# Designing an optimal sequence of non-pharmaceutical interventions for controlling COVID-19

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June 28, 2021

**Abstract**

The COVID-19 pandemic has had an unprecedented impact on global health and the economy since its inception in December, 2019 in Wuhan, China. Non-pharmaceutical interventions (NPI) like lockdowns and curfews have been deployed by affected countries for controlling the spread of infections. In this paper, we develop a Mixed Integer Non-Linear Programming (MINLP) epidemic model for computing the optimal sequence of NPIs over a planning horizon, considering shortages in doctors and hospital beds, under three different lockdown scenarios. We analyse two strategies - centralised (homogeneous decisions at the national level) and decentralised (decisions differentiated across regions), for two objectives separately - minimization of infections and deaths, using actual pandemic data of France. We linearize the quadratic constraints and objective functions in the MINLP model and convert it to a Mixed Integer Linear Programming (MILP) model. A major result that we prove analytically, is that the optimal sequence of NPIs always follows a decreasing severity pattern. Using this property, we further simplify the MILP model into an Integer Linear Programming (ILP) model, reducing computational time up to 99%. Our numerical results show that a decentralised strategy is more effective in controlling infections for a given severity budget, yielding up to 20% lesser infections, 15% lesser deaths and 60% lesser shortages in healthcare resources.

**Keywords**: COVID-19 ; Non-Pharmaceutical Interventions; Epidemiologic model ; Multi-periodic planning ; Scheduling ; Integer Programming

## 1 Introduction

Influenza outbreaks have always posed significant challenges to societies because of its adverse impact on public health and economic development, resulting in widespread loss of lives and unemployment. The novel coronavirus 2019 (SARS - CoV-2) originated in Wuhan, China in December, 2019 (WHO 2020a) and was declared a pandemic by WHO on 11th March, 2020 after 118 countries were affected with the virus. Coronaviruses (CoV) belong to a large family of viruses that cause a wide range of illnesses from the common cold to respiratory disorders and more severe diseases. The novel coronavirus (nCoV) is a virus strain that has been newly identified in humans and has been subsequently named the 'COVID-19 virus' (WHO 2020b).

COVID-19 has infected more than 180 million people across 213 countries as of 23rd June, 2021, with more than 3.9 million reported deaths (Worldometers 2021) with a projected economic loss of 3.94 trillion USD (Statista 2020). The last time the world witnessed a pandemic of such large proportions was back in 1918. The 1918 influenza pandemic, better known as 'The Spanish Flu' was caused by an H1N1 virus with genes of avian origin. The deadly virus outbreak originated in USA during the spring of 1918 and resulted in approximately 500 million infections, 50 million deaths across the world (CDC 2019), lowering the average life expectancy in the United States by 12 years (Jester et al. 2018). Other notable infectious disease outbreaks which have emerged in the last century are severe acute respiratory syndrome (SARS), Middle East respiratory syndrome (MERS), Ebola, Human Immunodeficiency Virus (HIV).

In this paper, we look at the policymaker's problem of managing an epidemic outbreak, balancing its sanitary and economic impacts, by factoring in finite healthcare capacity. In the absence of a vaccine, it is imperative for affected countries to implement containment measures to check the epidemic spread. These interventions which do not involve any kind of immunization or medical action are called non-pharmaceutical interventions (NPIs). CDC (2020) defines NPI as "actions, apart from getting vaccinated and taking medicine, that people and communities can take to help slow the spread of illnesses like pandemic influenza". NPIs can range from mild measures like self-isolation of symptomatic individuals to more severe ones like travel restrictions, ban on public gatherings, school closures, service closures, with the most stringent being a complete lockdown. Each of these containment measures come with varying economic and social costs. There are immediate direct costs like lost revenue and profits and far-reaching indirect effects like increased unemployment, suicide rates. The stringency of these measures has an impact on the infection rate and helps in controlling the rapid multiplication in the virus. The basic reproduction number, $R_0$, pronounced "R naught", also called the basic reproduction ratio, is an epidemiological metric used to describe the contagiousness of infected individuals (Delamater et al. 2019). $R_0$ is one of the most fundamental metrics for the study of infectious disease dynamics (Pellis et al. 2012). Dietz et al. (1993) defines $R_0$ as "the number of secondary cases one case would produce in a completely susceptible population."When the $R_0$ for an outbreak is greater than 1, it leads to an epidemic because of the exponential growth of the pathogen in the population (Diekmann et al. 1990). The objective of deploying NPIs is to reduce the $R_0$ as much as possible. The trade-off lies in the fact that the most draconian NPIs with the highest reduction in $R_0$ are also the most expensive to execute and sustain.

The timing of implementing NPIs play a crucial role in epidemic decision making. NPIs at the early and later stages of the epidemic have different levels of impact in controlling the infection spread. Pei et al. (2020) reported that if a lockdown had been imposed a week earlier in USA, there would have been 36000 fewer deaths due to COVID-19. Most countries have faced the extremely difficult dilemma of safeguarding lives versus livelihoods in controlling the pandemic, without derailing the economy. This excruciating dilemma is real and unavoidable (Loayza and Pennings 2020). Different nations have adopted varying strategies for tackling this challenging situation. While countries like South Korea, Taiwan have resorted to extensive testing and contact tracing for isolating the infected, European countries, namely France, Germany, Spain, Italy have imposed restrictive lockdowns for checking the spread. On the other hand, countries like Brazil, Sweden, USA have been late to react to the epidemic situation and allowed it to increase rapidly. Another important consideration for policymaking has been the implementation of decentralised (localised) and centralised containment measures. Lockdowns are not only economically expensive, but lead to social fatigue (Ouardighi et al. 2020) and political instability, as has been observed during the first wave of the COVID-19 pandemic. During the second wave, UK and Germany have enforced localised restrictions in order to allow

the least affected regions to mobilize the economy. Hence, the comparative effectiveness of decentralised and centralised measures will serve as an important insight for decision making for policymakers, till the finalization of a vaccine.

In this paper, we propose a mixed integer linear programming (MILP) formulation for modelling the progression and control of the epidemic. We develop optimization models to address two strategies - centralised and decentralised, and compare the effectiveness of both in controlling the epidemic. The models combine the epidemiological dynamics with operations research for devising an optimal policy for managing the COVID-19 pandemic. The research questions that we are exploring are the following:

1. What is the optimal sequence of non-pharmaceutical interventions (NPIs) for controlling an epidemic, considering economic constraints and healthcare resource shortages?

2. What is the benefit of a decentralised strategy (differentiated decisions across regions) in terms of reduction of infections and deaths?

Based on the study of extant literature, the unique contributions of this paper are the following:

1. This paper is the first to implement a Mixed Integer Linear Programming (MILP) model for scheduling an optimal sequence of non-pharmaceutical interventions (NPIs) for epidemic control. We also consider various restrictions on lockdowns implemented based on social fatigue and economic viability. Contrary to other papers evaluating a restricted subset of scenarios or policies, we consider all potential sequences of NPIs over the planning horizon.

2. We compare the decentralised and centralised strategies for epidemic control and evaluate their effectiveness in checking the growth of infections under multiple scenarios, using the data of the 13 French regions.

3. We factor in the phenomenon of the infection spread affecting healthcare professionals as a part of the epidemiological model and consider shortages in healthcare capacity in terms of regular beds and Intensive Care Unit (ICU) beds. This is an important consideration related to COVID-19 as it has been observed in many countries at the peak of their infection spread that a high number of doctors and medical workers have succumbed to the infection or have been inactive from service. On April 2, 2020, Spain was reported to have around 15,000 medical workers infected with COVID-19, making up 14% of the confirmed cases then (Nugent 2020). They were self-isolating and unable to render their services.

4. We show that the optimal sequence of NPIs necessarily follows a decreasing severity pattern, i.e., the most severe measures like lockdowns are scheduled first in the optimal sequence for a given severity budget. Based on this property, we simplify the MILP to an Integer Linear Program (ILP), which is computationally more efficient and accessible to the policymaker.

The remainder of the paper is organized as follows. In Section 2, we review the literature in two parts, we first look at papers with optimization models addressing operational challenges related to epidemic management and then analyse recent COVID-19 papers on containment strategies specifically. In Section 3, we introduce the epidemic compartmental model and notations. In Section 4, we explain the Time-based optimization models for the decentralised and centralised strategies. In Section 5, we discuss the Sequence-based optimization model. In Section 6, we report the numerical results and the relative performance of

both strategies considering data from France. In Section 7, we discuss the managerial implications of the strategies to inform policy-making. In Section 8, we summarize our findings and propose future extensions of our work.

# 2 Literature Review

Here, we review the literature in two different sections, first we focus on epidemic logistics papers in the field of Operations Research, encompassing various operational issues faced during an epidemic outbreak and second, we discuss papers exclusive to COVID-19 control.

## 2.1 Epidemic Logistics Models

A couple of key papers provide an exhaustive review of the different types of problems and methodologies implemented in epidemic logistics research. Dasaklis et al. (2012) review the literature for epidemic containment and characterize three major streams of research: first, where pharmaceutical containment like vaccines is planned and executed, second, where non-pharmaceutical interventions like school closure, lockdowns are imposed, third, where a combination of both are deployed. The authors also classify the research streams based on pre-epidemic, post-epidemic and integrated while looking at the various logistical problems like facility location, network design, vehicle routing, transportation of medical supplies, encompassing different solution approaches like mathematical programming, game theory, queueing theory, data simulation etc. for stochastic and deterministic problems. Most of these papers look at compartmental models based on the classical SIR framework proposed by Kermack and McKendrick (1932). Dimitrov and Meyers (2010) provide a detailed comparison of various mathematical approaches for analysing epidemic spreads. They infer that compartmental models are less complex computationally, compared to contact network models and agent-based simulations, although the latter are more accurate in capturing the progression of the epidemic.

Facility location layout, distribution and logistics related problems during an epidemic have been widely covered in this area of research for both influenza epidemics and man-made epidemics (bio-terrorism). Jia et al. (2007) analyse a maximal covering facility location problem of a large scale emergency response for medical supplies distribution by implementing different optimization solution approaches. A multi-objective stochastic optimization model using genetic algorithms and Monte Carlo simulations is presented by Wang et al. (2009) for constructing an emergency epidemic logistics network. Liu and Liang (2013) propose a three level dynamic optimization model for allocating medical resources in epidemic controlling, using the epidemic diffusion model for forecasting demand. Ekici et al. (2014) develop a food distribution network model using Mixed Integer Linear Programming during an influenza pandemic. A novel epidemic control objective function is addressed by He and Liu (2015), where they minimize psychological suffering for infected individuals who remain undiagnosed or untreated because of relief supply shortage, using an epidemic optimization model. Liu and Zhang (2016) model medical resource allocation in the light of an influenza outbreak (time discretized SEIR model) and divide their analysis in different phases for forecasting, planning and execution.Anparasan and Lejeune (2019) determine the optimal number, size and location of medical facilities and deployment of resources to attend medically ill patients, based on the cholera outbreak of Haiti in 2010. Other types of epidemic outbreak include planned bio-terrorist attacks where a pathogen is deliberately released in a population, for eg. smallpox (Kaplan et al. 2002, Dasaklis et al. 2017), anthrax (Craft et al. 2005, Wanying et al. 2016), influenza (Longini Jr et al. 2004).

There has been very few papers analysing non-pharmaceutical interventions for controlling an epidemic. Yaesoubi and Cohen (2011) develop a Markov Decision Process framework for dynamic policymaking for handling an influenza outbreak considering social distancing and vaccines as the two major types of interventions. The paper extends the SEIR epidemiological model with transition probabilities and defines actions as NPIs along with a reward function which captures the health benefits and economic costs owing to infections and interventions.

The non-linearity of the epidemic compartmental model poses challenges for mathematical programming approaches and has been approximated linearly for tackling epidemic logistics problems. Büyüktahtakın et al. (2018) devise a Mixed Integer Linear Programming (MILP) model to study the spatial spread of the Ebola epidemic and derive the optimal number, time and location of healthcare facilities under budget constraints. This paper relaxes the bilinear transmission dynamics of susceptible and infected individuals and uses a linear epidemic equation avoiding the computational complexities arising out of non-linearity. Liu et al. (2020) use a similar relaxation of the epidemic equations as Büyüktahtakın et al. (2018) and propose a Mixed Integer Linear Programming model for an H1N1 epidemic where the decision variables are the number of healthcare facilities to open and close for minimizing fatalities, subject to budget constraints.

We build on the simplified epidemic equations as introduced by Büyüktahtakın et al. (2018) and furthered by Liu et al. (2020), because it enables tractability in the MILP approach. Although we borrow this linear approximation of Buyuktahtakin's model, a lot of the other attributes are different and unique. We model the implementation of NPIs for controlling the epidemic, instead of modelling facility location decisions. We do not assume any spatial element in the population in terms of influx and exodus from other regions, but consider differing transmission rates for the different regions based on population density. We also assume a common national budget for managing the economic costs of implementing NPIs for all regions combined. There are other papers in the past which have linearized the epidemic equations for ensuring tractability of the models. Reveller et al. (1969) study the resource allocation problem for minimizing infections for a tuberculosis outbreak by implementing an integer programming model. They linearize the epidemic equations by assuming an average number of active infected people during the planning horizon. Zaric and Brandeau (2001) model interventions for controlling an epidemic spread and maximizing quality-adjusted life years of the population under budget constraints. The authors argue that the non-linear functions lead to increased computational complexity and hence propose some approximations and heuristics to tackle them.

## 2.2 COVID-19 Containment

We now look at some of the influential papers which have analysed the impact of various containment strategies in the context of the COVID-19 pandemic. Ferguson et al. (2020) carry out a comparative study between a counterfactual, unmitigated model, mitigation and suppression for the population of UK for different scenarios. They consider the implementation of five NPI measures individually and in combination. For the mitigation approach, the healthcare capacity is overwhelmed multiple times. For suppression, the combination of all NPIs at the same time yields the highest impact on transmission reduction. Walker et al. (2020) from the same Imperial College team extend the analysis to contrast mitigation and suppression strategies for Low, Middle and High Income countries. The authors argue that owing to a bigger unorganized sector in low income countries, the compliance to stringent lockdowns will be lower in comparison to higher income countries. Loayza and Pennings (2020) contends that indiscriminate lockdowns can be counter-productive leading to displacement of labourers and further aggravation of the spread. Also, when compliance to lockdowns is

low, it results in low marginal gains and eventually leads to second and third waves of infections. Barnett-Howell and Mobarak (2020) point out that the economic value generated by comparable social distancing measures is approximately 240 times larger for the United States, 70 times larger for Germany compared to the value created in Pakistan or Nigeria, according to their analysis.

Eichenbaum et al. (2020) analyse the interaction between epidemic modelling and economic decisions using an agent based modelling technique for various containment scenarios. They look at "smart containment" where the infection condition of the entire population is known to the policymaker with the help of extensive testing and hence enables the maximization of social welfare of the population by using a tailored intervention policy. Alvarez et al. (2020) analyse the policymaker's problem to handle the trade-off between output costs of lockdown policies and fatality costs because of infections by combining the SIR epidemiology framework in an optimal control model. The paper looks at various degrees of lockdown from 60% of the population being confined to 10% of the population with a parameterized effectiveness of the lockdown policy, and the subsequent impact on the economy in terms of GDP contraction. They argue that if the lockdown is not very effective, healthcare capacity is not ramped up, the severity and duration of the lockdown should be lowered. Ouardighi et al. (2020) compare the effectiveness of mobility restrictions and secured social interactions for epidemic control, factoring in ill-preparedness of the policymaker and population social fatigue, whereas Caulkins et al. (2021) model the optimal duration of lockdown using optimal control. Charpentier et al. (2020) model the optimal lockdown level, level of testing efforts for Type 1 (antigen) and Type 2 (antibody) using optimal control, by considering the combined impact of the epidemic on healthcare, economic and social costs.

Mehrotra et al. (2020) propose a multi-period mixed integer linear programming model for allocation of ventilators to different regions from the centre, based on stochastic demand for the COVID-19 pandemic. Acemoglu et al. (2020) study a Multi-Group SIR model for three different demographic groups- young, middle-aged and old and devise a customized lockdown strategy for minimizing economic damage and infections. The authors suggest a differential lockdown policy with targeted shielding of the elderly and testing, tracing, isolation of all symptomatic patients for improving the economic and health impacts of lockdowns. Di Domenico et al. (2020) propose a stochastic age structured epidemic transmission model for capturing the progression of the epidemic in the Ile-de-France region in France, evaluating possible exit strategies after the lockdown. Davies et al. (2020) carry out a similar study for United Kingdom, and analyse various NPIs for controlling the epidemic across 186 counties. They compare local and national control strategies and consider various triggers for initiating lockdowns based on healthcare capacity.

Note that in the above papers, when government policies are evaluated for controlling Covid-19 (lockdowns or other NPIs), only a few scenarios are generated, evaluated (often with optimal control techniques) and compared, but no optimization approach has been proposed so far for designing the best sequence (or optimal mix) of NPIs over a planning horizon among the huge set of all potential sequences. We did not find either a comparison of decentralised vs centralised strategies for epidemic control. In the next section we present our epidemic and decision models.

# 3 Epidemic model and notations

## 3.1 Epidemic model

In this section, we introduce the compartmental model for computing the epidemic spread of COVID-19 in a given population, based on classical model of Kermack and McKendrick (1932). The entire population is subdivided into the compartments of Susceptible, Infected, Critical, Recovered, Dead for a given time period $t$. The Susceptible compartment comprises of individuals who are vulnerable to the virus but have not been infected yet; Infected are the active infections in the population at time $t$; Critical represents the number of individuals who require any kind of hospitalization; Recovered constitutes the number of people who have been cured, Dead are the fatalities owing to COVID-19 specifically. The epidemic model considered here is discrete-time and deterministic. Each time unit has been assumed to be equivalent to one week. We assume a weekly review and situational assessment from the policymaker's perspective.

Figure 1: Epidemic Compartmental Model

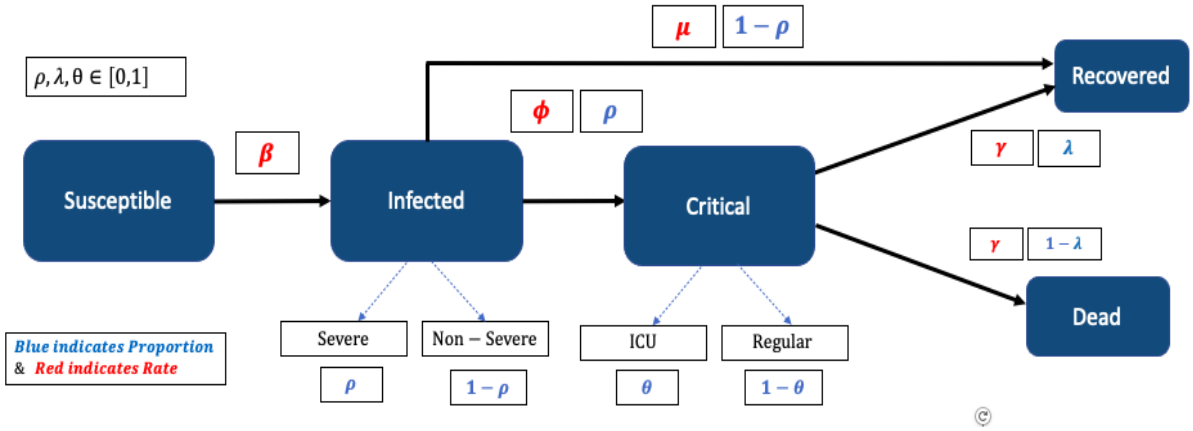


Figure 1 captures the compartmental model schematically. The individuals in the susceptible compartment transition into the Infected compartment at a rate of $\beta$. The infected population comprises of non-severe (mild or asymptomatic) and severe infections, where the latter require hospitalization. The proportion of severe infections is denoted by $\rho$. Severe infections enter the Critical or hospitalised state at a rate of $\phi$ per week, from symptom onset. Non-severe infections recover at a rate of $\mu$ per week. From the Infected compartment, people transition into either the Critical compartment for hospitalisation or to the Recovered compartment directly, if non-severe. Individuals in the Critical compartment who are hospitalized, recover at a rate of $\gamma$. We assume that when a patient requires hospitalisation (Critical), he is admitted to a regular bed or an ICU bed (subject to availability). $\lambda$ is the proportion of the hospitalised individuals who recover weekly and $1 - \lambda$ is the proportion of people who die weekly. For the schematic model, we represent the rates and proportions for ICU and non-ICU patients to be the same. However, in the numerical analysis we consider different rates based on empirical observations. The Dead and Recovered are terminal compartments with no exit. Conversely, the Susceptible compartment has no inflow from other compartments.

The assumptions for the epidemic compartmental model are the following:

- The transmission between Susceptible and Infected is assumed to be dependent only on the Infected population of the previous period, for linearizing the epidemic state equation (Büyüktahtakın et al.

2018).

- The infection transmission rate $\beta$ is a function of the population density of the region. Hence, densely populated clusters will have a higher rate of spread than sparsely populated regions. (Gerritse 2020) empirically shows that a population density multiplier of two leads to a 0.06 point increase in transmission rate.
- When an infected individual recovers, he is no longer susceptible to the virus, since the proportion of reinfection cases with respect to the total is negligible (less than 1 %) according to recent findings (Hall et al. 2020).
- Patients requiring hospitalisation avail either a regular bed or a Intensive Care Unit (ICU) bed, and not both (Di Domenico et al. 2020).
- A shortage of regular beds and doctors delays the recovery process and a shortage of ICU beds leads to deaths. Lin (2021) reports that COVID-19 mortality rates are strongly associated with ICU bed availability from an empirical study and we base our assumption on this.
- Deaths are accounted for only the infections which require hospitalization (Di Domenico et al. 2020).
- A given proportion of severe symptom infections self-isolate. Based on a recent study in the UK (Smith et al. 2021), it was reported that only 18% of COVID-19 patients self-isolated themselves.

## 3.2 Model notations

Table 1 summarizes the various NPI levels for controlling the pandemic. In Table 2, we report the optimization model notations and constant parameters. Table 3 lists the primary and auxiliary decision variables for the model.

Table 1: NPI List

| **Parameter** | **Description** |
|---|---|
| Self-Isolation (I) | Isolation or Quarantine of vulnerable/ symptomatic people for 7 days |
| Travel Restrictions (T) | Travel restrictions within and across regions based on distance. |
| School Closure (S) | Closure of schools and universities - primary, secondary, UG, PG. |
| Public Gathering Ban (G) | Restriction on gatherings for public events >50 people |
| Full Lockdown (L) | Full service closure, curfew, mass restriction of movement |
| | |
| **NPI Combinations** | |
| I | Level 1 NPIs |
| I+T, I+G, I+S | Level 2 NPIs |
| I+T+S, I+T+G, I+S+G | Level 3 NPIs |
| I+T+S+G. | Level 4 NPIs |
| L | Level 5 indicates Lockdown |

Table 2: Optimization Model Notations

| Parameter | Description |
|---|---|
| **Indices** | |
| $t$ | Index for time period (weeks), $t = \{1, ..., T\}$ |
| $i$ | Index for NPI level, $i \in N = \{1, 2, 3, 4, 5\}$ |
| $k$ | Index for region, $k = \{1, ..., K\}$ |
| $r$ | Index for medical resource,$r \in Res = \{1, 2, 3\}$,1 = Doctors,2 = ICU,3 = Regular (Beds) |
| **Epidemic parameters** | |
| $\beta_k$ | Weekly transmission rate of infections per infected person for region $k$ |
| $\beta_k^H$ | Weekly transmission rate of infections per doctor for region $k$ |
| $\rho$ | Percentage of infected people who become Critical (hospitalised) |
| $\mu$ | Weekly rate at which a non-severe infection recovers |
| $\phi$ | Weekly rate at which an Infected person enters the hospital |
| $\gamma$ | Weekly Rate at which a Critical infection leaves the hospital |
| $\lambda$ | Percentage of Critical people who recover |
| **Other parameters** | |
| $\bar{\tau}$ | Average time to recover after being infected. |
| $c_i$ | Severity cost of implementing NPI $i$ per period and per person. |
| $\bar{\alpha}$ | Maximum reduction in transmission rate, $\bar{\alpha}$=$Max\{\alpha_i \| i \in N\}$ |
| $m$ | Number of patients attended by one doctor |
| $b_{2,k}$ | Number of ICU beds in region $k$ |
| $b_{3,k}$ | Number of Regular beds in region $k$ |
| $\theta$ | Proportion of Critical patients requiring ICU beds |
| $\eta_{rkt}$ | Proportion of resource $r$ allocated for non-COVID cases in period $t$, region $k$ |
| $l_0$ | Maximum number of changeovers of NPIs |
| $l_1$ | Maximum number of weeks of lockdown over the $T$ weeks |
| $l_2$ | Maximum number of consecutive weeks of lockdown |
| $l_2'$ | Minimum number of weeks between two successive blocks of lockdowns |
| $B$ | Upper bound on the average severity of NPIs per individual (budget). |
| $V_{kt}$ | Theoretical Lower bound of Infections for period $t$, region $k$ |
| $I_{max}$ | Theoretical Upper bound of Infections for all periods. |
| $\epsilon$ | % of Critical patients who do not self isolate. |
| $P$ | Total population of the country |
| $p_k$ | Population in region $k$ |
| $I_{k1}, C_{k1}, H_{k1}$ | Epidemic state values for period 1 for region $k$, $\forall k \in \{1, .., K\}$ |

Table 3: Decision variables for MINLP and MILP models

| Variables | Description |
|---|---|
| **PRIMARY** | |
| $x_{kit}$ | =1 if NPI level $i$ is selected for week $t$ and region $k$, 0 otherwise |
| **AUXILIARY** | |
| **Epidemic** | |
| $I_{kt}$ | Number of Infected people at week $t$ in region $k$ |
| $C_{kt}$ | Number of Critical people at week $t$ in region $k$ |
| $H_{kt}$ | Number of active hospital doctors at week $t$ in region $k$ |
| **Shortage** | |
| $s_{rkt}$ | Shortage between demand and capacity of resource $r$ at time $t$ in region $k$ |
| $z_{0,kt}$ | =1 if shortage of doctors is less than regular beds, 0 otherwise, at time $t$, region $k$. |
| $z_{rkt}$ | =1 if resource $r$ demand exceeds supply, 0 otherwise, at time $t$ and region $k$ |
| $Q_{0,kt}$ | Max of doctor shortage and regular beds shortage at time $t$ and regon $k$ |
| $Q_{rkt}$ | Min of resource $r$ supply and demand at time $t$ in region $k$ |
| **Linearization of quadratic terms** | |
| $X_{kit}$ | Replacement for $x_{kit}((1-\alpha_i)(1-\rho\epsilon)I_{kt}$ |
| $Y_{0,kt}$ | Replacement for $H_{kt}z_{1,kt}$ |
| $Y_{rkt}$ | Replacement for $C_{kt}z_{rkt}$ |
| $U_{rkt}$ | Replacement for $s_{rkt}z_{0,kt}, \forall r \in \{1,3\}$ |
| **Changeover** | |
| $W_{kit}$ | =1, if NPI level i is not selected in week $t-1$ and selected in week $t$, 0 otherwise |

As indicated in Table 1, we group the NPIs in different levels, from 1 to 5. For each level (for Level 2,3,4 since Level 1 and 5 have only one combination), we assign a single combination corresponding to the best reduction factor based on our estimations and other empirical studies (Ferguson et al. 2020, Haug et al. 2020, Lin 2021).

Before delving into the model further, we take a closer look at the variables used, as listed in Table 3. The primary decision variable is the NPI level, $x_{kit}$. The epidemic state variables defined by the compartmental model are the auxiliary variables. Out of the state variables, we consider only Infected and Critical (Dead can be expressed as a function of Critical), since the rest do not directly impact the policy maker's decisions. We also model the epidemic impact on doctor availability with variable $H_{kt}$. Variable $s_{rkt}$ captures the shortage between the demand and supply of medical resource $r \in Res$ at time period $t$ and region $k$. The three forms of unserviced healthcare demand, in terms of doctors, ICU beds, regular beds indicate the number of untreated infected patients for all three cases. Hence, $s_{1,kt}$ indicates the shortage of doctors, $ms_{1,kt}$ is the number of COVID-19 patients untreated due to lack of doctors, where $m$ is the number of patients 1 doctor

treats on average. The combination of the $z_{rkt}$ and $Q_{rkt}$ variables are used to ensure that $s_{rkt}$ is exactly equal to the shortage between demand and supply of the medical resource $r$.

For analysing the optimal policy making for containing the spread of COVID-19, we first introduce a Mixed Integer Non Linear Programming (MINLP) optimization model. We linearize the bilinear terms in the MINLP model subsequently to convert it into a Mixed Integer Linear Programming (MILP) model. We further simplify it using model properties into an Integer Linear Programming (ILP) model and compare the computational efficiency of both in the numerical section. We refer to the MINLP and MILP formulations as 'time-based' because the NPI decision variables are associated with each week, whereas we refer to the ILP formulation as 'sequence-based' because entire sequences of NPI choices spanning the planning horizon are decided.

# 4 Time-based optimization models

## 4.1 Decentralised MINLP model

Here, we define the Time-based Mixed Integer Non-Linear Programming (MINLP) model for the decentralised strategy, with the aim to minimize the total number of unique infections:

$$Minimize \quad \sum_{k=1}^{K}\sum_{i\in N}\sum_{t=1}^{T}\beta_k((1-\alpha_i)(1-\rho\epsilon))x_{kit}I_{kt} \tag{1}$$

Subject to:

$$\sum_{i\in N} x_{kit} = 1. \qquad \forall t\in\{1,...,T\}, \quad \forall k\in\{1,...,K\} \tag{2}$$

$$\frac{1}{P.T}\sum_{k=1}^{K}\sum_{i\in N}\sum_{t=1}^{T} c_i p_k x_{kit} \leqslant B \tag{3}$$

$$s_{1,kt} = \frac{C_{kt}z_{1,kt}}{m} - (1-\eta_{1,kt})H_{kt}z_{1,kt} \qquad \forall t\in\{1,...,T\}, \quad \forall k\in\{1,...,K\} \tag{4}$$

$$s_{2,kt} = \theta C_{kt}z_{2,kt} - (1-\eta_{2,kt})b_{2,k}z_{2,kt} \qquad \forall t\in\{1,...,T\}, \quad \forall k\in\{1,...,K\} \tag{5}$$

$$s_{3,kt} = (1-\theta)C_{kt}z_{3,kt} - (1-\eta_{3,kt})b_{3,k}z_{3,kt} \qquad \forall t\in\{1,...,T\}, \quad \forall k\in\{1,...,K\} \tag{6}$$

$$Q_{1,kt} = (1-\eta_{1,kt})H_{kt}z_{1,kt} + \frac{C_{kt}(1-z_{1,kt})}{m} \qquad \forall t\in\{1,...,T\}, \quad \forall k\in\{1,...,K\} \tag{7}$$

$$Q_{1,kt} \leqslant (1-\eta_{1,kt})H_{kt} \qquad \forall t\in\{1,...,T\}, \quad \forall k\in\{1,...,K\} \tag{8}$$

$$Q_{1,kt} \leqslant \frac{C_{kt}}{m} \qquad \forall t\in\{1,...,T\}, \quad \forall k\in\{1,...,K\} \tag{9}$$

$$Q_{2,kt} = (1-\eta_{2,kt})b_{2,k}z_{2,kt} + \theta C_{kt}(1-z_{2,kt}) \qquad \forall t\in\{1,...,T\}, \quad \forall k\in\{1,...,K\} \tag{10}$$

$$Q_{2,kt} \leqslant (1-\eta_{2,kt})b_{2,k} \qquad \forall t\in\{1,...,T\}, \quad \forall k\in\{1,...,K\} \tag{11}$$

$$Q_{2,kt} \leqslant \theta C_{kt} \qquad \forall t\in\{1,...,T\}, \quad \forall k\in\{1,...,K\} \tag{12}$$

$$Q_{3,kt} = (1-\eta_{3,kt})b_{3,k}z_{3,kt} + (1-\theta)C_{kt}(1-z_{3,kt}) \qquad \forall t\in\{1,...,T\}, \quad \forall k\in\{1,...,K\} \tag{13}$$

$$Q_{3,kt} \leqslant (1-\eta_{3kt})b_{3,k} \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{14}$$

$$Q_{3,kt} \leqslant (1-\theta)C_{kt} \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{15}$$

$$Q_{0,kt} = ms_{1,kt}(1-z_{0,kt}) + s_{3,kt}z_{0,kt} \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{16}$$

$$Q_{0,kt} \geqslant ms_{1,kt} \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{17}$$

$$Q_{0,kt} \geqslant s_{3,kt} \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{18}$$

$$I_{k,t+1} = I_{kt} + \beta_k \sum_{i \in N}((1-\alpha_i)(1-\rho\epsilon)x_{kit}I_{kt} - (\rho\phi + \mu(1-\rho))I_{kt} \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{19}$$

$$C_{k,t+1} = C_{kt} + \rho\phi I_{kt} - (\gamma\lambda + (1-\lambda))C_{kt} + Q_{0,kt} - s_{2,kt} \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{20}$$

$$H_{k,t+1} = H_{kt} - \beta_k^H(Q_{2,kt} + Q_{3,kt} - \frac{1-\rho(1-\gamma\lambda)}{\bar{\tau}}(Q_{2,k,t-1} + Q_{3,k,t-1})) \qquad ,\forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{21}$$

$$x_{kit} \in \{0,1\} \qquad \forall k \in \{1,...,K\}, \quad i \in N, \quad t \in \{1,...,T\} \tag{22}$$

$$I_{kt}, C_{kt}, H_{kt} \geqslant 0 \qquad \forall k \in \{1,...,K\}, \quad t \in \{1,...,T\} \tag{23}$$

$$s_{rkt}, Q_{0,kt}, Q_{rkt} \geqslant 0 \qquad \forall r \in Res, \quad \forall k \in \{1,...,K\}, \quad \forall t \in \{1,...,T\} \tag{24}$$

In our model, we capture the shortage between healthcare demand and capacity and evaluate its impact on the epidemiological state variables. The shortage in capacity, in terms of doctors, regular beds, ICU beds gets added to number of critical and dead patients, influencing the NPI level selected by the policymaker subsequently.

The objective function in (1) minimizes the sum of unique infections over the $T$ weeks. Constraint (2) implies that exactly one NPI can be chosen every week $t$ in each region $k$. Constraint (3) ensures that the average severity of NPIs over the whole population and the horizon of $T$ weeks stays within the national severity budget $B$. This severity budget is an indicator of both the permissible loss in productivity that the policymaker is willing to afford for the country, and compliance of the population with measures limiting their freedom. The metric can be interpreted as labour hours lost due to containment measures. We assume that the economic impact of the contact reduction $\alpha_i$ is a convex, quadratic function of $\alpha_i$ as analysed by (Charpentier et al., 2020), i.e., $c_i = \alpha_i^2$ . Constraints (4)-(6) imply that the shortage due to unserviced healthcare demand of medical resources $r$ in terms of doctors, ICU Beds, regular beds is defined by max(Resource demand – Resource supply, 0). To ensure that the binary variables $z_{rkt}$ truly indicate if the demand of resource $r$ exceeds supply, we introduce another set of continuous variables $Q_{rkt}$ defined as the minimum between demand and supply for resource $r$. We add three constraints to achieve this as in (Büyüktahtakın et al. 2018). The first constraint computes $Q_{rkt} = z_{rkt}$ * Supply + (1-$z_{rkt}$)* Demand. The second and third constraints imply $Q_{rkt}$ is less than the supply and demand, respectively. Constraints (7) to (9),(10) to (12), (13) to (15) are the system of constraints for medical resources' demand and supply for all resources $\forall r \in Res$. Constraints (16) to (18) are introduced to compute the maximum between doctor shortage and regular bed shortage. Constraints (19) to (21) are the epidemic state equations based on the Kermack and McKendrick (1932) compartmental model. The choice of NPI level $x_{kit}$ for time period $t$ and region $k$, reduces the new infections by a reduction factor $\alpha_i$ for level $i$. A proportion $\epsilon$ of people with severe infections self isolate and the rest are subjected to the NPI (self-isolation is common to all NPIs). The Critical (C) compartment increases by the extent of the maximum between untreated patients due to shortage of doctors and shortage

of regular beds, whereas the number of deaths increases by the extent of untreated patients due to shortage in ICU beds. This implies that patients who are unable to avail regular beds or doctors due to shortages continue to stay in the critical state and their recovery is delayed, and patients who do not get ICU Beds, die. Constraint (21) captures the state equation for hospital doctors for time period $t$ and region $k$. In constraint (21), we assume that the infections in doctors are directly impacted by the number of actual hospitalizations in regular and critical beds and the number $m$ of patients attended per doctor. We consider the minimum of demand and capacity to be precise, as captured by variables $Q_{2,kt}$ and $Q_{3,kt}$ respectively. The rate of infection for doctors is given by $\beta_k^H$. Hence, the number of recovered doctors from the last period is added back to the constraint based on the effective recovery rate $\frac{1-\rho(1-\gamma\lambda)}{\bar{\tau}}$. Constraints (22), (23) and (24) are integrality and non-negativity constraints on variables.

We also test another objective function of minimizing the total number of deaths caused by the pandemic. We compute deaths due to the epidemiological progression, and due to ICU shortage captured by variable $s_{2,kt}$. The time horizon considered is $T+3$ weeks because the impact of the NPI in week $T$ ($x_{kiT}$) impacts the Infected state in $T+1$, Critical infections $C_{k,T+2}$ of week $T+2$, which in turn affects the deaths in week $T+3$. The objective function becomes:

$$Minimize \quad \sum_{k=1}^{K}\sum_{t=1}^{T+2}(1-\lambda)C_{kt} + s_{2,kt} \tag{25}$$

For the centralised model, the NPI decision variables $x_{kit}$ are replaced by $x_{it}$ because of homogeneous decisions across regions, the assignment constraint (2) is no more for each region $k$ and the sum over $k$ disappears in the budget constraint (3). The rest of the constraints are identical with the decentralised model. Infection and Critical variables and the corresponding shortage variables are still computed at a regional level $k$ because of differentiated infection rate and capacities across regions.

## 4.2 MILP reformulation

In order to convert the former MINLP model into a Mixed Integer Linear Programming (MILP) model, we linearize the quadratic parts of the model in the following way. We define four sets of auxiliary variables $X_{kit}, Y_{0,kt}, Y_{rkt}, U_{rkt}$ for linearizing the products of $x_{kit}(1-\alpha_i)(1-\rho\epsilon)I_{kt}, H_{kt}z_{1,kt}, C_{kt}z_{rkt}$, $s_{rkt}z_{0,kt}$ respectively.

Linearization of $(1-\alpha_i)(1-\rho\epsilon)x_{kit}I_{kt} = X_{kit}$:

$$X_{kit} \geqslant V_{kt}x_{kit} \qquad \forall i \in N, \quad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{26}$$

$$X_{kit} \leqslant I_{max}x_{kit} \qquad \forall i \in N, \quad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{27}$$

$$X_{kit} \geqslant ((1-\alpha_i)(1-\rho\epsilon))I_{kt} - I_{max}(1-x_{kit}) \qquad \forall i \in N, \quad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{28}$$

$$X_{kit} \leqslant ((1-\alpha_i)(1-\rho\epsilon))I_{kt} - V_{kt}(1-x_{kit}) \qquad \forall i \in N, \quad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{29}$$

Linearization of $H_{kt}z_{1,kt} = Y_{0,kt}$:

$$Y_{0,kt} \geqslant 0 \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{30}$$

$$Y_{0,kt} \leqslant H_{k0} z_{1,kt} \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{31}$$

$$Y_{0,kt} \geqslant H_{kt} - H_{k0}(1 - z_{1,kt}) \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{32}$$

$$Y_{0,kt} \leqslant H_{kt} \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{33}$$

Linearization of $C_{kt} z_{rkt} = Y_{rkt}$:

$$Y_{rkt} \geqslant \rho V_{kt} z_{rkt} \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\}, \quad \forall r \in Res \tag{34}$$

$$Y_{rkt} \leqslant \rho I_{max} z_{rkt} \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\}, \quad \forall r \in Res \tag{35}$$

$$Y_{rkt} \geqslant C_{kt} - \rho I_{max}(1 - z_{rkt}) \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\}, \quad \forall r \in Res \tag{36}$$

$$Y_{rkt} \leqslant C_{kt} - \rho V_{kt}(1 - z_{rkt}) \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \quad \forall r \in Res \tag{37}$$

Linearization of $s_{rkt} z_{0,kt} = U_{rkt}$:

$$U_{rkt} \geqslant 0 \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\}, \quad \forall r \in \{1,3\} \tag{38}$$

$$U_{rkt} \leqslant I_{max} z_{0,kt} \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\}, \quad \forall r \in \{1,3\} \tag{39}$$

$$U_{rkt} \geqslant s_{rkt} - I_{max}(1 - z_{0,kt}) \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\}, \quad \forall r \in \{1,3\} \tag{40}$$

$$U_{rkt} \leqslant s_{rkt} \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \quad \forall r \in \{1,3\} \tag{41}$$

For linearization of bi-linear terms in the objective function and Constraint (19), constraints (26) to (29) have been used. Glover (1975) linearization method, as implemented in Büyüktahtakın et al. (2018) has been used, where $n$ auxiliary variables and $4n$ constraints are introduced. The lower bound and upper bound for the corresponding continuous variable $I_{kt}$ has been defined as $V_{kt}$, $I_{max}$ respectively. The rationale behind defining the lower bound as time dependent is to make it tighter and improve the computational time. Hence, the lower bound for total active infected people at week $t$ is derived by considering that the strongest NPI (Level 5) is applied in all periods leading up to $t$. The auxiliary variable $X_{kit}$ is a replacement of $x_{kit}(1-\alpha_i)(1-\rho\epsilon)I_{kt}$. The lower bound for $I_{kt}$ is given by $V_{k,t+1} = V_{kt}(1 + \beta_k(1 - \bar{\alpha} - (\rho - \mu(1-\rho)))$. Constraints (30) to (33) are implemented for linearizing the bilinear terms corresponding to available doctors in region $k$ and time $t$, given by $H_{kt} z_{1,kt}$. Constraints (34) to (37) are introduced for linearizing the product of $C_{kt}$ and $z_{rkt}$. Constraints (38) to (41) are introduced for linearizing the product of shortage variables $s_{rkt}$ and the indicator variable $z_{0,kt}$. The lower bound for $C_{kt}$ is assigned as $\rho V_{kt}$ and the upper bound as $\rho I_{max}$ for each time period $t$ and region $k$.

## 4.3 Additional constraints on NPI sequences

In this subsection we introduce three scenarios S0, S1 and S2, of additional policy constraints in order to make the containment policy enforceable and sustainable from an implementation point of view.

**Scenario S0:**

**Restricted changeovers.** No more than $l_0$ changeovers across $T$ weeks:

$$x_{kit} + \sum_{j \in N \setminus \{i\}} x_{kj,t-1} \geqslant 2W_{kit} \qquad \forall t \in \{1, ..., T\}, \quad \forall k \in \{1, ..., K\} \tag{42}$$

$$x_{kit} + \sum_{j \in N \setminus \{i\}} x_{kj,t-1} - W_{kit} \leqslant 1 \qquad \forall t \in \{1, ..., T\}, \quad \forall k \in \{1, ..., K\} \tag{43}$$

$$\sum_{t=1}^{T} \sum_{i \in N} W_{kit} \leqslant l_0 \qquad \forall k \in \{1, ..., K\} \tag{44}$$

**Two week rule:** If an NPI is selected, it should be on for at least two consecutive weeks, whenever applied.

$$x_{ki,t+1} \geqslant W_{kit} \qquad \forall i \in N, \quad \forall t \in \{1, ..., T\}, \quad \forall k \in \{1, ..., K\} \tag{45}$$

$$x_{ki,t+1} \geqslant x_{kit} \qquad \forall i \in N, \quad \forall t \in \{1, T-1\}, \quad \forall k \in \{1, ..., K\} \tag{46}$$

**Scenario S1:** No more than $l_1$ weeks of lockdown

$$\sum_{t=1}^{T} x_{k5t} \leqslant l_1 \qquad \forall k \in \{1, ..., K\} \tag{47}$$

**Scenario S2:** Maximum $l_2$ weeks of consecutive lockdowns, Minimum gap of $l_2'$ weeks in between phases

$$\sum_{t=t'}^{t'+l_1} x_{k5t} \leqslant l_2 \qquad \forall t \in \{1, .., T\}, \quad \forall k \in \{1, ..., K\} \tag{48}$$

$$\sum_{t=t'}^{t'+l_2-2} x_{k5t} \leqslant (l_2' - 1)(2 - x_{k5,t-2} - \sum_{i=1}^{4} x_{ki,t-1}) \qquad \forall t \in \{1, .., T\}, \quad \forall k \in \{1, ..., K\} \tag{49}$$

Scenario $S0$ is the default model, where there are restrictions on NPIs in general. Scenario $S1$ and $S2$, impose additional restrictions on implementation of lockdowns specifically and are tested separately.

Constraints (42) to (44) limit the number of changeovers in NPIs across the planning horizon. This is done in order to make the policy practically sustainable. Switching between NPI modes from 1 to 5 rapidly will be chaotic to enforce and populations will find it difficult to comply with. Constraints (42) and (43) record the number of changeovers between NPIs. Constraint (44) puts an upper bound $l_0$ on the number of changeovers. Constraints (45) and (46) ensure that every NPI, whenever it is introduced, stays on for two weeks at least. For Scenario $S1$, constraint (47) ensures that the total number of lockdowns is less than $l_1$, as set by the policymaker. Constraints (48) and (49) together represent Scenario $S2$, which implies that there cannot be more than $l_2$ successive weeks of lockdowns (Constraint (48)), with a gap of at least $l_2'$ weeks between two phases of lockdowns. We test the centralised and decentralised strategy for all scenarios $S0$, $S1$ and $S2$ separately.

# 5 Sequence-based optimization model

In this section, we present a simplification of the MILP model based on nice properties of optimal solutions stated in Propositions 1 and 2, that are corollaries of the following lemmas.

**Lemma 1.** *For a given feasible solution, if for some region the NPIs at two periods $t_1$ and $t_2$, with $t_1 < t_2$, are swapped, then the infection level in period $t_2 + 1$ stays the same.*

**Lemma 2.** *If a solution satisfies $x_{k,i_1,t_1} = 1$ and $x_{k,i_2,t_2} = 1$ with $t_1 < t_2$ and $\alpha_{i_1} < \alpha_{i_2}$ (i.e NPI $i_1$ is less effective than $i_2$), then making the NPI swap $x_{k,i_2,t_1} = 1$ and $x_{k,i_1,t_2} = 1$ will strictly improve the objective value (for both minimization of infections and deaths).*

The proofs of the lemmas can be found in the Appendix. We now introduce the two propositions (corollaries of Lemma 2 that enable to turn the time-based model into a tractable sequence-based model.

**Proposition 1.** *For Scenario $S0$ and $S1$, the optimal NPI sequence is always ordered from higher severity to lower severity (for both minimization of infections and deaths).*

**Proposition 2.** *For Scenario $S2$, the optimal NPI sequence always follows an ordered sequence from higher to lower severity for every sub-sequence containing a lockdown, and for non-lockdown NPIs across sub-sequences (for both minimization of infections and deaths).*

Using Propositions 1 and 2, one can switch to a tractable 'sequence-based' model, where a pre-processing can easily compute the set $Seq$ of all sequences satisfying the property of descending severity, together with the budget constraint and the constraints of scenarios $S0, S1$ and $S2$. Doing so, the state variables and shortage variables become constants in the model. The cost of each sequence $s \in Seq$ is given by

$$c_s = \sum_{i \in N} \sum_{t=1}^{T} q_{sit} c_i$$

where binary parameter $q_{sit}$ is equal to 1 if NPI level $i$ is assigned to week $t$ in sequence $s$, 0 otherwise. $\bar{I}_{kst}, \bar{C}_{kst}, \bar{H}_{kst}$ represent the state values corresponding to sequence $s$ at week $t$ and region $k$. $\bar{Q}_{rkt}$, $\bar{Q}_{0,kt}$ and $\bar{s}_{rkt}$ represent the demand − supply values and shortage values for healthcare resources. As a result of this simplification, all the linearization variables of section 4.2 are discarded.

We discuss only the decentralised strategy, because this simplification turns the centralised optimization problem just into an enumeration problem (generate all sequences in $Seq$, evaluate the objective value of each and pick the best one). The sequence-based decentralised formulation is still an NP-hard Integer Linear Programming model, though much simpler than the earlier MILP formulation. For this ILP model we define a new decision variable:

$$y_{ks} = \begin{cases} 1 & \text{if sequence } s \in Seq \text{ is selected for region } k \\ 0 & \text{otherwise} \end{cases}$$

and the sequence-based model can be written as follows:

**Minimization of Infections**

$$Minimize \quad \sum_{k=1}^{K} \sum_{s \in Seq} \sum_{t=1}^{T} \beta_k (1 - \alpha_{st})(1 - \rho\epsilon) \bar{I}_{kst} y_{ks} \tag{50}$$

**Minimization of Deaths**

$$Minimize \quad \sum_{k=1}^{K} \sum_{s \in Seq} \sum_{t=1}^{T+2} ((1 - \lambda)\bar{C}_{kst} + \bar{s}_{2,kst}) y_{ks} \tag{51}$$

Subject to:

$$\sum_{s \in Seq} y_{ks} = 1 \qquad \forall k \in \{1, ..., K\} \tag{52}$$

$$\frac{1}{P.T} \sum_{k=1}^{K} \sum_{s \in Seq} p_k c_s y_{ks} \leqslant B \tag{53}$$

$$y_{ks} \in \{0, 1\} \qquad \forall s \in Seq, \quad k \in \{1, \ldots, K\} \tag{54}$$

The objective function (50) minimizes the sum of infections for the planning horizon, whereas objective (51) minimizes deaths. Constraint (52) implies that for each region $k$, only one sequence is assigned from the feasible set $Seq$. Similarly, Constraint (53) is the budget constraint, weighted by population, factoring in the NPI cost $c_s$ for each sequence $s$. We demonstrate the advantage of the sequence-based formulation over the time-based formulation in terms of computational efficiency in the next section.

# 6 Numerical analysis

In this section we discuss a case study based on the COVID-19 spread in the metropolitan region of France in Europe. We compare the containment models for both decentralised and centralised strategies by considering various lockdown constraints separately as scenarios ($S0$, $S1$, $S2$, as defined earlier). For the decentralised strategy, we study the epidemic spread in the 13 metropolitan regions of France. For our analysis, we have incorporated data from French national archives and databases for demographics and healthcare capacities. We consider population density, healthcare capacity in terms of regular beds, ICU beds, general practitioner for all 13 regions. Based on Salje et al. (2020), Di Domenico et al. (2020), we calibrate the various epidemic parameters pertaining to the regions and at a national level. Naturally, the decentralised model can be implemented at a finer granularity than the region level, e.g. at French "départements" or city level, however we chose a division of the country into regions since we had available data for the 13 regions, and the benefit of decentralisation is already shown with no ambiguity at this level.

## 6.1 Calibration of parameters

For quantifying the impact of various NPIs on the reduction of infection transmission and $R_0$, we resort to social contact matrices. Social contact matrices divide the population into age groups and define the per day contact between different age groups. Several empirical studies have been carried out in the past to record actual social contact patters in different countries. Prem et al. (2017), developed social contact matrices for 152 countries based on contact surveys and demographic data. Béraud et al. (2015) carried out one of the first large population contact based surveys for France and computed social contact matrices for modelling infection spread in the population.

Based on these studies we consider contact matrices for the French population for age groups (0-19, 20-39, 40-59, 60 -75, 75+) and modes of contacts such as home, school, work, other locations and calibrate the contacts for different NPIs. For consideration of reduction in contacts for different NPIs our assumptions are closely related to Di Domenico et al. (2020) and Davies et al. (2020). We report the final contact matrices considered for our analysis in Table 8 (See Appendix), in terms of resultant contacts after implementation of the corresponding level of NPI. For severity index of different NPIs, we have scaled the cost impacts of NPIs

on a scale of 0 to 10. Each of the levels correspond to the degree of combination of NPIs, from 1 to 4, with 5 indicating a Lockdown. Details of NPI calibration are explained in the Appendix. In Tables 9 and 10 (see Appendix) we report the epidemic parameter values for France, based on French government archives and empirical papers, as indicated.

In order to reduce the computational complexity for executing the models, we have not considered age specific variables. Instead, we have factored in weighted averages for the age-centric epidemic parameters and contact data. For $S1$ scenario we assume $l_1 = 0.5T$ and for $S2$ scenario we assume $l_2 = l_2' = 3$. For changeovers, we consider $l_0 = 0.2T$. For initiating the progression of the epidemic, we considered 0.004% of the population of each region to be infected already at the beginning of the planning horizon.

## 6.2 Gain of decentralization

The MILP and ILP models for decentralised and centralised strategies were executed on CPLEX 12.10 on Mac OSX, 8 GB RAM system. In Table 4 and Table 11 (Appendix), we report the results of the optimization model for minimization of infections and cumulative deaths respectively, for a time horizon of 8 and 10 weeks, three different severity budgets ($B = 5, 6.5, 8$), and for each scenarios $S0$, $S1$ and $S2$. Not surprisingly given its higher flexibility, the decentralised strategy consistently exhibits a better objective function value in all scenarios and instances compared to the centralised strategy, with a reduction in infections up to 20% and reduction in deaths up to 15%. We can also observe that the objective function values follow the relationship in all instances: $S0 \leqslant S1 \leqslant S2$. This is logical because in scenarios $S1$ and $S2$ we add restrictions on lockdowns which results in more infections and deaths in comparison to $S0$.

**Analysis of deaths due to ICU bed shortage**

In Table 5, we depict a particular instance, where we assume a skewed spread of initial infections, considering regions with higher population ($> 5$ millions) have a higher initial infections to population ratio in comparison to the rest. We consider the objective of minimization of deaths, for T=8, B=4 in this case. The table indicates the peak ICU demand to capacity ratio for the 8 weeks at a regional level. When it is more than 100%, it implies that there was a shortage in ICU beds, which resulted in as many deaths. In terms of NPI allocation, for the centralised strategy, we can see that every region gets the same allocation of severity index. As a result, there is a very high skew in demand to capacity ratio, as high as 231% for Ile-de-France, whereas in the decentralised model, the ratios stay within 150%. This leads to a much higher level of shortage in ICU beds in the centralised strategy, and hence more deaths. The total deaths due to shortage in the decentralised strategy is 1096, in comparison to 2704 for the centralised strategy, 60% lesser. The weighted peak demand to capacity ratio is 148.2% for Centralised and 119 % for Decentralised strategy. In terms of allocation of NPIs, if we observe Bretagne and Normandie regions, both have the same population of 3.3 millions, however, Bretagne's severity index of NPI is 3.4, in comparison to 2.62 for Normandie. This can be explained by the fact that, Bretagne has a population density of 122.8 people per square kilometres and an ICU density of 6.9 beds per 1000 people, in comparison to Normandie's population density of 111.6 people per square kilometres and ICU density of 8.3 beds per 1000 people. Population density directly impacts the infection spread, hence a higher population density coupled with lower ICU density indicates a greater risk, necessitating more stringent measures for safeguarding the sanitary situation. Hence, these kind of allocations at the regional level in the decentralised model result in a better management of the pandemic, with lesser infections and deaths, for the same economic loss, as compared to the centralised strategy.

## 6.3 Gain of sequence-based model

In Table 6, we report the number $|Seq|$ of sequences generated for the ILP model compliant with Proposition 1 for scenarios $S0, S1$, and with proposition 2 for scenario $S2$. the table shows there is a reduction of 99% from the total number $5^T$ of possible sequences. In Table 7, we compare the computational efficiency between the time-based (MILP) and sequence-based (ILP) formulations. We can check that both formulations yield the exact same objective value for infections. However, there is a drastic reduction in CPU time, as high as 99.7% in comparison to the time based MILP formulation.

## 6.4 Sensitivity analysis

In Figure 2, we highlight the optimal NPI sequences for scenarios $S0$ for both objectives of infections and deaths minimization, for decentralised and centralised strategies respectively. We observe that the optimal NPI sequences for the two objectives are different. For the minimization of deaths, the total number of lockdowns across regions are greater with a more drastic shift to lower severity NPIs towards the end of the planning horizon, in comparison to the minimization of infections. In Figure 3, we demonstrate the evolution of infections for scenarios $S0$ and $S2$ for the centralised strategy, performing a sensitivity analysis on NPI budget $B$ for $T = 14$ weeks. It can be clearly seen that the infections explode exponentially with a fall in budget $B$. An unlimited budget would naturally allow continuous lockdowns over the $T$ weeks, which would not be accepted by the population for a large $T$. Also, for $S2$, the pattern follows alternating rises and falls, owing to the consecutive lockdown condition, with higher peaks and troughs in comparison to $S0$. Finally, in Table 12 we perform a sensitivity analysis based on $\alpha$, the reduction factor, by varying it in a range $[-20\%, +20\%]$ of the estimated values. We test it for scenario $S0$, three budgets $B \in \{5, 6.5, 8\}$ and time horizon $T = 8$ weeks. These three budgets $B = 5, 6.5$ and 8 correspond to different values of average severity per individual, between Level 4 corresponding to $B = 5$, and Level 5 corresponding to $B = 10$ (the cost is non-linear with the NPI level; see Appendix C for more details on the scaling of severity costs $c_i$ and calibration of coefficients $\alpha_i$ used in the calculus of $c_i = \alpha_i^2$). We observe that the decentralised strategy still performs better for minimization of infections (up to 10%).

Figure 2: Differentiated NPI decisions across regions, $(S0$, B=6.5, T=10$)$

**Decentralised Model — Minimize Infections ( Inf*)**

| Pop | Region | W1 | W2 | W3 | W4 | W5 | W6 | W7 | W8 | W9 | W10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | Corse | 5 | 5 | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 |
| 2.8 | Bourgogne-Franche-Comté | 5 | 5 | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 |
| 6.0 | Nouvelle-Aquitaine | 5 | 5 | 5 | 5 | 5 | 3 | 3 | 3 | 2 | 2 |
| 5.9 | Occitanie | 5 | 5 | 5 | 5 | 5 | 3 | 3 | 3 | 2 | 2 |
| 2.6 | Centre-Val-de-Loire | 5 | 5 | 5 | 5 | 5 | 5 | 3 | 3 | 1 | 1 |
| 5.5 | Grand Est | 5 | 5 | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 |
| 8.0 | Auvergne-Rhône-Alpes | 5 | 5 | 5 | 5 | 5 | 3 | 3 | 3 | 2 | 2 |
| 3.3 | Normandie | 5 | 5 | 5 | 5 | 5 | 3 | 3 | 2 | 2 | 2 |
| 3.3 | Bretagne | 5 | 5 | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 |
| 3.8 | Pays de la Loire | 5 | 5 | 5 | 5 | 5 | 3 | 3 | 3 | 2 | 2 |
| 5.1 | Provence-Alpes-Côte d'Azur | 5 | 5 | 5 | 5 | 5 | 5 | 3 | 3 | 1 | 1 |
| 6.0 | Hauts-de-France | 5 | 5 | 5 | 5 | 5 | 5 | 3 | 3 | 1 | 1 |
| 12.3 | Île-de-France | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 2 | 2 | 2 |

**Centralised Model**

| Pop | Country | W1 | W2 | W3 | W4 | W5 | W6 | W7 | W8 | W9 | W10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 64.9 | France | 5 | 5 | 5 | 5 | 5 | 4 | 4 | 2 | 2 | 2 |

| Scenario S0 | | Centralised | | | Decentralised | | | Variation | |
|---|---|---|---|---|---|---|---|---|---|
| B | T | Inf* | Deaths | CPU Time (s) | Inf | Deaths | CPU Time (s) | I (Δ) | D (Δ) |
| 6.5 | 10 | 10906 | 308 | 0.84 | 10221 | 293 | 65.31 | -6% | -5% |

**Decentralised Model — Minimize Deaths ( Deaths*)**

| Pop | Region | W1 | W2 | W3 | W4 | W5 | W6 | W7 | W8 | W9 | W10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | Corse | 5 | 5 | 5 | 5 | 5 | 3 | 3 | 1 | 1 | 1 |
| 2.8 | Bourgogne-Franche-Comté | 5 | 5 | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 |
| 6.0 | Nouvelle-Aquitaine | 5 | 5 | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 |
| 5.9 | Occitanie | 5 | 5 | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 |
| 2.6 | Centre-Val-de-Loire | 5 | 5 | 5 | 5 | 5 | 5 | 3 | 3 | 1 | 1 |
| 5.5 | Grand Est | 5 | 5 | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 |
| 8.0 | Auvergne-Rhône-Alpes | 5 | 5 | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 |
| 3.3 | Normandie | 5 | 5 | 5 | 5 | 5 | 3 | 3 | 1 | 1 | 1 |
| 3.3 | Bretagne | 5 | 5 | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 |
| 3.8 | Pays de la Loire | 5 | 5 | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 |
| 5.1 | Provence-Alpes-Côte d'Azur | 5 | 5 | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 |
| 6.0 | Hauts-de-France | 5 | 5 | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 |
| 12.3 | Île-de-France | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 1 | 1 | 1 |

**Centralised Model**

| Pop | Country | W1 | W2 | W3 | W4 | W5 | W6 | W7 | W8 | W9 | W10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 64.9 | France | 5 | 5 | 5 | 5 | 5 | 5 | 2 | 2 | 1 | 1 |

| Scenario S0 | | Centralised | | | Decentralised | | | Variation | |
|---|---|---|---|---|---|---|---|---|---|
| B | T | Inf* | Deaths | CPU Time (s) | Inf | Deaths | CPU Time (s) | I (Δ) | D (Δ) |
| 6.5 | 10 | 11310 | 307 | 0.73 | 10291 | 291 | 65.97 | -9% | -5% |

Table 4: Gain of Decentralisation for Inf* (* = Optimization Criterion)

| Scenario $S0$ | | Centralised | | | Decentralised | | | Variation | |
|---|---|---|---|---|---|---|---|---|---|
| **B** | **T** | **#Inf*** | **#Deaths** | **Time(s)** | **#Inf*** | **#Deaths** | **Time(s)** | **Δ Inf** | **Δ Deaths** |
| 5 | 8 | 19370 | 473 | 0.55 | 18036 | 439 | 6.2 | -7% | -7% |
| 6.5 | 8 | 10826 | 290 | 0.6 | 9819 | 276 | 6.4 | -9% | -5% |
| 8 | 8 | 7146 | 223 | 0.49 | 7008 | 222 | 6.4 | -2% | -0.4% |
| 5 | 10 | 25746 | 621 | 1.12 | 22251 | 553 | 66.5 | -14% | -11% |
| 6.5 | 10 | 10906 | 308 | 0.84 | 10221 | 293 | 65.3 | -6% | -5% |
| 8 | 10 | 7432 | 238 | 0.83 | 7038 | 230 | 65.3 | -5% | -3% |
| Scenario $S1$ | | Centralised | | | Decentralised | | | Variation | |
| 5 | 8 | 19370 | 473 | 0.78 | 18036 | 439 | 6.4 | -7% | -7% |
| 6.5 | 8 | 10826 | 290 | 0.53 | 10028 | 284 | 6.2 | -7% | -2% |
| 8 | 8 | 8273 | 253 | 0.92 | 8273 | 253 | 6.0 | 0% | 0% |
| 5 | 10 | 25746 | 621 | 2.49 | 22251 | 553 | 66.5 | -14% | -11% |
| 6.5 | 10 | 10906 | 308 | 1.09 | 10537 | 303 | 68.2 | -3% | -1.6% |
| 8 | 10 | 8499 | 268 | 0.98 | 8499 | 268 | 67.2 | 0% | 0% |
| Scenario $S2$ | | Centralised | | | Decentralised | | | Variation | |
| 5 | 8 | 19370 | 473 | 0.73 | 18244 | 444 | 10.9 | -6% | -6% |
| 6.5 | 8 | 11591 | 321 | 0.8 | 10821 | 310 | 11.2 | -7% | -3% |
| 8 | 8 | 10219 | 301 | 0.88 | 8747 | 279 | 11.0 | -14% | -7% |
| 5 | 10 | 28736 | 688 | 2.37 | 23226 | 591 | 222.3 | -19% | -14% |
| 6.5 | 10 | 15103 | 420 | 1.41 | 12608 | 374 | 225.2 | -17% | -11% |
| 8 | 10 | 11236 | 362 | 1.07 | 9780 | 321 | 223.1 | -13% | -11.3% |

# 7 Managerial insights

Based on the output of the optimization model, we deduce some key takeaways for policymakers. Although there is a degree of ambiguity surrounding the epidemic parameter values and the exact economic and health impact of NPIs, we can identify some broad trends.

- The first key finding is with respect to the optimal sequence of NPIs. For both the decentralised and centralised strategies, the NPIs follow a pattern of decreasing severity across the planning horizon. This indicates that the policymaker needs to impose heavier restrictions in the beginning, in order to check the transmission in the population and utilise that time to ramp up healthcare capacity to prevent a health crisis. Initially, a few countries like UK had contemplated implementing the herd immunity strategy which would allow a sizeable proportion (around 70%) of the population to be infected and develop antibodies against it. However, the sharp increase in hospitalizations and overwhelming of healthcare capacity did not allow them to go ahead with it. Hence, the optimal strategy for controlling COVID-19 would be to put heavier NPIs in the beginning followed by less severe ones, for minimizing infections or deaths within a given budget constraint.

- The second takeaway is the benefit of a decentralised strategy. Based on our numerical analysis of the French regions, we observe that the decentralised strategy yields better results for all instances, with up to 20% lesser infections and 15% lesser deaths. It is natural that the objective value improves when giving more flexibility to the model, but the gain appears to be quite substantial here. This is an important consideration for policymaking in a pandemic, where vaccine development takes time and NPIs need to be sustained over a longer horizon to keep the infection transmission at a manageable level. Exhaustive confinement and lockdowns at a national level leads to social fatigue and is economically unsustainable. Given these limitations, it is pragmatic for the policymaker to follow a more customised strategy at a regional/ state/ city/ district level, considering the demographics, healthcare capacity

Table 5: Heterogeneity of NPI severity & ICU shortage across regions (Scenario $S0$, B=4, T=8, Deaths*)

| Regional Data | | | | | Peak Demand/Cap | | #Deaths (Shortage) | | Severity | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Pop.D** | **ICU.D** | **Pop** | **Region** | **Cap.** | **Cent.** | **Decent.** | **Cent.** | **Decent.** | **Cent.** | **Decent.** |
| 39.7 | 6.1 | 0.3 | Corse | 16 | 88% | 138% | 0 | 6 | 3.98 | 3.25 |
| 58.2 | 11.5 | 2.8 | Bourgogne-Franche-Comté | 240 | 59% | 106% | 0 | 14 | 3.98 | 2.95 |
| 71.4 | 7.9 | 6.0 | Nouvelle-Aquitaine | 400 | 141% | 140% | 162 | 190 | 3.98 | 3.85 |
| 81.5 | 10.0 | 5.9 | Occitanie | 480 | 116% | 116% | 79 | 75 | 3.98 | 3.85 |
| 65.4 | 6.8 | 2.6 | Centre-Val-de-Loire | 160 | 88% | 119% | 0 | 31 | 3.98 | 3.40 |
| 96.0 | 7.6 | 5.5 | Grand Est | 320 | 173% | 120% | 262 | 63 | 3.98 | 4.45 |
| 115.2 | 7.1 | 8.0 | Auvergne-Rhône-Alpes | 480 | 151% | 150% | 246 | 321 | 3.98 | 3.85 |
| 111.6 | 8.3 | 3.3 | Normandie | 240 | 57% | 123% | 0 | 55 | 3.98 | **2.62** |
| 122.8 | 6.9 | 3.3 | Bretagne | 160 | 101% | 138% | 2 | 60 | 3.98 | 3.40 |
| 118.5 | 7.5 | 3.8 | Pays de la Loire | 240 | 75% | 114% | 0 | 33 | 3.98 | 3.25 |
| 161.0 | 7.9 | 5.1 | Provence-Alpes-Côte d'Azur | 320 | 173% | 118% | 250 | 59 | 3.98 | 4.45 |
| 188.0 | 11.2 | 6.0 | Hauts-de-France | 560 | 121% | 83% | 119 | 0 | 3.98 | 4.45 |
| 1022.2 | 10.2 | 12.3 | Île-de-France | 1140 | 231% | 118% | 1584 | 189 | 3.98 | **4.79** |
| D= Density, Pop = Population (in millions) | | | | **4756** | **148.2%** | **119%** | **2704** | **1096** | **3.98** | **4.0** |

Table 6: Number of sequences generated for scenarios $S0$,$S1$,$S2$

| Time | #Total Sequences | #Feasible Sequences in $Seq$ | | |
|---|---|---|---|---|
| $T$ | **Total** $= 5^T$ | $S0$ | $S1$ | $S2$ |
| 8 | 390625 | 115 | 106 | 100 |
| 10 | 9765625 | 225 | 206 | 180 |

and the transmission dynamics observed initially. We conclude that a decentralised strategy not only results in better performance in terms of epidemic control but also aids in mobilising the economy wherever feasible. This conclusion holds regardless of the practical difficulty to manage differentiated measures from one region to another, but actually this geographical differentiation has already been implemented in France (in Ile-de-France region, the city of Nice, the "département" of Moselle, to cite a few examples), and in other countries.

- Finally, a very crucial aspect of pandemic management is to identify capacity risks and bottlenecks and take proactive measures. Our analysis of the decentralised and centralised strategies considering shortages in ICU beds, reveals how the former leads to a better allocation of NPIs, resulting in lesser shortages (up to 60 %) and consequently lesser deaths. This policymaking tool allows the policy think tank to identify capacity risks based on various scenarios and increase capacities accordingly to avoid an unfortunate overwhelming of the healthcare system.

# 8 Conclusion

In this paper, we develop an epidemic optimization model for controlling COVID-19 infections in a given population, considering budget constraints on the set of NPIs decided by the policymaker, and healthcare resource shortages. The classical SIR epidemic compartmental model of Kermack and McKendrick (1932) is incorporated with certain extensions in order to capture the dynamics of COVID-19. We factor in healthcare resource shortages in terms of doctors, ICU beds, regular beds. We linearize the initial MINLP time-based model, and then use a (proven) property of decreasing severity of NPIs over time to convert it into a 'sequence-based' ILP model, which can be solved in a much faster way (time reduction of up to 99%).

Table 7: Computational gain of the Sequence-based model (Inf*)

| Scenario $S0$ | | Time based | | Sequence based | | CPU Time (s) | | |
|---|---|---|---|---|---|---|---|---|
| **B** | **T** | **#Inf*** | **#Deaths** | **#Inf*** | **#Deaths** | **Time based** | **Sequence based** | **Reduction ($\Delta$)** |
| 7 | 6 | 8109 | 228 | 8109 | 228 | 11.4 | 3.32 | -70.9% |
| 6.5 | 6 | 9076 | 248 | 9076 | 248 | 44.9 | 3.29 | -92.7% |
| 6 | 6 | 10372 | 270 | 10372 | 270 | 1081.3 | 3.3 | -99.7% |
| 7 | 7 | 8341 | 242 | 8341 | 242 | 11.18 | 6.23 | -44.3% |
| 8 | 8 | 7008 | 222 | 7008 | 222 | 3814.2 | 11 | -99.7% |
| 8.5 | 10 | 6630 | 222 | 6630 | 222 | 1820.1 | 295.1 | -83.8% |

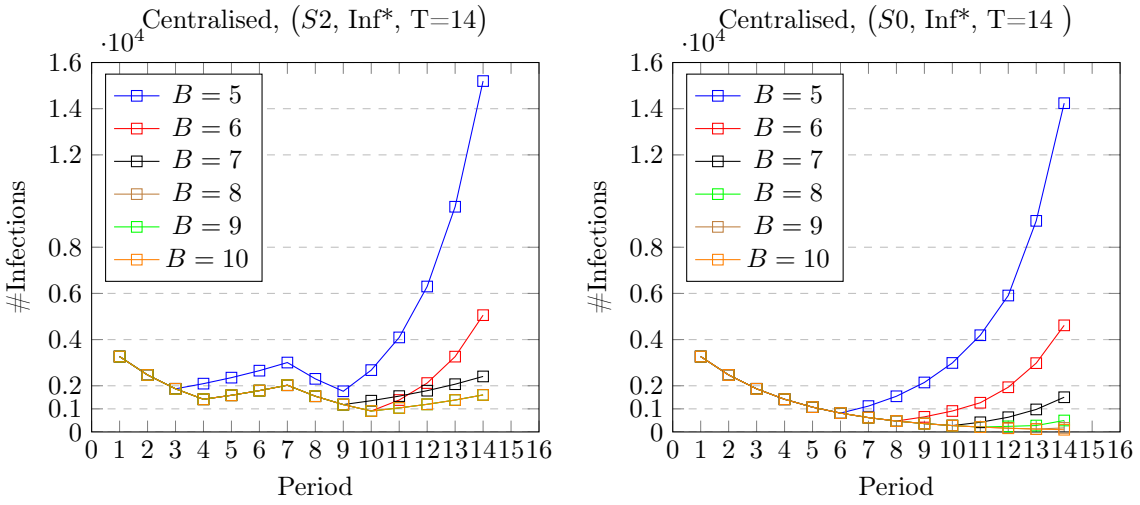


Figure 3: Evolution of the optimal number of infections with budget $B$

Two strategies are analysed - centralised and decentralised, where NPIs are modelled for the entire country for the former, and at a regional level for the latter. We use social contact matrices from empirical studies and actual epidemic and demographic data from French archives for developing a case study on the spread of COVID-19 in France. Our results indicate that the decentralised model performs better in terms of lesser infections (up to 20%), lesser deaths (up to 15 %), lesser shortages (up to 60 %) for a given budget. Despite the evolving nature of the pandemic and the uncertainty around data, these findings offer some direction to the policymaker for implementing NPIs optimally, weighting both the sanitary and economic impact of their decisions. We believe that this analysis will be valuable not only from the perspective of COVID-19, but also for managing future occurrences of epidemics of any kind, by leveraging some of the insights as rule of thumb. In terms of limitations of our research, although our analysis establishes the decentralized strategy to be better, compliance of populations with too many differential restrictions might pose challenges to the administration. Our motivation is to demonstrate the benefits of a decentralized strategy from a scientific standpoint, and the solutions obtained do not show too chaotic patterns across regions in any case.

We suggest some further extensions to this research. First, application of machine learning methods in combination with mathematical programming can be used for a more data driven approach to policy making, where an adaptive rolling plan of NPIs can be modelled based on population compliance data. Data obtained from the first wave of the pandemic can be used to train algorithms to predict compliance of the population to NPIs in subsequent waves and build more robust, predictive models for policymaking. Second, capacity

increase, facility location and layout, are some of the other dimensions of the epidemic control problem which can be explored. Third, for the sequence based model, column generation can be implemented when the number of various NPIs is large, in order to tackle the explosion of sequences.

Finally, this paper restricts to non-pharmaceutical interventions at the early stage of the spread a new virus like COVID-19, before efficient vaccines can be found. An extension to the MILP model can be made for considering vaccine administration along with NPIs simultaneously. A considerable number of approved vaccines have emerged since December, 2020 for inoculating the global population against COVID-19, namely Pfizer/BioNtech, Moderna, AstraZeneca, Johnson and Johnson, Sputnik V (Healthdata 2021). One could modify the objective function applying to the rate of infections a discount factor based on the percentage of an age group vaccinated with dose 1 and dose 2 at each week and region, along with the effectiveness of each dose (% of people getting sufficient immunity by dose 1 and 2). Older age groups should be prioritized ideally for preventing criticalities. The quantity of each dose made available at each time period in each region could be modelled as a supply chain problem, considering lead time, capacity, logistics and budget constraints. A new decision model could then indicate the optimal vaccine production and distribution plan along with the optimal NPI sequence.

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# Appendices

# Appendix A: Proof of model properties

### Proof of Lemma 1

*Proof.* Let $x$ be a feasible solution. Let us fix region $k$, and assume we swap two NPIs between time periods $t_1$ and $t_2$ for that region, such that $1 \leqslant t_1 \leqslant t_2 \leqslant T$. We skip index $k$ in the sequel and to ease the reading, by a slight abuse of notation we note $\alpha_t$ the reduction factor at week $t$ in solution $x$, i.e. the $\alpha_i$ such that $x_{kit} = 1$. So $\alpha_{t_1}$ and $\alpha_{t_2}$ are the reduction factors at weeks $t_1$ and $t_2$ in $x$ before the swap. Note that the swap that does not change the budget consumption, so the swapped solution remains feasible.

The infected state equation for the centralised model is given by:

$$I_{k,t+1} = I_{kt} + \beta_k \sum_{i \in N}((1-\alpha_i)(1-\rho\epsilon)x_{it}I_{kt} - (\rho\phi + \mu(1-\rho))I_{kt} \qquad \forall t \in \{1,...,T\}$$

which we can rewrite as:

$$I_{t+1} = I_t + \Omega_1(1-\alpha_t)I_t - \Omega_2 I_t \tag{55}$$

where $\Omega_1 = \beta(1-\rho\epsilon)$ and $\Omega_2 = (\rho\phi + \mu(1-\rho))$. Hence, we can write the general term of the state equation as follows:

$$I_{t+1} = I_1 \prod_{t'=1}^{t}(1 + \Omega_1(1-\alpha_{t'}) - \Omega_2) \tag{56}$$

Since $I_{t_2+1}$ is a product of the constant terms comprising the reduction factors from 1 to $t_2$, swapping NPIs between $t_1$ and $t_2$, keeps the overall product the same. Therefore, $I_{t_2+1}$ is the same for the original and swapped case. Additionally, we can say that for the period $T+1$, the infection level will be same for the swapped and original solutions. Since it is true for any region $k$, this concludes the proof of the lemma. □

### Proof of Lemma 2: Minimization of Infections

*Proof.* Let us start with some feasible solution $x$ and as in the proof of Lemma 1 we fix region $k$ and omit the index $k$ in the notation. Let us assume that we swap two NPIs between time periods $t_1$ and $t_2$, such that $1 \leqslant t_1 \leqslant t_2 \leqslant T$, with $\alpha_{t_1} < \alpha_{t_2}$ (same notations as in the proof of Lemma 1).

The unique infections generated each period $t$ is given by $\Omega_1(1-\alpha_t)I_t$. We define the set of infection state equations for the original solution and the swapped solution, by $I_t$ and $I_t^s$ respectively. For the original solution, the equations for $t_1+1$ and $t_2+1$ can be derived from (55)

Now, the swapped infection state equations are given by:

$$I^s_{t_1+1} = I_{t_1} + \Omega_1(1-\alpha_{t_2})I_{t_1} - \Omega_2 I_{t_1}$$

$$I^s_{t_2+1} = I^s_{t_2} + \Omega_1(1-\alpha_{t_1})I^s_{t_2} - \Omega_2 I^s_{t_2}$$

For period $t_1+1$, the relationship between the original and swapped equation is given by

$$I^s_{t_1+1} - I_{t_1+1} = \Omega_1(\alpha_{t_1} - \alpha_{t_2})I_{t_1} = \Delta_1 \tag{57}$$

For the general term, we define

$$\Delta_n = I^s_{t_1+n} - I_{t_1+n}$$

By induction we have for $n \geq 2$:

$$\Delta_n = \Omega_1(\alpha_{t_1} - \alpha_{t_2})I_{t_1} \prod_{t=t_1+1}^{t_1+n-1} (1 + \Omega_1(1-\alpha_t) - \Omega_2)$$

Now, using (55), we get:

$$\Delta_n = \Omega_1(\alpha_{t_1} - \alpha_{t_2})I_{t_1} \prod_{t=t_1+1}^{t_1+n-1} \frac{I_{t+1}}{I_t} \tag{58}$$

$$= \Omega_1(\alpha_{t_1} - \alpha_{t_2})I_{t_1} \left(\frac{I_{t_1+n}}{I_{t_1+1}}\right) \tag{59}$$

We now compare the sum of infections between $t_1$ and $t_2$, because $t_2+1$ onwards the sum is the same for both the original and swapped cases (Lemma 1) and before $t_1$, there is no swap, hence they are identical. For the original and swapped solutions, the sum of unique infections between period $t_1$ and $t_2$ are respectively given by:

$$U = \sum_{t=t_1}^{t_2} \Omega_1(1-\alpha_t)I_t$$

$$U^s = \Omega_1(1-\alpha_{t_2})I_{t_1} + \sum_{t=t_1+1}^{t_2-1} \Omega_1(1-\alpha_t)I^s_t + \Omega_1(1-\alpha_{t_1})I^s_{t_2}$$

Hence,

$$
\begin{aligned}
U^s - U &= \Omega_1 \Bigg[ (\alpha_{t_1} - \alpha_{t_2}) I_{t_1} + \sum_{t=t_1+1}^{t_2-1} (1-\alpha_t)(I_t^s - I_t) + (1-\alpha_{t_1}) I_{t_2}^s - (1-\alpha_{t_2}) I_{t_2} \Bigg] \\
&= \Omega_1 \Bigg[ (\alpha_{t_1} - \alpha_{t_2}) I_{t_1} + \sum_{n=1}^{t_2-t_1-1} (1-\alpha_{t_1+n}) \Delta_n + (1-\alpha_{t_1})(I_{t_2} + \Delta_{t_2-t_1}) - (1-\alpha_{t_2}) I_{t_2} \Bigg] \\
&= \Omega_1 \Bigg[ (\alpha_{t_1} - \alpha_{t_2})(I_{t_1} - I_{t_2}) + \sum_{n=1}^{t_2-t_1-1} \Big( (1-\alpha_{t_1+n}) \Delta_n \Big) + (1-\alpha_{t_1}) \Delta_{t_2-t_1} \Bigg]
\end{aligned}
$$

using $I_{t_1+n}^s - I_{t_1+n} = \Delta_n$ from (8). Making the substitution $\Omega_1(1-\alpha_{t_1+n}) = \frac{I_{t_1+n+1}}{I_{t_1+n}} - 1 + \Omega_2$ and using the expression (59) of $\Delta_n$ we get:

$$
(1-\alpha_{t_1+n})\Delta_n = \left( \frac{I_{t_1+n+1}}{I_{t_1+n}} - 1 + \Omega_2 \right) (\alpha_{t_1} - \alpha_{t_2}) I_{t_1} \frac{I_{t_1+n}}{I_{t_1+1}}
$$

$$
\begin{aligned}
U^s - U &= \Omega_1 \Bigg[ (\alpha_{t_1} - \alpha_{t_2})(I_{t_1} - I_{t_2}) + \sum_{n=1}^{t_2-t_1-1} \left( \big( \frac{I_{t_1+n+1}}{I_{t_1+n}} - 1 + \Omega_2 \big) (\alpha_{t_1} - \alpha_{t_2}) I_{t_1} \frac{I_{t_1+n}}{I_{t_1+1}} \right) + \big( \frac{I_{t_1+1}}{I_{t_1}} - 1 + \Omega_2 \big) (\alpha_{t_1} - \alpha_{t_2}) I_{t_1} \frac{I_t}{I_{t_1}} \\
&= \Omega_1 (\alpha_{t_1} - \alpha_{t_2}) \Bigg[ (I_{t_1} - I_{t_2}) + \sum_{n=1}^{t_2-t_1-1} \left( \big( \frac{I_{t_1+n+1}}{I_{t_1+n}} - 1 + \Omega_2 \big) I_{t_1} \frac{I_{t_1+n}}{I_{t_1+1}} \right) + \big( \frac{I_{t_1+1}}{I_{t_1}} - 1 + \Omega_2 \big) I_{t_1} \frac{I_{t_2}}{I_{t_1+1}} \Bigg] \\
&= \Omega_1 (\alpha_{t_1} - \alpha_{t_2}) \Bigg[ I_{t_1} + I_{t_1} \sum_{n=1}^{t_2-t_1-1} \left( \frac{I_{t_1+n+1} - I_{t_1+n}}{I_{t_1+1}} + \Omega_2 \frac{I_{t_1+n}}{I_{t_1+1}} \right) + \big( \Omega_2 - 1 \big) I_{t_1} \frac{I_{t_2}}{I_{t_1+1}} \Bigg] \\
&= \Omega_1 (\alpha_{t_1} - \alpha_{t_2}) \Bigg[ I_{t_1} + I_{t_1} \frac{I_{t_2} - I_{t_1+1}}{I_{t_1+1}} + I_{t_1} \sum_{n=1}^{t_2-t_1-1} \left( \Omega_2 \frac{I_{t_1+n}}{I_{t_1+1}} \right) + \big( \Omega_2 - 1 \big) I_{t_1} \frac{I_{t_2}}{I_{t_1+1}} \Bigg] \\
&= \Omega_1 (\alpha_{t_1} - \alpha_{t_2}) \Bigg[ I_{t_1} \sum_{n=1}^{t_2-t_1-1} \left( \Omega_2 \frac{I_{t_1+n}}{I_{t_1+1}} \right) + \Omega_2 I_{t_1} \frac{I_{t_2}}{I_{t_1+1}} \Bigg] \\
&= \Omega_1 (\alpha_{t_1} - \alpha_{t_2}) \Bigg[ I_{t_1} \Omega_2 \Big( 1 + \frac{\sum_{t=t_1+2}^{t_2} I_t}{I_{t_1+1}} \Big) \Bigg]
\end{aligned}
$$

Now, $(\alpha_{t_1} - \alpha_{t_2}) < 0$, and $I_{t_1} \Omega_2 \Big[ 1 + \frac{\sum_{t=t_1+2}^{t_1} I_t}{I_{t_1+1}} \Big] > 0$.

Therefore, $U^s < U$ when there is a swap of NPIs between $t_1$ and $t_2$ and $\alpha_{t_1} < \alpha_{t_2}$, which means the sum of total infections is strictly lower for the swapped case. This concludes the proof of the lemma.
□

**Proof of Proposition 1 (minimization of Infections)**

*Proof.* (i) For Scenario $S0$, the proof is a direct corollary from Lemma 2. A solution $x$ with $x_{k,i_1,t_1} = 1$ and $x_{k,i_2,t_2} = 1$ for some region $k$ with $t_1 < t_2$ and $\alpha_{t_1} < \alpha_{t_2}$ cannot be optimal. Hence, for all $(t_1, t_2)$, $t_1 < t_2$, we have $\alpha_{t_1} \geq \alpha_{t_2}$ for each region (or for the centralized model which corresponds to having a single index $k$). Therefore, the optimal sequence of NPIs always follows an ordered sequence from higher to lower severity, for both decentralised and centralised strategies.
(ii) For scenario $S1$, we restrict the total number of weeks of lockdown to $l_1$. For a given budget $B$, let $\mathbf{X}_B^0$ and $\mathbf{X}_B^1$ denote the set of feasible solutions for scenarios $S0$ and $S1$ respectively. We have $\mathbf{X}_B^1 \subseteq \mathbf{X}_B^0$.

Therefore, since Proposition 1 is true for any solution in $\mathbf{X}_B^0$ , then it is also true for any solution in $\mathbf{X}_B^1$.
□

**Proof of Proposition 1 (minimization of Deaths)**

*Proof.* The state equation for Critical can be written as:

$$\begin{aligned} C_{t+1} &= (1-\gamma)\lambda C_t + \rho\phi I_t \quad + \quad Q_{0,t} - s_{2,t} \\ &= (1-\gamma)\lambda C_t + \rho\phi I_t \quad + \quad \max(m(\max(\frac{C_t}{m} - H_t, 0)), \max((1-\theta)C_t - (1-\eta_{3,t})b_{3,t}, 0)) \\ &\qquad\qquad\qquad\quad - \quad \max(\theta C_t - (1-\eta_{2,t})b_{2,t}, 0) \end{aligned}$$

the last two terms of the sum corresponding to the maximum between doctor and non-ICU shortages, and to the shortage in ICU beds, respectively. The cumulative deaths at period $t+1$ can be written as:

$$D_{t+1} = (1-\gamma)\lambda C_t + \max(\theta C_t - (1-\eta_{2,t})b_{2,t}, 0)$$

Hence, the objective function for minimization of deaths we can rewritten as:

$$D_{T+3} = \sum_{t=1}^{T+2} \Big( (1-\gamma)\lambda C_t + \max(\theta C_t - (1-\eta_{2,t})b_{2,t}, 0) \Big) \tag{60}$$

Now let us start with some feasible solution $x$ and assume that for some region $k$ we swap two NPIs between time periods $t_1$ and $t_2$, such that $1 \leqslant t_1 \leqslant t_2 \leqslant T$ where $\alpha_{t_1} < \alpha_{t_2}$.

From the above recursive definition of $C_t$, we have that $C_t$ is an increasing function of vector $(I_1, \ldots, I_{t-1})$ associated with solution $x$. So the objective function (60) is also an increasing function of vector $(I_1, \ldots, I_{T+1})$. Therefore in order to prove the Proposition, it is sufficient to show that the swap between $t_1$ and $t_2$ gets a vector of infections $(I_1, \ldots, I_{t_1-1}, I_{t_1}^s \ldots, I_{T+1}^s)$ that is less than the initial vector $(I_1, \ldots, I_{T+1})$. This holds because:

- $I_{t_1+1}^s < I_{t_1+1}$ since $(1 + \Omega_1(1-\alpha_{t_2}) - \Omega_2)I_{t_1} < (1 + \Omega_1(1-\alpha_{t_1}) - \Omega_2)I_{t_1}$ with $\alpha_{t_1} < \alpha_{t_2}$
- $I_t^s < I_t$ for $t = t_1 + 1, \ldots, t_2$, similarly,
- $I_{t_2+1}^s = I_{t_2+1}$, due to Lemma 1, and the equality also holds for all weeks $t$ after the swap.

. We deduce that $I_t^s \leq I_t$ for each $t = 1, \ldots, T+1$ with a strict inequality $I_t^s < I_t$ for $t = t_1 + 1, \ldots, t_2$, so the objective (60) strictly decreases with the swap. Therefore, the solution we started out with cannot be optimal, and an optimal solution necessarily satisfies for each region $k$ (or for the centralized model): $\alpha_{t_1} \geq \alpha_{t_2}$ for $t_1 < t_2$, i.e. it follows a decreasing severity pattern. The same can be established for scenarios $S1$ and $S2$ scenarios using the same arguments as in the proof for minimization of infections.
□

**Proof of Proposition 2**

*Proof.* For the $S2$ scenario, we restrict the total number of consecutive weeks of lockdowns to $l_2$ and the minimum gap between two blocks of lockdowns to be $l_2'$ weeks. Let $x^*$ be an optimal solution for $S2$ for a budget $B$. Solution $x^*$ is necessarily composed of sub-sequences of NPIs such that each first NPI of the sub-sequence is a lockdown and at most $l_2$ weeks of lock-downs are placed at the beginning of each sub-sequence, followed by non-lockdown NPIs to complete the sub-sequence. If there is a sub-sequence of $x^*$ (except the

last one) with strictly less than $l_2$ lockdowns, then it cannot be optimal as swapping the first non-lockdown NPI of the sub-sequence and a lock-down of the next sub-sequence would strictly improve the objective value (since $\alpha_5 > \alpha_i$, $i \in \{1,2,3,4\}$), based on Proposition 1. So each sub-sequence but the last one contains exactly $l_2$ weeks of lockdowns. Furthermore, if the gap between two consecutive blocks of lockdowns were greater than $l_2'$ weeks, then the solution could be improved again by swapping the non-lockdown NPI at position $l_2' + 1$ after the lockdown block, and a lockdown of next sub-sequence, using again Proposition 1. Therefore an optimal solution $x^*$ is composed of $L$ sub-sequences such that each sub-sequence $q = 1, \ldots, L-1$ is composed of a block of exactly $l_2$ weeks of lockdowns followed by exactly $l_2'$ non-lockdown NPIs, and the last sub-sequence $q = L$ has a a block of at most $l_2$ weeks of lockdowns followed by at least $l_2'$ weeks of non-lockdown NPIs. Now let us suppose that there exists two subsequences $q$ and $q'$ ($q < q'$) with some non-lockdown NPI in sub-sequence $q$ being less effective (lower coefficient $\alpha$) than some non-lockdown NPI in $q'$. Again by Proposition 1 making the swap would stricty improve the value of the solution. Hence the non-lockdown NPIs in $x^*$ are necessarily in a non-increasing order of severity.
□

# Appendix B: Additional numerical tables

Table 8: Social Contact matrices for NPIs and Cost Impact

| Social Distancing (S.D) | | | | | Isolation (S.I) | Aggregated Effect | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Level** | **Others** | **Home** | **School** | **Work** | **Contacts** | **S.D** | **S.I** | **Cost** | **Scaled Cost** |
| Level 1 | 100% | 100% | 100% | 100% | 0% | 0% | 16.2% | 0.03 | 0.4 |
| Level 2 | 80% | 105% | 0% | 100% | 0% | 16% | 16.2% | 0.11 | 1.6 |
| Level 3 | 70% | 110% | 0% | 80% | 0% | 26% | 16.2% | 0.17 | 2.5 |
| Level 4 | 40% | 115% | 0% | 60% | 0% | 43% | 16.2% | 0.35 | 5.1 |
| Level 5 | 10% | 120% | 0% | 10% | 0% | 66% | 16.2% | 0.68 | 10 |

Table 9: Epidemic Parameters (Non Age Specific)

| **Metric** | Value | Source |
|---|---|---|
| R0 | 2.8 | SantePubliqueFrance (2021) |
| $\bar{u}$ | 0.09 | Prem et al. (2017) |
| $\beta$ | 1.4 | Derived |
| $\bar{c}$ | 2.21 | Prem et al. (2017) |
| $\phi$ | 1 | Salje et al. (2020) |
| $\mu$ | 0.4 | Di Domenico et al. (2020) |
| $\gamma_1$ | 0.33 | Di Domenico et al. (2020) |
| $\gamma_2$ | 0.7 | Salje et al. (2020) |

Table 10: Epidemic Parameters(Age Specific)

| **Parameter** | **0-19** | **20-39** | **40-59** | **60-75** | **75+** | **Source** |
|---|---|---|---|---|---|---|
| $\lambda_1$ | 0.8 | 0.76 | 0.76 | 0.54 | 0.54 | Di Domenico et al. (2020) |
| $\lambda_2$ | 0.99 | 0.98 | 0.95 | 0.82 | 0.61 | Di Domenico et al. (2020) |
| $\rho$ | 0.1 | 0.1 | 0.2 | 0.2 | 0.2 | Di Domenico et al. (2020) |
| $\theta$ | 0.1 | 0.24 | 0.24 | 0.24 | 0.24 | Di Domenico et al. (2020) |

Table 11: Gain of Decentralisation for Deaths*

| Scenario $S0$ | | Centralised | | | Decentralised | | | Variation | |
|---|---|---|---|---|---|---|---|---|---|
| **B** | **T** | **#Inf** | **#Deaths*** | **Time** | **#Inf** | **#Deaths*** | **Time** | **I ($\Delta$)** | **D ($\Delta$)** |
| 5 | 8 | 19370 | 473 | 0.55 | 18276 | 436 | 6.27 | -6% | -8% |
| 6.5 | 8 | 10826 | 286 | 0.52 | 10020 | 274 | 5.92 | -7% | -4% |
| 8 | 8 | 7146 | 223 | 0.54 | 7013 | 221 | 6.27 | -2% | -0.9% |
| 5 | 10 | 25746 | 621 | 1.04 | 22478 | 536 | 65.99 | -13% | -14% |
| 6.5 | 10 | 11310 | 307 | 0.73 | 10291 | 291 | 65.97 | -9% | -5% |
| 8 | 10 | 7432 | 238 | 0.88 | 7049 | 229 | 73.59 | 54% | -4% |
| Scenario $S1$ | | Centralised | | | Decentralised | | | Variation | |
| 5 | 8 | 19370 | 473 | 0.55 | 18442 | 436 | 6.06 | -5% | -8% |
| 6.5 | 8 | 10826 | 290 | 0.54 | 10125 | 280 | 5.69 | -6% | -3% |
| 8 | 8 | 8273 | 253 | 0.5 | 8273 | 253 | 12.67 | 0% | 0% |
| 5 | 10 | 25746 | 621 | 1.09 | 22478 | 536 | 65.2 | -13% | -14% |
| 6.5 | 10 | 10906 | 308 | 0.88 | 10663 | 302 | 66.4 | -2% | -2% |
| 8 | 10 | 8499 | 268 | 0.77 | 8499 | 268 | 67.52 | 0% | 0% |
| Scenario $S2$ | | Centralised | | | Decentralised | | | Comparison | |
| 5 | 8 | 19370 | 473 | 0.64 | 18389 | 442 | 10.82 | -5% | -7% |
| 6.5 | 8 | 11591 | 321 | 0.64 | 10821 | 310 | 11.03 | -7% | -3% |
| 8 | 8 | 10219 | 301 | 0.55 | 8747 | 279 | 10.79 | -14% | -7.3% |
| 5 | 10 | 28736 | 688 | 0.96 | 23621 | 582 | 226.13 | -18% | -15% |
| 6.5 | 10 | 15103 | 420 | 0.84 | 12608 | 374 | 237.53 | -17% | -11% |
| 8 | 10 | 11236 | 362 | 0.8 | 9814 | 321 | 238.59 | -13% | -11.3% |

Table 12: Sensitivity analysis of reduction factor $\alpha$ for Inf*

| $S0$ | $B=5, T=8$ | | | | $B=6.5, T=8$ | | | | $B=8, T=8$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | Centralised | | Decentralised | | Centralised | | Decentralised | | Centralised | | Decentralised | |
| **Range** | **#Inf*** | **#D** | **#Inf*** | **#D** | **#Inf*** | **#D** | **#Inf*** | **#D** | **#Inf*** | **#D** | **#Inf*** | **#D** |
| 0.8 | 44460 | 1008 | 39803 | 933 | 27126 | 642 | 23964 | 607 | 17667 | 475 | 16857 | 466 |
| 0.9 | 30344 | 710 | 27760 | 671 | 17481 | 436 | 15707 | 415 | 11313 | 327 | 10925 | 322 |
| 1.0 | 19370 | 473 | 18036 | 439 | 10826 | 290 | 9819 | 276 | 7146 | 223 | 7008 | 222 |
| 1.1 | 13203 | 337 | 12391 | 315 | 7299 | 210 | 6835 | 204 | 5236 | 172 | 5174 | 171 |
| 1.2 | 8589 | 246 | 7580 | 225 | 4693 | 160 | 4564 | 158 | 3862 | 144 | 3846 | 144 |

# Appendix C: Calibration of NPI reduction coefficients $\alpha_i$

For calibrating NPIs we use social contact matrices of the French population. We use the data from Prem et al. (2017), where contacts are recorded for four different location categories - home, school, work, others, based on age groups. We define five age groups - 0-19, 20-39, 40-59, 60-75, 75+. We start with the baseline matrix, when there is no NPI in place, and modify these matrices by adjusting the contacts by a reduction factor based on the type of NPI (Di Domenico et al. 2020, Davies et al. 2020). For example, for school closure, we assume 100% of school contacts are reduced and 5% of home contacts increase, because of children spending more time at home. We then apply a weight for each category (home, school etc.) in the final matrix for each NPI level $i$. The notation is as follows:

Table 13: NPI Calibration based on Social Contact Matrices

| **Parameter** | **Description** |
|---|---|
| $g, g'$ | Index for age group, $g, g' \in \{1, 2, 3, 4, 5\}$ |
| $i$ | Index for NPI level, $i \in N = \{1, 2, 3, 4, 5\}$ |
| $d_i$ | Percentage change in contact for NPI level $i$ from baseline. |
| $a$ | Index for location category,$a \in A = \{1, 2, 3, 4\}$,1 = Home,2 = School, 3 = Work, 4 = Others |
| $o$ | Baseline contact. |
| $w_a$ | Weight of contact location $a$, $a \in A$. |
| $c_{igg'}$. | Number of contacts between age group $g$ and $g'$ for NPI level $i$ |
| $c^o_{agg'}$ | Number of contacts between age group $g$ and $g'$ in baseline scenario $o$, for location $a$. |
| $p_g$ | Population of age group $g$ |

$$c_{igg'} = \sum_{a \in A} d_i w_a c^o_{agg'} \tag{61}$$

$$\alpha_i = \sum_{g=1}^{5} \left(1 - \frac{\sum_{g'=1}^{5} c_{igg'}}{\sum_{a \in A} \sum_{g'=1}^{5} w_a c^o_{agg'}}\right)\left(\frac{p_g}{P}\right) \tag{62}$$

From equation (61) we calculate the effective number of contacts between age group $g$ and $g'$ for NPI level $i$, $(g, g' \in \{1, 2, 3, 4, 5\})$, summed across all contact locations $a \in A$, based on the weights $w_a$ and the change in contacts from baseline, given by $d_i$. From equation (62), we compute the effective reduction factor corresponding to NPI level $i$. We first sum up the contacts between a given age group $g$ and all age groups $g' \in \{1, 2, 3, 4, 5\}$ for NPI level $i$ and divide it by the sum of contacts between age group $g$ and all age groups $g'$ across all contact locations $a \in A$, in the baseline scenario. This is basically the ratio of total contacts of age group $g$ for NPI level $i$ corresponding to the baseline. Hence, by subtracting this ratio from 1, we get the effective reduction in contacts for age group $g$. We multiply the this reduction ratio by the population ratio of age group $g$ and sum for all $g \in \{1, 2, 3, 4, 5\}$. This gives us the weighted reduction factor for NPI level $i$ across all age groups.