

# Heuristics for Home Appliances Scheduling Problems With Energy Consumption Bounds

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## Abstract

Many existing papers on the topic of smart grids work on scheduling of appliances considering an unlimited energy supply. However, the size of the power grid necessary to operate on periods of peak demand of the day would be large, and that would not be exploited during most of the day. Because of this, electricity providers could establish energy consumption bounds, which modify considerably appliance scheduling problems. In this paper we formulate a mathematical model for the problems we tackle, which include energy consumption bounds that depend on the time of the day. We prove that these problems are NP-hard and we later describe several heuristics we designed, explaining their foundations. Finally, we analyze the computational results obtained for instances we generated and present the best algorithms to employ according to different priorities the users might have.

*Keywords:* smart grids; day-ahead price; residential power scheduling; interruptible appliances; energy consumption bounds; heuristics.

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## 1. Introduction

The rational use of electric energy is essential for the socio-economic development of any society. The growing energy demand, in a context of system deficiency and progressive deterioration of the environment, makes the development of new paradigms in the energy system a priority. These paradigms encourage the use of renewable energy sources, which reduces the emission of carbon dioxide, the main cause of climate change. They also aim to increase efficiency in the use of energy in the phases of production, transmission and consumption and to guarantee a sustainable energy system that provides high levels of quality, efficiency and security of electric energy supply.

However, that is no simple task. Energy demand varies greatly throughout the year, the day of the week and even depends on what time of the day it is. Furthermore, there can be many climatic conditions

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as well as consumer profiles (industrial, commercial, residential). Energy supplying companies must satisfy peak demands, which results in a high infrastructure cost to enlarge grids that are used below their capacities during time of lower demand.

This scenario led to the concept of smart grids to emerge in the last decade. Smart grids are electricity networks that ensure sustainability of power systems, as well as efficiency and security of supply, by integrating all parts connected to them (producers, consumers and prosumers). Some of their more specific goals are:

- The reduction of the total energy consumption, benefiting both the environment and the consumers.
- The decrease in the maximum demand levels.
- The change in demand habits to adapt to available offer, specially in regions with a high presence of renewable energy sources.
- The reduction of the overloads to the electric power distribution system.

To achieve the proposed goals, not only does the smart grid need to be equipped with smart devices but it also relies on residential-level computational optimization tools that can manage the houses in the smart grids and that solve the new optimization problems that arise, in order to assist in the decision-making process, which involves, for example, deciding times or powers of operation for appliances.

In the last few years, optimization for smart grids has been approached by many researchers who have proposed models for many scenarios and objectives. As an example, reducing the peak-to-average-ratio (PAR) or the electricity bill, among others.

In Hussain et al. (2018), the authors propose an efficient home energy management controller (EHMC) that considers daily energy consumption bounds and uses real-time pricing (RTP) and critical peak pricing (CPP) as pricing schemes. In their model, as in Javaid et al. (2018), *discomfort*, which is the unpleasantness the users feel when the appliances do not operate as would be ideally desired, is only a result of delaying appliances and no discomfort is considered as a result of choosing the powers of operation.

An algorithm capable of generating an optimal energy consumption schedule based on the user-assigned priorities of the appliances subject to fixed budgets is presented by the authors in Khan et al. (2018). In Khan et al. (2019), the objective function to be minimized is the electricity cost but thresholds are defined for the energy consumption in each interval to restrict PAR. Appliances with flexible powers are not considered and metaheuristic algorithms are used. Energy consumption bounds for each interval are also considered in Fanti et al. (2019), where the authors formulate an integer linear programming (ILP) model for a problem on cooperative distributed energy scheduling to minimize the total cost, avoid power peaks and consider the preferences that users have on the operation of interruptible and non-interruptible time-shiftable appliances.

The aim of Vaziri et al. (2019) is to implement a demand dispatch program. Renewable energy sources (RES) are used to lower the cost of buying electricity from the grid and the dissatisfaction, which is related to delays. Energy can also be sold to the grid.

While the authors in Chiu et al. (2019) deal with a multiobjective optimization problem (MOP) that includes RESs, for which they work with the concept of Pareto optimality and develop a multiobjective evolutionary algorithm, in Jordehi (2019) discomfort is not considered in the model and a grey wolf optimization (GWO) algorithm is used for the purpose of optimally scheduling home appliances to minimize the electricity bill. In the latter, results are shown for scenarios with two different pricing

schemes along with incentives: RTP and Time of Use (ToU).

In Samadi et al. (2020), a model for a HEMS is presented in which the aim is to maximize satisfaction and to minimize the electricity cost resulting from a ToU pricing scheme. There are tasks and appliances. Tasks can involve many appliances and are divided into different categories. The mixed-integer nonlinear programming (MINLP) problem is solved using commercial software.

The problem considered in this paper is that of scheduling residential appliances considering energy consumption bounds for each interval. Decisions are made under a day-ahead pricing scheme which belongs to a demand response program. There are two groups of appliances. For those of the first type, scheduling consists on deciding in which intervals they are active given that they are interruptible, delayable and there is a discomfort factor that arises from the time waited by the user for the finalization of their operation. Instead, the operation window is fixed for those appliances of the second type, and the decisions to be made relate to their variable powers of operation as discomfort in this case is a result of the difference between the powers desired by the users and the actual ones. Besides from intending to minimize discomfort, the aim is to also minimize the economic expense. As far as we are aware, no other paper deals with the problems approached in this one, as they either work with distinct pricing schemes or employ other models that differ in their goals (such as minimizing PAR), formulations (for example, for discomfort) or consider other scenarios (such as no energy consumption bounds for each interval).

The rest of this paper is organized as follows. In Section 2 we present the model considered and a mathematical formulation for it is provided in Section 3. Section 4 contains a proof of the NP-hardness of the problems that we deal with and different families of heuristics are extensively described in Section 5. Computational results for instances we designed are exhibited and discussed in Section 6, considering various metrics. We finalize this paper with conclusions and ideas to continue our work, explained in Section 7.

Contributions of this paper are as follows:

- Two residential appliance scheduling problems were extended to model situations of practical interest.
- Mathematical formulations for these new problems were presented and shown to be NP-hard.
- Several heuristics were designed for these problems.

## 2. Description of Home Appliances Scheduling Problems With Bounds

The authors in Ma et al. (2016) propose a model regarding the operation of two types of residential electric appliances. Each appliance of the first type, grouped in the set  $\mathcal{A}_1$ , has a fixed power and operates continuously but with a flexible start time. Each appliance of the other type, included in the set  $\mathcal{A}_2$ , must be active in all subintervals of a predefined interval but the powers for the different subintervals could potentially be different.

However, in Taboh et al. (2020) we argue that it is beneficial to modify that model, allowing the operation of appliances of  $\mathcal{A}_1$  to be interrupted as many times as desired. Knowing the electricity prices  $p^t$  of consuming one kW during each interval  $t$  between two instants  $t$  and  $t + 1$  of a certain time window  $\mathcal{T} = \{0, \dots, T\}$ , we decide to distinguish each 10-minute interval of a day, horizon in which the user will schedule their appliances, to take advantage of our proposal of including interruptions.

While each appliance  $a \in \mathcal{A}_1$  operates with a fixed power  $r_a$  exactly  $T_a$  subintervals within a certain interval  $[\alpha_a, \beta_a)$  and its activity would ideally start at  $\alpha_a$  according to the user's preferences, each

appliance  $a \in \mathcal{A}_2$  is active throughout the whole interval  $[\alpha_a, \beta_a)$  and the power for each interval  $t$  can be chosen within the range  $[\tilde{r}_{a,t}^{min}, \tilde{r}_{a,t}^{max}]$ .

The goal of the decision-making process is to minimize the electricity bill paid by the users and the discomfort perceived by them, according to a desired-tradeoff.

If  $x_a^t$  designates the power of an appliance  $a \in \mathcal{A}_1 \cup \mathcal{A}_2$  in an interval  $t \in \mathcal{T}$ , the electricity bill emerges from the price and the energy used in each interval in the following way:

$$V_p = \sum_{t \in \mathcal{T}} \left( p^t \sum_{a \in \mathcal{A}_1 \cup \mathcal{A}_2} x_a^t \right)$$

As we define in Taboh et al. (2020), the discomfort for a certain appliance  $a \in \mathcal{A}_1$  relates to the number of hours of delay and is given by

$$V_a = \rho_a \left[ \left( 1 + \frac{t_a - (\alpha_a + T_a)}{iph} \right)^{k_a} - 1 \right]$$

where  $\rho_a \in (0, 1)$  and  $k_a \in \mathbb{R}_{\geq 1}$  are constants that depend on the operational characteristics of  $a$ , and  $iph$  designates the number of intervals included in one hour according to the chosen discretization.

For appliances in  $\mathcal{A}_2$ , the discomfort results from the difference between their powers of operation and the normal power consumption ( $\hat{x}_a^t$ ), and is expressed through the Taguchi loss function Taguchi et al. (2007) using the following formula

$$V_a = \sum_{t=\alpha_a}^{\beta_a-1} \omega_a^t (x_a^t - \hat{x}_a^t)^2$$

where  $0 < \omega_a^t$  and  $\hat{x}_a^t \leq r_a^{max}$  are constants that arise from the operational characteristics of  $a$  in each interval  $t$ .

Given that powers above  $\hat{x}_a^t$  would only lead to more economic expense and more discomfort, resulting unfavourable, we will use that value as an upper bound on  $x_a^t$ .

The objective function to minimize is a convex combination of the electricity bill and the discomfort. In other words, the goals have weights  $\alpha_1, \alpha_2 \in \mathbb{R}_{\geq 0}$ ,  $\alpha_1 + \alpha_2 = 1$ , and the objective is to minimize the weighted sum, which is a non-linear function that is not necessarily quadratic.

As can be seen, finding the optimal solutions for all appliances may lead to peak energy demands, because caring for the electricity bill could result in operating many appliances in cheap intervals and caring for comfort could result in great levels of energy consumption. This would be unfavorable for the energy supplying companies as enlarging the electricity grid to fulfill peak demands would not be exploited during the rest of the day. Therefore, the companies could impose energy consumption bounds for each interval. In this scenario, the possibility of interrupting appliances could make the difference between being able to schedule the appliances in a feasible way and not being able to do so, reason for which the incorporation of interruptions becomes even more relevant. Nonetheless, as users may possess appliances with technologies for which interruptions are not possible, in this paper we deal with both scenarios.

We shall call the problems extended with energy consumption bounds Two Types Appliance Scheduling for Economic and Discomfort Minimization With Bounds (2TASEDMWB) and Two Types Ap-

pliance Scheduling for Economic and Discomfort Minimization With Interruptions With Bounds (2TASEDMWIWB).

### 3. 2TASEDMWB and 2TASEDMWIWB Mathematical Formulations

Having described the 2TASEDMWB and 2TASEDMWIWB problems, now we formulate them as non-linear programming problems with linear restrictions. We start by defining the variables used in the models.

For each appliance  $a \in \mathcal{A}_1$ , we define the variables  $f_a^t \in \{0, 1\}$ ,  $\forall a \in \mathcal{A}_1, t \in \mathcal{T}$ , that take a value of 1 if and only if  $a$  operates in the interval  $t$ . We also introduce the variables  $e_a^{t+1} \in \{0, 1\}$ ,  $\forall a \in \mathcal{A}_1, t \in \mathcal{T}$ , that take a value of 1 if and only if  $a$  ends its operation definitively before the start of interval  $t + 1$ ; that is, it completes  $T_a$  intervals of operation on that instant and that does not happen earlier.

For the second type of appliances, the variables are the surpluses over the minimum required powers for each appliance  $a \in \mathcal{A}_2$  in each interval  $t \in \mathcal{T}$ , denoted by  $\tilde{x}_a^t$ .

In addition and only for the purpose of clarity, for each appliance  $a \in \mathcal{A}_1$  we shall define the variables  $t_a \in \mathbb{Z}_{\geq 0}$ , the ending time of operation, and the variables  $x_a^t$ , the powers of operation for each interval  $t \in \mathcal{T}$ , which will either be 0 or  $r_a$ . For each appliance  $a$ , the values of these variables can be determined by the values of  $e_a^{t+1}$  and by the fixed power of the appliance.

Next, we describe the restrictions to which the variables are subject.

Restrictions 1 and 2 establish the bounds for the ending times. The fact that each appliance of the first type operates the required amount of time is ensured by restriction 3. Restriction 4 states that there must be one and only one ending time for each appliance of the first type and restriction 5 establishes the relationship between the variables  $t_a$  and the variables  $e_a^{t+1}$ .

We reason that an appliance  $a \in \mathcal{A}_1$  ended its operation definitively for the start of the interval  $t$  if the last interval of activity was  $t - 1$  and it had completed its operation time:

$$\left( f_a^{t-1} = 1 \wedge \sum_{h=t}^{\beta_a-1} f_a^h = 0 \right) \iff e_a^t = 1$$

To formulate this logical condition with linear inequalities, remembering that restrictions 3 and 4 hold, we add inequalities 6 and 7.

If appliances in  $\mathcal{A}_1$  were not interruptible, it would follow that if  $a \in \mathcal{A}_1$  ended its operation at the start of interval  $t$ , it would be active in the  $T_a$  previous intervals (as it operates continuously). Therefore,  $e_a^t = 1 \iff f_a^{t-T_a} = \dots = f_a^{t-1} = 1$ , so restrictions 6 and 7 would be replaced by the following restrictions:

$$T_a e_a^t - \sum_{j=1}^{T_a} f_a^{t-j} \leq 0, \quad a \in \mathcal{A}_1, t \in \{\alpha_a + T_a, \dots, \beta_a\}$$

Restriction 8 expresses that appliances of  $\mathcal{A}_1$  are not active in intervals excluded from their time windows of possible operation and the fact that appliances do not end their operation when it is not possible is imposed by restriction 9.

Restriction 10 guarantees that when an appliance of the first type operates, its power is  $r_a$ , otherwise its power is zero. The possible values of the powers of appliances of the second type are bounded by restriction 11.

In restriction 12, which imposes that the energy consumption bounds are not exceeded,  $\mathcal{A}_2^t$  designates the set of appliances of  $\mathcal{A}_2$  that operate in the interval  $t$ , formally defined as  $\{a \in \mathcal{A}_2 \mid \alpha_a \leq t < \beta_a\}$ .

$$\min \sum_{a \in \mathcal{A}_1} \left[ \alpha_1 \left( \sum_{t \in \mathcal{T}} p^t x_a^t \right) + \alpha_2 \rho_a \left( \left( 1 + \frac{t_a - (\alpha_a + T_a)}{iph} \right)^{k_a} - 1 \right) \right] + \sum_{a \in \mathcal{A}_2} \sum_{t=\alpha_a}^{\beta_a-1} (\alpha_1 p^t \tilde{x}_a^t + \alpha_2 \omega_a^t ((\tilde{r}_{a,t}^{min} + \tilde{x}_a^t) - \hat{x}_a^t)^2)$$

$$\text{s.t.} \quad -t_a \leq -(\alpha_a + T_a) \quad a \in \mathcal{A}_1 \quad (1)$$

$$t_a \leq \beta_a \quad a \in \mathcal{A}_1 \quad (2)$$

$$\sum_{t=\alpha_a}^{\beta_a-1} f_a^t = T_a \quad a \in \mathcal{A}_1 \quad (3)$$

$$\sum_{t=\alpha_a+T_a}^{\beta_a} e_a^t = 1 \quad a \in \mathcal{A}_1 \quad (4)$$

$$t_a - \sum_{t=\alpha_a+T_a}^{\beta_a} t e_a^t = 0 \quad a \in \mathcal{A}_1 \quad (5)$$

$$e_a^{t+1} - f_a^t \leq 0 \quad a \in \mathcal{A}_1, t \in \mathcal{T} \quad (6)$$

$$T_a e_a^{t+1} + \sum_{h=t+1}^{\beta_a-1} f_a^h \leq T_a \quad a \in \mathcal{A}_1, t \in \mathcal{T} \quad (7)$$

$$f_a^t = 0 \quad a \in \mathcal{A}_1, t \in \mathcal{T} - \{\alpha_a, \dots, \beta_a - 1\} \quad (8)$$

$$e_a^t = 0 \quad a \in \mathcal{A}_1, t \in [1, \alpha_a + T_a - 1] \cup [\beta_a + 1, T + 1] \quad (9)$$

$$x_a^t - f_a^t r_a = 0 \quad a \in \mathcal{A}_1, t \in \mathcal{T} \quad (10)$$

$$\tilde{x}_a^t \leq \hat{x}_a^t - \tilde{r}_{a,t}^{min} \quad a \in \mathcal{A}_2, t \in \{\alpha_a, \dots, \beta_a - 1\} \quad (11)$$

$$\sum_{a \in \mathcal{A}_1} x_a^t + \sum_{a \in \mathcal{A}_2^t} \tilde{x}_a^t \leq c^t - \sum_{a \in \mathcal{A}_2^t} \tilde{r}_{a,t}^{min} \quad t \in \mathcal{T} \quad (12)$$

$$\text{variables} \quad e_a^{t+1} \in \{0, 1\} \quad a \in \mathcal{A}_1, t \in \mathcal{T}$$

$$f_a^t \in \{0, 1\} \quad a \in \mathcal{A}_1, t \in \mathcal{T}$$

$$x_a^t \in \mathbb{R}_{\geq 0} \quad a \in \mathcal{A}_1, t \in \mathcal{T}$$

$$\tilde{x}_a^t \in \mathbb{R}_{\geq 0} \quad a \in \mathcal{A}_2, t \in \{\alpha_a, \dots, \beta_a - 1\}$$

$$t_a \in \mathbb{Z}_{\geq 0} \quad a \in \mathcal{A}_1$$

#### 4. 2TASEDMWB and 2TASEDMWIWB in NP-hard

The imposition of energy bounds for each interval modifies substantially the possible ways of solving the problems: the independence between appliances that was exploited when solving 2TASEDM and 2TASEDMWI transforms into a strong dependence given by restriction 12. Moreover, the associated decision problems belong to the complexity class NP-hard, as we prove next.

Given a set of  $n$  items with sizes  $a_1, \dots, a_n \in \mathbb{Z}_{>0}$  and bins with capacity  $C \in \mathbb{Z}$ , the bin packing problem consists of finding the minimum number of bins needed to assign all the items without exceeding the capacities. The decision problem for bin packing, which is NP-complete Garey and Johnson (1979), is: given the additional parameter  $k \in \mathbb{Z}$ , is there any solution that uses  $k$  bins at most?

Solving the minimization problem 2TASEDMWIWB in particular solves the feasibility problem with no appliances of  $\mathcal{A}_2$  which, as we argue next, can be obtained from the decision problem for bin packing through a polynomial-time reduction.

Given the question of the decision problem for bin packing, we set up an instance of the 2TASEDMWIWB problem with exactly  $k$  intervals, all of them with an energy consumption bound  $C$ , and  $n$  appliances with fixed powers  $a_1, \dots, a_n$  that may be active in any of the  $k$  intervals and must be active in exactly one. The instance is feasible if and only if each of the appliances can be scheduled to function in exactly one interval such that for each interval the energy consumption bound is not exceeded. This is true if and only if the answer to the decision problem for bin packing is affirmative.

In view of the fact that each appliance operates in exactly one interval, this proof covers both scenarios, with and without interruptions.

#### 5. Heuristics

Due to the apparent difficulty of 2TASEDMWB and 2TASEDMWIWB, exact algorithms are, as far as we currently know, inefficient because of properties inherent to these problems. Therefore, in this section we propose 4 families of heuristics. Except for particular cases, these are described for the scenario with interruptions as well as for that without interruptions. First we introduce them by presenting the central idea for each family.

The first one, the  $\mathcal{A}_1\mathcal{A}_2$  family, is based on solving the appliances of  $\mathcal{A}_1$  in some order and then solving the appliances of  $\mathcal{A}_2$  using quadratic programming. The second family,  $\mathcal{A}_2\mathcal{A}_1$ , consists on solving the appliances of  $\mathcal{A}_2$  and then those of  $\mathcal{A}_1$ .

There is a family of heuristics that solve the problem ignoring energy consumption bounds and then try to adjust the conditions of operation of the appliances so that the bounds are not exceeded. This family of heuristics is called rearrangers family.

Lastly, the mixed integer quadratic programming (MIQP) family is composed of one heuristic that changes the objective function of the model so that it becomes a MIQP problem, hoping that solving it will lead to a good solution for the original problem.

### 5.1. $\mathcal{A}_1\mathcal{A}_2$ Family

The foundation for this family is the desire to prioritize the decisions on the operation for the appliances of  $\mathcal{A}_1$  over the ones for the appliances of  $\mathcal{A}_2$ . This is due to the fact that, in certain scenarios, the time at which some devices are used is more important than the operation powers of some other appliances. For example, maybe the user does not feel the difference of one degree on the AC but it is very important to use the computer at a certain time.

Therefore, the general idea consists on solving the appliances of  $\mathcal{A}_1$  in some order and then efficiently determining the optimal powers for the appliances of  $\mathcal{A}_2$  with a solver, using the reduced quadratic programming model that results of having fixed the variables for the appliances of  $\mathcal{A}_1$ .

Each appliance of  $\mathcal{A}_1$  is solved optimally according to the powers that remain after solving the previous appliances of  $\mathcal{A}_1$ . To this end, we modify, from the algorithms presented in Taboh et al. (2020) that determine, for scenarios without energy consumption bounds, in which intervals an appliance should be active, the one that uses less memory.

The algorithm uses a dictionary of key-value pairs which contains energy prices of intervals as keys and sets of intervals with a same energy price as values, which is accompanied by a value called *economic cost* that is the sum of the energy prices of those intervals. The algorithm starts by finding the  $T_a$  earliest intervals in  $[\alpha_a, \beta_a)$  that have enough power for the appliance  $a$  to operate in them. At each step, it adds the next available interval to the dictionary after removing an interval that has the highest price among those in the dictionary. Each time a change in the dictionary is done, the global cost is calculated (efficiently) and, if a new best global cost is found, the optimal time of ending is registered. Finally, the procedure will be repeated until the optimal time of ending.

The algorithm for interruptible appliances can be implemented with a time complexity of

$$\mathcal{O}\left(\beta_a + T_a + (\beta_a - \alpha_a) * \log(T_a)\right)$$

The different orderings we consider for the appliances of  $\mathcal{A}_1$  are:

- random,
- decreasing energy consumption,
- decreasing discomfort of  $k$  hours of delay,
- decreasing operation duration.

The quadratic programming problem for  $\mathcal{A}_2$  may be solved by finding the solutions to different sub-problems that arise of considering each interval separately, given that they are independent. Moreover, these problems for each interval  $t$  and the one that considers them all together are all quadratic programming problems with convex objective functions, so they can be solved in polynomial time Vavasis (2001).

The algorithm for the random ordering option can be implemented with a run-time of the order of

$$\sum_{a \in \mathcal{A}_2} (\beta_a - \alpha_a) + |\mathcal{A}_1| + \sum_{a \in \mathcal{A}_1} (\beta_a - \alpha_a) * \log(T_a) + \text{cost}(\text{solving QP})$$

For the other alternatives the run-time is of the order of

$$\sum_{a \in \mathcal{A}_2} (\beta_a - \alpha_a) + |\mathcal{A}_1| * \log(|\mathcal{A}_1|) + \sum_{a \in \mathcal{A}_1} (\beta_a - \alpha_a) * \log(T_a) + \text{cost}(\text{solving QP})$$

In all cases, it is a polynomial time complexity as the cost of solving the quadratic programming problems is polynomial.

However, there is one consideration that needs to be made. Even though it would be reasonable to think that a random ordering is disadvantageous because it is not justified by any criteria, its nature allows the algorithm to yield very different results for different executions. Therefore, using this algorithm would be reasonable if many orderings were considered, which would imply that the run-time exposed should be multiplied by the number of orderings tried.

In this paper we chose to set the number of random orderings to 1000, and the option that ordered appliances by decreasing discomfort did so according to the discomfort of 2 hours of delay.

We will refer to the heuristics of this family, depending on the ordering criteria for the appliances of  $\mathcal{A}_1$ , as *A1-Rand+A2* (random), *A1-Pow+A2* (power), *A1-Disc+A2* (discomfort) and *A1-Dur+A2* (duration).

## 5.2. $\mathcal{A}_2\mathcal{A}_1$ Family

Contrary to what happens in the family  $\mathcal{A}_1\mathcal{A}_2$ , in this one the operation conditions of the appliances of  $\mathcal{A}_2$  are favoured at the expense of those of the appliances of  $\mathcal{A}_1$ . This decision is based on the idea that, in certain situations, the way in which some appliances operate is more important than the time in which some others are active. For example, if a treadmill moves slowly it may not be useful to achieve the desired training. Therefore, the appliances of  $\mathcal{A}_2$  will be solved before those of  $\mathcal{A}_1$ . To decide on the operation of the devices with flexible power, we present 2 options.

The first one consists on optimally solving the appliances active in each interval in an order that is decreasing in the cost that they would have using half of the normal power consumption:

$$\alpha_1 p^t \frac{\hat{x}_a^t}{2} + \alpha_2 \omega_a^t \frac{\hat{x}_a^t}{4}$$

That is, for each interval, the power of each appliance is determined by minimizing its cost function according to the available power for that interval.

The second one finds a solution to the mixed integer quadratic programming problem that arises of disregarding the term of the objective function that corresponds to the appliances of  $\mathcal{A}_1$ . In this way, given that the restrictions related to those devices are considered, it is guaranteed that the decisions made for the appliances of  $\mathcal{A}_2$  are optimal among those that allow a solution for the appliances of  $\mathcal{A}_1$ , contrary to what happens with the previous option.

Afterwards, the appliances of  $\mathcal{A}_1$  are solved with the available powers that emerge from the decisions made for the appliances of  $\mathcal{A}_2$ . Again, we consider 2 options.

The first one is to randomly sort the appliances and then solve them, doing this 3000 times given that the time complexity is polynomial.

Another possible choice is to solve the integer linear programming problem that corresponds to  $\mathcal{A}_1$  when the objective function is linearized in the following way

$$\alpha_1 \sum_{t \in \mathcal{T}} \sum_{a \in \mathcal{A}_1} p^t x_a^t + \alpha_2 \sum_{a \in \mathcal{A}_1} \rho_a 10 k_a \frac{t_a}{iph}$$

Then, the real costs are compared for a certain amount of solutions in a certain neighbourhood of an optimal one.

Of the 4 combinations, the only one that we are certain that presents a polynomial time complexity with the algorithms known nowadays is the one that uses the first options for both types of appliances. Because of this, for both the MIQP problem and the ILP problem, we established an execution time bound of 10 minutes.

We named the heuristics of this family according to the solution methods for each class of appliances. For  $\mathcal{A}_2$  we have *A2-C* (cost sorting) and *A2-QP* (MIQP problem). For  $\mathcal{A}_1$  we have *A1-Rand* (random orders) and *A1-MIP* (MIP problem). The combinations are *A2-C+A1-Rand*, *A2-C+A1-MIP*, *A2-QP+A1-Rand* and *A2-QP+A1-MIP*.

### 5.3. *Rearrangers Family*

An idea that leads to another family of heuristics is that good results could be achieved by optimally solving the problem for the scenario without energy consumption bounds and then transforming the solution into a feasible one.

These algorithms make changes on those intervals for which the energy consumption bounds are exceeded. To enforce feasibility on an interval, we considered two ways that only work for the scenario with interruptions.

One way, that of the *Feasibility Rearranger* heuristic, focuses on achieving feasibility. To that end, the consumptions for the appliances of  $\mathcal{A}_2$  are gradually reduced until the energy consumption bound is not exceeded or all powers reach their required minimums. Then, if the bound is still exceeded, appliances of  $\mathcal{A}_1$  are transferred from that interval to others until the bound holds.

The other way is that of the *Balanced Rearranger* heuristic, which prioritizes balance. It does not start reducing the powers of the appliances of  $\mathcal{A}_2$  as it does not distinguish both groups at the moment of deciding with appliances to modify. Once the appliance has been selected, if it has flexible power then the power is reduced, and if it is fixed, the appliance is transferred, as in the previous way.

In both heuristics, the order in which the appliances are selected considers which are the ones with higher *excess* of their own. This is, how far their powers are from the minimum powers they could have in that interval. The only exception to this occurs in the *Feasibility Rearranger* when all appliances with flexible powers have already been set to their minimums. In that case, an appliance of  $\mathcal{A}_1$  whose power is the most similar to the consumption excess of the interval among those higher than the excess will

be removed if possible. If no appliance had a power greater than the excess, the power of the appliance removed will also be the most similar to the excess.

It is clear that even though the first way more easily leads to a feasible solution, it also presents an unbalance between both groups of appliances, being worse for those of  $\mathcal{A}_2$ .

### 5.3.1. Implementation

For both heuristics, the first step is to solve the appliances as if there were no energy consumption bounds with the algorithms presented in Taboh et al. (2020). Then, the intervals for which the bounds are exceeded are registered in an ordered set.

There is also an ordered associative container that, for each interval (not necessarily with an excess), has another ordered associative container with the different values of the excesses of the appliances active in that interval. Recall that the *excess* of an appliance  $a$  in an interval  $t$  is the difference between its power in  $t$  and the minimum power it could use in  $t$ . For the appliances of  $\mathcal{A}_1$  this value is  $r_a$  and for those of  $\mathcal{A}_2$  it is  $\tilde{x}_a^t$ . For each excess value in the ordered associative container of the interval  $t$  there is a set with the appliances that share that excess in said interval.

In both heuristics, the powers of the appliances of  $\mathcal{A}_2$  are gradually reduced by 7% of the excess at the moment of the reduction and updating the excess. Given that the sequence of excesses defined by  $e_n = 0.93 e_{n-1}$ , being  $e_1$  the original excess, has only positive terms if  $e_1 > 0$ , a solution would not be achieved if it were necessary to operate an appliance at its minimum required power as its excess would never be 0. Due to this, once the reduction reaches 90% of the original excess, the reduction will not be gradual but absolute. Moreover, the fact that  $\lim_{n \rightarrow \infty} e_n = 0$  guarantees that this will eventually occur.

The intervals are *fixed* in decreasing order, so the latest intervals are treated first. Because of this, when searching another interval for an appliance of  $\mathcal{A}_1$  instead of the interval  $t$ , an interval  $t' > t$  will be an option if and only if the power available in  $t'$  is greater than or equal to the power of the appliance. This will not be required for intervals  $t' < t$  given that they could be fixed in the future.

As explained before, the *Feasibility Rearranger* only modifies the operation of the appliances of  $\mathcal{A}_1$  if it did not reach a feasible solution by changing only the operation of the appliances of  $\mathcal{A}_2$ . Its decision on which appliance of  $\mathcal{A}_1$  to change is based on which presents a consumption most similar to the excess of the interval among those whose power is not lesser than the excess. If all appliances have lesser powers, the one with the closest energy consumption is selected.

On the other hand, the *Balanced Rearranger* does not distinguish between both groups of appliances when deciding which appliance to change and always selects the one with the largest excess.

The only missing explanation is how the intervals of operation of the appliances of  $\mathcal{A}_1$  are changed. Briefly explained, if the goal is to transfer an appliance  $a$  from an exceeded interval  $t$  to another, then the nearest interval to  $t$  in which  $a$  is not already active is selected. Considering that intervals are explored moving away from  $t$  in both directions, the interval which produces the cheapest change among those closest to  $t$  is selected. To avoid cycling in the process of *fixing* intervals, for intervals later than  $t$  we also request that there is enough available power for  $a$ .

#### 5.4. MIQP Family

Given that CPLEX can be used to solve the problem for values of  $k_a = 1, 2$ , a possible heuristic is:

- for each combination of values of  $k_a$  for the different appliances where each one was transformed into 1 or 2 according to a certain criterion
  - solve the problem using CPLEX and gather all solutions in a neighbourhood of the optimal one,
  - compare the values of the objective function for those solutions considering the original values of the  $k_a$ s,
  - if needed, update the optimal solution found up to the moment as a result of considering all combinations.

An upper bound for the cost of this algorithm is

$$\sum_{i=1}^{2^{|\mathcal{A}_1|}} (\text{cost}(\text{solve MIQP}) + \text{cost}(\text{analyze solutions for this combination}) + \text{cost}(\text{update optimum}))$$

In the worst case scenario, this idea leads to an exponential amount of combinations of values of  $k_a$  and, therefore, to a great computational cost. One possible reduction emerges from transforming  $k_a$  only to 2 when  $k_a > 2$ . Also, for the cases in which  $k_a$  is close to the number that would result of rounding it, one could only consider combinations with that value. Even though there could be a parameter for the range near 1.5 within which both possibilities 1 and 2 were considered, this could again lead to an exponential amount of combinations.

In consequence, we decided to replace the discomfort for the appliances of  $\mathcal{A}_1$  in the objective function with

$$\rho_a 10 k_a \left[ \left( 1 + \frac{t_a - (\alpha_a + T_a)}{iph} \right)^{\tilde{k}_a} - 1 \right]$$

selecting values of  $c \in \mathbb{R}$ ,  $1 \leq c \leq 2$  and defining  $\tilde{k}_a = 1$  if  $k_a \leq c$  and  $\tilde{k}_a = 2$  otherwise.

The algorithm, called *MIQP*, performs this procedure for 3 values of  $c$ :  $c = 1.25$ ,  $c = 1.50$  and  $c = 1.75$ .

We granted 10 minutes for each value of  $c$ , resulting in an upper bound of the execution time close to 30 minutes.

## 6. Computational Results

In Taboh et al. (2020) we detailed a diverse instance set we generated to achieve a realistic analysis. There were 3 types of houses: the diurnal house, which belonged to people that left the house to work during the day and where appliances operated mostly in the evening; the nocturnal house, which was habited by people that left the house to work during the night and where appliances operated mostly

during the morning and afternoon; and the full day house, which was occupied all day by people that did not work outside their houses and where appliances operated throughout the whole day. Each of these houses had 13 schedulable appliances, 8 that were time-shiftable and 5 that were power-shiftable. The appliances of the first type were a robot vacuum, a robotic pool cleaner, a computer, a television, a washing machine, an iron, an electric stove and a water heater. The appliances with a flexible power were lights, an air conditioner, a dishwasher, a treadmill and an hydromassage table. The houses were replicated with random slight changes to the parameters of their appliances to achieve neighbourhoods of variable sizes, which were composed by 70% of diurnal houses, 20% of nocturnal houses and 10% of full day houses.

In this section we compare the results obtained for the different heuristics for the scenario with energy consumption bounds, allowing and not allowing interruptions. We also make a comparison with the results achieved for the scenario without energy consumption bounds to understand the impact they have. To this purpose, we extend the instances belonging to houses presented in Taboh et al. (2020) with energy consumption bounds for each interval. According to the industrial activity hours and the main energy consumption habits of the society from Buenos Aires, Argentina, the bounds are set to 2.1 kW between 12 a.m. and 8 a.m., to 1.5 kW between 8 a.m. and 4 p.m. and to 1.8 kW between 4 p.m. and 12 a.m. of the following day. For the neighbourhoods, the bound for each interval is multiplied by the number of houses.

As in Ma et al. (2016), three operation modes according to different preferences are considered. The first one disregards discomfort and sets  $\alpha_1 = 1$  and  $\alpha_2 = 0$ , so we call it “Economic” (E) mode. The “Comfort” (C) mode answers to a concern only based on discomfort and sets  $\alpha_1 = 0$  and  $\alpha_2 = 1$ . Finally, there is a third operation mode which is called “Balance” (B) and cares equally about the economic expense and the discomfort, setting  $\alpha_1 = \alpha_2 = \frac{1}{2}$ .

To evaluate the heuristics we executed the algorithms for 5 randomly generated instances for neighbourhoods with each of the following amounts of houses: 100, 250, 380, 500, 630, 750, 880 and 1000. In Taboh et al. (2020), many metrics were studied: the value of the objective function, the electricity bill, the total discomfort, the peak energy demand in one interval, the total energy consumption, the peak-to-average ratio and the execution time. In this paper we also study the discomfort for each group of appliances and the number of instances solved for each algorithm and for each instance size. This last metric is essential due to the fact that the instances of a certain size are randomly generated and each value of the metrics for a certain size is the mean of the results obtained for the instances of that size that were solved. Therefore, no conclusions could be drawn from the metrics for the different algorithms as the differences could be the result of which specific instances had been solved. In order to guarantee that differences do not arise from the instances solved, for each heuristic we only consider the values of the metrics for the sizes for which all instances are solved.

For all the experiments presented in this paper we used a computer with *Intel Core i7 7700*, 3.60GHz and 16 GB RAM and all mathematical models were solved using CPLEX 12.7.1.

### *6.1. Number of Instances for Which Solutions Were Found*

The results for this metric are of great importance, reason for which it is the first one to be analyzed. Not only is it important in itself to know how many instances can be solved by an algorithm, but it is also a

mainstay of the analysis of the rest of the metrics as we explained before.

As we observed, the heuristics *AI-Rand+A2*, *A2-C+AI-Rand* and *A2-QP+AI-Rand* were not able to solve all instances for many sizes of various scenarios. This fact can be explained analyzing how restrictive it is to only consider solutions that result of optimizing appliances in a certain order and, even more, to try a fixed amount of orders.

Having  $n$  appliances of  $\mathcal{A}_1$ , the number of possible orders is  $n!$ . Therefore, as  $n$  grows and the amount of orders tried remains fixed, the ratio of orders explored to total becomes lower, making it more likely that no solutions are found. This theoretical fact is consistent with the results shown in figures 1 to 3, where  $n$  does not grow from 100 to 1000, but from 800 to 8000 as each house has 8 appliances of  $\mathcal{A}_1$ .

Moreover, the maximum numbers of possible solutions for the scenarios are

$$2TASEDWB: \prod_{a \in \mathcal{A}_1} ((\beta_a - T_a) - \alpha_a + 1) \qquad 2TASEDWIWB: \prod_{a \in \mathcal{A}_1} \binom{\beta_a - \alpha_a}{T_a}$$

Even though no comparison can be made between these numbers and  $n!$  as they depend on the values of  $\alpha_a$ ,  $\beta_a$  and  $T_a$  of each appliance, it can be said that these numbers consider all possible solutions and not just those in which appliances work optimally according to the decisions made up to that moment that they are solved.

Another conclusion that can be drawn from this analysis is that if it cannot be guaranteed that  $n!$  is lesser than or equal to the maximum number of possible solutions, it is possible that unnecessary computational work is being carried out. For example, if there were two appliances  $a_1$  and  $a_2$  with  $[\alpha_1, \beta_1] \cap [\alpha_2, \beta_2] = \emptyset$ , solving  $a_1$  and then  $a_2$  would lead to the same results as doing it in the opposite way.

The *MIQP* heuristic struggled to find solutions when the models had more variables, as can be seen for the scenario with interruptions in balance and comfort mode (figures 2 and 3) for bigger instances. In economic mode with interruptions it did find solutions for all instances of all sizes because the variables for the powers of the appliances of  $\mathcal{A}_2$  were, in some way, fixed values, as there would be no intelligent decision but to set them to their minimums (figure 1). This also leads to greater consumption bounds which allow more feasible configurations for the appliances of  $\mathcal{A}_1$ . However, in the scenario without interruptions for all three operation modes, this heuristic found solutions for all instances given that the variables  $f_a^t$  and  $e_a^t$  are strongly linked and the polyhedra are greatly reduced.

For the scenario without interruptions in comfort mode, the  $\mathcal{A}_2\mathcal{A}_1$  family presents poor results due to the fact that the appliances of  $\mathcal{A}_2$  consume a large fraction of the available power and, not being interruptible, the appliances of  $\mathcal{A}_1$  are left with few feasible configurations. Even though *A2-QP+AI-MIP* solved all instances of sizes 100 and 1000 for that scenario, it will not be considered because of its poor global performance.

Lastly, the heuristics *AI-Pow+A2* and *AI-Dur+A2* solved all instances in both scenarios in comfort mode but very few in the other operation modes. For *AI-Dur+A2*, we think this might be because it does not consider how many possibilities there are for an appliance of  $\mathcal{A}_1$ . That is, if one appliance operates one interval and its window of possible operation has only two intervals, it does not seem

smart to prioritize other appliances with large windows of possible operation and short operation times. The reasons for the unsatisfactory results of *A1-Pow+A2* are not clear to us, although it is clear that sorting the appliances based only on their powers of operation ignores data relative to their operational characteristics.

## 6.2. Objective Function Value

For the scenario without interruptions in economic mode, for sizes up to 380 houses no differences can be noticed between the results of the different algorithms, and also with the results for the scenario without energy consumption bounds. From 500 houses onward, the differences become clearer for the heuristics *A2-C+A1-MIP*, *A2-QP+A1-MIP* and *MIQP*, whose values are 7% higher. This can be explained considering that they all solve integer programming problems that involve more variables as the size of the instances increases. The heuristic *A1-Disc+A2*, instead, achieves results very close to those of the scenario without energy consumption bounds. This could suggest that the operation of the appliances of  $\mathcal{A}_2$  at their minimum powers left enough available power for the appliances of  $\mathcal{A}_1$ , virtually cancelling the effect of the bounds. However, that is not true. It was verified that the PAR is approximately 15% higher with unlimited energy, and this is a consequence of the fact that the peak demand is approximately 15% higher for that scenario given that the total energy consumption is, reasonably, the same. Then, the similarity between the values of the objective function would be related to the existence of various intervals with similar prices among those of the windows of possible operation of the appliances of  $\mathcal{A}_1$ .

For the instances with interruptions in economic mode, the *MIQP* heuristic finds solutions that present objective function values very similar to those of the same instances without energy consumption bounds, and those are the best ones found for all sizes, even though the differences with the other algorithms are really small. As before, even when the values of the objective function are similar for the instances with and without unlimited available energy, the solutions are very different, as we will see with the analysis of the other metrics.

In comfort mode with or without interruptions, not having consumption bounds leads to null values for the objective function because the economic objective is not considered and nothing prevents the appliances from operating in a way that there is no discomfort. Incorporating energy consumption bounds drastically increases the objective function value given that it imposes restrictions on the operation of the appliances.

With or without interruptions, the *MIQP* heuristic reaches the best results among all algorithms for those sizes for which it solves all instances. Moreover, when there are no interruptions, this occurs for all sizes. This is not the case when having interruptions, as from 750 houses onward the heuristics of the  $\mathcal{A}_1\mathcal{A}_2$  family are the best choices, presenting a large difference with those of the Rearrangers family and those of the  $\mathcal{A}_2\mathcal{A}_1$  family (figure 7).

In balance mode without interruptions, once more the heuristics *A1-Rand+A2* and *A1-Disc+A2* from the  $\mathcal{A}_1\mathcal{A}_2$  family provide results similar to those for the scenario without energy bounds. As in the economic mode without interruptions, the algorithms *A2-C+A1-MIP*, *A2-QP+A1-MIP* and *MIQP* obtained results slightly worse than the best found. It is notable that for neighbourhoods of 880 and 1000 houses, the *MIQP* heuristic finds much worse solutions than those for smaller sizes and, as it happened in economic mode, it becomes less convenient that those of the  $\mathcal{A}_2\mathcal{A}_1$  family.

Allowing interruptions does not change much the overall picture. The rearranger heuristics tie with those of the  $\mathcal{A}_1\mathcal{A}_2$  family and *MIQP* does not reach solutions for all instances from 630 houses onward.

### 6.3. Execution Time

The results regarding the execution times were, generally speaking, those expected, as shown partially in figures 4 to 6.

The algorithms for the scenario without bounds and those from the  $\mathcal{A}_1\mathcal{A}_2$  family that tried a unique order presented extremely low execution times. This shows that the procedure performed by *AI-Disc+A2* could be carried out sorting according to the discomfort with different values for the hours of delay. *AI-Rand+A2* took more time because of the number of random orders it tried, leading to a slow growth that seems linear in the number of houses.

In the same way, the execution times for the rearranger heuristics are not surprising, as the phase prior to the rearrangement consists in using the algorithms for the scenario without bounds, and the remaining part is not computationally complex.

In contrast, *A2-C+AI-MIP*, *A2-QP+AI-MIP* and *MIQP* had much higher execution times that answer to the fact that they solve integer programming problems. The execution times close to 10 minutes or to 30 minutes are a consequence of the time limit imposed. For *MIQP* in economic mode, the execution times were lower than 30 minutes for almost all instances due to the ease of finding the powers for the appliances of  $\mathcal{A}_2$  in said operation mode.

For the scenario with interruptions in economic mode (figure 4), the algorithms from the  $\mathcal{A}_2\mathcal{A}_1$  family that solved the appliances of  $\mathcal{A}_1$  trying random orders took more time than those that did so solving an ILP problem. We think this might be a consequence of the large number of orders considered. However, for the economic mode without interruptions, this was not the case. The execution times were extremely low, and this could be explained by how restrictive it results to disallow interruptions for the appliances of  $\mathcal{A}_1$ . Once again, the dependence with the number of houses was linear, with and without interruptions.

The result we found most shocking was the variation in the execution times for the *MIQP* heuristics for the scenario without interruptions in comfort mode. There was a sustained decline from 250 houses to 630 houses and then a sustained growth up to 1000 houses. We can think of no other explanation but the randomness of the instances combined with the operation mode and the heuristic used.

### 6.4. Electricity Bill

Part of the analysis of the electricity bill for the economic mode is the same as the one made for the value of the objective function, given that in said mode both metrics match. We notice that, in this mode, the incorporation of interruptions reduces the electricity bill as it occurs without energy consumption bounds. It also leads to similar results for the different algorithms.

For the comfort mode, it can be observed that the values of the electricity bill for the heuristics used to solve the instances with consumption bounds are lower than those for instances without bounds. We

think this difference arises from the limitation to the available power for the appliances of  $\mathcal{A}_2$  given that, when they are prioritized, as they are in the algorithms of the  $\mathcal{A}_2\mathcal{A}_1$  family, the cost is much more similar to that of the scenario without bounds than when they are not, as it occurs for the  $\mathcal{A}_1\mathcal{A}_2$  family.

In balance mode, the best results were obtained by *A1-Rand+A2* and *A1-Disc+A2*, with values 8% lower than those of the worst heuristics. Even though in the scenario with interruptions the results for all algorithms except for the *MIQP* heuristic were practically the same, the difference is notorious when there are no interruptions, scenario in which the heuristics *A2-C+A1-MIP*, *A2-QP+A1-MIP* and *MIQP* obtain much worse electricity bills. On another note, the variation with the incorporation of interruptions when having energy consumption bounds is almost null, as it happens without said bounds.

Regarding the comparison between operation modes, even when in the comfort mode there were solutions with cheaper electricity bills than others, even for those the difference with the balance mode was significant, being approximately 30% more expensive for neighbourhoods of 1000 houses. Likewise, the electricity bill for the balance mode was roughly 35% more expensive than that for the economic mode, as it happened for the scenarios without energy consumption bounds.

### 6.5. Discomfort

When analyzing the results for the economic mode we observe that, for all instances, the solutions found by the different algorithms incur in the same discomfort for the appliances of  $\mathcal{A}_2$ . The reason for this is clear: in the economic mode there is no benefit from using more power for the appliances of  $\mathcal{A}_2$  than the strictly required, so all algorithms set them to their minimum powers. Therefore, the differences in the discomfort for the appliances of  $\mathcal{A}_1$  translate directly to differences in the total discomfort. Even though these are small for the case with interruptions, that is not the case for bigger sizes without interruptions, scenario in which the discomfort for the worse heuristics is approximately 66% higher than that of the best, which is *A1-Disc+A2*. This last fact is not surprising considering that heuristic prioritizes the appliances of  $\mathcal{A}_1$  and solves them according to their discomfort. What is surprising is that those differences dissipate when allowing interruptions, although this can be explained taking into account that interruptions mitigate the effect the energy consumption bounds have on the operation of the appliances of  $\mathcal{A}_1$ : if an interval does not have enough available power for an appliance, interruptions make it possible to avoid it without changing the operation on the previous intervals. It is also worth mentioning that the increase in the minimum discomfort obtained for the appliances of  $\mathcal{A}_1$  when adding interruptions to the scenario with energy bounds is the same as that observed when there are no bounds.

For neighbourhoods up to 750 houses without interruptions in balance mode, the *MIQP* heuristic achieved the best results for the total discomfort and the discomfort of the appliances of  $\mathcal{A}_1$ ; but there is an increase larger than 400% for the neighbourhoods of 880 and 1000 houses, leaving it as the worst of all algorithms. If the discomfort from the appliances of  $\mathcal{A}_2$  is prioritized, the best heuristics are, reasonably, *A2-C+A1-MIP* and *A2-QP+A1-MIP*, from the  $\mathcal{A}_2\mathcal{A}_1$  family, obtaining results equal to those for the instances without energy consumption bounds.

For the instances with interruptions in balance mode, *A2-C+A1-MIP* and the *Balanced Rearranger* got the best values of global discomfort, approximately 10% lower than the worst obtained by the other heuristics. Moreover, the *Balanced Rearranger* lead to less discomfort for the appliances of  $\mathcal{A}_2$ , which

is the same for instances without energy consumption bounds.

If only the discomfort for the appliances of  $\mathcal{A}_1$  is analyzed, the *MIQP* heuristic offers the lowest discomfort for neighbourhoods up to 250 houses; for sizes from 380 up to 750 houses the best are *A2-C+AI-MIP* and *A2-QP+AI-MIP*; and for larger sizes the best choices are the *Feasibility Rearranger* and *AI-Disc+A2*, matching the results for the scenario without energy consumption bounds. It is not surprising that the heuristics from the  $\mathcal{A}_2\mathcal{A}_1$  family turned out to be better than *AI-Disc+A2* for the discomfort of the appliances of  $\mathcal{A}_1$  if one considers that, in balance mode, decisions also take into account the economic objective. It could happen that the heuristics of the  $\mathcal{A}_2\mathcal{A}_1$  family had higher energy consumptions in certain intervals, leading the appliances of  $\mathcal{A}_1$  to operate earlier than they would if the optimal decisions were made for them as it occurs in *AI-Disc+A2*.

The global discomfort for the comfort mode was already analyzed when studying the objective function, given that both metrics match in this mode. Briefly, we shall remember that the best heuristics are *MIQP* and, slightly worse, those of the  $\mathcal{A}_1\mathcal{A}_2$  family. We are missing the analysis of the composition of the total discomfort, that is, how much came from each group of appliances.

For the scenario without interruptions, the discomfort for the appliances of  $\mathcal{A}_1$  was null for all algorithms except for the *MIQP* heuristic, opposed to what happened for the scenario with interruptions, where it was the one that offered the best results among those that solved the instances with energy consumption bounds (figure 8). It is curious how the discomfort for the appliances of  $\mathcal{A}_1$  increased dramatically when instead of considering 880 houses, 1000 were considered.

When analyzing the scenario with interruptions, it can be seen in figures 8 and 9 that the best results for each group of appliances were obtained by the heuristics that prioritized each group respectively. That is, the  $\mathcal{A}_1\mathcal{A}_2$  family reduces the discomfort for the appliances of  $\mathcal{A}_1$  at the expense of the discomfort for the appliances of  $\mathcal{A}_2$  and the  $\mathcal{A}_2\mathcal{A}_1$  family benefits the appliances of  $\mathcal{A}_2$  at the expense of those of  $\mathcal{A}_1$ . For neighbourhoods with sizes up to 630 houses, the *MIQP* heuristic presents balanced results between both classes with the disadvantage of a slight increase in the global discomfort with respect to that found by other algorithms.

## 6.6. Peak Demand

The results related to the peak demand are, in some way, the same for all scenarios: the imposition of energy consumption bounds reduces significantly the peak demand for all instance sizes, proving it effective to avoid oversizing the electricity grid. We also noticed that the results for the different heuristics were very similar. Only in the economic mode there were slightly larger variations, being the heuristics of the  $\mathcal{A}_2\mathcal{A}_1$  family the advantageous ones.

In economic and balance mode, the decrease for the neighbourhoods of 1000 houses occurred from values close to 2000 kW for the scenario without energy consumption bounds to values close to 1750 kW, 12.5% lower. For instances of the same size in comfort mode, the decrease was roughly 30%, going from approximately 2600 kW to near 1800 kW.

If we recall the values of the energy consumption bounds for the three time periods of the day, it seems that the peak demands for the instances with energy consumption bounds are determined by the bound from 4 p.m. to 12 a.m. This is reasonable considering how the neighbourhoods are composed and the hours of higher activity of the appliances for the different house profiles. Specifically, the diurnal houses

are the most abundant ones, and in them the appliances operate predominantly when users leave their jobs, which happens in the evening, reaching the bound.

### 6.7. Total Energy Consumption

The energy consumption in economic mode is the same allowing or not allowing interruptions and having or not having energy consumption bounds, due to the fact that the powers for the appliances of  $\mathcal{A}_1$  are fixed and those for the appliances of  $\mathcal{A}_2$  are their required minimums. Likewise, it is the same for all heuristics.

In balance mode there are no significant differences, neither between the four scenarios nor between the heuristics, and the explanation is not as straightforward as it was for the economic mode. We can say that this is a consequence of the inexistence of significant variations for the powers of the appliances of  $\mathcal{A}_2$  as for those of  $\mathcal{A}_1$  the consumption is always the same. Therefore, the equilibrium may arise from the fact that, depending on which intervals are selected for the operation of the appliances of  $\mathcal{A}_1$ , the appliances of  $\mathcal{A}_2$  consume more energy in some intervals or do so in others.

For the comfort mode, having energy consumption bounds leads to a reduction for almost all heuristics, which does not happen for the other operation modes. The difference between the consumption for the instances without bounds and the minimum consumption achieved by the heuristics constitutes a reduction close to 7%.

All heuristics get the same results when there are no interruptions, but this changes when they are allowed, scenario where the  $\mathcal{A}_2\mathcal{A}_1$  family has the highest consumptions, which are the same as those for instances without bounds, followed by the *Balanced Rearranger* with consumptions 3% lower and finally by the *Feasibility Rearranger* and the heuristics of the  $\mathcal{A}_1\mathcal{A}_2$  family. These differences are linear on the number of homes.

As was expected, the difference in the energy consumption for the various operation modes is still significant even when the energy consumption bounds were useful to lessen the power consumption in the comfort mode. For larger instances, the values were close to  $0.95 \cdot 10^4$  kWh for the economic mode,  $1.1 \cdot 10^4$  kWh for the balance mode and  $1.3 \cdot 10^4$  kWh for the comfort mode.

### 6.8. Peak-to-Average Ratio (PAR)

Based on the results for the peak energy demand and for the total energy consumption, one obtains those for the peak-to-average ratio (PAR), which suggest how reasonable it is to enlarge the electricity grid so that it can withstand higher energy demands.

The general picture, partially shown in figures 10 to 12, is that the energy consumption bounds decrease drastically the value for this metric in all scenarios. Moreover, for instances with bounds, the *A2-QP+A1-MIP* heuristic provides the best results except in some cases, followed by *A2-C+A1-MIP*. In comfort mode without interruptions, those heuristics did not solve all instances and the results are virtually the same for those that were considered. The differences that can be observed between the algorithms for various scenarios arise from the variations already explained for the two metrics that compose the

PAR, considering that differences that seem negligible for total energy consumption translate to notable differences in PAR.

The economic mode presents a slightly lower energy consumption than the other operation modes and the worst results for this metric. Even though the comfort mode leads to higher peak demands, it also generates much higher levels of energy consumption, so it turns out to have the best values of PAR.

### 6.9. Integrative Situational Analysis

With the goal of having a more complete picture regarding the behaviour of the different heuristics, we summarize the results for the distinct scenarios, mentioning which heuristics would be best according to priorities on the metrics.

In economic mode without interruptions, *A1-Disc+A2* is advantageous for many metrics: among those that solve all instances, it presents almost null execution times, the cheapest prices for the electricity bill and the lowest discomfort for both groups of appliances (and, therefore, global). Its only drawback is that its peak demand is slightly higher than those of other heuristics and this translates to a difference in the PAR, but these do not seem to be valid reasons for not selecting it.

For the scenario with interruptions in economic mode, the heuristics that solve all instances can be grouped into two sets according to their results. Both present almost indistinguishable electricity bills and the same levels of energy consumption, but the values for the other metrics are not the same.

On one hand, we have the set of *A1-Disc+A2* and the rearranger heuristics, which exhibit negligible execution times and the best values of discomfort for both classes of appliances (and, therefore, global). On the other hand, there is the group of *A2-C+A1-MIP*, *A2-QP+A1-MIP* and the *MIQP* heuristic, that present much higher execution times and have slight worse levels of discomfort for the appliances of  $\mathcal{A}_1$ , but provide lower peak demands and, in consequence, the lowest values of PAR.

For the scenario without interruptions in balance mode, it seems wise to choose once more the *A1-Disc+A2* heuristic. In spite of the fact that its discomfort levels for the appliances of  $\mathcal{A}_1$  are slightly worse than those of the *MIQP* heuristic (except for larger instances) and this leads to the same comparison in global discomfort, and also that it present slightly higher peak demands for larger instances, it is beneficial regarding the other metrics and it appears to be more stable, not involving discomfort peaks as occurred with the *MIQP* heuristic.

Regarding the instances with interruptions in balance mode, the best choice appears to be the *Balanced Rearranger*. Its execution times are negligible, its values for electricity bill are roughly the same as those of the other heuristics and it sustainedly presents values of global discomfort that are close to the minimum values obtained. Even though *A2-C+A1-MIP* and *A2-QP+A1-MIP* provide lower discomfort for the appliances of  $\mathcal{A}_1$  for most instance sizes, this is not the case for larger instances and the differences are significant. Moreover, *Balanced Rearranger* offers the lowest levels of discomfort for the group  $\mathcal{A}_2$ . The peak demand, as well as the total energy consumption, are very similar for all heuristics.

If the scenario without interruptions in comfort mode is analyzed, the *MIQP* heuristic leads to a notable lower total discomfort with respect to the  $\mathcal{A}_1\mathcal{A}_2$  family. Although its values for the discomfort of the appliances of  $\mathcal{A}_1$  are reasonable worse, its advantage for the group  $\mathcal{A}_2$  is drastically more significant.

The electricity bill, as well as the peak demand, the total energy consumption and, therefore, the PAR, are equal for the different options. Apart from its higher execution time, it has no other disadvantages.

Lastly, for the scenario with interruptions in comfort mode, the  $\mathcal{A}_1\mathcal{A}_2$  family provides the best results globally speaking. Its heuristics solve all instances even when they are large and, except for *A1-Rand+A2*, they do so with low execution times. In addition, its electricity bills are the cheapest ones achieved by all heuristics and its global discomfort is close to the lowest one obtained considering the other algorithms. Even when *A2-C+A1-MIP* and *A2-QP+A1-MIP* present values for the discomfort of the appliances of  $\mathcal{A}_2$  that are beneath those of the heuristics of the  $\mathcal{A}_1\mathcal{A}_2$  family, the discomfort for the appliances of  $\mathcal{A}_1$  for those alternatives is dramatically higher. Furthermore, the peak demand is the same for the different algorithms but the heuristics of the mentioned family result considerably better for the environment than the rest because of their reduced levels of energy consumption. That is the reason for its worse PAR results.

## 7. Conclusions

In this paper we extended two residential appliance scheduling problems with energy consumption bounds as it would be expected that the energy supplying companies imposed these bounds to users to avoid oversizing the electricity grids. We presented mathematical formulations for these new problems and then we proved that the incorporation of energy consumption bounds transforms these problems into ones of the complexity class NP-hard. For that reason, we designed several heuristics and evaluated them on a diverse set of instances, achieving good results regarding instance solvability (number of instances solved and execution time) and solution quality (electricity bill, discomfort, among others). To better assess the quality of the solutions of the heuristics, we intend to focus our future research on the design of exact algorithms that solve these problems.

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## 8. Appendix

### 8.1. Number of Instances for Which Solutions Were Found

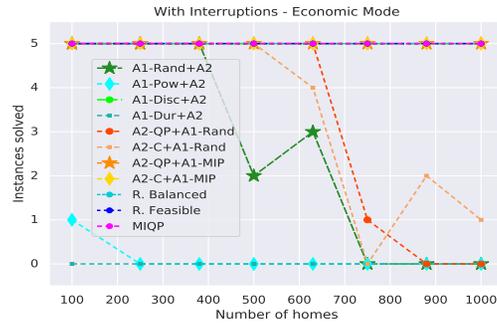


Fig. 1: Number of instances with found solutions: With Interruptions - Economic mode.

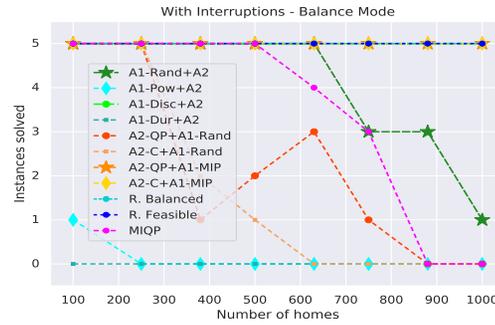


Fig. 2: Number of instances with found solutions: With Interruptions - Balance mode.

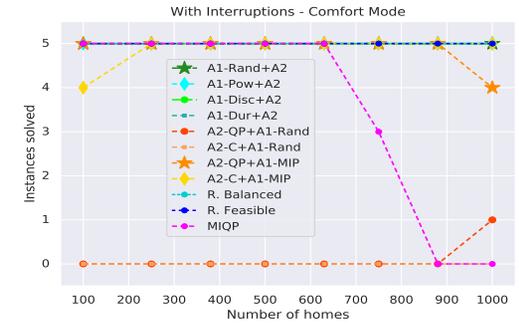


Fig. 3: Number of instances with found solutions: With Interruptions - Comfort mode.

### 8.2. Execution Time

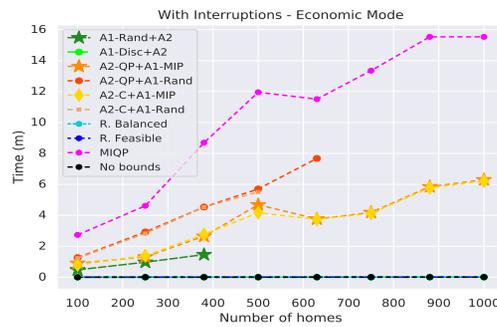


Fig. 4: Execution time: With Interruptions - Economic mode.

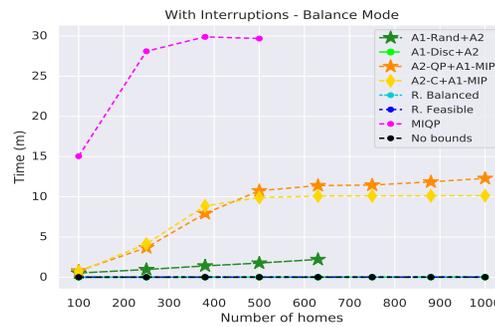


Fig. 5: Execution time: With Interruptions - Balance mode.

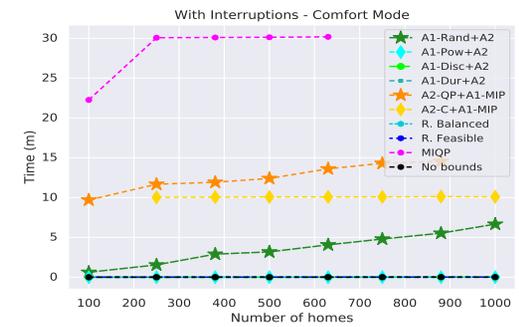


Fig. 6: Execution time: With Interruptions - Comfort mode.

### 8.3. Discomfort

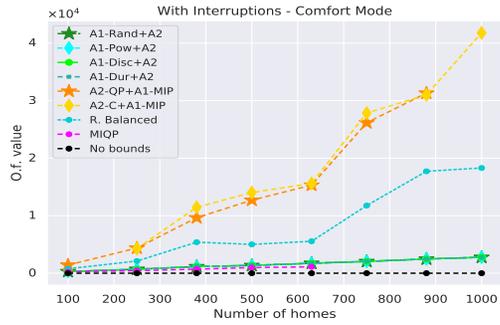


Fig. 7: Total discomfort:  
With Interruptions - Comfort mode.  
R. Feasible is not shown due to its poor results.

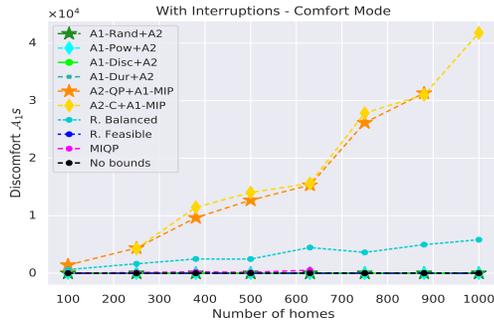


Fig. 8: Discomfort for  $\mathcal{A}_1$ :  
With Interruptions - Comfort mode.

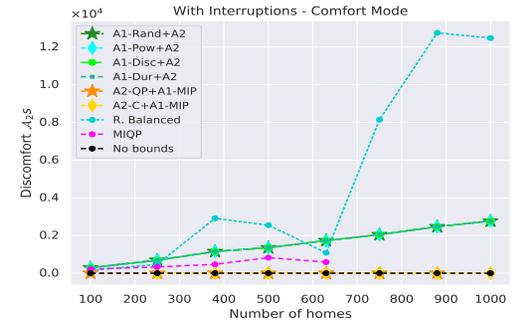


Fig. 9: Discomfort for  $\mathcal{A}_2$ :  
With Interruptions - Comfort mode.  
R. Feasible is not shown due to its poor results.

### 8.4. Peak-to-Average Ratio

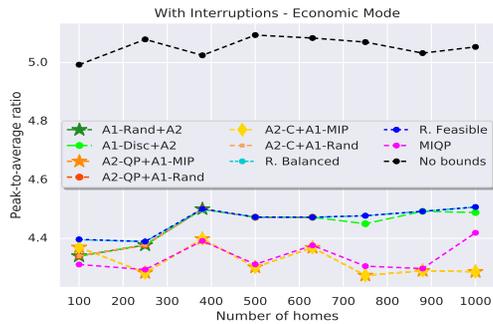


Fig. 10: Peak-to-average ratio:  
With Interruptions - Economic mode.

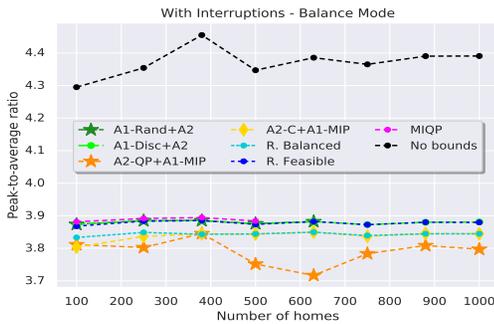


Fig. 11: Peak-to-average ratio:  
With Interruptions - Balance mode.

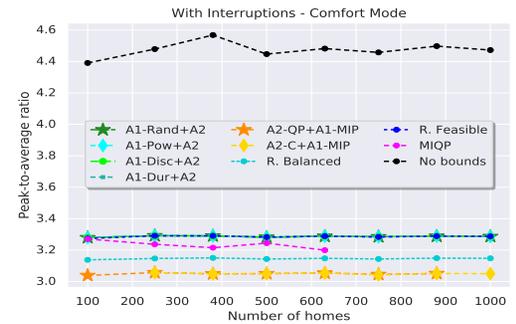


Fig. 12: Peak-to-average ratio:  
With Interruptions - Comfort mode.