

Network Migration Problem: A Hybrid Logic-based Benders Decomposition Approach

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Telecommunication networks frequently face technological advancements and need to upgrade their infrastructure. Adapting legacy networks to the latest technology requires synchronized technicians responsible for migrating the equipment. The goal of the network migration problem is to find an optimal plan for this process. This is a defining step in the customer acquisition of telecommunications service suppliers, and its outcome directly impacts the network owners' purchasing behaviour. We propose the first exact method for the network migration problem, a logic-based Benders decomposition approach that benefits from a hybrid constraint programming-based column generation in its master problem and a constraint programming model in its subproblem. This integrated solution technique is applicable to any integer programming problem with similar structure, most notably the vehicle routing problem with node synchronization constraints. Comprehensive evaluation of our method over instances based on six real networks demonstrates the computational efficiency of the algorithm in obtaining quality solutions. We also show the merit of each incorporated optimization paradigm in achieving this performance.

Key words: Logic-based benders decomposition, Constraint programming, Column generation, Network migration, Optical networks, Synchronized vehicle routing problem

1. Introduction

In telecommunication industries, network migration is the process of upgrading the existing infrastructure of a deployed network. Network migration can deal with smaller parts of the network i.e., addressing capacity exhaustion by gradually increasing bit-rate of the wavelengths or modernize the whole network and adapt to the latest technology (e.g., migrating to IP-centric networks from SONET/SDH infrastructures.) A telecommunication network is composed of a set of sites (demand points), and circuits that transmit the traffic between the sites. A circuit corresponds to a path created from a source to a destination over physical

fibers in order to transmit traffic. Every circuit can be viewed as a transport equipment that consists of a set of topological links and provisioned cross connects to switch traffic from one node to another. Migration of such a network is performed by upgrading the circuits one by one. In order to upgrade every circuit, two synchronized technicians migrate its two endpoints within a time window. The goal of the *network migration problem* (NMP) is to find the upgrade order of these circuits such that the associated costs are minimized. Migration of a network is a strategic decision that can lead to immense savings of 10 to 100 times in power and space as reported by Ciena (2013). Nevertheless, the process is quite costly and complex, with some circuits stretching over a continent. Furthermore, every circuit migration comes with a disruption as the endpoints disconnect from the equipment, thus affecting the efficiency of the network and the customers' satisfaction of the migration solution, i.e., the frequency of disruptions they experience during the upgrade. Consequently, a well-crafted plan is critical for the success of the migration. In this paper, we present the first exact solution method for the NMP, a *logic-based benders decomposition* (LBBDD) algorithm that integrates Benders Decomposition (BD), Column Generation (CG) and Constraint Programming (CP), enabling us to decompose the decision space into smaller subsets, each amenable to one of these solution frameworks.

The NMP can also be stated in the context of the *vehicle routing problem with synchronized constraints* (VRPS). In the VRPS, at least one vertex or arc requires simultaneous visits of vehicles, or successive visits resulting from some precedence constraints (Eksioglu et al. 2009). The synchronization constraints can be over the arcs (the synchronized arc routing problem, SARP), or the nodes (the synchronized node routing problem, SNRP). In the NMP, vehicles and nodes correspond to technicians and sites, respectively. Since the tasks (i.e., upgrading the circuit endpoints) are defined over the nodes, the NMP is a special case of the SNRP and an application of the VRPS in the telecommunications domain. In the SNRP, typically some nodes need to be visited by more than one vehicle because the personnel do not have the same expertise. Some examples are: Bredstrom and Rönnqvist (2007) for a vehicle routing and scheduling problem, Reinhardt et al. (2013) for the airport transportation, Labadie et al. (2014) and Hashemi Doulabi et al. (2020) for home healthcare systems, Hojabri et al. (2018) for an SNRP with precedence constraints and synchronization of two types of vehicles, and Li et al. (2020) for a variation of the SNRP where customers have multiple options for time windows. There are also studies considering both the SARP and the SNRP at the same

time. [Salazar-Aguilar et al. \(2013\)](#) studied the road marking operations as a synchronized arc and node routing problem such that several capacitated vehicles are used to paint the lines on the roads and a tank vehicle is used to replenish the painting vehicles. The interested reader may refer to [Drexl \(2012\)](#) for a review on VRPS problems and their classifications. In regards to the applicability of these studies to the NMP, an important issue is their *local* synchronisation assumption, meaning that the arrival of vehicles is synchronized at the same node. However, the NMP involves several sets of technicians distributed over multiple regions which can be synchronized with several other technicians from the same or other regions based on the location of the circuit endpoints. Besides, [Li et al. \(2020\)](#) is the only exact method that considers multiple depots and time windows (the same as the NMP), but the size of the instances it can solve is up to 40 customers with 3 time windows which is far from the need of telecommunication networks. Therefore, the existing works in the literature of the VRPS are not suited for the NMP's level of complexity.

As for the telecommunications literature, there are different cost models and strategies on the suitable time and technology to migrate a telecommunication network ([Podhradsky 2004](#), [Almughales and Alsaih 2010](#), [Türk et al. 2012](#), [Poularakis et al. 2019](#)). Moreover, study of the operational aspects of the NMP has recently gained momentum. [Bley et al. \(2013\)](#) studied the migration of a network as the problem of finding the order of link upgrades that minimizes the total disruption time. With no travels allowed between the sites, their problem involves no routing decisions. [Jaumard et al. \(2016\)](#) proposed a CG-based heuristic for the NMP that decides on the technician-to-circuit assignments, as well as travel paths for the technicians. [Pouya and Jaumard \(2017\)](#) observed that the prior mathematical formulations suffer from the highly symmetric nature of the circuits and technicians. A new symmetry-breaking model was developed, leading to significant gains in computational effort. Yet again there is no guarantee that the obtained solutions are indeed optimal/feasible for the individual circuit assignments. Subsequently, [Jaumard and Pouya \(2018\)](#) designed a two-phase CG-based heuristic for the planning of real-size networks. A greedy algorithm was proposed by [Dawadi et al. \(2021\)](#) for the cost minimization of a multi-technology migration problem. With the objective of minimizing the number of out-of-service sites and travels, [Javad-Kalbasi and Valaee \(2021\)](#) modeled the NMP as a binary quadratic program. They derived heuristic solutions using Digital Annealer and solved instances with up to 64 circuits.

In this work, we develop an exact decomposition framework for the NMP. To the best of our knowledge, both in the context of the telecommunications problems and as a VRPS, this is the first method that exactly solves the NMP with a certificate of optimality/infeasibility. We decompose the NMP into three problems and link them all by designing an LBBD algorithm. By doing so, we are able to leverage the power of different solution techniques for linear programming and combinatorial optimization, and delegate the task of solving each problem to the most suitable optimization paradigm. Numerical experiments defined over various real networks demonstrate the effectiveness of our algorithm in obtaining quality solutions with reasonable computational effort. Given that the NMP can be viewed as an SNRP, our method can also be adapted to a wide class of integer programming problems.

Contributions. The contributions of our work are summarized as follows.

- We develop the first exact solution method for the network migration problem in order to find the optimal planning solutions, i.e., the order of circuit upgrades, along with the technician assignment and routing decisions. Although our solution framework is developed for the NMP, it is also applicable for the VRP with node synchronization.
- In order to reduce the computational effort of the LBBD master problem, we further decompose it via the Benders decomposition, resulting in a mixed-integer program (MIP) as the master problem, and a CP-based CG formulation as its subproblem. Additionally, we augment this CG formulation with an auxiliary MIP subproblem, leading to a hybrid CP/MIP-based CG model that significantly improves its performance.
- Considering the planning nature of our LBBD subproblem, we propose a CP model that, given the number of migrated circuits in a maintenance window, decides on the optimal technician assignments, order of circuit upgrades, and travel routes, if any.
- For our LBBD-based decomposition framework, we design valid feasibility and optimality cuts that guarantee the convergence of the algorithm. We also characterize a set of solutions other than the candidate for which our optimality cuts are tight.
- We evaluate the proposed LBBD algorithm on instances defined over six real backbone and regional networks and provide detailed algorithmic analysis and discussions on the implementation choices, along with managerial insights on the trade-offs among the migration cost, resource usage, and the duration of the migration.

The remainder of the paper is organized as follows. In Section 2 we present the problem statement and its mathematical formulation. In Section 3 we develop an LBBD solution

framework for solving the NMP. In Section 4 we evaluate the performance of our algorithm on benchmark networks and provide managerial insights. Section 5 concludes the paper.

2. Problem Description

In this section, we formally describe the problem and introduce the sets and parameters. Next, we formulate the NMP as a CG-based integer linear program (ILP).

2.1. Problem Statement

The network migration problem is defined on a telecommunication network represented by a set of *sites* \mathcal{S} (or nodes) and a set of *circuits* \mathcal{C} (corresponding to edges) between the site pairs $\{s, s'\} \in \mathcal{S}_p$. Every site s is located in a geographical *region* $r \in \mathcal{R}$ (e.g., a city). To each region r , η_r^{TECH} number of *technicians* are assigned. Technicians assigned to a given region r can only work in that region. Considering that migrations often occur during low traffic periods (mostly nights), and the distance between the regions may require long-haul flights, employing local technicians is the safest option to minimize unforeseen impacts of the travels on migration planning. A circuit is migrated by disconnecting its endpoints from the old equipment and connecting them to the new one. These operations are performed by two technicians within the same maintenance window, each working at one endpoint. Additionally, every circuit migration requires an *engineer* that coordinates the technicians remotely and does not need to be present in the working site. There are at most η^{ENG} engineers available, and every engineer can coordinate up to α^{ENG} technicians. Figure 1 demonstrates an example of such a network with 5 regions, 9 sites, and 20 circuits. Migration of the network is performed during a *maintenance window*, which is a period of time usually at night or a specially low traffic time on the network and also the roads (in case of travel between the sites). Every maintenance window $w \in \mathcal{W}$ has a predefined duration e.g., 8 hours, and all operations have to be completed within this duration. Time required to migrate a circuit $c \in \mathcal{C}$ is θ . Since migrating every circuit results in a short disruption in the network and the number of disruptions cannot violate clients' Service Level Agreement (SLA) (Fawaz et al. 2004), there is a limit η^{CIR} on the number of migrated circuits per maintenance window.

A technician working in a given region r during maintenance window w is responsible for a *shift*. A shift is defined as a set of circuit endpoints migrated by a single technician during a maintenance window, together with any travels between the sites. In addition to staff shortage in a given region, the need for a specific expertise is another reason for forcing

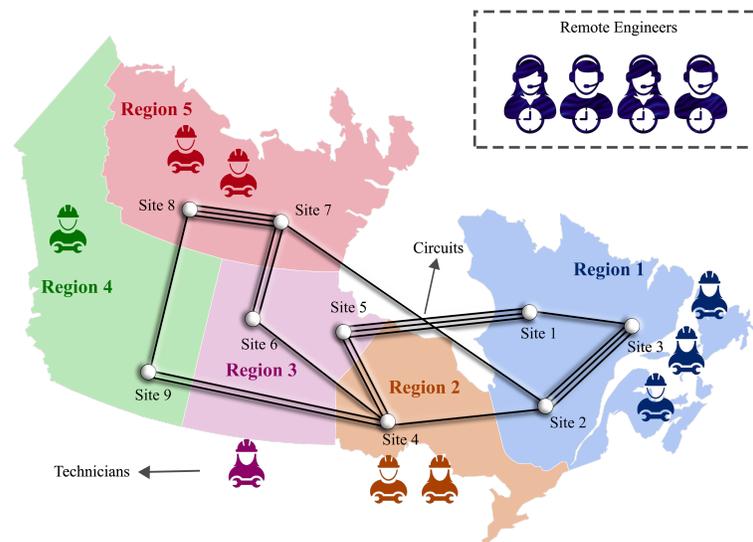


Figure 1 An example of a telecommunication network for the network migration problem

a technician to travel to another region. Figure 2 represents a subset of possible shifts as the solution of the NMP. This solution considers 3 shifts (for 3 technicians) in region 1, 1 shift in region 2, 1 shift in region 3 that includes a travel from site 6 to site 5, 1 shift in region 4 and 2 shifts in region 5. An example of such possible shifts for one maintenance

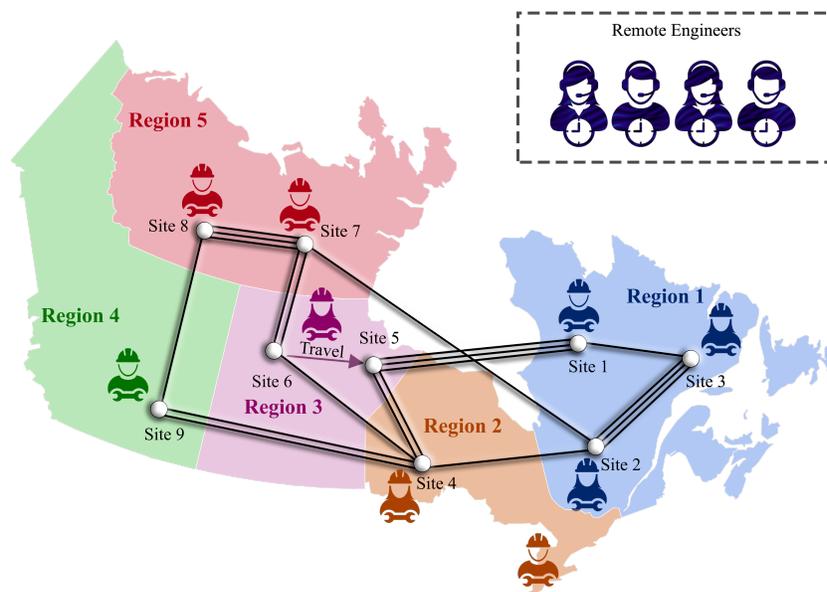


Figure 2 A possible solution as a subset of shifts

window is depicted in Figure 3. A set of boxes in front of a technician t shows its shift in this maintenance window. Every box includes a site s and the number of circuit endpoints $n_{ss'}$ that t migrates in this shift. Δ is a given set of possible shift durations, e.g. $\{6h, 8h\}$. One

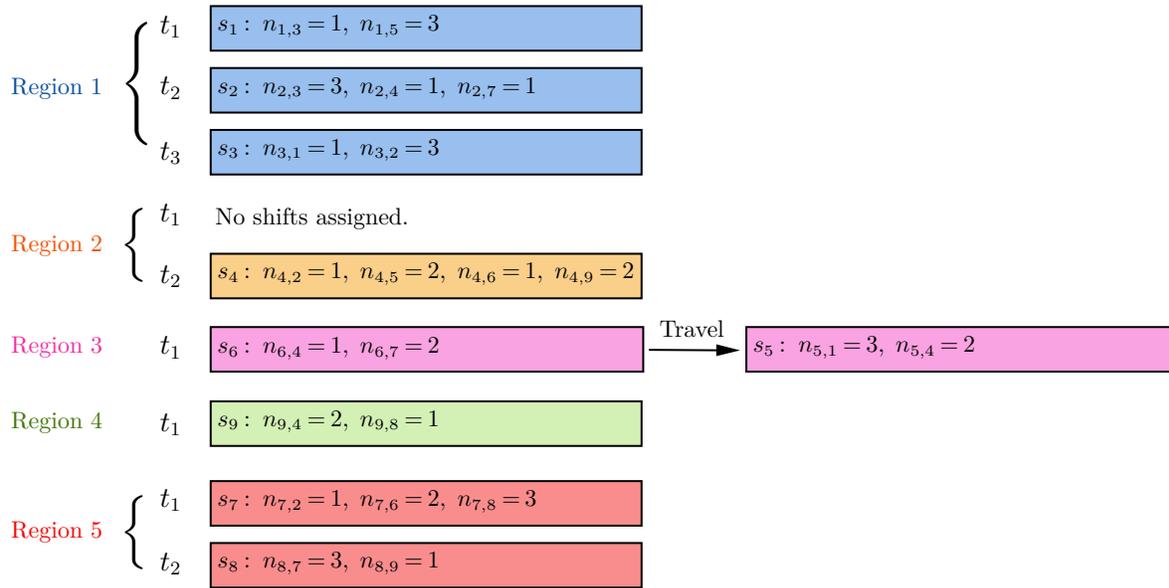


Figure 3 An example of possible shifts for the solution presented in Figure 2

reason for having multiple durations is related to the payment policy. Technicians should be paid for a minimum number of hours per shift. For example, if a technician works for any time less than 6 hours, they will be paid for the full 6 hours, while another technician working longer than 6 hours will be paid for 8 hours. In addition, access to the sites and the time spent at the sites should be within the SLA. Having multiple shift durations helps to avoid requesting unnecessary long access periods. Shift duration is determined based on SLA and is a practical aspect of the problem.

Migration costs include payments to the technicians and engineers. The network migration problem is to determine the order of upgrading the circuits in order to minimize the migration costs, i.e., building a set of minimum-cost plans for the technicians. Table 1 presents the parameters and notations used in our model.

REMARK 1. Vectors, matrices, and scalars are represented by bold (\mathbf{a}), capital (A) and lower-case with regular font (a), respectively. We use $(\cdot)^\top$ for the transpose operator, $|\cdot|$ for the size of a set, $\mathbf{1}$ and $\mathbf{0}$ respectively, for vectors of 1s and 0s, adjusted to the required size. We use relational operators for element-wise comparison of two vectors.

Table 1 Sets and parameters of the NMP

Notation	Description
<i>Sets:</i>	
\mathcal{S}	Set of sites (indexed by s)
\mathcal{S}_p	Set of site pairs $\{s, s'\}$ with at least one circuit between them
\mathcal{R}	Set of regions (indexed by r)
\mathcal{S}_r	Set of sites in region r
\mathcal{W}	Set of available maintenance windows (indexed by w)
\mathcal{C}	Set of circuits (indexed by c)
$\mathcal{C}_{ss'}$	Set of circuits between sites s and s'
\mathcal{C}_r	Set of circuits with at least one endpoint in the sites $s \in \mathcal{S}_r$
Δ	Ordered set of possible durations for a shift (e.g., 360 or 480 minutes) with $\max(\Delta) = \underline{\Delta}$, indexed by δ
<i>Parameters:</i>	
$\phi_{ss'}$	Number of circuits between sites s and s' ($\phi_{ss'} = \phi_{s's}$)
η_r^{TECH}	Max. number of technicians available in region r in a maintenance window
η^{CIR}	Max. number of circuits allowed to be migrated in a maintenance window
η^{ENG}	Max. number of engineers available in a maintenance window
$\text{COST}^{\text{TECH}}$	Hourly cost of a technician
COST^{ENG}	Hourly cost of an engineer
$T_{ss'}$	Travel time between sites s and s'
θ	Time required to migrate one endpoint of a circuit
α^{ENG}	The number of technicians supported by one engineer

2.2. Problem Formulation

We model the NMP as an ILP that returns a planning solution consisting of a set of shifts. The proposed formulation is amenable to the LBBDD framework, meaning that we can decouple the problem into smaller subproblems that are easier to solve.

Denote by $m_{ss'w} \in \mathbb{Z}_+$, a decision variable that determines the number of circuits between $\{s, s'\}$ migrated during maintenance window w . Let Γ be the set of all possible shifts (in Section 3.2.1 we *implicitly* enumerate this set). For a shift $\gamma \in \Gamma$, the decision variable $z_\gamma \in \mathbb{Z}_+$

determines the number of times γ is assigned to the technicians. Every shift γ is characterized by (i) $\Delta_{\text{SHIFT}}^\gamma$ its duration, (ii) $n_{ss'}^\gamma$ the number of circuit endpoints migrated between the pair of sites $\{s, s'\}$, and (iii) n_{CIR}^γ the total number of migrated circuit endpoints in the shift. The total set of shifts is denoted by $\Gamma = \bigcup_{w \in \mathcal{W}} \Gamma_w = \bigcup_{r \in \mathcal{R}, w \in \mathcal{W}} \Gamma_{rw}$, where Γ_w is the set of shifts for a maintenance window w and Γ_{rw} is the set of shifts for a technician located in region r during maintenance window w . The NMP can be formulated as:

$$\min \text{ COST} \sum_{\gamma \in \Gamma} \Delta_{\text{SHIFT}}^\gamma z_\gamma \quad (1a)$$

$$\text{s.t.} \quad \sum_{w \in \mathcal{W}} m_{ss'w} \geq \phi_{ss'} \quad \{s, s'\} \in \mathcal{S}_p, s < s' \quad (1b)$$

$$m_{ss'w} = m_{s'w} \quad \{s, s'\} \in \mathcal{S}_p, s < s', w \in \mathcal{W} \quad (1c)$$

$$\sum_{\{s, s'\} \in \mathcal{S}_p} m_{ss'w} \leq 2\eta^{\text{CIR}} \quad w \in \mathcal{W} \quad (1d)$$

$$\sum_{\gamma \in \Gamma_w} n_{ss'}^\gamma z_\gamma = m_{ss'w} \quad \{s, s'\} \in \mathcal{S}_p, w \in \mathcal{W} \quad (1e)$$

$$\sum_{\gamma \in \Gamma_{rw}} z_\gamma \leq \eta_r^{\text{TECH}} \quad r \in \mathcal{R}, w \in \mathcal{W} \quad (1f)$$

$$\sum_{\gamma \in \Gamma_w} z_\gamma \leq \alpha^{\text{ENG}} \eta^{\text{ENG}} \quad w \in \mathcal{W} \quad (1g)$$

$$\mathbf{z} \in \mathbb{Z}_+^{|\Gamma|}, \quad \mathbf{m} \in \mathbb{Z}_+^{|\mathcal{S}_p| \times |\mathcal{W}|}, \quad (1h)$$

where $\text{COST} = \text{COST}^{\text{TECH}} + \frac{\text{COST}^{\text{ENG}}}{\alpha^{\text{ENG}}}$. The objective function (1a) is the cost of the NMP, which is defined as the total technician and engineer costs over the duration of the migration. Constraints (1b) assure that all circuits between every two sites s and s' are migrated. Constraints (1c) enforce the number of migrated circuits from s to s' in w to be equal to the number of circuits migrated from s' to s . Constraints (1d) establish the bound on the number of migrated circuits at every maintenance window. Through constraints (1e), variables $m_{ss'w}$ are determined by aggregating over the number of migrated circuits between $\{s, s'\}$ during the shifts at w . Constraints (1f) and (1g) ensure that at w , the number of available technicians and engineers are respected. Constraints (1h) define the variable domains.

The size of Γ , the set of all shifts, is an exponential function of the number of circuits, hence it is not reasonable (or even possible) to include them all in solving the model (1). Column generation is a method for implicitly enumerating such a large set of columns that

relies on the duality theory for linear programming (LP). In the presence of integer decision variables, the *branch-and-price* (B&P) algorithm combines the branch-and-bound framework for solving a MIP with the CG procedure. The performance of a B&P depends on the strength of the LP bound, as well as the employed branching and search strategy. It has been observed that branching on the variables of the master problem associated with the generated columns is not efficient and results in an unbalanced tree (Vance 1998, Vanderbeck 2011). Branching on the aggregate variables of the original formulations, in case of identical subproblems, is not typically sufficient to eliminate all fractional solutions. Although this branching scheme theoretically does not guarantee the integrality of the solution, it experimentally returns the integral solution for some instances. Vanderbeck (2011) proposes a generic branching scheme based on the aggregated value of the original variables when returning to non-identical systems. In our preliminary experiments though, specially for small to medium-size instances, the quality of the lower bound was poor and did not improve adequately as the B&P proceeded. Therefore, since branching on neither the subproblem nor the master problem variables for the NMP solved any of our instances, we concluded that a pure B&P is not suitable to obtain exact solutions for the NMP.

In the next section, we develop an LBBDD framework that entails multiple levels of decomposition, with a proof of optimality if one exists, or infeasibility when it does not.

3. Solution Method

In what follows, we first discuss the general LBBDD framework for the NMP, then we provide detailed discussions on each step of the algorithm.

3.1. Logic-based Benders Decomposition

In the problem formulation (1), decision variables \mathbf{z} and \mathbf{m} are only linked through (1e). Furthermore, except for the constraints (1b), the problem is decomposable by maintenance windows. Using these two points, we propose the following decomposition. Denote by $\text{SP}_w^{\text{LBBDD}}(\mathbf{m}_w)$, the problem of generating a set of shifts with minimum cost for a given \mathbf{m}_w . Let η_w be the migration cost at w . We can reformulate the model (1) as below:

$$\min \left\{ \sum_{w \in \mathcal{W}} \eta_w : (1b) - (1d), \eta_w \geq \text{SP}_w^{\text{LBBDD}}(\mathbf{m}_w), w \in \mathcal{W}, \mathbf{m} \in \mathbb{Z}_+^{|\mathcal{S}_p| \times |\mathcal{W}|} \right\}. \quad (2)$$

Formulation (2) has the structure of a two-stage problem: the first-stage (master) problem decides on the number of migrated circuits between the site pairs at each maintenance

window, along with an estimation on the cost of such a plan; the second-stage (recourse) problems verify if it is feasible to migrate the assigned number of circuits with the available resources, and if so what is the actual cost of this migration.

The *Benders decomposition* (Benders 1962) is a well-established solution method for two-stage linear programs with continuous second-stage variables. After decomposing the problem into a master problem and a subproblem, it iteratively approximates the optimal solution to the recourse problem via Benders *feasibility* and *optimality* cuts (see Section 3.2 for more details), derived using the LP duality theory. For two-stage problems with mixed-integer recourse decisions, logic-based Benders decomposition (Hooker and Ottosson 2003) is a generalization of the Benders decomposition (including its special case in the context of stochastic programming, the integer L-shaped method; see Laporte and Louveaux (1993), Angulo et al. (2016)). For the NMP, the LBB D master problem is:

$$\text{MP}^{\text{LBB D}} = \min \sum_{w \in \mathcal{W}} \eta_w \tag{3a}$$

$$\text{s.t. (1b) – (1d)} \tag{3b}$$

$$(\eta_w, \mathbf{m}_w) \in \Lambda_w^{\text{FEAS}}, \quad w \in \mathcal{W} \tag{3c}$$

$$(\eta_w, \mathbf{m}_w) \in \Lambda_w^{\text{OPT}}, \quad w \in \mathcal{W} \tag{3d}$$

$$\mathbf{m} \in \mathbb{Z}_+^{|\mathcal{S}_p| \times |\mathcal{W}|}, \boldsymbol{\eta} \geq \mathbf{0}. \tag{3e}$$

where Λ_w^{FEAS} and Λ_w^{OPT} are sets of feasibility and optimality cuts, respectively, and together represent an LBB D subproblem $\text{SP}_w^{\text{LBB D}}(\mathbf{m}_w)$. The LBB D starts by $\Lambda_w^{\text{FEAS}} = \Lambda_w^{\text{OPT}} = \emptyset$ and at each iteration expands these sets with cuts if necessary until they are representative of the subproblem. The LBB D relies on logical reasoning for obtaining feasibility and optimality cuts, and as long as we can have feasibility and optimality certificates, the LBB D subproblem can have any form. Having such a generic framework, the LBB D depends on the modeler for designing problem-specific cuts and, unlike the Benders decomposition, does not have a readily available cut development mechanism.

The strength of the LBB D lies in the fact that it can integrate various optimization paradigms, most notably mixed-integer programming and constraint programming (Jain and Grossmann 2001, Hooker et al. 2012). Our first-stage problem $\text{MP}^{\text{LBB D}}$ is a variant of an assignment problem, which is suitable for a MIP-based solution approach. On the other hand, our second-stage problem takes $\overline{\mathbf{m}}_w$ as an argument and looks for the best plan for

such an assignment. CP is an optimization paradigm particularly powerful for planning and scheduling problems. Accordingly, in our LBBB solution framework for the NMP, we have a MIP model as the master problem, and $|\mathcal{W}|$ many CP models as the subproblems. The overall solution framework is presented in Figure 4 and explained in the remaining of this section. We first discuss the MP^{LBBB} and develop two other levels of decomposition to make it more informed. Then, for a master problem solution $\bar{\mathbf{m}}_w$, we formulate the $\text{SP}_w^{\text{LBBB}}(\bar{\mathbf{m}}_w)$ as a CP model, followed by the description of valid LBBB feasibility and optimality cuts that guarantee the convergence of the algorithm to an optimal solution.

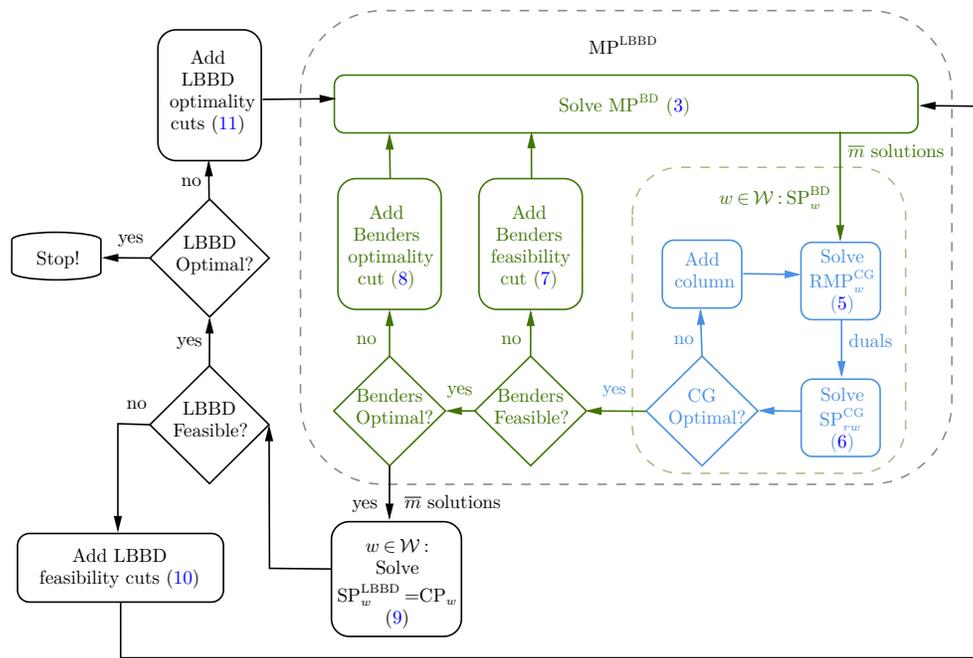


Figure 4 Overall framework of the LBBB method for the NMP.

3.2. The LBBB Master Problem: A Benders Decomposition

In its current form, MP^{LBBB} is oblivious to the structure of the NMP. In this section, our goal is to make the solutions of MP^{LBBB} more intelligent before passing them on to the $\text{SP}_w^{\text{LBBB}}(\bar{\mathbf{m}}_w)$. In the literature of LBBB, it has been observed that adding a relaxation of the subproblem considerably improves the performance of the algorithm (Elci and Hooker 2020). Often, this subproblem relaxation is in the form of an analytical expression based on the structure of the problem and is added as a bound to the master problem. We, however, resort to the LP relaxation of the subproblems to obtain valid inequalities for the MP^{LBBB} .

By relaxing the integrality constraints of the subproblems, we now have integer first-stage and continuous second-stage decision variables and the new problem is amenable to the Benders decomposition, with $\text{MP}^{\text{BD}} = \text{MP}^{\text{LBBDD}}$ as its master problem and the following subproblem for a maintenance window w :

$$\text{SP}_w^{\text{BD}}(\bar{\mathbf{m}}_w) = \min \text{ COST} \sum_{\gamma \in \Gamma_w} \Delta_{\text{SHIFT}}^\gamma z_\gamma \quad (4a)$$

$$\text{s.t.} \quad \sum_{\gamma \in \Gamma_w} n_{ss'}^\gamma z_\gamma \geq \bar{m}_{ss'w} \quad \{s, s'\} \in \mathcal{S}_p, w \in \mathcal{W} \quad (4b)$$

$$(1f) - (1g) \quad (4c)$$

$$\mathbf{z} \geq \mathbf{0}. \quad (4d)$$

Considering that (4a) has an exponential number of variables, in the next section we apply Dantzig-Wolfe decomposition principles to develop a column generation procedure for systematically adding them to the set of columns. For a review on the Dantzig-Wolfe decomposition and column generation, unfamiliar reader may refer to [Chvatal et al. \(1983\)](#).

3.2.1. The Benders Subproblem: A Column Generation Method. A CG solution method starts with solving the *restricted master problem*, defined as the original problem with a (potentially empty) subset of all the columns. Then, the optimal dual solutions are passed to a pricing problem (the CG subproblem) that checks their feasibility in the LP dual of the original problem, and if not, adds an improving column to the master problem. Once the pricing problem determines that a feasible dual solution is found, the CG stops as we have reached the optimality. By design, the pricing problem implicitly considers all the columns by using the properties that define valid columns of the master problem. Although the Dantzig-Wolfe decomposition lays out a precise scheme for decomposing a problem for the CG method, the master and pricing problems are often built by problem-specific modeling practices. In defining the “shifts” and z_γ as the decision variables we have used such an approach which can directly be translated into the pricing problems.

The CG master problem. Master problem of the NMP selects the best set of shifts for the technicians among a subset of columns $\Gamma' \subseteq \Gamma$, and its pricing problems generate improving shifts. The CG master problem is:

$$\text{RMP}_w^{\text{CG}} = \min \text{ COST} \sum_{\gamma \in \Gamma'} \Delta_{\text{SHIFT}}^\gamma z_\gamma \quad (5a)$$

$$\text{s.t. } \sum_{\gamma \in \Gamma'_w} n_{ss'}^\gamma z_\gamma \geq \bar{m}_{ss'w} \quad \{s, s'\} \in \mathcal{S}_p \quad (5b)$$

$$\sum_{\gamma \in \Gamma'_{rw}} z_\gamma \leq \eta_r^{\text{TECH}} \quad r \in \mathcal{R} \quad (5c)$$

$$\sum_{\gamma \in \Gamma'_w} z_\gamma \leq \alpha^{\text{ENG}} \eta^{\text{ENG}} \quad (5d)$$

$$\mathbf{z} \geq \mathbf{0}. \quad (5e)$$

Constraints of RMP_w^{CG} correspond to the constraints (1e) - (1h), except for the set of columns Γ' , and the fact that they are for a single maintenance window w .

The CG pricing problem. In the definition of the CG subproblems, for the sake of brevity, we drop the index γ from the decision variables. Denote by $\bar{\pi}^{(5b)}$, $\bar{\pi}^{(5c)}$, $\bar{\pi}^{(5d)}$, the optimal dual solutions associated with constraints (5b), (5c) and (5d), respectively. The pricing problem generating a shift for region r and maintenance window w is as follows:

$$\text{SP}_{rw}^{\text{CG}} = \min \text{COST} \Delta_{\text{SHIFT}} - \sum_{s \in \mathcal{S}_r} \sum_{s' \in \mathcal{S}} n_{ss'} \bar{\pi}_{ss'}^{(5b)} - \bar{\pi}_{rw}^{(5c)} - \bar{\pi}_w^{(5d)} \quad (6a)$$

$$\text{s.t. } \{\text{Constraints defining a valid shift}\} \quad (6b)$$

$$n_{ss'} \in \mathbb{Z}_+ \quad s \in \mathcal{S}_r, \{s, s'\} \in \mathcal{S}_p \quad (6c)$$

$$\Delta_{\text{SHIFT}} \geq 0. \quad (6d)$$

Objective function (6a) is the reduced-cost that determines if RMP_w^{CG} is at optimality. If not, the pricing problem generates a column that corresponds to a “valid” shift, and is characterized by its duration Δ_{SHIFT} and the number of circuits migrated between each site pair. A shift is made of a sequence of site visits by the technician. So constraints (6b) define a valid shift as a connected path over the sites in the region r such that the duration of the shift does not exceed the maximum possible duration of the maintenance window. Furthermore, as the endpoints of a circuit should be migrated by two technicians in the same maintenance window, at most one of the endpoints of every circuit $c \in \mathcal{C}_{ss'}$ can be migrated in a shift. In Section 3.3, we present a constraint programming model for generating a set of valid shifts in a maintenance window w , for all the regions and their technicians. Our preliminary experiments revealed that, because of the routing decisions in the pricing problems and the presence of loop elimination constraints, $\text{SP}_{rw}^{\text{CG}}$ as a CP model performs much better than a MIP. Therefore, in lieu of model (6), we fix the region in the CP model of Section 3.3 and solve it for one technician, with the objective function (6a).

REMARK 2. To accelerate the solution process, we use a hybrid CG, where first an auxiliary pricing problem generates improving columns with “ordered paths” that only have (s, s') , $s < s'$ links (model description is given in the e-companion). After the auxiliary problem converges, we solve $\text{SP}_{rw}^{\text{CG}}$ to verify the optimality. We observed that this two-subproblem strategy greatly improves the performance of the algorithm. The reason is that on backbone networks, it is quite possible that many regions have a few number of sites. If a region has up to two sites, the auxiliary problem alone guarantees the optimality.

Let $\overline{\text{SP}}_{rw}^{\text{CG}}$ be the optimal solution of the $\text{SP}_{rw}^{\text{CG}}$. If $\overline{\text{SP}}_{rw}^{\text{CG}} \geq 0, \forall r \in \mathcal{R}$, then the CG procedure stops. Otherwise, for each $r \in \mathcal{R}$ with $\overline{\text{SP}}_{rw}^{\text{CG}} < 0$ we add the generated column to Γ' and repeat the process. As a result of having the constraints (5b), we require an initial set of columns $\Gamma' \neq \emptyset$ that make the RMP_w^{CG} feasible. The usual approach for generating such Γ' is to go through an initial phase (INIT) where an artificial non-negative decision variable ρ is added to each “ \geq ” constraint with a positive right-hand-side, and the objective function is replaced with $\mathbf{1}^\top \rho$. The INIT phase iteratively adds columns to Γ' until no improving column with negative reduced-cost can be found. If the CG procedure for the new problem stops with an optimal value equal to zero, then the artificial decision variables are removed from the problem, the original objective function is brought back and the generated columns are selected as Γ' . Otherwise, if the optimal objective value is positive, we can conclude that the original problem is infeasible.

3.2.2. Benders Cuts. Depending on the status of the RMP_w^{CG} after solving, we might need to add feasibility (optimality) cuts to Λ_w^{FEAS} (Λ_w^{OPT}). Benders decomposition provides us with off-the-shelf cuts through the dual solutions of the subproblems. For our problem, because the subproblems are solved via CG, the feasibility cuts are not immediately clear. Next, we present the Benders feasibility and optimality cuts for the NMP and show that they are valid, despite being obtained from a restricted set of columns in the $\text{SP}_{rw}^{\text{CG}}$.

Benders feasibility cut. Assume that, at the end of the INIT phase, the CG procedure stops with $\rho \neq \mathbf{0}$, and $\overline{\pi}^{(5b)}, \overline{\pi}^{(5c)}, \overline{\pi}^{(5d)}$ are returned from the (modified) RMP_w^{CG} . The Benders feasibility cut to be added to Λ_w^{FEAS} is as follows:

$$0 \geq \sum_{\{s,s'\} \in \mathcal{S}_p} \overline{\pi}_{ss'}^{(5b)} m_{ss'w} + \sum_{r \in \mathcal{R}} \overline{\pi}_r^{(5c)} \eta_r^{\text{TECH}} + \overline{\pi}^{(5d)} \alpha^{\text{ENG}} \eta^{\text{ENG}} \quad (7)$$

To show that (7) is indeed a feasibility cut for the MP^{BD} and cuts off the infeasible solution $\overline{\mathbf{m}}_w$, we first prove that $(\overline{\pi}^{(5b)}, \overline{\pi}^{(5c)}, \overline{\pi}^{(5d)})$ constitutes a certificate of infeasibility for the

MP^{BD} , even if it is derived from the RMP_w^{CG} with $\Gamma' \subseteq \Gamma$. In the following theorem, we show that this is true for any Dantzig-Wolfe decomposition at the end of the INIT phase.

Theorem 1. Consider $\bar{P}(\mathcal{I}')$, the restricted master problem of a Dantzig-Wolfe decomposition at the end of the INIT phase, and $P(\mathcal{I}', \mathbf{c}_{\mathcal{I}'})$, the problem that is obtained by removing the artificial variables $\boldsymbol{\rho}$ from $\bar{P}(\mathcal{I}')$ and bringing back the original objective function:

$$\begin{aligned} \bar{P}(\mathcal{I}') = \min \quad & \mathbf{1}^\top \boldsymbol{\rho} & P(\mathcal{I}', \mathbf{c}_{\mathcal{I}'}) = \min \quad & \mathbf{c}_{\mathcal{I}'}^\top \mathbf{x}_{\mathcal{I}'} \\ \text{s.t.} \quad & A_{\mathcal{I}'} \mathbf{x}_{\mathcal{I}'} + \boldsymbol{\rho} \geq \mathbf{b} & \text{s.t.} \quad & A_{\mathcal{I}'} \mathbf{x}_{\mathcal{I}'} \geq \mathbf{b} \\ & \mathbf{x}_{\mathcal{I}'}, \boldsymbol{\rho} \geq \mathbf{0}, & & \mathbf{x}_{\mathcal{I}'} \geq \mathbf{0}, \end{aligned}$$

where $\emptyset \neq \mathcal{I}' \subseteq \mathcal{I}$, and $\mathbf{c}_{\mathcal{I}'}, A_{\mathcal{I}'}$ are the cost vector and columns associated with \mathcal{I}' . If $\boldsymbol{\pi}$ is a certificate of infeasibility for $P(\mathcal{I}', \mathbf{c}_{\mathcal{I}'})$, then $\boldsymbol{\pi}$ is a certificate of infeasibility for $P(\mathcal{I}, \mathbf{c}_{\mathcal{I}})$.

Proof. We first find the certificate of infeasibility for $P(\mathcal{I}', \mathbf{c}_{\mathcal{I}'})$. Then we show that it is also a certificate of infeasibility for $P(\mathcal{I}, \mathbf{c}_{\mathcal{I}})$. Let $(\bar{\mathbf{x}}_{\mathcal{I}'}, \bar{\boldsymbol{\rho}})$ denote the optimal solution of $\bar{P}(\mathcal{I}')$.

1. From (a variant of) the Farkas Lemma we know that exactly one of the following system of inequalities has a solution (Matousek and Gärtner 2007):

$$(I) \ A_{\mathcal{I}'} \mathbf{x}_{\mathcal{I}'} \geq \mathbf{b}, \mathbf{x}_{\mathcal{I}'} \geq \mathbf{0}, \quad (II) \ \boldsymbol{\pi}^\top A_{\mathcal{I}'} \leq \mathbf{0}, \boldsymbol{\pi}^\top \mathbf{b} > 0, \boldsymbol{\pi} \geq \mathbf{0}.$$

If $\bar{\boldsymbol{\rho}} \neq \mathbf{0}$, then $\{\mathbf{x} : A_{\mathcal{I}'} \mathbf{x} \geq \mathbf{b}, \mathbf{x} \geq \mathbf{0}\} = \emptyset$ and $P(\mathcal{I}', \mathbf{c}_{\mathcal{I}'})$ is infeasible. Therefore, there exists a $\boldsymbol{\pi}_0$ that satisfies the inequalities of (II) and is a certificate of infeasibility for $P(\mathcal{I}', \mathbf{c}_{\mathcal{I}'})$. Consider the dual of $\bar{P}(\mathcal{I}')$ as follows:

$$\bar{D}(\mathcal{I}') = \max\{\boldsymbol{\pi}^\top \mathbf{b} : \boldsymbol{\pi}^\top A_{\mathcal{I}'} \leq \mathbf{0}^\top, \boldsymbol{\pi} \leq \mathbf{1}, \boldsymbol{\pi} \geq \mathbf{0}\}.$$

From $\bar{\boldsymbol{\rho}} \neq \mathbf{0}$, we have $\mathbf{1}^\top \bar{\boldsymbol{\rho}} = \bar{\boldsymbol{\pi}}^\top \mathbf{b} > 0$, with $\bar{\boldsymbol{\pi}}$ the optimal solution of $\bar{D}(\mathcal{I}')$. Clearly $\bar{\boldsymbol{\pi}}$ satisfies (II) and we can set $\boldsymbol{\pi}_0 = \bar{\boldsymbol{\pi}}$. So $\bar{\boldsymbol{\pi}}$ is the Farkas certificate of $P(\mathcal{I}', \mathbf{c}_{\mathcal{I}'})$.

2. Let $D(\mathcal{I}, \mathbf{0}) = \max\{\boldsymbol{\pi}^\top \mathbf{b} : \boldsymbol{\pi}^\top A_{\mathcal{I}} \leq \mathbf{0}^\top, \boldsymbol{\pi} \geq \mathbf{0}\}$ be the dual of $P(\mathcal{I}, \mathbf{0})$. Since $\bar{P}(\mathcal{I}')$ is the last restricted master problem at phase one, no improving column with negative reduced-cost is found through the pricing problem with the dual solution of $\bar{P}(\mathcal{I}')$, i.e., $\mathbf{0}^\top - \bar{\boldsymbol{\pi}}^\top A_{\mathcal{I}} \geq 0$ meaning that $\bar{\boldsymbol{\pi}}$ is feasible for $D(\mathcal{I}, \mathbf{0})$, and consequently for inequalities of (II) with the full matrix $A_{\mathcal{I}}$. Using the Farkas Lemma one more time implies that $\{\mathbf{x} : A_{\mathcal{I}} \mathbf{x} \geq \mathbf{b}, \mathbf{x} \geq \mathbf{0}\} = \emptyset$, hence $\bar{\boldsymbol{\pi}}$ is a certificate of infeasibility for $P(\mathcal{I}, \mathbf{c}_{\mathcal{I}})$. \square

From the above discussions, $(\bar{\pi}^{(5b)}, \bar{\pi}^{(5c)}, \bar{\pi}^{(5d)})$ is a proof of infeasibility for MP^{BD} with $0 < \sum_{\{s,s'\} \in \mathcal{S}_p} \bar{\pi}_{ss'}^{(5b)} \bar{m}_{ss'w} + \sum_{r \in \mathcal{R}} \bar{\pi}_w^{(5c)} \eta_r^{\text{TECH}} + \bar{\pi}^{(5d)} \alpha^{\text{ENG}} \eta^{\text{ENG}}$ and can be removed using the inequality (7). Here we should mention that, we can also add the LBB D feasibility cuts (Section 3.4.1) for cutting infeasible solutions. However, as will be discussed later, our LBB D feasibility cuts are costly and inequalities (7) improve the overall efficiency of the method.

Benders optimality cut. If at the end of the INIT phase, $\rho = \mathbf{0}$, then $\text{SP}_{rw}^{\text{CG}}$ is feasible. Now we should check if $\bar{\eta}_w$ is an accurate estimate of the $\overline{\text{SP}}_w^{\text{BD}}(\bar{\mathbf{m}}_w)$. Because the CG procedure stops at optimality, we can treat the missing columns in Γ' as non-basic variables. Therefore it is clear that the optimal dual solutions from solving RMP_w^{CG} with Γ' are the same as the optimal dual solutions of $\text{SP}_w^{\text{BD}}(\bar{\mathbf{m}}_w)$. If $\bar{\eta}_w \geq \overline{\text{SP}}_{rw}^{\text{CG}}$, then $(\bar{\eta}_w, \bar{\mathbf{m}}_w)$ is feasible in MP^{BD} . Otherwise, we cut it off by adding the following inequality to Λ_w^{OPT} :

$$\eta_w \geq \overline{\text{SP}}_{rw}^{\text{CG}} - \sum_{\{s,s'\} \in \mathcal{S}_p} \bar{\pi}_{ss'}^{(5b)} (\bar{m}_{ss'w} - m_{ss'w}). \quad (8)$$

3.3. The LBB D Subproblem: A Constraint Programming Model

With its roots in logic, CP is a modeling framework for combinatorial problems and has proven quite powerful for making planning, scheduling and routing decisions (Ciré et al. 2016). Integration of CP and mathematical programming models through LBB D (Hooker et al. 2000), and CG (Rousseau et al. 2004), often results in solution algorithms that outperform methods relying solely on either of the two. Hooker and van Hoesve (2018) provide a review on integration of CP and Operations Research. In this section, considering the presence of planning and routing decisions, we formulate the $\text{SP}_w^{\text{LBB D}}(\bar{\mathbf{m}}_w)$ as a CP model.

For a maintenance window w , a CP model serving as our LBB D subproblem creates a *plan*, i.e., a set of shifts corresponding to a set of technicians working during the maintenance window w . For example, assume that the solution illustrated in Figure 2 is planned for one maintenance window. A plan for this solution consists of determining for each technician, a connected path along with the number of circuit endpoints migrated between each site pair and the set of shifts in Figure 3 is one such plan. We see that in the CP model we need to treat the technicians as individual entities, unlike the engineers and circuit endpoints. Therefore, for each region r at maintenance window w , we define $\mathcal{T}_{rw} = \{1, \dots, \eta_r^{\text{TECH}}\}$ as its set of available technicians, and the CP model generates shifts for the technicians that belong to $\bigcup_{r \in \mathcal{R}} \mathcal{T}_{rw}$. It is clear that the newly introduced technician symmetry stays within

the CP model, since the only link between MP^{LBBDD} and its subproblems is through the \mathbf{m} decision variables defined for site pairs and independent of the technician (see Section 3.4).

Next, we describe the variable types, functions and global constraints used in the CP model, followed by the model description. While we use names and conventions of the CP Optimizer (Laborie et al. 2018), equivalent notions exist for other notable CP solvers such as the CP-SAT from Google OR Tools (Perron and Furnon 2019).

- IntegerVar() An *integer* decision variable. If defined as *Optional*, it can be absent from the CP solution.
- IntervalVar() An *interval* decision variable, modeling a time interval characterized by start, end and length. It can be optional.
- SequenceVar(\mathcal{X}) A *sequence* decision variable defining a total order over a set of interval variables \mathcal{X} . If all members of \mathcal{X} are absent, the sequence becomes empty.
- PresenceOf(a) Returns 1 if a is present in the solution, 0 otherwise.
- EndOf(a) Returns the finish time of the task if the interval variable a is present in the solution.
- LengthOf(a) Returns the time spent on the task if the interval variable a is present in the solution.
- Span(a, \mathcal{X}) Enforces the interval variable a to span the set of interval variables \mathcal{X} .
- NoOverlap(ℓ, T) Enforces the intervals in the sequence ℓ to be disjoint, while taking into account the transition times T .
- IfThen(p, q) Implements the logical constraint $p \rightarrow q$.

In addition to the sets and parameters described in Section 2, we need the variables defined in Table 3. Note that, although the variables are initially defined for all technicians, they are mostly “optional”, meaning that if no shift is assigned to a technician t from a region r , then its associated variables x_{rts} , $WTIME_{rt}$, Δ_{SHIFT}^{rt} , n_{rts}^S and $n_{rtss'}^{SP}$ are absent from the solution and vice-versa. For interval variables x_{rts} , $WTIME_{rt}$, the initial domain of their “start” and “end” times are $[0, \underline{\Delta}]$, the maximum time interval possible for a maintenance window. The initial domain of SEQ_{rt} is $\mathfrak{S}(\{x_{rts} : s \in \mathcal{S}_r\})$, the set of permutations from the interval variables x_{rts} . With the objective of minimizing the cost of a plan, the model $SP_w^{LBBDD}(\overline{\mathbf{m}}_w) = CP_w(\overline{\mathbf{m}}_w)$ is:

$$\min \text{ COST} \sum_{r \in \mathcal{R}} \sum_{t \in T_{rw}} \Delta_{SHIFT}^{rt} \quad (9a)$$

Table 3 Decision variables of the constraint programming model

Variable	Type	Initial domain	Optional	Description
x_{rts}	IntervalVar	$[[0, \underline{\Delta}], [0, \underline{\Delta}]]$	✓	Time plan for tech t of r at site s
SEQ_{rt}	SequenceVar	$\mathfrak{S}(\{x_{rts} : s \in \mathcal{S}_r\})$	×	Order of sites visited by tech t of r
$WTIME_{rt}$	IntervalVar	$[[0, \underline{\Delta}], [0, \underline{\Delta}]]$	✓	Working time of tech t of r
Δ_{SHIFT}^{rt}	IntegerVar	Δ	✓	Shift duration of tech t of r
n_{rts}^S	IntegerVar	$[0, \sum_{s'} \bar{m}_{ss'w}]$	✓	# of endpoints migrated by tech t of r at site s
$n_{rtss'}^{SP}$	IntegerVar	$[0, m_{ss'w}]$	✓	# of endpoints migrated by tech t of r at site s for $\{s, s'\}$

$$\text{s.t. NoOverlap}(SEQ_{rt}, T) \quad t \in \mathcal{T}_{rw}, r \in \mathcal{R} \quad (9b)$$

$$\text{Span}(WTIME_{rt}, \{x_{rts} : s \in \mathcal{S}_r\}) \quad t \in \mathcal{T}_{rw}, r \in \mathcal{R} \quad (9c)$$

$$\text{IfThen}(\text{PresenceOf}(WTIME_{rt}), \text{EndOf}(WTIME_{rt}) \leq \Delta_{SHIFT}^{rt}) \quad t \in \mathcal{T}_{rw}, r \in \mathcal{R} \quad (9d)$$

$$\text{PresenceOf}(x_{rts}) = \text{PresenceOf}(n_{rts}^S) \quad s \in \mathcal{S}_r, t \in \mathcal{T}_{rw}, r \in \mathcal{R} \quad (9e)$$

$$\text{IfThen}(\text{PresenceOf}(x_{rts}), \text{LengthOf}(x_{rts}) = \theta n_{rts}^S) \quad s \in \mathcal{S}_r, t \in \mathcal{T}_{rw}, r \in \mathcal{R} \quad (9f)$$

$$\sum_{s': \{s, s'\} \in \mathcal{S}_p} n_{rtss'}^{SP} = n_{rts}^S \quad s \in \mathcal{S}_r, t \in \mathcal{T}_{rw}, r \in \mathcal{R} \quad (9g)$$

$$n_{rtss'}^{SP} = n_{rts's}^{SP} \quad \{s, s'\} \in \mathcal{S}_p, t \in \mathcal{T}_{rw}, r \in \mathcal{R} \quad (9h)$$

$$\sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}_{rw}} n_{rtss'}^{SP} = \bar{m}_{ss'w} \quad \{s, s'\} \in \mathcal{S}_p \quad (9i)$$

$$\sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}_{rw}} \text{PresenceOf}(WTIME_{rt}) \leq \alpha^{ENG} \eta^{ENG}. \quad (9j)$$

By default, the interval variables in a SequenceVar are allowed to have overlaps. Using constraints (9b), we make sure that the time intervals a technician t spends at different sites are disjoint, and moreover these interval are defined apart enough such that there is time for the technician to travel from one site to another (as given by the matrix of travel times T). Constraints (9c) make the working time of the technicians consistent with the planned activities. For the working technicians in the current plan, constraints (9d) determine their shift duration from Δ . Constraints (9e) and (9f) together ensure that the circuit endpoints

are only distributed among the working technicians and the time a technician spends at a site is the amount of time it needs for migrating the endpoints. Constraints (9g) determine the site pairs to which the migrated endpoints belong, while constraints (9h) prevent the technician to move both ends of the circuits for the same site pair. Via constraints (9i), the number of planned circuit migrations between the site pairs, as given by the master problem, is respected in the CP solution. Through constraints (9j), the number of technicians to be used in each plan is bounded above by the maximum number of available engineers.

3.4. LBBB Cuts

Depending on the status of the $CP_w(\bar{\mathbf{m}}_w)$, we might need LBBB feasibility/optimalty cuts to proceed. Next, we design these cuts and discuss their merits and drawbacks.

3.4.1. LBBB Feasibility Cuts. Infeasibility of $CP_w(\bar{\mathbf{m}}_w)$ implies that there is not enough resources (technicians and/or engineers) to migrate $\bar{\mathbf{m}}_w$ circuit endpoints in w . Using LBBB feasibility cuts, we remove this solution from the master problem MP^{LBBB} , along with any other solution that requires at least the same amount of resources as $\bar{\mathbf{m}}_w$. MP^{LBBB} solutions are general integer, so the usual type of (strengthened) no-good cuts are not applicable to our problem. As such, for each generated $\bar{m}_{ss'w}$ we create a new binary variable to compare the solution $m_{ss'w}$ with $\bar{m}_{ss'w}$. At iteration τ of the LBBB algorithm, we add the following LBBB feasibility cuts to Λ_w^{FEAS} :

$$m_{ss'w} - (\phi_{ss'} + 1)\kappa_{ss'w}^\tau \leq \bar{m}_{ss'w} - 1, \quad \{s, s'\} \in \mathcal{S}_p : \bar{m}_{ss'w} > 0 \quad (10a)$$

$$\sum_{\substack{\{s, s'\} \in \mathcal{S}_p \\ \bar{m}_{ss'w} > 0}} \kappa_{ss'w}^\tau \leq |\mathcal{S}_p| - 1 \quad (10b)$$

$$\kappa_w^\tau \in \{0, 1\}^{|\mathcal{S}_p|} \quad (10c)$$

As the number of iterations grows, the number of binary variables created via (10) and the additional constraints can grow prohibitively large. However, as seen in our numerical experiments, the fact that the MP^{LBBB} solutions first go through the Benders decomposition phase (with its own feasibility checks), refines them and reduces the number of required feasibility cuts. Additionally, (10) cut-off not just one solution, but a potentially large set of solutions, further reducing the chance of needing new feasibility cuts. Proof of Proposition 1 explains the logic behind the design of our LBBB feasibility cuts.

Proposition 1. *The set of inequalities (10) are valid LBBB feasibility cuts for MP^{LBBB} .*

Proof. We introduce a vector of binary variables $\kappa_{ss'w}^\tau, \{s, s'\} \in \mathcal{S}_p$ that takes 1 if $m_{ss'w} \geq \bar{m}_{ss'w}$. We have the following logical relationships:

$$\begin{aligned}
 m_{ss'w} \geq \bar{m}_{ss'w} \implies \kappa_{ss'w}^\tau = 1 &\equiv \kappa_{ss'w}^\tau = 0 \implies m_{ss'w} < \bar{m}_{ss'w} && \text{(contraposition)} \\
 &\equiv \kappa_{ss'w}^\tau = 0 \implies m_{ss'w} \leq \bar{m}_{ss'w} - 1 && (m_{ss'w} \in \mathbb{Z}_+) \\
 &\equiv m_{ss'w} - (\phi_{ss'} + 1)\kappa_{ss'w}^\tau \leq \bar{m}_{ss'w} - 1 && (m_{ss'w} \leq \phi_{ss'})
 \end{aligned}$$

The last inequality becomes redundant if $\kappa_{ss'w}^\tau = 1$. In order to cut-off any solution $\mathbf{m}_w \geq \bar{\mathbf{m}}_w$, it is sufficient to make sure that the number of migrated circuit endpoints between at least one site pair $\{s, s'\}$ is less than $\bar{m}_{ss'w}$, which completes the proof. \square

3.4.2. LBB D Optimality Cuts. Assuming that $\text{CP}_w(\bar{\mathbf{m}}_w)$ is feasible and solved to optimality, let $\overline{\text{CP}}_w(\bar{\mathbf{m}}_w)$ be its optimal objective value. If $\bar{\eta}_w \geq \overline{\text{CP}}_w(\bar{\mathbf{m}}_w)$ for a $w \in \mathcal{W}$, then $(\bar{\mathbf{m}}, \bar{\eta}_w)$ is feasible for $\text{MP}^{\text{LBB D}}$. If $\bar{\eta}_w \geq \overline{\text{CP}}_w(\bar{\mathbf{m}}_w)$ for all $w \in \mathcal{W}$, then $\bar{\mathbf{m}}$ is optimal for the problem and the algorithm stops. Otherwise, for any w with $\bar{\eta}_w < \overline{\text{CP}}_w(\bar{\mathbf{m}}_w)$, we cut-off the solution pair $(\bar{\eta}_w, \bar{\mathbf{m}}_w)$. At iteration τ , we add the following LBB D optimality cuts to Λ_w^{OPT} :

$$(10a), (10c) \tag{11a}$$

$$\eta_w \geq \overline{\text{CP}}_w(\bar{\mathbf{m}}_w) \left(1 - \sum_{\substack{\{s,s'\} \in \mathcal{S}_p \\ \bar{m}_{ss'w} > 0}} (1 - \kappa_{ss'w}^\tau) \right) \tag{11b}$$

As with the feasibility cuts (10), we are creating a large number of new binary variables and constraints. Once more, our numerical analysis show that, for the considered instances, the number of iterations required for the algorithm to converge to a reasonable gap is small enough that $\text{MP}^{\text{LBB D}}$ remains scalable despite the large number of binary decision variables and constraints added at each iteration.

Proposition 2. *The set of inequalities (11) are valid LBB D optimality cuts for $\text{MP}^{\text{LBB D}}$.*

Proof. The LBB D optimality cut is valid if (i) it can determine whether the solution $\bar{\mathbf{m}}_w$ is optimal, (ii) returns a valid bound for the solutions other than $\bar{\mathbf{m}}_w$, and (iii) it is tight at $\bar{\mathbf{m}}_w$. Because the $\text{CP}_w(\mathbf{m})$'s cost function is non-decreasing, for any $\mathbf{m}_w \geq \bar{\mathbf{m}}_w$ we have $\overline{\text{CP}}_w(\mathbf{m}) \geq \overline{\text{CP}}_w(\bar{\mathbf{m}}_w)$. With the binary variables $\kappa_{ss'w}^\tau$ as defined in the proof of Proposition 1, it is clear that inequalities (11) are tight at $\bar{\mathbf{m}}_w$. It also imposes a negative bound on $\mathbf{m}_w < \bar{\mathbf{m}}_w$, and $\overline{\text{CP}}_w(\bar{\mathbf{m}}_w)$ if $\mathbf{m}_w > \bar{\mathbf{m}}_w$, which completes the proof. \square

We can also characterize a set of solutions other than $\overline{\mathbf{m}}_w$ for which the cuts (11) are tight: If $\hat{m}_{ss'w} - \overline{m}_{ss'w} > 0$, then at least $\theta(\hat{m}_{ss'w} - \overline{m}_{ss'w})$ minutes should be added to WTIME_{rt} for some technician t . Let $\overline{n}_{rts}^s, \overline{\text{WTIME}}_{rt}, \overline{\Delta}_{\text{SHIFT}}^{rt}$ be the solutions from the CP model. Define a bipartite graph $\mathcal{G} = (\mathcal{V}_1 \cup \mathcal{V}_2, \mathcal{E})$, with the following set of nodes:

$$\mathcal{V}_1 = \bigcup_{r \in \mathcal{R}} \mathcal{T}_{rw} \setminus \{t \in \mathcal{T} : \overline{n}_{rts}^s = 0\}, \quad \mathcal{V}_2 = \{\{s, s'\} \in \mathcal{S}_p : \hat{m}_{ss'w} - \overline{m}_{ss'w} > 0\}.$$

In other words, \mathcal{V}_1 is the set of working technicians and \mathcal{V}_2 is the set of site pairs that need extra circuit migration in the new solution. We build the set of links as follows:

$$\mathcal{E} = \{(t, \{s, s'\}) \in \mathcal{V}_1 \times \mathcal{V}_2 : \exists t \in \mathcal{T}_{s,w}, \overline{\text{WTIME}}_{rt} + \theta(\hat{m}_{ss'w} - \overline{m}_{ss'w}) \leq \overline{\Delta}_{\text{SHIFT}}^{rt}\},$$

i.e., we create a link between a technician that is already planned to work at s and has enough spare time within its shift to migrate the remaining circuits. If there exists a maximum matching of size $|\mathcal{V}_2|$, then $\overline{\text{CP}}_w(\overline{\mathbf{m}}_w)$ does not change and (11) is tight at $\hat{\mathbf{m}}_w$. Note that, the proposition is not biconditional.

Algorithm 1 presents the detailed steps of solving the NMP. Our LBBDD method converges with an exact solution, or returns “infeasible” if the original problem is infeasible. This follows from the integrality of \mathbf{m} , which makes the set of feasible solutions for MP^{LBBDD} finite. Using the feasibility and optimality cuts (10) and (11), we either cut-off the solution passed by MP^{LBBDD} or stop the algorithm if no action is needed after checking the feasibility and optimality conditions. When the algorithm stops, if we do not have any incumbent $\overline{\mathbf{m}}$, the original problem is infeasible, otherwise the algorithm has reached to optimality.

4. Numerical Results

In this section we have evaluated our LBBDD method with the NMP instances that are built over six networks of different sizes, with some deployed over a continent and another serving a region. In Section 4.1, we provide some details on the characteristics of the instances. This is followed by algorithmic and cost analysis discussions in Sections 4.3 and 4.4, respectively.

4.1. Data Sets

In Figure 5 we have reproduced the topology of six real optical fiber networks as reported by Knight et al. (2011). Considering the nodes of the graphs as the sites of the network, we use the nodes' coordinates to partition the sites into regions, using the quality threshold clustering algorithm with 80km threshold. Detailed characteristics of the base networks are

Algorithm 1 Solution algorithm for solving the NMP

```

1: while LBBB not optimal do
2:   while Benders not optimal do
3:     Solve  $MP^{BD}$  (3)
4:     Get  $\bar{m}$  solutions from  $MP^{BD}$ 
5:     for  $w \in \mathcal{W}$  do ▷  $SP_w^{BD}$ 
6:       Solve  $RMP_w^{CG}$  (5) with  $\bar{m}$ 
7:       for  $r \in \mathcal{R}$  do
8:         Solve  $SP_{rw}^{CG}$  (6)
9:         if  $\overline{SP}_{rw}^{CG} > 0$  then
10:           Add an improving column to  $RMP_w^{CG}$ 
11:         if  $\rho \neq 0$  then
12:           Add Benders feasibility cuts (7)
13:         else
14:           for  $r \in \mathcal{R}$  do
15:             if  $\bar{\eta}_w < \overline{SP}_{rw}^{CG}$  then
16:               Add Benders optimality cuts (8)
17:         for  $w \in \mathcal{W}$  do ▷  $SP_w^{LBBB}$ 
18:           Solve  $CP_w$  (9) with  $\bar{m}$ 
19:           if  $CP_w$  not feasible then
20:             Add LBBB feasibility cuts (10)
21:           else if  $\bar{\eta}_w < \overline{CP}_w(\bar{m}_w)$  then
22:             Add LBBB optimality cuts (11)
23: return  $\bar{m}$  from  $MP^{BD}$  and  $\bar{z}$  from  $RMP_w^{CG}$ s
    
```

Table 4 Characteristics of base networks

Characteristics	EUNETWORKS	NEXTGEN	PIONIER	SAGO	SAVVIS	VISIONNET
$ \mathcal{S} $	15	16	21	18	19	22
$ \mathcal{R} $	11	15	16	10	19	16
$ \mathcal{S}_r ^{\max}$	4	2	3	3	1	3
Geographic scope	Continent	Country	Country	Region	Country	Region
Coverage	Western Europe	Australia	Poland	Florida & Georgia	USA	Montana

given in Table 4, including the number of sites, regions, maximum number of sites in a region, their geographic extent and location. Let $\mu_{EP/s}$ and $\sigma_{EP/s}$ be the mean and standard deviation of the number of circuit endpoints at a site, respectively. Using a Lognormal distribution with parameters $\mu_{EP/s}$ and $\sigma_{EP/s}$, we have generated 114 NMP instances over

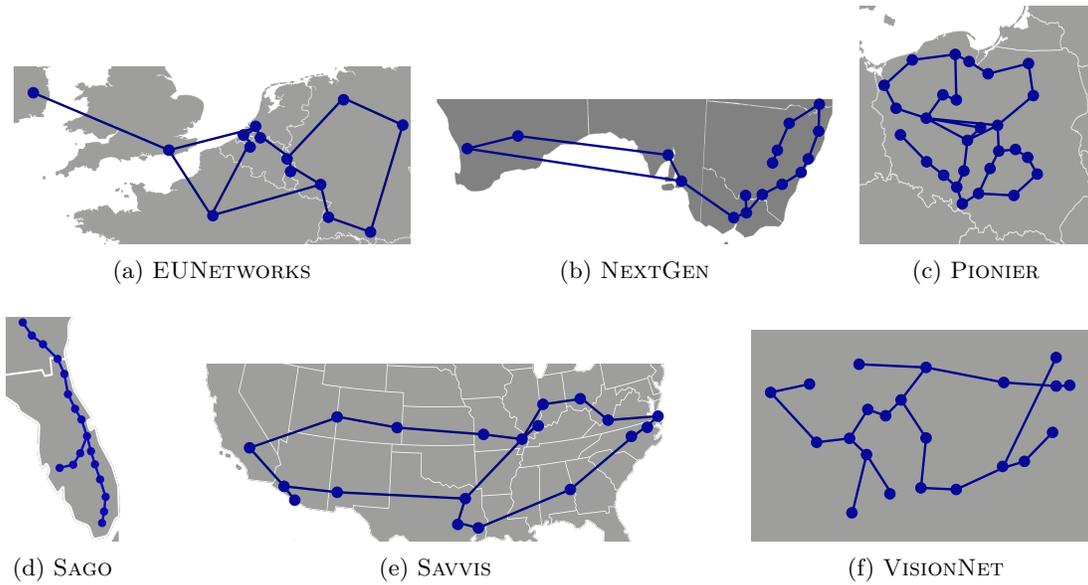


Figure 5 Topology of the base networks

the base networks by creating six instances per network and $(\mu_{EP/s}, \sigma_{EP/s})$. The considered values for $\mu_{EP/s}$ are comparable with $\mu_{EP/s} = 6.18$ of a real data set studied in Pouya et al. (2017) (Ciena data set I). The instances then are characterized by their base network, the number of circuit endpoints and site pairs with at least one circuit. Table 5 presents a summary of the characteristics of the instances, and the details are provided in the e-companion. Technicians may work in shifts of $\Delta_{\text{SHIFT}} \in \{6, 8\}$ hours and an engineer can coordinate 5 technicians at a time. These instance parameters are based on the inputs of a real data set analyzed in Pouya (2018).¹

4.2. Implementation Details

Programs are written in Python and run on Niagara supercomputer servers (Loken et al. 2010, Ponce et al. 2019) using Gurobi 9.1.1 as the MIP solver (Gurobi Optimization, LLC 2021) and CP Optimizer 20.1 as the CP solver (IBM® ILOG® CP® Optimizer 2021). For implementation of the Benders decomposition, we have used Gurobi’s callback feature such that, while solving the MP^{BD} , at every integer node of the branch-and-bound tree, we separate the Benders optimality and feasibility cuts by solving the CG subproblems. In implementing the LBBD method, we have opted for a cutting plane framework where every time

¹ Networks and instances, along with the detailed results, are available for download at <https://github.com/mdaryalal/Migration> (accessed January 13, 2023).

Table 5 Characteristics of the NMP instances

Dataset	#Instances	$ \mathcal{W} $	η^{CR}	$\mu_{EP/s}$	$\sigma_{EP/s}$	Endpoints		$ \mathcal{S}_p $	
						min	max	min	max
EUNETWORKS5	6	3	30	5	2.5	46	66	22	32
EUNETWORKS6	6	3	30	6	3.0	70	88	28	34
EUNETWORKS7	6	4	40	7	3.5	80	98	34	40
EUNETWORKS8	6	4	40	8	4.0	94	116	38	43
EUNETWORKS9	6	4	40	9	4.5	104	150	41	46
NEXTGEN5	6	3	30	5	2.5	52	74	23	31
NEXTGEN6	6	3	30	6	3.0	62	96	26	39
NEXTGEN7	6	3	30	7	3.5	78	92	31	37
NEXTGEN8	6	4	40	8	4.0	92	126	35	47
NEXTGEN9	6	5	40	9	4.5	98	134	41	51
PIONIER5	6	3	30	5	2.5	70	98	35	43
PIONIER6	6	3	30	6	3.0	96	112	40	51
SAGO5	6	3	30	5	2.5	64	90	29	39
SAGO6	6	3	40	6	3.0	80	94	32	40
SAVVIS5	6	3	30	5	2.5	54	102	25	42
SAVVIS6	6	3	30	6	3.0	84	106	38	41
SAVVIS7	6	3	30	7	3.5	88	120	37	47
VISIONNET5	6	3	30	5	2.5	82	104	36	47
VISIONNET6	6	4	40	6	3.0	106	126	47	52

after the convergence of the Benders decomposition method, we solve a CP problem to separate the violated LBBD optimality and feasibility cuts, if any. The reason for employing the cutting plane framework in this case is that our LBBD feasibility and optimality cuts introduce new binary variables. This is not allowed in the Gurobi’s callback implementation, since introducing new integer variables changes the structure of the branch-and-bound tree.

The programs stop whenever the algorithm reaches an optimality gap under 10%, or the running time passes 3 hours, whichever happens first. Our preliminary experiments showed that at the beginning of the solution process, early stopping of the Benders decomposition step reduces the computational effort, as it provides the MP^{LBBD} with informative cuts even without having to solve it to optimality. Therefore, in solving the MP^{LBBD} (i.e., the Benders decomposition phase) we start with a MIP gap of 10%, and reduce this by 5% at each iteration. Note that in these steps we should use the best lower bound on the solution in measuring the gap, which gives us a valid lower bound on the NMP instances. We have also made the following implementation choices and analyzed their impacts in the next section:

- In solving the CG subproblems, we use the hybrid CG as explained in Remark 2.

- Despite the fact that the CG subproblems are frequently solved during the algorithm, we do not remove the generated columns from Γ' .
- Considering the symmetry between the maintenance windows, we propagate the Benders optimality cuts, i.e., every time a Benders optimality cut is separated for a certain maintenance window w , we also modify and apply it for all $w' \in \mathcal{W} \setminus \{w\}$.

4.3. Algorithmic Analysis

In the following sections, we first examine our implementation choices, and based on the best obtained settings, solve the NMP instances. We break down the algorithmic analysis of the method into each solution paradigm and study their share of the computational effort.

4.3.1. Performance of the Hybrid CG. We have solved the first instances of EUNETWORKS5, NEXTGEN5, PIONIER5, SAGO5, SAVVIS5 and VISIONNET5 with a pure CP-based CG, a pure MIP-based CG, and a hybrid version where first a MIP is solved to generate improving columns with *ordered paths*, and then the CP pricing problem (6) continues the process by generating columns with general paths or verifies the optimality. The pure MIP-based CG fails to solve any of the instances to acceptable accuracy (final gaps are given in the e-companion), thus we remove it from the rest of the analysis. The results of Table 6 show that the hybrid CG reduces the computational effort of solving the MP^{LBBDD} by orders of magnitude. Most significantly, in 3 hours the pure CP-based method is unable to achieve a gap below 10% for PIONIER5, SAGO5 and VISIONNET5, while the hybrid CG in most cases closes the gap in a short amount of time. As mentioned in Section 3.2.1, in our backbone networks many regions have one or two sites, so the MIP pricing problem with ordered paths is already sufficient to guarantee their optimality. This greatly benefits the geographically larger networks EUNETWORKS, NEXTGEN and SAVVIS to the point that the instances based on the last two networks can be solved to optimality by only solving the MIP subproblem.

The comparatively remarkable performance of the hybrid CG can further be explained by a common practice in the column generation literature, where sub-optimal columns are also added to the restricted master problem in order to make it more descriptive and obtain more informative dual solutions for the pricing problem. From Table 6 we clearly see that the number of generated columns in the hybrid CG is generally much larger, meaning that the MIP subproblem is returning many columns with sub-optimal reduced-costs, and we are practically following the mentioned strategy in increasing the number of columns.

Table 6 Impact of the hybrid CG on the overall performance of the method

Dataset	CP-sub				CP/MIP-sub			
	# Columns	Time (s)		Gap (%)	# Columns	Time (s)		Gap (%)
		CG	MP ^{LBB}			CG	MP ^{LBB}	
EUNETWORKS5	138	4184.5	4192.2	0.0%	509	627.1	643.0	0.0%
NEXTGEN5	45	304.2	305.8	0.0%	45	5.2	10.4	0.0%
PIONIER5	493	Timeout	Timeout	47.4%	993	1039.4	1181.1	0.8%
SAGO5	314	Timeout	Timeout	31.6%	141	655.2	692.5	0.0%
SAVVIS5	57	1219.7	1257.3	0.0%	57	11.7	78.2	0.0%
VISIONNET5	512	Timeout	Timeout	50.0%	1738	1878.2	2010.4	3.0%

4.3.2. CG Column Management The CG subproblems are solved frequently, at every integer node of the branch-and-bound tree every time the algorithm reaches the Benders decomposition stage. Furthermore, the models are modified constantly, with different right-hands sides in each node. It is not immediately clear whether keeping all the columns leads to a speedup of the method or overloading the RMP_w^{CG} . For the same instances of Section 4.3.1, we have evaluated the impact of removing the columns after processing an integer node of the tree. Results in Table 7 show that the overall solution process indeed benefits from previously generated columns, and we are still able to efficiently solve the RMP_w^{CG} . However, the results also show that solving the CG models with an initial set $\Gamma' = \emptyset$ is still manageable and if the size of the instances leads to poor performance of the RMP_w^{CG} , we can switch to removing the columns or design a hybrid column management strategy.

4.3.3. Propagation of the Optimality Cuts. The symmetry among the maintenance windows $w \in \mathcal{W}$ in the NMP signifies that if an inequality $\eta_w \geq \mathbf{g}(\bar{\eta}_w, \bar{\mathbf{m}}_w) \mathbf{m}_w$ cuts off the solution pair $(\bar{\eta}_w, \bar{\mathbf{m}}_w)$ from the solution space, a similar inequality $\eta_{w'} \geq \mathbf{g}(\hat{\eta}_{w'}, \hat{\mathbf{m}}_{w'}) \mathbf{m}_{w'}$ removes $(\hat{\eta}_{w'}, \hat{\mathbf{m}}_{w'})$ if $\hat{\eta}_{w'} = \bar{\eta}_w, \hat{\mathbf{m}}_{w'} = \bar{\mathbf{m}}_w$. Therefore, any optimality cut obtained for one w

Table 7 Performance of the LBBD method under two column management strategies

Dataset	CG w/o previous columns			CG w/ all generated columns		
	#ITER.	Time (s)		#ITER.	Time (s)	
		CG	MP ^{LBB}		CG	MP ^{LBB}
EUNETWORKS5	4	727.3	1294.1	4	627.1	643.0
NEXTGEN5	2	307.6	308.8	14	5.2	10.4
PIONIER5	6	1162.1	2132.9	5	1039.4	1181.1
SAGO5	10	1601.3	2134.5	10	655.2	692.5
SAVVIS5	3	74.0	139.8	4	11.7	78.2
VISIONNET5	14	2572.0	7933.0	7	1878.2	2010.4

Table 8 Performance of the LBBB method with and without propagation of the Benders optimality cuts

Dataset	LBBB with local Benders cuts				LBBB with propagated Benders cuts			
	#ITER.	#Cut _{OPT} ^{BD}	#Cut _{OPT} ^{LBBB}	Time (s)	#ITER.	#Cut _{OPT} ^{BD}	#Cut _{OPT} ^{LBBB}	Time (s)
EUNETWORKS5	3	49	81	624.9	4	42	139	643.0
NEXTGEN5	4	16	170	8.8	14	32	579	10.4
PIONIER5	9	241	567	1841.3	5	90	269	1181.1
SAGO5	22	726	1297	3507.9	10	91	553	692.5
SAVVIS5	3	147	101	71.8	4	45	174	78.2
VISIONNET5	13	723	836	5835.2	7	202	359	2010.4

can be modified to use for all maintenance windows. This is called Benders cut propagation (Roshanaei et al. 2017). In Table 8 we have examined the effect of propagating the Benders optimality cuts in solving the instances of Section 4.3.1 and compared the results with a version of the method that only uses the classic local Benders optimality cuts. Columns #Cut_{OPT}^{BD} and #Cut_{OPT}^{LBBB} respectively report the number of the Benders and LBBB optimality cuts at the end of the solution process. The results indicate that, the cut propagation is quite effective in accelerating the solution process of the instances that are based on geographically smaller networks PIONIER, SAGO and VISIONNET, while its performance is comparable to the classic version for the networks stretched over a larger geographic scope EUNETWORKS, NEXTGEN and SAVVIS. Note that, the same arguments apply to the LBBB optimality cuts, yet any gain from propagating the cuts (11) is outweighed by their numbers. As a result, we only propagate the Benders optimality cuts in the LBBB method.

4.3.4. Computational Performance. In order to study the performance of the designed LBBB method for the NMP, we first compare it against a compact formulation (the CP-based model is given in the e-companion). We have randomly generated 5 toy instances with $\mathcal{W} = 2$ and $\eta^{\text{CIR}} = 10$ and solved them within a time limit of 5 minutes using the compact model and our LBBB method. The results are reported in Table 9, showing that while the compact formulation fails to solve any of the toy instances in the time limit, our method solves all to optimality.

With the implementation choices fixed by the analyses of Sections 4.3.1 - 4.3.3, we have solved the NMP instances of Table 5 and reported the confidence intervals on the optimality gaps, solution costs, the number of LBBB iterations and solution times in Table 10. In all instances, the LBBB method returns quality solutions with small gaps in a reasonable amount of time. The small number of iterations in the LBBB stage is a result of multiple

Table 9 Performance of the compact formulation

Instance	Endpoints	$ \mathcal{S}_p $	Compact		LBB D	
			Time (s)	Gap (%)	Time (s)	Gap (%)
TOY1	10	3	Timeout	58%	34	0%
TOY2	12	3	Timeout	43%	66	0%
TOY3	14	3	Timeout	75%	111	0%
TOY4	16	4	Timeout	76%	81	0%
TOY5	18	4	Timeout	85%	197	0%

Table 10 Solution and computational effort

Dataset	Gap		Cost (\$)		#ITER.		Time (s)	
	Mean	CI width	Mean	CI width	Mean	CI width	Mean	CI width
EUNETWORKS5	0.6%	1.1%	10472.0	448.7	3.5	0.6	1124.1	268.6
EUNETWORKS6	1.0%	1.9%	11424.0	923.4	10.5	4.3	4607.1	2309.7
EUNETWORKS7	4.7%	1.9%	11877.3	648.9	6.5	2.0	1459.0	509.7
EUNETWORKS8	4.9%	2.9%	12693.3	611.3	7.3	1.6	2582.6	539.9
EUNETWORKS9	3.7%	3.0%	13328.0	615.6	10.8	6.1	5109.8	3376.7
NEXTGEN5	1.2%	2.2%	13192.0	954.9	6.0	3.0	699.2	560.3
NEXTGEN6	2.6%	2.1%	14960.0	1443.7	4.8	2.3	675.7	374.2
NEXTGEN7	2.4%	1.7%	15640.0	243.3	5.3	2.0	997.9	415.8
NEXTGEN8	1.8%	1.3%	16048.0	615.6	8.2	4.1	2445.6	399.6
NEXTGEN9	4.1%	2.5%	16864.0	721.8	8.0	2.9	3120.4	1939.1
PIONIER5	1.9%	1.7%	15912.0	821.6	5.7	1.6	1817.2	796.4
PIONIER6	3.2%	2.1%	17136.0	533.1	9.2	2.0	4929.4	1385.8
SAGO5	1.3%	1.5%	11832.0	625.1	8.3	2.3	2387.6	697.1
SAGO6	5.2%	2.4%	12648.0	498.7	5.7	2.3	3635.1	2640.4
SAVVIS5	1.3%	1.4%	18224.0	1109.8	4.5	1.4	619.6	475.2
SAVVIS6	3.1%	2.1%	19312.0	721.8	6.0	1.6	855.6	460.3
SAVVIS7	4.2%	2.2%	19856.0	721.8	6.0	3.7	2472.6	2731.3
VISIONNET5	4.9%	1.8%	17272.0	877.3	5.8	3.5	2259.2	899.9
VISIONNET6	3.8%	1.9%	17952.0	377.0	7.3	3.1	3308.0	769.4

levels of decomposition in the LBB D master problem, giving it enough information to pass on quality first-stage solutions to the subproblems. This is quite valuable, considering the expensive design of the LBB D feasibility and optimality cuts. In Figure 6, we have further inspected the solution times. The box plots in this figure represent the distribution of the time that each step of the algorithm spends for solving the six instances of each dataset. We observe that in the smaller instances the $MP^{LBB D}$ run time is essentially dominated by the effort to solve the CG models. However, in larger instances with more endpoints, the time spent in solving the MP^{BD} apart from the CG models becomes increasingly significant. In contrast, as the number of the endpoints increases, with the exception of EUNETWORKS9,

the computational effort by the CP models remains relatively stable with a mild increase. The exponential growth in the solution time of the MP^{LBBB} is expected, considering the need to add more feasibility and optimality cuts, as reported in the e-companion. It is worth noting that, none of the steps of the LBBB method dominates the others in all the studied instances. Accordingly, all solution paradigms play a determining role in the solution process and enhancements in any of them can lead to overall improvement of the method.

4.4. Network Migration Cost Analysis

Figure 7 demonstrates the impact of various parameters on the solutions of the EUNETWORKS5 (similar analyses for two other networks are provided in the e-companion). As shown in Figure 7a, although the objective function has not benefited from engaging more technicians, increasing η^{CIR} from 30 to 40 lowers the number of the shifts and migration costs. Finding the right amount of workforce is one of the main concerns of the network operators as it comes with logistics and hidden considerations. While a solution with higher cost does not necessarily imply having more shifts, fewer shifts might be considered an advantage as it requires fewer personnel and less human traffic in the sites. The distribution of the shifts, illustrated in Figure 7b, shows that the majority of the technicians are assigned shorter than 2-hour shifts if possible. This is the result of either shortage of technicians or circuits in the targeted regions. Since offering more technicians has not improved the migration costs, we can conclude that the large number of the 2-hour shifts is due to the availability of the circuit endpoints in the region of the working technicians. Once the 2-hour shifts are not offered, 4-hour shifts become the obvious choice. By moving to 4-hour shifts, the number of the shifts, as expected, declines and we deal with fewer number of the technicians during

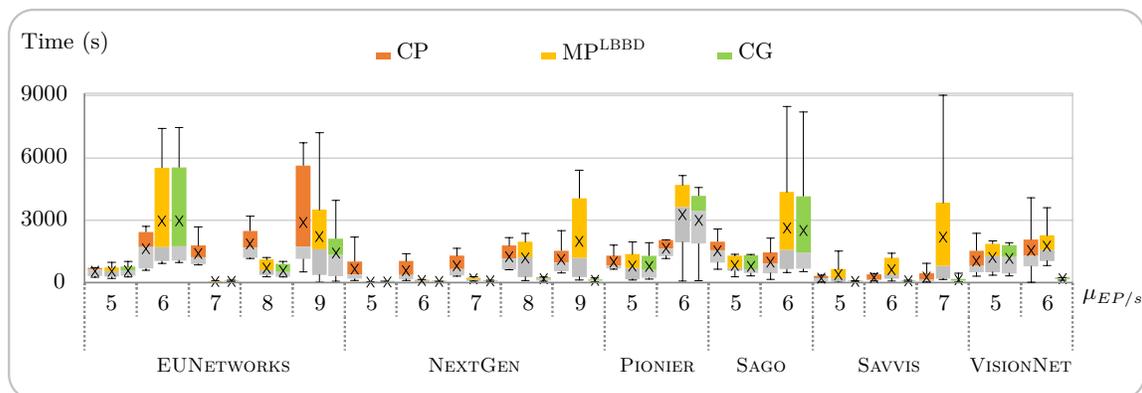


Figure 6 Breakdown of the solution times spent at each step of the decomposition. The time to solve the MP^{LBBB} includes the total solution time of the CG models.

the migration. However, as mentioned before, this does not mean a decrease in the migration costs since the shifts are the indicators of the payments, not the working time. Figure 7c displays the average working time (migrating circuit endpoints or traveling between the sites) of the technicians during the shifts. We notice that, by setting the minimum shift duration to 4 hours, the working time percentage is between 40% - 50% of a shift, while this number is above 60% for the settings with minimum shift duration of 2 hours. The increase of the costs show that 4-hour shifts are on average less busy than 2-hour shifts. It should be mentioned that there are a few 6-hour shifts as well. Considering the negative impact of longer maintenance windows on the Service-Level-Agreement (SLA) between the network operators and the customers, the analysis indicates that having 6-hour shifts does not help the migration process in neither costs nor the number of the shifts. As seen in Figure 7, we can build equivalent solutions with shorter shifts and ignore the 6-hour ones.

5. Concluding Remarks

Migrating legacy telecommunication networks to the latest technology involves planning synchronized technicians. We propose the first exact method for the network migration problem, a logic-based Benders decomposition method augmented by column generation and

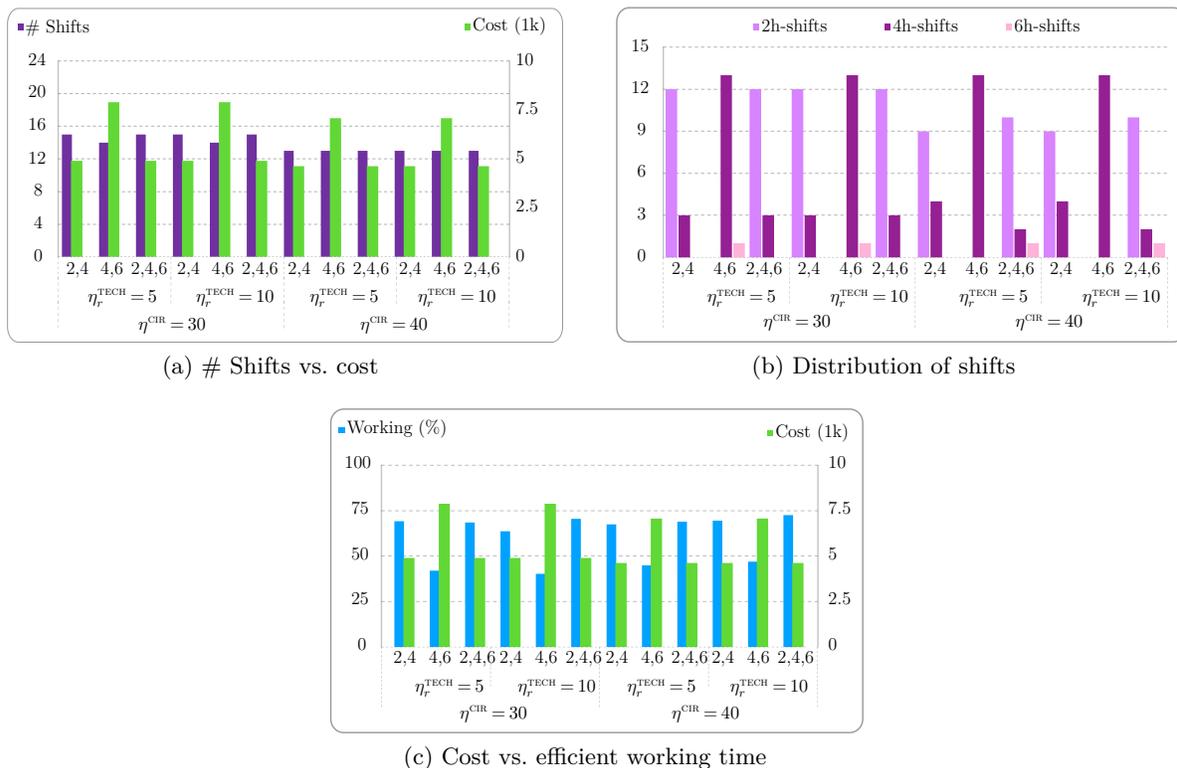


Figure 7 Cost analysis for EUNetworks

constraint programming models. We also make several algorithmic enhancements and considerably improve the basic version of the algorithm with the classical adaption of Benders decomposition and column generation. The proposed solution method can further be adapted to any vehicle routing problem with node synchronization constraints. Our evaluations on the instances generated over six real networks show that our method is effective in obtaining quality solutions, and that all the solution paradigms are contributing to the efficiency of the method. Future work includes improving the scalability of the algorithm for the networks with a larger number of circuit endpoints, particularly the design of less expensive LBBB optimality and feasibility cuts. Furthermore, new methodological developments are needed to build migration plans in the presence of uncertainty, which is an inherent feature of such planning problems.

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Appendix A: Detailed Models

A.1. Compact Formulation For the NMP

With the parameters and decision variables described in Sections 2.2 and 3.3, the compact formulation for the NMP modeled as a CP is as follows:

$$\begin{aligned}
\min \quad & \text{COST} \sum_{w \in \mathcal{W}} \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}_{rw}} \Delta_{\text{SHIFT}}^{rt} \\
\text{s.t.} \quad & \sum_{w \in \mathcal{W}} m_{ss'w} \geq \phi_{ss'} && \{s, s'\} \in \mathcal{S}_p, s < s' \\
& m_{ss'w} = m_{s'sw} && \{s, s'\} \in \mathcal{S}_p, s < s', w \in \mathcal{W} \\
& \sum_{\{s, s'\} \in \mathcal{S}_p} m_{ss'w} \leq 2\eta^{\text{CIR}} && w \in \mathcal{W} \\
& \text{NoOverlap}(\text{SEQ}_{rt}, T) && t \in \mathcal{T}_{rw}, r \in \mathcal{R}, w \in \mathcal{W} \\
& \text{Span}(\text{WTIME}_{rt}, \{x_{rts} : s \in \mathcal{S}_r\}) && t \in \mathcal{T}_{rw}, r \in \mathcal{R}, w \in \mathcal{W} \\
& \text{IfThen}(\text{PresenceOf}(\text{WTIME}_{rt}), \text{EndOf}(\text{WTIME}_{rt}) \leq \Delta_{\text{SHIFT}}^{rt}) && t \in \mathcal{T}_{rw}, r \in \mathcal{R}, w \in \mathcal{W} \\
& \text{PresenceOf}(x_{rts}) = \text{PresenceOf}(n_{rts}^s) && s \in \mathcal{S}_r, t \in \mathcal{T}_{rw}, r \in \mathcal{R}, w \in \mathcal{W} \\
& \text{IfThen}(\text{PresenceOf}(x_{rts}), \text{LengthOf}(x_{rts}) = \theta n_{rts}^s) && s \in \mathcal{S}_r, t \in \mathcal{T}_{rw}, r \in \mathcal{R}, w \in \mathcal{W} \\
& \sum_{s' : \{s, s'\} \in \mathcal{S}_p} n_{rts s'}^{\text{SP}} = n_{rts}^s && s \in \mathcal{S}_r, t \in \mathcal{T}_{rw}, r \in \mathcal{R}, w \in \mathcal{W} \\
& n_{rts s'}^{\text{SP}} = n_{rts' s}^{\text{SP}} && \{s, s'\} \in \mathcal{S}_p, t \in \mathcal{T}_{rw}, r \in \mathcal{R}, w \in \mathcal{W} \\
& \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}_{rw}} n_{rts s'}^{\text{SP}} = \bar{m}_{ss'w} && \{s, s'\} \in \mathcal{S}_p, w \in \mathcal{W} \\
& \sum_{r \in \mathcal{R}} \sum_{t \in \mathcal{T}_{rw}} \text{PresenceOf}(\text{WTIME}_{rt}) \leq \alpha^{\text{ENG}} \eta^{\text{ENG}} && w \in \mathcal{W}.
\end{aligned}$$

As stated in the main text, the problem can also be formulated as a MIP, which includes a large number of loop elimination constraints.

A.2. Auxiliary MIP Pricing Problem

In addition to $n_{ss'}$ and n_{CIR} described in Section 2.2, we define the new decision variables in Table EC.1. Denote by $\bar{\pi}^{(5b)}$, $\bar{\pi}^{(5c)}$, $\pi^{(5d)}$, the optimal dual solutions associated with constraints (5b), (5c), (5d), respec-

Table EC.1 New decision variables for the CG subproblem

Variable	Type	Description
$h_{ss'}$	Binary	= 1 if the technician migrates at least one circuit endpoint in site s with the other endpoint in site s' .
h_s	Binary	= 1 if the technician works in site s in this shift, 0 otherwise.
$t_{ss'}$	Binary	= 1 if a travel from site s to site s' occurs in the shift under construction, 0 otherwise.
x_δ	Binary	= 1 if the length of the shift under construction is equal to Δ_δ , 0 otherwise.

tively. The pricing problem generating a shift for (r, w) is:

$$\text{SP}'_{rw}{}^{\text{CG}} = \min \text{COST} \Delta_{\text{SHIFT}} - \sum_{s \in \mathcal{S}_r} \sum_{s' \in \mathcal{S}} n_{ss'} \bar{\pi}_{ss'}^{(5b)} - \bar{\pi}_{rw}^{(5c)} - \bar{\pi}_w^{(5d)} \quad (\text{EC.1a})$$

$$\text{s.t. } h_{ss'} + h_{s's} \leq 1 \quad s, s' \in \mathcal{S}_r, s \neq s' \quad (\text{EC.1b})$$

$$h_s \leq \sum_{s' \in \mathcal{S}} h_{ss'} \leq M h_s \quad s \in \mathcal{S}_r \quad (\text{EC.1c})$$

$$h_{ss'} \leq n_{ss'} \leq M h_{ss'} \quad s \in \mathcal{S}_r, s' \in \mathcal{S}, s \neq s' \quad (\text{EC.1d})$$

$$\sum_{s \in \mathcal{S}_r} t_{s_{\text{SRC}}, s} = 1, \quad \sum_{s \in \mathcal{S}_r} t_{s, s_{\text{DST}}} = 1 \quad (\text{EC.1e})$$

$$t_{ss'} \leq \frac{1}{2}(h_s + h_{s'}) \quad s, s' \in \mathcal{S}_r, s' > s \quad (\text{EC.1f})$$

$$\sum_{\substack{s' \in \mathcal{S}_r^+ \\ s' > s}} t_{ss'} + \sum_{\substack{s' \in \mathcal{S}_r^+ \\ s' < s}} t_{s's} = 2h_s \quad s \in \mathcal{S}_r \quad (\text{EC.1g})$$

$$\sum_{s \in \mathcal{S}_r} \sum_{\substack{s' \in \mathcal{S}_r \\ s' > s}} t_{ss'} = \sum_{s \in \mathcal{S}_r} h_s - 1 \quad (\text{EC.1h})$$

$$\theta \sum_{\substack{s \in \mathcal{S}_r \\ s' \in \mathcal{S} \\ s \neq s'}} n_{ss'} + \sum_{\substack{s \in \mathcal{S}_r \\ s' \in \mathcal{S}_r \\ s' > s}} T_{ss'} t_{ss'} \leq \Delta_{\text{SHIFT}} \quad (\text{EC.1i})$$

$$\sum_{\delta \in \Delta} x_\delta = 1 \quad (\text{EC.1j})$$

$$\sum_{\delta \in \Delta} \Delta_\delta x_\delta = \Delta_{\text{SHIFT}} \quad (\text{EC.1k})$$

$$n_{ss'}, h_{ss'} \in \mathbb{Z}_+, h_s \in \{0, 1\} \quad s \in \mathcal{S}_r, s' \in \mathcal{S}, s \neq s' \quad (\text{EC.1l})$$

where $M \geq 0$ is a large number and $\mathcal{S}^+ = \mathcal{S} \cup \{s_{\text{SRC}}, s_{\text{DST}}\}$, $\mathcal{S}_r^+ = \mathcal{S}_r \cup \{s_{\text{SRC}}, s_{\text{DST}}\}$. s_{SRC} and s_{DST} are two dummy sites introduced as the beginning and end of a path. Objective function (EC.1a) is the reduced-cost. Constraints (EC.1b) ensure that at most one of the endpoints of every circuit $c \in \mathcal{C}_{ss'}$ can be migrated in every shift. Constraints (EC.1c) determine the sites where a technician works in the current shift. Constraints (EC.1d) assure that all migrated circuit endpoints are from the sites where the technician works in the current shift. Thanks to constraints (EC.1e), generated path in this configuration starts from dummy site s_{SRC} and ends in dummy site s_{DST} . Constraints (EC.1f) guarantee that if a travel occurs between the two sites s and s' , the technician works in both. Constraints (EC.1g) determine the sites visited before and after every site $s \in \mathcal{S}_r$. Constraint (EC.1h) ensures that we have a path linking all visited sites. Constraint (EC.1i) makes sure that the shift duration does not exceed the maximum predefined duration. Constraints (EC.1j) and (EC.1k) together determine the duration of the shift. The rest of the constraints determine the variables domain.

Appendix B: Detailed Numerical Results

Table EC.2 Characteristics of instances

Dataset	Instance											
	1		2		3		4		5		6	
	Endpoints	$ \mathcal{S}_p $										
EUNETWORKS5	66	27	60	26	46	22	66	26	66	30	66	32
EUNETWORKS6	70	30	72	32	84	31	88	34	80	33	74	28
EUNETWORKS7	98	40	84	36	94	34	80	35	90	39	86	35
EUNETWORKS8	116	41	94	40	110	43	102	38	106	42	108	39
EUNETWORKS9	110	46	150	44	112	45	112	41	104	42	104	42
NEXTGEN5	54	26	52	23	56	26	74	31	66	31	56	26
NEXTGEN6	86	39	62	26	88	34	80	34	96	39	74	35
NEXTGEN7	82	36	86	35	90	35	92	37	92	35	78	31
NEXTGEN8	122	46	126	47	108	44	92	35	118	44	98	40
NEXTGEN9	134	45	126	51	106	41	124	45	114	45	98	44
PIONIER5	78	38	82	39	70	35	90	39	98	43	74	36
PIONIER6	112	48	112	48	106	51	106	47	96	40	104	42
SAGO5	76	35	64	29	88	39	74	33	82	35	90	39
SAGO6	86	37	94	40	94	40	80	32	86	38	90	40
SAVVIS5	72	35	98	42	102	41	80	36	54	25	86	40
SAVVIS6	104	40	90	40	84	40	106	41	88	38	94	40
SAVVIS7	120	46	106	44	88	37	114	47	114	45	92	39
VISIONNET5	86	40	100	47	104	45	88	38	102	44	82	36
VISIONNET6	116	52	106	51	120	52	120	47	126	52	110	49

Table EC.3 Impact of the CG subproblem formulation on the overall performance of the method

Dataset	CP-sub			MIP-sub		
	Time (s)		Gap (%)	Time (s)		Gap (%)
	CG	MP ^{LBBD}		CG	MP ^{LBBD}	
EUNETWORKS5	4184.5	4192.2	0.0%	Timeout	Timeout	18.0%
NEXTGEN5	304.2	305.8	0.0%	Timeout	Timeout	21.0%
PIONIER5	Timeout	Timeout	47.4%	Timeout	Timeout	47.4%
SAGO5	Timeout	Timeout	31.6%	Timeout	Timeout	52.5%
SAVVIS5	1219.7	1257.3	0.0%	Timeout	Timeout	26.0%
VISIONNET5	Timeout	Timeout	50.0%	Timeout	Timeout	23.1%

Table EC.4 Breakdown of solution times

Dataset	Iterations		Sol Time		LBB D MP Time		CG Time		CP Time	
	Mean	CI width	Mean	CI width	Mean	CI width	Mean	CI width	Mean	CI width
EUNETWORKS5	3.5	0.6	1124.1	268.6	548.2	198.3	540.1	196.0	575.8	147.6
EUNETWORKS6	10.5	4.3	4607.1	2309.7	2953.3	1929.8	2939.2	1928.9	1653.5	680.5
EUNETWORKS7	6.5	2.0	1459.0	509.7	45.7	23.4	27.6	13.7	1413.2	497.5
EUNETWORKS8	7.3	1.6	2582.6	539.9	693.8	271.9	531.6	225.2	1888.4	563.5
EUNETWORKS9	10.8	6.1	5109.8	3376.7	2201.9	1889.0	1384.5	1000.8	2907.4	1817.8
NEXTGEN5	6.0	3.0	699.2	560.3	18.0	16.1	4.6	2.7	680.9	561.0
NEXTGEN6	4.8	2.3	675.7	374.2	94.8	42.8	17.6	6.3	580.8	361.1
NEXTGEN7	5.3	2.0	997.9	415.8	177.6	63.9	32.9	15.0	820.1	381.1
NEXTGEN8	8.2	4.1	2445.6	399.6	1182.4	626.9	135.9	66.9	1262.8	433.9
NEXTGEN9	8.0	2.9	3120.4	1939.1	1980.7	1546.1	67.1	42.6	1139.5	543.4
PIONIER5	5.7	1.6	1817.2	796.4	812.9	505.1	726.2	475.5	1004.1	322.2
PIONIER6	9.2	2.0	4929.4	1385.8	3267.2	1306.1	2961.9	1167.6	1661.8	272.9
SAGO5	8.3	2.3	2387.6	697.1	836.1	293.6	748.0	319.1	1551.2	480.3
SAGO6	5.7	2.3	3635.1	2640.4	2630.0	2194.2	2473.5	2132.5	1005.0	490.1
SAVVIS5	4.5	1.4	619.6	475.2	400.5	408.2	27.1	12.1	218.8	94.4
SAVVIS6	6.0	1.6	855.6	460.3	608.2	393.2	36.1	15.3	247.1	108.7
SAVVIS7	6.0	3.7	2472.6	2731.3	2184.8	2503.3	96.2	108.1	287.3	251.1
VISIONNET5	5.8	3.5	2259.2	899.9	1217.2	523.3	1112.4	507.2	1041.8	541.1
VISIONNET6	7.3	3.1	3308.0	769.4	1759.4	718.7	139.2	36.3	1548.3	996.6

Table EC.5 The number of cuts and columns generated during the solution process

Dataset	#Cut _{OPT} ^{LBB D}		#Cut _{FEAS} ^{LBB D}		#Cut _{OPT} ^{BD}		#Columns	
	Mean	CI width	Mean	CI width	Mean	CI width	Mean	CI width
EUNETWORKS5	121.8	35.3	27.3	11.2	33.0	11.6	408.8	180.2
EUNETWORKS6	566.8	273.0	84.0	30.5	79.2	43.0	951.2	375.3
EUNETWORKS7	361.2	101.1	66.3	44.1	76.8	36.8	1350.8	565.1
EUNETWORKS8	463.7	114.1	88.3	29.0	170.8	62.0	3693.8	1120.1
EUNETWORKS9	919.2	605.7	119.3	104.5	480.7	464.9	7318.7	4824.6
NEXTGEN5	218.3	136.2	59.2	23.6	35.3	28.4	45.5	0.9
NEXTGEN6	237.5	167.3	42.7	25.4	115.8	43.8	159.8	162.7
NEXTGEN7	267.7	130.6	61.5	40.4	220.5	50.3	274.8	411.2
NEXTGEN8	434.3	148.6	66.0	31.4	386.2	164.1	1405.5	646.1
NEXTGEN9	622.3	317.5	103.7	32.8	666.5	405.6	1456.2	991.8
PIONIER5	289.0	96.7	76.0	20.5	110.0	73.0	741.5	462.9
PIONIER6	666.8	159.0	124.8	36.9	308.2	70.9	2101.8	1108.5
SAGO5	476.8	185.9	78.3	36.1	232.7	165.6	1026.8	691.0
SAGO6	307.7	161.8	37.0	17.9	365.5	260.5	1024.3	582.1
SAVVIS5	215.8	83.0	50.0	29.9	176.0	107.9	57.0	0.0
SAVVIS6	343.3	137.5	72.2	13.6	219.0	83.0	57.0	0.0
SAVVIS7	375.7	320.2	65.2	42.6	661.0	728.0	57.0	0.0
VISIONNET5	385.5	322.0	63.3	43.9	136.7	53.6	1772.0	1091.4
VISIONNET6	571.3	328.8	115.8	53.0	411.5	102.8	4607.8	1571.0

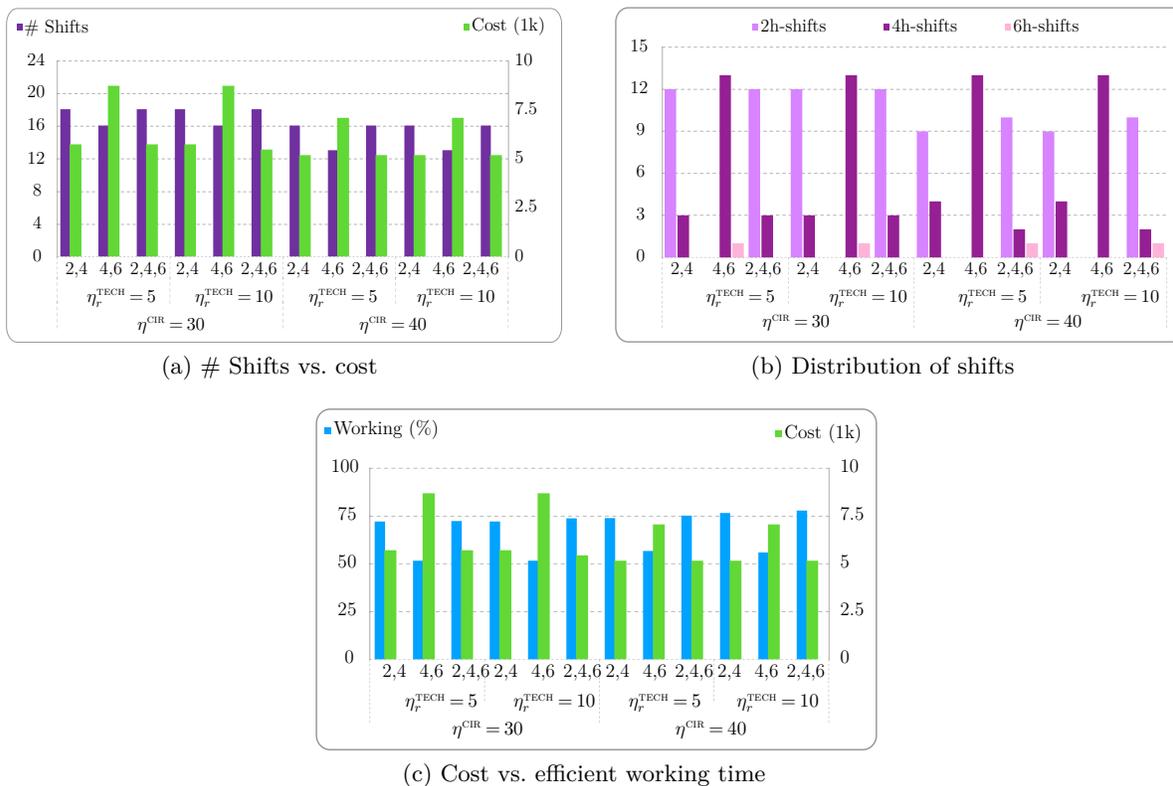


Figure EC.1 Cost analysis for Sago

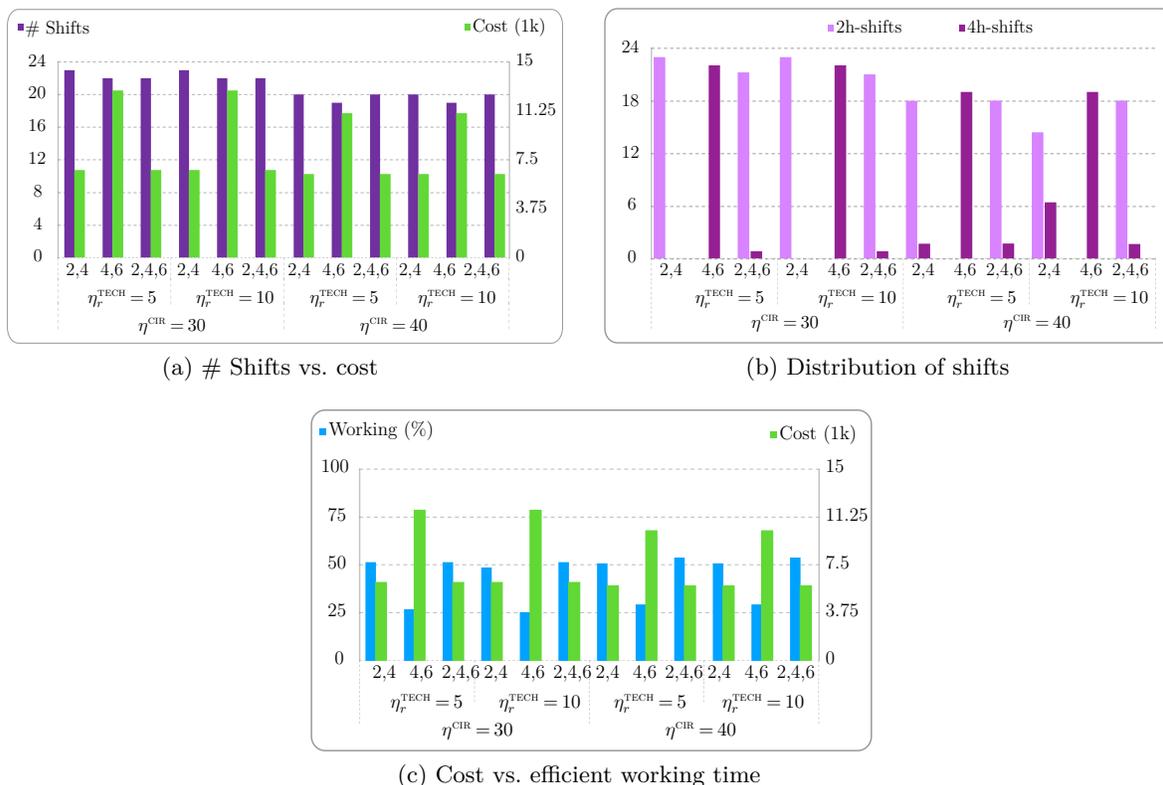


Figure EC.2 Cost analysis for Savvis