Line		Two Extra Open Lines			One Extra Open Line		Two Extra Open Lines	
Initial Failure	**Avg.Cost**	**Std. Dev.**	**Avg.Cost**	**Std. Dev.**	**Initial Failure**	**Avg.Cost**	**Std. Dev.**	**Avg.Cost**	**Std. Dev.**
1	1987	17.52	1862	17.52	94	1924	0.02	1799	0.02
2	2049	26.19	1925	26.19	95	1927	1.89	1802	1.89
3	1976	10.49	1851	10.49	96	1924	0.02	1799	0.02
4	1926	0.36	1802	0.32	97	1941	4.59	1816	4.6
5	1927	2.21	1802	2.21	98	1934	3.53	1809	3.52
6	1987	16.04	1862	16.03	99	1924	0.02	1799	0.02
7	1938	7.46	1813	7.46	100	1924	0.02	1799	0.02
8	1961	14.07	1836	14.07	101	1933	8.65	1808	8.64
9	1926	0.36	1802	0.32	102	1924	0.01	1799	0.01
10	1953	12.83	1829	12.82	103	1924	0.0053	1799	0.004
11	1939	5.59	1815	5.59	104	1925	0.0037	1801	0.0036
12	2016	0.37	1883	0.46	105	1925	0.0069	1801	0.07
13	2131	0.17	2005	0.12	106	1925	0.05	1801	0.08
14	1945	0.76	1816	0.83	107	1950	8.75	1825	8.74
15	2131	1.21	2005	0	108	1929	0.04	1804	0.04
16	1927	2.21	1802	2.2	109	1929	0.06	1804	0.05
17	1978	14.03	1852	14.05	110	1923	0.05	1796	0.04
18	1933	3.8	1808	3.8	111	1915	0.35	1778	0.33
19	1941	6.55	1816	6.55	112	1927	0.14	1802	0.14
20	3915	0.0061	3791	0.0028	113	2263	64.86	2153	63.81
21	1984	14.29	1859	14.29	114	1969	11.53	1844	11.52
22	3318	0.0052	3194	0.0044	115	2284	55.51	2073	55.51
23	1945	6.89	1821	6.89	116	2411	53.54	2371	53.54
24	2037	26.48	1913	26.48	117	20227	1.07	20090	1.31
25	1941	5.93	1816	5.94	118	1939	10.42	1815	10.41
26	4413	0.0027	4288	0.0045	119	1961	6.28	1836	6.28
27	2045	26.98	1921	26.98	120	2278	9.77	2118	9.74
28	1954	8.94	1829	8.95	121	1931	2.29	1806	2.27
29	1948	6.62	1823	6.62	122	1980	21.25	1855	21.23
30	7897	0.0043	7772	0.0021	123	1921	0.29	1794	0.25
31	2066	27.41	1941	27.41	124	1969	7.4	1844	7.39
32	2031	26.33	1907	26.33	125	2523	0.0069	2398	0.0035
33	1970	8.41	1846	8.41	126	1927	0.14	1803	0.12
34	1972	7.7	1848	7.71	127	1922	2.07	1795	2.06
35	1952	5.14	1827	5.14	128	2039	21.28	1915	21.27
36	1921	0.0011	1792	0.0009	129	2197	43.8	2073	43.74
37	1921	0.01	1792	0.01	130	2010	22.23	1886	22.22
38	1928	0.05	1804	0.07	131	1972	1.76	1848	1.75
39	1923	0.09	1795	0.1	132	1818	3.06	1674	3.21
40	1933	3.75	1808	3.75	133	1858	2.75	1700	3.11
41	1925	0.006	1802	0.006	134	2555	77.55	2490	77.71
42	1923	0.42	1795	0.5	135	1864	4.24	1705	4.28
43	1973	0.09	1849	0.09	136	1832	1.88	1660	1.82
44	1925	0.0015	1801	0.001	137	2340	12.7	2106	6
45	1925	0.01	1800	0.01	138	2491	29.9	2176	14.66
46	1969	10.26	1844	10.27	139	2119	12.84	1940	12.39
47	1941	6.52	1816	6.52	140	2102	12.61	1924	12.24
48	1925	0.01	1801	0.001	141	2272	0.88	2233	0.61
49	4314	7.95	4189	0.0014	142	12758	0.43	12732	0.38
50	1939	6.17	1810	6.18	143	2598	92.1	2474	92.64
51	1941	0.21	1816	0.21	144	2279	47.92	2119	47.26
52	1941	6.25	1816	6.51	145	2484	59.81	2323	59.39
53	1932	2.09	1808	2.09	146	2223	43.55	2080	43.57
54	1941	3.73	1816	3.73	147	3218	22.58	1896	8.86
55	1970	10.69	1845	10.7	148	2435	49.35	2201	48.5
56	1977	13.27	1852	13.28	149	2462	56.38	2267	56.55
57	1924	0.01	1799	0.05	150	4760	0.6	2825	2.41
58	1945	6.31	1816	6.3	151	2122	30.98	1973	29.28
59	1963	11.52	1838	11.51	152	8350	148.52	6182	160.5
60	1980	14.13	1855	14.12	153	1892	20.07	1718	15.07
61	1958	9.84	1834	9.84	154	5203	11.74	5066	8.66
62	1925	0.03	1801	0.032	155	2193	21.67	2025	10.83
63	1920	0.02	1793	0.06	156	7895	72.18	5457	79.58
64	1915	0.09	1778	0.16	157	2005	13.34	1881	13.31
65	1934	5.15	1809	5.17	158	1928	12.88	1804	12.85
66	1986	15.74	1862	15.73	159	1979	12.15	1853	12.31
67	1925	0.0059	1801	0.006	160	1981	12.02	1856	12.01
68	1945	9.11	1821	9.12	161	1964	1.73	1839	1.72
69	1925	0.22	1801	0.2	162	1932	17.44	1808	17.42
70	1925	0.22	1802	0.22	163	1840	4.29	1763	3.14
71	1954	7.4	1829	7.41	164	2154	2.69	1927	1.4
72	1942	5.61	1817	5.61	165	1878	14.81	1802	14.61
73	1925	0.0034	1801	0.003	166	2394	22.9	2149	10.83
74	1925	0.21	1801	0.2	167	2080	14.38	1955	14.37
75	1925	0.03	1800	0.04	168	2023	13.13	1899	13.11
76	1927	2.21	1802	2.23	169	1989	16.04	1864	16.03
77	1925	0.22	1802	0.22	170	1929	1.2	1804	1.21
78	1940	2.57	1818	2.01	171	1929	1.2	1804	1.2
79	1969	9.79	1844	9.79	172	1929	1.2	1804	1.19
80	1952	7.47	1827	7.48	173	1980	11.4	1855	11.39
81	1929	2	1804	2.01	174	1925	0	1801	0.001
82	1929	2.57	1804	2.43	175	1925	0	1801	0
83	1929	2.57	1805	2.58	176	2043	29.35	1919	29.35
84	1922	0.22	1795	0.35	177	1925	0	1801	0.001
85	1922	0.22	1795	0.21	178	1925	0	1801	2.01
86	1938	2.55	1815	1.99	179	2028	23.47	1904	23.48
87	1961	7.33	1836	7.32	180	1931	1.12	1806	1.12
88	1963	9.31	1838	9.33	181	1925	0	1801	0
89	1957	7.53	1832	7.52	182	1930	1.08	1805	1.07
90	2055	16.14	1930	16.13	183	2067	29.58	1942	29.59
91	1971	10.7	1846	10.75	184	2054	29.4	1930	29.42
92	1925	0.0073	1800	0.011	185	8577	0	8455	0
93	1927	1.89	1802	1.9	186	1953	5.62	1829	5.61

Table 3 This tables gives the average cost and standard error for the solutions assuming an initial failure of each line for the optimal transmission switching problems with at most one additional open line using the (MC-SAA) model for 1000 scenarios, the IRS-SAA model, and the model assuming only a single line failure (i.e., the initial line failure).

Model:	MC-SAA		IRS-SAA		$N-1$		MC-SAA		IRS-SAA		$N-1$
Initial Failure	**Avg.Cost**	**Std. Dev.**	**Avg.Cost**	**Std. Dev.**	**Cost**	**Initial Failure**	**Avg.Cost**	**Std. Dev.**	**Avg.Cost**	**Std. Dev.**	**Cost**
1	1987	17.52	1932	9.85	1925	94	1924	0.02	1924	0.01	1924
2	2049	26.19	2037	24.96	1925	95	1927	1.89	1924	0.003	1925
3	1976	10.49	1973	10.13	1925	96	1924	0.02	1924	0.01	1924
4	1926	0.36	1926	0.36	1925	97	1941	4.59	1938	4.17	1925
5	1927	2.21	1945	6.61	1925	98	1934	3.53	1935	3.74	1925
6	1987	16.04	1927	0.4	1925	99	1924	0.02	1924	0.01	1924
7	1938	7.46	1930	4.32	1925	100	1924	0.02	1924	0.01	1924
8	1961	14.07	1926	0.22	1925	101	1933	8.65	1924	0.01	1924
9	1926	0.36	1926	0.38	1925	102	1924	0.01	1924	0.02	1924
10	1953	12.83	1976	17.18	1925	103	1924	0.0053	1924	0.005	1924
11	1939	5.59	1948	7.09	1925	104	1925	0.0037	1924	0.005	1925
12	2016	0.37	2015	0.52	2015	105	1925	0.0069	1924	0.007	1925
13	2131	0.17	2130	0.56	2131	106	1925	0.05	1925	0.02	1925
14	1945	0.76	1945	0.71	1941	107	1950	8.75	1925	0.004	1925
15	2131	1.21	2131	0	2131	108	1929	0.04	1929	0.03	1929
16	1927	2.21	1945	6.5	1925	109	1929	0.06	1929	0.06	1929
17	1978	14.03	1925	0.01	1925	110	1923	0.05	1923	0.04	1923
18	1933	3.8	1948	6.36	1925	111	1915	0.35	1915	0.35	1913
19	1941	6.55	1975	11.22	1925	112	1927	0.14	1929	0.2	1927
20	3915	0.0061	3915	0.005	3915	113	2263	64.86	2175	63.81	1968
21	1984	14.29	1993	15.05	1925	114	1969	11.53	1951	8.72	1925
22	3318	0.0052	3273	8.21	3318	115	2284	55.51	2253	55	2076
23	1945	6.89	1953	7.49	1925	116	2411	53.55	2569	77.07	2268
24	2037	26.48	1987	18.12	1925	117	20227	1.07	20226	1.16	20222
25	1941	5.93	1936	4.08	1925	118	1939	10.42	1913	1.67	1906
26	4413	0.0027	4304	17.76	4413	119	1961	6.28	1949	4.78	1929
27	2045	26.98	1960	12.17	1925	120	2278	9.77	2220	3.56	2229
28	1954	8.94	1974	11.37	1924	121	1931	2.29	1929	1.88	1923
29	1948	6.62	1948	7.89	1925	122	1980	21.25	1921	0.04	1920
30	7897	0.0043	7758	30.07	7897	123	1921	0.29	1925	3.32	1920
31	2066	27.41	1952	13.25	1925	124	1969	7.4	1945	4.35	1923
32	2031	26.33	1991	20.88	1925	125	2523	0.0069	2523	0.008	2523
33	1970	8.41	1940	5.23	1926	126	1927	0.14	1980	19.86	1928
34	1972	7.7	1940	4.48	1927	127	1922	2.07	1928	11.34	1913
35	1952	5.14	1930	2.47	1927	128	2039	21.28	1907	0.3	1905
36	1921	0.0011	1921	0.03	1921	129	2197	43.8	1935	1.63	1929
37	1921	0.01	1921	0.02	1921	130	2010	22.23	1976	20.13	1912
38	1928	0.05	1931	2.65	1929	131	1972	1.76	1970	1.56	1970
39	1923	0.09	1922	0.04	1923	132	1818	3.06	1829	3.61	1800
40	1933	3.75	1925	1.45	1923	133	1858	2.75	1863	7.45	1835
41	1925	0.006	1925	0.09	1925	134	2555	77.55	2301	35.12	2194
42	1923	0.42	1921	0.14	1921	135	1864	4.24	1935	25.37	1860
43	1973	0.09	1973	0.26	1973	136	1832	1.88	1896	16.15	1823
44	1925	0.0015	1926	2.65	1925	137	2340	12.7	2309	18.49	2311
45	1925	0.01	1924	0.01	1925	138	2491	29.9	2391	20.73	2332
46	1969	10.26	1924	2.1	1925	139	2119	12.84	2078	4.15	2062
47	1941	6.52	1943	7	1925	140	2102	12.61	2060	3.23	2051
48	1925	0.01	1924	0.01	1925	141	2272	0.88	2274	0.9	2268
49	4314	7.95	4287	8.35	4314	142	12758	0.43	12450	59.23	12755
50	1939	6.17	1941	6.75	1924	143	2598	92.1	2336	61.25	2095
51	1941	0.21	1941	0.18	1942	144	2279	47.92	2159	41.08	2015
52	1941	6.25	1956	9.14	1925	145	2484	59.81	2277	45.2	2077
53	1932	2.09	1929	1.62	1925	146	2223	43.55	2087	24.34	2003
54	1941	3.73	1924	0.01	1925	147	3218	22.58	3108	16.58	3050
55	1970	10.69	1945	7.7	1925	148	2435	49.35	2147	15.66	2083
56	1977	13.27	1924	0.005	1925	149	2462	56.38	2193	35.21	2041
57	1924	0.01	1924	0.02	1924	150	4760	0.6	4762	1.03	4758
58	1945	6.31	1925	0.01	1925	151	2122	30.98	2208	38.64	1991
59	1963	11.52	1976	13.17	1925	152	8350	148.52	7478	28.65	7527
60	1980	14.13	1953	10.28	1925	153	1892	20.07	2100	76.9	1800
61	1958	9.84	1925	0.02	1925	154	5203	11.74	5069	26.17	5131
62	1925	0.03	1925	0.03	1925	155	2193	21.67	2791	166	2097
63	1920	0.02	1920	0.025	1920	156	7895	72.18	7603	62.92	7765
64	1915	0.09	1915	0.47	1914	157	2005	13.34	2004	15.87	1936
65	1934	5.15	1955	9.39	1925	158	1928	12.88	1933	16.74	1849
66	1986	15.74	1925	0.01	1925	159	1979	12.15	2004	16.39	1921
67	1925	0.0059	1924	0.01	1925	160	1981	12.02	1994	15.71	1924
68	1945	9.11	1976	14.65	1924	161	1964	1.73	1974	3.97	1961
69	1925	0.22	1924	0.03	1924	162	1932	17.44	1858	1.1	1854
70	1925	0.22	1924	0.04	1924	163	1840	4.29	1818	2.57	1801
71	1954	7.4	1938	5.25	1925	164	2154	2.69	2124	1.8	2136
72	1942	5.61	1929	3.06	1924	165	1878	14.81	1836	9.58	1801
73	1925	0.0034	1924	0.006	1925	166	2394	22.9	2355	16.17	2289
74	1925	0.21	1924	0.04	1924	167	2080	14.38	2048	9.9	2024
75	1925	0.03	1925	0.02	1925	168	2023	13.13	2010	11.31	1983
76	1927	2.21	1924	0.002	1924	169	1989	16.04	1929	2.19	1919
77	1925	0.22	1924	0.04	1924	170	1929	1.2	1929	0.95	1925
78	1940	2.57	1937	1.22	1932	171	1929	1.2	1929	1.3	1925
79	1969	9.79	1951	7.61	1924	172	1929	1.2	1935	5.51	1925
80	1952	7.47	1925	0.07	1926	173	1980	11.4	1932	4.1	1923
81	1929	2	1925	0.04	1926	174	1925	0	1924	0	1925
82	1929	2.57	1925	0.03	1926	175	1925	0	1924	0	1925
83	1929	2.57	1925	0.03	1926	176	2043	29.35	1976	19.49	1925
84	1922	0.22	1921	0.05	1921	177	1925	0	1924	0	1925
85	1922	0.22	1924	3.1	1921	178	1925	0	1931	5.44	1925
86	1938	2.55	1935	1.04	1931	179	2028	23.47	1924	0	1925
87	1961	7.33	1941	5.29	1925	180	1931	1.12	1924	0	1925
88	1963	9.31	1925	0.01	1925	181	1925	0	2001	0	1925
89	1957	7.53	1925	0.005	1925	182	1930	1.08	1927	1.02	1925
90	2055	16.14	1948	6.85	1925	183	2067	29.58	1967	16.61	1925
91	1971	10.7	1945	7.16	1925	184	2054	29.4	2017	24.34	1935
92	1925	0.0073	1924	0.006	1925	185	8577	0	8577	0.05	8577
93	1927	1.89	1925	1.33	1925	186	1953	5.62	1947	6.49	1925

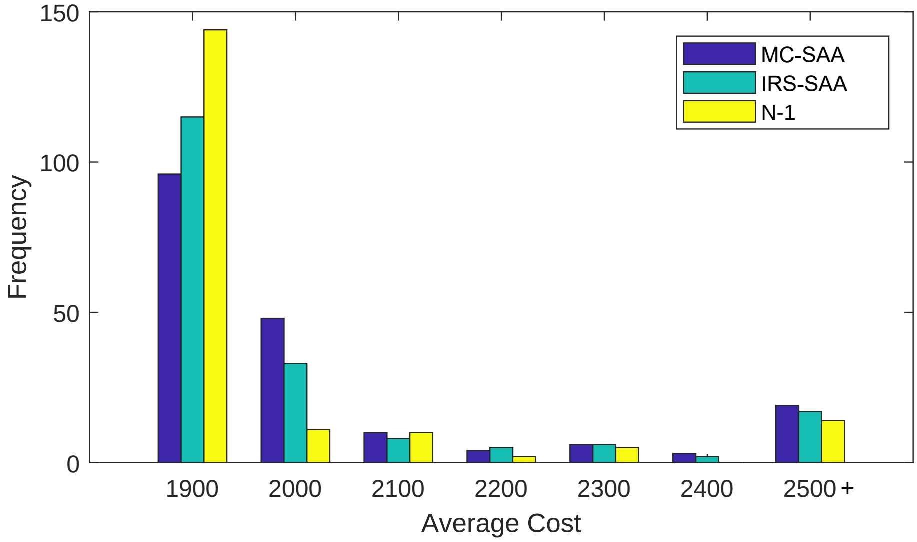

Figure 3 **Histogram of the average costs of the model with transmission switching allowing one open line in addition to failed lines in** (3) **using Markov chain Monte Carlo to generate scenarios (SAA/MCMC) with the independent relay-state model (IRS-SAA, Chen et al. (2005)) and the cost using the** $(N-1)$**-contingency dispatch (**$N-1$**).**