On the Fairness of Aggregator's Incentives in Residential Demand Response

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Abstract

The main motivation of this work is to provide an optimization-based tool for an aggregator involved in residential demand response (DR) programs. The proposed tool comply with the following requirements, which are widely accepted by the residential DR literature: (i) the aggregated consumption should be optimized under a particular utility's target, such as the minimization of the Peak-to-Average Ratio, (ii) incentives can be used to retain prosumers in the DR program, (iii) the participants' profit/cost should not be worse off due to their enrollment in the program and (iv) prosumers' comfort and privacy must be preserved. To that end, an existing optimization framework involving two phases is revisited. First, by taking into account both the limitations of their smart-home components and their comfort preferences, each prosumer optimizes their self-generation and household consumption. Then, in a subsequent phase, the aggregator coordinates all prosumers responses by solving a mixed-integer linear programming problem. As a salient feature, new constraints based on the concept of the Shapley Value are devised for their incorporation into the problem, yielding a fair allocation of the aggregator's incentives while respecting the above DR requirements. The resulting problem is solved in a decentralized fashion by applying Dantzig-Wolfe decomposition, which is embedded in the proposed solution methodology to accommodate a large number of prosumers and their specific characteristics and needs. Computational results highlight the advantages of the proposed tool in terms of the Peak-to-Average-Ratio, fairness of the incentives' allocation and scalability.

Keywords: Dantzig–Wolfe decomposition; Fairness; Mixed-Integer Linear Programming; Privacy; Residential Demand Response; Shapley Value.

Nomenclature

Sets

N	Set of prosumers.
T	Set of time intervals.
$\mathcal{F}_n$	Feasible space of all variables for prosumer n.
$\mathcal{I}_n$	Feasible space of incentives for prosumer n.

Parameters

$\hat{C}^{DA}$	Day-ahead energy cost under no coordination [\$].
$\hat{C}^{DV}$	Load deviation cost under no coordination [\$].
D_n^{max}	Maximum discomfort level for prosumer n.
$\hat{E}_{n,t}^B$	Electric power bought by prosumer n at time t under no coordination [W].
$\hat{E}_{n,t}^S$	Electric power sold by prosumer n at time t under no coordination [W].
$\hat{f}^A$	Aggregator's costs under no coordination [\$].
$\hat{f}_n^P$	Profit of prosumer n under no coordination [\$].

Δ_t	Duration of time interval t [h].
λ_t^{DA}	Day-ahead market price for energy at time t [\$/Wh].
λ_t^S	Selling price of energy for prosumers at time t [\$/Wh].
λ_t^T	Electricity tariff for prosumers at time t [\$/Wh].
μ_t^{DV}	Price to adjust the load deviation at time t [\$/Wh].
$\hat{E}_t^{des}$	Computed value of the aggregated power consumed at time t [W].
δ	Percentage of the aggregator's savings invested on prosumers' incentives.

Variables

$C_{n,t}^B$	Cost of purchasing electricity incurred by prosumer n at time t [\$].
C_t^{DA}	Day-ahead energy cost at time t under coordination [\$].
C_t^{DV}	Load deviation cost at time t under coordination [\$].
E_t^{des}	Target value of the aggregated power consumed at time t [W].
$C_{n,t}^O$	Other costs incurred by prosumer n at time t [\$].
$E_{n,t}^B$	Electric power bought by prosumer n at time t [W].
$E_{n,t}^S$	Electric power sold by prosumer n at time t [W].
e_t^+, e_t^-	Positive and negative power imbalance at time t [W].

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f^A	Aggregator's total cost under coordination [$].
f_n^P	Profit of prosumer n under coordination [$].
$R_{n,t}^S$	Revenue of prosumer n for selling energy at time t [$].
$\boldsymbol{y}_n^P$	Vector of decision variables related to prosumer n.
κ_n	Incentive paid to prosumer n [$].

Function

$g_n^P(\boldsymbol{y}_n^P)$ Discomfort level of prosumer n.

1. Introduction

In recent years, Demand Response (DR) programs have received a great deal of attention worldwide [1], with particular emphasis on their application to the residential sector [2]. The success of DR programs relies on the fact that customers are able to change their electricity consumption in response to electricity tariffs warnings or incentive payments in such a way that the resulting aggregated load profile follows a particular load shape. Accordingly, DR programs are categorized in the literature into price-based or incentive-based DR (IBDR) programs [3]. In the first category, users optimize their consumption by responding to the price signals issued by the electric utility, while in the second one, users are paid to shift and/or reduce their consumption over a given period of time. However, both DR categories are not exempt from drawbacks, such as the difficulties for changing users' behavior [4] and the origination of rebound peaks [5] related to price-based approaches, or the negative impact on users' comfort and the limitations to retain/lure users associated with incentive-based approaches [6]. Note that, while price-based approaches may not be efficient on their own, incentive-based programs highly depend on the amount of incentives available to captivate users.

In addition to the above issues, the implementation of any residential DR program should preserve data privacy for the enrolled users. Private data include users' information related to their flexibility/comfort levels and consumption habits that could be used to identify and monitor behavior patterns or preferences [7]. Note that Direct Load Control (DLC), a class of IBDR programs in which the appliances/equipment of a given user are remotely controlled, certainly violates users' privacy since utilities need access to users' sensitive information. These concerns on privacy together with other computational issues related to DLC-based programs has been reported in [8]. Note that current home energy management systems (HEMS) overcome these limitations not only by allowing residential users to directly control their own loads but also by reducing the computational burden, which can be distributed among them. In such a case, the utility needs a coordination strategy to be applied in conjunction with householders. It is also noteworthy that these HEMS can handle quite accurate and specific models of users' appliances/equipment. The relevance of considering detailed models

was addressed in [9] by analyzing the trade-off between the complexity and accuracy of different users' models to take dispatch or investment decisions. The conclusions drawn indicate that more complex dispatch models generally leads to more accurate results. Additionally, from a user's perspective, [10] illustrated that more detailed models increases users' profitability while guaranteeing their levels of comfort.

Another topic currently under discussion is how incentives should be allocated among users in IBDR programs. While it is obvious that non-contributing or inflexible users shouldn't receive any incentive, it is not straightforward to determine a fair allocation of incentives among flexible users contributing to the program at different intensity levels. In some works [11] the fairness of users' incentives is evaluated through several metrics that quantifies users' contribution to the DR program. Note that a fair allocation guarantees the equity of the DR program [11], which is key to attract new participants [12] or to maintain users enrolled [13]. Hence, from a user' perspective, DR requests should be devised with a lower impact on their comfort and their privacy through a fair allocation of incentives.

To overcome the drawbacks derived from the limited responsiveness of residential users, the notion of the aggregator has recently emerged as an intermediary agent that allows them participating in wholesale electricity markets [14]. In the residential sector, the role of the aggregator is to coordinate consumers' electricity consumption to optimize a specific system- or utility-wide target. The optimization should be driven by the above mentioned aspects related to users' privacy and a fair allocation of incentives taking into account that users' costs or their comfort levels cannot be deteriorated due to the adopted coordination strategy. Similarly, the aggregator's costs cannot be worse off by overcompensating the enrolled users. In addition, the optimization-based tool should be scalable, i.e., it should not create an impractical computational burden when the number of enrolled users grows. Note that increasing this number is one of the simplest mechanisms to strengthen the aggregator's position in the electricity market.

Within the context of residential demand response, this paper addresses the challenges above-described placing the focus on the fairness of incentives' allocation among users, which is not thoroughly analyzed in the DR literature. In this paper, a decomposition-based approach is devised, which allows optimizing users' incentives while fulfilling the specific requirements of both the aggregator and the coordinated users with moderate computational effort.

2. Related Work

For the purpose of comparison with previous work on residential DR programs, the following requirements are analyzed: (i) the aggregated consumption is driven by a desired load shape, (ii) prosumers are kept enrolled in the IBDR program by receiving a fair amount of incentives,

(iii) the participation in the program cannot negatively impact on either the participants' profits, their levels of comfort or their privacy, (iv) the program is scalable in such a way that a larger number of prosumers can be enrolled, and (v) the program is able to consider accurate models for a variety of users' appliances/equipments, which are personalized for each user. Furthermore, for those works based on mathematical programming, the guarantee of convergence to an optimal solution is also studied. Hereinafter, programs meeting all the above requirements are denoted as aDR.

Except for fairness, the mentioned aDR requirements have been extensively studied in the DR coordination strategies available in the literature. Hence, the literature review is focused on DR programs implementing a coordination strategy in which the fairness issue is addressed. Note that there exists a scarcity of works discussing this issue as evidenced by [15].

A fair billing mechanism for residential users involved in DR was proposed in [12]. In this work, the aggregator minimizes utility-wide costs considering the flexibility provided by users' shiftable loads. An approach based on game theory was devised to optimize users' bills according to their consumption levels and their contributions to the DR program. However, the proposed approach breaches users' privacy since the aggregator requires information on the time windows of users' shiftable loads. This work was extended in [11] to address the privacy issue so that each user needs to receive the aggregated load demand of other users. Although the proposed secure-sum method preserves users' privacy, this coordination mechanism features the following disadvantages. First, the probability of communication interruptions increases due to the large amount of information exchanged between users. Second, the iterative process requires that users solve their own problems sequentially, hindering its scalability.

Similarly to [12], the approaches proposed in [16] and in [15] promote fairness on incentives, but users' privacy is violated since the aggregator requires data on the operating time ranges of the users' time-shiftable appliances, which is sensitive information. Privacy concerns also arise in [17] by considering an agent that requires private data from users as one of the inputs for an optimization model that was solved in a centralized manner.

In order to obtain a DR program accounting for the limitations of the power grid, the distribution network was represented by its power flow equations in [18]. Although a mechanism for charging users equitably was devised, specific users' characteristics are disregarded in this work.

The problem of multiple aggregators supplied by the same Load Serving Entity (LSE) was addressed in [19]. In this work, a fairness function is defined to guarantee that aggregators are rewarded with coupons according to their contribution to the LSE's targets. Likewise, users' rewards issued by one specific aggregator are proportional to their load reductions so that fairness is preserved. However, users are just characterized by their particular load profiles, which simplifies their consumption requirements and may jeopardize the accuracy of results.

In addition to smart homes, the fairness issue has also been addressed in the context of microgrids or virtual energy communities. A framework relying on model predictive control for microgrids' coordination was proposed in [20]. In this work, each microgrid is considered as a combination of shiftable loads, distributed generators, renewable generation resources, storage systems and/or cogeneration plants. Detailed models for microgrids are included, in which building thermal dynamics, the operation of thermal and electrical components as well as users' preferences are specifically characterized. In addition, intertemporal constraints are considered. In order to solve the problem, a multi-step optimization-based method was devised, which consisted in splitting the original problem into several subproblems to solve them sequentially. Considering also those mentioned details, a similar solution approach was proposed in [21] relying also on sequential optimization, which has no proof of convergence to the optimum [21].

The privacy of microgrids' operators was addressed in [22]. Although privacy is not preserved in the preliminary bi-level optimization problem, the combination of two optimization-based techniques allows separating the decisions taken by a central operator in the upper level from those adopted by each migrogrid's operator in their corresponding lower-level problems. Consequently, microgrids' operators do not share any sensitive information between them or with the central operator. However, for the proposed hybrid methodology, the authors do not provide any proof of convergence to a global optimum.

The decision making of a distribution network operator in the presence of autonomous microgrids was formulated in [23] as a bi-level optimization problem. In this work energy is exchanged between the microgrids and the network operator. In the upper level problem, the operator's costs are minimized, while the profits of each microgrid are maximized in the lower level problems. Each microgrid is composed of a combination of solar panels, wind turbines generation, and a given demand, which is characterized by aggregating several loads. Karush-Kuhn-Tucker conditions are applied to reformulate the bi-level problem into a single-level equivalent. Since a centralized solution of the single-level problem is carried out, privacy concerns arise on the participants' sensitive data.

Within the context of virtual energy communities, an IBDR program was proposed in [24]. In this work, each community receives an electricity bill in proportion to its relative consumption with respect to the total consumption of all communities. Likewise, each user within a community is charged in proportion to its consumption with respect to the community's total consumption. Communities can receive incentives if load curtailment is implemented on their members' loads. Although fairness is analyzed in detail, some issues arise in this work related to the low accuracy of users' models, the violation of users'

privacy and the lack of convergence proof to optimality of the proposed solution algorithm.

In [8], a bargaining cooperative game framework was proposed. The game players consist of several DR aggregators and a distribution company (Disco) with self-owned generators. The Disco is also a retailer who maximizes its profits by procuring energy from the main grid at wholesale prices to sell it to the consumers at retail prices. Aggregators, as independent business agents, coordinate flexible loads to bargain electricity prices (or payments) with the Disco. The aggregators' objective functions are formulated by a weighted sum approach that considers the trade-off between monetary and dissatisfaction costs. A distributed implementation relying on Augmented Lagrangian techniques is applied to decompose the original problem into one sub-problem per player, which guarantees players' privacy. From the analysis of proposed framework, two main disadvantages can be identified, namely, specific users' characteristics are disregarded and the representation of discomfort or dissatisfaction functions is cumbersome. Another research relying on cooperative game theory can be found in [13], in which a reinforcement learning algorithm was devised to estimate the Shapley Value (SV). It is important to emphasize that SV is acknowledged here as fair mechanism to distribute incentives among DR participants. As shown in this paper, one of the main disadvantages of the SV approach is the computational burden.

An alternative IBDR framework was proposed in [25] by considering a fair reduction on users' profits in such a way that utility requirements were met. However, it should be noted that this kind of approaches, in which users are penalized for participating in DR programs, may cause that either existing users leave the program or that potential users are discouraged from enrolling into it.

Recently, a two-phase day-ahead framework combining price- and incentive-based DR programs has been proposed in [7]. In a first phase, responsive users optimize their own household consumption, considering their appliances and equipment and their comfort preferences under a price-based DR program. Subsequently, the aggregator exploits in a second phase this preliminary non-coordinated solution by solving a bi-level optimization problem in which an IBDR coordination strategy is carried out. In this phase, aggregator's costs are minimized considering that users' consumptions are coordinated to follow a utility desired load shape while guaranteeing that nor the aggregator, nor users are worse off in either monetary or comfort terms. The framework preserves the privacy of each user by implementing a distributed solution approach based on Dantzig-Wolfe Decomposition (DWD). This exact optimization method allows each user and the aggregator to solve their own optimization problems. Simulation results considering up to 10,000 users, prove the scalability of the proposed solution methodology. Therefore, the proposed framework meet the requirements on privacy, scalability, optimality and impact on users' comfort levels or

on the participants' costs. However, the fairness issue is disregarded, since all users receive the same amount of incentives regardless their levels of contribution to the DR program.

In summary, DR coordination strategies addressing the fairness issue in the literature exhibit at least one drawback related to privacy, scalability, reduction on profits or comforts' levels, low accuracy of users' model or lack of optimality. To fill this research gap, this paper extends the work in [7] by allocating users' incentives under fairness criteria. Differently from [7], specific constraints accounting for fairness on incentives are included in this optimization framework. This new constraints are inspired on the concept of the Shapley Value from game theory [26], which requires nonlinear expressions. To avoid nonlinearities in the optimization, a linear approximation is derived, which also preserves incentives' fairness as backed by the simulation results. Additionally, the application of DWD is adapted to accommodate the new constraints. Thus, the main contribution of this work is to provide an optimization-based tool that fulfills all the requirements mentioned for aDR programs.

The remainder of this paper is organized as follows. Section 3 summarizes the framework and the models introduced in [7]. Section 4 presents the nonlinear expressions derived to allocate the users' incentives and their corresponding linear approximations. The solution methodology is explained in Section 5. In Section 6, numerical results are shown placing the focus on the fairness issue. Finally, concluding remarks are provided in Section 7.

3. Two-Phase Framework

The two-phase framework proposed in [7] is extended in this work to assure fairness on the allocation of incentives among prosumers. In Phase I, prosumers are not coordinated. Their consumption patterns are individually optimized under a price-based DR program. Using the results of Phase I, the coordination takes place in Phase II by imposing via incentives that neither the aggregator, nor the prosumers are worse off as compared to their situation in Phase I. Both phases are summarized next.

Phase I (no coordination). Based on [10], the general model for prosumer n is formulated in a compact way in (1)–(5). The resulting mixed-integer linear programming problem (MILP) can be solved to optimality by the prosumer' HEMS without interacting with the aggregator. Note that this formulation allows that a given HEMS decides whether a specific appliance/equipment is used for DR independently from the decisions taken by other HEMS.

$$\max_{\boldsymbol{y}_n^P} \quad f_n^P = \sum_{t \in T} \left(R_{n,t}^S - C_{n,t}^B \right) - C_n^O \tag{1}$$

subject to:

$$R_{n,t}^S = \lambda_t^S \Delta_t E_{n,t}^S \ \forall \ t \in T \tag{2}$$

$$C_{n,t}^B = \lambda_t^T \Delta_t E_{n,t}^B \ \forall \ t \in T \tag{3}$$

$$g_n^P(\boldsymbol{y}_n^P) \leq D_n^{max} \tag{4}$$

$$\boldsymbol{y}_n^P \in \mathcal{F}_n \tag{5}$$

The optimization goal in (1) is the maximization of the profits of prosumer n. The revenues from selling energy $R_{n,t}^S$ are defined in (2), where the electricity selling prices λ_t^S are given or estimated. The electricity costs $C_{n,t}^B$ are defined in (3), where the contracted tariffs λ_t^T are known with certainty ahead of time. $C_{n,t}^O$ represents other costs incurred by prosumer n. Constraint (4) sets the maximum on prosumer n' discomfort D_n^{max}, which is explicitly formulated in [27]. Finally, expression (5) provides a compact formulation to characterize the energy flow conservation, the balance between supply/demand, the appliances operation, the pricing policies and the energy limits. Note that symbols $C_{n,t}^O$, $E_{n,t}^S$ and $E_{n,t}^B$ are elements of the vector of decision variables $\boldsymbol{y}_n^P$, while f_n^P, $R_{n,t}^S$, and $C_{n,t}^B$ are included here for explanation purposes. The full list of variables in $\boldsymbol{y}_n^P$ and constraints in $\mathcal{F}_n$ as well as all the assumptions related to the prosumers' models can be found in [10]. Note that this formulation complies with the aDR requirement related to more representative models for a variety of users' appliances/equipment.

As a result of this phase, the optimal solution of problem (1)–(5) is obtained. In particular, for each prosumer n, the optimal values of the electrical energy to be bought, the electrical energy to be sold and their profits (respectively denoted by $\hat{E}_{n,t}^B$, $\hat{E}_{n,t}^S$ and $\hat{f}_n^P$) are required in the implementation of Phase II.

Phase II (coordination). In this phase, the aggregator coordinates prosumers' consumptions with its own targets and provides specific incentives to each prosumer to change its load pattern. Specifically, Phase II comprises the following two main steps.

Firstly, the aggregator takes the results from Phase I that do not involve private data and compute the following values assuming that no coordination takes place.

- The total cost of the electricity bought in the day-ahead market:

$$\hat{C}^{DA} = \sum_{t \in T} \lambda_t^{DA} \Delta_t \sum_{n \in N} \hat{E}_{n,t}^B. \tag{6}$$

Note that the hourly clearing prices λ_t^{DA} resulting from this market are assumed to be known or estimated a day ahead.

- The total cost associated with load deviations:

$$\hat{C}^{DV} = \sum_{t \in T} \mu_t^{DV} \Delta_t \sum_{n \in N} \left(\hat{E}_{n,t}^B - \hat{E}_t^{des} \right), \tag{7}$$

where $\hat{E}_t^{des}$, which is set by the aggregator to modify the aggregated consumption profile, either follows a given consumption pattern or any specific target, such as the Peak-to-Average Ratio (PAR). In the later case, the $\hat{E}_t^{des}$ can be computed with values from Phase I as follows:

$$\hat{E}_t^{des} = \frac{\sum_{n \in N} \sum_{t' \in T} \hat{E}_{n,t'}^B}{|T|} \ \forall \ t \in T \tag{8}$$

- The maximum attainable value of the costs incurred by the aggregator:

$$\hat{f}^A = \hat{C}^{DA} + \hat{C}^{DV}. \tag{9}$$

This value cannot be exceeded so that the aggregator is not worse off by implementing the coordination strategy in Phase II.

Secondly, using the values computed in (6)–(9) the following MILP problem is solved in a decentralized manner:

$$\min_{\substack{C_t^{DA}, C_t^{DV}, e_t^+, \\ e_t^-, f^A, \boldsymbol{y}_n^P, \kappa_n}} f^A = \sum_{t \in T} C_t^{DA} + \sum_{t \in T} C_t^{DV} + \sum_{n \in N} \kappa_n \tag{10}$$

subject to:

$$C_t^{DA} = \lambda_t^{DA} \Delta_t \sum_{n \in N} E_{n,t}^B \ \forall \ t \in T \tag{11}$$

$$C_t^{DV} = \mu_t^{DV} \Delta_t (e_t^+ + e_t^-) \ \forall \ t \in T \tag{12}$$

$$\sum_{n \in N} E_{n,t}^B + e_t^+ - e_t^- = E_t^{des} \ \forall \ t \in T \tag{13}$$

$$e_t^+, e_t^- \geq 0 \ \forall \ t \in T \tag{14}$$

$$f^A \leq \hat{f}^A \tag{15}$$

$$f_n^P + \kappa_n \geq \hat{f}_n^P \ \forall \ n \in N \tag{16}$$

Constraints $(2) - (5) \ \forall \ n \in N \tag{17}$

$$\kappa_n \in \mathcal{I}_n \ \forall \ n \in N \tag{18}$$

Aggregator's costs in (10) include the following terms to be minimized: (i) the costs of purchasing electricity in the day-ahead market, (ii) the variability cost of the aggregated energy consumed according to the utility-wide target, and (iii) the incentives that should be paid to prosumers for changing their consumption patterns. Note that f^A, C_t^{DA}, and C_t^{DV} are auxiliary variables only included for explanation purposes.

Equations (11) define the costs of purchasing electricity in the day-ahead market, while the costs associated with the deviation of the load profile respect to the desired consumption pattern are defined in (12). Equations (13) characterize the consumption variance from the utility's desired load at time step $t \in T$, namely E_t^{des}. Surplus variables e_t^+ and e_t^- are nonnegative as per (14) and quan-

tify the deviation from E_t^{des}. As done in [7], E_t^{des} in this work is an approximation for the PAR, which is computed as follows:

$$E_t^{des} = \frac{\sum\limits_{n \in N} \sum\limits_{t' \in T} E_{n,t'}^B}{|T|} \ \forall \ t \in T \quad (19)$$

Constraint (15) ensures that by coordinating users' consumption the aggregator does not incur financial losses. Thus, according to (15), the aggregator's total cost, f^A, associated with the implementation of the incentive-based DR program, cannot be greater than the total cost that would be incurred without any coordination $\hat{f}^A$. In other words, the incentives given by the aggregator cannot exceed the savings obtained by implementing the DR coordination. Likewise, constraints (16) guarantee that the profits obtained by prosumer n by participating in the proposed DR coordination, $f_n^P + \kappa_n$, are greater than or equal to the profit that would be achieved under no coordination, $\hat{f}_n^P$.

Constraints (17) are identical to constraints (2)-(5) imposed by each prosumer.

Finally, constraints (18) characterizes the allocation of incentives paid by the aggregator to prosumers. The incentives given to a prosumer are optimized by the aggregator and should remain private for other prosumers. Note that the main difference of problem (10)–(18) and the one addressed in [7] are constraints (18). Since these constraints constitute one of the main contribution of this work, their derivation is explained in the next section.

It should also be noted that both (16) and (17) contain private data specific to each to prosumer that cannot be shared with other prosumers or with the aggregator. These privacy-related constraints prevent the aggregator from solving problem (10)–(18) in a centralized fashion. This inconvenient is circumvented if Dantzig-Wolfe decomposition is applied as in [7]. In Section 5 all the details on the adaptation of this technique to the proposed problem are explained.

4. Allocation of Prosumers' Incentives

The derivation of constraints (18) relies on the following assumptions:

1. Prosumer' consumption is always supplied regardless their participation in DR programs.

2. Prosumers always receives nonnegative incentives. That is,

$$\kappa_n \geq 0 \ \forall \ n \in N \quad (20)$$

3. The average daily power consumption in both phases has the same order of magnitude. That is:

$$|N|^{-1} \sum_{t \in T} \sum_{n \in N} \hat{E}_{n,t}^B \approx |N|^{-1} \sum_{t \in T} \sum_{n \in N} E_{n,t}^B. \quad (21)$$

4. If two participants contribute equally to reduce their consumption they are rewarded equally, as assumed in [13].

5. If the daily consumption of prosumer n is either increased or remains the same from Phase I to Phase II, this prosumer should not receive any incentive, as assumed in [13]. That is:

$$\left(\sum_{t \in T} \hat{E}_{n,t}^B \leq \sum_{t \in T} E_{n,t}^B \right) \implies \kappa_n = 0.$$

6. If the daily consumption of prosumer n is reduced from Phase I to Phase II, this prosumer should receive incentives in proportion to the level of reduction, being κ_n determined by Phase II. That is:

$$\left(\sum_{t \in T} \hat{E}_{n,t}^B > \sum_{t \in T} E_{n,t}^B \right) \implies \kappa_n \propto \sum_{t \in T} \left(\hat{E}_{n,t}^B - E_{n,t}^B \right).$$

Considering the above assumptions, the allocation of users' incentives derived in this section is inspired on the concept of SV from the game theory. In particular, a fair allocation of incentives is guaranteed by the two last assumptions since prosumers are compensated in proportion to their load reduction from Phase I to Phase II. Furthermore, assuming that incentives are paid with the savings obtained by the aggregator from the coordination strategy, the contribution of prosumer n to aggregator's savings can be defined as follows:

$$S_n^A = (\hat{f}^A - f^{A*}) - (\hat{f}^A - f_{\backslash n}^{A*}), \quad (22)$$

which can be simplified as:

$$S_n^A = -(f^{A*} - f_{\backslash n}^{A*}), \quad (23)$$

where $\hat{f}^A$, already defined in (9), is the optimal value of the aggregator's costs considering that no prosumers contributes to Phase II and $f_{\backslash n}^{A*}$ represents the optimal value of the aggregator's costs assuming that prosumer n does not contribute to the coordination carried out in Phase II. Within the context of the proposed two-phase framework, the value of $f_{\backslash n}^{A*}$ can be obtained by fixing the daily consumption of prosumer n in Phase II to the corresponding optimum in Phase I. That is:

$$E_{n,t}^B \leftarrow \hat{E}_{n,t}^B \ \forall \ t \in T \quad (24)$$

It is worth emphasizing that the first term in parenthesis in (22) characterizes the maximum value of aggregator's savings. Furthermore, it can be concluded that aggregator's savings due to a specific prosumer are always nonnegative ($S_n^A \geq 0$) since the aggregator's costs resulting from disregarding the contribution to Phase II of this prosumer are greater than or equal to those associated

with an scenario in which this contribution is considered $(f_{\backslash n}^{A*} \geq f^{A*})$.

Once prosumer's contributions to aggregator's savings are computed in (22), incentives can be allocated ex-post among prosumers in proportion to their contribution. The following equation impose a fair allocation of incentives:

$$\frac{\kappa_n^{SV}}{\kappa_m^{SV}} = \frac{S_n^A}{S_m^A}, \tag{25}$$

where κ_n^{SV} and κ_m^{SV} denote the incentives given to prosumers n and m, respectively.

In addition to the above allocation rule, the incentives given to prosumers are limited by the maximum value of aggregators' savings. That is:

$$\sum_{n \in N} \kappa_n^{SV} \leq (\hat{f}^A - f^{A*}). \tag{26}$$

Furthermore, if the aggregator specifies the percentage of savings to be spent in prosumers' incentives, the above expression should be replaced by:

$$\sum_{n \in N} \kappa_n^{SV} = \delta(\hat{f}^A - f^{A*}), \tag{27}$$

with $0 \leq \delta \leq 1$, which is set by the aggregator to fix the maximum percentage of savings that can be invested on incentives.

Finally, taking into account that incentives are minimized in (10), the allocation rule in (25) and the budget for incentives in (27), a fair allocation of incentives is given by the following SV-based expression:

$$
\begin{aligned}
\kappa_n^{SV} &= \frac{S_n^A}{\sum_{m \in N} S_m^A} \delta(\hat{f}^A - f^{A*}) \\
&= \frac{-f^{A*} + f_{\backslash n}^{A*}}{-|N| f^{A*} + \sum_{m \in N} f_{\backslash m}^{A*}} \delta(\hat{f}^A - f^{A*})
\end{aligned} \tag{28}
$$

However, the computation of κ_n^{SV} in (28) should be made ex-post. First, the value of f^{A*} is obtained by solving problem (10)–(17) with $\kappa_n = 0 \ \forall \ n \in N$. Second, to obtain $f_{\backslash n}^{A*}$, the previous problem is solved, for each prosumer n, by fixing its consumption as in (24). Therefore, $|N| + 1$ optimization problems need to be solved. As can be seen, the resulting SV-based allocation methodology is inefficient from a computational standpoint when the number of prosumer grows. Moreover, this ex-post methodology does not consider incentives within the optimization process.

With the purpose of changing prosumers' behaviour via incentives, the allocation rule should be embedded in constraints (18). In particular, the proposed allocating rule complies with the fairness-related Assumptions 4-6 by imposing that prosumers' incentives are proportional to the level of reduction of their daily consumption from Phase I to Phase II:

$$\kappa_n = \frac{\sum_{t \in T} \left(\hat{E}_{n,t}^B - E_{n,t}^B \right)}{\sum_{t \in T} \sum_{n' \in N} \left(\hat{E}_{n',t}^B - E_{n',t}^B \right)} \delta(\hat{f}^A - f^{A*}) \ \forall \ n \in N \tag{29}$$

The above expressions assures that the incentives allocated to prosumer n should be a share of the aggregator's budget in proportion to its consumption reduction out of the total reduction. Furthermore, these expressions are in line with the SV-based allocation characterized in (28) since the aggregator's savings due to a given prosumer are proportional to the energy reduction implemented by this prosumer. That is:

$$S_n^A \propto \hat{E}_{n,t}^B - E_{n,t}^B \ \forall \ n \in N \tag{30}$$

Additionally, the proportionality associated with the incentives computed in 29 is consistent with the SV allocation rule in (28). That is:

$$\kappa_n \propto \kappa_n^{SV} \ \forall \ n \in N \tag{31}$$

Note that including (29) in problem (10)–(18) is a Catch-22 situation, since the optimal value f^{A*} can only be determined once the problem is solved. To circumvent this inconvenient, constraint (15) is considered to obtain an estimation of f^{A*} as follows:

$$f^{A*} \leq \hat{f}^A \to f^{A*} = \beta \hat{f}^A : 0 \leq \beta \leq 1, \tag{32}$$

where β is a constant conveniently set by the aggregator as explained in Section 5.

Regardless the above approximation, expressions (29) include nonlinearities, which precludes their incorporation into the MILP-based framework adopted in this work. To resolve this issue, a linear approximation of these constraints is derived next.

By considering the assumption formulated in (21), the total daily consumption of Phase II in the denominator of expressions (29) is approximated as follows:

$$|N|^{-1} \sum_{t \in T} \sum_{n \in N} E_{n,t}^B = \alpha |N|^{-1} \sum_{t \in T} \sum_{n \in N} \hat{E}_{n,t}^B : \alpha \approx 1, \tag{33}$$

where α is a constant conveniently set by the aggregator as explained in Section 5.

Approximations (32) and (33) allow transforming (29) into the following linear expressions:

$$\kappa_n \approx \frac{\sum_{t \in T} \left(\hat{E}_{n,t}^B - E_{n,t}^B \right)}{\sum_{t \in T} \sum_{n' \in N} \hat{E}_{n',t}^B (1 - \alpha)} \delta \hat{f}^A (1 - \beta) \ \forall \ n \in N \tag{34}$$

Taking into account that incentives are minimized in (10), expressions (34) should be incorporated into problem (10)–(18) through a greater-than-or-equal-to constraint, which

can be compactly reformulated as follows:

$$\kappa_n \geq \sum_{t \in T} \left(\hat{E}_{n,t}^B - E_{n,t}^B \right) \delta \Gamma \ \forall \ n \in N \quad (35)$$

where Γ is a constant defined as:

$$\Gamma = \frac{\hat{f}^A (1 - \beta)}{\sum_{t \in T} \sum_{n' \in N} \hat{E}_{n',t}^B (1 - \alpha)} \quad (36)$$

From (36) it can be noted that the value of Γ depends on the choice of constants α and β. Furthermore, for a given Γ there exists an infinite number of combinations of α and β. If a combination is chosen in such a way that β is too high, prosumers may not receive any incentives. Conversely, if the selected combination results in a low value of β, savings can be underestimated. Hence, the most interesting combination from a practical perspective is the one that better approximates the aggregator's savings to be allocated among prosumers. The algorithm for selecting such a practical combination of α and β is detailed in Section 5, while the validation of the proposed approximations is illustrated in Section 6.

It is important to point out that constraints (35) allows a fair allocation of incentives among householders capable of modifying their consumption patterns, which is the main contribution of this work over [7]. Furthermore, different from the existing allocation methodologies, the proposed formulation considers all the requirements in aDR. The union of these constraints and (20) defines $\mathcal{I}_n$ in (18). Thus, the problem to be solved by the aggregator and prosumers in Phase II is comprised by (10)–(17), (20), and (35), which is hereinafter denoted as P^{aDR}.

5. Solution methodology

The proposed solution algorithm is summarized next. In Phase I, prosumers solve independently problem (1)–(5) using available MILP solvers in their own HEMS. In Phase II, problem P^{aDR} is decomposed to be solved by both the aggregator and prosumers in an iterative way. In this section, the solution methodology of [7] is extended to match all aDR requirements.

To that end, the value of β in 36 is iteratively updated for a fixed value of α consistent with (33). The value of β is improved using a divide-and-conquer searching method in such a way that a solution of problem P^{aDR} that either overestimates (high value of β) or underestimates (low value of β) the aggregator's savings is discarded. Thus, for a specific instance of problem P^{aDR}, k subproblems are created and solved for different values of β using DWD as in [7]. At each iteration, the continuous interval $[\beta^{min}, \beta^{max}]$ for β is reduced by half. When the amplitude of this interval is lower than or equal to ϵ, the iterative process stops returning the lower endpoint of the interval as the most practical value for β. Finally, a fair

allocation of incentives is obtained by solving P^{aDR} with the last value of β.

Algorithm 1 - Solution methodology.

1: $\beta^{min} \leftarrow 0$
2: $\beta^{max} \leftarrow 0.5$
3: $AUX \leftarrow 1$
4: **while** $\beta^{max} - \beta^{min} \leq \epsilon$ **do**
5: $\quad \beta \leftarrow \beta^{max}$
6: $\quad$ Compute Γ in 36 considering $\alpha \in [0.9, 0.999]$
7: $\quad$ Solve problem P^{aDR} using DWD as in [7]
8: $\quad$ **if** $\kappa_n^* = 0 \ \forall \ n \in N$ **then**
9: $\quad\quad AUX \leftarrow \beta^{max}$
10: $\quad\quad \beta^{max} \leftarrow (\beta^{min} + \beta^{max})/2$
11: $\quad$ **else**
12: $\quad\quad \beta^{min} \leftarrow \beta^{max}$
13: $\quad\quad \beta^{max} \leftarrow (AUX + \beta^{max})/2$
14: $\quad$ **end if**
15: **end while**
16: Stop: $\beta \leftarrow \beta^{min}$
17: Compute Γ in 36 considering $\alpha \in [0.9, 0.999]$
18: Solve problem P^{aDR} using DWD as in [7]

Taking into account that the interval of β is reduced by half in each iteration and the given stopping criteria ϵ, the number of subproblems k to be solved can be determined using the following expression:

$$\arg \min_{k \in \mathbb{N}} 2^k : 2^k \geq \left(\frac{0.5}{\epsilon} \right) \quad (37)$$

Note that for the particular case of $\epsilon = 1\%$, the number of subproblems solved is 6 ($k = 6$). Compared with an ex-post method that requires $|N| + 1$ calls of the optimization procedure to compute κ_n^{SV} in equations 28, it can be concluded that the proposed approach shows a lower computational burden when $|N| + 1 > k + 1$.

Note also that the selection of a practical value for β and thus for Γ, is crucial for the proposed aDR framework. Furthermore, if β is too high, incentives may not be required to flat the aggregated load profile. This effect may take place for a specific combination of aggregator's costs in (10), the aggregator's budget in (15) and the availability for energy storage of end-users. In particular, aggregator's savings are overestimated if $\beta > \beta^{max}$, setting incentives to 0 with all prosumers increasing or maintaining their daily consumption with respect to Phase I, that is, $\sum_{t \in T} \left(\hat{E}_{n,t}^B - E_{n,t}^B \right) \leq 0 \ \forall \ n \in N$. This counter-intuitive situation occurs only if prosumers have enough storage capability and the aggregator is not worse off by coordinating prosumers. Without this flexibility, the coordination strategy would be unnecessary yielding the same results as those obtained in Phase I.

6. Results

This section analyses the effect of considering different instances of constraints (18) in problem P^{aDR} and the impact of the fairness-related constraints on the results. Firstly, the intractability of the nonlinear constraints (29) in $\mathcal{I}_n$ is evidenced. Then, the quality of the approximation (21) is discussed. Additionally, results show how prosumers' incentives are allocated following the SV-inspired criteria. Finally, the impact of the new constraints (35) in the CPU time and in the PAR is analyzed.

The case studies considered for comparisons are taken from [7], in which real data for prosumers were extracted from [10]. The new required parameters are initialized as follows: α, δ, and ϵ are set to 0.9, 1, and 5% respectively.

The time horizon considered comprises one day divided in intervals of 10 minutes, i.e., $|T| = 144$. For the MILP instance the commercial solver CPLEX 12.7.1 is used, while KNITRO 10.3.0 and BARON 21.1.7 are the solvers used to handle the nonlinear instances of problem P^{aDR}.

6.1. Intractability of nonlinear constraints

For problem (10)–(18), a nonlinear instance is solved if constraints (29) rather than (35) are considered. This instance is classified as a nonlinear mixed-integer programming problem, for which the convergence to the optimal solution cannot be guaranteed. Furthermore, for a small case considering only 10 prosumers, none of the two selected solvers were able to find a feasible solution within 24 hours, which is inefficient to be implemented in practice.

In order to reduce the degree of nonlinearity of the above instance, which, in turn, reduces the associated computational burden, an alternative instance, referred to as P^{NL}, is considered. In this instance, the aggregators' savings are fixed in (29) using the approximation (32), where β is selected using Algorithm 1. Table 1 summarizes the results for problems P^{aDR} and P^{NL}, in which the nonlinear model was solved using KNITRO 10.3.0. As can be noted, the difference of the optimal values of P^{aDR} and P^{NL} is neglible while the CPU time for P^{aDR} is significantly lower.

Table 1: Comparison of results for P^{aDR} and P^{NL}.

	P^{aDR}	P^{NL}
f^{A*} [$]	36.2	36.3
CPU Time [s]	15	1349

6.2. Practicality of the assumption on the average daily consumption

The assumption regarding the total daily consumption stated in (21) is validated with the simulations summarized in Table 2. In this table, $\mathbf{1}^{1 \times |N||T|}$ denotes a vector of ones while $\hat{\boldsymbol{E}}^{|N||T| \times 1}$ and $\boldsymbol{E}^{|N||T| \times 1}$ are vectors comprising the optimal values of $\hat{E}_{n,t}^B$ and $E_{n,t}^B$, respectively.

Table 2: Comparison of the average daily power consumption on phases I and II (kW).

$|N|$	$\frac{\mathbf{1}\hat{\boldsymbol{E}}}{|N|}$	$\frac{\mathbf{1}\boldsymbol{E}}{|N|}$	$\frac{(\mathbf{1}\hat{\boldsymbol{E}} - \mathbf{1}\boldsymbol{E})}{|N|}$
10	342.27	340.41	1.85
50	356.63	354.83	1.80
500	349.26	348.15	1.12
1000	354.45	352.19	2.26
5000	353.91	350.93	2.98
10000	353.05	350.38	2.66

The columns of Table 2 show the following information: (i) the number of prosumers considered, (ii) the average energy consumption obtained in Phase I, (iii) the average energy consumption obtained in Phase II and (iv) the difference of the values in the second and the third column. The results in the last column corroborates that Assumption in (21) is valid in practice. Note that the largest difference in the average consumption is lower than 3kW, which represents a relative error of 0.8%.

6.3. Assessment of fairness

The following simulations are intended to assess the fairness of the proposed approach on allocating incentives. For 10 prosumers, Fig. 1 depicts the amount of incentives allocated to each prosumer in a bar chart format and the reduction on their daily consumption from Phase I to Phase II in a line chart. It can be noted that each householder receives incentives according to their contribution to the coordinated DR program, i.e., their daily consumption reduction for Phase I to Phase II. These results shows that the obtained allocation meets Assumption 6 in Section 4.
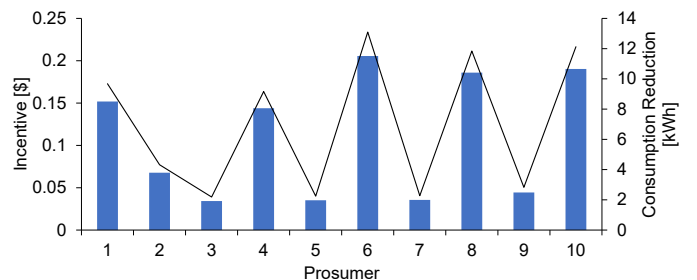

Figure 1: Prosumers' incentives and contribution to DR for $|N| = 10$.

The former case, considered hereinafter as the base case, is slightly modified to impose that a specific prosumer, referred to as U1, does not contribute to the DR program. In practice, this situation may occur if this prosumer has no flexibility to change its behavior in a particular day. Thus, once U1 executes Phase I, this prosumer should inform the aggregator not only on its daily consumption but also on its unavailability to contribute to the coordinated solution of Phase II. To provide a consistent comparison, the remaining 9 prosumers remain identical to those in the base case. The amount of incentives allocated to each prosumer is illustrated for both case studies

in Fig. 2. Additionally, the aggregated consumption for the two cases is depicted in Fig. 3. From the results, two main conclusions can be drawn. First, consistently with Assumption 5 in Section 4, U1 does not receive any incentive since it does not contribute to the DR program, as shown in Fig. 2. Note that the incentives received by the remaining prosumers in this case are slightly higher although aggregator's savings are lower. The reduction on savings is explained by the fact that PAR is reduced from 1.895 to 1.858 when prosumer U1 is not contributing to Phase II. Second, as shown in Fig. 3, the aggregated load patterns for both cases are quite similar. It is worth emphasizing that the above conclusions depends on the characteristics of the inflexible prosumers as well as on the flexibility of the remaining ones.
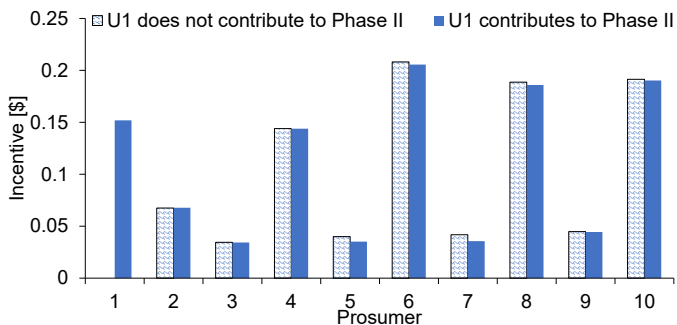

Figure 2: Prosumers' incentives for $|N| = 10$ - base case vs case with an inflexible prosumer.
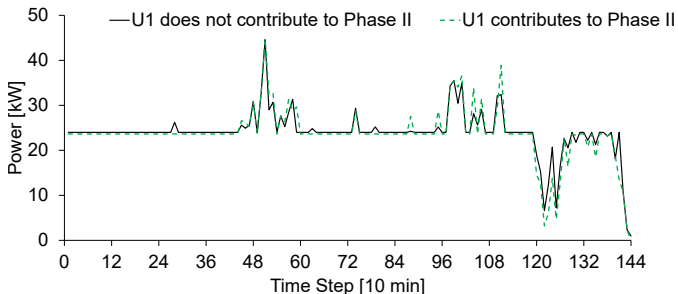

Figure 3: Aggregated load profiles for $|N| = 10$ - base case vs case with an inflexible prosumer.

Fig. 4 further illustrates the allocation of incentives for a case with 500 prosumers. The minimum and the maximum amount of incentives given are, respectively, \$ 0.01 and \$ 0.30. In this case, incentives also vary according to the contribution of each prosumer to the DR program.

Additionally, Fig. 5 compares the incentives obtained from the proposed approach with κ_n^{SV} in (28), obtained using the ex-post method. From this figure, it can be concluded that the incentives from the proposed framework are quite similar to those from a SV-based methodology, which further validates the assumptions and the approximations adopted in the formulation of P^{aDR}.

6.4. Efficiency in terms of PAR and CPU time

In this section, a comparison of the proposed approach (with F) and the approach from [7] (w/o F), which does
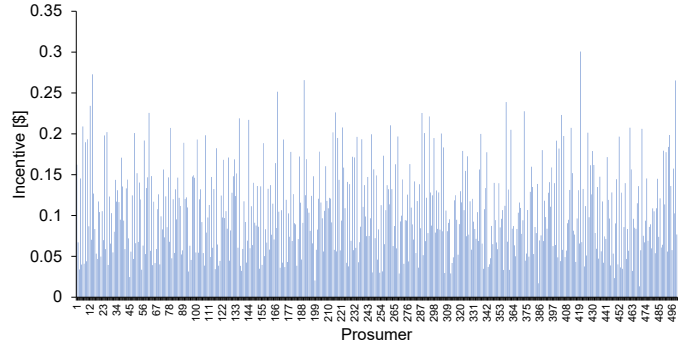

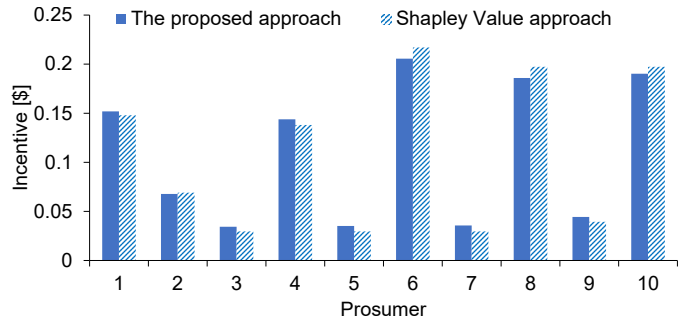Figure 4: Prosumers' incentives for $|N| = 500$.

Figure 5: Prosumers' incentives for $|N| = 10$ - comparison with the Shapley-based methodology.

not address the fairness on incentives' allocation, is carried out. Six case studies of an increasing number of prosumers are analyzed to highlight the differences between the two approaches in terms of PAR results and their computational burden.

First, the Peak-to-Average ratios of both approaches are shown in Table 3. It can be seen that the proposed approach increases this ratio as compared with the corresponding cases in [7]. However, the increment is limited to 9.6% for 10,000 prosumers. Thus, a fair allocation of incentives can be achieved at the cost of slightly deteriorating this measure.

Table 3: Peak-to-Average-Ratio.

$|N|$	w/o F	with F
10	1.79	1.89
50	1.33	1.47
500	1.14	1.31
1000	1.15	1.25
5000	1.17	1.30
10000	1.18	1.29

It is worth emphasizing that the results shown in Table 3 can be improved if additional constraints to link the current and the next day are included in both formulations. Without these constraints, the optimal strategy consists in nullifying prosumers' energy consumption at the end of the time horizon which increases PAR. This nullifying process is evidenced in Fig. 6 for the proposed approach and the one in [7], starting at time step 125 s. Note that this inconvenient can be circumvented in practical applications by using a rolling horizon methodology.

10

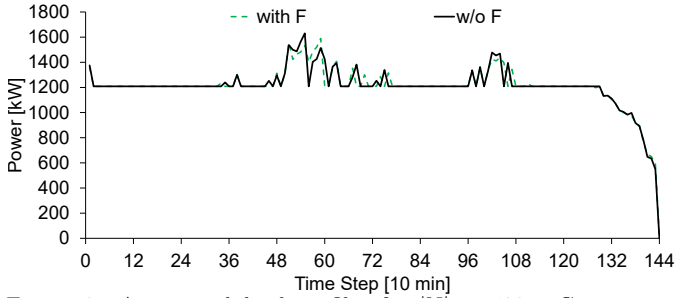Hence, it is expected that PAR results are lower in practice than those presented in Table 3.

Figure 6: Aggregated load profiles for $|N| = 500$ - Comparison with [7].

Next, a comparison in terms of the computational burden for both approaches is presented. To that end, estimated computation times are reported in Table 4. This is estimation is based on the assumption that a parallel implementation of the proposed approach is carried out using all prosumers' HEMS and the aggregator's EMS, as assumed in [7]. For the proposed solution methodology, problem P^{aDR} is solved $k + 1$ times with k obtained from (37). This explains the fact that the fairness-focused approach shows CPU times approximately $k + 1$ times larger than those in [7] for a similar problem. Notwithstanding the increment on CPU times, the proposed approach is still promising for being implemented on a day-ahead setting.

Table 4: Estimated CPU times per prosumer.

$|N|$	w/o F [s]	with F [s]
10	51.00	421.00
50	17.14	76.90
500	69.00	286.17
1000	89.63	421.33
5000	117.77	559.82
10000	130.54	638.66

7. Conclusions

The optimization tool presented in this work is useful for an aggregator that implements a DR program with the following targets: (i) to meet the utility's desired load shape, (ii) to retain/attract prosumers in/to the DR program via a fair allocation of incentives, (iii) to avoid a negative impact on either participants' profits or on prosumers' privacy or on their comfort levels, and (iv) to be a tool with scalability, i.e., that covers a considerable amount of prosumers. The proposed framework coordinates the prosumers' consumption in such a way that targets of the aggregator and the prosumers are aligned.

This work represents an extension of a previous one in which the fairness of prosumers' incentives was disregarded. Addressing this issue not only requires the reformulation of the aggregator's problem but also an adaptation of the Dantzig–Wolfe decomposition proposed in the previous work, to accommodate the privacy and scalability needs. Furthermore, the proposed solution algorithm overcomes the computational drawbacks of other SV-based methodologies addressing incentives' fairness. Simulation results corroborates that the proposed framework is capable to efficiently implement DR programs with all the mentioned features. Different cases studies with real-life data are presented to illustrate how fairness is achieved. As compared to the extended work, the cases analyzed show that both PAR and CPU times are increased under the proposed solution methodology, but cases with up to 10,000 prosumers can be still efficiently solved.

To extend this work, stochastic or robust optimization-based models will be formulated to characterize the uncertainty in both aggregator- and users-related parameters.

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