

# Designing reliable future energy systems by iteratively including extreme periods in time-series aggregation

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## Abstract

Generation Capacity Expansion Planning (GCEP) requires high temporal resolution to account for the volatility of renewable energy supply. Because the GCEP optimization problem is often computationally intractable, time-series input data are often aggregated to representative periods using clustering. However, clustering removes extreme events, which are important to achieve reliable system designs. We present a method to include extreme periods into time-series aggregation for GCEP that guarantees reliable system designs on the full input data even though only the reduced data set is used for system design. Our method iteratively adds extreme periods to the set of representative periods based on information from the optimization problem itself until the energy system provides power reliably.

We perform a comprehensive analysis on several case studies of both German and Californian energy systems and show that our method leads to meeting electricity demand at all times, whereas when clustering without extreme periods, lost load is between 1.9%-13.5% of total system load.. We show that our method outperforms the state-of-the-art method of adding a pre-defined number of extreme periods based on statistical properties of the data itself.

*Keywords:* Clustering, Energy systems, Time-series aggregation, Temporal resolution, Extreme periods, Optimization

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## 1. Introduction

CO<sub>2</sub> emissions from power generation are a main driver of climate change. The electricity sector alone accounts for around 25% of global CO<sub>2</sub> emissions (1). It is thus of utmost importance to design future power generation systems with low CO<sub>2</sub> emissions. Designing these systems is usually done using generation capacity expansion planning (GCEP) (2–4). These are computational optimization problems that minimize overall system cost, including both the design with its associated capital costs, and the operations with its associated operational costs.

GCEPs merge long-term decisions that are on the scale of decades (i.e. generation capacity investment decisions) and short term decisions on the scale of hours (i.e. scheduling of power generation) (3). This makes GCEPs computationally complex optimization problems (5), and they are often computationally intractable for the full set of time-varying input data. Time-varying input data to GCEPs are most commonly electricity demand, wind availability, and solar availability.

To reduce the computational complexity of the problem, time-series aggregation methods have been developed in the literature (6–8). These methods reduce the number of variables and constraints in the optimization problem by using representative periods instead of the full time-varying input data (e.g., representing the 365 days of a year of data by 5 representative

days), or by aggregating multiple consecutive time-steps into few.

Various methods have been developed in the literature to find representative periods (7; 9; 10). The most common method is time-series clustering, which groups periods that are similar. Teichgraeber and Brandt (11) provide a framework for clustering including three categories: normalization, assignment, and representation. Methods in the literature mostly differ in their assignment and representation. In terms of assignment, partitional clustering algorithms such as k-means (12–25), k-medoids (24; 26–29), and hierarchical clustering using Ward’s method (3; 20; 30–33) have been used. A cluster found in the assignment can either be represented by the cluster centroid, which is calculated by taking the mean of all periods that belong to a cluster, or by the cluster medoid, which is calculated by taking the closest real period to the cluster centroid. In the optimization problem, each representative period is weighted by the number of periods its cluster represents.

Because the representative periods found by clustering are cluster averages or close to cluster averages, clustering results in smoothed periods. However, reliable system designs in GCEPs are largely influenced by extremes (e.g. a combination of maximum electricity demand and minimum renewable energy availability). This is especially the case in systems that are constrained to have low carbon emissions, i.e. a high share of renewable generation capacity (34). Therefore, several studies use extreme periods additional to the periods found by clustering. According to the decision framework presented by Teichgraeber et al. (35), including extreme periods involves mul-

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multiple decisions concerning the selection process, the identification measure, and representation modification. Most commonly to date, in a “simple” selection process, a pre-defined number of extreme periods have been added by use of statistical identification measures from the data itself. For example, one might add the period with maximum demand (12), or the three periods with maximum demand, minimum wind availability, and minimum solar availability (20; 36). These extreme periods can be added to the set of clustered representative periods prior to evaluating the GCEP optimization problem. While often improving results, the addition of these extreme periods does not guarantee that the ensuing system design reliably meets demand on the problem solved for the full time series.

Several authors have developed methods that guarantee reliability on the full time series for various optimization problem applications. These methods are based on the idea that extreme periods are selected iteratively until a reliable system design is found, and the extreme periods are identified based on information from the optimization problem itself. Bahl et al. (16; 37) present a method to include extreme periods for the design and operations of an industrial energy supply system. They find an optimal design based on clustered periods, and they evaluate that design on the optimization problem with fixed design on the full time series (e.g. 365 periods). Because the periods are operationally independent, they identify which periods are infeasible for a given system design, add these extreme periods to the set of representative periods, and find a new optimal design based on the design and operations problem with the new set of representative periods. By iteratively adding periods until no periods are infeasible on the operations problem with fixed design, they can guarantee a reliable system design. Teichgraber et al. (35) present a method to iteratively add extreme periods based on slack variables they add in the optimization problem with fixed design and operations on the full time series. They apply their method to a residential energy supply system, where periods are also operationally independent. Their method is as follows: A design is found based on clustered periods and evaluated on the operations problem with full time series input data and fixed design. They add slack variables to the demand constraints of the operational optimization problem and to the objective function. These slack variables correspond to lost load, i.e. the demand that could not be satisfied by the current system design. The day with highest slack variable value is chosen as the next extreme period to be added to the set of representative periods, and this process is continued iteratively until a reliable system design is found where all slack variables are zero.

However, the iterative methods using information from the optimization problem itself that have been developed to date can only be applied to optimization problems where periods are separable operationally. If there are complicating constraints in the operational part of the optimization problem (constraints binding multiple periods such as a yearly limit on CO<sub>2</sub> emissions or seasonal storage), the previously introduced methods may not be able to identify extreme periods that yield reliable system designs. For adding infeasible periods, this is the case because infeasibility cannot be determined for every period independently if complicating constraints exist in the operational

part of the optimization problem. For the slack-variable based approach, this is the case because the operational optimization may result in high lost load that substitutes CO<sub>2</sub>-emitting generation in certain periods to satisfy the CO<sub>2</sub> emissions constraint. Adding the corresponding period as an extreme period may not result in a system design that reduces lost load and the iterative method may thus not converge to a reliable system design.

In this work, we propose a method that iteratively selects extreme periods and results in reliable GCEP system designs. We add a slack variable to the emissions constraint (“excess emissions”), and based on the combination of “lost load” and “excess emissions” slack variables, we are able to identify and iteratively add extreme periods that ultimately result in reliable system designs. We define reliable here as having no lost load. We furthermore measure the resulting “excess emissions”. Reducing them is beyond the scope of this work. Our work makes the following key contributions:

- Our method is able to identify reliable system designs for GCEP with the aforementioned complicating constraints.
- Our method enables the use of time-series aggregation for the planning of future electricity systems with low carbon emissions.
- We perform a comprehensive analysis of our iterative extreme period inclusion method and test it on multiple regions and multiple years of data. We show that our method has superior performance compared to conventionally used methods such as “simple” extreme period selection.

The rest of the paper is organized as follows: Section 2 provides the formulation of the GCEP, Section 3 provides a description of the iterative extreme period selection method we propose in this work, Section 4 provides the results, and Section 5 concludes and points to ideas for future work.

## 2. Capacity Expansion Planning

The GCEP formulation in Section 2.1 jointly optimizes system design and operations on a set of representative periods. We refer to the GCEP formulation as  $D_K \& O_K$ . This indicates that it is designed for  $K$  days and operated for  $K$  days. The formulation in Section 2.2 validates the solution by using the resulting design and optimizing its operations on the full set of temporal input data. We refer to the validation formulation as  $D_K \& O_{full}$ . This indicates that it is designed for  $K$  days and operated for the full input data.

### 2.1. Capacity Expansion

The GCEP formulation minimizes capital cost of capacity buildout  $C$  per generator  $g \in \mathcal{G}$  (with fixed cost  $\hat{c}$ ) and operating cost provided by each generator at each node  $n \in \mathcal{N}$ , time step  $t \in \mathcal{T}$  and period  $k \in \mathcal{K}$  (with variable cost  $\tilde{c}$ ). The set of generators includes dispatchable generators  $\mathcal{G}_{\mathcal{T}}$ , non-dispatchable

generators  $\mathcal{G}_R$  and storage  $\mathcal{G}_S$ :  $\mathcal{G} = \mathcal{G}_T \cup \mathcal{G}_R \cup \mathcal{G}_S$ .  $\omega$  assigns weights to different operating scenarios based on cluster assignments.

$$\min_{\substack{C, E, \dot{E}^{in} \\ \dot{E}^{out}, SOC, f}} \sum_{g \in \mathcal{G}} \sum_{n \in \mathcal{N}} \hat{c}_g C_{gn} + \sum_{k \in \mathcal{K}} \omega_k \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}_T} \tilde{c}_g E_{gnkt} \quad (1a)$$

subject to

$$\sum_{g \in \mathcal{G}_T} E_{gnkt} + \sum_{g \in \mathcal{G}_S} (\dot{E}_{gnkt}^{out} - \dot{E}_{gnkt}^{in}) \Delta t + \sum_{g \in \mathcal{G}_R} C_g \bar{a}_{gnkt} \Delta t \geq \bar{D}_{nkt} + \sum_{m \in \mathcal{N}} f_{nm}(1 - l_{nm}) \Delta t - \sum_{m \in \mathcal{N}} f_{mn}(1 - l_{mn}) \Delta t \quad \forall n \in \mathcal{N}, \forall k \in \mathcal{K}, t \in \mathcal{T} \quad (1b)$$

$$0 \leq E_{gnkt} \leq C_{gn} \Delta t \quad \forall g \in \mathcal{G}_T, n \in \mathcal{N}, k \in \mathcal{K}, t \in \mathcal{T} \quad (1c)$$

$$0 \leq \dot{E}_{gnkt}^{in} \leq C_{gn} \quad \forall g \in \mathcal{G}_S, n \in \mathcal{N}, k \in \mathcal{K}, t \in \mathcal{T} \quad (1d)$$

$$0 \leq \dot{E}_{gnkt}^{out} \leq C_{gn} \quad \forall g \in \mathcal{G}_S, n \in \mathcal{N}, k \in \mathcal{K}, t \in \mathcal{T} \quad (1e)$$

$$0 \leq SOC_{gnkt} \leq \gamma C_{gn} \quad \forall g \in \mathcal{G}_S, n \in \mathcal{N}, k \in \mathcal{K}, t \in \mathcal{T} \cup \{T+1\} \quad (1f)$$

$$0 \leq f_{mn} \leq F_{mn} \quad \forall n \in \mathcal{N}, m \in \mathcal{N} \quad (1g)$$

$$SOC_{gnkt+1} = SOC_{gnkt}(1 - \eta_g^{self}) + \Delta t(\eta_g^{in} \dot{E}_{gnkt}^{in} - \dot{E}_{gnkt}^{out} / \eta_g^{out}) \quad \forall g \in \mathcal{G}_S, n \in \mathcal{N}, k \in \mathcal{K}, t \in \mathcal{T} \quad (1h)$$

$$SOC_{gnkT+1} = SOC_{gnk1} \quad \forall g \in \mathcal{G}_S, n \in \mathcal{N}, k \in \mathcal{K} \quad (1i)$$

$$\sum_{g \in \mathcal{G}} \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} e_g E_{gnkt} \leq e^{target} \sum_{g \in \mathcal{G}} \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} E_{gnkt} \quad (1j)$$

where (1b) is the demand balance with clustered input data  $\bar{a}$  (renewable availability factors) and  $\bar{D}$  (electricity demand) and the time step length  $\Delta t$  is one hour.  $f$  is the power flow through the transmission lines, and  $l$  is a loss factor that is pre-calculated based on line length. (1c) limits power generation by generator capacity  $C$ , and (1d-1f) limit storage power inflow ( $\dot{E}^{in}$ ), outflow ( $\dot{E}^{out}$ ), and state of charge ( $SOC$ ), where  $\gamma$  is the hours of storage capacity. We assume four hours of storage capacity in this study. (1g) limits the power flow  $f$  to existing transmission line limits  $F$ . (1h) is the storage balance over time, with storage efficiencies  $\eta$ , and (1i) is the storage cycling condition per period. We only consider intra-period storage. Inter-period storage is beyond the scope of this work, and formulations have been developed in the literature (23; 38). (1j) enforces that average carbon emissions of the system stay below  $e^{target}$ , where  $e_g$  are the marginal emissions of each generator.

(1j) is a complicating constraint. Complicating constraints are constraints that if removed, the problem can be decomposed into simpler subproblems. They are also referred to as coupling constraints because they couple variables across multiple sets that usually occur independently of each other in the constraint formulation. In the case of (1j), the constraint couples variables across all time steps and periods, whereas they usually are only

coupled for time steps and are decomposable by period. The method presented in this paper applies iterative extreme periods to problems with an emissions constraint.

While this formulation includes intra-period storage, complicating constraints such as inter-period storage would require additional considerations to ensure that the presented method converges to zero lost load. Other complicating constraints such as ramping constraints could be included in the above formulation, but have been left out to focus on the core problem. Ramping constraints would be applied to every time step within each period.

The GCEP problem formulation in Equation (1) jointly optimizes design and operations for  $K = |\mathcal{K}|$  periods. We refer to this problem as  $D_K \& O_K$  in the remainder of this paper. Note that for  $D_K \& O_K$ ,  $\mathcal{K}$  is the set of representative periods:  $\mathcal{K} = \mathcal{R}$ . By reducing the number of periods, both the number of variables and constraints is reduced, which results in reduced computational complexity. Also note that the time varying input data all occurs in the demand constraint (1b).

## 2.2. Operational validation

After obtaining generation capacity  $C$  by running  $D_K \& O_K$ , we validate the operations of this candidate design on the full input data. We refer to the problem with fixed design and full operations as  $D_K \& O_{full}$  in the remainder of this paper. In Section 3.2, we describe how we use information from  $D_K \& O_{full}$  to identify extreme periods.

The formulation of  $D_K \& O_{full}$  is similar to the formulation in Equation (1), but with the following adjustments:

(i) The generation capacities are fixed to the capacities  $C^*$  obtained from  $D_K \& O_K$  by adding the constraint

$$C_{gn} = C_{gn}^* \quad \forall g \in \mathcal{G}, n \in \mathcal{N} \quad (2)$$

(ii) The set  $\mathcal{K}$  is the set of all periods contained in the full dataset:  $\mathcal{K} = \mathcal{P}$ . For instance, for a year's worth of data,  $|\mathcal{K}| = 365$ .

(iii) We add slack variables "lost load"  $LL$  to the right-hand side of the demand constraint (1b). This is necessary because the system design that is found based on few representative periods  $K$  may not be able to satisfy demand at all times. The right-hand side of (1b) becomes

$$\bar{D}_{nkt} + LL_{nkt} + \sum_{m \in \mathcal{N}} f_{nm}(1 - l_{nm}) \Delta t - \sum_{m \in \mathcal{N}} f_{mn}(1 - l_{mn}) \Delta t \quad (3)$$

(iv) We add a slack variable "excess emissions"  $EE$  to the right-hand side of the emissions constraint (1j). System designs based on time-series aggregation may overshoot emissions targets when operated on the full input data because wind is challenging to aggregate (34) (see Section 4.2 for a more detailed elaboration on excess emissions). This constraint allows for the operational problem to be feasible, which allows us to determine the amount of lost load that occurs, which in turn allows us to find extreme periods that result in reliable system designs. The right-hand side of (1j) becomes

$$e^{target} \sum_{g \in \mathcal{G}} \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} E_{gnkt} + EE \quad (4)$$

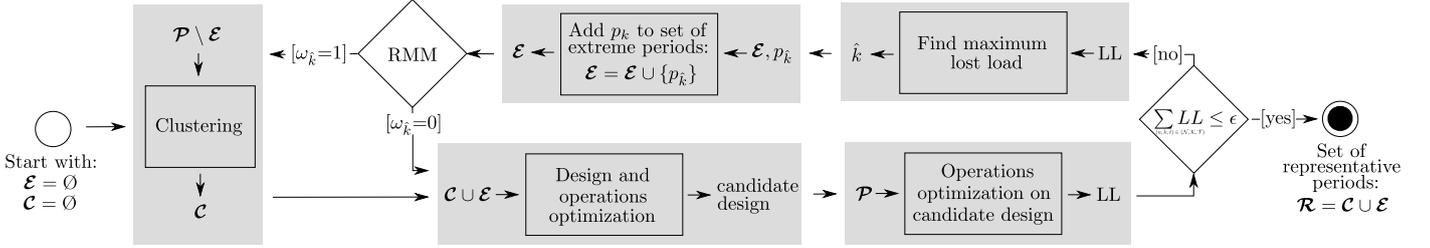


Figure 1: Hybrid clustering approach: The set of all periods  $\mathcal{P}$  is reduced to the set of representative periods  $\mathcal{R}$ , which consists of the set of clustered periods  $\mathcal{C}$  and the set of extreme periods  $\mathcal{E}$ . The members of  $\mathcal{E}$  are found as the days of maximum lost load  $LL$  of the optimization problem. RMM stands for representation modification method.

(v) We add both excess emissions and lost load with associated costs  $c^{LL}$  and  $c^{EE}$  to the objective function (1a). It is changed to

$$\min_{\substack{C, E, \tilde{E}^{in} \\ E^{out}, SOC, f}} \sum_{g \in \mathcal{G}} \sum_{n \in \mathcal{N}} \hat{c}_g C_{gn} + \sum_{k \in \mathcal{K}} \omega_k \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}_{\mathcal{T}}} \tilde{c}_g E_{gnkt} + \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} c^{LL} LL_{nkt} + c^{EE} EE \quad (5)$$

The values for  $c^{LL}$  and  $c^{EE}$  have to be chosen so that each slack variable acts as needed.  $LL$  should be used only when demand cannot be met.  $EE$  should be used when the emissions target cannot be met. Because no emissions are associated with  $LL$ , it could theoretically also be used to satisfy the emissions constraint by replacing power generation from  $CO_2$ -emitting generators. However, we want to ensure that the effects of lost load and excess emissions are separable in order to identify periods that are most important for the reliability of the system for our iterative extreme period inclusion method. It is thus important to ensure that  $EE$  and not  $LL$  is used when the emissions target cannot be met. This can be done by having the cost of excess emissions lower than the cost of lost load considering the dirtiest generator:

$$c^{EE} < \frac{c^{LL}}{\max_g e_g} \quad (6)$$

Because the shadow prices on (3) and (4) are not only influenced by  $c^{LL}$  and  $c^{EE}$ , we recommend to choose  $c^{EE}$  orders of magnitude lower than this bound. At the same time, we need to make sure that the emissions constraint is met to the best extent possible, i.e. if the emissions constraint is not met and a more expensive, but cleaner generator is available, we want the dispatch to run that generator, because this is also how  $D_K \& O_K$  would dispatch the generators. To do so, the cost of  $EE$  has to be high enough so that it is never economical to run a cheaper, but dirtier generator:

$$c^{EE} > \max_{i,j} \frac{\tilde{c}_i - \tilde{c}_j}{e_j - e_i} \quad (7)$$

where  $i \in \mathcal{G}_{\mathcal{T}}$  is a more expensive, but cleaner generator than  $j \in \mathcal{G}_{\mathcal{T}}$ . In this paper, based on the considerations outlined above, we use  $c^{LL} = 10,000 \text{ \$/MWh}$  and  $c^{EE} = 1 \text{ \$/kgCO}_2$ .

Separating the effects of the emissions constraint, which is a complicating constraint in the operational problem  $D_K \& O_{full}$ ,

from the use of the lost load slack variables is a major contribution of this work.

### 3. Time series aggregation

In this paper, we propose a hybrid clustering approach that finds out of all original periods  $\mathcal{P}$  a set of representative periods  $\mathcal{R}$  that results in reliable system designs. Figure 1 illustrates our approach.  $\mathcal{R} = \mathcal{C} \cup \mathcal{E}$  consists of clustered periods  $\mathcal{C}$  and iteratively added extreme periods  $\mathcal{E}$ . In this section, we present the clustering method used, our extreme period inclusion methods, and a simple extreme period inclusion method for comparison.

#### 3.1. Clustering

We follow the framework introduced by (11) when describing the clustering method used in this paper. We normalize all data types  $d$  (electricity demand, solar availability, wind availability) per node  $n$  individually using z-normalization, where data is normalized to mean  $\mu_{nd}=0$  and standard deviation  $\sigma_{nd}=1$ . We use hierarchical clustering using Ward's algorithm (39) and use the cluster centroid as cluster representation. The weight  $\omega_k$  associated with each cluster centroid is the number of periods represented by that centroid.

Many other clustering methods exist that have been used in energy systems optimization. Previous work has shown that for problems where time-series occur in constraints that are binding, once a certain number of clusters is part of the set of representative periods (35), the most benefit in terms of optimization outcome is gained by adding relevant extreme periods. An initial analysis confirmed this for our work, and we thus use  $|\mathcal{C}|=5$  clusters using the clustering method described above throughout this paper.

#### 3.2. Proposed iterative extreme period inclusion method

Figure 1 illustrates our iterative approach. After finding the clusters  $\mathcal{C}$ , we run the design and operations optimization  $D_K \& O_K$  to obtain a candidate design, i.e. generation capacities. We use these generation capacities to run the operations optimization  $D_K \& O_{full}$ . We obtain lost load  $LL_{nkt}$  for each constraint in Equation 3, i.e., for each node  $n$ , cluster  $k$ , and time step  $t$ .

After obtaining  $LL$ , we can check the termination criterion. It evaluates whether the cumulative lost load is below a certain threshold  $\epsilon$ :

$$\sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} LL_{nkt} \leq \epsilon \quad (8)$$

If (8) is true, the set of representative periods that yields a reliable system design is  $\mathcal{R} = \mathcal{C} \cup \mathcal{E}$ . If (8) is not true, further iterations are needed to modify  $\mathcal{R}$  to eventually yield a reliable system design.

This is done by adding the period  $p_{\hat{k}} \in \mathcal{P}$  with the maximum lost load to the set of extreme periods  $\mathcal{E}$ . There are several ways to define maximum lost load. In this paper, we consider two. First, we consider the day with absolute maximum lost load in an hour:

$$\hat{k} = \operatorname{argmax}_k \max_{n,t} LL_{nkt} \quad (9)$$

Second, we consider the day with integrally the highest lost load across all hours:

$$\hat{k} = \operatorname{argmax}_k \sum_{n,t} LL_{nkt} \quad (10)$$

We then add the identified period to the set of extreme periods:

$$\mathcal{E} = \mathcal{E} \cup \{p_{\hat{k}}\} \quad (11)$$

The weight associated with the extreme period is either  $\omega_{\hat{k}}=0$  (often referred to as ‘‘feasibility steps’’ in the literature) or  $\omega_{\hat{k}}=1$  (often referred to as ‘‘append’’ in the literature) (35). ‘‘Feasibility steps’’ are periods that are not assigned a weight in the objective function, but only enter the constraint body. They ensure that the design variables are chosen such that demand constraints associated with the extreme period are satisfied, but the extreme periods do not enter the objective function directly. The ‘‘append’’ method adds every extreme period as a separate cluster with weight 1. If there are  $n$  extreme periods, the clustering algorithm then chooses the remaining clusters based on the remaining  $K - n$  periods. This new set  $\mathcal{E}$  is then used to modify the existing set of representative periods  $\mathcal{R}$ . If the representation modification is carried out with  $\omega_{\hat{k}}=0$ ,  $\mathcal{R} = \mathcal{C} \cup \mathcal{E}$  is the new set of representative periods and is evaluated on the design and operations optimization  $D_K \& O_K$ , which starts a new iteration of the algorithm. If the representation modification is carried out with  $\omega_{\hat{k}}=1$ , the cluster representations  $\mathcal{C}$  have to be recalculated. In order for the representative periods to have weighted averages equal to the original data, the extreme periods with weight 1 are excluded from the clustering process. For example, if we aim to obtain clusters from 365 days of data and have identified two of these days as extreme periods, we need to cluster the 363 remaining days. After recalculating the cluster representations, we can proceed with a new iteration of the design and operations optimization  $D_K \& O_K$ .

Note that as per algorithm design with separable lost load and excess emissions, a period will never occur twice as part of  $\mathcal{E}$ . If it is part of  $\mathcal{E}$ , the candidate design obtained from  $D_K \& O_K$  will have to satisfy the demand constraint (1b), and will thus result in zero lost load on the demand constraint (3) of  $D_K \& O_{full}$ . Because a period will at most occur once in  $\mathcal{E}$ , the algorithm

will converge to zero lost load. In the worst case, this will be the case with 365 representative periods, but in practice, many constraints of (3) of  $D_K \& O_{full}$  are non-binding and the associated periods are thus not required as part of  $\mathcal{E}$  to guarantee zero lost load.

Overall, according to the decision framework presented in (35), our approach can be classified as an iterative selection process with optimization-based identification measure and  $\omega_{\hat{k}}=0$  or  $\omega_{\hat{k}}=1$  as representation modification.

### 3.3. Simple extreme periods

The most-commonly used method in the literature (referred to as ‘‘simple’’ extreme period selection) is to use a pre-defined number of extreme periods that are identified based on statistical properties of the data itself (20; 24; 35; 36). Because we compare our iterative method to this method, we present ‘‘simple’’ extreme period inclusion in this section. Prior to clustering and evaluating the optimization problem, extrema of the electricity price  $\bar{D}$ , solar availability  $\bar{a}^s$ , and wind availability  $\bar{a}^w$  data are identified and used to select extreme periods. The period with the absolute maximum demand can be identified by

$$\hat{k} = \operatorname{argmax}_k \max_{n,t} \bar{D}_{nkt} \quad (12)$$

and the period with the integrally highest demand can be identified by

$$\hat{k} = \operatorname{argmax}_k \sum_{n,t} \bar{D}_{nkt} \quad (13)$$

The extrema for wind and solar availability can be similarly identified as the minima of  $\bar{a}_{nkt}^s$  and  $\bar{a}_{nkt}^w$ . Typically, one extreme period (absolute or integrally maximum electricity demand) is added to the set of representative periods, or three extreme periods (absolute or integrally maximum electricity demand, integrally minimum solar availability, and integrally minimum wind availability) are added to the set of representative periods. This is done either with weight  $\omega_{\hat{k}} = 0$  or with  $\omega_{\hat{k}} = 1$ , and for the latter, the extreme periods are excluded from the clustering process.

## 4. Results

We first evaluate the iterative extreme period inclusion method proposed in this work on a single case study. We then compare different identification measures and representation modifications for multiple case studies, study the cost implications of eliminating lost load, and then compare the proposed method to the most commonly used simple extreme period inclusion methods. Throughout our studies, we use  $|\mathcal{C}|=5$  representative periods found by clustering and the additional extreme periods. We evaluate the iterative extreme period inclusion method to find reliable system designs that result in  $\sum LL = 0$ .

### 4.1. GCEP Case Studies Explored

We consider a 10-node system of California, and a 16-node system of Germany. Each case study uses a year’s worth of data, and we consider emission limits of 500 kg $CO_2$ /MWh and

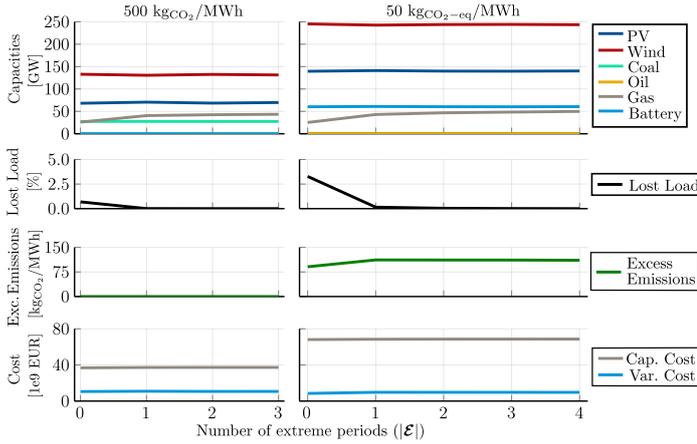


Figure 2: Capacity expansion results for a Germany case study using our iterative approach with  $C = 5$ ,  $\omega_{\hat{k}} = 0$ , and absolute identification measure, for emission targets of 500 kgCO<sub>2</sub>/MWh and 50 kgCO<sub>2</sub>/MWh.

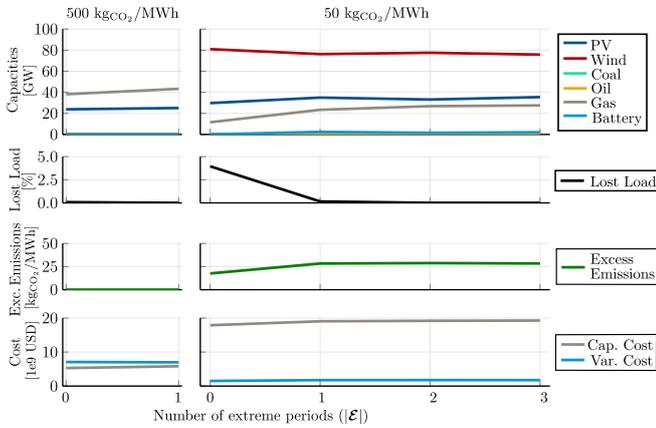


Figure 3: Capacity expansion results for a California case study using our iterative approach with  $C = 5$ ,  $\omega_{\hat{k}} = 0$ , and absolute identification measure, for emission targets of 500 kgCO<sub>2</sub>/MWh and 50 kgCO<sub>2</sub>/MWh.

50 kgCO<sub>2</sub>/MWh (Germany currently is at around 500 kgCO<sub>2</sub>/MWh (40)). While we use the units of kgCO<sub>2</sub>/MWh, we note that these numbers are technically CO<sub>2</sub> equivalent, because they include other emissions occurring during construction that are converted to CO<sub>2</sub> equivalent units. For California, we use data for the years 2014-2017, and for Germany, we use data for the years 2006-2016. Both the implementation and data are available online as the open-source software packages “TimeSeriesClustering.jl”<sup>1</sup> (41) and “CapacityExpansion.jl”<sup>2</sup> (42).

#### 4.2. Detailed Case Study Evaluation

Here, we consider the case study of a 16-node system of Germany with one year of data from 2016. Figure 2 shows the generation capacities from  $D_K \& O_K$ , and the lost load, excess emissions, and capital and variable cost of the operations problem with full input data  $D_K \& O_{full}$ , with extreme periods identi-

fied using the absolute maximum lost load and a representation modification method of  $\omega_{\hat{k}}=0$ .

For both the emissions limit of 500 kgCO<sub>2</sub>/MWh and of 50 kgCO<sub>2</sub>/MWh, we observe that electricity demand cannot be met when only five clustered representative periods ( $|\mathcal{E}|=0$ ) are used: lost load is at 0.7% and 3.3% of total demand, respectively. As we add extreme periods iteratively, periods that have the highest impact on system design are identified in order to reduce lost load. The first period alone that is added ( $|\mathcal{E}|=1$ ) results in reductions of lost load to 0.007% and 0.2% of total demand, respectively. The additional power is provided mostly through an increase in gas capacity. Because the load that was previously not satisfied is provided by a CO<sub>2</sub>-emitting generator, system emissions increase, and if the CO<sub>2</sub> emissions constraint (Equation (1j)) is binding in  $D_K \& O_K$ , this may result in an increase in excess emissions as we observe for the case of 50 kgCO<sub>2</sub>/MWh. The iteratively added periods thereafter result in a reliable system design with zero lost load. While the study here reduces lost load to zero<sup>3</sup>, one could set the threshold  $\epsilon$  to an amount acceptable to the decision maker and stop adding extreme periods as soon as the threshold is reached.

Overall, we observe that the iterative extreme period inclusion method realizes its goal of achieving a reliable system design by adding only a few extreme periods. We are able to reduce the number of representative periods for the case of 50 kgCO<sub>2</sub>/MWh from 366 to 9, which results in reduction of computational complexity of at least two orders of magnitude (7).

We furthermore observe that time-series aggregation results in exceeding the CO<sub>2</sub> emissions target by almost 100%. This is a phenomenon described and analyzed in detail in (34). Because wind availability data is smoothed during clustering and looks more similar to baseload in its aggregated form,  $D_K \& O_K$  results in high wind power capacities, at around 5-10% higher than if optimized on the full number of hours, and also in higher wind power utilization. However, because the wind data in its original form as run on  $D_K \& O_{full}$  is highly variable, there are times when gas generators are used in  $D_K \& O_{full}$  even though  $D_K \& O_K$  was planning on using wind power during the design phase. These gas generators have low utilization factors in  $D_K \& O_K$ , but higher utilization in  $D_K \& O_{full}$ . Because gas generators have higher emissions than wind turbines, this results in excess emissions. As the iterative method adds extreme periods and reduces lost load, we observe that excess emissions increase. This is because previously unserved load is now served by dispatchable generation, which in the case of this study is CO<sub>2</sub>-emitting. Thus, excess emissions increase when lost load is reduced. Note that in this paper, for reasons of simplicity, the only dispatchable generator available are fossil gas turbines. However, future systems will also likely include zero-emissions or negative-emissions dispatchable generators, which could be dispatched instead. In a case where these technologies are included and prices  $c_{EE}$  are high enough, excess emissions may not be present because the variable generation could be covered without additional CO<sub>2</sub> emissions. Exploration of these cases

<sup>1</sup><https://github.com/holgerteichgraeber/TimeSeriesClustering.jl>

<sup>2</sup><https://github.com/YoungFaithful/CapacityExpansion.jl>

<sup>3</sup>We set  $\epsilon=0.1$  MWh to account for numerical error.

Region	CO <sub>2</sub> Limit	Year	RMM: IM:	$\omega_k = 0$ Absolute	$\omega_k = 0$ Integral	$\omega_k = 1$ Absolute	$\omega_k = 1$ Integral
CA	50	2017		243	245, 243	243	245, 243
CA	50	2016		262, 281, 282	301, 281, 282, 262	262, 281, 282, 319	301, 281, 282, 319, 262
CA	50	2015		263, 285	9, 263, 285	263, 285	9, 263, 285
CA	50	2014		254, 276	211, 277, 276	254, 277, 276	211, 343, 277, 276
GER	50	2016		340, 328, 315, 349	299, 349	340, 315, 300, 349, 299, 67	299, 349
GER	50	2015		35, 20	20	35, 20	20
GER	50	2014		21, 323, 309	118, 324, 309	21, 323, 309	118, 21, 324, 309
GER	50	2013		25, 330, 16	330, 16	25, 16	330, 16
GER	50	2012		34, 320	61, 320	34, 320	61, 320
GER	50	2011		326, 31	31	326, 31	31
GER	50	2010		337, 355, 340, 330, 329	320, 330, 329	337, 355, 340, 330	320, 330, 329
GER	50	2009		335, 348, 35, 49	89, 35, 49	335, 35	89, 35
GER	50	2008		303, 351, 345	351, 345	303, 351, 345	351, 345
GER	50	2007		37, 353	38, 354, 353	37, 319, 354, 353	38, 354, 353
GER	50	2006		352, 34	33, 3, 34	352, 34	33, 355, 3, 34
CA	500	2017		244	244	244	244
CA	500	2016		208	208	208	208
CA	500	2015		253	253	253	253
CA	500	2014		258	259, 258	258	259, 258
GER	500	2016		340, 341, 21	347, 341, 21	340, 341, 21	347, 341, 21
GER	500	2015		20	20	20	20
GER	500	2014		338, 324	21, 324	338, 324	21, 324
GER	500	2013		330	23, 330	330	23, 330
GER	500	2012		34, 320	34, 320	34, 320	34, 320
GER	500	2011		326, 31, 321	31, 326, 321	326, 31, 321	31, 326, 321
GER	500	2010		337, 329, 341	330, 341, 329	337, 329, 341	330, 341, 329
GER	500	2009		335, 348, 35	35	335, 348, 35	35
GER	500	2008		303, 351	303, 351	303, 351	303, 351
GER	500	2007		353, 354	353, 354	353, 354	353, 354
GER	500	2006		352, 34, 32	33, 32, 34	352, 34, 32	33, 34, 32

Table 1: The indices  $k$  of the extreme periods selected by the different implementations of the iterative extreme value identification method to reach  $\sum LL = 0$  for different regions, CO<sub>2</sub> limits and years. Indices listed in the order of selection. RMM stands for representation modification method, and IM stands for identification measure.

in future work is recommended. Kuepper et al. (34) describe two challenges to be addressed for time-series aggregation for low carbon energy systems (e.g.  $\text{CO}_2$  emissions limits of  $50 \text{ kg}_{\text{CO}_2}/\text{MWh}$ ): (i) To reduce lost load in order to design reliable energy systems, and (ii) to design energy systems that emit the amount of  $\text{CO}_2$  that they were planned for. In this study, we address the first challenge. The second challenge is beyond the scope of this paper and left for future work.

Comparing how system designs change as we move to low-emissions systems, we observe that overall cost and capacities are higher, and that no coal capacity and more renewable and battery capacity is built in the  $50 \text{ kg}_{\text{CO}_2}/\text{MWh}$  than in the  $500 \text{ kg}_{\text{CO}_2}/\text{MWh}$  case. Furthermore, the cost structure is dominated by capital cost, and variable cost only plays a small role (21). The importance of capital cost also explains why extreme periods become much more important as we design low carbon emission systems: The size of generation capacity is mainly influenced by the demand constraints (1b), most importantly by the ones that are binding. Time series occur exclusively in the demand constraints, and the binding ones are binding due to extremes. The quality of the representation of extremes thus directly influences capital cost.

Figure 3 shows a similar study for the California case study with one year of data from 2016. The observations are qualitatively similar to the ones described above for the Germany case study.

#### 4.3. Comparison Across Methods

We have seen that the iterative extreme period inclusion method provides reliable systems designs. Now, we analyze whether there exists a combination of identification measure and representation modification that performs best. In Table 1, we report the extreme indices of the periods that were selected for the different case studies and the different combinations of identification measure and representation modification. The indices correspond to the day of the year, and we report the extreme indices in the order in which they were added: For instance, the four periods added for the case of  $50 \text{ kg}_{\text{CO}_2}/\text{MWh}$  in Figure 2 are 340, 328, 315, and 349. All combinations have zero lost load after the addition of the reported extreme periods.

Table 1 shows that over the full set of studies, all methods quickly converge to reliable system designs. Most often, one to three extreme periods are required, and the maximum is five. Some years, independent of the specific method used, require fewer extreme periods, and some more. This depends on the characteristics of the data, but we cannot predict it a priori. Furthermore, as mentioned above, the more stringent the  $\text{CO}_2$  emissions limit is, the more important extreme periods become, i.e., more extreme periods are required to achieve reliable system designs.

We also observe that for a given case study, there are different sets of periods that result in reliable system designs. For example, in the case of Germany for 2014 with  $50 \text{ kg}_{\text{CO}_2}/\text{MWh}$  emissions limit, only period 309 is shared by two different sets; the other periods are different. The fact that period 309 is chosen indicates that it is important to system design, and we would like to note that “simple” extreme period inclusion methods (see

Section 3.3) do not identify period 309 as an extreme period. Based on additional studies we have conducted but do not report here, we also observe that for the same case study, different methods may lead to reliable system designs with entirely disjoint sets of extreme periods.

Overall, we observe that no method is in all cases superior to all others concerning the number of extreme periods required, neither in terms of identification measure (absolute vs. integral), nor in terms of representation modification method ( $\omega_{\bar{k}}=0$  vs.  $\omega_{\bar{k}}=1$ ).

There are cases where identification based on absolute maximum requires fewer extreme periods than identification based on integral (e.g. California,  $50 \text{ kg}_{\text{CO}_2}/\text{MWh}$ , 2016), and vice versa (e.g. Germany,  $50 \text{ kg}_{\text{CO}_2}/\text{MWh}$ , 2016). One plausible explanation for the performance differences of different identification methods could be that when storage plays an important role in the solution of the optimization problem, integral identification may be favorable, because it finds the extremum per period within which storage can move loads around temporally. On the other hand, if storage does not play an important role, absolute extremes may be more successful in capturing characteristics relevant to the GCEP optimization problem.

Concerning representation modification, we observe that for most of the cases we study,  $\omega_{\bar{k}}=0$  achieves reliable system designs with fewer or the same number of extreme periods than  $\omega_{\bar{k}}=1$ . However, there are cases where the opposite is true as well, so this is not a general property.

The representation modification method has one important effect on extreme period selection:  $\omega_{\bar{k}}=1$  requires recalculation of the clusters, whereas  $\omega_{\bar{k}}=0$  does not. This has implications for extreme period selection. Adding extreme periods iteratively with  $\omega_{\bar{k}}=0$  at each step of the iteration results in designs that in  $D_K \& O_{full}$  at least satisfy the same load as the design from the previous step, plus the load of the extreme period added. However, this is not the case for  $\omega_{\bar{k}}=1$ : Because at each step of the iteration, the extreme period is excluded from the clustering and  $|C|$  is recalculated, the design in that iteration will in  $D_K \& O_{full}$  satisfy the load of the extreme period but not necessarily the same load in all other periods as it does in the  $D_K \& O_{full}$  of the previous iteration. An example of this can be seen for California 2016 with  $50 \text{ kg}_{\text{CO}_2}/\text{MWh}$  and absolute identification measure. Both choose periods 262, 281, 282 as the first three extreme periods, and the problem with  $\omega_{\bar{k}}=0$  results in zero lost load. But because the clusters  $|C|$  look different for  $\omega_{\bar{k}}=1$ , an additional period is required to get to zero lost load for that method.

Reliable system designs of different methods yield almost identical results with regard to generation capacity for the same year. Comparing results from reliable system designs across years, there are differences in capacity buildout. These differences are mostly in terms of how much backup dispatchable generation is built. This is likely because different years have different sets of extreme periods, some more extreme than others, as also shown by Pfenninger (20). This leads to the conclusion that for long-term investment planning used for decision-making, the current method should be applied to multiple years of data.

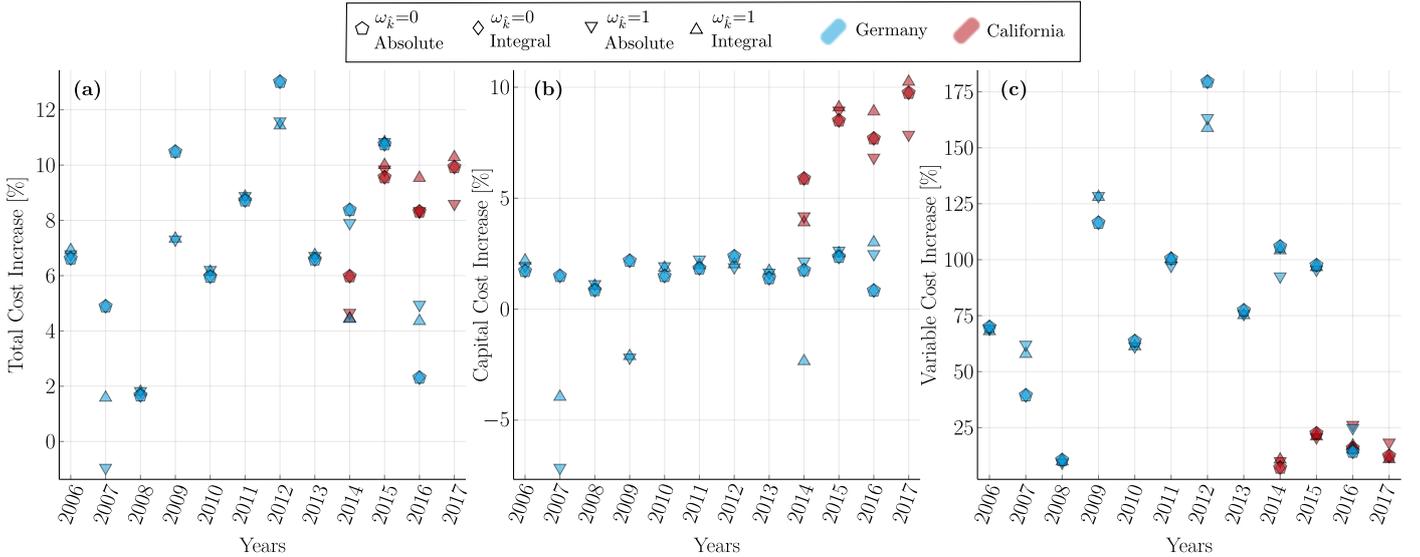


Figure 4: Relative increase in (a) total cost, (b) capital cost, and (c) variable cost of the system resulting from the last iteration of the iterative extreme period identification method ( $\sum LL = 0$ ) compared to the system resulting from clustering only ( $|\mathcal{E}| = 0$ ). The emissions limit is  $50 \text{ kg}_{CO_2}/\text{MWh}$ .

Overall, we observe that the iterative extreme period inclusion method presented in this paper results in the design of reliable energy systems on many different case studies with few extreme periods required. No identification measure and no representation modification method is generally superior. It depends on the data and case study at hand and is difficult to predict a priori. Importantly, all methods result in reliable energy system designs, and differences in terms of how many extreme periods are required are small for a given case study (1-2 periods difference). Thus, we cannot make a general recommendation, but emphasize that  $\omega_k=0$  has the advantage that it does not require reclustering of  $|C|$ .

#### 4.4. Comparison to clustering alone

Figure 4 shows the relative increase in total cost, capital cost, and variable cost of the system resulting from the last iteration of the iterative extreme period identification method ( $\sum LL = 0$ ) compared to the system resulting from clustering only ( $|\mathcal{E}| = 0$ ). Total cost is the sum of capital cost and variable cost, where capital cost is the annualized cost to install the capacity, and variable cost is the cost to operate the generators on the full input data. Note that these costs do not include the cost of lost load and excess emissions.

In order to achieve reliable system designs for  $50 \text{ kg}_{CO_2}/\text{MWh}$ , the average increase in total cost among all Germany case studies is 6.8% and is 8.3% among all California case studies. The average increase in capital cost among all Germany case studies is 1.2% and is 7.7% among all California case studies, and the average increase in variable cost among all Germany case studies is 79.7% and is 15.6% among all California case studies.

We observe that for representation modification of  $\omega_k=0$ , the cost are the same for integral and absolute identification measures. Because for both methods,  $|C|$  is the same, they yield the same system design when achieving zero lost load, and thus result in the same capital and variable cost. For the same reason,

we observe that for representation modification of  $\omega_k=0$ , capital cost are always increasing when adding extreme periods.

On the other hand, for  $\omega_k=1$ , results can differ between different identification measures because  $|C|$  can be different depending on which extreme periods have been chosen. In the case of  $\omega_k=1$ , we also observe that for some case studies, capital cost and as a result total cost are decreasing. This is because by having a different  $|C|$  for the last iteration with extreme periods compared to the result with clustering only, it is possible to find a solution that reduces capital cost and still meets overall demand on the full operations problem.

Variable costs are increasing for all reported cases. This is because lost load that occurs in the operations based on a system designed with clustering only is substituted mostly by natural gas, which has fuel cost associated with it. Overall however, the effect on total cost is small, because variable cost make up for a much smaller amount of total cost than capital cost (see Figure 2).

#### 4.5. Comparison to simple extreme period inclusion

We compare the performance of the state-of-the-art method of adding “simple” extreme periods based statistical properties of the data itself to the iterative extreme period inclusion method proposed in this paper. We explore two simple extreme period inclusion methods: (i) To add a single period based on the maximum absolute electricity demand, and (ii) to add three periods, one based on the minimum integral solar availability, one based on the minimum integral wind availability, and one based on the maximum absolute electricity demand. We add the extreme periods for both methods with  $\omega_k=0$  to the set of clusters.

Figure 5 shows the lost load on  $D_K \& O_{full}$  as percentage of total system load resulting from designs based on clustering without extreme periods, one simple extreme period, and three

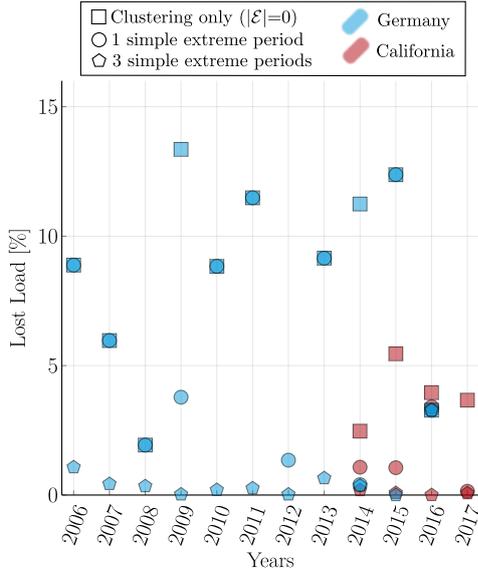


Figure 5: Lost load as a percentage of total system load for an emissions limit of  $50 \text{ kg}_{\text{CO}_2}/\text{MWh}$ . Simple extreme period inclusion reduces lost load, but not to zero. The proposed iterative extreme period inclusion method (not shown here) results in zero lost load.

simple extreme periods. Designed without extreme periods, lost load is between 13.5% and 1.9% of total system load.

With a single simple extreme period, lost load is reduced in some cases, and lost load is the same as for a system designed without the extreme period in other cases. Overall, adding a single extreme period based on maximum demand is not a satisfactory strategy to achieve reliable system designs.

Adding three simple extreme periods is able to reduce lost load to 0.0003% - 3.3% of total system load, with most cases in the range of 0.1%-1.0%. Even though a significant improvement in terms of reliability, these designs do not meet common reliability criteria. A common reliability metric is normalized expected unserved energy (normalized EUE) (43), which is the expected amount of load not served divided by the total system load. Our relative lost load can be interpreted as similar and compared to this metric. Acceptable normalized EUE is usually on the order of 0.001% (43), and all but one of the case studies considered fall above that threshold. Thus, we find that it is not sufficient to use simple extreme periods, and the only method to consistently achieve reliable system designs is the proposed iterative extreme period inclusion method.

While simple extreme periods reduce lost load, they, similar to iterative extreme periods in Figure 2 and Figure 3, result in increased excess emissions compared to the case with clustered data only. Iterative extreme periods result in an increase in excess emissions because the reduction in lost load is served by dispatchable generation, which is  $\text{CO}_2$ -emitting. Simple extreme periods reduce some portion of lost load as well, and this load is served by dispatchable generation. Adding simple extreme periods thus results in an increase in excess emissions proportional to a reduction in lost load.

We initially conducted experiments for other methods to find simple extreme periods, such as using a combination of

wind, solar, and demand (i.e. residual load such as in (44)). These methods resulted in qualitatively similar results to the ones presented here.

## 5. Conclusion

GCEP are formulated as optimization problems that are often computationally intractable when using the full set of time-series input data. The challenge for low  $\text{CO}_2$  emission studies is that extremes in the data play an important role, and that they are challenging to identify a priori to evaluating the optimization itself. In this work, we present a hybrid method that generates a set of representative periods partially based on clustering, and partially based on iteratively adding extreme periods. The proposed method separates the effects of system reliability and the operationally complicating constraints, i.e., a yearly  $\text{CO}_2$  emissions limit. Thus, the proposed method can identify extreme periods that are relevant for reliable system design.

Our analysis shows that the proposed iterative extreme period inclusion method realizes its goal of achieving a reliable system design by adding only a few extreme periods. We show that independent of the specific implementation that is used to identify extreme periods (either based on absolute or integrally maximum slack variables), and independent of the way extreme periods are assigned a weight as part of the set of representative periods (either weight  $\omega_k=0$  or  $\omega_k=1$ ), the method converges to a reliable system design. We show that this is the case on a total of 30 case studies in two regions, for  $\text{CO}_2$  emissions limits of  $50 \text{ kg}_{\text{CO}_2}/\text{MWh}$  and  $500 \text{ kg}_{\text{CO}_2}/\text{MWh}$ .

We furthermore show that a pre-selected number of simple extreme periods identified based on statistical properties is not sufficient to meet common reliability criteria.

This paper provides a method to include extreme periods in time-series aggregation when designing future energy systems with low  $\text{CO}_2$  emissions. Avenues for future research are to develop methods to find extreme periods when seasonal storage is part of the GCEP formulation. Furthermore, one challenge that was previously described in the literature and that we confirm in this study is that time-series aggregation can result in exceeding  $\text{CO}_2$  emissions limits. It is thus of interest to develop methods that address this challenge by reducing excess emissions. Further research may explore integrating multiple years of input data and developing out-of-sample validation methods.

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