MILP models for the continuous Berth Allocation and Quay Crane Assignment Problem considering crane movement and setup times

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Abstract

In this technical report we present several Mixed Integer Linear Programming (MILP) models for the Berth Allocation and Quay Crane Assignment Problem (BACASP) considering crane movement and setup time (namely, BACASP-S). First, we propose a MILP for the time-invariant BACASP in which both berthing time and position variables are continuous. Then, we extend this MILP to take into account crane movement and setup times. Finally, we also propose a MILP for the variable-in-time BACASP-S with both continuous time and space variables.

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1. MILP model for the time-invariant BACASP on continuous time and space

1.1. Description of the problem

We consider a set of ships arriving at a port that will be scheduled over a planning horizon. The vessels must be processed at the quay, to be loaded and unloaded. These tasks are carried out by a number of cranes, available on the quay. No crane can process more than one vessel at a time. The number of cranes assigned to a vessel remains constant throughout the vessel's processing (time invariant). The schedule will explicitly consider the setup and travel times required for cranes between two consecutive vessels.

More specifically, we assume that each vessel:

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- has a known arrival time and a desired departure time. Waiting times before mooring and delays with respect to departure times are penalized.

- requires a number of cranes to be served on the quay, between a minimum and a maximum value fixed for each vessel. The handling time of a vessel depends on the number of cranes assigned.

- has a desired position on the quay. Separations from that position are penalized.

Moreover, we consider that cranes are heterogeneous, in the sense that the speed at which they move on the quay can be different. This is an important aspect, since one of the novelties of this work is that we consider a cost associated with the setup of a crane between consecutive vessels. This setup has a fixed value (time for cleaning, adjustments, etc.) plus a variable value that depends on the distance between the two vessels in the quay. We call this problem the Berth Allocation and Crane Assignment Problem with Setups (BACASP-S).

1.2. Input data

The following sets are defined:

- $V = \{1, ..., n\}$ is the set of vessels. For modelling purposes, let $V^0 = V \cup \{0\}$, with vessel 0 being a dummy vessel. Vessels are indexed by i and j.

- $K = \{1, ..., Q\}$ is the set of available cranes, indexed by k, k' (which refer to specific cranes) and q (which refers to the number of cranes assigned to a vessel). Cranes are numbered from 1 to Q, increasingly with their distance to the origin of the quay. As the cranes can move along the quay but cannot cross each other, their relative position remains constant.

- L is the length of the quay.

The following input data are assumed to be known and deterministic. For each vessel $i \in V$ we know:

- Length: $l_i \in [0, L]$

- Arrival time: a_i

- Cost per unit of time waiting for berthing after the expected arrival time: C_i^w

- Desired departure time: s_i

- Cost per unit of time of delay after the desired departure time: C_i^d

- Desired position in the quay: $b_i \in [0, L - l_i]$

- Cost per unit of length away from the desired position at the quay: C_i^p

- Minimum and maximum number of cranes that can be assigned to the vessel: $q_i^{\min}, q_i^{\max}$, with $q_i^{\min} \leq q_i^{\max} \leq Q$.

- We define $K_i = \{1, \ldots, Q - q_i^{\min} + 1\} \subseteq K, \forall i \in V$, as the set of cranes that can be the first crane assigned to vessel i. Cranes assigned to a vessel are consecutive so the first crane assigned to vessel i cannot be greater than $Q - q_i^{\min} + 1$.

- Let $Q_i = \{q_i^{\min}, \ldots, q_i^{\max}\}$ be the set of all the numbers of cranes admitted for vessel i.

- Processing time if q cranes are assigned to it: $u_{iq},\ q \in Q_i$. The processing time of a vessel can vary linearly with the number of cranes or can be related to the number of cranes by a more complex expression, considering cranes interference. It may have a non-integer value.

When considering crane movement and setup time, for each crane $k \in K$ it is also known:

- α_k: the speed of crane k, in order to measure how long it will take it to move from one position on the quay to another.

- β_k: the fixed time that crane k requires to set up after serving another vessel.

If cranes are homogeneous, let $\alpha = \alpha_k$ and $\beta = \beta_k,\ \forall k \in K$. Otherwise, $\alpha = \max_{k \in K}(\alpha_k)$ and $\beta = \max_{k \in K}(\beta_k)$.

1.3. Variables

In this section we present a new mixed integer linear programming model for the BACASP problem, with the novelty that it considers both continuous time and space, as opposed to the more usual discretization of these dimensions. This model uses the following variables:

- $t_i \in [a_i, T - u_{iq_{min}}]$ is the berthing time of vessel i, where T is an upper bound on the total time required to berth all the vessels. Note that we impose that vessel i cannot be moored before its arrival time.

- $p_i \in [0, L - l_i]$ is the berthing position for vessel i on the quay, measured as the shortest distance from the vessel to the origin of the quay.

- $d_i \geq 0$ is the delay incurred in the handling of vessel i with respect to its desired departure time.

- $e_i \geq 0$ is the deviation of the berthing position of vessel i with respect to its desired position at the quay.

- $\sigma_{ij} = 1$ if the departure time of vessel i is prior to the berthing time of vessel j, 0 otherwise.

- $\delta_{ij} = 1$ if vessel i is completely below vessel j, i.e. $p_i + l_i \leq p_j$, 0 otherwise.
- $r_{iq} = 1$ if vessel i is served by q cranes, 0 otherwise.
- $w_{ik} = 1$ if k is the first crane of the sequence of consecutive cranes assigned to i, 0 otherwise.

1.4. Objective function and constraints

Using the input data and the variables defined above, we propose the following MILP:

$$\min \sum_{i \in V} \left(C_i^w (t_i - a_i) + C_i^d d_i + C_i^p e_i \right) \tag{1}$$

subject to

$$
\begin{aligned}
& e_i \geq p_i - b_i, && \forall i \in V && (2)\\
& e_i \geq -(p_i - b_i), && \forall i \in V && (3)\\
& d_i \geq t_i + \sum_{q \in Q_i} u_{iq} r_{iq} - s_i, && \forall i \in V && (4)\\
& p_j \geq p_i + l_i - L(1 - \delta_{ij}), && \forall i, j \in V, i \neq j && (5)\\
& t_j \geq t_i + \sum_{q \in Q_i} u_{iq} r_{iq} - T(1 - \sigma_{ij}), && \forall i, j \in V, i \neq j && (6)\\
& \sigma_{ij} + \sigma_{ji} + \delta_{ij} + \delta_{ji} \geq 1, && \forall i, j \in V, i \neq j && (7)\\
& \sum_{q \in Q_i} r_{iq} = 1, && \forall i \in V && (8)\\
& \sum_{k \in K_i} w_{ik} = 1, && \forall i \in V && (9)\\
& \sum_{k \in K_i} k w_{ik} + \sum_{q \in Q_i} q r_{iq} \leq Q + 1, && \forall i \in V && (10)\\
& \sum_{k \in K_i} k w_{jk} \geq \sum_{k' \in K} k' w_{ik} + \sum_{q \in Q_i} q r_{iq} - Q(\delta_{ji} + \sigma_{ij} + \sigma_{ji}), && \forall i, j \in V, i \neq j && (11)\\
& w_{ik} \in \{0, 1\}, && \forall i \in V, k \in K_i && (12)\\
& \sigma_{ij}, \delta_{ij} \in \{0, 1\}, && \forall i, j \in V, i \neq j && (13)\\
& r_{iq} \in \{0, 1\}, && \forall i \in V, q \in Q_i && (14)\\
& p_i \in [0, L - l_i], d_i \geq 0, t_i \in [a_i, T - u_{iq_{min}}], && \forall i \in V && (15)
\end{aligned}
$$

The objective function (1) sums up the total cost due to waiting for berthing after arrival times, the total cost of delay after desired departure times, and the total cost of deviations with respect to desired positions at the quay. Constraints (2) and (3) define the deviation of vessels with respect to their desired position at the quay and constraints (4) the delay with respect to the desired departure time. Constraints (5), (6), and (7) ensure that vessels do not overlap. If δ_{ij} or δ_{ji} take value 1, vessels i and j are separated in space by (5). If σ_{ij} or σ_{ji} take value 1, vessels are separated in time by (6). By (7), at least one of these variables takes value one. Constraints (8) force the number of cranes assigned to vessel i to be in its allowed range and constraints (9) set the number of the crane with the lowest number assigned to each vessel. Together, (8) and (9) fix the number of cranes assigned to each vessel. By constraints (10), the numbers of the cranes assigned to a vessel cannot exceed the number of available cranes. Constraints (11) ensure that the cranes assigned to each pair of vessels i and j do not cross each other. Finally, (12) to (15) define the ranges of the variables involved in the model.

2. Model for the time-invariant BACASP with crane movement and setup time

The model for the BACASP-S is based on the BACASP model presented in Section 1.

We define the following additional variables:

- $v_{ij} = 1$ if vessel i is below vessel j and the crane with the highest number among those assigned to vessel i (its last crane) has a number lower than the number of the crane with the lowest number among those assigned to j (its first crane); 0 otherwise.
- f_{ij} is the distance between vessels i and j, measured as the distance between their middle positions.

The model for the BACASP-S consists of minimizing (1) subject to constraints (2)–(15) and (16)–(21):

$$f_{ij} \geq \left(p_i + \frac{l_i}{2}\right) - \left(p_j + \frac{l_j}{2}\right), \qquad i, j \in V, i \neq j \quad (16)$$

$$f_{ij} \geq \left(p_j + \frac{l_j}{2}\right) - \left(p_i + \frac{l_i}{2}\right), \qquad i, j \in V, i \neq j \quad (17)$$

$$t_j \geq t_i + \sum_{q \in Q_i} u_{iq} r_{iq} + \frac{f_{ij}}{\alpha} + \beta - T(1 - \sigma_{ij} + v_{ij} + v_{ji}), \quad i, j \in V, i \neq j \quad (18)$$

$$v_{ij} \geq \delta_{ij} - \sigma_{ji} - \sigma_{ij}, \qquad i, j \in V, i \neq j \quad (19)$$

$$\sum_{k \in K} k w_{jk} \geq \sum_{k' \in K} k' w_{ik} + \sum_{q \in Q_i} q r_{iq} - Q(1 - v_{ij}), \qquad i, j \in V, i \neq j \quad (20)$$

$$v_{ij} \in \{0, 1\}, \qquad i, j \in V, i \neq j \quad (21)$$

Constraints (16) and (17) define the distance between vessels i and j. Constraints (18) extend constraints (6) by including between the starting times of vessels i and j not only the handling time of vessel i but also the setup times of the cranes that are assigned to both of them consecutively. Constraints (19) link the new variables v_{ij} with variables δ_{ij} and σ_{ij} defined to avoid overlaps. Constraints (20) ensure that the cranes assigned to each pair of vessels i and j do not cross. Constraints (20) define the possible values for variables v_{ij}.

Now constraints (11) become redundant with (20) and can be eliminated, although they may be kept as additional cuts.

3. MILP model for the variable-in-time BACASP on continuous time and space

In this problem, the number of cranes serving a vessel may vary along its processing time.

3.1. Assumption

Cranes can serve a vessel once and only once. In other words, no crane can serve a vessel, move to another vessel and then return to the first vessel to serve it again.

3.2. Input data

- $V = \{1, ..., N\}$ is the set of vessels to schedule. Vessels are indexed by i and j.
- $K = \{1, ..., Q\}$ is the set of the cranes available on the quay, indexed by letters k and k'.
- $K_0 = K \cup \{0\}$.
- KG is the set containing all valid crane groups. A group consists of a number of consecutive cranes, with cardinality varying from 1 to Q. There will be Q groups composed of 1 crane, $Q-1$ groups of 2 cranes: $\{1, 2\}, \{2, 3\}, \ldots$; $Q-2$ groups of 3 cranes, and so on; the final group is composed of all the Q cranes.
- L is the length of the quay.
- T is an upper bound on the total time required to berth all the vessels.

  For each vessel $i \in V$:

  - Length: l_i.
  - Arrival time: a_i.
  - C_i^w cost per period of waiting for berthing after the expected arrival time: C_i^w.
  - Desired departure time: s_i.
  - Cost per period of delay after the desired departure time: C_i^d
  - Desired position in quay: b_i.
  - Cost per unit away from the desired position at the quay: C_i^p.
  - Minimum and maximum number of concurrent cranes serving the vessel: $q_i^{\min}, q_i^{\max}$.
  - Workload of a vessel i in crane-hours: u_i.
  - Exponent used to reduce crane productivity: $\mu \in (0, 1]$.

– Let $Q_i = \{q_i^{\min}, \ldots, q_i^{\max}\}$ be the set of all the numbers of cranes admitted for vessel i.

– MKG_i is a set containing all the minimal crane groups $G \in KG$ satisfying $|G| > q_i^{\max}$.

When considering crane movement and setup time, for each crane $k \in K$ it is also known:

- α_k: the speed of crane k, in order to measure how long it will take it to move from one position in the quay to another.
- β_k: the fixed time that crane k requires after serving a vessel to be ready to serve another vessel.

If cranes are homogeneous, let $\alpha = \alpha_k$, and $\beta = \beta_k,\ \forall k \in K$.

3.3. Variables

- $t_i \geq a_i$ is the berthing time of vessel i.
- $o_i > t_i$ is the finishing time of vessel i.
- $p_i \in [0, L]$ is the berthing position (bottom point) for vessel i on the quay.
- $d_i \geq 0$ is the delay incurred in the handling of vessel i with respect to its desired departure time.
- $e_i \geq 0$ is the deviation of berthing position of vessel i with respect to its desired position at the quay.
- $\sigma_{ij} = 0$ if $t_j < o_i, \quad i, j \in V$.
- $\delta_{ij} = 0$ if $p_j < p_i + l_i, \quad i, j \in V$.
- $w_{ik} = 1$ if crane k serves vessel i at some time.
- τ_{ki} = instant in which crane k starts to handle vessel i.
- ϕ_{ki} = instant in which crane k finishes the handling of vessel i.
- $\gamma_{kgij} = 1$ if $\tau_{gj} \geq \phi_{ki},\ i \in V, g, k \in K$, 0 otherwise.
- $\theta_{kgi} = 1$ if $\tau_{gi} \leq \phi_{ki} \wedge \tau_{gi} \geq \tau_{ki},\ i \in V, g, k \in K$, 0 otherwise.

3.4. Objective function and constraints

The following MILP is based on the input data and the variables defined above:

$$\min \sum_{i \in V} \left(C_i^w (t_i - a_i) + C_i^d d_i + C_i^p e_i \right) \tag{22}$$

subject to

$$p_i + l_i \leq L, \qquad \forall i \in V \tag{23}$$

$$e_i \geq p_i - b_i, \qquad \forall i \in V \tag{24}$$

$$e_i \geq b_i - p_i, \qquad \forall i \in V \tag{25}$$

$$t_i \geq a_i, \qquad \forall i \in V \tag{26}$$

$$t_i \leq T, \qquad \forall i \in V \tag{27}$$

$$o_i \geq t_i, \qquad \forall i \in V \tag{28}$$

$$o_i \leq T, \qquad \forall i \in V \tag{29}$$

$$d_i \geq o_i - s_i, \qquad \forall i \in V \tag{30}$$

$$p_j \geq p_i + l_i - L(1 - \delta_{ij}), \qquad \forall i, j \in V, i \neq j \tag{31}$$

$$t_j \geq o_i - T(1 - \sigma_{ij}), \qquad \forall i, j \in V, i \neq j \tag{32}$$

$$\sigma_{ij} + \sigma_{ji} + \delta_{ij} + \delta_{ji} \geq 1, \qquad \forall i, j \in V, i \neq j \tag{33}$$

$$\tau_{ki} \geq t_i - T(1 - w_{ik}), \qquad \forall i \in V, \forall k \in K \tag{34}$$

$$\phi_{ki} \leq T \cdot w_{ik}, \qquad \forall i \in V, \forall k \in K \tag{35}$$

$$\phi_{ki} \geq \tau_{ki}, \qquad \forall i \in V, \forall k \in K \tag{36}$$

$$\phi_{ki} \leq o_i, \qquad \forall i \in V, \forall k \in K \tag{37}$$

$$\sum_{k \in K} (\phi_{ki} - \tau_{ki}) \geq u_i, \qquad \forall i \in V \tag{38}$$

$$\phi_{ki} \leq \tau_{gj} + T(1 - \gamma_{kgij}), \qquad \forall i, j \in V, \forall k, g \in K, i \neq j \vee k \neq g \tag{39}$$

$$\tau_{gj} \leq \phi_{ki} + T\gamma_{kgij}, \qquad \forall i, j \in V, \forall k, g \in K, i \neq j \vee k \neq g \tag{40}$$

$$\sum_{h_1=k}^{g} \sum_{h_2=h_1+1 | h_1 \neq k \vee h_2 \neq g}^{g} \gamma_{h_1 h_2 ii} + \gamma_{h_2 h_1 ii} \leq M(\gamma_{kgii} + \gamma_{gkii}), \qquad \forall i \in V,$$

$$\forall k, g \in K, k + 1 < g < k + q_i^{\max}, n = g - k + 1, M = \frac{n(n-1)}{2} - 1 \tag{41}$$

$$\gamma_{kgij} + \gamma_{gkji} + \delta_{ij} + 2 - w_{ik} - w_{gj} \geq 1, \quad \forall i, j \in V, i \neq j, k, g \in K, g \geq k \quad (42)$$
$$\tau_{gj} \leq \phi_{ki} + T(1 - \theta_{kgi}), \quad \forall i \in V, k, g \in K, k \neq g \quad (43)$$
$$\tau_{ki} \leq \tau_{gj} + T(1 - \theta_{kgi}), \quad \forall i \in V, k, g \in K, k \neq g \quad (44)$$
$$\tau_{gj} \geq \phi_{ki} - T\theta_{kgi}, \quad \forall i \in V, k, g \in K, k \neq g \quad (45)$$
$$\sum_{g \in K_0, g \neq k} \theta_{gki} = w_{ki}, \quad \forall i \in V, k \in K \quad (46)$$
$$\sum_{g \in K_0, g \neq k} \theta_{kgi} = w_{ki}, \quad \forall i \in V, k \in K \quad (47)$$
$$\theta_{kgi} + \theta_{gki} \leq 1, \quad \forall i \in V, k, g \in K, k \neq g \quad (48)$$
$$\tau_{ki} \leq t_{ki} + T(1 - \theta_{0ki}), \quad \forall i \in V, k \in K \quad (49)$$
$$o_{ki} \leq \phi_{ki} + T(1 - \theta_{k0i}), \quad \forall i \in V, k \in K \quad (50)$$
$$\sum_{k \in K} \theta_{0ki} \geq q_i^{\min}, \quad \forall i \in V, k \in K \quad (51)$$
$$\sum_{k \in K} \theta_{k0i} \geq q_i^{\min}, \quad \forall i \in V, k \in K \quad (52)$$
$$\sum_{k \in G} \sum_{g \in G} (\gamma_{kgij} + \gamma_{gkij}) \geq 1, \quad \forall G \in MKG \quad (53)$$
$$\gamma_{kgij} \in \{0, 1\}, \quad i, j \in V, k, g \in K, i \neq j \vee k \neq g \quad (54)$$
$$\theta_{kgi} \in \{0, 1\}, \quad \forall i \in V, k, g \in K, k \neq g \quad (55)$$
$$w_{ik} \in \{0, 1\}, \quad \forall i \in V, k \in K \quad (56)$$
$$\sigma_{ij}, \delta_{ij} \in \{0, 1\}, \quad \forall i, j \in V, i \neq j \quad (57)$$
$$p_i, d_i \geq 0, \quad \forall i \in V \quad (58)$$
$$\tau_{ki}, \phi_{ki} \geq 0, \quad \forall i \in V, k \in K \quad (59)$$

The objective function (22) sums up the total cost resulting from waiting for berthing after arrival times, the total cost of delay after desired departure times, and the total cost of deviations with respect to desired positions on the quay. Constraints (23) ensure vessels fit within the quay. Constraints (24) and (25) define the position deviation with respect to their desired position on the quay and (26)–(29) set the proper bounds to berthing and departure times. Constraints (30) define the delays with respect to the desired departure times. Constraints (31), (32), and (33) ensure that vessels do not overlap. If δ_{ij} or δ_{ji} take value 1, vessels i and j are separated in space by (31). If σ_{ij} or σ_{ji} take value 1, vessels are separated in time by (32). By (33), at least one of these variables takes value 1. Constraints (34)–(37) define variables w_{ik} and bound starting and finishing times of each crane serving a vessel. Constraints (38) ensure that the number of crane-hours worked on a vessel is enough to complete the workload of the vessel, estimated in crane-hours. Together, (39) and (40) define variables γ_{kgij}. By constraints (41), the order of the cranes serving a vessel is guaranteed so that they do not cross each other, while constraints (42) prevent incorrect crane assignments in which cranes cross other cranes when they serve different vessels. Moreover, they also prevent that a crane serves two or more vessels concurrently. Constraints (43)–(45) define variables θ_{kgi}. Constraints (46) and (47) determine the antecessor crane and the successor crane of each crane that serves a vessel, thus creating circuits of cranes from and to a dummy crane with id 0, while (48) prevent cycles of real cranes. Constraints (49) and (50) establish crane circuits subject to the order of starting service time of the cranes serving the vessel. By setting a minimum number of crane circuits for each vessel, constraints (51) and (52) ensure that each vessel is served by at least a minimum number of concurrent cranes along its processing. Constraints (53) limit the number of concurrent cranes to a maximum value by preventing any set of cranes with a greater cardinal to be a clique on concurrency relations. Finally, constraints (54)–(59) define the admissible values of the variables.

3.5. Considering non-linear vessel processing time

In order to consider non-linear vessel processing time functions on the number of different cranes assigned to the vessel, instead of constraint (38), we can introduce the following variables and constraints:

Additional variables:

- $r_{iq} = 1$ if vessel i is served by a number of q different cranes during its processing, being $q_i^{\min} \leq q \leq Q$.

Additional constraints:

$$r_{iq} \in \{0,1\}, \qquad \forall i \in V, q_i^{\min} \leq q \leq Q \qquad (60)$$

$$\sum_{q=q_i^{\min}}^{q=Q} r_{iq} = 1, \qquad \forall i \in V \qquad (61)$$

$$\sum_{q=q_i^{\min}}^{q=Q} q \cdot r_{iq} = \sum_{k \in K} w_{ik}, \qquad \forall i \in V \qquad (62)$$

$$\sum_{k \in K} (\phi_{ki} - \tau_{ki}) \geq u_i + \sum_{q=q_i^{\min}}^{q=Q} q^{\mu} \cdot r_{iq}, \qquad \forall i \in V \qquad (63)$$

Some alternatives to (63) are the following:

$$\sum_{k \in K} (\phi_{ki} - \tau_{ki}) \geq \sum_{q=q_i^{\min}}^{q=Q} {u_i}^{q^{\mu}} \cdot r_{iq} - 1, \qquad \forall i \in V \qquad (64)$$

$$\sum_{k \in K} (\phi_{ki} - \tau_{ki}) \geq u_i \cdot \sum_{q=q_i^{\min}}^{q=Q} q^{\mu} \cdot r_{iq} - 1, \qquad \forall i \in V \qquad (65)$$

$$\sum_{k \in K} (\phi_{ki} - \tau_{ki}) \geq \sum_{q=q_i^{\min}}^{q=Q} u_i^{q} \cdot r_{iq}, \qquad \forall i \in V \qquad (66)$$

The constraint to prevent vessel processing interruption now can be expressed as:

$$\sum_{k \in K} \sum_{g \in K/\{k\}} \gamma_{kgii} \leq \sum_{q=q_i^{\min}}^{q=Q} r_{iq} \frac{(q-1)^2 - (q-1)}{2}, \qquad \forall i \in V \tag{67}$$

Therefore, we could also drop the requirement of a minimum number of cranes and thus remove variables $\theta_{kgi}, \forall i \in V, k, g \in K$ and the constraints in which they are involved.

4. Model for the variable-in-time BACASP with crane movement and setup time

The previous model can be extended to consider crane movement and setup time by adding the following variables and constraints:

Additional variables:

- f_{ij} is the distance between vessels i and j, measured as the distance between their middle positions.

Additional constraints:

$$f_{ij} \geq \left(p_i + \frac{l_i}{2}\right) - \left(p_j + \frac{l_j}{2}\right), \qquad \forall i, j \in V, i \neq j \tag{68}$$

$$f_{ij} \geq \left(p_j + \frac{l_j}{2}\right) - \left(p_i + \frac{l_i}{2}\right), \qquad \forall i, j \in V, i \neq j \tag{69}$$

$$\tau_{ki} \geq \phi_{kj} + \frac{f_{ij}}{\alpha_k} + \beta_k - T(1 - \gamma_{kkij}), \qquad \forall i, j \in V, i \neq j, k \in K \tag{70}$$