Determining locations and layouts for parcel lockers to support supply chain viability at the last mile

Michael Kahr

University of Vienna, Research Network Data Science, Vienna, Austria.
m.kahr@univie.ac.at

July 13, 2022

Abstract

The pandemic caused by the corona virus SARS-CoV-2 raised many new challenges for humanity. For instance, governments imposed regulations such as lockdowns, resulting in supply chain shocks at different tiers. Additionally, delivery services reached their capacity limits because the demand for mail orders soared temporarily during the lockdowns. We argue that one option to support supply chain viability at the last-mile delivery tier is to use (outdoor) parcel lockers through which customers can collect their orderings 24/7 while ensuring physical distancing. The location planning of such lockers is known to be of utmost importance for their success. Another important topic to address is that the design of the compartment structure of the parcel lockers should meet the (uncertain) customer demand for different commodities. Both of the latter planning issues are combined into one optimization problem in this article. The objective is to maximize a linear function (e.g., expected profits) of the covered demand, given a budget an operator is willing to invest. An integer linear programming formulation is proposed, and a reformulation based on Benders decomposition is derived. It is shown that the Benders cuts can be separated in linear time. The developed algorithms enable solving of large-scale problem instances demonstrated by a performance analysis of computational experiments. The impact of different problem parameters on the obtained solutions is demonstrated by a sensitivity analysis. A case study based on real-world data from Austria is presented. The results show that using parcel lockers can support supply chain viability at the last-mile delivery tier. Moreover, the relatively small investment cost yields promising returns. The results further indicate that small-sized and medium-sized compartments should be preferred over large and x-large ones in the parcel locker compartment design.

Keywords: Parcel locker location; Parcel locker layout; Supply chain viability; Integer linear programming; Benders decomposition

1 Introduction

The pandemic proclaimed by the World Health Organization due to the proliferation of the corona virus SARS-CoV-2 in the year 2020 posed many new challenges for humanity. To decelerate the spread of the virus and to avoid overloading existing healthcare capacities, governments all over the world imposed regulations such as closed borders, partial lockdowns of the economy, stay-at-home orders, and physical distancing. These regulations shocked different tiers of local and global supply

chains (SCs) while demand and supply ripples were observed [46]. On one hand, customers changed their buying behavior such that the demand for mail orders soared in most economies during the lockdowns [1]. Many industries were consequently affected by the *bullwhip effect* in which small variations in demand have large recurrent effects on SCs. On the other hand, many industries faced production disruptions that propagate through SCs downstream; a phenomenon known as the *ripple effect* [10]. Both of the latter effects occurred simultaneously during the pandemic, which resulted in a disruption of the overall economy, and in substantial reverberations in some economies [46]. Scarpin et al. [47], however, recently proposed that the observed SC disruptions caused by the bullwhip effect were statistically insignificant within the first 3.5 months after the pandemic was declared. The pandemic was nevertheless a strong test of the *viability* of SCs and demonstrated that their adaption capabilities play a crucial role in such situations [23]. SC viability is defined by Ivanov et al. [24] as the "ability to survive and exist after a disruption [...] with the re-design of the supply chain structure and re-planning economic performance with long-term impacts". Ivanov [22] also recently proposed the *viable SC model* suggesting that SCs should quickly adapt between agility, resilience, and sustainability.

One option to support SC viability is to consider alternative modes of transport at the last mile, because conventional door delivery is labor-intensive and time-consuming [46]. The aforementioned soaring in demand for mail orders during the lockdowns therefore challenged courier, express and parcel (CEP) delivery services, particularly with respect to their capacity limits. Recently considered alternative last-mile delivery concepts that support relieving the delivery workforce include the use of autonomous vehicles such as delivery robots or drones; see [5, 45] for comprehensive surveys. The latter concepts are, however, accompanied with legal aspects such as liability issues, which typically cannot be resolved in the short term [21]. Starting to implement last-mile delivery networks of robots or drones during SC shocks is therefore likely to fail. Also crowd-shipping concepts in which private people act as the last-mile delivery workforce are increasingly studied [30] and implemented in practice (e.g., amazon flex [2]). Crowd-shipping is, however, similar to door delivery, hardly supporting physical distancing during pandemics. Moreover, deliverers may be affected by stay-at-home orders (or quarantine).

A promising last-mile delivery concept during pandemics is the use of automatic self-service (outdoor) parcel lockers. CEP deliverers can be relieved hereby because they can deliver several shipments at a stroke. The short-term usage of parcel lockers therefore enables expanding of delivery capacities while maintaining the existing delivery workforce. Additionally, the overall delivery (and environmental) cost can be thereby reduced [35, 38, 54]. Using parcel lockers for last-mile delivery further poses less legal issues than using autonomous vehicles (i.e., parcel lockers can be set up comparably fast). Further favorable properties of parcel lockers include that they (i) support physical distancing, (ii) are easy to build and maintain, (iii) enable collection (and return) of mail orders 24/7, (iv) enable location-specific (modular) design, (v) can be powered by batteries or solar panels, (vi) are operable under most geographic and climatic conditions, (vii) exist in variants suitable for (frozen) food, (viii) can be located at many places in urban areas (e.g., parking areas), and (ix) are available as mobile variants (e.g., locker trucks [49]). Last-mile delivery via parcel lockers has therefore appealing properties compared to different modes of transport so that their usage is not limited to pandemics.

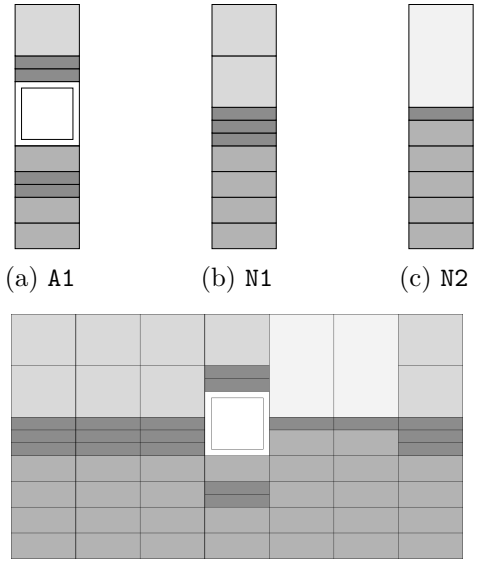
Although parcel lockers can be set up relatively fast, starting to implement them from the scratch during SC shocks might be nevertheless too slow to support SC viability at the last mile. One option to overcome this is to warehouse a backup fleet of parcel lockers, which can be set up

quickly in case of a soaring demand for mail orders. The warehousing of parcel lockers may be performed, e.g., by large CEP delivery operators such as Amazon or by (local) authorities. In the latter case, one option is to provide *white-label parcel lockers* that are publicly (or privately) owned, and each CEP operator may use them (against a service fee) [42]. Alternatively, one can resort to parcel locker rental services; however, this option is not safe because rental services may run out of stock.

Challenges arising when planning distribution networks of parcel lockers include the decision on where to locate them. The location planning for such lockers is known to be the utmost important factor for success by reason of the customers' limited willingness to invest travel time to collect their orderings [5]. Moreover, deciding on the size and the compartment structure of parcel lockers is challenging but crucial to meet the customer demand for different commodities. For instance, small parcels are ordered more often than large ones (cf., Section 6). Another challenging aspect to consider is the uncertain customer demand, especially during a pandemic. The aforementioned challenges are addressed in this article from the perspective of an operator (e.g., CEP delivery operator or authority) aiming to maximize a linear function (e.g., expected profits) of the customer demand for different commodities covered by parcel lockers (within one replenishment period). The focus is on determining optimal locations for parcel lockers together with their location-specific *locker configuration*. A locker configuration consists of locker modules providing compartments of (possibly) different sizes and features (e.g., cooling). An illustration of such modules is given in Figure 1. Figure 1a shows a base module containing a control unit, Figures 1b and 1c depict two optional modules, and Figure 1d illustrates a locker configuration. The locker modules contain compartments with four different sizes indicated by different shade. The base module is typically mandatory whereas optional modules can be arbitrarily combined up to a limit on the number of modules set by the manufacturer.

Although there exists (limited) research on parcel locker location planning and on the compartment layout design separately, to our knowledge, no work combines both of the latter decisions into one optimization framework. Note that the importance of pursuing this research avenue is also emphasized by Boysen et al. [5] and by Rohmer and Gendron [44]. Based on the suggestions of the latter authors, we propose and study the novel *stochastic multi-compartment locker location problem* (SMCLLP) in which both location and layout decisions are combined. The main aims include (i) studying one option to support SC viability at the last-mile delivery tier, (ii) developing an optimization framework that enables solving large-scale problem instances, and (iii) deriving new insights for researchers and managerial implications for practitioners based on new real-world data. The contributions are as follows. In Section 2, we summarize the related literature, focusing on parcel locker location planning and compartment layout design. In Section 3, we define the SMCLLP in the spirit of multi-commodity multi-type facility location problems and prove its NP-hardness. In Section 4, we propose a (to our knowledge) novel integer linear programming (ILP) formulation based on the concept of locker configurations (i.e., multi-type facilities). We further prove that a set of used integer variables can be relaxed without harming the existence of optimal integral solutions. We use the latter result to derive a reformulation based on Benders decomposition [4] in the spirit of a recent successful approach for solving location problems [8]. We further prove that the derived Benders cuts can be separated in linear time. In Section 5, we outline the used algorithmic framework and propose a preprocessing procedure that supports avoiding solutions in which parcel lockers have excess idle capacity. In Section 6, we demonstrate the performance of the latter framework in an extensive computational study using (artificial and) real-world data. We

Figure 1: Illustration of parcel locker configuration consisting of different locker modules (`B1`, `N1`, `N2`).



(a) `A1` (b) `N1` (c) `N2`

(d) Exemplary locker configuration

Note. The locker modules provide four different compartment sizes indicated by different shade. The base module `A1` provides a control unit, and modules `N1` and `N2` are optional. The exemplary locker configuration (Figure 1d) consists of these locker modules.

further derive new insights for practitioners based on a sensitivity analysis of the obtained results. In Section 7, we present a case study in which Austrian census data and results from a representative survey we conducted are used. Finally, we draw conclusions from the presented work, discuss limitations of the proposed approach, and point to future research directions in Section 8.

2 Related literature

The SMCLLP belongs to the large family of facility location problems (FLPs), which are of high practical relevance in numerous applications. Examples of application areas include (humanitarian) supply chain logistics [13, 15], (COVID-19) vaccination planning [52], (preventive) disaster management [29], traffic infrastructure planning [27], agriculture [20], and social network analysis [26]. To classify the SMCLLP into the family of FLPs, it is a stochastic, single-objective (or pseudo multi-objective), capacitated, single-source, single-period, multi-commodity, multi-type FLP defined in a discrete action space. Note that the latter combination of FLP variants has not yet been studied to the best of our knowledge. For more examples of FLP variants and application areas, we refer to comprehensive surveys [14, 53] and proceed with discussing literature related to parcel lockers.

Although research on parcel lockers is growing, most articles are related to economic or environmental aspects of using parcel lockers (e.g., [36, 42]) or customer experience and adoption (e.g., [28, 58]). Research is also growing on using parcel lockers in vehicle routing problems (e.g., [11, 35, 37]). The related work on parcel locker location problems is, however, limited while literature on the locker layout design is very sparse. The remainder of this section focuses on the latter two streams of research.

Deutsch and Golany [9] considered a problem of determining the optimal number, location, and

size for parcel lockers to maximize the overall profit of an operator. Parcel lockers were assumed to have unlimited capacity (for one type of commodity). Customers were assumed to be willing to collect parcels from each potential location with their willingness negatively correlating with the distance to those locations. The authors showed that the considered problem can be formulated as a classical uncapacitated FLP formulation. A case study related to an operator located in Toronto, Canada, was presented in which 96 potential locations for parcel lockers were considered.

Luo et al. [33] studied a multi-objective parcel locker location problem. The objectives include maximizing the accessibility of parcel lockers to customers while minimizing the setup and procurement costs. The authors proposed a multi-objective ILP to solve the problem. Besides, the paper basically focused on the development of an active-learning Pareto evolutionary algorithm to tackle the problem. The authors demonstrated that the considered objectives are conflicting in extensive computational experiments (on artificial and real-world problem instances based on data from Shenyang, China). Che et al. [6] studied a multi-objective parcel locker location problem, aiming to identify optimal locations for homogeneous parcel lockers. The objectives include maximizing the demand coverage, minimizing the number of customers simultaneously covered by multiple parcel lockers, and minimizing the total idle capacity of lockers. The authors proposed a mixed ILP formulation, however, they used heuristics to solve their problem in the computational experiments. They focused on deriving the Pareto-frontiers of the considered multi-objectives. The authors further observed the existence of inefficient solutions involving parcel lockers with excess idle capacity. The latter issue was addressed by Rabe et al. [43] who considered a multi-period parcel locker location problem modeled as a multi-period capacitated FLP. The locker network can evolve over time depending on the estimated customer demand. Minimum locker utilization rates were enforced by constraints. The objective was to minimize the overall cost, including service costs and fixed costs (for setting up parcel lockers). The problem was solved deterministically for different demand scenarios. The reliability of the obtained solutions was subsequently cross-evaluated on demand scenarios generated by Monte Carlo simulations. Two other multi-period problems were proposed by Xu et al. [55] to minimize the cost of locating attended (e.g., convenience stores) and unattended delivery points (e.g., parcel lockers), respectively. In the "attended model", a given expected customer demand (estimated with machine learning models using real-world data) needs to be covered. In the "unattended model", the number of delivery points are fixed; however, the delivery points are allowed to move from one location to another one each period at a given relocation cost. The results of a computational study showed a negative correlation between operational cost and service distance (attended model) and a positive correlation between demand coverage and number of (movable) delivery points.

Lin et al. [31] studied a parcel locker location planning problem using a multinomial logit customer choice behavior model to maximize the overall service level. A non-linear (non-concave) programming formulation was developed, and a linearization using McCormick inequalities was derived. The presented computational results were, however, based on heuristics. The authors emphasized that their model had forced optimistic revenue estimation, which had lead to unnecessarily high investment cost in a follow-up paper [32]. Therein, the authors used the threshold Luce model, which enables relaxation of the aforementioned optimistic revenue estimation. Two profit-maximizing integer programming formulations with concave objective function, and a corresponding reformulation with linear objective function and second-order cone constraints were proposed. The results showed that using the threshold Luce model enables obtaining a more balanced use of investment cost. Peppel and Spinler [41] studied a parcel locker location problem

to minimize both CO_2 equivalent emissions and economic cost. In a first step, the authors used a multinomial logit model to obtain the fraction of customers (aggregated into demand points) who prefer parcel locker delivery to home delivery. The obtained fractions are subsequently used in the linear objective function of the proposed ILP formulation that covers the aforementioned objectives. Real-world data from Europe, provided by an logistic service provider, was used to derive managerial insights. The latter include (among others) that parcel lockers should be installed at locations with a densely populated neighborhood and at frequently used routes based on both economic and environmental considerations.

Avgerinos et al. [3] proposed an FLP motivated by minimizing the cost of last-mile delivery using multi-type pick-up points (which may include parcel lockers). The setup and operating costs were assumed to increase non-linearly with the number (and type) of facilities. The latter's non-linearity was approximated with a piece-wise linear function, and a corresponding ILP formulation was proposed. The authors further used (meta)heuristics in the performed computational experiments whose focus is on the analysis of the algorithms' performance.

Lyu and Teo [34] studied a parcel locker location problem with the objective of reducing last-mile delivery traffic in the central business district of Singapore. The authors proposed a robust second-order cone formulation, aiming to minimize the proportion of parcels not delivered to parcel lockers while considering real-world (uncertain) customer demand profiles. They further showed that the customers' willingness to collect parcels drops off fast if the travel distance for collecting parcels exceeds 250 meters. Yang et al. [56] proposed a bilevel parcel locker location problem. In the upper-level problem, an operator aims to identify optimal locations for homogeneous parcel lockers such that the customer demand is met at minimum cost. In the lower-level problem, customers aim to minimize travel costs for collecting parcels. The problem was solved with a genetic algorithm which was also used in the presented case study based on data from Hunan, China.

Faugère and Montreuil [16] are to our knowledge the only authors who considered the design of the compartment layout of parcel lockers, however, neglecting location decisions. They proposed two ILP-based models for the design of parcel lockers focusing on (i) bin-packing compartments, and (ii) bin-packing predefined locker modules in the available space at the considered location. The objective was to maximize the profit of an operator while stochastic demand scenarios were considered.

As outlined in this section, the related literature addresses either parcel locker location problems or those related to the design of their compartment layout. Combining location and layout decisions into one optimization framework has not yet been addressed to the best of our knowledge. The main novelty of the proposed SMCLLP is to fill this research gap by explicitly considering the compartment layout of the parcel lockers. Moreover, all the formulations of the exact parcel locker location problem proposed by the authors mentioned in the section (except [34]) are limited to solving problem instances up to a few hundred potential locations. In contrast, our proposed approach enables to solve large-scale problem instances to proven optimality. We further use new real-world Austrian data to derive novel practical implications on parcel locker design.

3 Problem definition and properties

The SMCLLP is defined on a finite two-dimensional action space representing the planning area, and a finite probability space Ω of demand scenarios. Probabilities $p^\omega \in (0, 1]$, $\forall \omega \in \Omega$ are given such that $\sum_{\omega \in \Omega} p^\omega = 1$. The set of potential parcel locker locations is denoted by I, and the set

of customers is denoted by J. The geographical coordinates of the potential locations and those of the customers are known. A locker configuration $k \in K(M)$ consists of at least one locker module $m \in M$, where M and $K(M)$ denote the sets of all available locker modules and all possible locker configurations, respectively. For brevity, set $K(M)$ will simply be denoted by K in the remainder of this paper. Each locker configuration $k \in K$ is associated a construction cost $r_k \in \mathbb{R}_+$ depending on the number (and type) of locker modules it contains. Let ℓ_k denote the number of locker modules contained in locker configuration $k \in K$, and let $\ell^{\max}$ denote an upper bound on the number of locker modules that can be combined (e.g., a technical restriction given by the manufacturer). Moreover, there exist location-specific upper bounds q_i, $\forall i \in I$ on the number of locker modules that fit at a certain location. Then, set $K_i = \{k \in K : \ell_k \leq \min\{q_i, \ell^{\max}\}\}$, $\forall i \in I$ denotes the set of all locker configurations that fit at location i. Each locker module $m \in M$ provides (different) compartments suitable for (different) commodities from set C. Examples for commodity $c \in C$ include small parcels, large parcels, and frozen food of particular size (cf., Section 1). Commodities are further associated location-specific weights v_{ic}, $\forall i \in I$, $\forall c \in C$, which may correspond to profits of delivering commodity $c \in C$ to location $i \in I$ (cf., Remark 1 for more details). Commodities that are unsuitable for parcel locker delivery (e.g., because of size) are assumed to be shipped by different modes of transport (e.g., conventional door delivery).

The number of compartments (i.e., supply) for commodity $c \in C$ provided by configuration $k \in K$ is denoted by $s'_{kc} \in \mathbb{Z}_+$, $\forall k \in K$, $\forall c \in C$. Each compartment is assumed to contain at most one commodity. It is further assumed that only $s_{kc} \coloneqq \lfloor \eta s'_{kc} \rfloor$, $\forall k \in K$, $\forall c \in C$, $\eta \in (0, 1]$ of commodities can be replenished in each replenishment period, i.e., $s'_{kc} - s_{kc}$, $\forall k \in K$, $\forall c \in C$ commodities have not been collected by customers before replenishment. Thus, the *replenishment rate* η determines the number of commodities that can be delivered to parcel lockers regularly (e.g., daily); cf., Remark 2 for details. The scenario-dependent customer demand for commodity $c \in C$ is denoted by $d^{\omega}_{jc} \in \mathbb{Z}_+$, $\forall j \in J$, $\forall c \in C$, $\forall \omega \in \Omega$. Each customer can be assigned to at most one parcel locker within a given coverage distance $\delta \in \mathbb{R}_+$ in each scenario $\omega \in \Omega$. Note that the latter requirement avoids multi-source pick-ups, which may cause customer dissatisfaction. Further note that distance δ may refer to any appropriate distance measure (e.g., Eucledian distance, Manhattan distance, or lengths of shortest paths in street networks). Finally, note that we do not consider transportation cost of serving customers via parcel lockers, because they are implicitly paid by the customers.

The SMCLLP aims to identify a set $I^* \subseteq I$ of locations for building parcel locker configurations $k^* \in K_i \subseteq K$, $\forall i \in I^*$, which maximizes the expected location-specific weights of the covered customer demand for commodities from set C within one replenishment period, i.e., excluding $s'_{kc} - s_{kc}$, $\forall k \in K$, $\forall c \in C$. Moreover, a given budget $b \in \mathbb{R}_+$ for buying the parcel lockers needs to be respected.

Remark 1 (Location-specific weights). The objective of the SMCLLP is intentionally defined generic, i.e., location-specific weights on commodities v_{ic} enable to maximize several objective function metrics including: (i) Profits considering, e.g., costs for transporting commodities from a depot to the locker locations, and location-specific rents for placing lockers. (ii) Cost savings compared to conventional door delivery, which may differ per location, e.g., depending on population density. (iii) Customer satisfaction, e.g., customers may prefer collecting small parcels over collecting large parcels from lockers. (iv) Multi-objectives combined in a weighted-sum fashion, e.g., considering both profit and customer satisfaction.

Remark 2 (Replenishment rate). The number of commodities that can be replenished in parcel

lockers on a regular basis is an important performance indicator for CEP delivery operators. The latter number is clearly at least the number of parcels collected within one replenishment period. For instance, Schnieder et al. [48] and Lyu and Teo [34] report that 70% and 75% of parcels are collected within 24 hours, respectively. The replenishment rate η is, however, typically larger because additional parcels that reside longer than one replenishment period in a locker may be collected. Based on computational experiments, the latter rate seemingly converges fast to $\eta = 1/(1+x)$, where $x \in [0, 1)$ is the fraction of parcels that reside longer than one replenishment period in a parcel locker. For instance, if a replenishment period is 24 hours and 70% of the delivered parcels are collected within that period, we have $\eta = 1/1.3 \approx 77\%$.

Remark 3 (Number of locker configurations). By assumption, there exists an upper bound $\ell^{\max}$ on the number of locker modules that can be combined given by the manufacturer, and a finite set of (off-the-shelf) locker modules M. The number of locker configurations of size $q \leq \ell^{\max}$, $q \in \mathbb{Z}_+$ is therefore equal to the binomial coefficient

$$\binom{q + |M| - 1}{q},$$

i.e, the number of possibilities to assign $|M|$ locker modules to q bins while neglecting the order. The number of possible locker configurations respecting $\ell^{\max}$ is therefore

$$|K| = \sum_{q=1}^{\ell^{\max}} \binom{q + |M| - 1}{q}. \tag{1}$$

Note that set K, and thus $|K|$, can be trivially adapted in case the manufacturer requires a lower bound on the size of a locker configuration (e.g., at least three locker modules).

Remark 4 (Mobile lockers). Parcel lockers are available in the form of parcel locker vans that could be warehoused as a backup fleet to support SC viability. The SMCLLP can be easily adapted for determining the fleet size and parking locations for locker vans (given a set of potential locations). For this purpose, only the locker configurations that fit exactly into a van need to be considered, i.e., set K needs to be adapted accordingly. To obtain a homogeneous fleet of locker vans (if desired), the corresponding constraints need to be enforced additionally.

Finally, we conclude this section by showing the NP-hardness of the SMCLLP in the following Theorem 1.

Theorem 1. *The SMCLLP is NP-hard.*

Proof. Consider an instance of the SMCLLP in which $|K| = |C| = |\Omega| = \eta = 1$, and $s_{kc} = |J|$, $k \in K$, $c \in C$ (i.e., large enough to be able to cover the overall demand). Moreover, let configuration cost be $r_k = 1$, $k \in K$, weights $v_{ic} = 1$, $\forall i \in I, c \in C$, customer demands $d^{\omega}_{jc} = 1$, $\forall j \in J$, $c \in C$, $\omega \in \Omega$, and budget $b > 0$. Then, this instance corresponds to an instance of the generalized maximum covering location problem (GMCLP) [7] with zero assignment costs and with potential locations I' and customers J' whereby $I = I'$ and $J = J'$. The GMCLP seeks a subset of locations $I'^{*} \subseteq I'$ while respecting budget b' that maximizes that covered demand. Thus, if $b = b'$, the aforementioned instance of the SMCLLP is an instance of the GMCLP, which is known to be NP-hard. □

4 Models for solving the SMCLLP

In this section, we propose a stochastic ILP formulation (in the form of its deterministic equivalent) for solving the SMCLLP (Section 4.1). Moreover, a reformulation based on Benders decomposition is derived (Section 4.2). For convenience, we use $J_i \subseteq J$, $\forall i \in I$ to denote the set of customers within distance δ from location $i \in I$. Analogously, $I_j \subseteq I$, $\forall j \in J$ denotes the set of locations $i \in I$ within distance δ from customer $j \in J$.

4.1 Integer linear programming formulation

Let $y_{ik} \in \{0,1\}$, $\forall i \in I$, $\forall k \in K_i$ indicate whether or not locker configuration k is built at location i. Variables $z^{\omega}_{ij} \in \{0,1\}$, $\forall i \in I$, $\forall j \in J_i$, $\forall \omega \in \Omega$ indicate whether or not customer $j \in J_i$ is covered by location $i \in I$ in scenario $\omega \in \Omega$. Integer variables $x^{\omega}_{ijc} \in \mathbb{Z}_+$, $\forall i \in I$, $\forall j \in J_i$, $\forall c \in C$, $\forall \omega \in \Omega$ correspond to the demand for commodity $c \in C$ of customer $j \in J_i$ covered by location $i \in I$ in scenario $\omega \in \Omega$. Then, the SMCLLP can be formulated as

$$\max \quad \sum_{\omega \in \Omega} p^{\omega} \sum_{i \in I} \sum_{j \in J_i} \sum_{c \in C} v_{ic} x^{\omega}_{ijc} \tag{2a}$$

$$\text{s.t.} \quad \sum_{i \in I} \sum_{k \in K_i} r_k y_{ik} \leq b \tag{2b}$$

$$\sum_{k \in K_i} y_{ik} \leq 1 \qquad \forall i \in I, \tag{2c}$$

$$z^{\omega}_{ij} \leq \sum_{k \in K_i} y_{ik} \qquad \forall i \in I, \forall j \in J_i, \forall \omega \in \Omega, \tag{2d}$$

$$\sum_{i \in I_j} z^{\omega}_{ij} \leq 1 \qquad \forall j \in J, \forall \omega \in \Omega, \tag{2e}$$

$$\sum_{j \in J_i} x^{\omega}_{ijc} \leq \sum_{k \in K_i} s_{kc} y_{ik} \qquad \forall i \in I, \forall c \in C, \forall \omega \in \Omega, \tag{2f}$$

$$x^{\omega}_{ijc} \leq d^{\omega}_{jc} z^{\omega}_{ij} \qquad \forall i \in I, \forall j \in J_i, \forall c \in C, \forall \omega \in \Omega, \tag{2g}$$

$$x^{\omega}_{ijc} \in \mathbb{Z}_+ \qquad \forall i \in I, \forall j \in J_i, \forall c \in C, \forall \omega \in \Omega, \tag{2h}$$

$$y_{ik} \in \{0,1\} \qquad \forall i \in I, \forall k \in K_i, \tag{2i}$$

$$z^{\omega}_{ij} \in \{0,1\} \qquad \forall i \in I, \forall j \in J_i, \forall \omega \in \Omega\,. \tag{2j}$$

The objective function (2a) maximizes the expected weight of the covered customer demand (within one replenishment period). The number of parcel lockers that can be built is limited by budget in constraint (2b). Constraints (2c) ensure that at most one locker configuration per location can be built. Inequalities (2d) guarantee that customers can only be covered by a location in a certain scenario if a parcel locker is installed there. Constraints (2e) ensure that each customer is assigned at most one parcel locker per scenario. Inequalities (2f) bind the customer demand covered by a certain locker to its available supply for each commodity in each scenario. Constraints (2g) ensure that customers are supplied at most their demand of the corresponding commodities in each scenario.

4.2 Reformulation based on Benders decomposition

The number of variables in formulation (2) grows rapidly with the problem instance size, which impedes to solve practically relevant real-world instances with that model. A computationally more tractable reformulation of (2) is derived in this section. First observe that variables $\mathbf{x}$ can be relaxed in formulation (2) as shown in Theorem 2.

Theorem 2. *There exists an optimal integral solution* $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*)$ *of* (2) *if the coordinates of* $\mathbf{x}$ *are relaxed, i.e.,* $x^{\omega}_{ijc} \in \mathbb{R}_+$, $\forall i \in I, \forall j \in J_i$ $\forall c \in C$, $\forall \omega \in \Omega$.

Proof. Recall that each customer is assigned to at most one parcel locker in each scenario by constraints (2e). First focus on one specific scenario $\omega \in \Omega$, an arbitrary installed locker configuration $y^*_{ik} = 1$ at location $i \in I^* \subseteq I$ that supplies an arbitrary commodity $c \in C$ with $s_{kc} \geq 0$ to the covered customers $J^{\omega*}_i := \{j \in J_i, i \in I^* : z^{\omega*}_{ij} = 1\}$. Suppose that the demand of the covered customers in scenario ω is at most the locker supply, i.e, $\sum_{j \in J^{\omega*}_i} d^{\omega}_{ijc} \leq s_{kc}$. Then, the corresponding coordinates of variables $\mathbf{x}^*$ are integral because the respective constraints (2g) are tight, i.e., $x^{\omega*}_{ijc} = d^{\omega}_{jc} \in \mathbb{Z}_+$, $\forall j \in J^{\omega*}_i$. Conversely, suppose that the demand of the covered customers exceeds the locker supply, i.e., $\sum_{j \in J^{\omega*}_i} d^{\omega}_{ijc} > s_{kc}$. Then, the corresponding constraint (2f) is tight so that $\sum_{j \in J^{\omega*}_i} x^{\omega*}_{ijc} = s_{kc} \in \mathbb{Z}_+$. Thus, in case of fractional values at the respective $\mathbf{x}^*$-coordinates, an optimal solution $\mathbf{x}'^*$ with integral values can be constructed without changing the objective value. By repeating the argument for each location, each commodity and each scenario the statement follows. □

The result of Theorem 2 invites the derivation of a reformulation based on Benders decomposition. To this end, we use a similar approach as proposed in [8]. Particularly, we decompose the SMCLLP into one Benders master problem in the $(\mathbf{u}, \mathbf{y}, \mathbf{z})$-space while projecting out continuous variables $\mathbf{x}$ into $|I| \times \sum_{i \in I} |K_i| \times |C| \times |\Omega|$ linear subproblems in which the scenario-specific coverage of the customer demand is performed. Variables $u^{\omega}_{ikc} \in \mathbb{R}_+$, $\forall i \in I, \forall k \in K_i, \forall c \in C, \forall \omega \in \Omega$ denote upper bounds on the demand for commodity c covered by configuration k at location i in scenario ω. For convenience, let

$$P(\mathbf{y}, \mathbf{z}) := \left\{ (y_{ik}, z^{\omega}_{ij}) \in \{0,1\}^2, \forall i \in I, \forall k \in K_i, \forall j \in J_i, \forall \omega \in \Omega : (2b) - (2e) \right\}. \tag{3}$$

A reformulation of (2) is

$$\max \quad \sum_{\omega \in \Omega} p^{\omega} \sum_{i \in I} \sum_{k \in K_i} \sum_{c \in C} v_{ic} u^{\omega}_{ikc} \tag{4a}$$

$$\text{s.t.} \quad u^{\omega}_{ikc} \leq \Phi^{\omega}_{ikc}(\bar{\mathbf{y}}, \bar{\mathbf{z}}) \qquad \forall (\bar{\mathbf{y}}, \bar{\mathbf{z}}) \in P(\mathbf{y}, \mathbf{z}), \forall i \in I, k \in K_i, \forall c \in C, \forall \omega \in \Omega, \tag{4b}$$

$$(\mathbf{y}, \mathbf{z}) \in P(\mathbf{y}, \mathbf{z}) \tag{4c}$$

where

$$\Phi^{\omega}_{ikc}(\bar{\mathbf{y}}, \bar{\mathbf{z}}) \quad := \quad \max \quad \sum_{j \in J_i} x^{\omega}_{ijc} \tag{5a}$$

$$\text{s.t.} \quad \sum_{j \in J_i} x^{\omega}_{ijc} \leq s_{kc} \bar{y}_{ik} \qquad (\alpha^{\omega}_{ikc}) \tag{5b}$$

$$x^{\omega}_{ijc} \leq d^{\omega}_{jc} \bar{z}^{\omega}_{ij} \qquad (\beta^{\omega}_{ijc}) \qquad \forall j \in J_i, \tag{5c}$$

$$x^{\omega}_{ijc} \in \mathbb{R}_+ \qquad \forall j \in J_i, \tag{5d}$$

The objective function (4a) maximizes the expected weight of the covered demand. Constraints (4b) are Benders optimality cuts while constraints (4c) ensure feasibility. Note that no Benders feasibility cuts are needed. Terms $\Phi^{\omega}_{ikc}(\bar{\mathbf{y}}, \bar{\mathbf{z}})$, $\forall i \in I, \forall k \in K_i, \forall c \in C, \forall \omega \in \Omega$ correspond to the objective values of the Benders subproblems (5) given a candidate solution $(\bar{\mathbf{y}}, \bar{\mathbf{z}}) \in P(\mathbf{y}, \mathbf{z})$. Thus, a mixed ILP with an exponential number of constraints needs to be solved.

Benders optimality cuts (4b) are now rephrased explicitly. It is referred to, e.g., [8, 17, 18], for further details about the procedure. Note that indices $i \in I$, $k \in K_i$, $c \in C$, $\omega \in \Omega$ are fixed in (5). The derivation of the Benders optimality cuts is therefore shown for one specific set of the latter indices. For the sake of brevity, it is useful to omit them and to define $\bar{s} := s_{kc}\bar{y}_{ik}$, $\bar{d}_j := d^{\omega}_{jc}\bar{z}^{\omega}_{ij}$, $\forall j \in J_i$ at point $(\bar{\mathbf{y}}, \bar{\mathbf{z}}) \in P(\mathbf{y}, \mathbf{z})$, and $x_j := x^{\omega}_{ijc}$, $\forall j \in J_i$. Given the latter definitions, a specific Benders subproblem (5) can be written as

$$\Phi^{\omega}_{ikc}(\bar{\mathbf{y}}, \bar{\mathbf{z}}) = \max \left\{ \sum_{j \in J_i} x_j : \sum_{j \in J_i} x_j \leq \bar{s},\ x_j \leq \bar{d}_j, \forall j \in J_i, \mathbf{x} \geq \mathbf{0} \right\}, \tag{6}$$

and its dual as

$$\Theta^{\omega}_{ikc}(\bar{\mathbf{y}}, \bar{\mathbf{z}}) := \min \left\{ \bar{s}\alpha + \sum_{j \in J_i} \bar{d}_j \beta_j : \alpha + \beta_j \geq 1, \forall j \in J_i,\ (\alpha, \boldsymbol{\beta}) \geq (0, \mathbf{0}) \right\}, \tag{7}$$

where α and β_j, $\forall j \in J_i$ are dual variables associated with the constraints in (6), respectively. Note that by weak duality, the following sequence of inequalities holds

$$u^{\omega}_{ikc} \leq \Phi^{\omega}_{ikc}(\bar{\mathbf{y}}, \bar{\mathbf{z}}) \leq \Theta^{\omega}_{ikc}(\bar{\mathbf{y}}, \bar{\mathbf{z}}) \qquad \forall (\bar{\mathbf{y}}, \bar{\mathbf{z}}) \in P(\mathbf{y}, \mathbf{z}), \forall i \in I, k \in K_i, \forall c \in C, \forall \omega \in \Omega\,,$$

which is in fact a sequence of equations for each optimal pair $(\mathbf{y}^*, \mathbf{z}^*) \in P(\mathbf{y}, \mathbf{z})$ because strong duality holds by the linearity of subproblems (5). Moreover, variables $\mathbf{u}$ are forced to their bounds by the objective function (4a). An optimal solution $(\bar{\alpha}^*, \bar{\boldsymbol{\beta}}^*)$ of (7) at point $(\bar{\mathbf{y}}, \bar{\mathbf{z}}) \in P(\mathbf{y}, \mathbf{z})$ can be used to construct the Benders optimality cuts as follows. Let $\rho^{\omega}_{ikc}(\bar{\mathbf{y}}, \bar{\mathbf{z}}) := s_{kc}\bar{\alpha}^{\omega *}_{ikc}$, $\forall i \in I$, $k \in K_i$, $\forall c \in C$, $\forall \omega \in \Omega$ and $\varphi^{\omega}_{ijc}(\bar{\mathbf{y}}, \bar{\mathbf{z}}) := d^{\omega}_{jc}\bar{\beta}^{\omega *}_{ijc}$, $\forall i \in I$, $\forall j \in J_i$, $\forall c \in C$, $\forall \omega \in \Omega$, and note the back-substitution to the original index space using the dual variables associated with constraints (5b) and (5c), respectively. Then, Benders optimality cuts (4b) can be written explicitly as

$$u^{\omega}_{ikc} \leq \rho^{\omega}_{ikc}(\bar{\mathbf{y}}, \bar{\mathbf{z}}) y_{ik} + \sum_{j \in J_i} \varphi^{\omega}_{ijc}(\bar{\mathbf{y}}, \bar{\mathbf{z}}) z^{\omega}_{ij} \quad \forall (\bar{\mathbf{y}}, \bar{\mathbf{z}}) \in P(\mathbf{y}, \mathbf{z}), \forall i \in I, k \in K_i, \forall c \in C, \forall \omega \in \Omega\,. \tag{8}$$

Note that the number of cuts (8) is exponential, however, they can be separated on-the-fly in a branch-and-cut fashion. Moreover, each cut in (8) can be separated in linear time as shown in Theorem 3.

Theorem 3. *Each Benders optimality cut in* (8) *can be separated in linear time* $\mathcal{O}(|J|)$.

Proof. Observe that an optimal solution of (7) is $(\bar{\alpha}, \bar{\boldsymbol{\beta}}) \in \{(0, \mathbf{1}), (1, \mathbf{0})\}$ depending on whether or not $\bar{s} > \sum_{j \in J_i} \bar{d}_j$, respectively, given $(\bar{\mathbf{y}}, \bar{\mathbf{z}}) \in P(\mathbf{y}, \mathbf{z})$. The statement follows from $J_i \subseteq J$, $\forall i \in I$. □

Corollary 1. *Each Benders cut can be separated in linear time $\mathcal{O}(|J|)$ for fractional values of $(\bar{\mathbf{y}}, \bar{\mathbf{z}})$.*

Proof. The statement follows from substituting integer polyhedron $P(\mathbf{y}, \mathbf{z})$ with its relaxed version

$$P'(\mathbf{y}, \mathbf{z}) \coloneqq \left\{ (y_{ik}, z_{ij}^{\omega}) \in [0,1]^2, \forall i \in I, \forall k \in K_i, \forall j \in J_i, \forall \omega \in \Omega : (2b) - (2e) \right\} ,$$

in the proof of Theorem 3. □

5 Algorithmic framework

This section summarizes the algorithmic framework used in all computational experiments. Note that we restrict ourselves to present obtained results using the reformulation based on Benders decomposition (4) because the compact formulation (2) is incapable of solving large-scale problem instances as observed in preliminary tests.

5.1 Preprocessing

In a first step, the set of locker configurations K is computed. As long as there exist different locker configurations $\{k, k'\} \in K$ with equivalent supply for each compartment type, configuration k' is removed. Note that this preprocessing step needs to be performed only once for a given set of locker modules M.

Sets I_j, $\forall j \in J$, and J_i, $\forall i \in I$ are determined based on Euclidean distance δ, and unreachable customers are removed. We further remove inefficient locker configurations, i.e., those which obviously have excess idle capacity. More precisely, we adapt set K_i, $\forall i \in I$ as follows. From all locker configurations which are able to cover all reachable demand (per compartment and per scenario), only the locker configuration with the smallest excess supply is kept. Finally, we remove inefficient potential locations, i.e., locations for which the demand in the coverage area is less than 70% of the supply of a minimum locker configuration that consists of three locker modules (cf., Section 6.3).

5.2 Separation of Benders cuts

To avoid unboundedness in the LP relaxation of reformulation (4), initial cuts of the form

$$u_{ikc}^{\omega} \leq \bar{\rho}_{ikc}^{\omega} y_{ik} \qquad \forall i \in I, k \in K_i, \forall c \in C, \forall \omega \in \Omega \,, \tag{9}$$

are added. These cuts are derived from the trivial candidate solution $(\bar{\mathbf{y}}, \bar{\mathbf{z}}) = (\mathbf{0}, \mathbf{0})$, i.e., $\alpha = 1$ and $\boldsymbol{\beta} = \mathbf{0}$. Further Benders cuts are separated on-the-fly in a branch-and-cut fashion at each node of the branch-and-bound tree as long as no violated cuts are found. Beside integral Benders cuts (8), fractional Benders cuts are added (cf., Corollary 1). The separation of fractional cuts is stopped in some branch-and-bound node if the absolute decrease of the dual bound compared to the previous iteration falls below 0.1%. The separation routine is summarized in Algorithm 1.

6 Computational experiments and result analysis

In this section, we present the results of the performed computational experiments in which we solved artificial problem instances (generated based on real-world data). The parameter setting

Algorithm 1: Separation of the Benders cuts.

```
// Input:  Sets I, J, K, C, Ω, and current values of ū and (ȳ, z̄) ∈ P'(y, z)
// Output:  Set C of violated Benders cuts (8)
C ← ∅                                                  // set of violated Benders cuts
for i ∈ I, k ∈ K_i, c ∈ C, ω ∈ Ω do
    ρ^ω_ikc(ȳ, z̄) ← 0
    φ^ω_ijc(ȳ, z̄) ← 0, ∀j ∈ J_i
    Φ^ω_ikc(ȳ, z̄) ← ∞
    if s_kc ȳ_ik > Σ_{j∈J_i} d^ω_jc z̄^ω_ij then                    // supply > demand
        φ^ω_ijc(ȳ, z̄) ← 1, ∀j ∈ J_i
        Φ^ω_ikc(ȳ, z̄) ← Σ_{j∈J_i} d^ω_jc z̄^ω_ij
    else                                                           // supply ≤ demand
        ρ^ω_ikc(ȳ, z̄) ← 1
        Φ^ω_ikc(ȳ, z̄) ← s_kc ȳ_ik
    if ū^ω_ikc > Φ^ω_ikc(ȳ, z̄) then                   // found a violated Benders cut
        C ← C ∪ {u^ω_ikc ≤ ρ^ω_ikc(ȳ, z̄) y_ik + Σ_{j∈J_i} φ^ω_ijc(ȳ, z̄) z^ω_ij}   // add Benders cut
```

was mainly based on the literature, on Austrian census data, and on a survey we conducted in June 2020. The focus of the survey was on determining (i) the mail-order behavior of the respondents, and (ii) their willingness to use parcel lockers. A representative sample of the Austrian population ($n = 2000$) participated. We, however, reduced that data set such that only respondents living in an Austrian (provincial) capital were contained ($n = 801$).

6.1 Description of the artificial planning areas

Square-shaped grids (of different size) corresponding to the planning areas were generated. Small grids are of sides $\{1\,\text{km}, 2.5\,\text{km}\}$ in kilometers [km], medium grids of sides $\{5\,\text{km}, 7.5\,\text{km}\}$, and large grids of sides $\{10\,\text{km}, 12.5\,\text{km}\}$, respectively. Note that we will use the latter categorization scheme to refer to the problem instance size. Customers were aggregated into the square-shaped grid cells of side 100 m (in meters [m]), which is the smallest granularity for which Austrian census data is available. We will refer to the latter cells as *customer cells.* The geographical coordinates of the customer cells correspond to those of their centroids. The number of customers per customer cell was sampled from the Vienna census data (cf., Table 3 in Section 7.2) based on a uniform distribution. Note, however, that 9% of the respondents of the conducted survey denied using parcel lockers. The number of aggregated customers in each customer cell was therefore reduced by 9% (and rounded to the nearest integer).

The number of potential locations for parcel lockers was set to the number of square-shaped cells of the sides $\{250\,\text{m}, 500\,\text{m}\}$ that fit into the corresponding planning area. For instance, a cell of side 500 m in a planning area of side 1 km yields four potential locker locations whose coordinates were randomly generated based on a uniform distribution. Three different instance grids were generated for each of the aforementioned combinations of parameters. Details on the artificial planning areas are given in Table 1.

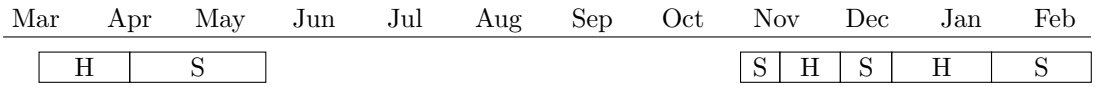
Table 1: Description of the artificial planning areas. The referred problem instance size, the sides (a) of the square-shaped planning areas in kilometers, the number of customers cells ($|J|$), and two different numbers $\cdot, \cdot$ of potential locker locations $|I|$ are reported.

Size	a [km]	$\|J\|$	$\|I\|$
small	1	100	4, 16
	2.5	625	25, 100
medium	5	2 500	100, 400
	7.5	5 625	225, 900
large	10	10 000	400, 1 600
	12.5	15 625	625, 2 500

6.2 Demand scenarios

To estimate the demand for mail orders during a pandemic, we used the historic data of the Austrian regulations from March 2020 until the end of February 2021. A distinction was made between "hard" and "soft" lockdowns of the economy. During hard lockdowns, only shops for basic supplies (e.g., supermarkets and pharmacies) were allowed to open. During soft lockdowns, all shops were allowed to open, however, with limitations on the number of customers allowed therein depending on the available selling space. Figure 2 illustrates the timeline of the considered period.

Figure 2: Timeline of hard (H) and soft lockdowns (S) of the Austrian economy from March 2020 until end of February 2021.



The base demand (i.e., in periods without lockdowns) was estimated based on the Viennese CEP delivery report 2020 [39], stating that each Viennese citizen received on average 36 parcels in the year 2019 in the X2C market. The annual increase in e-commerce was assumed to be the average of its increase in the year 2019, which was 6.2% based on data from Germany [51] (due to the lack of Austrian data). Thus, each customer was assumed to have an average base demand for 0.1047 parcels per day in periods without lockdowns. The latter value was multiplied with the number of citizens in each customer cell to obtain the base demand therein. The distribution of ordered parcel sizes was obtained from the conducted survey. The respondents reported that they had ordered 37% small parcels, 35% medium parcels, 19% large parcels, and 9% x-large parcels (within a three-month period). The base demand for each different parcel size in each customer cell was adapted accordingly (and rounded to the nearest integer).

To estimate the increase in mail orders during hard and soft lockdowns, we used the same data set as before [51]. The latter data contains the monthly increase in e-commerce compared to the corresponding month of the previous year and ranges from January 2018 to March 2021. For the demand during lockdowns, the weighted averages of the corresponding periods (cf., Figure 2) were computed and adjusted by the assumed annual increase in e-commerce. This yields an average increase in e-commerce of 20.6% and 24.4% during hard and soft lockdowns, respectively. Three

lockdown scenarios were used in the computational experiments:

(i) $|\Omega| = 1$: No lockdowns at all.

(ii) $|\Omega| = 2$: The government imposes soft lockdowns if the number of active COVID-19 infections exceeds 15 000. This was the case in Austria for 77% days in the year 2021. Thus, we set $p^1 = 23\%$ and $p^2 = 77\%$.

(iii) $|\Omega| = 3$: The scenario as shown in Figure 2. In total, 80 days of hard and 108 days of soft lockdowns were imposed by the Austrian government. Thus, we set $p^1 = {}^{188}/_{365} \approx 48\%$, $p^2 = {}^{80}/_{365} \approx 22\%$, and $p^3 = {}^{108}/_{365} \approx 30\%$.

6.3 Locker modules and locker configurations

The used locker modules are based on an Austrian manufacturer's products. A minimum locker configuration consists of one mandatory base module and two optional modules. Locker modules can provide up to four different compartments (with 1, 2, 4, and 8 height units of 9 cm, width 44 cm, and depth 61 cm, in centimeters [cm]). These compartments are suitable for commodities of sizes small (S), medium (M), large (L), and x-large (XL). For the computational experiments, we used two sets of locker modules. `Module set 1` is the off-the-shelf variant from the Austrian manufacturer as shown in Figure 1. The number of provided compartments (#S, #M, #L, #XL) is `B1`=(4,3,1,0), `N1` = (3,4,2,0), and `N2` = (1,5,0,1) for the base module and two optional modules, respectively. Based on the results of the conducted survey, we propose an alternative `Module set 2` with modules `A1` =(0,1,1,1), `M1` = (7,4,1,0), and `M2` = (1,3,1,1). Note that on one hand, the latter set of modules overrepresents small- and medium-sized compartments concerning the proportion of ordered parcel sizes reported by the respondents in the survey. On the other hand, the respondents' willingness to use parcel lockers was negatively correlated with the parcel sizes (cf., Section 6.4). The prices of the locker modules were set to €5 000 and €1 000 for the base module and for an optional module, respectively, based on personal communication with a responsible employee of the locker manufacturer.

6.4 Objective function metric

Like in the case study (cf., Section 7), we assumed that a (local) authority aims to set up a last-mile delivery network of white-label parcel lockers. We further assumed that each CEP delivery operator can use these lockers against a service fee related to their cost savings of using lockers compared to conventional door delivery (as suggested in [42]). The authority was assumed to be concerned about customer satisfaction to some extent. We therefore used a weighted sum multi-objective function combining service fees and the willingness of customers to use parcel lockers.

For estimating the service fees, we used prices of the Austrian branch of the CEP delivery operator Direct Parcel Distribution (DPD) who charge €4.3, €6.3, €8.3, and €12.3 for delivering small, medium, large, and x-large parcels, respectively. We assumed that DPD determines these prices based on mark-ups, i.e., the last-mile delivery cost and potential cost savings obtained from locker delivery are fractions of the prices charged by DPD. In the remainder of this section, we therefore use the latter prices as service fees directly (because it affects neither the solutions of the computational experiments nor the result analysis). We detail how to obtain meaningful scale factors in the case study (cf., Section 7).

To assess the willingness of customers to use parcel lockers, we took recourse to the data from the conducted survey. Each respondent was shown 10 randomized exemplary scenarios for collecting parcels from lockers. The parameters parcel size, weight, and walking distances varied in each scenario. The respondents were asked to assess their willingness to use parcel lockers on a 7-Likert scale (1 = worst and 7 = best) in each scenario shown. Each of the aforementioned parameters has a significant negatively correlated impact on the respondents' willingness to collect parcels (revealed by a regression model). However, even for the largest value of the parameter walking distance of 500 m (or more), the average willingness to collect parcels is 4.1 on the Likert scale. The average values for the willingness to collect parcels within 500 m are 4.4, 4.2, 4.1, and 4 for small, medium, large, and x-large parcels, respectively.

Finally, we normalized the obtained values of the willingness to collect parcels, and the service fees, and combined them linearly by *customer satisfaction rate* $\lambda \in [0, 1]$, i.e., if $\lambda = 0$ only the service fees are considered in the objective function.

6.5 Further parameter setting and summary of the used data

The maximum number of locker modules that can be combined to a locker configuration was set to $\ell^{\max} \in \{5, 10, 15\}$. The number of possible locker configurations $|K|$ is therefore at most 120, cf., (1). For each potential location $i \in I$, the maximum number of modules which can be placed was drawn randomly from the integer interval $[3 : \ell^{\max}]$ based on a uniform distribution. The available budget was computed via fractions of the maximum setup cost, i.e., the total cost of building the maximum number of modules (including one base module) at each location $i \in I$. The latter *budget fraction* b_{f} was set to $b_{\mathrm{f}} \in \{10, 25, 50, 100\}[\%]$. Note that the usage of budget fractions enables a meaningful comparison of the results of different problem instances. The replenishment rates were set to $\eta \in \{68, 77, 86\}[\%]$ based on Remark 2. The customer satisfaction rate was set to $\lambda \in \{0, 50, 100\}[\%]$. The coverage distance was set to $\delta \in \{300, 400, 500\}[\mathrm{m}]$, based on the questions of the conducted survey. Table 2 summarizes the data and problem parameters (discussed in Sections 6.2 – 6.5), which are used in the remainder of this section.

6.6 Performance analysis

Given the number of generated problem instance grids and the parameter setting outlined above in this section, the number of all combinations of the latter is 69 984. We therefore restricted ourselves to a randomly chosen subset of these combinations so that the performance analysis and the sensitivity analysis are based on 35 973 computational experiments. All algorithms were implemented in *Julia 1.6* together with the JuMP modeling language [12]. Moreover, IBM CPLEX 12.10 (with default settings) was used as ILP solver. Each experiment was performed on a single core of an Intel Xeon E5-2670v2 machine with 2.5 GHz. A memory limit of 16 GB RAM and a time limit of 1 000 seconds [s] ($\approx$ 17 minutes) were set. Note the we chose the latter time limit based on previous computational experiments (of the initial paper draft) in which a time limit of one hour was set. It turned out that 97% of the problem instances solved to optimality finished within 1 000 seconds. The shorter time limit comes, however, at the cost of larger average optimality gaps, which increased from 3.5% to 5.9%.

Figure 3 shows performance profiles including runtimes and optimality gaps of the small, medium, and large problem instances. Figure 3a shows that 86.3% of the small, 64.2% of the medium, and 53.3% of the large problem instances were solved to optimality within the set time

Table 2: Summary of the data and problem parameters discussed in Sections 6.2 – 6.5. Abbreviations S, M, L, and XL correspond to parcels or compartments of size small, medium, large, and x-large, respectively.

	Data description	Values
Demand scenarios	Relative demand: base, soft-, hard lockdowns	100%, 124.4%, 120.6%; (100% = 0.1047)
	Relative number of ordered parcel sizes	37% (S), 35% (M), 19% (L), 9% (XL)
	Scenario probabilities ($\|\Omega\| = 2$)	$p^1 = 23\%$, $p^2 = 77\%$
	Scenario probabilities ($\|\Omega\| = 3$)	$p^1 = 48\%$, $p^2 = 22\%$, $p^3 = 30\%$
Locker modules	`Module Set 1`	`B1`: 4 (S), 3 (M), 1 (L), 0 (XL)
		`N1`: 3 (S), 4 (M), 2 (L), 0 (XL)
		`N2`: 1 (S), 5 (M), 0 (L), 1 (XL)
	`Module Set 2`	`A1`: 0 (S), 1 (M), 1 (L), 1 (XL)
		`M1`: 7 (S), 4 (M), 1 (L), 0 (XL)
		`M2`: 1 (S), 3 (M), 1 (L), 1 (XL)
	Prices for base module and optional module	€5 000, €1 000
Obj. metric	Prices charged by DPD	€4.3 (S), €6.3 (M), €8.3 (L), €12.3 (XL)
	Willingness to collect parcels (7-Likert scale)	4.4 (S), 4.2 (M), 4.1 (L), 4 (XL)
Other parameters	Maximum number of locker modules $\ell^{\max}$	5, 10, 15
	Budget fraction b_{f}	10%, 25%, 50%, 100%
	Replenishment rate η	68%, 77%, 86%
	Customer care fraction λ	0%, 50%, 100%
	Coverage distance δ	300 m, 400 m, 500 m

limit. The optimality gaps of the remaining problem instances are depicted in Figure 3b. The latter gaps were computed by (UB - OV)/UB, where UB is the best known upper bound and OV the objective value of the best known integer solution. All problem instances with an optimality gap of 100% hit the set memory limit.

A more fine-grained analysis with focus on the impact of different problem parameters on the performance is provided in A. Figures 10-12 show a large volatility in performance with respect to the available budget b_{f}, the maximum size of a locker configuration (measured in number of modules) $\ell^{\max}$, and the ratio between number of customer cells and potential locations $|J|/|I|$. It can be observed that the optimality gaps are correlated with b_{f} and $\ell^{\max}$, and negatively correlated with $|J|/|I|$, i.e., the more the potential locations in a problem instance of the same size (measured in $|J|$) the larger the gaps. Parameters coverage distance δ and number of scenarios $|\Omega|$ have a minor impact on the algorithm performance. We remark, however, that parameter δ is seemingly negatively correlated to the performance. The impact of parameters replenishment rate η and customer satisfaction rate λ on the performance is negligible and therefore unreported.

Figure 4a shows the runtimes used for the separation of Benders cuts relative to the total runtime per solved problem instance. The latter fraction can be observed to be negatively correlated with the problem instance size. The figure also shows that the most time in the solving process is used for branching, which is a bottleneck compared to the separation of the Benders cuts. Figure 4b shows the number of explored branch-and-bound nodes clustered by different problem instance sizes.

Figure 3: Performance profiles of problem instance sizes small, medium, and large. Figure 3a shows the relative number of problem instances solved to optimality within the set time limit. Figure 3b shows the optimality gaps computed by (UB - OV)/UB where UB is the best known upper bound and OV is the objective value of the best known integer solution.

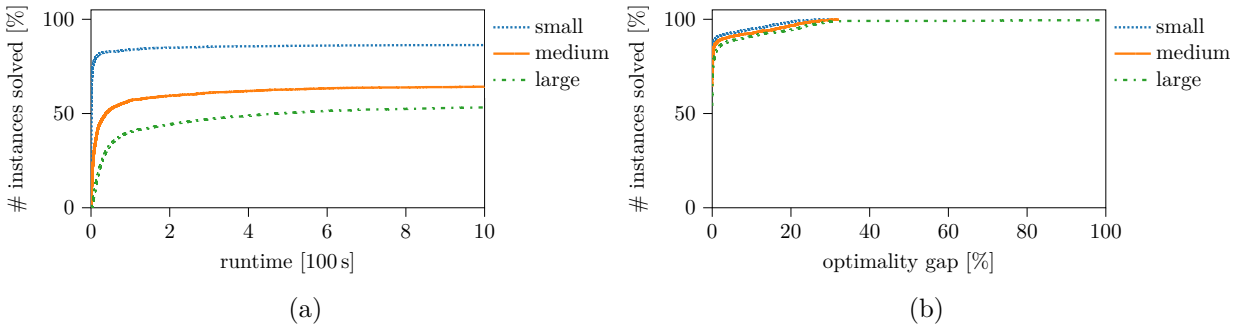
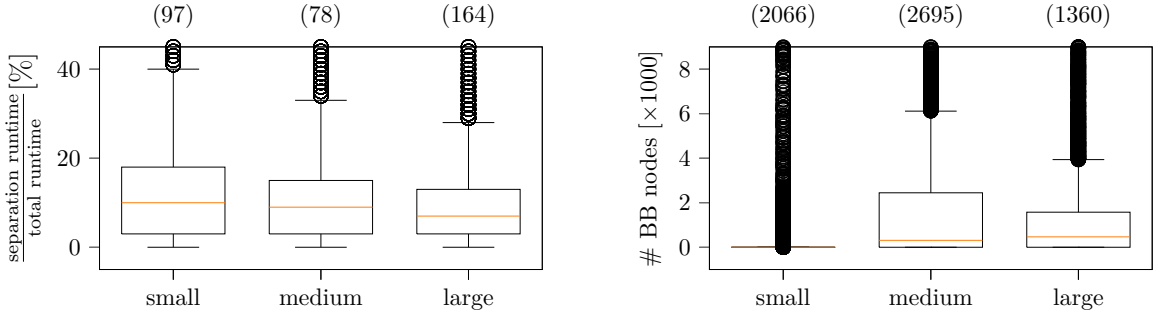


Figure 4: Runtime of separating Benders cuts relative to the total runtime of different problem instance sizes (Figure 4a). Number of explored branch-and-bound (# BB) nodes of different problem instance sizes (Figure 4b). The number of outliers that are not depicted is given above each boxplot in the brackets $(\cdot)$.



(a) Relative separation runtime

(b) Number of branch-and-bound nodes

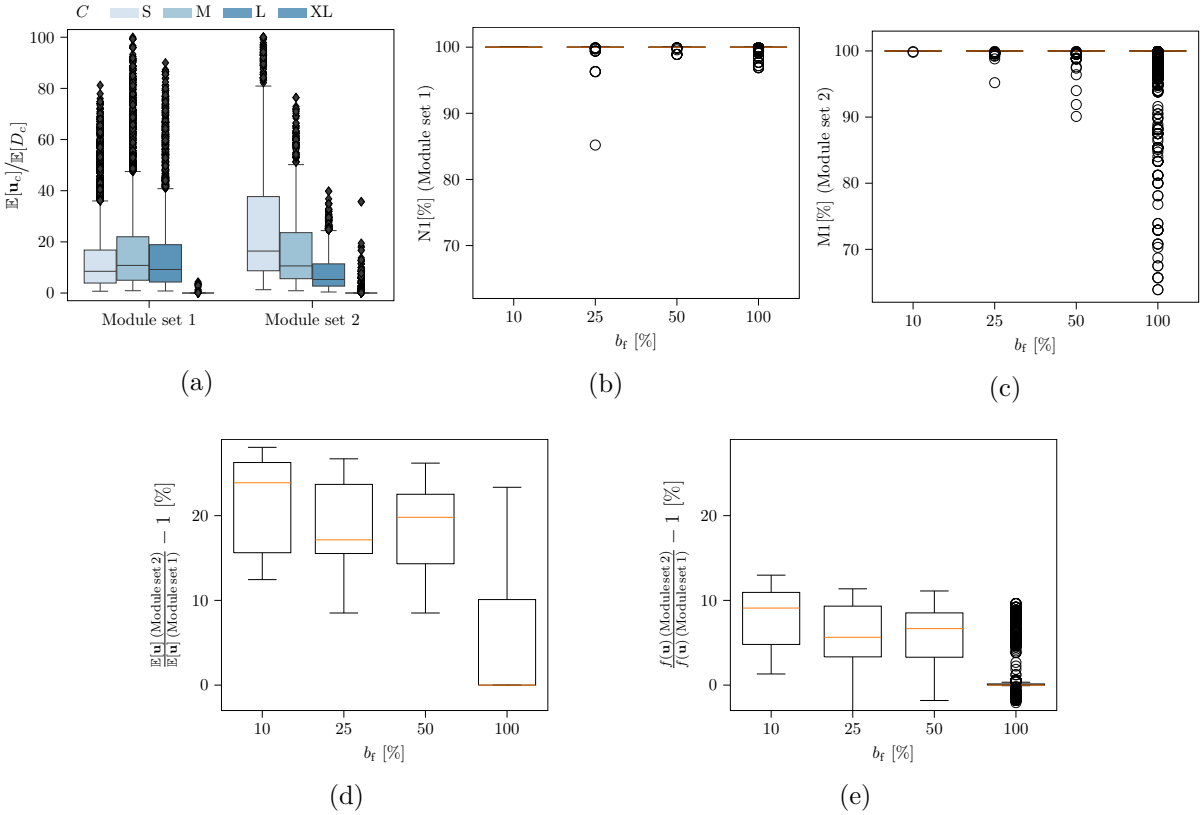
6.7 Sensitivity analysis

In this section, we investigate the sensitivity of the solutions with respect to different problem parameters. Only results of problem instances solved to optimality are reported unless stated otherwise.

We start by comparing the two chosen sets of locker modules `Module set 1` and `Module set 2` (cf., Figure 5). Slightly abusing notation, we denote the expected covered demand per commodity $c \in C$ by $\mathbb{E}[\mathbf{u}_c]$ and the expected total demand per commodity by $\mathbb{E}[D_c]$. Analogously, the expected total covered demand is denoted by $\mathbb{E}[\mathbf{u}]$ and objective function values by $f(\mathbf{u})$. Figure 5a shows the expected relative covered demand for each $c \in C := \{\mathrm{S}, \mathrm{M}, \mathrm{L}, \mathrm{XL}\}$. `Module set 1` can be observed to provide a more balanced coverage than `Module set 2`. Parcels of size XL are hardly covered by both module sets. The reason is that in both module sets, one type of module (`N1` and `M1`) is preferred in the solutions as shown in Figures 5b and 5c. The relative number of modules of type `N2` and `M2` used in the solutions is, however, correlated with the available budget. Note that the latter results strongly depend on the assumptions on the locker module design (cf.,

Section 6.3) and on the objective function weights (cf., Section 6.4). If we break down the assumed service fees per parcel to service fees per consumed height unit of a locker module, we note that the contribution to the objective function (per height unit) decreases with the compartment size. For instance, using one medium-sized compartment contributes less to the objective function than using two small-sized compartments does. The same holds true for the customers' willingness to collect parcels. The compartment structure of modules `N1` and `M1`, therefore, implies that using `Module set 2` enables covering of more customer demand (13% on average) and obtaining larger objective function values (4% on average) compared to `Module set 1`. Figures 5d and 5e show the latter effect and further reveal that its impact is negatively correlated with the available budget. Note that we only compare solutions of two module sets if they are optimal and all other problem parameter values are identical. In the remaining analysis, we restrict ourselves to results of problem instances using `Module set 2`.

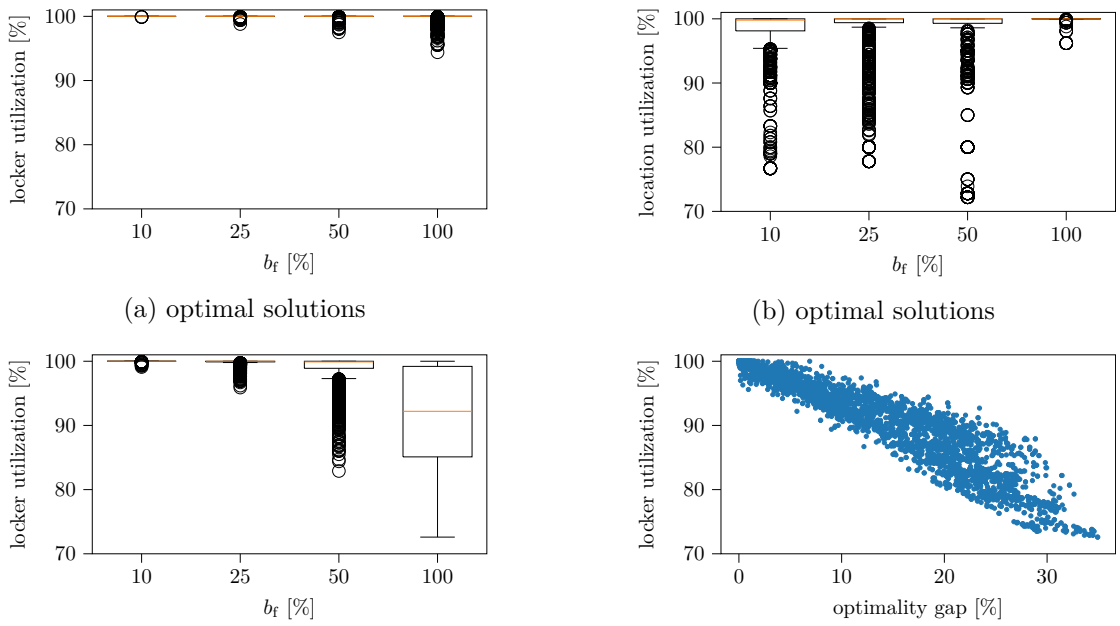
Figure 5: Comparison of `Module set 1` and `Module set 2` (w.r.t. different commodities C and budget fraction b_{f}). Figure 5a shows the relative covered demand per parcel size. Figures 5b and 5c show the relative number of locker modules of type `N1` (N1[%]) and `M1` (M1[%]) used in the solutions, respectively. Figure 5d shows the expected covered demand of solutions using `Module set 2` relative to solutions using `Module set 1`. Figure 5e shows the same comparison with respect to objective function values.



We proceed with an analysis of the locker and location utilization, i.e., the number of compartments used relative to the total number of compartments (per locker configuration) and the number of locker modules relative to the maximum number of modules that can be installed (per

location), respectively. Figure 6a shows the obtained locker utilization. It can be observed that there hardly exists any idle capacity, however, the latter is correlated with the available budget. The opposite trend can be observed in Figure 6b, showing the location utilization. The reason for these trends is that the model tends to increasingly choose inefficient locker configurations, which nevertheless contribute to the objective function (if budget is no restriction). The locker utilization rates are, however, quite large because (i) the demand in coverable areas is larger than the potential available supply in most cases, and (ii) obviously inefficient locker configurations were removed in the preprocessing. The locker utilization is also correlated with the optimality gap, i.e., the larger the gap, the larger the idle capacity in the obtained the solutions. This trend can be observed in Figures 6c and 6d in which results of problem instances solved with an optimality gap greater than zero are shown. A similar trend regarding the location utilization was not observable.

Figure 6: Locker and location utilization (w.r.t. budget fraction b_{f}) of problem instances solved to optimality (Figures 6b and 6b). Locker utilization (w.r.t. budget fraction and optimality gap) of solved problem instances with optimality gaps (Figures 6c and 6d).



(a) optimal solutions

(b) optimal solutions

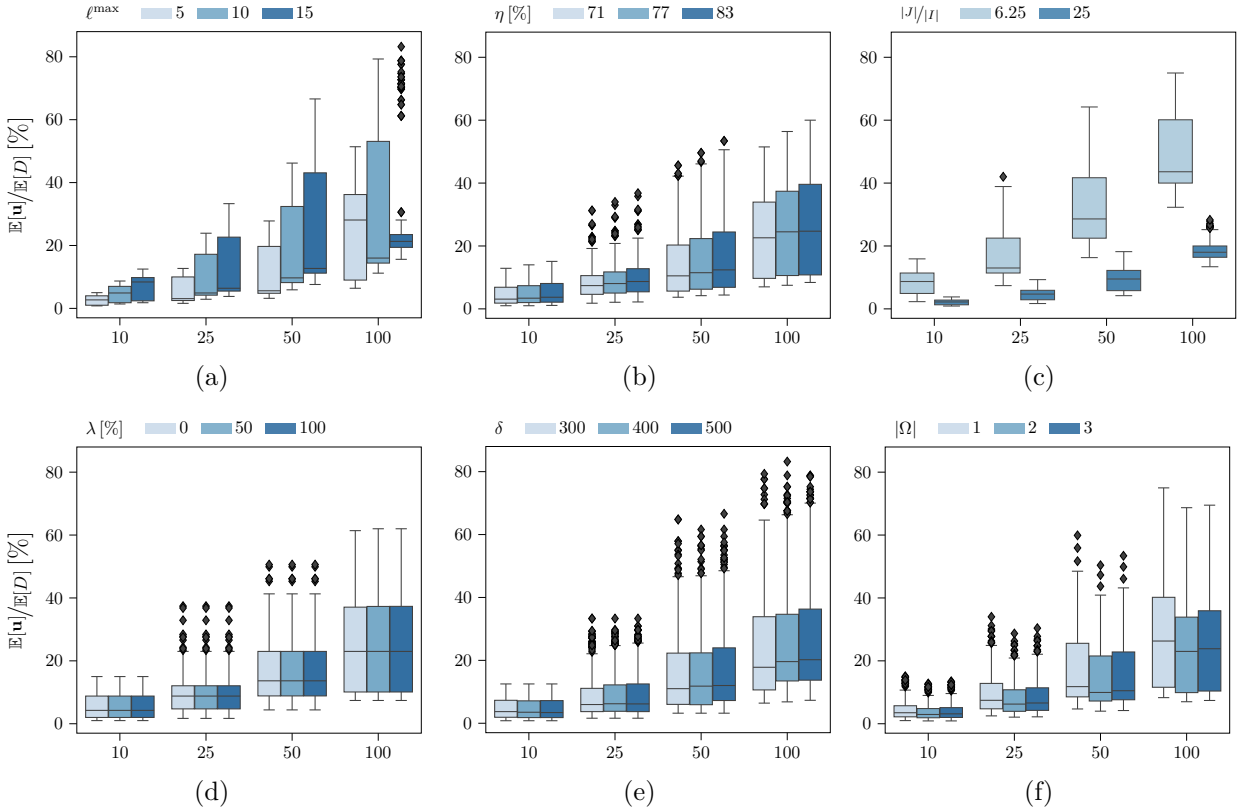
(c) non-optimal solutions

(d) non-optimal solutions

We proceed with an analysis of the impact of different problem parameters on the relative covered demand (cf., Figure 7). Note that we again consider only results for which optimal solutions for all values of the parameter of interest exist (whereas all other problem parameter values are identical). Figure 7a shows that the relative covered demand grows approximately linearly with the maximum number of modules that can be installed $\ell^{\max}$. This again indicates that there exist more demand in coverable areas than the potential supply in the problem instances. Note, however, that the aforementioned linearity seemingly breaks down if budget is not restricted, which again indicates the existence of idle compartments. A similar trend can be observed in Figure 7b, which shows that the relative covered demand changes approximately like the relative changes in replenishment rate η. The latter trend has, however, also less impact if budget is not restricted.

Figure 7c shows a notable impact of the ratio of customers and potential locations $|J|/|I|$ on the relative covered demand. In contrast, the impact of the customer care fraction λ is negligible (cf., Figure 7d) and in fact not significant at a confidence level of 95% revealed by an analysis of variance. The reason is that the customers' willingness to collect parcels decreases with parcel size and so does the profitability per consumed locker module height unit (as discussed above in the comparison of `Module set 1` and `Module set 2`). The impact of the coverage distance δ on the relative covered demand is quite low (cf., Figure 7e). This again indicates that there exists more demand than potential supply, i.e., most supply can be exhausted within a coverage distance of 300 m. Finally, Figure 7f shows the impact of the considered demand scenarios on the relative covered demand, which decreases notably in the very strict lockdown scenario ($|\Omega| = 2$). Although not directly observable from the figure, this decrease is 17% on average. For the less strict lockdown scenario ($|\Omega| = 3$), this decrease is only 10% on average. The latter comparison clearly shows that relying on existing parcel lockers will hardly help to cover a soaring increase in demand for mail orders as it was the case during the lockdowns. More promising options to do so include (i) augmenting the existing infrastructure with new parcel lockers, (ii) warehousing a backup fleet of parcel lockers, and (iii) ordering a parcel locker rental service.

Figure 7: Relative covered demand (w.r.t. budget fraction b_{f} [%] on the x-axis) in dependence of parameters maximum number of locker modules $\ell^{\max}$, customer-location ratio $|J|/|I|$, replenishment rate η, customer care fraction λ, coverage distance δ, and number of scenarios $|\Omega|$.



7 Case study

The case study presented in this section is based on census data comprising the number of citizens (and their geographical location) of Vienna and the two largest Austrian provincial capitals Graz and Linz. Further used data was sourced from the conducted survey (cf., Section 6) and from the literature. The results are based on computational experiments performed on the same machine as outlined in Section 6. The time limit was set to 24 hours and the memory limit to 64 GB.

7.1 Status in Austria and mode of operation

The parcel locker supply is currently limited but emerging in Austria. Parcel lockers are mainly in use at several post offices and nearby supermarkets. There exists also a pilot project aiming to test the usage of white-label outdoor parcel lockers in the 5th district of Vienna [42]. The results of the testing phase of the latter project are positive. Thus, there exists a large potential for building additional (white-label) parcel lockers in Austria. For the case study, we assumed that the government aims to invest in a backup fleet of such lockers that are only activated in case of a soaring increase of demand for mail orders (e.g., during lockdowns or around Christmas). Options to warehouse that parcel lockers include: (i) to store them at some (publicly owned) area outside the city and to transport them to the determined locations if needed, and (ii) to place them directly at the determined locations and to activate them only if needed. Note that the second option gives rise to support local artists who may design individual artworks for each parcel locker. Clearly, also renting out their surface for advertisement is an option which ensures cash flows if they are idle. We used the second option in the result analysis (neglecting revenues from rents for advertising).

7.2 Problem instance description

Real-world problem instances include the cities of Vienna, Graz, and Linz. The city areas were segmented into standardized square-shaped customer cells of side 100 m, which is the smallest granularity for which census data is available. The number of residents registered in each customer cell by 2020/01/01 is known from that census data. The geographical coordinates of each cell correspond to the coordinates of its centroid. Uninhabited customer cells (e.g., covering rivers) were removed. Potential parcel locker locations were assumed to be at certain points of interest (POI) provided by OpenStreetMap [40]. Depending on the availability of POI in the respective cities, locations I are chosen from a subset of POI in raw data set. The following POI were chosen (based on [25]) whereby the number in the brackets $(\cdot)$ indicates the assumed maximum number of locker modules that can be built there: subway stations (20), train stations (20), petrol stations (10), marketplaces (10), parks (10), parking garages (10), playgrounds (10), large supermarkets (Billa Plus, Eurospar, Interspar) (10), universities (10), hospitals (10), tram stops (7), and bus stops (5). Table 3 shows details about the used real-world problem instances.

Given the bounds on the number of locker modules defined for the POI, there exist $|K| = 207$ locker configurations (excluding those with less than three modules, which is a requirement imposed by the manufacturer). For each potential location $i \in I$, the maximum number of modules that can be installed was set to a randomly chosen integer ranging between (and including) three and the upper bound defined for the corresponding POI.

We set a coverage distance of $\delta = 300$ m, a replenishment rate $\eta = 77\%$ (based on [48]), and a customer satisfaction rate $\lambda = 20\%$. We further used `Module set 2` because it performed better

Table 3: Description of the real-world problem instances. The total number of inhabitants $|J'|$, the number customer cells $|J|$, the average number of inhabitants per customer cell $|J'|/|J|$, the number of potential locker locations $|I|$, and the number of customer cells per potential location $|J|/|I|$ are reported.

City name	$\lvert J'\rvert$	$\lvert J\rvert$	$\frac{\lvert J'\rvert}{\lvert J\rvert}$	$\lvert I\rvert$	$\frac{\lvert J\rvert}{\lvert I\rvert}$
Vienna	1 911 274	17 701	108.0	1 401	12.6
Graz	291 245	6 777	43.0	1 024	6.6
Linz	206 724	3 351	61.7	356	9.4

in the sensitivity analysis (cf., Section 6.7). The prices for the locker modules are the same as reported in Section 6.3, however, we assumed a 10% mark-up per year for service. As a demand scenario, we considered the Austrian lockdown scenario ($|\Omega| = 3$) described in Section 6.2.

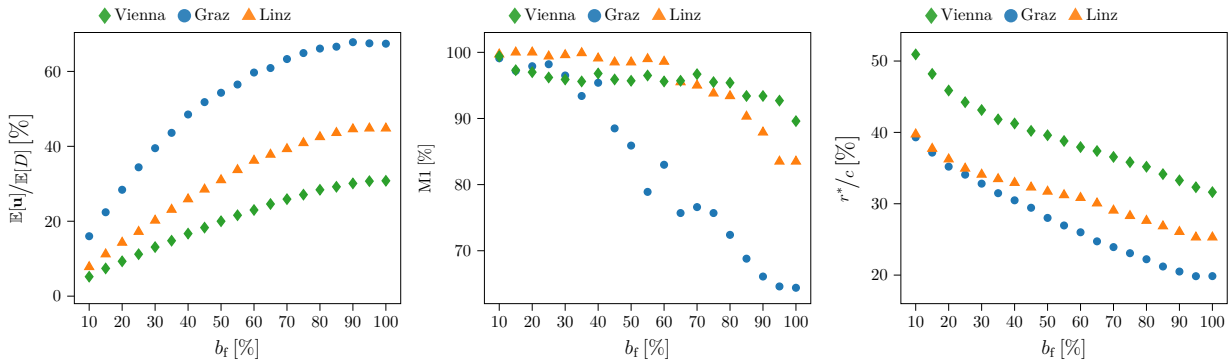
7.3 Results

For the analysis of the results, we scaled the prices charged by DPD (Section 6.4) as follows. We first discounted them by the value added tax (20%) and by an assumed mark-up (5%). We then took 41% of the latter values that correspond to the last-mile delivery cost (based on [32]). The cost savings of using parcel lockers compared to that of door delivery reported in the literature deviate. For instance, Iwan et al. [25] and Turkoglu and Genevois [53] report 16%, whereas Gevaers et al. [19] and Yu et al. [57] report 58%. We therefore used these percentages to compute lower and upper bounds on the latter cost savings. Finally, we deducted 30% of these values (to incentivize CEP operators to use the parcel lockers) and obtained lower and upper bounds on the net service fees. Given the aforementioned manipulations, we effectively scaled the prices charged by DPD by factors by 3.64% (lower bound) and 13.21% (upper bound). Note that we also used an average scale factor of 8.43% (particularly in Figure 8c).

An overview of the results is performed in Figure 8. Figure 8a shows the relative covered demand, depending on the available budget for Vienna, Graz, and Linz. We observe that the marginal relative covered demand decreases with the budget. This indicates that building large parcel lockers is the preferred option if budget is restrictive (because the price of the base locker module substantially exceeds the price of an optional module, and enough demand exists within the coverage distance). Moreover, there is a notable difference in the relative covered demand with respect to the cities. One reason for that difference is the volatility of the ratios between the number of customer cells and potential locations $|J|/|I|$ (cf., Table 3). Figure 8b again shows that using locker modules of type `M1` is the preferred option if the budget is restrictive. Figure 8c shows the average expected return on investment per year, i.e., the average annual net revenues r^* (using a scale factor of 8.43%) relative to the investment cost c. Note that we assumed that the parcel lockers are in use 161 days per year, i.e., the number of lockdown days (except Sundays) in the considered demand scenario. It can be observed that the average return on investment decreases with the available budget for the reasons mentioned above: (i) increasingly small parcel locker configurations are constructed if there exists plenty of budget, (ii) the relative number of the less profitable locker modules of type `M2` increases, and (iii) the optimality gaps increase, which is correlated with less efficient compartment utilization (cf., Table 4). Clearly, the return on investment will be substantially larger if the parcel lockers are in use the whole year (or rent out as advertising surface). In the latter case, the average return on investment over all problem

instances is 75.7%, which yields an average amortization time of 1.4 years. This is in line with the results of Seghezzi et al. [50], who report an amortization time of 1.3 years.

Figure 8: Results of the real-world problem instances. Figure 8a shows the relative covered demand ($\mathbb{E}[\mathbf{u}]/\mathbb{E}[D]$). Figure 8b shows the relative number of modules of type `M1` used (M1). Figure 8c shows the average return on investment r^*/c.



(a) (b) (c)

Table 4 reveals further details. Note that problem instances with a runtime lower than the set time limit and an optimality gap greater than zero hit the memory limit. We observe similar trends as in the sensitivity analysis (cf., Section 6.7). Columns *covered demand* show that small- and medium-sized locker compartments (i.e., locker module `M1`) are preferred over large and x-large compartments (particularly if budget is limited). We further observe that the compartment utilization is acceptable, however, decreasing with the available budget. Moreover, the investment cost of roughly €9 million (Vienna), €1.3 million (Graz), and €1 million (Linz) are sufficient to cover a soaring increase of demand for mail orders as observed during the lockdowns in Austria. The reported lower and upper bounds on the expected net revenues ($\underline{r}$ and $\overline{r}$) are again based on 161 days of use per year.

Finally, one exemplary solution for Vienna (with $b_f = 50\%$) is given in Figure 9. It can be observed that most parcel lockers cluster in the inner districts, which have the largest population density. An advantageous side-effect regards the relief of traffic in these districts, which have sparse traffic areas with many narrow alleys. Another interesting fact to remark is that most parcel lockers are located at tram stops (182), subway stations (148), parking garages (75), fuel stations (50), and train stations (45), where the number in the brackets $(\cdot)$ indicates the number of location types in Figure 9. The latter fact opens the possibility that customers can collect their parcels when commuting, e.g., between work and home.

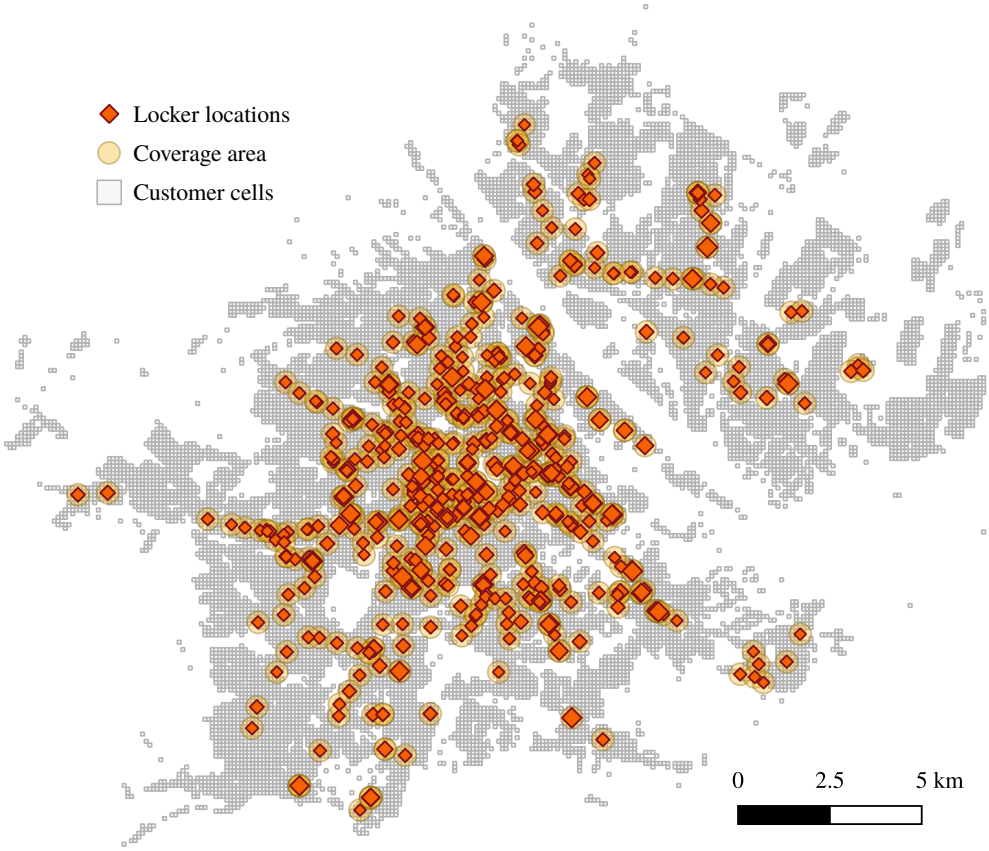
8 Conclusions

We proposed and studied the stochastic multi-compartment locker location problem (SMCLLP), which was proven to be NP-hard. An ILP formulation for the SMCLLP was proposed, and a reformulation based on Benders decomposition was derived. We showed that the Benders cuts can be separated in linear time. The performance of the developed algorithmic framework enables solving of large-scale problem instances to proven optimality, which was demonstrated by an analysis

Table 4: Results of the case study. The following values are reported: the budget fraction (b_{f}), the runtime (rt) in hours, the optimality gap (gap) in percent, and the relative covered demand for different parcel sizes including the total covered demand ($\mathbb{E}[\mathbf{u}]/\mathbb{E}[D]$) in percent, the relative compartment utilization including the total utilization (T) in percent; the relative number of locker module of type `M1` (M1), the location utilization (LU), the relative number of installed parcel lockers ($|I^*|/|I|$), the investment cost (c) in million Euro, a lower bound on the expected net revenues per year ($\underline{r}$) in million Euro, and an upper bound on the expected net revenues per year ($\overline{r}$) in million Euro.

				Covered demand [%]					Compartment utilization [%]										
	b_{f} [%]	rt [h]	gap [%]	S	M	L	XL	$\frac{\mathbb{E}[\mathbf{u}]}{\mathbb{E}[D]}$	S	M	L	XL	T	M1 [%]	LU	$\frac{\lvert I^*\rvert}{\lvert I\rvert}$ [%]	c [mio. €]	$\underline{r}$ [mio. €]	$\overline{r}$ [mio. €]
Vienna	5	0.2	0.0	4	3	1	0	3	100	100	100	100	100	100	99	2	0.750	0.169	0.613
	10	24.0	0.4	8	5	2	0	5	100	100	100	100	100	99	100	5	1.501	0.331	1.198
	15	10.7	3.8	11	7	4	0	7	96	99	100	100	98	97	97	8	2.252	0.469	1.702
	20	17.1	6.2	14	9	4	0	9	94	99	100	99	96	97	97	12	2.991	0.593	2.151
	25	18.1	6.8	17	11	5	0	11	93	98	100	100	96	96	96	16	3.753	0.718	2.602
	30	21.8	6.9	19	13	6	1	13	93	98	100	100	96	96	97	20	4.502	0.840	3.044
	35	18.1	7.4	22	15	7	1	15	94	97	100	100	96	96	97	25	5.253	0.950	3.444
	40	20.9	6.5	25	17	8	1	17	94	98	100	100	96	97	98	30	6.005	1.071	3.883
	45	16.9	7.3	27	18	9	1	18	93	97	100	100	95	96	98	34	6.755	1.175	4.258
	50	18.1	6.8	30	20	10	1	20	94	98	100	99	96	96	98	40	7.506	1.285	4.660
	55	15.8	6.8	32	22	11	1	22	94	97	100	99	95	96	98	45	8.256	1.385	5.021
	60	1.9	7.5	34	23	12	2	23	94	97	100	98	96	96	98	50	9.006	1.478	5.356
	65	11.8	6.8	36	24	12	2	25	94	97	100	98	96	96	99	56	9.758	1.578	5.719
	70	16.4	7.2	38	26	13	2	26	93	97	99	98	95	97	98	62	10.508	1.661	6.023
	75	18.9	7.4	40	27	14	2	27	94	97	100	99	96	96	98	68	11.256	1.744	6.321
	80	20.3	7.2	42	28	14	2	28	94	97	100	98	96	95	98	75	12.009	1.827	6.624
	85	19.0	8.4	42	29	15	3	29	93	96	99	97	95	93	98	81	12.759	1.884	6.831
	90	19.7	8.5	43	30	16	3	30	93	96	99	98	95	93	98	89	13.510	1.944	7.048
	95	24.0	9.0	44	31	16	4	31	93	96	98	97	94	93	99	95	14.261	1.993	7.224
	100	5.3	9.9	44	31	16	4	31	93	96	99	94	94	90	98	100	14.672	2.007	7.274
Graz	5	3.4	0.0	12	8	4	0	8	99	100	100	100	100	98	99	3	0.435	0.078	0.284
	10	24.0	0.0	24	15	8	0	16	99	100	100	100	99	99	99	7	0.870	0.148	0.536
	15	16.6	1.7	33	22	11	1	22	98	100	100	100	99	97	98	11	1.305	0.210	0.760
	20	22.4	1.9	41	27	14	1	28	98	99	100	100	98	98	99	16	1.740	0.265	0.960
	25	24.0	1.6	50	33	16	1	34	98	100	100	100	99	98	99	21	2.176	0.321	1.162
	30	24.0	3.1	57	39	19	3	40	97	99	100	100	98	96	98	25	2.610	0.370	1.343
	35	24.0	4.8	61	44	22	6	44	96	99	100	100	98	93	97	30	3.040	0.414	1.500
	40	24.0	5.3	68	49	25	4	48	94	99	100	100	97	95	98	36	3.481	0.459	1.663
	45	24.0	6.7	70	52	28	11	52	94	98	99	99	96	88	98	40	3.915	0.498	1.807
	50	13.2	9.3	72	56	31	12	54	91	97	100	96	94	86	97	46	4.351	0.527	1.911
	55	24.0	11.1	72	58	33	21	56	92	97	99	96	95	79	96	51	4.784	0.558	2.021
	60	24.0	12.1	76	63	36	17	60	90	97	100	95	94	83	96	57	5.221	0.587	2.128
	65	24.0	14.5	76	65	38	22	61	89	96	99	90	92	76	96	63	5.657	0.605	2.193
	70	4.5	15.6	78	67	40	22	63	87	94	99	91	91	77	96	69	6.088	0.630	2.283
	75	15.4	16.8	80	68	43	27	65	87	92	97	93	90	76	94	75	6.526	0.651	2.360
	80	24.0	17.9	80	70	45	31	66	86	91	98	92	89	72	95	81	6.961	0.669	2.425
	85	12.1	19.8	78	72	47	31	67	84	91	98	89	88	69	95	87	7.392	0.678	2.457
	90	24.0	20.8	79	72	48	37	68	84	88	96	89	87	66	96	92	7.832	0.694	2.517
	95	4.2	22.6	78	72	49	35	68	82	87	96	86	86	65	95	96	8.064	0.692	2.509
	100	2.6	22.4	77	72	49	36	67	82	87	96	87	86	64	95	96	8.070	0.693	2.512
Linz	5	0.1	0.0	6	4	2	0	4	99	100	100	100	100	99	100	3	0.151	0.027	0.100
	10	0.0	0.0	12	8	4	0	8	100	100	100	100	100	100	100	7	0.307	0.053	0.191
	15	0.0	0.0	17	11	5	0	11	100	100	100	100	100	100	100	11	0.460	0.075	0.272
	20	17.9	0.1	22	14	7	0	14	99	100	100	100	100	100	98	16	0.614	0.096	0.349
	25	11.0	0.2	26	17	8	0	17	100	100	100	100	100	99	99	21	0.767	0.116	0.420
	30	24.0	0.1	30	20	9	0	20	100	100	100	100	100	100	99	25	0.921	0.136	0.492
	35	24.0	0.0	35	23	10	0	23	100	100	100	100	100	100	99	30	1.074	0.155	0.564
	40	24.0	0.0	39	26	12	0	26	100	100	100	100	100	99	99	35	1.228	0.175	0.634
	45	24.0	0.0	42	28	13	1	28	99	100	100	100	99	98	100	40	1.381	0.193	0.699
	50	24.0	0.0	46	31	14	1	31	99	100	100	100	100	98	100	45	1.534	0.210	0.763
	55	24.0	0.1	50	33	16	1	34	100	100	100	100	100	99	100	51	1.688	0.228	0.826
	60	24.0	0.2	54	36	17	1	36	99	100	100	100	100	99	100	56	1.842	0.246	0.890
	65	22.8	1.6	55	38	19	3	38	98	100	100	100	99	96	99	61	1.994	0.259	0.940
	70	19.8	2.7	56	40	20	3	39	97	100	100	100	98	95	99	67	2.149	0.270	0.979
	75	3.8	3.1	59	41	21	4	41	97	99	100	98	98	94	99	73	2.302	0.282	1.021
	80	5.1	3.4	60	43	22	4	42	96	99	100	99	98	93	99	79	2.456	0.293	1.064
	85	4.7	4.0	61	44	23	6	44	96	99	100	98	98	90	98	85	2.610	0.303	1.099
	90	24.0	4.7	62	46	24	6	45	95	99	100	97	97	88	98	91	2.762	0.311	1.129
	95	2.9	5.5	61	46	25	8	45	95	98	99	94	96	84	98	96	2.880	0.315	1.142
	100	3.1	5.5	61	46	25	8	45	95	98	99	94	96	84	98	96	2.880	0.315	1.142

Figure 9: Results for Vienna ($b_{\mathrm{f}} = 50\%$). The shape of the city is composed of inhabited customer cells of side 100 m. The size of the diamonds indicates the number of locker modules of a locker configuration at the corresponding location.



of results of computational experiments on artificial problem instances (generated based on real-world data). In a sensitivity analysis of the latter results and in a case study (based on data from Vienna, Graz, and Linz), we have derived practical considerations for setting up a (white-label) parcel locker last-mile delivery network. For instance, we showed that (given our assumptions) small- and medium-sized locker compartments should be preferred over large and x-large sized compartments when designing parcel lockers. Moreover, we showed that the expected return on investment of using parcel lockers is promising, however, decreasing with the investment cost. We further demonstrated that parcel lockers can cover a soaring increase in demand for mail orders (as observed during the COVID-19 pandemic). Thus, SC viability can be supported if operators (e.g., governments) warehouse backup parcel lockers, which are activated if needed. In contrast, relying on parcel lockers that are permanently in use will hardly support SC viability because CEP delivery operators will adapt their resources accordingly. However, the SMCLLP enables, nevertheless, to determine locations and compartment layout for such parcel lockers. Benefits of using the proposed model include that decision support for parcel locker locations and their layout can be obtained simultaneously. The proposed approach further enables solving of large-scale problem instances.

The proposed models are accompanied by some limitations. Although we were able to solve large-scale problem instances, the computational performance heavily depends on some problem

parameters. One option to overcome this limitation is to solve the problem for smaller planning areas (e.g., districts) while increasing problem parameters that negatively affect the computational performance (e.g., bounds on the number of locker modules that can be combined). We observed that typically one type of locker module is preferable, which can imbalance coverage of demand for different commodities. The proposed models are not able to control a more balanced demand coverage (if desired). Also the presented case study is accompanied by some limitations. White-label parcel lockers are not yet a well-established last-mile delivery mode. Their usage needs to be more standardized to provide a sound interaction between sellers, CEP delivery operators, and customers (which is, however, out of the scope of this work). The potential locations for parcel lockers were further chosen by assumption. Moreover, although the proposed models fit well for private profit-maximizing operators (with limited budget), a probably more intuitive approach for determining a backup fleet of parcel lockers is minimizing investment cost while ensuring a certain demand coverage. Another option is to replace the budget constraint (2b) with a constraint that bounds the expected covered demand by a targeted fraction of the overall expected demand.

The latter issues are options for future work for which we give further examples in the following. Although we considered the willingness of customers to use parcel lockers in the objective function, the probability that they do so was neglected. One option to do so is to use multinomial logit customer choice behavior objective functions. Such functions are frequently concave, i.e., a solution approach using generalized Benders decomposition for solving large-scale problem instances seems promising. Including customer choice behavior also opens an avenue to investigate the effects of different pricing strategies, e.g., the effects of discounts for customers if they opt for parcel locker delivery. Another research direction involves the design of the parcel locker compartment layout. In the SMCLLP, we choose from a set of predefined locker modules. More effort is needed on the design of such modules. Alternatively, one can consider combining location decisions while simultaneously bin-packing compartments to those locations. If the order of the compartments in a locker configuration is neglected (which is not the case in [16]), the bin-packing procedure could be probably performed in knapsack subproblems.

Data availability

The used artificial planning areas (including population size per demand point) as well as the parcel locker module sets are available here: https://doi.org/10.13140/RG.2.2.36444.26241. The real-world instances are also included, however, the population size of each customer cell was randomly permuted to account for a license restriction. Note that the summary statistics of those instances remain unaffected.

Acknowledgments

This work has been supported by the Austrian Science Fund (project P-34151-N), which is greatly acknowledged. The author thanks the following scientists for discussions about the initial paper draft: Emilio J. Alarcon-Ortega, Immanuel M. Bomze, Karl F. Doerner, Margaretha Gansterer, and Sylvia Kritzinger. The support of Sylvia Kritzinger regarding the development of the conducted survey whose results were used in this paper is very appreciated. The author further thanks Ivana Ljubić for pointing to a refinement of the initial version of the derivation of the Benders cuts

in personal communication. Finally, the author thanks the guest editor and the two anonymous referees for their invaluable comments during the peer-review process.

References

[1] Joel Alcedo, Alberto Cavallo, Bricklin Dwyer, Prachi Mishra, and Antonio Spilimbergo. E-commerce during Covid: Stylized facts from 47 economies. *IMF Working Papers*, 2022(19), 2022.

[2] Amazon, 2022. https://flex.amazon.com/, last accessed on 2022/07/08.

[3] Ioannis Avgerinos, Ioannis Mourtos, and Georgios Zois. Multi-type facility location in printing and parcel delivery services. *Annals of Operations Research*, 309:365–393, 2022. doi:10.1007/s10479-021-04469-3.

[4] Jacques F. Benders. Partitioning procedures for solving mixed-variables programming problems. *Numerische Mathematik*, 4(1):238–252, 1962.

[5] Nils Boysen, Stefan Fedtke, and Stefan Schwerdfeger. Last-mile delivery concepts: A survey from an operational research perspective. *OR Spectrum*, 43:1–58, 2020. doi:10.1007/s00291-020-00607-8.

[6] Zhen-Hua Che, Tzu-An Chiang, and Yun-Jhen Luo. Multiobjective optimization for planning the service areas of smart parcel locker facilities in logistics last mile delivery. *Mathematics*, 10(3), 2022. doi:10.3390/math10030422.

[7] Reuven Cohen and Liran Katzir. The generalized maximum coverage problem. *Information Processing Letters*, 108(1):15–22, 2008. doi:10.1016/j.ipl.2008.03.017.

[8] Jean-François Cordeau, Fabio Furini, and Ivana Ljubić. Benders decomposition for very large scale partial set covering and maximal covering location problems. *European Journal of Operational Research*, 275(3):882–896, 2019. doi:10.1016/j.ejor.2018.12.021.

[9] Yael Deutsch and Boaz Golany. A parcel locker network as a solution to the logistics last mile problem. *International Journal of Production Research*, 56(1-2):251–261, 2018. doi:10.1080/00207543.2017.1395490.

[10] Alexandre Dolgui, Dmitry Ivanov, and Boris Sokolov. Ripple effect in the supply chain: An analysis and recent literature. *International Journal of Production Research*, 56(1-2):414–430, 2018. doi:10.1080/00207543.2017.1387680.

[11] Alina G. Dragomir, Tom Van Woensel, and Karl F. Doerner. The pickup and delivery problem with alternative locations and overlapping time windows. *Computers & Operations Research*, 143:105758, 2022. doi:10.1016/j.cor.2022.105758.

[12] Iain Dunning, Joey Huchette, and Miles Lubin. JuMP: A modeling language for mathematical optimization. *SIAM Review*, 59(2):295–320, 2017. doi:10.1137/15M1020575.

[13] Zehranaz Dönmez, Bahar Y. Kara, Özlem Karsu, and Francisco Saldanha da Gama. Humanitarian facility location under uncertainty: Critical review and future prospects. *Omega*, 102: 102393, 2021. doi:10.1016/j.omega.2021.102393.

[14] Reza Zanjirani Farahani, Nasrin Asgari, Nooshin Heidari, Mahtab Hosseininia, and Mark Goh. Covering problems in facility location: A review. *Computers & Industrial Engineering*, 62(1): 368–407, 2012. doi:10.1016/j.cie.2011.08.020.

[15] Reza Zanjirani Farahani, Shabnam Rezapour, Tammy Drezner, and Samira Fallah. Competitive supply chain network design: An overview of classifications, models, solution techniques and applications. *Omega*, 45:92–118, 2014. doi:https://doi.org/10.1016/j.omega.2013.08.006.

[16] Louis Faugère and Benoit Montreuil. Smart locker bank design optimization for urban omnichannel logistics: Assessing monolithic vs. modular configurations. *Computers & Industrial Engineering*, 139:105544, 2020. doi:10.1016/j.cie.2018.11.054.

[17] Matteo Fischetti, Domenico Salvagnin, and Arrigo Zanette. A note on the selection of Benders' cuts. *Mathematical Programming*, 124(1):175–182, 2010. doi:10.1007/s10107-010-0365-7.

[18] Matteo Fischetti, Ivana Ljubić, and Markus Sinnl. Redesigning Benders decomposition for large-scale facility location. *Management Science*, 63(7):2146–2162, 2017. doi:10.1287/mnsc.2016.2461.

[19] Roel Gevaers, Eddy Van de Voorde, and Thierry Vanelslander. Cost modelling and simulation of last-mile characteristics in an innovative B2C supply chain environment with implications on urban areas and cities. *Procedia - Social and Behavioral Sciences*, 125:398–411, 2014. doi:10.1016/j.sbspro.2014.01.1483.

[20] Jialin Han, Jiaxiang Zhang, Bing Zeng, and Mingsong Mao. Optimizing dynamic facility location-allocation for agricultural machinery maintenance using Benders decomposition. *Omega*, 105:102498, 2021. doi:10.1016/j.omega.2021.102498.

[21] Thomas Hoffmann and Gunnar Prause. On the regulatory framework for last-mile delivery robots. *Machines*, 6(3), 2018. doi:10.3390/machines6030033.

[22] Dmitry Ivanov. Viable supply chain model: Integrating agility, resilience and sustainability perspectives—lessons from and thinking beyond the COVID-19 pandemic. *Annals of Operations Research*, pages 1–21, 2020. doi:10.1007/s10479-020-03640-6.

[23] Dmitry Ivanov and Alexandre Dolgui. OR-methods for coping with the ripple effect in supply chains during COVID-19 pandemic: Managerial insights and research implications. *International Journal of Production Economics*, 232:107921, 2021. doi:10.1016/j.ijpe.2020.107921.

[24] Dmitry Ivanov, Alexandre Dolgui, and Boris Sokolov. Scheduling of recovery actions in the supply chain with resilience analysis considerations. *International Journal of Production Research*, 56(19):6473–6490, 2018. doi:10.1080/00207543.2017.1401747.

[25] Stanisław Iwan, Kinga Kijewska, and Justyna Lemke. Analysis of parcel lockers' efficiency as the last mile delivery solution – the results of the research in Poland. *Transportation Research Procedia*, 12:644–655, 2016. doi:10.1016/j.trpro.2016.02.018.

[26] Michael Kahr, Markus Leitner, Mario Ruthmair, and Markus Sinnl. Benders decomposition for competitive influence maximization in (social) networks. *Omega*, 100:102264, 2021. doi:10.1016/j.omega.2020.102264.

[27] Mouna Kchaou-Boujelben. Charging station location problem: A comprehensive review on models and solution approaches. *Transportation Research Part C: Emerging Technologies*, 132:103376, 2021. doi:10.1016/j.trc.2021.103376.

[28] Woojung Kim and Xiaokun Cara Wang. The adoption of alternative delivery locations in new york city: Who and how far? *Transportation Research Part A: Policy and Practice*, 158: 127–140, 2022. doi:https://doi.org/10.1016/j.tra.2022.02.006.

[29] Ömer Burak Kınay, Francisco Saldanha da Gama, and Bahar Y. Kara. On multi-criteria chance-constrained capacitated single-source discrete facility location problems. *Omega*, 83: 107–122, 2019. doi:10.1016/j.omega.2018.02.007.

[30] Tho V. Le, Amanda Stathopoulos, Tom Van Woensel, and Satish V. Ukkusuri. Supply, demand, operations, and management of crowd-shipping services: A review and empirical evidence. *Transportation Research Part C: Emerging Technologies*, 103:83–103, 2019. doi:10.1016/j.trc.2019.03.023.

[31] Yun Hui Lin, Yuan Wang, Dongdong He, and Loo Hay Lee. Last-mile delivery: Optimal locker location under multinomial logit choice model. *Transportation Research Part E: Logistics and Transportation Review*, 142:102059, 2020. doi:10.1016/j.tre.2020.102059.

[32] Yunhui Lin, Yuan Wang, Loo Hay Lee, and Ek Peng Chew. Profit-maximizing parcel locker location problem under threshold Luce model. *Transportation Research Part E: Logistics and Transportation Review*, 157:102541, 2022. doi:10.1016/j.tre.2021.102541.

[33] Rongjuan Luo, Shoufeng Ji, and Yuanyuan Ji. An active-learning pareto evolutionary algorithm for parcel locker network design considering accessibility of customers. *Computers & Operations Research*, 141:105677, 2022. doi:https://doi.org/10.1016/j.cor.2021.105677.

[34] Guodong Lyu and Chung-Piaw Teo. Last mile innovation: The case of the locker alliance network. *Manufacturing & Service Operations Management*, 2022. doi:10.1287/msom.2021.1000.

[35] Simona Mancini and Margaretha Gansterer. Vehicle routing with private and shared delivery locations. *Computers & Operations Research*, 133:105361, 2021. doi:10.1016/j.cor.2021.105361.

[36] Mia Meliaresti and Nahry. The externality of parcel locker service. *IOP Conference Series: Earth and Environmental Science*, 1000(1):012005, 2022. doi:10.1088/1755-1315/1000/1/012005.

[37] Santiago Nieto-Isaza, Pirmin Fontaine, and Stefan Minner. The value of stochastic crowd resources and strategic location of mini-depots for last-mile delivery: A Benders decomposition approach. *Transportation Research Part B: Methodological*, 157:62–79, 2022. doi:https://doi.org/10.1016/j.trb.2021.12.014.

[38] Ido Orenstein, Tal Raviv, and Elad Sadan. Flexible parcel delivery to automated parcel lockers: Models, solution methods and analysis. *EURO Journal on Transportation and Logistics*, 8(5): 683–711, 2019. doi:10.1007/s13676-019-00144-7.

[39] Wirtschaftskammer Österreich. KEP Branchenreport 2020 Wien, 2020. https://www.logistik2030.at/wp-content/uploads/2021/03/20210310_B_KEP-Branchenreport_SiS-web.pdf, last accessed on 2022/07/08.

[40] Kostas Patroumpas. World-scale OpenStreetMap POIs in CSV, 2019. http://www.slipo.eu/?p=1551, last accessed on 2022/07/08.

[41] Marcel Peppel and Stefan Spinler. The impact of optimal parcel locker locations on costs and the environment. *International Journal of Physical Distribution & Logistics Management*, 52 (4):324–350, 2022. doi:10.1108/IJPDLM-07-2021-0287.

[42] Matthias Prandtstetter, Clovis Seragiotto, Johannes Braith, Sandra Eitler, Bernhard Ennser, Georg Hauger, Nina Hohenecker, Reinhold Schodl, and Matthias Steinbauer. On the impact of open parcel lockers on traffic. *Sustainability*, 13(2):755, 2021. doi:10.3390/su13020755.

[43] Markus Rabe, Jorge Chicaiza-Vaca, Rafael D. Tordecilla, and Angel A. Juan. A simulation-optimization approach for locating automated parcel lockers in urban logistics operations. In *2020 Winter Simulation Conference (WSC)*, pages 1230–1241, 2020. doi:10.1109/WSC48552.2020.9384087.

[44] Sonja Rohmer and Bernard Gendron. *A guide to parcel lockers in last mile distribution: Highlighting challenges and opportunities from an OR perspective*. CIRRELT, 2020.

[45] Daniela Rojas Viloria, Elyn L Solano-Charris, Andrés Muñoz-Villamizar, and Jairo R Montoya-Torres. Unmanned aerial vehicles/drones in vehicle routing problems: A literature review. *International Transactions in Operational Research*, 28(4):1626–1657, 2021. doi:10.1111/itor.12783.

[46] Joseph Sarkis. Supply chain sustainability: Learning from the COVID-19 pandemic. *International Journal of Operations & Production Management*, 41(1):63–73, 2020. doi:10.1108/IJOPM-08-2020-0568.

[47] Marcia Regina Santiago Scarpin, Jorge Eduardo Scarpin, Nayane Thais Krespi Musial, and Wilson Toshiro Nakamura. The implications of COVID-19: Bullwhip and ripple effects in global supply chains. *International Journal of Production Economics*, 251:108523, 2022. doi:https://doi.org/10.1016/j.ijpe.2022.108523.

[48] Maren Schnieder, Chris Hinde, and Andrew West. Combining parcel lockers with staffed collection and delivery points: An optimization case study using real parcel delivery data (London, UK). *Journal of Open Innovation: Technology, Market, and Complexity*, 7(3):183, 2021. doi:10.3390/joitmc7030183.

[49] Stefan Schwerdfeger and Nils Boysen. Optimizing the changing locations of mobile parcel lockers in last-mile distribution. *European Journal of Operational Research*, 285(3):1077–1094, 2020. doi:10.1016/j.ejor.2020.02.033.

[50] Arianna Seghezzi, Chiara Siragusa, and Riccardo Mangiaracina. Parcel lockers vs. home delivery: A model to compare last-mile delivery cost in urban and rural areas. *International Journal of Physical Distribution & Logistics Management*, 53(3):213–237, 2022. doi:10.1108/IJPDLM-03-2020-0072.

[51] Statista, 2021. https://de.statista.com/statistik/daten/studie/579708/umfrage/monatliche-umsatzentwicklung-im-versand-und-internet-einzelhandel/, last accessed on 2022/07/08.

[52] Lianhua Tang, Yantong Li, Danyu Bai, Tao Liu, and Leandro C. Coelho. Bi-objective optimization for a multi-period COVID-19 vaccination planning problem. *Omega*, 110:102617, 2022. doi:https://doi.org/10.1016/j.omega.2022.102617.

[53] Derya Celik Turkoglu and Mujde Erol Genevois. A comparative survey of service facility location problems. *Annals of Operations Research*, pages 1–70, 2019. doi:10.1007/s10479-019-03385-x.

[54] J.H. Ron van Duin, Bart W. Wiegmans, Bart van Arem, and Yorick van Amstel. From home delivery to parcel lockers: A case study in Amsterdam. *Transportation Research Procedia*, 46: 37–44, 2020. doi:10.1016/j.trpro.2020.03.161.

[55] Xianhao Xu, Yaohan Shen, Wanying (Amanda) Chen, Yeming Gong, and Hongwei Wang. Data-driven decision and analytics of collection and delivery point location problems for online retailers. *Omega*, 100:102280, 2021. doi:10.1016/j.omega.2020.102280.

[56] Gege Yang, Yin Huang, Ying Fu, Biao Huang, Sishi Sheng, Lisha Mao, Shumin Huang, Yan Xu, Jia Le, Yun Ouyang, et al. Parcel locker location based on a bilevel programming model. *Mathematical Problems in Engineering*, 2020, 2020. doi:10.1155/2020/5263689.

[57] Yaoqin Yu, Feng Lian, and Zhongzhen Yang. Pricing of parcel locker service in urban logistics by a TSP model of last-mile delivery. *Transport Policy*, 114:206–214, 2021. doi:10.1016/j.tranpol.2021.10.002.

[58] Min Zhou, Lindu Zhao, Nan Kong, Kathryn S. Campy, Ge Xu, Guiju Zhu, Xianye Cao, and Song Wang. Understanding consumers' behavior to adopt self-service parcel services for last-mile delivery. *Journal of Retailing and Consumer Services*, 52:101911, 2020. doi:10.1016/j.jretconser.2019.101911.

A Detailed performance plots

Figure 10: Performance profiles of the *small* problem instances with respect to problem parameters relative available budget b_{f}, maximum number of locker modules that can be installed $\ell^{\max}$, potential location density $|J|/|I|$, coverage distance δ, and number of scenarios $|\Omega|$. The left column shows the relative number of problem instances solved to optimality within the runtime limit. The right column shows the optimality gaps of the remaining problem instances.

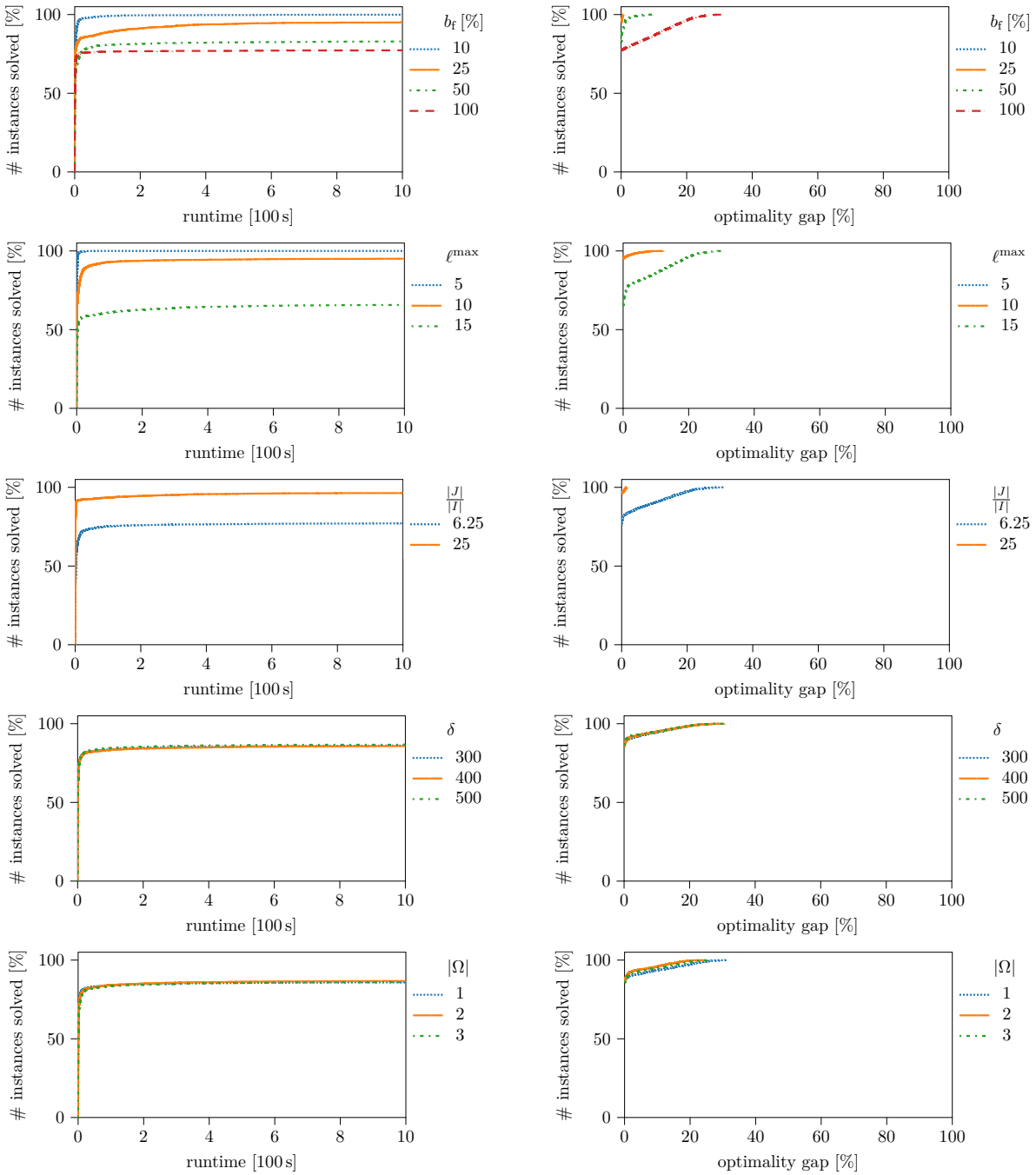


Figure 11: Performance profiles of the *medium* problem instances with respect to problem parameters relative available budget b_{f}, maximum number of locker modules that can be installed $\ell^{\max}$, potential location density $|J|/|I|$, coverage distance δ, and number of scenarios $|\Omega|$. The left column shows the relative number of problem instances solved to optimality within the runtime limit. The right column shows the optimality gaps of the remaining problem instances.

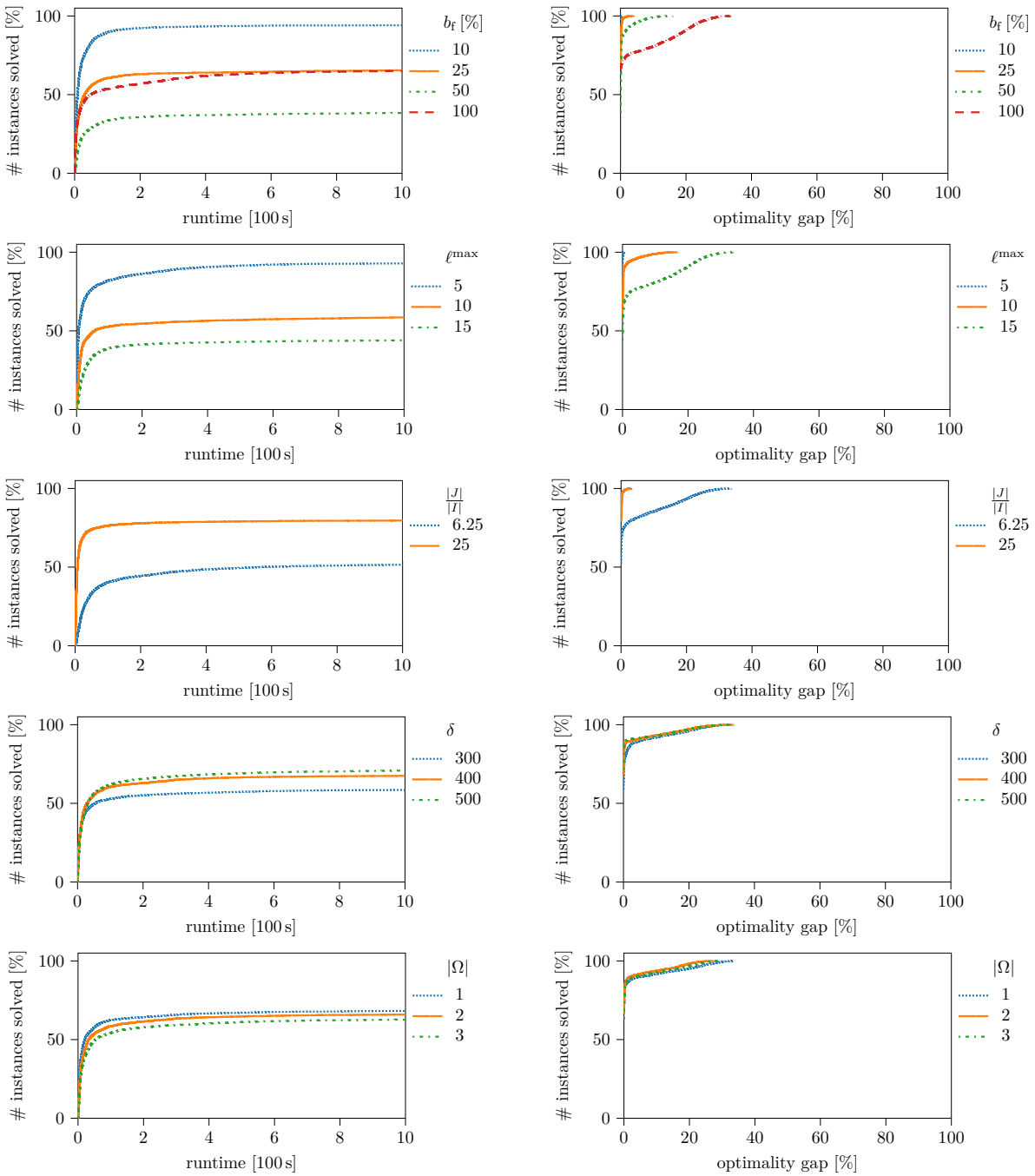


Figure 12: Performance profiles of the *large* problem instances with respect to problem parameters relative available budget b_{f}, maximum number of locker modules that can be installed $\ell^{\max}$, potential location density $|J|/|I|$, coverage distance δ, and number of scenarios $|\Omega|$. The left column shows the relative number of problem instances solved to optimality within the runtime limit. The right column shows the optimality gaps of the remaining problem instances.