

QCQP with Extra Constant Modulus Constraints: Theory and Applications on QoS Constrained Hybrid Beamforming for mmWave MU-MIMO

Xin He , *Member, IEEE*

Abstract—The constant modulus constraint is widely used in analog beamforming, hybrid beamforming, intelligent reflecting surface design, and radar waveform design. The quadratically constrained quadratic programming (QCQP) problem is also widely used in signal processing. However, the QCQP with extra constant modulus constraints was not systematically studied in mathematic programming and signal processing. For example, the multiple quality of service (QoS) constrained analog beamforming is rare, while the QoS constrained digital beamforming methods are abundant. We propose to tackle the QCQP with extra constant modulus constraints problem by solving a series of subproblems with linear programming (LP) under extra constant modulus constraints. Under mild condition, the strong duality between the LP with extra constant modulus constraints and its dual problem is established. Then, by using the optimal solutions from the subproblems, the QCQP with extra constant modulus constraints problem is solved with a monotonically converged algorithm, and all converged solutions are KKT points. As an application of the positive semidefinite quadratic form, the mean squared error (MSE) constrained hybrid beamforming is firstly proposed and solved in the past decades. As an application of the indefinite quadratic form, the signal-to-interference-plus-noise-ratio (SINR) constrained hybrid beamforming is solved. Simulation results show that the transmit power of the proposed method is similar to that of the semidefinite relaxation (SDR) method, while the computational time of proposed method is much faster than the SDR method.

Index Terms—QCQP, LP, Constant modulus, Unit modulus, MU-MIMO, MSE, SINR, Hybrid beamforming, Analog beamforming.

I. INTRODUCTION

The quadratically constrained quadratic programming (QCQP) problem is a well studied topic in mathematic programming [1]–[4]. The constant modulus constraint is a widely used constraint in signal processing, wireless communications, e.g., analog beamforming [5], hybrid beamforming [6], intelligent reflect surface (IRS) design [7], and the radar waveform design [8]. Therefore, the QCQP with extra constant modulus constraints problem has lots of real applications in 5G and 6G wireless communications.

In hybrid beamforming design, owing to the expensive radio frequency (RF) chains, the analog beamforming with constant

modulus is a necessary requirement. The semidefinite relaxation (SDR) was proposed in [5] to tackle the constant modulus analog beamforming constraint. By minimizing the mean squared error (MSE), an efficient majorization-minimization (MM) method was proposed for analog beamforming [9], [10]. The gradient projection (GP) method was proposed to tackle the unit-modulus least squares problem, and the computational complexity is lower than the SDR method [11]. Another efficient method to tackle the unit-modulus quadratic problem is the alternating direction method of multipliers (ADMM) method, but the nonconvexity makes the theoretic convergence property generally not guaranteed [12]. To maximize the sum rate of the multi-user multiple-input multiple-output (MU-MIMO), an approximation based efficient hybrid beamforming scheme was proposed in [6]. Based on matrix factorization, the manifold optimization was proposed to tackle the constant modulus constraint [13]. For the mentioned different methods [5], [6], [9]–[13], they all tend to maximize a certain total performance indicator with only the constant modulus constraint. The QCQP problems with constant-modulus constraints was proposed in [14], but there is only one quadratic constraint and the result cannot be generalized to multiple quadratic constraints. With multiple signal-to-interference-plus-noise-ratio (SINR) constraints, the analog beamforming in [15] did not impose the constant modulus constraint, the SINR constraints are not satisfied in [16], [17], only the recent paper with the SDR method guaranteed the SINR constraints [18]. In fact, owing to the difficult of the QCQP with extra constant modulus constraints, the multiple quality of service (QoS) constrained analog beamforming is rare, while the QoS constrained digital beamforming methods are abundant [19]–[25]. This difficult is the motivation for us to tackle the general QCQP with extra constant modulus constraint, and apply to the QoS constrained analog beamforming.

In IRS design, the modulus of the reflecting element in IRS is usually fixed as one [26], [27]. The leading approach to tackle the unit-modulus constraint is the SDR method [26]–[31]. However, the computational complexity of the SDR approach is large, and the simple method only exists for special case with a single user [26], [27]. To reduce the computational complexity, the complex circle manifold (CCM) method was proposed to solve the unit-modulus constraint [7]. Moreover, the penalty dual decomposition (PDD) method was proposed in [32], [33]. Although the subproblem of the PDD method is simple, the inner and outer loops and the non-monotonic convergence behavior make the iteration number

This work was supported by the National Natural Science Foundation of China under Grants 61807018.

Xin He was with the College of Electronics and Information Engineering, Shenzhen University, China. Xin He is with the College of Electronics and Information Engineering, Shantou University, China. Email: hx66hx@gmail.com.
ORCID iD: 0000-0002-2248-4742

become large. Furthermore, the CCM and PDD methods are suitable for the problem without constraints except the unit-modulus constraint. With unit-modulus constraint and only one extra power transfer constraint, the MM method and successive convex approximation (SCA) method were proposed in [34]. With unit-modulus constraint and multiple QoS constraints, a convex relaxation based relax-and-retract method was proposed for the joint IRS and MU-MIMO transceiver design [35], which has lower computational complexity than that of the SDR method. Similar to the hybrid beamforming design, the difficulty of the joint multiple QoS constraints and constant modulus constraint arises again in IRS design.

In radar waveform design, the constant modulus constraint is generally required in radar waveform design owing to the efficient and cheap nonlinear power amplifier. The SDR method was proposed to tackle the constant modulus and similarity constraints [8], [36]. To reduce the computational complexity, the successive quadratic refinement method was proposed in [37], and the ADMM method was proposed in [38]. The most efficient method to tackle the constant modulus and ℓ_∞ -norm based similarity constraint is the MM method [39]. However, when the similarity constraint is associated with the general ℓ_p -norm, the constant modulus and ℓ_p -norm similarity constraint becomes difficult again [40], [41].

In this paper, we propose to tackle the general QCQP with extra constant modulus constraints, and apply the obtained theory to QoS constrained hybrid beamforming. Firstly, the objective and constrained quadratic functions are approximated by their majorizers, which is a special upper bound. Under the constant modulus constraints, the subproblems become the linear programming (LP) problem with extra constant modulus constraints. By proving the strong duality, the optimal solution of LP with extra constant modulus constraints is obtained under mild condition. With optimal subproblem solutions, the objective value of QCQP with extra constant modulus constraints is monotonically converged, and all converged solutions are KKT points of the original problem. Therefore, the theoretic contributions of the paper are summarized as follows.

- The strong duality between the LP problem with extra constant modulus constraints and its dual problem is established under mild condition.
- The QCQP with extra constant modulus constraints is tackled by solving a series of LP problem with extra constant modulus constraints, the objective value of the proposed method is monotonically converged, and all converged solutions are KKT points of the original problem.

By using the obtained theory, we proposed to solve the QoS constrained hybrid beamforming in wireless communications. As an application in terms of positive semidefinite quadratic form, the MSE constrained hybrid beamforming is solved for the mmWave MU-MIMO system. As an application in terms of indefinite quadratic form, the SINR constrained hybrid beamforming is solved for the mmWave multi-user multiple-input single-output (MU-MISO) system. The transmit power of the proposed two schemes are monotonically

converged. Simulation results show that the computational time of proposed method is much smaller than the SDR method, while the transmit power is similar to that of the SDR method. Therefore, the contributions of the real applications are summarized as follows.

- For positive semidefinite quadratic form, the first MSE constrained hybrid beamforming is proposed and solved in the past decades.
- For indefinite quadratic form, the SINR constrained hybrid beamforming is solved.

Note that the proposed theory was also applied to the MSE constrained IRS design [42].

Notation: In this paper, $\mathbb{E}(\cdot)$, $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ denote statistical expectation, transposition, conjugate and Hermitian, respectively. The $\|\cdot\|_2$ denotes the norm of a vector, and $\|\cdot\|_F$ stands for the Frobenius norm of a matrix. The notation $\text{Tr}(\mathbf{A})$ is the trace of a square matrix \mathbf{A} , and $\text{vec}(\mathbf{G})$ is to vectorize the matrix \mathbf{G} into a column vector. The notation $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ stand for the real part and the imaginary part of a complex number, respectively. The notation \otimes and \oplus stand for the Kronecker product and the direct sum of different matrices, respectively. For a vector $\boldsymbol{\theta}$, $\boldsymbol{\theta}(n)$ is the n -th element of $\boldsymbol{\theta}$. For a matrix \mathbf{G} , $\mathbf{G}(i, j)$ is the element from the i -th row and j -th column of \mathbf{G} . For positive semidefinite matrix $\boldsymbol{\Sigma}$, we have $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}^{\frac{1}{2}} \boldsymbol{\Sigma}^{\frac{H}{2}}$. The \mathbf{I}_K is a $K \times K$ identity matrix. The elements in $\arg(\mathbf{z})$ are the phases of the input complex vectors. The special notation $\mathbf{p} \neq 0$ means that all elements in \mathbf{p} are not zero.

II. QCQP WITH EXTRA CONSTANT MODULUS CONSTRAINTS

The QCQP with extra constant modulus constraints problem is

$$\begin{aligned} \min_{\mathbf{x}} \quad & f_0(\mathbf{x}) \\ \text{s.t.} \quad & f_k(\mathbf{x}) \leq 0, \quad \forall k \in [1, K], \\ & |\mathbf{x}(n)| = 1, \quad \forall n \in [1, N], \end{aligned} \quad (1)$$

where the quadratic functions are $f_k(\mathbf{x}) := \mathbf{x}^H \mathbf{Q}_k \mathbf{x} + 2\text{Re}(\mathbf{p}_k^H \mathbf{x}) + c_k$, $k \in [0, 1, \dots, K]$, $\mathbf{Q}_k = \mathbf{Q}_k^H \in \mathbb{C}^{N \times N}$, $\mathbf{p}_k \in \mathbb{C}^N$, $c_k \in \mathbb{R}$, and the complex decision variable is $\mathbf{x} \in \mathbb{C}^N$.

For the nonconvex problem (1), the SDR method can be used to tackle it. However, the SDR method with a fixed randomization number is not guaranteed to provide feasible solution, and the its computational complexity order $\mathcal{O}(N^{6.5})$ is large. The proposed method is to tackle problem (1) sequentially. In particular, the majorizer (a special upper bound function) of the objective function is used to approximate the objective function, and the majorizers of the quadratic constraints are used to approximate the constraint functions. Firstly, with a feasible solution \mathbf{x}_i in the problem (1), one majorizer of the quadratic term is constructed as [43, e.q. (26)]

$$\begin{aligned} \mathbf{x}^H \mathbf{Q}_k \mathbf{x} \leq & 2\text{Re}(\mathbf{x}^H (\mathbf{Q}_k - t_k \mathbf{I}) \mathbf{x}_i) \\ & + t_k \mathbf{x}^H \mathbf{I} \mathbf{x} + \mathbf{x}_i^H (t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i \end{aligned} \quad (2)$$

where $t_k \mathbf{I} \succeq \mathbf{Q}_k$. A computational efficient way to choose t_k is illustrated in Appendix A.

Algorithm 1 iMM method for QCQP with extra constant modulus constraints (1).

- 1: **initialization:** set $i = 0$, \mathbf{x}_i is a feasible solution in (1).
- 2: **repeat**
- 3: Find optimal solution \mathbf{x} in problem (4), set $i = i + 1$, let $\mathbf{x}_i = \mathbf{x}$.
- 4: **until** $(f_0(\mathbf{x}_{i-1}) - f_0(\mathbf{x}_i))/|f_0(\mathbf{x}_i)| \leq \epsilon_1$

With the majorizer in (2), the majorizer of the quadratic function under $\{|\mathbf{x}(n)| = 1\}_{n=1}^N$ is constructed as

$$f_k(\mathbf{x}) \leq \overbrace{2\text{Re}(\mathbf{x}^H[(\mathbf{Q}_k - t_k\mathbf{I})\mathbf{x}_i + \mathbf{p}_k])}^{\bar{f}_k(\mathbf{x})} + \underbrace{2t_kN - \mathbf{x}_i^H\mathbf{Q}_k\mathbf{x}_i + c_k}_{\bar{c}_k} \quad (3)$$

where the majorizers $\{\bar{f}_k(\mathbf{x})\}_{k=0}^K$ are linear functions of \mathbf{x} .

After those constructions, the problem (1) can be solved by the proposed method in Algorithm 1 with a series of subproblems,

$$\begin{aligned} \min_{\mathbf{x}} \quad & \bar{f}_0(\mathbf{x}) \\ \text{s.t.} \quad & \bar{f}_k(\mathbf{x}) \leq 0, \forall k \in [1, K] \\ & |\mathbf{x}(n)| = 1, \forall n \in [1, N], \end{aligned} \quad (4)$$

which is a linear programming (LP) problem with extra constant modulus constraints. Strictly speaking, the proposed method is a combination of MM method [43], [44] and the inner approximation algorithm [45]. Since the objective function of problem (4) is an upper bound of that of problem (1), and the feasible set of problem (4) is a subset of that of problem (1), the proposed method is called as inner majorization-minimization (iMM) method.

The property of the objective values of problems (1) and (4) is described as follows.

Lemma 1. *If the problem (1) is feasible, the problem (4) is feasible, and the objective values of the problems (1) and (4) are bounded.*

Proof. Since one special property of the majorizer function is $f_k(\mathbf{x}_i) = \bar{f}_k(\mathbf{x}_i)$ under $\{|\mathbf{x}(n)| = 1\}_{n=1}^N$, if the problem (1) is feasible, the subproblem (4) is feasible. Owing the constant modulus constraints on the decision variable \mathbf{x} , the feasible sets of problems (1) and (4) are compact. The quadratic function $f_0(\mathbf{x})$ and linear function $\bar{f}_0(\mathbf{x})$ with bounded input lead to bounded output. \square

The property in Lemma 1 is a necessary component for our convergence analysis and strong duality results.

If the optimal solution of the subproblem (4) is found in every iteration, then we have $f_0(\mathbf{x}_i) = \bar{f}_0(\mathbf{x}_i) \geq \bar{f}_0(\mathbf{x}_{i+1}) \geq f_0(\mathbf{x}_{i+1})$, i.e., the iMM method in Algorithm 1 is monotonically nonincreasing. Furthermore, since the objective value of the problem (1) is bounded, Algorithm 1 guarantees to converge.

The subproblem (4) can be solved by its Lagrange dual problem,

$$\sup_{\{\nu_k \geq 0\}_{k=1}^K} \min_{\{|\mathbf{x}(n)|=1\}_{n=1}^N} \bar{f}_0(\mathbf{x}) + \sum_{k=1}^K \nu_k \bar{f}_k(\mathbf{x}), \quad (5)$$

where $\{\nu_k\}_{k=1}^K$ are the Lagrange dual variables and $\boldsymbol{\nu} = [\nu_1, \nu_2, \dots, \nu_K]^T$. since the Lagrange in (5) is a linear function of \mathbf{x} , the optimal dual function is obtained at

$$\mathbf{x}(\boldsymbol{\nu}) = \exp\left(\sqrt{-1} \arg\left(\sum_{k=1}^K [(t_k\mathbf{I} - \mathbf{Q}_k)\mathbf{x}_i - \mathbf{p}_k]\nu_k + (t_0\mathbf{I} - \mathbf{Q}_0)\mathbf{x}_i - \mathbf{p}_0\right)\right). \quad (6)$$

Therefore, the dual problem (5) is equivalent to

$$\sup_{\{\nu_k \geq 0\}_{k=1}^K} \bar{f}_0(\mathbf{x}(\boldsymbol{\nu})) + \sum_{k=1}^K \nu_k \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})), \quad (7)$$

where

$$\bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) = 2\text{Re}(\mathbf{x}(\boldsymbol{\nu})^H[(\mathbf{Q}_k - t_k\mathbf{I})\mathbf{x}_i + \mathbf{p}_k]) + \bar{c}_k, \quad (8)$$

and $k \in [0, 1, \dots, K]$.

We claim that some optimal solution $\{\nu_k\}_{k=1}^K$ of problems (5) and (7) comes from the solution of the equations,

$$\mathbf{x}(\boldsymbol{\nu}) = \exp\left(\sqrt{-1} \arg\left(\sum_{k=1}^K [(t_k\mathbf{I} - \mathbf{Q}_k)\mathbf{x}_i - \mathbf{p}_k]\nu_k + (t_0\mathbf{I} - \mathbf{Q}_0)\mathbf{x}_i - \mathbf{p}_0\right)\right), \quad (9)$$

$$0 \leq \nu_k \leq \infty, \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) \leq 0, \forall k \in [1, K], \quad (10)$$

$$\nu_k \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) = 0, \forall k \in [1, K], \quad (11)$$

The conditions (9) to (11) are used to enforce strong duality between the nonconvex primal problem (4) and its dual problem (7). In general, strong duality between a nonconvex (even convex) primal problem and its dual problem does not hold. However, we'll show that strong duality between the LP with extra constant modulus constraints and its dual problem holds at a certain condition.

Proposition 1. *If the solution of the equations (9) to (11) exists, then the strong duality between the nonconvex primal problem (4) and its dual problem (7) is held, and all solutions in (9) to (11) are optimal solutions of the problem (4).*

Proof. Since the solution $\{\nu_k\}_{k=1}^K$ in (9) to (11) exist, it has to be a subset of $\{\nu_k \geq 0\}_{k=1}^K$. Therefore, we can take the solution $\{\nu_k\}_{k=1}^K$ in (9) to (11) into the problem (7), a dual objective value is achieved as

$$\begin{aligned} \sup_{\{\nu_k\}_{k=1}^K} \quad & \bar{f}_0(\mathbf{x}(\boldsymbol{\nu})) + 0 \\ \text{s.t.} \quad & 0 \leq \nu_k \leq \infty, \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) \leq 0, \forall k \in [1, K], \\ & \nu_k \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) = 0, \forall k \in [1, K], \end{aligned} \quad (12)$$

which equals to one objective value of the primal problem (4). Therefore, the existence of the solution in (9) to (11) implies the strong duality.

Since any dual objective value under the exist solution in (9) to (11) equals to the primal objective value in problem (4), and the optimal values of problems (4) and (7) are unique, then all solutions in (9) to (11) are optimal solutions of the problem (4). \square

The condition (9) seems to be always exist, but it is undetermined when the number of $\nu_k = \infty$ is larger than or equal to two. Note that if only one $\nu_k = \infty$ and all elements in $(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k$ are not zero, which is represented as $(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k \neq 0$ for notational simplicity, then (9) becomes $\lim_{\nu_k \rightarrow \infty} \mathbf{x}(\boldsymbol{\nu}) = \exp(j \arg((t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k))$.

Although Proposition 1 suggest a possible way to solve the primal problem (4), solving the nonlinear equations in (9) to (11) is very difficult and the existence is also unknown. To tackle the dual problem, we propose to solve $\{\nu_k\}_{k=1}^K$ one by one with the coordinate ascend method, i.e., optimizing ν_k with other $\{\nu_i\}_{i \neq k}^K$ being fixed,

$$\sup_{\nu_k \geq 0} \bar{f}_0(\mathbf{x}(\boldsymbol{\nu})) + \sum_{k=1}^K \nu_k \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})). \quad (13)$$

We claim that some optimal solution ν_k of the problem (13) comes from the solution of the equations,

$$\mathbf{x}(\boldsymbol{\nu}) = \exp\left(\sqrt{-1} \arg\left(\sum_{k=1}^K [(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k] \nu_k + (t_0 \mathbf{I} - \mathbf{Q}_0) \mathbf{x}_i - \mathbf{p}_0\right)\right), \quad (14)$$

$$0 \leq \nu_k \leq \infty, \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) \leq 0, \quad (15)$$

$$\nu_k \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) = 0, \quad (16)$$

Before to prove the above statement, we need the following lemma.

Lemma 2. *If $\lim_{\nu_k \rightarrow \infty} \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) = 0$, $\{\nu_i\}_{i \neq k}^K$ are finite, and $(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k \neq 0$, then $\lim_{\nu_k \rightarrow \infty} \nu_k \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) = 0$.*

Proof. See Appendix B. \square

The limit result in Lemma 2 is the result of the special structure of the LP with extra constant modulus constraints in the problem (4). By using Lemma 2, the optimal solution of the dual subproblem (13) is obtained as follows.

Proposition 2. *If the problem (1) is feasible, $\{\nu_i\}_{i \neq k}^K$ are finite, and $(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k \neq 0$, the solutions in the equations (14) to (16) exist and they are the optimal solutions of the problem (13).*

Proof. Since the problem (1) is feasible and the subproblem (4) is constructed at a feasible solution, the primal problem (4) is feasible and its objective value is upper bounded due to Lemma 1. According to the weak duality theorem, the dual problem (13) is also upper bounded. Therefore, the following condition has to be satisfied,

$$\lim_{\nu_k \rightarrow \infty} \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) \leq 0. \quad (17)$$

Otherwise, the condition $\lim_{\nu_k \rightarrow \infty} \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) > 0$ leads to the result with $\lim_{\nu_k \rightarrow \infty} \nu_k \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) = \infty$, which contradicts the bounded property of dual problem (13). Therefore, the result in (17) means the existence of (15). Furthermore, the conditions $(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k \neq 0$, finite $\{\nu_i\}_{i \neq k}^K$, and the result in (17) means the existence of (14) and (15)

After proving the existence of (14) and (15), the existence of (16) is proved as the following three steps.

- 1) If $\bar{f}_k(\mathbf{x}(\boldsymbol{\nu}))|_{\nu_k=0} \leq 0$, then $\nu_k = 0$ is the solution of (14) to (16).
- 2) Otherwise, if $\lim_{\nu_k \rightarrow \infty} \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) = 0$, then $\nu_k = \infty$ is the solution of (14) to (16) due to Lemma 2.
- 3) Otherwise, the conditions $\bar{f}_k(\mathbf{x}(\boldsymbol{\nu}))|_{\nu_k=0} > 0$ and $\lim_{\nu_k \rightarrow \infty} \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) < 0$ ensure that there exist a finite ν_k such that $\bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) = 0$, owing to the continuity of the function $\bar{f}_k(\mathbf{x}(\boldsymbol{\nu}))$.

Therefore, the three steps proof means that the solution of (14) to (16) exists.

From (4) to (5), we know that the problem (13) is the dual problem of the following constructed primal problem,

$$\begin{aligned} \min_{\mathbf{x}} \bar{f}_0(\mathbf{x}) + \sum_{i=1, i \neq k}^K \nu_i \bar{f}_i(\mathbf{x}) \\ \text{s.t. } \bar{f}_k(\mathbf{x}) \leq 0, \\ |\mathbf{x}(n)| = 1, \forall n \in [1, N]. \end{aligned} \quad (18)$$

Since the solution ν_k in (14) to (16) exists, and they are a subset of $\{\nu_k \geq 0\}$, we can take the solution in (14) to (16) into the problem (13), a dual objective value is achieved as

$$\begin{aligned} \sup_{\nu_k} \bar{f}_0(\mathbf{x}(\boldsymbol{\nu})) + \sum_{i=1, i \neq k}^K \nu_i \bar{f}_i(\mathbf{x}(\boldsymbol{\nu})) + 0 \\ \text{s.t. } 0 \leq \nu_k \leq \infty, \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) \leq 0, \\ \nu_k \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) = 0, \end{aligned} \quad (19)$$

which equals to one objective value of the primal problem (18). Therefore, the strong duality between the primal problem (18) and dual problem (13) is held, and the optimal solution is obtained from (14) to (16) owing to the same reason in the proof of Proposition 1. \square

Although the existence of the solution in (14) to (16) is proved in in Proposition 2, we need the following lemma to efficiently compute the solution in (14) to (16).

Lemma 3.

- 1) $\bar{f}_k(\mathbf{x}(\boldsymbol{\nu}))$ is a monotonic nonincreasing function of ν_k if $\{\nu_l\}_{l \neq k}^K$ are finite.
- 2) $\bar{f}_k(\mathbf{x}(\boldsymbol{\nu}))$ is a strictly decreasing function of ν_k if $\{\nu_l\}_{l \neq k}^K$ are finite, $[(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k] \nu_k + \sum_{l=1, l \neq k}^K [(t_l \mathbf{I} - \mathbf{Q}_l) \mathbf{x}_i - \mathbf{p}_l] \nu_l + (t_0 \mathbf{I} - \mathbf{Q}_0) \mathbf{x}_i - \mathbf{p}_0 \neq 0$ for all $\nu_k \in [0, \infty)$, $(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k \neq 0$, and $[(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k] s \neq \sum_{l=1, l \neq k}^K [(t_l \mathbf{I} - \mathbf{Q}_l) \mathbf{x}_i - \mathbf{p}_l] \nu_l + (t_0 \mathbf{I} - \mathbf{Q}_0) \mathbf{x}_i - \mathbf{p}_0, \forall s \in \mathbb{R}$.

Algorithm 2 K -bisection method to find $\mathbf{x}(\boldsymbol{\nu})$ in (9).

```

1: initialization: set  $i_2 = 0$  and use the latest finite  $\boldsymbol{\nu}$  as  $\boldsymbol{\nu}[0]$ ,
   otherwise  $\boldsymbol{\nu}[0] = [0, 0, \dots, 0]$ . For notational simplicity,
    $\bar{f}_k(\mathbf{x}(a))$  with scalar input  $a$  means  $\bar{f}_k(\mathbf{x}(\boldsymbol{\nu}))|_{\nu_k=a}$ .
2: repeat
3:   for each  $k \in [1, K]$  do
4:     if  $\bar{f}_k(\mathbf{x}(0)) \leq 0$ , then
5:        $\nu_k = 0$ 
6:     else if  $|\lim_{\nu_k \rightarrow \infty} \bar{f}_k(\mathbf{x}(\boldsymbol{\nu}))| \leq \epsilon$  then
7:        $\nu_k = \infty$ . Stop Algorithm 2.
8:     else
9:       Let  $\nu_k^l = 0, \nu_k^u = 1$ 
10:      if  $\bar{f}_k(\mathbf{x}(1)) \leq 0$  then
11:         $\nu_k^u = 1$ 
12:      else
13:        repeat
14:           $\nu_k^u = 2\nu_k^l$ 
15:        until  $\bar{f}_k(\mathbf{x}(\nu_k^u)) \leq 0$ 
16:         $\nu_k^l = \nu_k^u/2$ 
17:      end if
18:      repeat
19:        Let  $\nu_k = \frac{\nu_k^l + \nu_k^u}{2}$ . If  $\bar{f}_k(\mathbf{x}(\nu_k)) > 0$ , then  $\nu_k^l = \nu_k$ .
        Otherwise,  $\nu_k^u = \nu_k$ .
20:      until  $|\bar{f}_k(\mathbf{x}(\nu_k)) + \epsilon/2| \leq \epsilon/2$ 
21:      end if
22:    end for
23:    Let  $\boldsymbol{\nu}[i_2+1] = [\nu_1, \nu_2, \dots, \nu_K]$ ,  $h[i_2] = \bar{f}_0(\mathbf{x}(\boldsymbol{\nu}[i_2])) + \sum_{k=1}^K \nu_k[i_2] \bar{f}_k(\mathbf{x}(\boldsymbol{\nu}[i_2]))$  and set  $i_2 = i_2 + 1$ .
24: until  $(h[i_2] - h[i_2 - 1])/|h[i_2]| \leq \epsilon_2$ 

```

Proof. See Appendix C □

By using the monotonic property of Lemma 3 and the three steps existence proof of Proposition 2, the ν_k in (14) to (16) can be found via the bisection method with a finite iteration number. The coordinate ascent method with K -bisections to solve the the dual problem (7) is described in Algorithm 2, where the precondition becomes that all elements of $\{(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k\}_{k=1}^K$ are not zero. The useful conditions (assumptions) of this paper are summarized as follows

Assumption 1. $\{\{(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k \neq 0\}_{k=1}^K\}_{i=0}^\infty$.

Assumption 2. For any $k \in [1, \dots, K]$, $\{[(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k] \nu_k + \sum_{l=1, l \neq k}^K [(t_l \mathbf{I} - \mathbf{Q}_l) \mathbf{x}_i - \mathbf{p}_l] \nu_l + (t_0 \mathbf{I} - \mathbf{Q}_0) \mathbf{x}_i - \mathbf{p}_0 \neq 0\}_{i=0}^\infty$ for all $\nu_k \geq 0$, $\{[(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k] s \neq \sum_{l=1, l \neq k}^K [(t_l \mathbf{I} - \mathbf{Q}_l) \mathbf{x}_i - \mathbf{p}_l] \nu_l + (t_0 \mathbf{I} - \mathbf{Q}_0) \mathbf{x}_i - \mathbf{p}_0, \forall s \in \mathbb{R}\}_{i=0}^\infty$.

Under Assumption 1, the optimal solution of the dual subproblem (13) guarantees that the objective value of the dual problem (7) is monotonically nondecreasing, i.e.,

$$\begin{aligned} & \bar{f}_0(\mathbf{x}(\boldsymbol{\nu}[i_2 - 1])) + \sum_{k=1}^K \nu_k[i_2 - 1] \bar{f}_k(\mathbf{x}(\boldsymbol{\nu}[i_2 - 1])) \\ & \leq \bar{f}_0(\mathbf{x}(\boldsymbol{\nu}[i_2])) + \sum_{k=1}^K \nu_k[i_2] \bar{f}_k(\mathbf{x}(\boldsymbol{\nu}[i_2])), \end{aligned} \quad (20)$$

where i_2 is the iteration number in Algorithm 2. Furthermore, the dual problem (7) is upper bounded due to the weak duality theorem and the bounded primal problem in Lemma 1. Therefore, Algorithm 2 is guaranteed to converge, i.e.,

$$\begin{aligned} & \lim_{i_2 \rightarrow \infty} \bar{f}_0(\mathbf{x}(\boldsymbol{\nu}[i_2 - 1])) + \sum_{k=1}^K \nu_k[i_2 - 1] \bar{f}_k(\mathbf{x}(\boldsymbol{\nu}[i_2 - 1])) \\ & = \lim_{i_2 \rightarrow \infty} \bar{f}_0(\mathbf{x}(\boldsymbol{\nu}[i_2])) + \sum_{k=1}^K \nu_k[i_2] \bar{f}_k(\mathbf{x}(\boldsymbol{\nu}[i_2])), \end{aligned} \quad (21)$$

Under Assumptions 1 and 2, Lemma 3 makes the K bisection results in Algorithm 2 being unique, i.e., the optimal solutions in dual subproblems (13) are unique according to Proposition 2. Therefore, the Algorithm 2 converges with strict inequalities in (20) until the converge condition (21) happens under the unique reason: $\boldsymbol{\nu}[i_2 - 1] = \boldsymbol{\nu}[i_2]$. Therefore, all the equations in (9) to (11) are simultaneously satisfied, and the optimal solution of the primal problem (4) is obtained as $\mathbf{x}(\boldsymbol{\nu})$ according to Proposition 1.

To tackle the QCQP and LP with extra constant modulus constraint, the major theorem of the proposed IMM method is summarized as follows.

Theorem 1. If \mathbf{x}_0 is a feasible solution of (1), under Assumptions 1 and 2, we have

- 1) The strong duality holds between problems (7) and (4), and its optimal solution is obtained from the limit solution of the K -bisection method in Algorithm 2¹.
- 2) The problem (1) is tackled by the IMM method in Algorithm 1 (with Algorithm 2 as the subproblem solver), which has a monotonically nonincreasing convergence property.

Proof. Under Assumption 1, the minimizer of $\bar{f}_k(\mathbf{x})$ under $\{|\mathbf{x}(n)| = 1\}_{n=1}^N$ is unique. Since problems (1) and (4) are feasible, under Assumption 1, the feasible set of (4) is either strictly feasible or a singleton.

If the feasible set of (4) is strictly feasible, then $\lim_{\nu_k \rightarrow \infty} \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) = \bar{f}_k(\exp(j \arg[(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k])) < 0$. This makes $\nu_k = \infty$ is not an optimal solution of (13) because any finite ν_k leads to finite dual objective value. Therefore, the assumptions on $\{\nu_k < \infty\}_{k=1}^K$ are satisfied. By proving the existence of (14) to (16) via Proposition 2, the strong duality ensures that the optimal solution of the dual subproblem (13) is obtained. By using the dual coordinate ascent method, the dual objective values in (7) are monotonically nondecreasing. Since the primal problem (4) is feasible and bounded, its dual problem (7) is also bounded. Therefore, the dual coordinate ascent method converges. Under Assumptions 1 and 2, Lemma 3 makes the objective value of the dual coordinate ascent process strictly increasing until it converges with $\boldsymbol{\nu}[i_2 - 1] = \boldsymbol{\nu}[i_2]$. Therefore, the conditions in (9) to (11) are simultaneously satisfied, and the strong duality holds between the problems (4) and (7) according to Proposition 1.

If the feasible set of (4) is a singleton, e.g., the set $\{\mathbf{x} | \bar{f}_k(\mathbf{x}) \leq 0, \{|\mathbf{x}(n)| = 1\}_{n=1}^N\}$ is a singleton

¹In theoretic analysis, the optimal solution is obtained with termination threshold $\epsilon = \epsilon_2 = 0$, and there is no numeric error.

$\{\exp(j \arg[(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k])\}$. Under Assumptions 1 and 2, the strictly decreasing property in Lemma 3 makes $\bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) \geq 0, \forall \nu_k \geq 0$ and only $\nu_k = \infty$ lead to $\bar{f}_k(\lim_{\nu_k \rightarrow \infty} \mathbf{x}(\boldsymbol{\nu})) = 0$. Therefore, Algorithm 2 will be terminated at the 7-th step and the primal optimal solution is returned as $\lim_{\nu_k \rightarrow \infty} \mathbf{x}(\boldsymbol{\nu}) = \exp(j \arg[(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k])$. Since the problem (4) is feasible, the feasible set of (4) is $\{\mathbf{x} | \{\bar{f}_k(\mathbf{x}) \leq 0\}_{k=1}^K, \{|\mathbf{x}(n)| = 1\}_{n=1}^N\} = \{\mathbf{x} | \mathbf{x} = \lim_{\nu_k \rightarrow \infty} \mathbf{x}(\boldsymbol{\nu})\}$. Therefore, the constraints $\{\bar{f}_l(\lim_{\nu_k \rightarrow \infty} \mathbf{x}(\boldsymbol{\nu})) \leq 0\}_{l \neq k}^K$ are satisfied. If $\bar{f}_l(\lim_{\nu_k \rightarrow \infty} \mathbf{x}(\boldsymbol{\nu})) = 0$, then the corresponding ν_l can be any nonnegative number. If $\bar{f}_l(\lim_{\nu_k \rightarrow \infty} \mathbf{x}(\boldsymbol{\nu})) < 0$, then the corresponding ν_l has to be zero. Therefore, the optimal dual objective value is $\bar{f}_0(\lim_{\nu_k \rightarrow \infty} \mathbf{x}(\boldsymbol{\nu})) + \lim_{\nu_k \rightarrow \infty} \nu_k \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) + 0 = \bar{f}_0(\lim_{\nu_k \rightarrow \infty} \mathbf{x}(\boldsymbol{\nu}))$, which is proved in Lemma 2. Therefore, strong duality holds between the problems (7) and (4).

For part 2, with the optimal solution of the subproblem (4), the problem (1) can be solved by the IMM method in Algorithm 1, which is guaranteed to be monotonically converged, as proved below Lemma 1. \square

For the converged solution $\mathbf{x}^* = \mathbf{x}_i|_{i=\infty}$, its property is summarized as follows.

Theorem 2. *All converged solutions are the KKT points of the problem (1). In particular, the properties of some special converged solutions are summarized as follows.*

- 1) If $\nu_k = \infty$, the solution is the unique optimal solution of problem (4), and a boundary solution of problem (1).
- 2) If $t_0 = 0, \mathbf{Q}_0 = \mathbf{0}$ and the converged solution \mathbf{x}^* satisfies the conditions $\{\bar{f}_k(\mathbf{x}^*) < 0\}_{k=1}^K$, the converged solution is a local optimum of the original problem (1).
- 3) If $\{t_k = 0, \mathbf{Q}_k = \mathbf{0}\}_{k=1}^K$, the converged solution is a Boulingand stationary point of the problem (1).

Proof. We first prove the KKT result. The KKT conditions of the problem (4) are

$$-(\nabla \bar{f}_0(\mathbf{x}) + \sum_{k=1}^K \nu_k \nabla \bar{f}_k(\mathbf{x})) \in \mathcal{N}_{\mathcal{C}}(\mathbf{x}), \quad (22)$$

$$0 \leq \nu_k \leq \infty, \quad \bar{f}_k(\mathbf{x}) \leq 0, \quad \forall k \in [1, K], \quad (23)$$

$$\nu_k \bar{f}_k(\mathbf{x}) = 0, \quad \forall k \in [1, K]. \quad (24)$$

where $\mathcal{C} := \{\mathbf{x} | |\mathbf{x}(n)| = 1, \forall n \in [1, N]\}$ and $\mathcal{N}_{\mathcal{C}}(\mathbf{x})$ is the normal cone of \mathcal{C} at \mathbf{x} . It is clear that the conditions (9) to (11) are one of the KKT conditions of the problem (4).

Let the converged solution be denoted as \mathbf{x}^* . The majorizer function $\bar{f}_k(\mathbf{x})$ in (3) has the following properties at the converged solution \mathbf{x}^* ,

$$\bar{f}_k(\mathbf{x}^*) = f_k(\mathbf{x}^*), \quad \forall k \in [0, 1, \dots, K], \quad (25)$$

$$\nabla \bar{f}_k(\mathbf{x}^*) = \nabla f_k(\mathbf{x}^*), \quad \forall k \in [0, 1, \dots, K], \quad (26)$$

Therefore, the converged solution \mathbf{x}^* satisfies the KKT conditions of problems (4) and (1) at the same time.

For part 1, the terminate criterion $\nu_k = \infty$ in the step 7 of Algorithm 2 is very special. That happens when $\lim_{\nu_k \rightarrow \infty} \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) = 0$, and note that the precondition is that

$(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k \neq 0$, then $\lim_{\nu_k \rightarrow \infty} \mathbf{x}(\boldsymbol{\nu}) = \exp(j \arg((t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k))$, which is the unique minimal point of $\bar{f}_k(\mathbf{x})$. Therefore, the feasible set of problems (18) and (4) is a singleton, which has to be \mathbf{x}_i itself. Since the majorizer has the property $f_k(\mathbf{x}_i) = \bar{f}_k(\mathbf{x}_i)$, then $f_k(\mathbf{x}_i) = 0$, i.e., the returned solution \mathbf{x}_i (or $\lim_{\nu_k \rightarrow \infty} \mathbf{x}(\boldsymbol{\nu})$ or $\exp(j \arg((t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k))$) is a boundary solution of the original problem (1).

For part 2, we first denote the feasible sets of problems (1) and (4) as \mathcal{X} and $\bar{\mathcal{X}}$, and let $\mathcal{B}(\mathbf{x}^*, \epsilon_0)$ be the local area of \mathbf{x}^* , i.e., $\mathcal{B}(\mathbf{x}^*, \epsilon_0) := \{\mathbf{x} | \|\mathbf{x} - \mathbf{x}^*\| \leq \epsilon_0\}$. Firstly, we have the conclusion that $\bar{\mathcal{X}} \subseteq \mathcal{X}$ due to the upper bound construction in (3). Secondly, since $\{\bar{f}_k(\mathbf{x}^*) < 0\}_{k=1}^K$, we have the following results,

$$\{f_k(\mathbf{x}^*) < 0\}_{k=1}^K, \quad (27)$$

$$\exists \epsilon_0 > 0 : \mathcal{B}(\mathbf{x}^*, \epsilon_0) \cap \bar{\mathcal{X}} = \mathcal{B}(\mathbf{x}^*, \epsilon_0) \cap \mathcal{X}, \quad (28)$$

i.e., the changes from the small feasible set $\bar{\mathcal{X}}$ to the larger feasible set \mathcal{X} does not change the local feasible set of \mathbf{x}^* in (28). Furthermore, since \mathbf{x}^* is the global optimal solution over the set $\bar{\mathcal{X}}$ with cost function $\bar{f}_0(\mathbf{x})$, then \mathbf{x}^* is the local optimal solution over the sets $\bar{\mathcal{X}}$ and \mathcal{X} with cost function $f_0(\mathbf{x})$, which is identical to the cost function $\bar{f}_0(\mathbf{x})$ due to the condition $t_0 = 0, \mathbf{Q}_0 = \mathbf{0}$.

For part 3, the condition $\{t_k = 0, \mathbf{Q}_k = \mathbf{0}\}_{k=1}^K$ makes the feasible sets in (1) and (4) become identical. Therefore, the proposed IMM method is degenerated to the classic MM method [43], [44], [46], instead of the inner approximation method [45]. Since the feasible set is a nonconvex set, the converged solution is a Boulingand stationary point of the problem (1) [46], [47]. \square

A special case in Theorem 1 is $\{t_k = 0, \mathbf{Q}_k = \mathbf{0}\}_{k=1}^K$, which makes the problem (1) degenerate to the quadratic programming (QP) with linear and constant modulus constraints. A closed-form solution can be obtained from the conditions (9) to (11) when the problem is further degenerated to QP with constant modulus constraints.

Theorem 3. (of the Alternative) *Let $\{t_k = 0, \mathbf{Q}_k = \mathbf{0}, \mathbf{p}_k = \mathbf{0}\}_{k=1}^K$. Either the problem (1) is infeasible, or the problem (4) is feasible with closed-form solution $\mathbf{x} = \exp(\sqrt{-1} \arg((t_0 \mathbf{I} - \mathbf{Q}_0) \mathbf{x}_i - \mathbf{p}_0))$, but not both.*

Proof. When $\{t_k = 0, \mathbf{Q}_k = \mathbf{0}, \mathbf{p}_k = \mathbf{0}\}_{k=1}^K$, all the quadratic constraints in (1) and linear constraints in (4) become $\{c_k \leq 0\}_{k=1}^K$. If those constant constraints are satisfied, the problems (1) and (4) are both feasible. Otherwise, the problems (1) and (4) are both infeasible. Therefore, both alternatives cannot happen at the same time.

If the problem (4) is feasible under the condition $\{t_k = 0, \mathbf{Q}_k = \mathbf{0}, \mathbf{p}_k = \mathbf{0}\}_{k=1}^K$, it is easy to check that the solutions $\{\nu_k = 0\}_{k=1}^K$ satisfy all the conditions in (9) to (11). Therefore, according to Proposition 1, the strong duality makes the solution $\mathbf{x}(\boldsymbol{\nu})|_{\{\nu_k=0\}_{k=1}^K} = \exp(\sqrt{-1} \arg((t_0 \mathbf{I} - \mathbf{Q}_0) \mathbf{x}_i - \mathbf{p}_0))$ be the optimal solution of the problem (4). \square

Assumptions 1 and 2 seem to be strong conditions, but they are mild conditions as explained in Appendix D. Since

the bisection iteration in Algorithm 2 needs to evaluate the inner product in (8), its computational complexity order is $\mathcal{O}(N^2)$. If the SDR method is used to tackle the problem (1), its computational complexity order is $\mathcal{O}(N^{6.5})$.

III. APPLICATION WITH POSITIVE SEMIDEFINITE QUADRATIC FORM: MSE CONSTRAINED HYBRID BEAMFORMING

A. System Model of Hybrid Beamforming in mmWave MU-MIMO

The MU-MIMO transceiver design with hybrid beamforming in downlink communication system is illustrated in Fig. 1. The base station (BS) is equipped with N transmit antennas, there are K active users, and the k -th user is equipped with M_k antennas. The shorter wavelength at mmWave frequencies enables more antennas to be packed in BS and user equipments, the transmit antenna number N is generally large, while expensive radio frequency (RF) chain makes the RF chain number N_A at BS is small, i.e., $N_A \ll N$. Similarly, the RF chain number at the k -th user M_{A_k} is smaller than the receiver antenna number M_k . Therefore, the transmitter is divided as digital transmitter $\mathbf{G}_D \in \mathbb{C}^{N_A \times L}$ and analog transmitter $\mathbf{G}_A \in \mathbb{C}^{N \times N_A}$, and the k -th receiver is divided into digital receiver $\mathbf{F}_{D_k} \in \mathbb{C}^{L_k \times M_{A_k}}$ and analog receiver $\mathbf{F}_{A_k} \in \mathbb{C}^{M_{A_k} \times M_k}$. The L_k independent data streams are transmitted to the k -th user, and the total data stream number is $\sum_{k=1}^K L_k = L$. The necessary conditions to guarantee data recovery in MU-MIMO system are

$$L_k \leq \min\{M_{A_k}, M_k\}, \quad L \leq \min\{N_A, N\}. \quad (29)$$

Let $\mathbf{s} \in \mathbb{C}^{L \times 1}$ be the transmitted symbol with zero-mean and $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_L$, the received signal at the k -th mobile user is

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{G}_A \mathbf{G}_D \mathbf{s} + \mathbf{n}_k, \quad (30)$$

where \mathbf{H}_k is the channel from BS to the k -th user, and the noise \mathbf{n}_k is circularly-symmetric complex Gaussian, its distribution information is described as $\mathbf{n}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_k)$ with covariance matrix $\mathbf{\Sigma}_k \succ \mathbf{0}$.

At the k -th mobile user's receiver, the analog receiver $\mathbf{F}_{A_k} \in \mathbb{C}^{M_{A_k} \times M_k}$ and digital receiver $\mathbf{F}_{D_k} \in \mathbb{C}^{L_k \times M_{A_k}}$ are used to filter the received signal \mathbf{y}_k . Then the recovered $L_k \times 1$ data stream is

$$\hat{\mathbf{s}}_k = \mathbf{F}_{D_k} \mathbf{F}_{A_k} \mathbf{H}_k \mathbf{G}_A \mathbf{G}_D \mathbf{s} + \mathbf{F}_{D_k} \mathbf{F}_{A_k} \mathbf{n}_k. \quad (31)$$

Since the transmitted data \mathbf{s} is independent of the noise \mathbf{n}_k , the symbol detection MSE of the k -th user is

$$\text{MSE}_k = \mathbb{E}_{\mathbf{s}, \mathbf{n}_k} \left(\text{Tr} \{ (\mathbf{D}_k \mathbf{s} - \hat{\mathbf{s}}_k) (\mathbf{D}_k \mathbf{s} - \hat{\mathbf{s}}_k)^H \} \right), \quad (32)$$

$$= \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \mathbf{H}_k \mathbf{G}_A \mathbf{G}_D - \mathbf{D}_k\|_F^2 + \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \mathbf{\Sigma}_k^{\frac{1}{2}}\|_F^2, \quad (33)$$

where the matrix $\mathbf{D}_k = [\mathbf{0}_{L_k \times \sum_{i=1}^{k-1} L_i} \quad \mathbf{I}_{L_k} \quad \mathbf{0}_{L_k \times \sum_{i=k+1}^K L_i}] \in \mathbb{R}^{L_k \times L}$ is to select the k -th user's data streams.

The MSE constrained hybrid beamforming is to minimize the transmit power at the BS, subject to K users' MSE

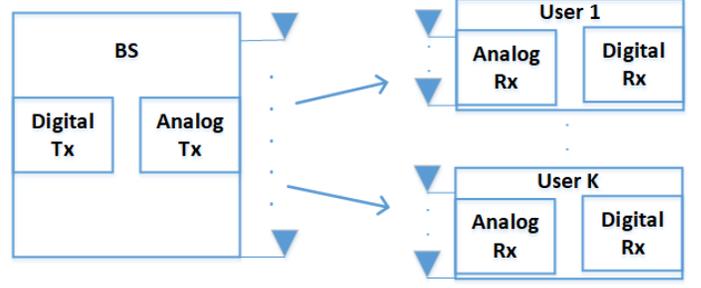


Fig. 1. Hybrid Beamforming in MU-MIMO communication system.

requirements and the constant modulus constraints on analog transmitter and analog receivers, i.e.,

$$\begin{aligned} \min_{\mathbf{G}_D, \mathbf{G}_A, \{\mathbf{F}_{A_k}, \mathbf{F}_{D_k}\}_{k=1}^K} & \|\mathbf{G}_A \mathbf{G}_D\|_F^2 \\ \text{s.t.} & \text{MSE}_k \leq \varepsilon_k, \quad \forall k, \\ & |\mathbf{G}_A(i, j)| = 1, \quad \forall i, j, \\ & |\mathbf{F}_{A_k}(i, j)| = 1, \quad \forall i, j, k, \end{aligned} \quad (34)$$

where $\varepsilon_k > 0$ is the k -th user's MSE target.

Note that the problem (34) is a hard nonconvex problem, the block coordinate descent (BCD) methodology is used to find $\mathbf{G}_D, \mathbf{G}_A, \{\mathbf{F}_{A_k}, \mathbf{F}_{D_k}\}_{k=1}^K$ sequentially. The BCD methodology requires an initial $\mathbf{G}_A, \{\mathbf{F}_{A_k}, \mathbf{F}_{D_k}\}_{k=1}^K$ to make the problem (34) feasible. Therefore, we propose a simple initialization scheme: the phases of elements in $\mathbf{G}_A, \{\mathbf{F}_{A_k}\}_{k=1}^K$ are generated uniformly and independently in $[0, 2\pi]$, and the digital receivers are

$$\{\mathbf{F}_{D_k} = \sqrt{\frac{\varepsilon_k}{2\text{Tr}(\mathbf{F}_{A_k} \mathbf{\Sigma}_k \mathbf{F}_{A_k}^H)}} \mathbf{I}_{M_{A_k}}(1 : L_k, :)\}_{k=1}^K, \quad (35)$$

where $\mathbf{I}_{M_{A_k}}(1 : L_k, :)$ is a matrix filled with the first L_k rows of identity matrix $\mathbf{I}_{M_{A_k}}$. If the elements in $\{\mathbf{H}_k\}_{k=1}^K$ are independent continuous random variables, then, with probability one, the problem (34) is feasible and all MSE constraints are strictly feasible under the initialization scheme (35) [42].

B. Analog Transmitter Design with iMM

With digital transmitter and receivers being fixed, the optimal analog transmitter is obtained from,

$$\begin{aligned} \min_{\mathbf{G}_A} & \|\mathbf{G}_A \mathbf{G}_D\|_F^2 \\ \text{s.t.} & \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \mathbf{H}_k \mathbf{G}_A \mathbf{G}_D - \mathbf{D}_k\|_F^2 + \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \mathbf{\Sigma}_k^{\frac{1}{2}}\|_F^2 \leq \varepsilon_k, \quad \forall k, \\ & |\mathbf{G}_A(i, j)| = 1, \quad \forall i, j, \end{aligned} \quad (36)$$

which is a QCQP with extra constant modulus constraints, and the cost and constraint functions are positive semidefinite quadratic forms on analog transmitter \mathbf{G}_A . Therefore, the proposed method in Section II can be applied.

To get the vectorized quadratic form, the objective function is reformulated as,

$$\|\mathbf{G}_A \mathbf{G}_D\|_F^2 = \|\text{vec}(\mathbf{G}_A \mathbf{G}_D)\|_2^2 \quad (37)$$

$$= \|(\mathbf{G}_D^T \otimes \mathbf{I}_N) \mathbf{g}\|_2^2 \quad (38)$$

where $\mathbf{g} := \text{vec}(\mathbf{G}_A)$. Similarly, the first term of the MSE function in (33) is reformulated as,

$$\|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \mathbf{H}_k \mathbf{G}_A \mathbf{G}_D - \mathbf{D}_k\|_F^2 \quad (39)$$

$$= \|\text{vec}(\mathbf{F}_{D_k} \mathbf{F}_{A_k} \mathbf{H}_k \mathbf{G}_A \mathbf{G}_D - \mathbf{D}_k)\|_2^2 \quad (40)$$

$$= \|\underbrace{(\mathbf{G}_D^T \otimes (\mathbf{F}_{D_k} \mathbf{F}_{A_k} \mathbf{H}_k))}_{\mathbf{C}_k} \text{vec}(\mathbf{G}_A) - \text{vec}(\mathbf{D}_k)\|_2^2 \quad (41)$$

$$= \|\mathbf{C}_k \mathbf{g} - \mathbf{d}_k\|_2^2 \quad (42)$$

where $\mathbf{d}_k := \text{vec}(\mathbf{D}_k)$. Therefore, the problem (36) is the same as

$$\begin{aligned} \min_{\mathbf{g}} \quad & \|(\mathbf{G}_D^T \otimes \mathbf{I}_N) \mathbf{g}\|_2^2 \\ \text{s.t.} \quad & \|\mathbf{C}_k \mathbf{g} - \mathbf{d}_k\|_2^2 + \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \Sigma_k^{\frac{1}{2}}\|_F^2 \leq \varepsilon_k, \forall k \\ & |\mathbf{g}(n)| = 1, \forall n \in [1, N_A N], \end{aligned} \quad (43)$$

The majorizers of the cost function and constraint functions are illustrated as follows. Firstly,

$$\|(\mathbf{G}_D^T \otimes \mathbf{I}_N) \mathbf{g}\|_2^2 = \mathbf{g}^H (\mathbf{G}_D^* \mathbf{G}_D^T \otimes \mathbf{I}_N) \mathbf{g} \quad (44)$$

$$\|\mathbf{C}_k \mathbf{g} - \mathbf{d}_k\|_2^2 = \mathbf{g}^H \mathbf{C}_k^H \mathbf{C}_k \mathbf{g} + \|\mathbf{d}_k\|_2^2 - 2\text{Re}(\mathbf{g}^H \mathbf{C}_k^H \mathbf{d}_k). \quad (45)$$

Secondly, at the latest feasible point \mathbf{g}_i , one majorizer is constructed as [43, e.q. (26)]

$$\begin{aligned} \mathbf{g}^H (\mathbf{G}_D^* \mathbf{G}_D^T \otimes \mathbf{I}_N) \mathbf{g} &\leq 2\text{Re}(\mathbf{g}^H (\mathbf{G}_D^* \mathbf{G}_D^T \otimes \mathbf{I}_N - t_0 \mathbf{I}) \mathbf{g}_i) \\ &\quad + t_0 \mathbf{g}^H \mathbf{I} \mathbf{g} + \mathbf{g}_i^H (t_0 \mathbf{I} - \mathbf{G}_D^* \mathbf{G}_D^T \otimes \mathbf{I}_N) \mathbf{g}_i \end{aligned} \quad (46)$$

$$\begin{aligned} \mathbf{g}^H \mathbf{C}_k^H \mathbf{C}_k \mathbf{g} &\leq 2\text{Re}(\mathbf{g}^H (\mathbf{C}_k^H \mathbf{C}_k - t_k \mathbf{I}) \mathbf{g}_i) \\ &\quad + t_k \mathbf{g}^H \mathbf{I} \mathbf{g} + \mathbf{g}_i^H (t_k \mathbf{I} - \mathbf{C}_k^H \mathbf{C}_k) \mathbf{g}_i \end{aligned} \quad (47)$$

where $t_0 = N \text{Tr}(\mathbf{G}_D^* \mathbf{G}_D^T)$, $t_k = \text{Tr}(\mathbf{C}_k^H \mathbf{C}_k)$. Therefore, a majorizer of the MSE function under $\{|\mathbf{g}(n)| = 1\}_{n=1}^{N_A N}$ is constructed as

$$\text{MSE}_k \leq \overbrace{2\text{Re}(\mathbf{g}^H [(\mathbf{C}_k^H \mathbf{C}_k - t_k \mathbf{I}) \mathbf{g}_i - \mathbf{C}_k^H \mathbf{d}_k]) + c_k}^{\overline{\text{MSE}}_k} \quad (48)$$

where $c_k = \|\mathbf{d}_k\|_2^2 + 2N N_A t_k - \mathbf{g}_i^H \mathbf{C}_k^H \mathbf{C}_k \mathbf{g}_i + \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \Sigma_k^{\frac{1}{2}}\|_F^2$. After those constructions, the problem (43) can be solved by a series of subproblems,

$$\begin{aligned} \min_{\mathbf{g}} \quad & 2\text{Re}(\mathbf{g}^H (\mathbf{G}_D^* \mathbf{G}_D^T \otimes \mathbf{I}_N - t_0 \mathbf{I}) \mathbf{g}_i) + c_0 \\ \text{s.t.} \quad & \overline{\text{MSE}}_k \leq \varepsilon_k, \forall k \in [1, K] \\ & |\mathbf{g}(n)| = 1, \forall n \in [1, N_A N], \end{aligned} \quad (49)$$

where $c_0 = 2N N_A t_0 - \mathbf{g}_i^H (\mathbf{G}_D^* \mathbf{G}_D^T \otimes \mathbf{I}_N) \mathbf{g}_i$.

According to Theorem 1, under the mild condition $\{\{(t_k \mathbf{I} - \mathbf{C}_k^H \mathbf{C}_k) \mathbf{g}_i \neq 0\}_{k=1}^K\}_{i=0}^\infty$, the optimal solution of problem (49) is obtained from the solution of the equations,

$$\begin{aligned} \mathbf{g}(\nu) = \exp\left(\sqrt{-1} \arg\left(\sum_{k=1}^K [(t_k \mathbf{I} - \mathbf{C}_k^H \mathbf{C}_k) \mathbf{g}_i + \mathbf{C}_k^H \mathbf{d}_k] \nu_k \right.\right. \\ \left.\left. + (t_0 \mathbf{I} - \mathbf{G}_D^* \mathbf{G}_D^T \otimes \mathbf{I}_N) \mathbf{g}_i\right)\right), \end{aligned} \quad (50)$$

$$0 \leq \nu_k \leq \infty, \overline{\text{MSE}}_k(\nu_k) \leq \varepsilon_k, \forall k \in [1, K], \quad (51)$$

$$\nu_k (\overline{\text{MSE}}_k(\nu_k) - \varepsilon_k) = 0, \forall k \in [1, K], \quad (52)$$

Algorithm 3 iMM method to find analog transmitter \mathbf{G}_A in problem (36).

-
- 1: **initialization:** set $i = 0$, \mathbf{g}_i is the latest feasible solution
 - 2: **repeat**
 - 3: Use Algorithm 2 to find $\mathbf{g}(\nu)$ in (50) to (52), and denote the objective value of problem (43) as $f[i + 1]$. Set $i = i + 1$, let $\mathbf{g}_i = \mathbf{g}(\nu)$.
 - 4: **until** $i \geq 2$ and $(f[i - 1] - f[i])/f[i] \leq \epsilon_3$
-

where

$$\overline{\text{MSE}}_k(\nu_k) = 2\text{Re}(\mathbf{g}(\nu)^H [(\mathbf{C}_k^H \mathbf{C}_k - t_k \mathbf{I}) \mathbf{g}_i - \mathbf{C}_k^H \mathbf{d}_k] + c_k). \quad (53)$$

With optimal solution of the subproblem (49), the iMM method to solve problem (43) is described in Algorithm 3, which is guaranteed to converge as explained in Theorem 1.

C. Analog Receiver Design with MM

With transmitter and digital receivers being fixed, the k -th user's analog receiver is obtained by minimizing the k -th user's MSE,

$$\begin{aligned} \min_{\mathbf{F}_{A_k}} \quad & \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \mathbf{H}_k \mathbf{G}_A \mathbf{G}_D - \mathbf{D}_k\|_F^2 + \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \Sigma_k^{\frac{1}{2}}\|_F^2 \\ \text{s.t.} \quad & |\mathbf{F}_{A_k}(i, j)| = 1, \forall i, j, \end{aligned} \quad (54)$$

which is a degenerated form of the problem (1), and the closed-form solution in Theorem 3 for the linearized subproblem can be used to solve it.

To get the vectorized quadratic form, the first term of the objective function is reformulated as,

$$\|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \mathbf{H}_k \mathbf{G}_A \mathbf{G}_D - \mathbf{D}_k\|_F^2 \quad (55)$$

$$= \|\underbrace{((\mathbf{H}_k \mathbf{G}_A \mathbf{G}_D)^T \otimes \mathbf{F}_{D_k})}_{\tilde{\mathbf{C}}_k} \text{vec}(\mathbf{F}_{A_k}) - \text{vec}(\mathbf{D}_k)\|_2^2 \quad (56)$$

$$= \|\tilde{\mathbf{C}}_k \mathbf{f}_k - \mathbf{d}_k\|_2^2, \quad (57)$$

where $\mathbf{f}_k := \text{vec}(\mathbf{F}_{A_k})$. Similarly, the second term of the objective function is reformulated as,

$$\|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \Sigma_k^{\frac{1}{2}}\|_F^2 = \|(\Sigma_k^{\frac{T}{2}} \otimes \mathbf{F}_{D_k}) \mathbf{f}_k\|_2^2. \quad (58)$$

Therefore, the problem (54) is the same as

$$\begin{aligned} \min_{\mathbf{f}_k} \quad & \|\tilde{\mathbf{C}}_k \mathbf{f}_k - \mathbf{d}_k\|_2^2 + \|(\Sigma_k^{\frac{T}{2}} \otimes \mathbf{F}_{D_k}) \mathbf{f}_k\|_2^2 \\ \text{s.t.} \quad & |\mathbf{f}_k(n)| = 1, \forall n \in [1, M_{A_k} M_k]. \end{aligned} \quad (59)$$

The majorizer of the cost function is illustrated as follows. Firstly,

$$\begin{aligned} \|\tilde{\mathbf{C}}_k \mathbf{f}_k - \mathbf{d}_k\|_2^2 + \|(\Sigma_k^{\frac{T}{2}} \otimes \mathbf{F}_{D_k}) \mathbf{f}_k\|_2^2 &= \|\mathbf{d}_k\|_2^2 - 2\text{Re}(\mathbf{f}_k^H \tilde{\mathbf{C}}_k^H \mathbf{d}_k) \\ &\quad + \mathbf{f}_k^H (\tilde{\mathbf{C}}_k^H \tilde{\mathbf{C}}_k + \Sigma_k^T \otimes (\mathbf{F}_{D_k}^H \mathbf{F}_{D_k})) \mathbf{f}_k. \end{aligned} \quad (60)$$

Secondly, at the latest feasible point $\mathbf{f}_{k,i}$, a majorizer of the objective function is constructed as [43, e.q. (26)]

$$\begin{aligned} \|\tilde{\mathbf{C}}_k \mathbf{f}_k - \mathbf{d}_k\|_2^2 + \|(\Sigma_k^{\frac{T}{2}} \otimes \mathbf{F}_{D_k}) \mathbf{f}_k\|_2^2 &\leq \|\mathbf{d}_k\|_2^2 - 2\text{Re}(\mathbf{f}_k^H \tilde{\mathbf{C}}_k^H \mathbf{d}_k) \\ &\quad + 2\text{Re}(\mathbf{f}_k^H (\tilde{\mathbf{C}}_k^H \tilde{\mathbf{C}}_k + \Sigma_k^T \otimes (\mathbf{F}_{D_k}^H \mathbf{F}_{D_k}) - \tilde{t}_k \mathbf{I}) \mathbf{f}_{k,i}) \\ &\quad + \tilde{t}_k \mathbf{f}_k^H \mathbf{I} \mathbf{f}_k + \mathbf{f}_{k,i}^H (\tilde{t}_k \mathbf{I} - \tilde{\mathbf{C}}_k^H \tilde{\mathbf{C}}_k - \Sigma_k^T \otimes (\mathbf{F}_{D_k}^H \mathbf{F}_{D_k})) \mathbf{f}_{k,i} \end{aligned} \quad (61)$$

where $\tilde{t}_k = \text{Tr}(\tilde{\mathbf{C}}_k^H \tilde{\mathbf{C}}_k + \Sigma_k^T \otimes (\mathbf{F}_{D_k}^H \mathbf{F}_{D_k}))$. Under the constraint $\{|\mathbf{f}_k(n)| = 1\}_{n=1}^{N_R}$, the quadratic term $\tilde{t}_k \mathbf{f}_k^H \mathbf{I} \mathbf{f}_k$ turns into a constant term, which is unrelated to \mathbf{f}_k . After removing the terms unrelated to the optimization variable \mathbf{f}_k in the majorizer (61), the problem (59) can be tackled by a series of subproblems,

$$\begin{aligned} \min_{\mathbf{f}_k} \text{Re}(\mathbf{f}_k^H [(\tilde{\mathbf{C}}_k^H \tilde{\mathbf{C}}_k + \Sigma_k^T \otimes (\mathbf{F}_{D_k}^H \mathbf{F}_{D_k})) - \tilde{t}_k \mathbf{I}] \mathbf{f}_k - \tilde{\mathbf{C}}_k^H \mathbf{d}_k) \\ \text{s.t. } |\mathbf{f}_k(n)| = 1, \forall n \in [1, M_{A_k} M_k], \end{aligned} \quad (62)$$

The optimal solution of the problem (62) is

$$\mathbf{f}_k = \exp\left(\sqrt{-1} \arg([\tilde{t}_k \mathbf{I} - \tilde{\mathbf{C}}_k^H \tilde{\mathbf{C}}_k - \Sigma_k^T \otimes (\mathbf{F}_{D_k}^H \mathbf{F}_{D_k})] \mathbf{f}_k + \tilde{\mathbf{C}}_k^H \mathbf{d}_k)\right), \quad (63)$$

which coincides with the prediction in Theorem 3. The iMM method (degenerated to the classic MM method [43]) to solve problem (54) is described in Algorithm 4, its monotonic convergence property is guaranteed.

Algorithm 4 iMM method to find analog receivers $\{\mathbf{F}_{A_k}\}_{k=1}^K$ in problem (54).

- 1: **initialization:** set $i = 0$, $\mathbf{f}_{k,i}$ is the latest feasible analog receiver.
 - 2: **repeat**
 - 3: Calculate (63), and denote the objective value of problems (59) as $f[i+1]$. Set $i = i + 1$, let $\mathbf{f}_{k,i} = \mathbf{f}_k$.
 - 4: **until** $i \geq 2$ and $(f[i-1] - f[i])/f[i] \leq \epsilon_4$
-

D. Digital Transmitter Design with ADMM

With analog transmitter and receivers being fixed, the optimal digital transmitter \mathbf{G}_D is obtained as

$$\begin{aligned} \min_{\mathbf{G}_D} \|\mathbf{G}_A \mathbf{G}_D\|_F^2 \\ \text{s.t. } \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \mathbf{H}_k \mathbf{G}_A \mathbf{G}_D - \mathbf{D}_k\|_F^2 + \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \Sigma_k^{\frac{1}{2}}\|_F^2 \leq \epsilon_k, \forall k, \end{aligned} \quad (64)$$

which is a convex QCQP problem, it can be solved by the standard interior point method with computational complexity order $\mathcal{O}(N^{3.5})$ [48], [49]. To reduce the computational complexity, the ADMM method in [42] is extended to solve the problem (64) as follows.

For better presentation, the first term of the MSE function in (33) is reformulated as,

$$\underbrace{\|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \mathbf{H}_k \mathbf{G}_A \mathbf{G}_D - \mathbf{D}_k\|_F^2}_{\mathbf{B}_k} = \|\mathbf{B}_k \mathbf{G}_D - \mathbf{D}_k\|_F^2 \quad (65)$$

Therefore, the problem (64) is the same as

$$\begin{aligned} \min_{\mathbf{G}_D} \|\mathbf{G}_A \mathbf{G}_D\|_F^2 \\ \text{s.t. } \|\mathbf{B}_k \mathbf{G}_D - \mathbf{D}_k\|_F^2 \leq \epsilon_k - \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \Sigma_k^{\frac{1}{2}}\|_F^2, \forall k \end{aligned} \quad (66)$$

With slack variables $\{\mathbf{E}_k \in \mathbb{C}^{L_k \times L_k}\}_{k=1}^K$, the problem (66) is equivalent to

$$\begin{aligned} \min_{\mathbf{G}_D, \{\mathbf{E}_k\}_{k=1}^K} \|\mathbf{G}_A \mathbf{G}_D\|_F^2 \\ \text{s.t. } \|\mathbf{E}_k\|_2^2 \leq \epsilon_k - \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \Sigma_k^{\frac{1}{2}}\|_F^2, \forall k \in [1, K] \\ \mathbf{E}_k = \mathbf{B}_k \mathbf{G}_D - \mathbf{D}_k, \forall k \in [1, K]. \end{aligned} \quad (67)$$

Furthermore, we can construct an indicator function $\mathbb{I}_k(\mathbf{E}_k)$ to represent the k -th MSE constraint as

$$\mathbb{I}_k(\mathbf{E}_k) = \begin{cases} 0, & \text{If } \|\mathbf{E}_k\|_F^2 \leq \epsilon_k - \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \Sigma_k^{\frac{1}{2}}\|_F^2, \\ +\infty, & \text{Otherwise.} \end{cases} \quad (68)$$

Therefore, the problem (67) is equivalent to

$$\begin{aligned} \min_{\mathbf{G}_D, \{\mathbf{E}_k\}_{k=1}^K} \|\mathbf{G}_A \mathbf{G}_D\|_F^2 + \sum_{k=1}^K \mathbb{I}_k(\mathbf{E}_k) \\ \text{s.t. } \mathbf{E}_k = \mathbf{B}_k \mathbf{G}_D - \mathbf{D}_k, \forall k \in [1, K], \end{aligned} \quad (69)$$

which is a standard two-block convex ADMM formulation [50].

With Lagrange multiplier $\{\Lambda_k \in \mathbb{C}^{L_k \times L_k}\}_{k=1}^K$, the augmented Lagrange of the problem (69) is

$$\begin{aligned} \mathcal{L}(\mathbf{G}_D, \{\mathbf{E}_k, \Lambda_k\}_{k=1}^K) \\ = \|\mathbf{G}_A \mathbf{G}_D\|_F^2 + 2 \sum_{k=1}^K \text{Re}(\text{Tr}(\Lambda_k^H (\mathbf{B}_k \mathbf{G}_D - \mathbf{D}_k - \mathbf{E}_k))) \\ + \sum_{k=1}^K \mathbb{I}_k(\mathbf{E}_k) + \mu \sum_{k=1}^K \|\mathbf{B}_k \mathbf{G}_D - \mathbf{D}_k - \mathbf{E}_k\|_F^2. \end{aligned} \quad (70)$$

To update the primal and dual variables sequentially, the ADMM iteration steps are

$$\mathbf{G}_D \leftarrow \arg \min_{\mathbf{G}_D} \mathcal{L}(\mathbf{G}_D, \{\mathbf{E}_k, \Lambda_k\}_{k=1}^K), \quad (71)$$

$$\mathbf{E}_k \leftarrow \arg \min_{\mathbf{E}_k} \mathcal{L}(\mathbf{G}_D, \{\mathbf{E}_k, \Lambda_k\}_{k=1}^K), \quad (72)$$

$$\Lambda_k \leftarrow \Lambda_k + \mu (\mathbf{B}_k \mathbf{G}_D - \mathbf{D}_k - \mathbf{E}_k), \quad (73)$$

where the left arrow notation $b \leftarrow a$ means that the numeric value of a is assigned to the symbol b . The closed-form solutions of (71) and (72) are listed as follows,

$$\mathbf{G}_D \leftarrow \underbrace{(\mathbf{G}_A^H \mathbf{G}_A + \mu \sum_{k=1}^K \mathbf{B}_k^H \mathbf{B}_k)^{-1}}_{\mathbf{B}} \sum_{k=1}^K (\mathbf{B}_k^H (\mu (\mathbf{D}_k + \mathbf{E}_k) - \Lambda_k)), \quad (74)$$

$$\mathbf{E}_k \leftarrow \begin{cases} \hat{\mathbf{E}}_k, & \text{If } \|\hat{\mathbf{E}}_k\|_F^2 \leq \epsilon_k - \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \Sigma_k^{\frac{1}{2}}\|_F^2 \\ \hat{\mathbf{E}}_k / \|\hat{\mathbf{E}}_k\|_F \cdot \sqrt{\epsilon_k - \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \Sigma_k^{\frac{1}{2}}\|_F^2}, & \text{Otherwise} \end{cases}, \quad (75)$$

where $\hat{\mathbf{E}}_k = \mathbf{B}_k \mathbf{G}_D - \mathbf{D}_k + \Lambda_k / \mu$. The termination criteria of the ADMM method are $\{\|\mathbf{B}_k \mathbf{G}_D - \mathbf{D}_k - \mathbf{E}_k\|_F \leq \epsilon^p\}_{k=1}^K$, and the Frobenius norm of the difference between two \mathbf{G}_D in successive iterations is smaller than ϵ^d , where ϵ^p and ϵ^d are the tolerances.

Therefore, the proposed ADMM method is listed in Algorithm 5. If the transmitter problem (64) is strictly feasible,

Algorithm 5 ADMM method to find transmitter \mathbf{G} in the problem (64).

- 1: **initialization:** set $\{\mathbf{E}_k, \mathbf{\Lambda}_k\}_{k=1}^K$ as the previous values, otherwise $\{\mathbf{E}_k = 0, \mathbf{\Lambda}_k = 0\}_{k=1}^K$. Precompute $\mathbf{B} = (\mathbf{G}_A^H \mathbf{G}_A + \mu \sum_{k=1}^K \mathbf{B}_k^H \mathbf{B}_k)^{-1}$.
- 2: **repeat**
- 3: Update \mathbf{G}_D in (74) with precomputed \mathbf{B}
- 4: Update \mathbf{E}_k in (75) for all $k \in [1, K]$
- 5: Update $\mathbf{\Lambda}_k$ in (73) for all $k \in [1, K]$
- 6: **until** The termination criteria below (75) are satisfied.

Algorithm 6 Joint analog and digital transceiver for MSE constrained MU-MIMO system.

- 1: **initialization:** use initialization scheme (35), and set $i_6 = 0$.
- 2: **repeat**
- 3: Use Algorithm 5 to find digital transmitter \mathbf{G}_D , denote the transmit power as $P[i_6 + 1]$.
- 4: Use Algorithm 3 to find analog transmitter \mathbf{G}_A .
- 5: Use Algorithm 4 to find analog receivers $\{\mathbf{F}_{A_k}\}_{k=1}^K$.
- 6: Use (77) as the digital receiver, and set $i_6 = i_6 + 1$.
- 7: **until** $i_6 \geq 2$ and $(P[i_6 - 1] - P[i_6])/P[i_6] \leq \epsilon_6$

the two functions in the objective function of (69) are closed, proper, and convex. Furthermore, the strong duality makes the unaugmented Lagrangian of (69) has a saddle point. Therefore, the limit solution of the proposed ADMM method converges to the optimal solution under the mild strictly feasible condition [50, p.16].

Since there is only one matrix inversion to calculate \mathbf{B} in (74), the major computational complexity comes from the matrix multiplications. Therefore, the computational complexity order of the proposed ADMM approach is $\mathcal{O}(N^2 L)$, which is lower than that of the interior point method with $\mathcal{O}(N^{3.5})$.

E. Digital Receiver Design with Closed-Form Solution

With transmitter and analog receivers being fixed, the k -th user's digital receiver is obtained by minimizing the k -th user's MSE,

$$\min_{\mathbf{F}_{D_k}} \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \mathbf{H}_k \mathbf{G}_A \mathbf{G}_D - \mathbf{D}_k\|_F^2 + \|\mathbf{F}_{D_k} \mathbf{F}_{A_k} \Sigma_k^{\frac{1}{2}}\|_F^2. \quad (76)$$

Since the MSE function is a convex quadratic function of \mathbf{F}_{D_k} , its optimal solution occurs at the stationary point,

$$\mathbf{F}_{D_k} = \mathbf{D}_k \mathbf{H}_{E_k}^H (\mathbf{H}_{E_k} \mathbf{H}_{E_k}^H + \mathbf{F}_{A_k} \Sigma_k \mathbf{F}_{A_k}^H)^{-1}, \quad (77)$$

where $\mathbf{H}_{E_k} = \mathbf{F}_{A_k} \mathbf{H}_k \mathbf{G}_A \mathbf{G}_D$ is an effective channel. The computational complexity order of the receiver design is $\mathcal{O}(M_{A_k}^3)$.

F. Joint Analog and Digital Transceiver Algorithm

The algorithm to solve the joint problem (34) is described in Algorithm 6. The individual MSE minimizations in (54) and (76) make the resulting MSEs are smaller than the MSE

TABLE 1
COMPUTATIONAL COMPLEXITY ORDERS OF DIFFERENT METHODS

subproblem	Digital Tx	Analog Tx	Analog Rx	Digital Rx
ADMM+iMM	$\mathcal{O}(N_A^2 L)$	$\mathcal{O}((N_A N)^2)$	$\mathcal{O}((M_{A_k} M_k)^2)$	$\mathcal{O}(M_{A_k}^3)$
interior point (QCQP+SDR)	$\mathcal{O}(N_A^{3.5})$	$\mathcal{O}((N_A N)^{6.5})$	$\mathcal{O}((M_{A_k} M_k)^{6.5})$	$\mathcal{O}(M_{A_k}^3)$

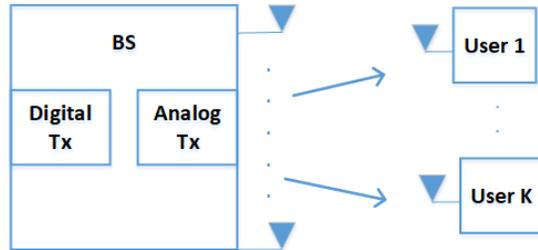


Fig. 2. Hybrid Beamforming in MU-MISO communication system.

targets $\{\epsilon_k\}_{k=1}^K$. Therefore, the MSE minimizations in (54) and (76) make the successive transmitter design have more space for the digital and analog transmitter to reduce the transmission power, i.e., $P[i_6 + 1] \leq P[i_6]$. Since the transmit power is monotonically decreasing and the transmit power is bounded below from zero, Algorithm 6 converges.

The computational complexity orders of the proposed method are compared with the interior point method in Table 1. It can be seen from Table 1 that the combination of the proposed ADMM+iMM method has lower computational complexity order than the interior point method.

IV. APPLICATION WITH INDEFINITE QUADRATIC FORM: SINR CONSTRAINED HYBRID BEAMFORMING

A. System Model of Hybrid Beamforming in mmWave MU-MISO

In MU-MISO system, a single antenna is equipped at all K mobile users, every user has a single data stream, and there is no beamforming operation at receivers. Only the digital and analog beamformings are implemented at the transmitter, and Fig. 2 is used to illustrate this system. At this setting, without complex beamforming operation at the receivers, the mobile users can have every cheap equipment.

With digital transmitter $\mathbf{G}_D \in \mathbb{C}^{N_A \times K}$ and analog transmitter $\mathbf{G}_A \in \mathbb{C}^{N \times N_A}$, the received signal at the k -th mobile user is

$$y_k = \mathbf{h}_k^H \mathbf{G}_A \mathbf{G}_D \mathbf{s} + n_k, \quad (78)$$

where $\mathbf{s} \in \mathbb{C}^{K \times 1}$ is the transmitted symbol with zero-mean and $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_K$, $\mathbf{h}_k \in \mathbb{C}^{N \times 1}$ is the k -th user's channel, and the noise n_k is circularly-symmetric complex Gaussian i.e., $n_k \sim \mathcal{CN}(0, \sigma_k^2)$. Since the transmitted symbol is independent with the noise, the SINR of the k -th user is

$$\text{SINR}_k = \frac{\mathbb{E}_{\mathbf{s}}(|\mathbf{h}_k^H \mathbf{G}_A \mathbf{G}_D \mathbf{s}(k)|^2)}{\mathbb{E}_{\mathbf{s}}(\sum_{l=1, l \neq k}^K |\mathbf{h}_k^H \mathbf{G}_A \mathbf{G}_D \mathbf{s}(l)|^2) + \mathbb{E}_{n_k}(|n_k|^2)}, \quad (79)$$

$$= \frac{|\mathbf{h}_k^H \mathbf{G}_A \mathbf{g}_{D_k}|^2}{\sum_{l=1, l \neq k}^K |\mathbf{h}_k^H \mathbf{G}_A \mathbf{g}_{D_l}|^2 + \sigma_k^2}, \quad (80)$$

where \mathbf{g}_{D_l} is the l -th column vector of \mathbf{G}_D , i.e., $\mathbf{G}_D = [\mathbf{g}_{D_1}, \dots, \mathbf{g}_{D_K}]$.

The SINR constrained hybrid beamforming in MU-MISO system is to minimize the transmit power at the BS, subject to K users' SINR requirements and constant modulus constraints on analog transmitter,

$$\begin{aligned} \min_{\mathbf{G}_D, \mathbf{G}_A} \quad & \|\mathbf{G}_A \mathbf{G}_D\|_F^2 \\ \text{s.t.} \quad & \text{SINR}_k \geq \gamma_k, \quad \forall k, \\ & |\mathbf{G}_A(i, j)| = 1, \quad \forall i, j, \end{aligned} \quad (81)$$

where $\gamma_k > 0$ is the k -th user's SINR target.

The methodology to tackle problem (81) is the coordinate decent method. With fixed analog transmitter \mathbf{G}_A , the method to obtain the optimal digital transmitter is well-studied, and widely known as the convex SOCP method [51] or the fixed point method [19]. Therefore, we focus on the analog transmitter design, i.e.,

$$\begin{aligned} \min_{\mathbf{G}_A} \quad & \|\mathbf{G}_A \mathbf{G}_D\|_F^2 \\ \text{s.t.} \quad & \text{SINR}_k \geq \gamma_k, \quad \forall k, \\ & |\mathbf{G}_A(i, j)| = 1, \quad \forall i, j, \end{aligned} \quad (82)$$

The objective and constraint functions of problem (82) can be transformed into definite and indefinite quadratic form of \mathbf{G}_A , respectively. Firstly, the objective function is

$$\|\mathbf{G}_A \mathbf{G}_D\|_F^2 = \|(\mathbf{G}_D^T \otimes \mathbf{I}_N) \mathbf{g}\|_2^2 \quad (83)$$

$$= \mathbf{g}^H \mathbf{Q}_0 \mathbf{g} \quad (84)$$

where $\mathbf{g} := \text{vec}(\mathbf{G}_A)$ and $\mathbf{Q}_0 = (\mathbf{G}_D^* \mathbf{G}_D^T) \otimes \mathbf{I}_N$. Secondly, the SINR constraint is reformulated as

$$\sum_{l=1, l \neq k}^K |\mathbf{h}_k^H \mathbf{G}_A \mathbf{g}_{D_l}|^2 - \frac{1}{\gamma_k} |\mathbf{h}_k^H \mathbf{G}_A \mathbf{g}_{D_k}|^2 + \sigma_k^2 \leq 0, \quad (85)$$

which is the same as

$$\mathbf{g}^H \left(\sum_{l=1, l \neq k}^K \mathbf{Q}_l - \frac{1}{\gamma_k} \mathbf{Q}_k \right) \mathbf{g} + \sigma_k^2 \leq 0, \quad (86)$$

where $\{\mathbf{Q}_l = (\mathbf{g}_{D_l}^* \mathbf{g}_{D_l}^T) \otimes (\mathbf{h}_k \mathbf{h}_k^H)\}_{l=1}^K$. Therefore, the problem (82) is equivalent to

$$\begin{aligned} \min_{\mathbf{g}} \quad & \mathbf{g}^H \mathbf{Q}_0 \mathbf{g} \\ \text{s.t.} \quad & \mathbf{g}^H \left(\sum_{l=1, l \neq k}^K \mathbf{Q}_l - \frac{1}{\gamma_k} \mathbf{Q}_k \right) \mathbf{g} + \sigma_k^2 \leq 0, \quad \forall k, \\ & |\mathbf{g}(n)| = 1, \quad \forall n, \end{aligned} \quad (87)$$

which is an indefinite QCQP with extra constant modulus constraint. The method in Section II can be directly applied to tackle this problem.

At the latest feasible point \mathbf{g}_i , the majorizers are constructed as [43, e.q. (26)]

$$\begin{aligned} \mathbf{g}^H \mathbf{Q}_0 \mathbf{g} \leq \bar{f}_0(\mathbf{g}) := & 2\text{Re}(\mathbf{g}^H (\mathbf{Q}_0 - t_0 \mathbf{I}) \mathbf{g}_i) \\ & + t_0 \mathbf{g}^H \mathbf{I} \mathbf{g} + \mathbf{g}_i^H (t_0 \mathbf{I} - \mathbf{Q}_0) \mathbf{g}_i \end{aligned} \quad (88)$$

$$\begin{aligned} \mathbf{g}^H \left(\sum_{l=1, l \neq k}^K \mathbf{Q}_l - \frac{1}{\gamma_k} \mathbf{Q}_k \right) \mathbf{g} + \sigma_k^2 \leq \bar{f}_k(\mathbf{g}) := & t_k \mathbf{g}^H \mathbf{I} \mathbf{g} \\ & + 2\text{Re}(\mathbf{g}^H \left(\sum_{l=1, l \neq k}^K \mathbf{Q}_l - \frac{1}{\gamma_k} \mathbf{Q}_k - t_k \mathbf{I} \right) \mathbf{g}_i) \\ & + \mathbf{g}_i^H \left(t_k \mathbf{I} - \left(\sum_{l=1, l \neq k}^K \mathbf{Q}_l - \frac{1}{\gamma_k} \mathbf{Q}_k \right) \right) \mathbf{g}_i + \sigma_k^2 \end{aligned} \quad (89)$$

where $t_0 = \text{Tr}(\mathbf{Q}_0) = N \text{Tr}(\mathbf{G}_D^* \mathbf{G}_D^T)$, $t_k = \text{Tr}(\sum_{l=1, l \neq k}^K \mathbf{Q}_l)$. Therefore, the majorizers of the objective and constraints functions under $\{|\mathbf{g}(n)| = 1\}_{n=1}^{N_A N}$ become

$$\bar{f}_0(\mathbf{g}) = 2\text{Re}(\mathbf{g}^H (\mathbf{Q}_0 - t_0 \mathbf{I}) \mathbf{g}_i) + 2N N_A t_0 - \mathbf{g}_i^H \mathbf{Q}_0 \mathbf{g}_i \quad (90)$$

$$\begin{aligned} \bar{f}_k(\mathbf{g}) = 2\text{Re}(\mathbf{g}^H \left(\sum_{l=1, l \neq k}^K \mathbf{Q}_l - \frac{1}{\gamma_k} \mathbf{Q}_k - t_k \mathbf{I} \right) \mathbf{g}_i) + & 2N N_A t_k \\ - \mathbf{g}_i^H \left(\sum_{l=1, l \neq k}^K \mathbf{Q}_l - \frac{1}{\gamma_k} \mathbf{Q}_k \right) \mathbf{g}_i + \sigma_k^2 \end{aligned} \quad (91)$$

As explained in Section II, the problem (87) can be solved by a series of subproblems,

$$\begin{aligned} \min_{\mathbf{g}} \quad & \bar{f}_0(\mathbf{g}) \\ \text{s.t.} \quad & \bar{f}_k(\mathbf{g}) \leq 0, \quad \forall k \in [1, K] \\ & |\mathbf{g}(n)| = 1, \quad \forall n \in [1, N_A N], \end{aligned} \quad (92)$$

According to Theorem 1, under the mild condition $\{ \{(t_k \mathbf{I} - \sum_{l=1, l \neq k}^K \mathbf{Q}_l + \frac{1}{\gamma_k} \mathbf{Q}_k) \mathbf{g}_i \neq 0\}_{k=1}^K \}_{i=0}^\infty$, the optimal solution of (92) comes from the solution of the equations,

$$\begin{aligned} \mathbf{g}(\nu) = \exp \left(\sqrt{-1} \arg \left(\sum_{k=1}^K [(t_k \mathbf{I} - \sum_{l=1, l \neq k}^K \mathbf{Q}_l + \frac{1}{\gamma_k} \mathbf{Q}_k) \mathbf{g}_i] \nu_k \right. \right. \\ \left. \left. + (t_0 \mathbf{I} - \mathbf{Q}_0) \mathbf{g}_i \right) \right), \end{aligned} \quad (93)$$

$$0 \leq \nu_k \leq \infty, \quad \bar{f}_k(\mathbf{g}(\nu)) \leq 0, \quad \forall k \in [1, K], \quad (94)$$

$$\nu_k \bar{f}_k(\mathbf{g}(\nu)) = 0, \quad \forall k \in [1, K]. \quad (95)$$

With optimal solution of the subproblem (92), the iMM method to solve problem (87) is described in the inner loop of Algorithm 7. Since analog transmitter in the iMM method monotonically decreases the transmit power, and the optimal digital transmitter also find the minimal transmit power with fixed analog transmitter, the Algorithm 7 is guaranteed to converge.

Algorithm 7 Joint analog and digital transmitter for SINR constrained MU-MISO system.

- 1: **initialization:** the elements in \mathbf{g}_0 is generated from the unit circle independently, and set $i_7 = 0$.
- 2: **repeat**
- 3: Use fixed point or SOCP method [19], [51] to find digital transmitter \mathbf{G}_D .
- 4: **initialization:** set $i = 0$, \mathbf{g}_i is the latest feasible point
- 5: **repeat**
- 6: Using Algorithm 2 to find $\mathbf{g}(\boldsymbol{\nu})$ in (93) to (95), and denote the objective value of problem (87) as $f[i+1]$. Set $i = i + 1$, let $\mathbf{g}_i = \mathbf{g}(\boldsymbol{\nu})$.
- 7: **until** $i \geq 2$ and $(f[i-1] - f[i])/f[i] \leq \epsilon_{72}$
- 8: Denote the transmit power as $P[i_7 + 1]$, and set $i_7 = i_7 + 1$.
- 9: **until** $i_7 \geq 2$ and $(P[i_7 - 1] - P[i_7])/P[i_7] \leq \epsilon_7$

V. SIMULATION RESULTS

The simulation parameters of different algorithms are described as follows. In Algorithm 2, the bisection accuracy level is $\epsilon = 10^{-6}$, and the termination threshold is $\epsilon_2 = 10^{-3}$. In Algorithm 3, the iMM termination threshold is $\epsilon_3 = 2 \times 10^{-4}$. In Algorithm 4, the MM termination threshold is $\epsilon_4 = 10^{-3}$. In Algorithm 5, the ADMM tolerances are $\epsilon^p = \epsilon^d = 10^{-6}$, and the penalty factor is $\mu = 20$. In Algorithms 6 and 7, the power termination thresholds are $\epsilon_6 = 10^{-2}$, $\epsilon_7 = 10^{-2}$, respectively. The inner termination threshold in Algorithm 7 is $\epsilon_{72} = 2 \times 10^{-4}$.

All the simulation results are averaged over 100 random mmWave channel realizations, which is generated from the standard setting in [6]. All optimization problems are solved on a laptop PC with 3.6 GHz CPU and 16GB RAM, and the software is Matlab. The results of the SDR method [26], [35] with interior point algorithm are obtained with the solver CVX [52], and the Gaussian randomization number is 10^5 (with efficient vectorized multiplication without any loop). The computational times are measured by the Matlab functions *tic* and *toc*.

A. MSE Constrained Hybrid Beamforming for MU-MIMO

For the MU-MIMO system, the transmit antenna number is $N = 10$ unless otherwise specified, the receiver antenna numbers are $\{M_k = 10\}_{k=1}^K$, the transmit and receive RF chain numbers are $N_A = 4, \{M_{A_k} = 4\}_{k=1}^K$, respectively, the user number is $K = 2$ and the data stream numbers are $\{L_k = 1\}_{k=1}^K$. Unless specified otherwise, the MSE targets are fixed as $\{\epsilon_k = 0.1\}_{k=1}^K$. The noise covariance matrices are $\{\boldsymbol{\Sigma}_k = 0.01\mathbf{I}_{M_k}\}_{k=1}^K$.

The transmit powers, $10 \log_{10}(\|\mathbf{G}_A \mathbf{G}_D\|_F^2 / \boldsymbol{\Sigma}_1(1, 1))$ (dB), of the proposed method and the SDR approach [35] are compared in Fig. 3. It can be seen that the transmit power of the proposed iMM method is monotonically decreasing, which validates the convergence analysis for Algorithm 6. Furthermore, the transmit powers of the proposed method with different MSE targets are comparable to that of the SDR

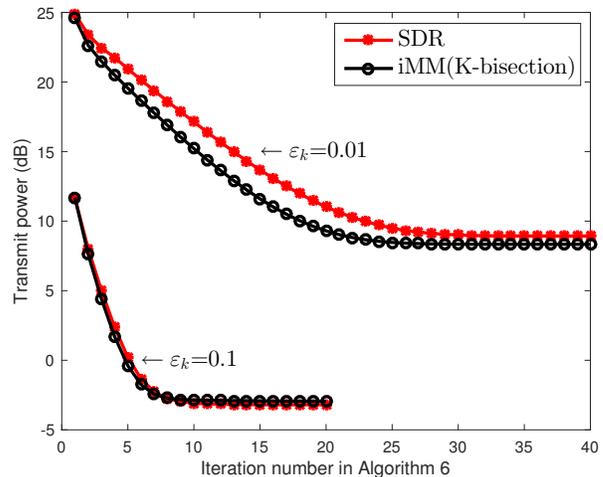


Fig. 3. Transmit powers under different methods (MSE constrained).

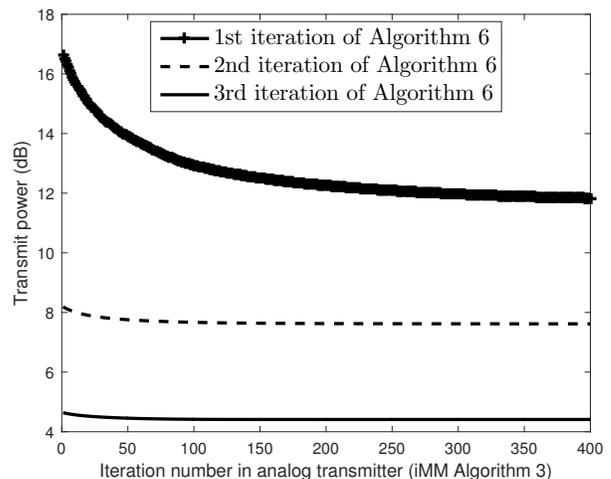


Fig. 4. Convergence performance of the iMM method (MSE constrained).

approach, which reveals the good performance of the proposed method.

In the analog transmitter design, the convergence performance of the iMM method is depicted in Fig. 4. It can be seen from Fig. 4 that the transmit powers of the proposed iMM method are monotonically decreasing at different outer iteration loops, which confirms the monotonic convergence property in Theorem 1 with positive semidefinite quadratic forms. Fig. 4 also shows that the iteration number of the iMM method is large at the first outer iteration, while that of the iMM method is just tens iterations at the second (or later) outer iterations. This phenomena reveals the the analog beamforming directions are basically determined at the first outer iteration, i.e., the analog beamforming direction is quickly decided, while the power gains in the latter outer iteration are majorly due to the digital transmitter and the coordination between transmitter and receivers.

To compare the computational times, the proposed method

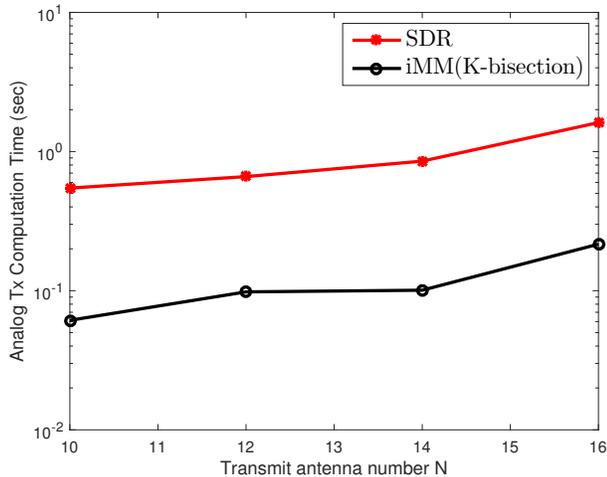


Fig. 5. Analog transmitter computational times under different methods (MSE constrained).

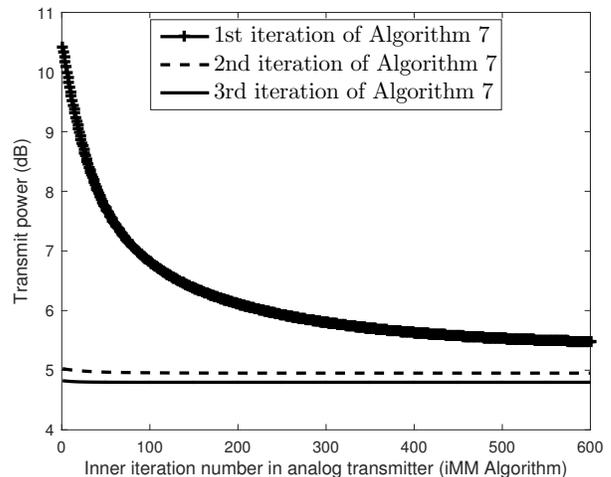


Fig. 7. Convergence performance of the iMM method (SINR constrained).

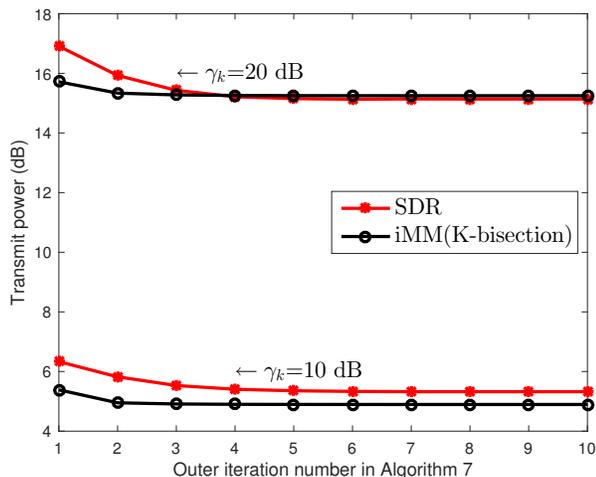


Fig. 6. Transmit powers under different methods (SINR constrained).

is compared with the SDR method in Fig. 5. It can be seen from Fig. 5 that the computational times of the iMM method are almost 10 times faster than that of the SDR method, which validates the complexity analysis in Table 1. Therefore, Figs. 3 and 5 reveal that compared to the SDR method, the proposed iMM method achieves the similar transmit power but with a much lower computational complexity.

B. SINR Constrained Hybrid Beamforming for MU-MISO

For the MU-MISO system, the transmit antenna number is $N = 10$ unless otherwise specified, the transmit RF chain numbers is $N_A = 4$, the user number is $K = 2$. Unless specified otherwise, the SINR targets are fixed as $\{\gamma_k = 10 \text{ dB}\}_{k=1}^K$. The noise variances are $\{\sigma_k^2 = 0.01\}_{k=1}^K$.

The transmit powers, $10 \log_{10}(\|\mathbf{G}_A \mathbf{G}_D\|_F^2 / \sigma_1^2)$ (dB), of the proposed method and the SDR approach are compared in Fig. 6. It can be seen that the transmit power of the proposed

iMM method is monotonically decreasing, which validates the convergence analysis for Algorithm 7. Furthermore, the transmit powers of the proposed method with different SINR targets are comparable to that of the SDR approach, which reveals the good power performance of the proposed method. From Figs. 3 and 6, we know the outer iteration number is very small for SINR constrained beamforming, while that of the MSE constrained beamforming is relative large. This reason is that the MSE constrained coordination between the four components, i.e., digital transmitter, analog transmitter, K analog receivers and K digital receivers, need lots efforts. But the SINR constrained coordination between the digital transmitter and analog transmitter converges quickly.

For the analog transmitter design, the convergence performance of the iMM method is illustrated in Fig. 7. It can be seen from Fig. 7 that the transmit powers of the proposed iMM method are monotonically decreasing at different outer iteration loops, which confirms the monotonic convergence property in Theorem 1 with indefinite quadratic forms. Fig. 7 also shows that the iteration number of the iMM method is large at the first outer iteration, while that of the iMM method is just tens iterations at the second (or later) outer iterations. This phenomena reveals the the analog beamforming directions are basically determined at the first outer iteration, while the later analog beamformings are just small refinements.

To compare the computational times, the proposed method is compared with the SDR method in Fig. 8. It can be seen from Fig. 8 that the computational times of the iMM method are almost 10 times faster than that of the SDR method, which validates the complexity analysis in Table 1. Therefore, Figs. 6 and 8 reveal that compared to the SDR method, the proposed iMM method achieves similar transmit powers but with a much lower computational complexity.

VI. CONCLUSIONS

In this paper, the QCQP with extra constant modulus constraints problem is solved by a series of subproblems

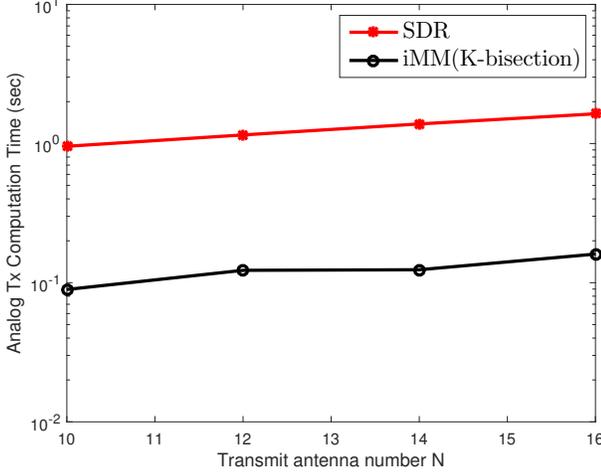


Fig. 8. Analog transmitter computational times under different methods (SINR constrained).

with LP under extra constant modulus constraints. Under mild condition, the strong duality between the LP with extra constant modulus constraints and its dual problem is established. Then, by using the optimal solutions from the subproblems, the QCQP with extra constant modulus constraints problem is solved by a monotonically converged algorithm. For the positive semidefinite quadratic form, the MSE constrained hybrid beamforming is firstly proposed and solved in the past decades. For the indefinite quadratic form, the SINR constrained hybrid beamforming is solved. Simulation results show that the transmit power of the proposed method is similar to that of the SDR method, while the computational time of proposed method is much faster than the SDR method.

APPENDIX A

AN EFFICIENT METHOD TO CHOOSE t_k

Since $t_k \mathbf{I} \succeq \mathbf{Q}_k$ is needed, the widely used method to choose t_k is $t_k = \lambda_{\max}(\mathbf{Q}_k)$, which is the maximal eigenvalue of \mathbf{Q}_k . However, its computational complexity order $\mathcal{O}(N^3)$ is relatively high. To reduce the computational complexity, we propose the following simple method with complexity order $\mathcal{O}(N)$, "sum of positive eigenvalues", to choose t_k in Table 2,

TABLE 2
DIFFERENT METHODS TO CHOOSE t_k

\mathbf{Q}_k	$\mathbf{Q}_k \succeq 0$	$\mathbf{Q}_k \preceq 0$	Indefinite	complexity order
$t_k =$	$\lambda_{\max}(\mathbf{Q}_k)$	$\lambda_{\max}(\mathbf{Q}_k)$	$\lambda_{\max}(\mathbf{Q}_k)$	$\mathcal{O}(N^3)$
$t_k =$	$\text{Tr}(\mathbf{Q}_k)$	0	$\sum \lambda_i^+$	$\mathcal{O}(N), \mathcal{O}(1)$

When \mathbf{Q}_k is indefinite in Table 2, the method with $t_k = \sum \lambda_i^+$ seems to be more complex than the method with $t_k = \lambda_{\max}(\mathbf{Q}_k)$. However, if the structure of the indefinite \mathbf{Q}_k is special, the computational complexity of the method with $t_k = \sum \lambda_i^+$ is $\mathcal{O}(N)$, e.g.,

If $\mathbf{Q}_k = \mathbf{Q}_k^+ \oplus \mathbf{Q}_k^-$ with $\mathbf{Q}_k^+ \succeq 0, \mathbf{Q}_k^- \preceq 0$, then $t_k = \text{Tr}(\mathbf{Q}_k^+)$ (96)

If $\mathbf{Q}_k = \mathbf{Q}_k^+ + \mathbf{Q}_k^-$ with $\mathbf{Q}_k^+ \succeq 0, \mathbf{Q}_k^- \preceq 0$, then $t_k = \text{Tr}(\mathbf{Q}_k^+)$ (97)

The special structures in (96) and (97) are widely used in SINR based beamforming in wireless communications and radar waveform designs.

APPENDIX B

PROOF OF THE LIMIT RESULT IN LEMMA 2

If $\lim_{\nu_k \rightarrow \infty} \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) = 0$, we have

$$\lim_{\nu_k \rightarrow \infty} \nu_k \bar{f}_k(\mathbf{x}(\boldsymbol{\nu})) = \lim_{\nu_k \rightarrow \infty} \frac{\bar{f}_k(\mathbf{x}(\boldsymbol{\nu}))}{1/\nu_k} \quad (98)$$

$$= \lim_{\nu_k \rightarrow \infty} \frac{(\bar{f}_k(\mathbf{x}(\boldsymbol{\nu})))'}{(1/\nu_k)'} \quad (99)$$

where the notation $(\cdot)'$ stands for the operation of taking derivative respect to ν_k . For notional simplicity, we define

$$\hat{\mathbf{a}} = (t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k, \quad (100)$$

$$\hat{\mathbf{b}} = \sum_{l=1, l \neq k}^K [(t_l \mathbf{I} - \mathbf{Q}_l) \mathbf{x}_i - \mathbf{p}_l] \nu_l + (t_0 \mathbf{I} - \mathbf{Q}_0) \mathbf{x}_i - \mathbf{p}_0 \quad (101)$$

$$\hat{a}_n = \hat{\mathbf{a}}(n), \hat{b}_n = \hat{\mathbf{b}}(n), j = \sqrt{-1}, \quad (102)$$

which are valid only in this proof. Since $\{\nu_i\}_{i \neq k}^K$ are finite, the elements of $\hat{\mathbf{b}}$ are finite. Then, we have

$$\lim_{\nu_k \rightarrow \infty} \nu_k (\bar{f}_k(\mathbf{x}(\boldsymbol{\nu}))) = \lim_{\nu_k \rightarrow \infty} \frac{(\bar{f}_k(\mathbf{x}(\boldsymbol{\nu})))'}{(1/\nu_k)'} \quad (103)$$

$$= \lim_{\nu_k \rightarrow \infty} \frac{(2\text{Re}(-\hat{\mathbf{a}}^H \exp(j \arg(\hat{\mathbf{a}}\nu_k + \hat{\mathbf{b}}))))'}{-1/\nu_k^2} \quad (104)$$

$$= \lim_{\nu_k \rightarrow \infty} \frac{(2\text{Re}(\hat{\mathbf{a}}^H \exp(j \arg(\hat{\mathbf{a}}\nu_k + \hat{\mathbf{b}}))))'}{1/\nu_k^2} \quad (105)$$

$$= \lim_{\nu_k \rightarrow \infty} \frac{\sum_{n=1}^N (\text{Re}([\text{Re}(\hat{a}_n) - j\text{Im}(\hat{a}_n)] \frac{\text{Re}(\hat{a}_n \nu_k + \hat{b}_n) + j\text{Im}(\hat{a}_n \nu_k + \hat{b}_n)}{\sqrt{\text{Re}(\hat{a}_n \nu_k + \hat{b}_n)^2 + \text{Im}(\hat{a}_n \nu_k + \hat{b}_n)^2}}))'}{1/\nu_k^2/2} \quad (106)$$

$$= \lim_{\nu_k \rightarrow \infty} \frac{2 \sum_{n=1}^N (\frac{\text{Re}(\hat{a}_n) \text{Re}(\hat{a}_n \nu_k + \hat{b}_n) + \text{Im}(\hat{a}_n) \text{Im}(\hat{a}_n \nu_k + \hat{b}_n)}{\sqrt{\text{Re}(\hat{a}_n \nu_k + \hat{b}_n)^2 + \text{Im}(\hat{a}_n \nu_k + \hat{b}_n)^2}})'}{1/\nu_k^2} \quad (107)$$

$$= \lim_{\nu_k \rightarrow \infty} 2\nu_k^2 \sum_{n=1}^N \left[|\hat{a}_n|^2 \sqrt{\text{Re}(\hat{a}_n \nu_k + \hat{b}_n)^2 + \text{Im}(\hat{a}_n \nu_k + \hat{b}_n)^2} - (\text{Re}(\hat{a}_n \nu_k + \hat{b}_n)^2 + \text{Im}(\hat{a}_n \nu_k + \hat{b}_n)^2)^{-\frac{1}{2}} \cdot (\text{Re}(\hat{a}_n) \text{Re}(\hat{a}_n \nu_k + \hat{b}_n) + \text{Im}(\hat{a}_n) \text{Im}(\hat{a}_n \nu_k + \hat{b}_n))^2 \right] / [\text{Re}(\hat{a}_n \nu_k + \hat{b}_n)^2 + \text{Im}(\hat{a}_n \nu_k + \hat{b}_n)^2] \quad (108)$$

$$= \lim_{\nu_k \rightarrow \infty} 2 \sum_{n=1}^N \left[|\hat{a}_n|^2 \sqrt{\text{Re}(\hat{a}_n \nu_k + \hat{b}_n)^2 + \text{Im}(\hat{a}_n \nu_k + \hat{b}_n)^2} - (\text{Re}(\hat{a}_n \nu_k + \hat{b}_n)^2 + \text{Im}(\hat{a}_n \nu_k + \hat{b}_n)^2)^{-\frac{1}{2}} \cdot (\text{Re}(\hat{a}_n) \text{Re}(\hat{a}_n \nu_k + \hat{b}_n) + \text{Im}(\hat{a}_n) \text{Im}(\hat{a}_n \nu_k + \hat{b}_n))^2 \right] / |\hat{a}_n|^2 \quad (109)$$

$$= \lim_{\nu_k \rightarrow \infty} 2 \sum_{n=1}^N \frac{(\text{Im}(\hat{a}_n) \text{Re}(\hat{b}_n) - \text{Re}(\hat{a}_n) \text{Im}(\hat{b}_n))^2}{|\hat{a}_n|^2 \sqrt{\text{Re}(\hat{a}_n \nu_k + \hat{b}_n)^2 + \text{Im}(\hat{a}_n \nu_k + \hat{b}_n)^2}} \quad (110)$$

$$= 0 \quad (111)$$

where the last equation is obtained due to the conditions with \hat{b}_n being finite, and $\hat{\mathbf{a}} = (t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k \neq \mathbf{0}$, i.e., $\{\hat{a}_n \neq 0\}_{n=1}^N$.

APPENDIX C

PROOF OF THE MONOTONIC PROPERTY IN LEMMA 3

The first part of the following proof is adapted from [34] under a sum rate maximization problem. Let the Lagrange of the problem (5) be denoted as

$$\begin{aligned} \mathcal{L}(\mathbf{x}, \nu_k) &= \bar{f}_0(\mathbf{x}) + \sum_{k=1}^K \nu_k \bar{f}_k(\mathbf{x}) \\ &= \bar{f}_0(\mathbf{x}) + \nu_k \bar{f}_k(\mathbf{x}) + \sum_{l \neq k} \nu_l \bar{f}_l(\mathbf{x}) \end{aligned} \quad (112)$$

Let ν_k^a, ν_k^b be two fixed values with $\nu_k^a > \nu_k^b \geq 0$, and denote $\mathbf{x}(\nu_k^a), \mathbf{x}(\nu_k^b)$ as the optimal solutions of $\min_{\{\|\mathbf{x}(n)\|=1\}_{n=1}^N} \mathcal{L}(\mathbf{x}, \nu_k)$ with $\nu_k = \nu_k^a, \nu_k = \nu_k^b$, respectively. For notational simplicity, the $\mathbf{x}(\nu_k^a)$ is the simple notation of $\mathbf{x}(\nu)|_{\nu_k=\nu_k^a}$. Therefore, we have the following inequalities,

$$\mathcal{L}(\mathbf{x}(\nu_k^a), \nu_k^a) \leq \mathcal{L}(\mathbf{x}(\nu_k^b), \nu_k^a), \quad (113)$$

$$\mathcal{L}(\mathbf{x}(\nu_k^b), \nu_k^b) \leq \mathcal{L}(\mathbf{x}(\nu_k^a), \nu_k^b). \quad (114)$$

After taking the expressions of $\mathcal{L}(\mathbf{x}, \nu_k)$ of (112) into the above equalities, we have

$$\begin{aligned} &\bar{f}_0(\mathbf{x}(\nu_k^a)) + \nu_k^a \bar{f}_k(\mathbf{x}(\nu_k^a)) + \sum_{l \neq k} \nu_l \bar{f}_l(\mathbf{x}(\nu_k^a)) \\ &\leq \bar{f}_0(\mathbf{x}(\nu_k^b)) + \nu_k^a \bar{f}_k(\mathbf{x}(\nu_k^b)) + \sum_{l \neq k} \nu_l \bar{f}_l(\mathbf{x}(\nu_k^b)), \end{aligned} \quad (115)$$

$$\begin{aligned} &\bar{f}_0(\mathbf{x}(\nu_k^b)) + \nu_k^b \bar{f}_k(\mathbf{x}(\nu_k^b)) + \sum_{l \neq k} \nu_l \bar{f}_l(\mathbf{x}(\nu_k^b)) \\ &\leq \bar{f}_0(\mathbf{x}(\nu_k^a)) + \nu_k^b \bar{f}_k(\mathbf{x}(\nu_k^a)) + \sum_{l \neq k} \nu_l \bar{f}_l(\mathbf{x}(\nu_k^a)). \end{aligned} \quad (116)$$

By adding the above equalities, we have

$$(\nu_k^a - \nu_k^b) \bar{f}_k(\mathbf{x}(\nu_k^a)) \leq (\nu_k^a - \nu_k^b) \bar{f}_k(\mathbf{x}(\nu_k^b)) \quad (117)$$

and using the condition $\nu_k^a > \nu_k^b \geq 0$, we have the conclusion,

$$\bar{f}_k(\mathbf{x}(\nu_k^a)) \leq \bar{f}_k(\mathbf{x}(\nu_k^b)) \quad (118)$$

Therefore, the proof of the first conclusion is completed.

For the second conclusion, if $\{\nu_l\}_{l \neq k}^K$ are finite and $[(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k] \nu_k + \sum_{l=1, l \neq k}^K [(t_l \mathbf{I} - \mathbf{Q}_l) \mathbf{x}_i - \mathbf{p}_l] \nu_l + (t_0 \mathbf{I} - \mathbf{Q}_0) \mathbf{x}_i - \mathbf{p}_0 \neq \mathbf{0}$ for all $\nu_k \in [0, \infty)$, then the optimal solution of the problem $\min_{\{\|\mathbf{x}(n)\|=1\}_{n=1}^N} \mathcal{L}(\mathbf{x}, \nu_k)$ is unique. Furthermore, if $(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k \neq \mathbf{0}$, $[(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k] s \neq$

$\sum_{l=1, l \neq k}^K [(t_l \mathbf{I} - \mathbf{Q}_l) \mathbf{x}_i - \mathbf{p}_l] \nu_l + (t_0 \mathbf{I} - \mathbf{Q}_0) \mathbf{x}_i - \mathbf{p}_0, \forall s \in \mathbb{R}$, then $\mathbf{x}(\nu_k^a) \neq \mathbf{x}(\nu_k^b)$. Therefore, the inequalities in (113)-(118) become the strict inequalities, and the proof of the strictly decreasing property is completed.

APPENDIX D

EXPLAINING THE MILD CONDITIONS IN ASSUMPTIONS 1 AND 2

We can construct special cases to make $(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k = \mathbf{0}$, e.g.,

- 1) $t_k = 0, \mathbf{Q}_k = \mathbf{0}, \mathbf{p}_k = \mathbf{0}$
- 2) $\{t_k = 0, \mathbf{Q}_k = \mathbf{0}, \mathbf{p}_k = \mathbf{0}\}_{k=1}^K$

Firstly, the condition $t_k = 0, \mathbf{Q}_k = \mathbf{0}, \mathbf{p}_k = \mathbf{0}$ makes the quadratic function degenerated to a constant, then problem (1) is infeasible or degenerated to a problem with $K - 1$ quadratic constraints, which can be tackled by the proposed method again.

Secondly, when $\{t_k = 0, \mathbf{Q}_k = \mathbf{0}, \mathbf{p}_k = \mathbf{0}\}_{k=1}^K$, the proposed problem, if it is feasible, is degenerated to the QP problem with constant modulus constraint. And the optimal solution in (9) to (11) is a closed-form solution with $\{\nu_k = 0\}_{k=1}^K$ as shown in Theorem 3.

In fact, we can always set t_k such that $t_k \mathbf{I} \succ \mathbf{Q}_k$, as illustrated in Table 2 from Appendix A. Furthermore, the feasible solution \mathbf{x}_i lies on the unit circles, then $(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i \neq \mathbf{0}$, this makes the condition $(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k \neq \mathbf{0}$ become a very mild condition from the numeric computing aspect.

For the condition $\{[(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k] \nu_k + \sum_{l=1, l \neq k}^K [(t_l \mathbf{I} - \mathbf{Q}_l) \mathbf{x}_i - \mathbf{p}_l] \nu_l + (t_0 \mathbf{I} - \mathbf{Q}_0) \mathbf{x}_i - \mathbf{p}_0 \neq \mathbf{0}\}_{i=0}^\infty$ with $\nu_k \geq 0$, we can take following example to illustrate the weak condition. For notational simplicity, we define

$$\hat{\mathbf{a}} = (t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k, \quad (119)$$

$$\hat{\mathbf{b}} = \sum_{l=1, l \neq k}^K [(t_l \mathbf{I} - \mathbf{Q}_l) \mathbf{x}_i - \mathbf{p}_l] \nu_l + (t_0 \mathbf{I} - \mathbf{Q}_0) \mathbf{x}_i - \mathbf{p}_0 \quad (120)$$

$$\hat{a}_n = \hat{\mathbf{a}}(n), \hat{b}_n = \hat{\mathbf{b}}(n), \quad (121)$$

then the condition becomes $\hat{a}_n \nu_k \neq -\hat{b}_n$ for all $\nu_k \geq 0$. Note that both \hat{a}_n and \hat{b}_n are complex numbers, the counter example only happens when the phases of \hat{a}_n and $-\hat{b}_n$ are exactly the same, which is rare. The assumption $[(t_k \mathbf{I} - \mathbf{Q}_k) \mathbf{x}_i - \mathbf{p}_k] s \neq \sum_{l=1, l \neq k}^K [(t_l \mathbf{I} - \mathbf{Q}_l) \mathbf{x}_i - \mathbf{p}_l] \nu_l + (t_0 \mathbf{I} - \mathbf{Q}_0) \mathbf{x}_i - \mathbf{p}_0, \forall s \in \mathbb{R}$ means $\hat{\mathbf{a}} s \neq \hat{\mathbf{b}}, \forall s \in \mathbb{R}$. The counter example only happens when all phases of $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are exactly the same or reversed, which is rare.

The purpose of introducing Assumption 2 is to make sure $\nu[i_2 - 1] = \nu[i_2]$, we can also introduce two different stronger conditions to get the same conclusion,

- 1) The dual problem (7) is strongly convex.

$$2) \lim_{i_2 \rightarrow \infty} \nu[i_2 - 1] - \nu[i_2] = \mathbf{0}.$$

Although the dual problem (7) is convex, its hessian matrix is very difficult to calculate. Furthermore, it is also numerically difficult to judge whether the hessian matrix is positive definite. Similar assumptions on $\lim_{i_2 \rightarrow \infty} \nu[i_2 - 1] - \nu[i_2] = \mathbf{0}$ are widely used in nonconvex ADMM convergence proof, e.g., the

theorem 1 in references [12], [53]. It is clear that the equality in $\lim_{i_2 \rightarrow \infty} \nu[i_2 - 1] - \nu[i_2] = \mathbf{0}$ only happens with one case, while the inequalities in Assumptions 1 and 2 can occur at infinite many different cases. Therefore, Assumptions 1 and 2 are mild conditions.

REFERENCES

- [1] C. Audet, P. Hansen, B. Jaumard, and G. Savard, "A branch and cut algorithm for nonconvex quadratically constrained quadratic programming," *Mathematical Programming*, vol. 87, no. 1, pp. 131–152, 2000.
- [2] A. Beck and Y. C. Eldar, "Strong duality in nonconvex quadratic optimization with two quadratic constraints," *SIAM Journal on Optimization*, vol. 17, no. 3, pp. 844–860, 2006.
- [3] K. M. Anstreicher, "Semidefinite programming versus the reformulation linearization technique for nonconvex quadratically constrained quadratic programming," *Journal of Global Optimization*, vol. 43, no. 2-3, pp. 471–484, 2009.
- [4] Z.-Q. Luo, N. D. Sidiropoulos, P. Tseng, and S. Zhang, "Approximation bounds for quadratic optimization with homogeneous quadratic constraints," *SIAM Journal on Optimization*, vol. 18, no. 1, pp. 1–28, 2007.
- [5] L. Jiang and H. Jafarkhani, "Multi-user analog beamforming in millimeter wave MIMO systems based on path angle information," *IEEE Trans. on Wireless Communications*, vol. 18, no. 1, pp. 608–619, 2019.
- [6] F. Sotrobiani and W. Yu, "Hybrid digital and analog beamforming design for large-scale antenna arrays," *IEEE Journal of Selected Topics in Signal Processing*, vol. 10, no. 3, pp. 501–513, 2016.
- [7] C. Pan, H. Ren, K. Wang, W. Xu, M. El-kashlan, A. Nallanathan, and L. Hanzo, "Multicell MIMO communications relying on intelligent reflecting surfaces," *IEEE Transactions on Wireless Communications*, vol. 19, no. 8, pp. 5218–5233, 2020.
- [8] G. Cui, H. Li, and M. Rangaswamy, "MIMO radar waveform design with constant modulus and similarity constraints," *IEEE Transactions on Signal Processing*, vol. 62, no. 2, pp. 343–353, 2014.
- [9] A. Arora, C. G. Tsinos, B. Shankar Mysore R, S. Chatzinotas, and B. Ottersten, "Majorization-minimization algorithms for analog beamforming with large-scale antenna arrays," in *2019 IEEE Global Conference on Signal and Information Processing (GlobalSIP)*, 2019, pp. 1–5.
- [10] A. Arora, C. G. Tsinos, B. S. M. R. Rao, S. Chatzinotas, and B. Ottersten, "Hybrid transceivers design for large-scale antenna arrays using majorization-minimization algorithms," *IEEE Transactions on Signal Processing*, vol. 68, pp. 701–714, 2020.
- [11] J. Tranter, N. D. Sidiropoulos, X. Fu, and A. Swami, "Fast unit-modulus least squares with applications in beamforming," *IEEE Transactions on Signal Processing*, vol. 65, no. 11, pp. 2875–2887, 2017.
- [12] C. G. Tsinos and B. Ottersten, "An efficient algorithm for unit-modulus quadratic programs with application in beamforming for wireless sensor networks," *IEEE Signal Processing Letters*, vol. 25, no. 2, pp. 169–173, 2018.
- [13] X. Yu, J. Shen, J. Zhang, and K. B. Letaief, "Alternating minimization algorithms for hybrid precoding in millimeter wave MIMO systems," *IEEE Journal of Selected Topics in Signal Processing*, vol. 10, no. 3, pp. 485–500, 2016.
- [14] X. Xue, Y. Wang, L. Dai, and C. Masouros, "Relay hybrid precoding design in millimeter-wave massive MIMO systems," *IEEE Transactions on Signal Processing*, vol. 66, no. 8, pp. 2011–2026, 2018.
- [15] G. Zang, Y. Cui, H. V. Cheng, F. Yang, L. Ding, and H. Liu, "Optimal hybrid beamforming for multiuser massive MIMO systems with individual SINR constraints," *IEEE Wireless Communications Letters*, vol. 8, no. 2, pp. 532–535, 2019.
- [16] Q. Song, S. Zhao, and Q. Shi, "Secure satellite-terrestrial transmission via hybrid analog-digital beamforming," in *2018 10th International Conference on Wireless Communications and Signal Processing (WCSP)*, 2018, pp. 1–6.
- [17] M. A. Vazquez, L. Blanco, and A. Perez-Neira, "Multiuser downlink hybrid analog-digital beamforming with individual SINR constraints," in *WSA 2017; 21th International ITG Workshop on Smart Antennas*, 2017, pp. 1–6.
- [18] Z. Lin, M. Lin, B. Champagne, W.-P. Zhu, and N. Al-Dhahir, "Secrecy-energy efficient hybrid beamforming for satellite-terrestrial integrated networks," *IEEE Transactions on Communications*, pp. 1–1, 2021.
- [19] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Transactions on Vehicular Technology*, vol. 53, no. 1, pp. 18–28, 2004.
- [20] D. P. Palomar, M.-A. Lagunas, and J. M. Cioffi, "Optimum linear joint transmit-receive processing for MIMO channels with QoS constraints," *IEEE Trans. on Signal Processing*, vol. 52, no. 5, pp. 1179–1197, 2004.
- [21] S. Shuying, M. Schubert, and H. Boche, "Downlink MMSE transceiver optimization for multiuser MIMO systems: MMSE balancing," *IEEE Transactions on Signal Processing*, vol. 56, no. 8, pp. 3702–3712, 2008.
- [22] K. Wang, N. Jacklin, Z. Ding, and C. Chi, "Robust MISO transmit optimization under outage-based QoS constraints in two-tier heterogeneous networks," *IEEE Transactions on Wireless Communications*, vol. 12, no. 4, pp. 1883–1897, April 2013.
- [23] N. Vucic and H. Boche, "Robust QoS-constrained optimization of downlink multiuser MISO systems," *IEEE Transactions on Signal Processing*, vol. 57, no. 2, pp. 714–725, 2009.
- [24] X. He and Y. Wu, "Tight probabilistic SINR constrained beamforming under channel uncertainties," *IEEE Transactions on Signal Processing*, vol. 63, no. 13, pp. 3490–3505, 2015.
- [25] X. He and Y.-C. Wu, "Set squeezing procedure for quadratically perturbed chance-constrained programming," *IEEE Transactions on Signal Processing*, vol. 69, pp. 682–694, 2021.
- [26] Q. Wu and R. Zhang, "Intelligent reflecting surface enhanced wireless network via joint active and passive beamforming," *IEEE Transactions on Wireless Communications*, vol. 18, no. 11, pp. 5394–5409, 2019.
- [27] B. Zheng and R. Zhang, "Intelligent reflecting surface-enhanced OFDM: Channel estimation and reflection optimization," *IEEE Wireless Communications Letters*, vol. 9, no. 4, pp. 518–522, 2020.
- [28] Q. Wu and R. Zhang, "Weighted sum power maximization for intelligent reflecting surface aided SWIPT," *IEEE Wireless Communications Letters*, pp. 1–1, 2019.
- [29] M. Cui, G. Zhang, and R. Zhang, "Secure wireless communication via intelligent reflecting surface," *IEEE Wireless Communications Letters*, vol. 8, no. 5, pp. 1410–1414, 2019.
- [30] Z. Chu, W. Hao, P. Xiao, and J. Shi, "Intelligent reflecting surface aided multi-antenna secure transmission," *IEEE Wireless Communications Letters*, vol. 9, no. 1, pp. 108–112, 2020.
- [31] X. Guan, Q. Wu, and R. Zhang, "Intelligent reflecting surface assisted secrecy communication: Is artificial noise helpful or not?" *IEEE Wireless Communications Letters*, pp. 1–1, 2020.
- [32] M. M. Zhao, Q. Wu, M. J. Zhao, and R. Zhang, "Intelligent reflecting surface enhanced wireless network: Two-timescale beamforming optimization," *arXiv preprint arXiv:1912.01818*, 2019.
- [33] M. M. Zhao, Q. Wu, M. J. Zhao, and R. Zhang, "Exploiting amplitude control in intelligent reflecting surface aided wireless communication with imperfect CSI," *arXiv preprint arXiv:2005.07002*, 2020.
- [34] C. Pan, H. Ren, K. Wang, M. El-kashlan, A. Nallanathan, J. Wang, and L. Hanzo, "Intelligent reflecting surface aided MIMO broadcasting for simultaneous wireless information and power transfer," *IEEE J. on Selected Areas in Communications*, vol. 38, no. 8, pp. 1719–1734, 2020.
- [35] X. He, L. Huang, and J. Wang, "Novel relax-and-retract algorithm for intelligent reflecting surface design," *IEEE Transactions on Vehicular Technology*, vol. 70, no. 2, pp. 1995–2000, 2021.
- [36] X. He and L. Huang, "Joint MIMO communication and MIMO radar under different practical waveform constraints," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 12, pp. 16342–16347, 2020.
- [37] O. Aldayel, V. Monga, and M. Rangaswamy, "Successive QCQP refinement for MIMO radar waveform design under practical constraints," *IEEE Transactions on Signal Processing*, vol. 64, no. 14, pp. 3760–3774, 2016.
- [38] X. Yu, G. Cui, J. Yang, and L. Kong, "MIMO radar transmit-receive design for moving target detection in signal-dependent clutter," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 1, pp. 522–536, 2020.
- [39] L. Wu, P. Babu, and D. P. Palomar, "Transmit waveform/receive filter design for MIMO radar with multiple waveform constraints," *IEEE Transactions on Signal Processing*, vol. 66, no. 6, pp. 1526–1540, 2018.
- [40] X. He, L. Huang, and J. Wang, "MIMO radar design with joint constant-modulus and any p -norm similarity constraints," *submitted to IEEE Transactions on Vehicular Technology*, 2021.
- [41] X. He, "MIMO radar optimization with constant-modulus and any p -norm similarity constraints," 2021. [Online]. Available: http://www.optimization-online.org/DB_HTML/2021/05/8420.html
- [42] X. He, "Low complexity design of intelligent reflecting surface and MU-MIMO transceiver," *under preparation*.
- [43] Y. Sun, P. Babu, and D. P. Palomar, "Majorization-minimization algorithms in signal processing, communications, and machine learning," *IEEE Trans. on Signal Processing*, vol. 65, no. 3, pp. 794–816, 2017.
- [44] M. Razaviyayn, M. Hong, and Z.-Q. Luo, "A unified convergence analysis of block successive minimization methods for nonsmooth

- optimization,” *SIAM Journal on Optimization*, vol. 23, no. 2, pp. 1126–1153, 2013. [Online]. Available: <https://doi.org/10.1137/120891009>
- [45] B. R. Marks and G. P. Wright, “A general inner approximation algorithm for nonconvex mathematical programs,” *Operations Research*, vol. 26, no. 4, pp. 681–683, 1978. [Online]. Available: <http://www.jstor.org/stable/169728>
- [46] L. Wu, P. Babu, and D. P. Palomar, “Transmit waveform/receive filter design for MIMO radar with multiple waveform constraints,” *IEEE Transactions on Signal Processing*, vol. 66, no. 6, pp. 1526–1540, 2018.
- [47] J. S. Pang, “Partially b-regular optimization and equilibrium problems,” *Mathematics of Operations Research*, vol. 32, no. 3, pp. 687–699, 2007. [Online]. Available: <http://www.jstor.org/stable/25151818>
- [48] A. Nemirovski, *Interior Point Polynomial Time Methods in Convex Programming*. Lecture Notes, 1996.
- [49] Z.-Q. Luo and W. Yu, “An introduction to convex optimization for communications and signal processing,” *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 8, pp. 1426–1438, 2006.
- [50] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, “Distributed optimization and statistical learning via the alternating direction method of multipliers,” *Foundations & Trends in Machine Learning*, vol. 3, no. 1, pp. 1–122, 2010.
- [51] A. Wiesel, Y. Eldar, and S. Shamai, “Linear precoding via conic optimization for fixed MIMO receivers,” *IEEE Transactions on Signal Processing*, vol. 54, no. 1, pp. 161–176, 2006.
- [52] M. Grant and S. Boyd, “CVX: Matlab software for disciplined convex programming, version 2.1,” <http://cvxr.com/cvx>, Mar. 2014.
- [53] K. Huang and N. D. Sidiropoulos, “Consensus-ADMM for general quadratically constrained quadratic programming,” *IEEE Transactions on Signal Processing*, vol. 64, no. 20, pp. 5297–5310, 2016.