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Solving a Multi-product, Multi-period, Multi-modal Freight Transportation Problem Using Integer Linear Programming

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Abstract We consider a real-world multimodal freight transportation problem that arises in a food grain organization in India. This problem aims to satisfy the demand for a set of warehouses for different types of food grains from another set of warehouses with surplus quantities over multiple periods of time by rail and road, while minimizing the total cost of transportation. We first present some examples that show that the existing method can lead to suboptimal solutions. Motivated by the need to efficiently solve such an important problem, we propose an integer programming formulation for this problem and solve it using state-of-art solvers. We highlight that the use of the proposed mathematical model gives significant cost savings over the existing method. Our computational experiments on real instances from historical data show that the proposed model is effective for such instances and could solve them optimally in a reasonably short time (less than a minute). Moreover, our results show that this model is also suitable for much larger instances. We also analyze the effects of changes in model parameters on the size of the proposed mathematical model. We emphasize that the proposed model can be easily adapted to other multimodal freight transportation problems that arise in different industries.

Keywords: Multimodal transportation, Freight consolidation, Volume discount, Integer programming

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1 INTRODUCTION

We consider a real-world multimodal freight transportation problem faced by a food organization in India (FOI for short, an organization that is responsible for storing, distributing, and transporting food grains between different warehouses, from where these products are transported to other distributors to meet demand in different regions of the country). In multimodal freight transportation, goods are transported using at least two different modes of transport. The two most common and widespread modes for transporting bulk goods are rail and road. India (a country in South Asia that is the seventh largest country in the world by area) has one of the largest rail and road networks in the world, and much of its bulk freight is transported by a combination of these two modes of transport. The economies of scale and reach resulting from multimodal transport have made it a potential driver for the logistics industry and is gaining more attention for further infrastructure investment and development [35].

We consider the problem of transporting food grains from a number of warehouses with surplus quantities to other warehouses across many regions in the country. In a country as large as India, transporting bulk commodities such as food grains to geographically dispersed regions is challenging because it requires extensive storage and transportation infrastructure. Transportation modes are expensive, sometimes their reach is geographically limited, and their availability (especially railways) depends on demand from other users in the country. Because food grain transportation and distribution are critical to food security in the country, they must be carried out within a given time horizon while minimizing the overall cost of transportation.

Currently, FOI handles about 85% of food grain transportation through a combination of two modes of transport, rail and road. On the one hand, rail offers economic advantages, as it allows bulk goods to be transported over long distances at a cheaper transport rate than road. On the other hand, road provides the final leg of transportation to places that cannot otherwise be reached by rail. The overall efficiency of the transport system is decisively determined by the combined use of rail and road.

This paper presents a real-world freight transportation problem that the FOI faces for transporting food grains between a network of locations. The problem aims to satisfy the warehouses' demand for multiple grains from other warehouses with the surplus quantities over multiple periods by rail and road, while minimizing the total cost of transportation. Thus, it is a multi-product, multi-period, and multi-modal transportation problem. Multimodal freight transportation problems are known to be theoretically and computationally difficult problems. In this paper, we formulate the considered problem as a mathematical model that can be solved to optimality in significantly less amount of time. We allow freight consolidation in order to take advantage of the volume discount offered by the railways for bulk transport. We also handle congestion that can occur in rail-connected locations due to the simultaneous arrival of several rakes by limiting the number of allocation at these locations at any period of time. These aspects are not commonly addressed in the literature.

Currently, FOI creates a transportation plan manually based on certain rules of thumb and experience gained over time. This existing method used by the FOI is presented in more detail in Section 6. We note that this seemingly simple approach is cumbersome to apply to networks with many locations. Moreover, it typically leads to solutions whose objective value falls short of that of an optimal solution. We propose to solve this problem using the well-known integer programming technique. In the last few decades, integer programming has witnessed a tremendous advancement in its ability to solve difficult practical optimization problems. We present a mathematical formulation of the problem under consideration and analyze the impact of changing certain parameters on the size of model. Next, we present a computational study of the performance of the proposed model on real instances of different sizes drawn from historical data.

Next, we assess the scalability of the model as the size of an instance increases. Our results show that with the proposed mathematical model, instances of practical sizes can be solved optimally within a minute. We emphasize that the proposed model can be easily applied to other multimodal (rail and road) freight transportation problems that arise in different industries.

The outline of the paper is as follows. Section 2 briefly introduces the literature on multimodal freight transportation and integer programming. Section 3 formally describes the freight transportation problem at hand. Section 4 presents the mathematical formulation of this problem as an integer program. A comparison of the performance of the proposed mathematical model and the existing method used by the FOI is presented in the Section 5. Section 6 reports the computational results using the proposed model and its scalability. Section 7 presents the conclusions and some future directions.

2 LITERATURE REVIEW

Research in multimodal freight transport has increased significantly since the 1990s with financial support from international organizations such as the European Union [2]. Various aspects of multimodal freight planning have been addressed in the literature. For example, the problem of the choice of transport mode and location of distribution facilities when planning a supply chain network is presented in [15]. A multiobjective optimization of multimodal transportation considering transportation costs and emission-related factors is presented in [23]. [36] considers minimizing the impact of various disturbances on an intermodal network and a decision support system for multimodal transportation routing is presented in [22].

A very important decision in multimodal transport planning is the choice of intermediate terminals for transshipment. In that context, [26] deals with the selection of optimal locations for rail and road transshipment terminals in the European network. In [14], the freight rate and transshipment locations are jointly optimized to minimize the total transportation cost.

Research in multimodal transportation can be categorized by commodity groups, which include food grains. Long-distance and bulk transportation of food grains using a combination of rail and truck has recently received much attention. The work reported in the context of India in [27] and [31] underscores the importance of efficient food grain distribution in ensuring food security. Also, many bulk transportation problems involve multi-period planning which adds to the complexity of finding exact solutions to such problems as reported in [29],[28],[32],[33],[24]).

One of the key factors in reducing freight transportation costs is freight consolidation, which enables economies of scale ([1],[4],[13],[30]). Literature contains several works addressing the impact of using full and partially loaded (less than truck load) truck on transportation costs ([18], [7], [5], [6]). However, a very few works have carried out similar studies in the context of rail transportation. In this paper, we consider freight consolidation in multiperiod multiproduct multimodal transportation with discounts based on the quantity transported using rail. We employ an exact solver based approach using integer programming to solve the considered transportation problem.

Integer programming is a technique for solving optimization problems in which all decision variables can take only integer values. Such problems are called integer programs and are known to be NP-complete [11]. Integer programs appear in a wide variety of practical applications like train scheduling, airline crew scheduling, production scheduling, agriculture, transportation, telecommunications, power generation, health care, etc. ([37], [8], [10], [3]). Advances in mathematical modeling language and computer architecture, as well as the availability of fast and reliable linear programming software, have contributed to the tremendous growth in the ability of integer programming solvers to solve challenging practical problems. The main algorithmic principles for solving integer problems are branch-and-bound [25], cutting planes [19], and branch-and-cut [34]. Integer programming is a well-researched area of study, and interested readers are encouraged to refer to widely used textbooks and surveys such as [9], [37], [21], [12]. There are many open-source and commercial solvers for integer programs. The success of these solvers is due to subroutines for presolving the problem, generating cutting planes, and heuristics that provide good quality solutions, thus reducing the size of the branch-and-bound tree and the overall solution time. In this work, we use two solvers CPLEX [20] and SCIP [16] to solve the instances in our test set.

3 PROBLEM DESCRIPTION

We consider a multiproduct, multiperiod, multimodal freight transportation problem (called MPPMFT problem for short). The considered modes of transport are rail and road (with trucks). There are three products consisting of two types of grains: wheat and rice, and there are two types of rice - boiled and raw. So the three products are wheat, boiled rice, and raw rice. The time horizon is considered in the number of weeks. FOI currently generates monthly transportation plan by dividing the planning horizon into four weeks.

The problem consists of a set of warehouses from which these products are to be transported to satisfy the demand of another set of warehouses using a set of transshipment points. The first set of warehouses are called the source warehouses, and the other is called destination warehouses. A source warehouse can store only one type of grain (wheat or rice). However, a source warehouse that stores rice can contain both types of rice. Some destination warehouses are connected to source warehouses by rail, while others are not. The former warehouses are referred to as rail-connected warehouses, while the latter are referred to as road-connected warehouses. Since road-connected warehouses do not have a rail connection to a source warehouse, a product to be transported must first be transported by rail to a transshipment point. From this transshipment point, the product is then transported by road to the road-connected warehouse. There are dedicated transshipment points that are used only for unloading product from rail and loading trucks. These transshipment locations do not have any storage capacity or demand for a product. Rail-connected warehouses also act as transshipment points. A product can be transported between rail-connected warehouses using trucks.

At any period of time, a rail-connected location can handle either a full train load (called full rake) or a half train load (referred to as half rake). This is called the terminal capacity of a rail-connected location. And, a rake or a truck can hold only one product at a time. That is, at any given time, a full-rake or truck can carry either wheat or one of the rice varieties but not a combination of two products. A source warehouse can transport a full rake of the same product for two destinations (rail-connected warehouses or transshipment points), half a rake for each. We enable the consolidation of freight in order to benefit from the volume discounts that the rail offers for bulk transportation. There are different rates for full and half rakes. We also avoid congestion that can occur at rail-connected locations due to the simultaneous arrival of multiple rakes. We achieve this by limiting the number of rakes allowed at any given time. The other types of costs are the handling costs associated with unloading the rakes and loading the trucks at rail-connected locations, and the fixed costs associated with the placement of rakes to rail-connected warehouses. The objective of the problem is to satisfy the demand of the destination warehouses for two types of grains from the source warehouses using rail and road transportation within the given time horizon while reducing the total cost of transportation. Figure 1 presents a schematic diagram of the transportation network under consideration.

4 SOLUTION APPROACH

We model the MPPMFT problem as an integer linear program. First, we present the mathematical formulation along with a description of the parame-

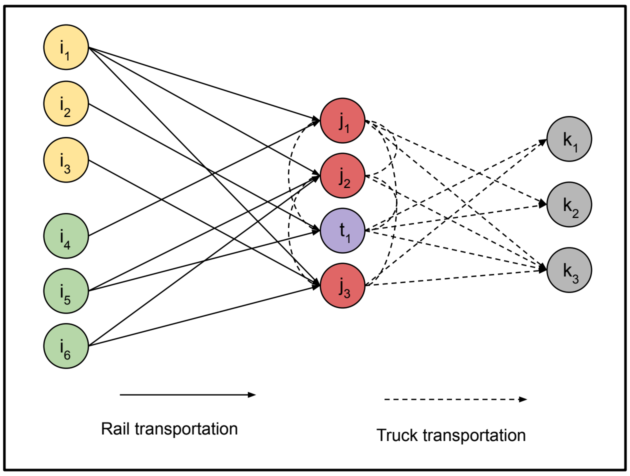

Fig. 1: A pictorial representation of the network underlying the MPPMFT problem. It shows different warehouses, transshipment point, and possible connections between them for transportation. $\{i_1, i_2, i_3\}$ and $\{i_4, i_5, i_6\}$ are the source warehouses containing wheat and rice, respectively. The sets of rail-connected and road-connected warehouses are given by $\{j_1, j_2, j_3\}$ and $\{k_1, k_2, k_3\}$, respectively. t_1 is a dedicated transshipment location. Connections without arrows indicate that transportation is allowed in both directions.

ters, decision variables, the objective function, and the constraints. This model is then solved using state-of-art solvers for integer programs. We conclude this section by showing how increasing several useful parameters affects the size of the model.

4.1 Mathematical Model

Parameters

W_1 : Set of source warehouses containing wheat

W_2 : Set of source warehouses containing rice

P : Set of products $\{wh, br, rr\}$, where wh denotes wheat, br is boiled rice, and rr indicates raw rice

P_2 : Set of product type rice $\{br, rr\}$

D_1 : Set of rail-connected warehouses and transshipment points

D_2 : Set of road-connected warehouse

T : Set $\{1, \ldots, W\}$, where W is the number of weeks

$\bar{c}, \tilde{c}$: Carrying capacity of a truck and half a rake, respectively, in tons

$\bar{h}, \tilde{h}$: Handling cost of a truck and a wagon, respectively

a_i^w : Quantity (in tons) of wheat available at warehouse $i \in W_1$

a^r_{ij} : Quantity (in tons) of rice type $j \in P_2$ available at warehouse $i \in W_2$

$\bar{f}_i$: Fixed cost of placing a rake (full or half) at location $i \in D_1$

$\hat{t}_{ij}$: Rail transportation cost for a full rake from warehouse $i \in W_1$ to $j \in D_1$

$\tilde{t}_{ij}$: Rail transportation cost for a half rake from warehouse $i \in W_1$ to $j \in D_1$

$\bar{t}_{ij}$: Truck transportation cost from $i \in D_1$ to $j \in D_1 \cup D_2, i \neq j$

d_{ip} : Total demand (in tons) of product $p \in P$ at location $i \in D_1 \cup D_2$

$\bar{a}_{ij,j>i}$: 1 if a rake can carry the same product for locations $i, j \in D_1$. Otherwise, 0

$\bar{n}$: Number of wagons in a half rake

$\hat{c}_i$: Terminal capacity of $i \in D_1$

$\tilde{n} = \left\lceil \frac{2 \times \bar{n} \times \tilde{c}}{\bar{c}} \right\rceil$: Maximum number of trucks that can be allocated by any $i \in D_1$

The parameter $\bar{c}_{ij}$ specifies whether a source warehouse is allowed to assign a full rake of a single product for the two locations i and j, with half a rake assigned to each location. The terminal capacity can take value 1 or 2. $\hat{c}_i = 2, i \in D_1$ indicates that location i can accommodate a full rake, and 1 indicates that it can accommodate only half a rake. A location that can accommodate a full rake can also accommodate half a rake, but not the other way around.

Variables

We use superscripts w and r to indicate that the variable corresponds to the product type wheat and rice, respectively.

$\eta_{ijpt} \in \mathbb{Z}_+$: Number of trucks assigned from $i \in D_1$to $j \in D_1 \cup D_2$ for $p \in P$ in week $t \in T, where j \neq i$

$$\alpha_{ijpt} = \begin{cases} 1, & \text{if } \eta_{ijpt} > 0, i, j \in D_1, i \neq j, p \in P, t \in T \\ 0, & \text{otherwise.} \end{cases}$$

$$x^w_{ijt} = \begin{cases} 1, & \text{if full rake is allocated from } i \in W_1 \text{ to } j \in D_1 \text{ in } t \in T \\ 0, & \text{otherwise.} \end{cases}$$

$$y^w_{ijt} = \begin{cases} 1, & \text{if half rake is allocated from } i \in W_1 \text{ to } j \in D_1 \text{ in } t \in T \\ 0, & \text{otherwise.} \end{cases}$$

$$x^r_{ijpt} = \begin{cases} 1, & \text{if full rake is allocated from } i \in W_2 \text{ to } j \in D_1 \text{ in } t \in T \\ & \text{for product } p \in P_2 \\ 0, & \text{otherwise.} \end{cases}$$

$$y^r_{ijpt} = \begin{cases} 1, & \text{if half rake is allocated from } i \in W_2 \text{ to } j \in D_1 \text{ in } t \in T \\ & \text{for product } p \in P_2 \\ 0, & \text{otherwise.} \end{cases}$$

$$z^w_{it} = \begin{cases} 1, & \text{if a warehouse in } W_1 \text{ transport to } \ i \in D_1 \ \text{ in } \ t \in T \\ 0, & \text{otherwise.} \end{cases}$$

$$z^r_{ipt} = \begin{cases} 1, & \text{if a warehouse in } W_2 \text{ transport product } p \in P_2 \text{ to } \ i \in D_1 \\ & \text{in } \ t \in T \\ 0, & \text{otherwise.} \end{cases}$$

$$\nu^w_{ijkt} = \begin{cases} 1, & \text{if warehouse } i \in W_1 \text{ combines quantities for } j,k \in D_1, j < k \\ & \text{in } \ t \in T \\ 0, & \text{otherwise.} \end{cases}$$

$$\nu^r_{ijkpt} = \begin{cases} 1, & \text{if warehouse } i \in W_2 \text{ combines product } p \in P_2 \text{ for } j,k \in D_1, j < k \\ & \text{in } \ t \in T \\ 0, & \text{otherwise.} \end{cases}$$

Objective

$$\begin{aligned} \text{Minimize} & \sum_{i \in D_1, t \in T} z^w_{it} \times \bar{f}_i + \sum_{i \in D_1, p \in P_2, t \in T} z^r_{ipt} \times \bar{f}_i + \\ & \sum_{i \in D_1, j \neq i \in D_1 \cup D_2, p \in P, t \in T} \eta_{ijpt} \times (\bar{t}_{ij} + \bar{h}) + \sum_{i \in W_1, j \in D_1, t \in T} \Big[\hat{t}_{ijt} \times \\ & (2 \times x^w_{ijt} \times \bar{n} \times \tilde{c}) + \tilde{t}_{ijt} \times (y^w_{ijt} \times \bar{n} \times \tilde{c}) + \\ & (2 \times x^w_{ijt} + y^w_{ijt}) \times \bar{n} \times \tilde{h} \Big] + \sum_{i \in W_2, j \in D_1, p \in P_2, t \in T} \Big[\hat{t}_{ij} \times \\ & (2 \times x^r_{ijpt} \times \bar{n} \times \tilde{c}) + \tilde{t}_{ij} \times (y^r_{ijpt} \times \bar{n} \times \tilde{c}) + \\ & (2 \times x^r_{ijpt} + y^r_{ijpt}) \times \bar{n} \times \tilde{h} \Big]. \end{aligned} \tag{1}$$

The objective function (1) represents the total transportation cost, which consists of variable costs and handling costs associated with the use of rakes and trucks, as well as fixed costs from using rail to transport between source warehouses and rail-connected warehouses and transshipment locations.

Constraints

$$\sum_{j \in D_1, t \in T} (2 \times x^w_{ijt} + y^w_{ijt}) \times \bar{n} \times \tilde{c} \le a^w_i, \ i \in W_1, \tag{2}$$

$$\sum_{j\in D_1,t\in T}(2\times x^r_{ijpt}+y^r_{ijpt})\times\bar{n}\times\tilde{c}\le a^r_{ip},\ \ i\in W_2,p\in P_2 \tag{3}$$

$$\sum_{i\in W_1}x^w_{ijt}+y^w_{ijt}\le 1,\ \ j\in D_1,t\in T \tag{4}$$

$$\sum_{i\in W_2}x^r_{ijpt}+y^r_{ijpt}\le 1,\ \ j\in D_1,p\in P_2,t\in T, \tag{5}$$

$$\sum_{i\in W_1}\left(2\times x^w_{ijt}+y^w_{ijt}\right)+\sum_{i\in W_2,p\in P_2}\left(2\times x^r_{ijpt}+y^r_{ijpt}\right)\le 4,\ \ j\in D_1,t\in T \tag{6}$$

The constraint (2) states that the total amount of wheat transported from a source warehouse (containing wheat) to all rail-connected locations over the entire time horizon must not exceed the quantity available at the source warehouse. A similar constraint for source warehouses with rice is given by (3). Constraints (4) and (5) state that a source warehouse (in W_1 or W_2, respectively) can allocate either a full or a half rake of wheat or rice, respectively, to a rail-connected location in D_1 at any time. Constraint (6) ensures that congestion does not occur at a location $i\in D_1$ by limiting the maximum number of half-rakes that can be placed at it at any time-period to 4. Note that a full rake consists of two half rakes.

$$\sum_{i\in W_1,t\in T}(2\times x^w_{ijt}+y^w_{ijt})\times\bar{n}\times\tilde{c}+\sum_{i\neq j\in D_1,t\in T}(\eta_{ijpt}-\eta_{jipt})\times\bar{c}$$
$$-\sum_{i\in D_2,t\in T}\eta_{jipt}\times\bar{c}=d_{jp},\ \ j\in D_1,p=wh, \tag{7}$$

$$\sum_{i\in W_2,t\in T}(2\times x^r_{ijpt}+y^r_{ijpt})\times\bar{n}\times\tilde{c}+\sum_{i\neq j\in D_1,t\in T}(\eta_{ijpt}-\eta_{jipt})\times$$
$$\bar{c}-\sum_{i\in D_2,t\in T}\eta_{jipt}\times\bar{c}=d_{jp},\ \ j\in D_1,p\in P_2 \tag{8}$$

$$\sum_{i\in W_1}(2\times x^w_{ijt}+y^w_{ijt})\times\bar{n}\times\tilde{c}+\sum_{i\neq j\in D_1}\eta_{ijpt}\times\bar{c}\ge\sum_{i\neq j\in D_1}\eta_{jipt}\times$$
$$\bar{c}+\sum_{i\in D_2}\eta_{jipt}\times\bar{c},\ \ j\in D_1,p=wh,t\in T \tag{9}$$

$$\sum_{i\in W_2}(2\times x^r_{ijpt}+y^r_{ijpt})\times\bar{n}\times\tilde{c}+\sum_{i\neq j\in D_1}\eta_{ijpt}\times\bar{c}\ge\sum_{i\neq j\in D_1}\eta_{jipt}\times$$
$$\bar{c}+\sum_{i\in D_2}\eta_{jipt}\times\bar{c},\ \ j\in D_1,p\in P_2,t\in T \tag{10}$$

Constraints (7) and (8) concern the demand satisfaction of locations in D_1 for different products. The first constraint states that the net amount of wheat allocated to location $i\in D_1$ from warehouses in W_1 and other locations in D_1, and from i to other locations in D_1 and D_2 satisfy the demand at i over the entire time horizon. Constraint (8) models the same demand satisfaction condition for each rice type for each location $i \in D_1$. The constraints (9) and (10) state that the inflow quantity of a product (wheat and rice, respectively) to a location in D_1 must always be greater than or equal to the outflow of that product at any time-period.

$$\eta_{ijpt} \leq \alpha_{ijpt} \times \tilde{n}, i, j \in D_1, i \neq j, p \in P, t \in T, \tag{11}$$
$$\alpha_{ijpt} \leq \eta_{ijpt}, i, j \in D_1, i \neq j, p \in P, t \in T, \tag{12}$$
$$\alpha_{ijpt} + \alpha_{jipt} \leq 1, i, j \in D_1, i \neq j, p \in P, t \in T, \tag{13}$$
$$\sum_{i \in D_1, t \in T} \eta_{ijpt} \times \bar{c} = d_{jp}, j \in D_2, p \in P, \tag{14}$$

The condition (11) states that the number of trucks assigned to a road-connected warehouse $j \in D_2$ with product p at time t from a location at $i \in D_1$ must not exceed the maximum possible number of trucks available at i. In the present scenario, there is always enough number of trucks available at any rail-connected location. And if no truck is allocated from i to j for the product p at time t, then the binary variable α_{ijpt} is set to 0 as indicated by (12). The condition (13) ensures that two locations in D_1 may not transport the same product to each other at the same time t. Constraint (14) models the demand satisfaction of road-connected warehouses in D_2 for different products.

$$z^w_{jt} \leq \sum_{i \in W_1} x^w_{ijt} + y^w_{ijt}, j \in D_1, t \in T, \tag{15}$$
$$z^w_{jt} \geq \frac{\sum_{i \in W_1} x^w_{ijt} + y^w_{ijt}}{|W_1|}, j \in D_1, t \in T, \tag{16}$$
$$z^r_{jpt} \leq \sum_{i \in W_2} x^r_{ijpt} + y^r_{ijpt}, j \in D_1, p \in P_2, t \in T, \tag{17}$$
$$z^r_{jpt} \geq \frac{\sum_{i \in W_2} x^r_{ijpt} + y^r_{ijpt}}{|W_2|}, j \in D_1, p \in P_2, t \in T, \tag{18}$$

Constraints (15) and (16) model the condition that a location in D_1 receives any quantity of wheat at time t if at least one warehouse in W_1 transports either a full or a half rake of that product to it at time t. Constraints (17) and (18) indicate similar conditions for locations in D_1 for rice.

$$y^w_{ijt} = \sum_{k > j \in D_1} \nu^w_{ijkt}, i \in W_1, j \in D_1, t \in T, \tag{19}$$
$$\nu^w_{ijkt} \leq \bar{a}_{ij}, i \in W_1, j, k \in D_1, j < k, t \in T, \tag{20}$$
$$y^r_{ijpt} = \sum_{k > j \in D_1} \nu^r_{ijkpt}, i \in W_2, j \in D_1, p \in P_2, t \in T, \tag{21}$$
$$\nu^r_{ijkpt} \leq \bar{a}_{ij}, i \in W_2, j, k \in D_1, j < k, p \in P_2, t \in T, \tag{22}$$

Constraints (19) and (20) model the condition that a warehouse can form a full rake by combining the quantities of wheat for two locations in D_1 that are allowed to be combined as half rakes at time t. A similar condition for warehouses in W_2 is given by (21) and (22).

$$x^w_{ijt} \leq \frac{\hat{c}_j}{2}, i \in W_1, j \in D_1, t \in T, \tag{23}$$

$$y^w_{ijt} \leq \hat{c}_j, i \in W_1, j \in D_1, t \in T, \tag{24}$$

$$x^r_{ijpt} \leq \frac{\hat{c}_j}{2}, i \in W_2, j \in D_1, p \in P_2, t \in T, \tag{25}$$

$$y^r_{ijpt} \leq \hat{c}_j, i \in W_2, j \in D_1, p \in P_2, t \in T, \tag{26}$$

A source warehouse can assign either a full rake or a half rake to a location in D_1 depending upon its terminal capacity. This condition for different warehouses and products are modeled in (23) to (26). The following remaining constraints specify the type of variables and the values they can take.

$$\eta_{ijpt} \in \mathbb{Z}_+, i \in D_1, j \in D_1 \cup D_2, i \neq j, p \in P, t \in T, \tag{27}$$

$$\alpha_{ijpt} \in \{0, 1\}, i, j \in D_1, i \neq j, p \in P, t \in T, \tag{28}$$

$$x^w_{ijt}, y^w_{ijt} \in \{0, 1\}, i \in W_1, j \in D_1, t \in T, \tag{29}$$

$$x^r_{ijpt}, y^r_{ijpt} \in \{0, 1\}, i \in W_2, j \in D_1, p \in P_2, t \in T, \tag{30}$$

$$z^w_{it} \in \{0, 1\}, i \in D_1, t \in T, \tag{31}$$

$$z^r_{ipt} \in \{0, 1\}, i \in D_1, p \in P_2, t \in T, \tag{32}$$

$$\nu^w_{ijkt} \in \{0, 1\}, i \in W_1, j, k \in D_1, j < k, t \in T, \tag{33}$$

$$\nu^r_{ijkpt} \in \{0, 1\}, i \in W_2, j, k \in D_1, j < k, p \in P_2, t \in T. \tag{34}$$

The above mathematical model has $3|T||D_1|(|D_2| + |D_1 - 1|)$ number of integer variables, $|T||D_1|(3|D_1-1|+2|W_1|+4|W_2|+3)+\binom{|D_1|}{2}|T|(|W_1|+2|W_2|)$ number of binary variables, and $|W_1| + 2|W_2| + |D_1||T|(13 + 3|W_1| + 6|W_2| + 9|D_1-1|)+3(|D_1|+|D_2|)+\binom{|D_1|}{2}|T|(|W_1|+2|W_2|)$ number of constraints, where $\binom{|D_1|}{2}$ is the binomial coefficient indicating the number of 2-combinations of the set $|D_1|$.

We can analytically identify the impact of adding an extra warehouse (source or destination) or a transshipment point on the size of the mathematical model in terms of increase in the number of variables and constraints as shown in the Table 1. We have not considered increasing the number of products and the time horizon, as these two parameters are unlikely to change in the real scenario.

From Table 1 it can be seen that adding a source warehouse containing rice results in twice the number of binary variables and constraints than adding a warehouse containing wheat. This is obvious because the second set of warehouses also contains twice the number of products. However, adding a source

parameter	int. var	bin. var	cons.
W_1	0	$\|T\|\Big(2\|D_1\| + \binom{\|D_1\|}{2}\Big)$	$1 + \|T\|\Big(3\|D_1\| + \binom{\|D_1\|}{2}\Big)$
W_2	0	$2\|T\|\Big(2\|D_1\| + \binom{\|D_1\|}{2}\Big)$	$2\Big[1 + \|T\|\Big(3\|D_1\| + \binom{\|D_1\|}{2}\Big)\Big]$
D_1	$3\|T\|(\|D_1\| + \|D_2\|)$	$\|T\|\Big(\|D_1\|(3 + \|W_1\| + 2\|W_2\|)+$ $2\|W_1\| + 4\|W_2\| + 3\Big)$	$\|T\|\Big(\|D_1\|(9 + \|W_1\| + 2\|W_2\|)+$ $3\|W_1\| + 6\|W_2\| + 13\Big)$
D_2	$3\|T\|\|D_1\|$	0	3

Table 1: Given the set of source warehouses (W_1 and W_2) and destinations (D_1 and D_2), increase in the number of integer variables, binary variables, and constraints when the parameter in the first column increases by 1 is shown in column second, third, and fourth, respectively.

warehouse does not contribute to the number of integer variables, since integer variables are associated with truck allocation to road-connected warehouses. Adding a rail-connected warehouse or a transshipment point contributes to both the integer and binary variables and constraints, which is also clear since these locations are associated with both source warehouses and destination locations. Depending on the values of the source and destination locations, an increase in W_2 or D_1 may dominate the other. Finally, increasing the number of road-connected warehouses results in the smallest increase in model size compared to the other parameters considered. This information can help the decision maker in expanding the transportation network.

5 COMPARISON OF METHODS

We briefly describe the existing method employed by the FOI for solving the MPPMFT problem instances. We then provide some examples to show that this method can lead to a lower quality solution (in terms of the total transportation cost) than the one provided by the proposed model in this paper. The current FOI approach (referred to as method M_0) manually generates a transportation plan by following certain rules of thumb based on the experience gained over time. One such rule is to select the transshipment points for road-connected warehouses based on their distance. Generally, the closest transshipment point is used. Another rule is to assign the source warehouses to rail-connected warehouses based on the lowest rail transportation cost.

Assume that for a road-connected warehouse k, the rail-connected warehouse j is the closest location that can serve as a transshipment point. And j has the lowest rail transportation cost from the source warehouse i. However, location j cannot serve its demand and the demand of k in the same time period depending upon its terminal capacity. Similarly, i may not be able to serve all of j's demand, which includes k's demand in the same time period. Thus, another decision is how much to transport between two locations at any period of time. Also, if the transshipment locations and the sources warehouses, chosen in a greedy manner, do not have enough capacity to satisfy the demand, one must choose a different combination of such locations, and several such combinations are possible. It can be seen that the adverse effects of such local choices can very easily lead to a solution where demand at the destinations is satisfied in an inefficient manner. In most cases, the feasibility of the solution is ensured by the understanding and experience of the person in charge. We point out that the manual creation of such a plan is cumbersome and time-consuming. Moreover, even such a feasible solution can still be suboptimal, as the following three examples demonstrate.

5.1 Impact of Fixing Transshipment Locations

The first example shows that fixing transshipment locations for road-connected warehouses based on shortest distance can lead to suboptimal solutions. This example is depicted in Figure 2. There is one source warehouse (i_1) containing boiled rice, two rail-connected warehouses (j_1 and j_2), and one road-based warehouse (k_1). The demand for j_1, j_2, and k_1 is 0, 1740, and 1740 tons, respectively. Moreover, j_1 and j_2 have terminal capacities of 1 and 2, respectively. The carrying capacity of a half rake is 1740 tons and that of a truck is 20 tons. Since k_1 is a road-connected warehouse, its demand can be satisfied with 87 trucks from either j_1 or j_2. Values of different cost related parameters

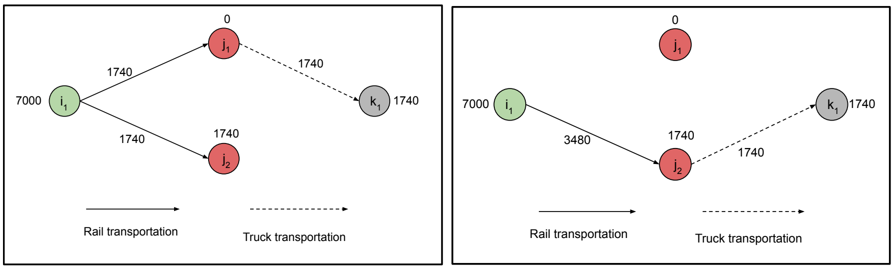

Fig. 2: An example showing that prefixing the transshipment terminals for road-connected warehouses may lead to suboptimal solution. Transportation plan by the method M_0 and the mathematical model are depicted on the left and the right side, respectively.

are listed in Table 2. In this example, the handling costs are not considered, since the corresponding cost share in the total costs is the same for both the methods.

Location j	$\hat{t}_{ij}$	$\tilde{t}_{ij}$	$\bar{t}_{jk}$	$\bar{f}_j$
j_1	1875	1786	6443	13410
j_2	1755	1671	8687	23915

Table 2: Values for the parameters related to the fixed and variable rail and truck transportation costs. Here i is the source warehouse i_1 and k is the road-connected warehouse k_1.

While solving this example, method M_0 fixes j_1 as the transshipment location for warehouse k_1 beforehand. Thus, in the final solution reported by this method, j_1 and j_2 are assigned a half rake each, and then the product is transported from j_1 to k_1 by trucks. The total transportation cost is 6914066 (variable costs for rail (1875x1740 + 1755x1740) and truck (87x6443), and fixed cost for rail (13410 + 23915)). Solving this example using the proposed integer programming technique yields a total cost of 6594764 (variable costs for rail (1671x3480) and truck (87x8687), and fixed cost for rail (23915)). In the solution obtained by solving the proposed model, the demand of k_1 is satisfied by a full-rake allocation to j_2, which allows the use of volume discounts on full-rake orders. In addition, j_2 is located closer to the source warehouse i_1. It is found that the lower rail transportation costs offset the additional cost of trucking from j_2 to k_1, ultimately resulting in cost savings. Figure 2 shows a pictorial depiction of the solutions from the two methods.

5.2 Impact of Fixing Source Warehouses

Next, we illustrate the adverse effects that result when the decision to allocate source warehouses to rail-connected destinations is based solely on the cost of rail transportation. We take the example of allocating boiled rice from a set of six source warehouses (i_1 to i_6) to a set of nine destinations consisting of seven rail-connected warehouses (j_1 to j_7) and two transshipment locations (t_1 and t_2) over a period of four weeks. Let all the destinations can handle only a half rake at any time-period. The rail transportation cost per ton for a half rake from a source warehouse to a destination is given in Table 3. The capacity of a half rake is 1740 tons; the total cost of transporting a half rake is obtained by multiplying the values given in Table 3 by 1740.

	j_1	j_2	t_1	j_3	j_4	j_5	j_6	j_7	t_2	a^r_{ip}
i_1	1755	1755	1635	1713	1635	1513	1513	1513	1513	17400
i_2	1875	1875	1755	1755	1755	1635	1635	1635	1635	3480
i_3	2377	2377	2353	2377	2377	2353	2206	2353	2206	5220
i_4	1875	1755	1635	1755	1755	1635	1513	1635	1513	1740
i_5	1875	1755	1635	1755	1755	1635	1513	1635	1513	1740
i_6	1875	1875	1755	1875	1875	1635	1635	1755	1635	1740
d_j	6960	3480	3480	3480	1740	1740	1740	5220	3480	

Table 3: Rail transportation cost for a half rake, parameter $\tilde{t}_{ij}$, for $i \in \{i_1, \ldots, i_6\}$ and $j \in \{j_1, \ldots, j_7, t_1, t_2\}$ are given in the first six rows and nine columns. The quantity of boiled rice ($p = br$) available at source warehouses is shown in the last column and the demand of the destinations in the set $\{j_1, \ldots, j_7, t_1, t_2\}$ are indicated in the last row.

Table 4 shows the number of half rakes allocated from the source warehouses to the destinations using the method employed by FOI. Table 5 shows the optimal allocation resulting from the proposed model for the same example. Comparing only the rail transportation costs (fixed costs and handling costs are the same in both cases, since the allocation quantity and the number of allocations are the same) corresponding to these two solutions, we see that the M_0 method leads to a suboptimal solution with a higher cost of 55,579,080 than the optimal cost of 54,848,280 obtained by solving the proposed integer programming model.

	j_1	j_2	t_1	j_3	j_4	j_5	j_6	j_7	t_2
i_1	2	1	1	1	1	1	1	1	1
i_2	1	0	0	1	0	0	0	0	0
i_3	1	1	1	0	0	0	0	0	0
i_4	0	0	0	0	0	0	0	1	0
i_5	0	0	0	0	0	0	0	1	0
i_6	0	0	0	0	0	0	0	0	1

Table 4: The number of half rakes allocated from source warehouses $i \in \{i_1, \ldots, i_6\}$ to the destinations $j \in \{j_1, \ldots, j_7, t_1, t_2\}$ over the period of four weeks by using method M_0 of the FOI.

	j_1	j_2	t_1	j_3	j_4	j_5	j_6	j_7	t_2
i_1	0	1	2	0	1	1	0	3	2
i_2	0	0	0	2	0	0	0	0	0
i_3	3	0	0	0	0	0	0	0	0
i_4	0	1	0	0	0	0	0	0	0
i_5	0	0	0	0	0	0	1	0	0
i_6	1	0	0	0	0	0	0	0	0

Table 5: The number of half rakes allocated from source warehouses $i \in \{i_1, \ldots, i_6\}$ to the destinations $j \in \{j_1, \ldots, j_7, t_1, t_2\}$ over the period of four weeks from solving the proposed integer programming model.

Finally, we present an example instance that demonstrates the combined adverse effects of specifying source warehouses and transshipment points beforehand. The full description of this instance is given in Appendix 7. The transportation plans generated using method M_0 and the proposed model are shown in Table 6 and Table 7, respectively. In the solution using method M_0, the demand at all destinations is satisfied only with half rakes, while in the solution using the proposed model, full and half rakes are used in an optimal way. Thus, the latter solution benefits from volume discounts due to freight consolidation and results in lower total fixed costs. The solution resulting from method M_0 uses more number of half rake allocations, resulting in higher fixed costs and a total transportation cost of 87,414,816. The total cost corresponding to the optimal solution is 82,187,219. In total, a huge saving of 5,227,597 can be obtained with the proposed model.

	$T=1$	$T=2$	$T=3$	$T=4$
wh	- -	$i_1 \to j_2, j_3$ (HR) $j_2 \to k_2$, 87 trucks	$i_1 \to j_2, j_3$ (HR) $j_2 \to k_2$, 87 trucks	$i_1 \to j_2, j_3$ (HR) $j_2 \to k_2$, 87 trucks
br	$i_3 \to j_1, j_2$ (HR) -	$i_3 \to j_2, j_3$ (HR) $j_2 \to k_1$, 87 trucks	$i_4 \to j_2, j_3$ (HR) $j_2 \to k_1$, 87 trucks	$i_4 \to j_2, j_3$ (HR) $j_2 \to k_1$, 87 trucks
rr	$i_3 \to j_1, j_2$ (HR) -	$i_3 \to j_2, j_3$ (HR) $j_2 \to k_1$, 87 trucks	$i_4 \to j_2, j_3$ (HR) $j_2 \to k_1$, 87 trucks	$i_4 \to j_2, j_3$ (HR) $j_2 \to k_1$, 87 trucks

Table 6: The solution obtained by applying the existing method M_0 to the instance in Appendix 7. The columns correspond to the allocation in four different weeks that form the planning horizon. Each row corresponds to the transportation of a product in $\{wh, br, rr\}$ between different locations. In a row, the top entries indicate the allocation from the source warehouse to rail-connected locations using a rake. The lower entries indicate the allocation from rail-connected locations to road-connected warehouses. For example, the first entry in the first row, second column states that in week 2, source warehouse i_1 allocates a half rake (HR) of wheat to location j_2 and another half rake to location j_3. Then, location j_2 transports 87 trucks of wheat to road-connected warehouse k_2. Note that in this solution, all rail allocations are made using half rakes only. The entry $-$ means no allocation.

	$T=1$	$T=2$	$T=3$	$T=4$
wh	- -	$i_2 \to j_3$ (FR) -	$i_2 \to j_3$ (FR) $j_3 \to k_2$, 174 trucks	$i_2 \to j_2, j_3$ (HR) $j_2 \to k_2$, 87 trucks
br	$i_3 \to j_3$ (FR) -	$i_4 \to j_1$ (FR) $j_1 \to k_1$, 87 trucks	$i_4 \to j_1$ (FR) $j_1 \to k_1$, 174 trucks	$i_3 \to j_2, j_3$ (HR) -
br	$i_3 \to j_3$ (FR) -	$i_4 \to j_1$ (FR) $j_1 \to k_1$, 87 trucks	$i_4 \to j_1$ (FR) $j_1 \to k_1$, 174 trucks	$i_3 \to j_2, j_3$ (HR) -

Table 7: An optimal solution obtained from solving the proposed model for the instance in Appendix 7. The columns correspond to the allocation in four different weeks that form the planning horizon. Each row corresponds to the transportation of a product in $\{wh, br, rr\}$ between different locations. In a row, the top entries indicate the allocation from the source warehouse to rail-connected locations using a rake. The lower entries indicate the allocation from rail-connected locations to road-connected warehouses. For example, the first entry in the second row, third column states that in week 3, source warehouse i_4 allocates a full rake (FR) of boiled rice to location j_1. Then, location j_1 transports 174 trucks of boiled rice to road-connected warehouse k_1. Note that in this solution, full and half rakes are used in an optimal way. The value $-$ means that no allocation is made.

6 COMPUTATIONAL STUDY

In this section, we present a computational study of the performance of the proposed mathematical formulation of the MPPMFT problem on real instances of different sizes drawn from historical data. The details of these instances are presented in Table 8. A sample instance used in Section 5 is provided in Appendix 7. Furthermore, we have solved the instances up to double the current real-life size to test the scalability of the proposed model.

All the computational experiments have been carried out on a system with two 64-bit Intel(R) Xeon(R) E5-2670 v2, 2.50GHz CPUs having 10 cores each and sharing 128GB RAM. For solving the mathematical model, optimization solvers CPLEX version 20.1.0.0 and SCIP version 7.0.3 are used with a time limit of 3600 seconds and relative optimality gap of 10^{-6}. The rest of the solvers' parameters are used at their default values.

S. No	$\|W_1\|$	$\|W_2\|$	$\|D_1^1\|$	$\|D_1^2\|$	$\|D_2\|$	# vars	# cons
1	2	2	2	1	2	(216, 324)	687
2	2	2	3	1	2	(240, 528)	1096
3	3	2	3	2	3	(420, 860)	1711
4	3	3	5	3	3	(960, 2352)	4346
5	4	4	6	4	5	(1680, 4320)	7417
6	4	5	7	4	5	(1980, 5764)	9522
7	5	5	7	5	6	(2448, 7128)	11565
8	5	6	8	4	7	(2592, 7848)	12386
9	6	6	8	4	7	(2592, 8208)	12795
10	6	7	9	4	9	(3276, 10348)	15738
11	7	7	9	4	8	(3120, 10764)	16204
12	7	8	9	5	9	(3696, 13300)	19608
13	8	8	10	4	10	(3864, 13776)	20144
14	9	9	10	6	9	(4608, 19488)	27718
15	10	10	10	6	10	(4800, 21312)	29740
16	15	15	15	10	15	(11700, 70500)	90565
17	20	20	20	12	20	(19584, 146688)	179672

Table 8: Description of test instances. The columns from left to right represents the serial number of the instance, the number of warehouses containing wheat and rice, the number of rail-connected warehouses and transshipment locations ($D_1 = D_1^1 \cup D_1^2$), and the number of road-connected warehouses, total number of variables (integer, binary), and total number of constraints, respectively. Instance no. 15 is the size of the real instance that is required to be solved by FOI.

The mathematical model is written using the modeling language AMPL [17] and solved using two different solvers: CPLEX and SCIP. These two solvers are used with the intention of presenting the solvability of proposed model using a commercial (CPLEX) solver and an open-source (SCIP) solver. The solution time and optimal value for solving test instances by these solvers are reported in Table 9. Our computational results show that both the solvers could solve all the instances. CPLEX could solve them in very short time - the instance of the real-life size could be solved in less than 15 seconds and an instance of double the size could be solved in less than 60 seconds. These results indicate that the proposed model is very efficient for solving real-life instances and scales well for even larger instances, like instance 16 and 17. These two instances are much bigger than the real-life instances solved by the FOI and are created by increasing the size of the existing transportation network.

7 CONCLUSIONS AND FUTURE WORK

We consider a real-life multimodal freight transportation problem with freight consolidation to take advantage of volume discounts offered by the railways. First, through examples, we demonstrate that the method currently in practice to solve this problem is cumbersome and results in suboptimal solutions. We propose an integer programming formulation of this problem and solve it using state-of-art solvers. Our computational results show that using this technique one could solve practical sized problems to optimality in reasonably short time

S. No	CPLEX	SCIP	Obj. val
1	0.02	0.06	28881191
2	0.03	0.17	35122897
3	0.15	0.27	57033617
4	0.16	0.78	64323122
5	0.19	1.39	77328788
6	0.53	2.08	95156660
7	0.45	2.25	104847329
8	0.85	3.17	129924127
9	4.41	6.59	131148742
10	3.17	21.74	159501103
11	5.87	30.74	145976461
12	9.44	34.31	144545611
13	7.20	35.06	156205051
14	20.04	89.32	154961159
15	12.21	39.26	158843229
16	12.74	367.56	623336991
17	40.70	1490.65	785511977

Table 9: The first column represents the serial number of the instances as shown in Table 8. The second and the third columns show the solution time (in seconds) by CPLEX and SCIP respectively. The last column contains the optimal objective values as commonly reported by both the solvers.

and obtain considerable cost savings. The proposed method also performs well on larger instances.

Varying the number of source warehouses containing rice and locations connected by rail (warehouses or transshipment points) changes the size of the mathematical model the most. The smallest change is caused by varying the number of road-connected warehouses. This information is helpful in tactical decisions regarding the design of the transportation network. The proposed model captures general operational constraints associated with rail and road transportation and can be easily adapted to similar problems encountered in industries such as cement, steel, etc. that involve bulk freight transportation.

One possible extension of the model is to include more layers of the transportation network such as the centers serving the source warehouses and the distributors connecting the destination warehouses to the end users. Also, waterways can be included as another mode of transportation. This will allow the optimization of the entire transportation system in a holistic way.

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APPENDIX

Test Instance

We provide the AMPL data statements for an instance of the MPPMFT problem.

data;
set W_1 := i_1 i_2;
set $W_2 := i_3\ i_4$;
set $P := wh\ br\ rr$;
set $D_1 := j_1\ j_2\ j_3$;
set $D_2 := k_1\ k_2$;
set $T := 1, 2, 3, 4$;
param $\tilde{h} := 3600$;
param $\bar{h} := 1200$;
param $\tilde{c} := 60$;
param $\bar{c} := 20$;
param $\bar{n} := 29$;

param d :	wh	br	rr :=
j_1	0	1740	1740
j_2	0	1740	1740
j_3	5220	5220	5220
k_1	0	5220	5220
k_2	5220	0	0;

param: a^w :=

i_1	24750
i_2	43410;

param a^r :

	br	rr :=
i_3	7707	8130
i_4	9300	43000;

param $\hat{t}$:

	j_1	j_2	j_3 :=
i_1	2904	2904	2867
i_2	2747	2867	2747
i_3	923	1096	1038
i_4	1671	1897	1897;

param $\tilde{t}$:

	j_1	j_2	j_3 :=
i_1	3049	3049	3010
i_2	2884	3010	2884
i_3	969	1151	1090
i_4	1755	1992	1992;

param $\bar{t}$:

	j_1	j_2	j_3	k_1	k_2 :=
j_1	0	21590	26180	13005	30090
j_2	21590	0	5219	12784	14688
j_3	26180	5219	0	17170	16201;

param:	$\bar{f}$	$\hat{c}$:=
j_1	22127	2
j_2	13410	1
j_3	39783	2;

param $\bar{a}$:

	j_1	j_2	j_3 :=
j_1	0	1	1
j_2	1	0	1
j_3	1	1	0;