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Non-anticipative risk-averse analysis with effective scenarios applied to long-term hydrothermal scheduling

Karla Cristiane Arsie · Clóvis C. Gonzaga · Elizabeth W. Karas · Gislaine A. Periçaro

Abstract In this paper, we deal with long-term operation planning problems of hydrothermal power systems by considering scenario analysis and risk aversion. This is a stochastic sequential decision problem whose solution must be non-anticipative, in the sense that a decision at a stage cannot use a perfect knowledge of the future. We propose strategies to reduce the number of scenarios in such way that the decision obtained by solving the non-anticipative risk-averse problem considering the subset of effective scenarios is as reliable as the decision from the whole set of scenarios. Numerical experiments are presented for validation of the techniques proposed by solving the problem for the Brazilian interconnected system with real data. The results obtained by simulation indicate that our approach can help the decision maker to prevent for the occurrence of critical events in the future.

Keywords Non-anticipative scenario analysis · Stochastic programming · Nonlinear optimization · Hydrothermal power systems

Mathematics Subject Classification (2020) 90C06 · 90C15 · 90C90

Karla Cristiane Arsie
Pontifícia Universidade Católica do Paraná, Curitiba, Brazil
E-mail: karla.arsie@grupomarista.org.br

Clóvis Caesar Gonzaga
This is his last article as co-author, since he passed way in August, 2021.

Elizabeth Wegner Karas
Uniersidade Federal do Paraná, Curitiba, Brazil
E-mail: ewkaras@ufpr.br

Gislaine Aparecida Periçaro
Universidade Estadual do Paraná, Campo Mourão, Brazil
E-mail: gpericaro@gmail.com

1 Introduction

In this paper, we address large-scale stochastic multistage problems that include non-linear constraints. We are interested in non-anticipative solutions, in the sense that a decision at a stage cannot use a perfect knowledge of the future [17]. Our approach follows the ideas discussed recently in [14], based on scenario analysis with the additional inclusion of a risk-averse measure.

The study is motivated by the optimal operation planning of a hydro-thermal power system applied to the Brazilian interconnected system, which will be described in Sec. 5. A main feature of the problem is that the amount of energy generated by the hydroelectric plants depends on the water inflows which are non deterministic but are described by stochastic models based on known historical data.

This problem, known in the literature as Long-Term Hydrothermal Scheduling (LTHS) problem, is often modelled as a linear multistage stochastic program. The linear model is obtained by simplifications in the hydropower production function (HPF), which is naturally a nonlinear function of net water head and water discharge. A detailed discussion about some strategies to represent HPF in the LTHS problem is presented in [9]. The LTHS problem is frequently solved by Stochastic Dual Dynamic Programming (SDDP), a technique proposed in [13] or by Progressive Hedging decomposition method [20], as in [7]. In general, a large number of scenarios are necessary to represent accurately the stochastic inflows. A Scenario Optimal Reduction technique using probability metrics is proposed in [3]. An overview of the LTHS problem is presented in [4], by discussing the model formulations, classical and heuristics methods of solution and other topics, such as the inclusion of risk measures. Risk-averse modeling approaches for the LTHS problem, with application to the Brazilian power system, are presented in [10–12].

In this context, our approach considers a model with individual hydro plants, a non-linear model for the HPF and scenario analysis to treat the uncertainties, a technique where the uncertainty is modeled by a set of scenarios, also used in [5, 8]. We use completely independent and equiprobable inflow scenarios generated by a CARMA model [2,6]. In the non-anticipative approach considered in this paper, the decision taken at a stage must minimize a risk measure for the whole horizon, which may be a mean value (risk-neutral solution), or a measure like worst cost or *CVaR* (risk-averse solution), a concept described in detail, for instance, in [18,19]. In order to improve the tractability of our risk-averse approach, we study a simplification of *CVaR* minimization using the concept of ineffective scenarios, described in [1,16]. The detection of these scenarios may lead to a significant reduction in the problem size, as we shall demonstrate in the application to the operation problem.

The paper is organized as follows. Sec. 2 simplifies the concepts of the risk measures *VaR* and *CVaR*, developed in [19], for the particular case of interest in which the random variables are discrete and the events are equiprobable. Sec. 3 describes the deterministic, non-anticipative risk-neutral and non-anticipative risk-averse problems. To reduce the size of this last problem, we propose in Sec. 4 an algorithm that identifies ineffective scenarios, based in [1,16]. In Sec. 5, we apply the proposed methodology for solving the long-term operation problem of the Brazilian hydro-thermal power system. Conclusions are presented in Sec. 6.

2 Risk measures

Consider a set of L independent equiprobable events and a discrete random variable that associates a scalar cost to each event. So, if the events are indexed by $i = 1, 2, \ldots, L$, the random variable can be represented by a vector $Z = [Z_1, Z_2, \ldots, Z_L]$. It may happen that different events have the same cost, so we assume that

$$Z_1 \leq Z_2 \leq \ldots \leq Z_L. \tag{1}$$

In this case, Z_L represents the worst case and for a given $p \in \{1, \ldots L\}$, the p-th worst cost is given by Z_{L-p+1}. The p events with greatest costs are called expensive events and the other $L - p$ ones, cheap events. The mean value is denoted by $E[Z]$. The percentage of cheap events is given by

$$\alpha = \frac{L - p}{L} \tag{2}$$

and consequently, $p = (1 - \alpha)L$ and α is a multiple of $1/L$. With this notation, the risk measures *VaR* and *CVaR*, defined in [19], are given by

$$VaR_\alpha(Z) = Z_{L-p+1} \qquad \text{as the } p\text{-th worst event}, \tag{3}$$

and

$$CVaR_\alpha(Z) = \frac{1}{p} \sum_{i=L-p+1}^{L} Z_i \qquad \text{as the mean of the } p \text{ expensive events.} \tag{4}$$

Note that the cases in which $p = 1$ and $p = L$ refer to the study of the worst case and of the mean value, respectively. So, these cases will not be considered from now on. We could denote VaR_p and $CVaR_p$, but we prefer to keep the traditional notation, as described in details in [19], that considers events Z_i with any probability p_i such that $\sum_{i=1}^{L} p_i = 1$ and $\alpha \in [0, 1]$. For our particular case, it holds that

$$P[Z < VaR_\alpha(Z)] = \alpha \quad \text{and} \quad P[Z \geq VaR_\alpha(Z)] = 1 - \alpha.$$

In these definitions, we have assumed that the order of the costs (1) is known. But, in general, this is not true. Next theorem states how to compute *VaR* and *CVaR* without the knowledge of the order of the costs. This theorem is proved in [19] for the general case, but we include also the proof for our particular case because it is simple and original.

Theorem 1 *Consider a set of L equiprobable events and the random variable that assumes the values* $[Z_1, Z_2, \ldots, Z_L]$*. Then, for* $p \in \{2, \ldots, L - 1\}$ *and* $\alpha = (L - p)/L$*, we have*

$$CVaR_\alpha(Z) = \min_{\lambda \in \mathbb{R}} \left\{ \lambda + \frac{1}{p} \sum_{i=1}^{L} [Z_i - \lambda]^+ \right\}, \tag{5}$$

where $[a]^+ = \max\{0, a\}$.

Proof Suppose, without loss of generality, that $Z_1 \leq Z_2 \leq \ldots \leq Z_L$, since the order of the costs is irrelevant for (5). Consider the convex function $g : \mathbb{R} \to \mathbb{R}$ defined by

$$g(\lambda) = \lambda + \frac{1}{p}\sum_{i=1}^{L}[Z_i - \lambda]^+ \tag{6}$$

and assume that $\bar{\lambda}$ is one of its minimizers. Define q^+ and q as the first index such that $Z_i - \bar{\lambda} > 0$ and $Z_i - \bar{\lambda} \geq 0$, respectively. So, $L - q^+ + 1$ and $L - q + 1$ are the number of terms such that $Z_i - \bar{\lambda} > 0$ and $Z_i - \bar{\lambda} \geq 0$, respectively. Furthermore, $q^+ \geq q$ and $Z_{q-1} - \bar{\lambda} < 0$, if this term exists. We will now prove that $L - q^+ + 1 \leq p \leq L - q + 1$, and consequently

$$Z_i \geq \bar{\lambda}, \quad \text{for} \quad i \geq L-p+1 \geq q \qquad \text{and} \qquad Z_i < \bar{\lambda}, \quad \text{for} \quad i < L-p+1 \leq q^+. \tag{7}$$

(i) Suppose by contradiction that $L - q^+ + 1 > p$. Let $\varepsilon = (Z_{L-p+1} - \bar{\lambda})/2 > 0$. Then,

$$\begin{aligned} g(\bar{\lambda} + \varepsilon) &= \bar{\lambda} + \varepsilon + \frac{1}{p}\left(\sum_{i=q^+}^{L}(Z_i - \bar{\lambda}) - (L - q^+ + 1)\varepsilon\right) \\ &= \bar{\lambda} + \frac{1}{p}\sum_{i=q^+}^{L}(Z_i - \bar{\lambda}) + (p - (L - q^+ + 1))\,\frac{\varepsilon}{p} \\ &< g(\bar{\lambda}), \end{aligned}$$

which contradicts the optimality of $\bar{\lambda}$ and proves that $L - q^+ + 1 \leq p$.
(2) Now, suppose by contradiction that $L - q + 1 < p$. Let $\varepsilon = -(Z_{q-1} - \bar{\lambda})/2$ which is positive since $Z_{q-1} - \bar{\lambda} < 0$. Thus, we have,

$$\begin{aligned} g(\bar{\lambda} - \varepsilon) &= \bar{\lambda} - \varepsilon + \frac{1}{p}\left(\sum_{i=q}^{L}(Z_i - \bar{\lambda}) + (L - q + 1)\varepsilon\right) \\ &= \bar{\lambda} + \frac{1}{p}\sum_{i=q}^{L}(Z_i - \bar{\lambda}) + (L - q + 1 - p)\,\frac{\varepsilon}{p} \\ &< g(\bar{\lambda}), \end{aligned}$$

which contradicts the optimality of $\bar{\lambda}$ and proves that $p \leq L - q + 1$.

Computing g, defined in (6), at $\bar{\lambda}$, we have

$$g(\bar{\lambda}) = \bar{\lambda} + \frac{1}{p}\sum_{i=1}^{L-p}[Z_i - \bar{\lambda}]^+ + \frac{1}{p}\sum_{i=L-p+1}^{L}[Z_i - \bar{\lambda}]^+,$$

and using (7),

$$g(\bar{\lambda}) = \bar{\lambda} + \frac{1}{p}\sum_{i=L-p+1}^{L}(Z_i - \bar{\lambda}) = \frac{1}{p}\sum_{i=L-p+1}^{L} Z_i = CVaR_\alpha(Z),$$

completing the proof.

3 The problems

We are interested in stochastic sequential decision problems. These problems involve making decisions subject to uncertainty along several stages. The uncertainties are dealt with through scenario analysis and the decisions for one stage must be made without deterministic knowledge of future events, that is, decisions must be non-anticipative. Furthermore, decision-making can be risk-neutral or risk-averse. In this section we present the problems of interest.

Consider a scenario $A = [A_1, A_2, \dots, A_T]$ that is defined as a sequence of T random vectors $A_t \in \mathbb{R}^R$. The multistage optimization problem is built on a structure $x = [x_1, x_2, \dots, x_T]$ where $x_t \in \mathbb{R}^n$ are vectors of decisions, for all $t \in \{1, \dots, T\}$. For a given scenario A, a feasible set $\Gamma(A)$ is defined by problem constraints, and a cost $f(x)$ is associated with each decision.

Deterministic problem. Given a scenario A, consider the deterministic problem

$$\begin{aligned} &\text{minimize } f(x) \\ &\text{subject to } x \in \Gamma(A), \end{aligned} \tag{8}$$

where $f : \mathbb{R}^{nT} \to \mathbb{R}$ is a differentiable function and $\Gamma \subset \mathbb{R}^{nT}$ represents the feasible set associated with the scenario A. A solution of this problem, denoted by $\bar{x}(A)$, is composed by a sequence of decisions $\bar{x}_t, t = 1, \dots, T$, which defines a policy associated with the scenario A.

Consider now a set $\mathcal{A} = [A^1, A^2, \dots, A^L]$ of L independent scenarios. For each scenario A^i of the set $\mathcal{A}$, we have a deterministic problem. By solving the L independent deterministic problems, we obtain L solutions $\bar{x}(A^i) \in \mathbb{R}^{nT}$, one for each problem (8) with $A = A^i$.

Non-anticipative problem. Assume now that the L scenarios coincide at the first stage, i.e, $A_1^i = \bar{A}_1 \in \mathbb{R}^R$, for all $i = 1, \dots, L$. The non-anticipative problem is given by

$$\begin{aligned} &\text{minimize } \frac{1}{L}\sum_{i=1}^{L} f(x^i) \\ &\text{subject to } x^i \in \Gamma(A^i), \quad i = 1, \dots, L, \\ &\qquad\qquad x_1^i = x_1^j, \quad i, j = 1, \dots, L. \end{aligned} \tag{9}$$

We have L scenarios and a solution of this problem is a cloud of scenario-solutions, with identical decisions in the first stage. The number of variables is L times the number of variables of each member of the cloud of solutions. This number attains, normally, one million of variables. We will introduce a notation of the cost for each of these members:

$Z_i(x^i) = f(x^i)$	cost of a policy associated with scenario A^i
$Z(x) \;= [Z_1(x^1), Z_2(x^2), \dots, Z_L(x^L)]$	vector of costs associated with the L scenarios of the cloud.

The objective function of the problem (9) is the mean value $E[Z(x)]$ of $Z(x)$. The vector Z can be seen as a discrete random equiprobable variable associated with the scenarios, which will be discussed ahead.

Non-anticipative risk-averse problem. Given $p \in \{2, \dots, L-1\}$ and $\alpha = (L-p)/L$, the non-anticipative risk-averse problem is given by

$$\begin{array}{lll} \text{minimize} & CVaR_\alpha(Z(x)) & \\ \text{subject to} & x^i \in \Gamma(A^i), & i = 1, \dots, L, \\ & x_1^i = x_1^j, & i, j = 1, \dots, L. \end{array} \tag{10}$$

This problem has the same structure as (9), but they differ from the objective function. While in (9) the objective function is $E[Z(x)]$ that is easy to compute, here it is necessary to use Theorem 1. Thus, the problem (10) is rewritten as

$$\begin{array}{lll} \text{minimize} & \lambda + \frac{1}{p} \sum_{i=1}^{L} \left[Z_i(x^i) - \lambda \right]^+ & \\ \text{subject to} & x^i \in \Gamma(A^i), & i = 1, \dots, L, \\ & x_1^i = x_1^j, & i, j = 1, \dots, L \\ & \lambda \geq 0. & \end{array} \tag{11}$$

Note that this problem involves the variables $x^i \in \mathbb{R}^{nT}$, for $i = 1, \dots, L$, and $\lambda \in \mathbb{R}$. Since $[Z_i(x^i) - \lambda]^+ = max\{0, Z_i(x^i) - \lambda\}$, by introducing variables g_i, with $i = 1, \dots, L$, this problem can be rewritten as

$$\begin{array}{lll} \text{minimize} & \lambda + \frac{1}{p} \sum_{i=1}^{L} g_i & \\ \text{subject to} & x^i \in \Gamma(A^i), & i = 1, \dots, L, \\ & x_1^i = x_1^i, & i, j = 1, \dots, L \\ & g_i - Z_i(x^i) + \lambda \geq 0, & i = 1, \dots, L \\ & g_i \geq 0, & i = 1, \dots, L. \end{array} \tag{12}$$

One of the greatest difficult of this problem is in the constraints $x_1^i = x_1^j$, for all $i, j \in \{1, \dots, L\}$, called *coupling constraint*. Without these constraints, the problem consists, essentially, in computing a cloud of scenario-solution, i.e., in solving the L deterministic problems (with distinct decisions for the first stage).

4 Ineffective scenarios

A great reduction of work to solve (12) can be obtained with the use of the concept of ineffective scenarios [1,16], which we are going to develop now. The following definition can be interpreted as: a scenario that did not participate in the cloud in solving the problem is ineffective if its incorporation into the cloud does not change the optimal solution obtained.

Definition 1 Assume that

(i) The problem (10) with a cloud of scenarios $\mathcal{A} = \{A^1, \dots, A^L\}$ was solved generating a cloud of scenario-solution $[x^1, \dots, x^L]$ with costs $[Z_1, \dots, Z_L]$ with $x_1^i = x_1^j$, for all $i, j \in \{1, \dots, L\}$. Define $\hat{x}_1 = x_1^1$.

(ii) $VaR_\alpha(Z)$ is the p-th worst case of Z.
(iii) Given a scenario $\bar{A} \notin \mathcal{A}$, it is solved the deterministic problem

$$\begin{aligned} &\text{minimize } f(x) \\ &\text{subject to } x \in \Gamma(\bar{A}), \\ &\qquad\qquad x_1 = \hat{x}_1. \end{aligned} \tag{13}$$

Consider that problem has solution $\bar{x}$ and denote its optimal cost by $\bar{Z} = f(\bar{x})$. If $\bar{Z} \leq VaR_\alpha(Z)$, then the scenario $\bar{A}$ is *ineffective* for the problem (10).

If a scenario $\bar{A}$ is ineffective for the problem (10), then it can be included in the set $\mathcal{A}$ without to modify the result. In fact, the solution $\bar{x}$ satisfies the constraints of (10) and it provides a cost less than the other p expensive scenario-solution.

If it is possible to detect the ineffectiveness of a scenario before to solve (10), we can simplify the problem. For that, we propose the following algorithm.

Algorithm 1. For identifying ineffective scenarios

Data: Cloud of scenarios $\mathcal{A}$ with indices $I = \{1, \ldots, L\}$, $p < L$.
Choose $\mathcal{A}' \subset \mathcal{A}$ with indices $I' \subset I$ with $L' > p$ elements.
Set $\alpha' = \dfrac{L' - p}{L'}$ and $\bar{I} = \emptyset$
Solve (10), using the cloud $\mathcal{A}'$ with α' and L' scenarios of indices I'. The results are $\bar{x}_1 = x_1^i$, for $i \in I'$, with costs $Z_{I'}$ and the value $VaR_{\alpha'}(Z_{I'})$.
Verify if the scenarios in the set $\mathcal{A} \backslash \mathcal{A}'$ are ineffective as follows:
  For all $j \in I \backslash I'$
    Solve the problem (13) with $\bar{A} = A^j$, and obtain $Z_j = \bar{Z}$.
    If $Z_j \leq VaR_{\alpha'}(Z_{I'})$,
      then A^j is ineffective.
    Else
      Set $\bar{I} = \bar{I} \bigcup \{j\}$.
  If $\bar{I} = \emptyset$, then
    all scenarios of $\mathcal{A} \backslash \mathcal{A}'$ are ineffective,
  Else
    restart the algorithm with $I' = I' \bigcup \bar{I}$.

If all scenarios of $\mathcal{A} \backslash \mathcal{A}'$ are ineffective, the results of the whole problem (10) will be the same: the p scenarios more expensive, the solution x_1^i and the values *VaR* and *CVaR* are identical (less tie between costs of the expensive scenarios). Furthermore, by (2) and the definition of α', we have

$$(1 - \alpha')L' = (1 - \alpha)L.$$

The question now is how to choose the cloud $\mathcal{A}' \subset \mathcal{A}$ candidate to effective scenarios? We have two suggestions:

– Ordering the cloud of deterministic solutions: as the cost variation in the deterministic cloud is very large, an error in the decision of the first stage (that is, ignoring the coupling constraints) does not have a very big effect on the costs of the optimum deterministic. So, the strategy is to compute the L deterministic costs and to choose the index set I' of the L' worst scenarios for constructing the cloud $\mathcal{A}'$.

- Solving a simplification of the problem (10) by considering, for example, a linearization of the functions involved and then by choosing the L' worst scenarios.

5 Application

In this section we apply the methodology discussed above to solve the long-term operation problem of the Brazilian hydro-thermal power system, which is a large-scale stochastic optimization problem whose the mathematical model used here is described in [14]. The system considered in this paper is composed by 141 hydroelectric and 105 thermoelectric plants, distributed in 5 interconnected subsystems.

As described in [14], the objective function of the deterministic problem (8) represents the total operation cost given by the sum of the thermal generation cost, energy deficits (when demand is not met) and the cost of energy interchange among subsystems. On the other hand, the non-anticipative problem (9) minimizes the expected operation costs. From the initial volume of the reservoirs $V_0 \in \mathbb{R}^R$ and an inflow scenario $A^i \in \mathbb{R}^R \times \mathbb{R}^T$ with T stages, the feasible set Γ is given by

$$\Gamma(A^i) = \Gamma(V_0, A^i) = \{x \in \mathbb{R}^{nT} \mid \ell \leq x \leq u,\ h(x, V_0, A^i) = 0 \text{ and } g(x) = 0\}, \tag{14}$$

where $\ell, u \in \mathbb{R}^{nT}$ represent the operational bounds for the decision variables; $h : \mathbb{R}^{nT} \to \mathbb{R}^p$ is a linear function that represents the water balance constraint; and $g : \mathbb{R}^{nT} \to \mathbb{R}^m$ corresponds to the demand constraint. The linear function h depends on the initial volume V_0 and the inflow scenario A. On the other hand, the function g involves the generation at each hydroelectric plant that depends of the plant volume and total outflow and can be nonlinear due to the head computation in plants.

We considered synthetic scenarios of direct inflows provided by the authors of [6]. These scenarios are generated by the CARMA model [2] from the natural inflows at all hydro plants available in Brazil for 88 years, ranging from 1931 to 2019, in monthly time steps.

The non-anticipative problem, with 50 synthetically generated inflow scenarios and a horizon of 60 months, amounts to about one million variables and 15000 nonlinear constraints. As these constraints have sparse and easily computed second derivatives, the second order approximations for the Lagrangians are exactly computed. Thus, the problem is efficiently solved by the filter algorithm presented in [15] with sequential quadratic programming iterations that minimize quadratic Lagrangian approximations using exact hessians in L_∞ trust regions in a standard 2020 notebook computer.

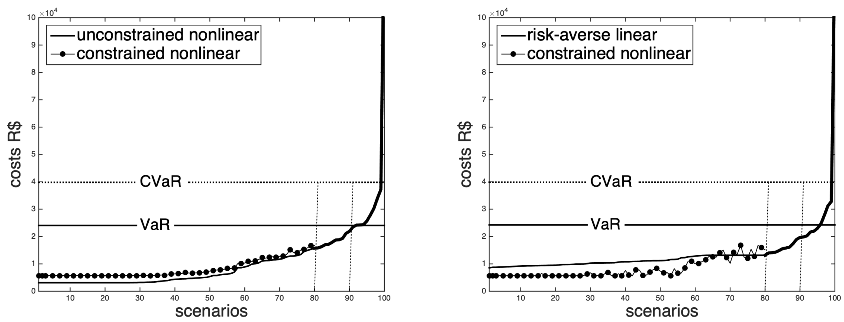
5.1 Ineffective scenarios

In this section we present numerical experiments related to Algorithm 1 by considering a relative small horizon $T = 12$ in order to highlight the influence of the decision of the first stage.

We considered a cloud $\mathcal{A}$ of $L = 100$ synthetic scenarios and we fixed $p = 10$. We took subsets $\mathcal{A}' \subset \mathcal{A}$ with $L' = 20$ scenarios chosen by the two strategies described at the end of Section 4:

- ordering the cloud of deterministic solutions;
- solving the problem (10) with linear demand constraint g in (14), obtained by taking as constant the head of the hydro plants.

Figure 1 shows the system planning costs associated with the $L = 100$ scenarios. The solid line corresponds to the scenario costs used to choose the effective candidate scenarios $\mathcal{A}'$. The difference between both graphs is the way these costs were obtained. In Figure 1a, the solid line represents the ordered costs from the deterministic problem (8) for each scenario of $\mathcal{A}$. On the other hand, in Figure 1b, the solid line represents the ordered costs obtained by solving a linearized version of the problem (10). In both cases, $L' = 20$ expensive scenarios were selected to the set $\mathcal{A}'$ of effective candidate scenarios. Then, the values of *VaR* and *CVaR* and the decision $\bar{x}_1$ were obtained by solving the problem (10) with the scenarios of $\mathcal{A}'$. Finally, the dotted line represents the costs obtained from the deterministic problems (13) for each scenario of $\mathcal{A}\backslash\mathcal{A}'$ with the additional constraint included by fixing the decision of the first stage $\bar{x}_1$. Evidently, these constrained costs are higher than the unconstrained ones as illustrated in Figure 1a. Both figures show that the scenarios in $\mathcal{A}\backslash\mathcal{A}'$ are ineffective for both strategies since their costs are smaller than *VaR*.



(a) Ordering the cloud of deterministic solutions (b) Solving a linearized problem

Fig. 1: Total costs related to the scenarios

5.2 Simulations

In real life it is impossible to know about the future. Long-term optimization only provides the goals for the first stage. To obtain a policy with a horizon $H > 1$, it is necessary to fix a simulation scenario $\bar{A} = [\bar{A}_1, \bar{A}_2, ..., \bar{A}_H]$, with $\bar{A}_i \in \mathbb{R}^R$, $i = 1, \dots, H$, and compute for this scenario what would happen in practice, over H stages.

So, to obtain a non-anticipative policy associated with the simulation scenario $\bar{A}$ and horizon H, we sequentially solve a non-anticipative problem followed by an one-stage problem. This sequential process is performed as many times as the simulation horizon is. At each stage of the sequential process, from the state variable (the initial volume of the reservoirs), a non-anticipative problem is solved and a decision is obtained for the first stage. The sequence of decisions obtained defines the policy for the given simulation scenario.

We describe below a scheme to generate an operation policy using in every stage any long-term optimization method, for example, optimization with or without risk aversion, with linear or nonlinear model, optimization for an average scenario, or even, SDDP. Besides the data related to the planning, we need the following additional information:

- $H \leq T$: simulation horizon;
- $\bar{A} = [\bar{A}_1, \bar{A}_2, ..., \bar{A}_H]$: simulation scenario;
- T_h: horizon for each simulation stage $h = 1, 2, \dots, H$.

Now we present the simulation algorithm that consists of obtaining the decisions of each month in which, the past is completely disregarded, with the exception of the state generated in the previous stage.

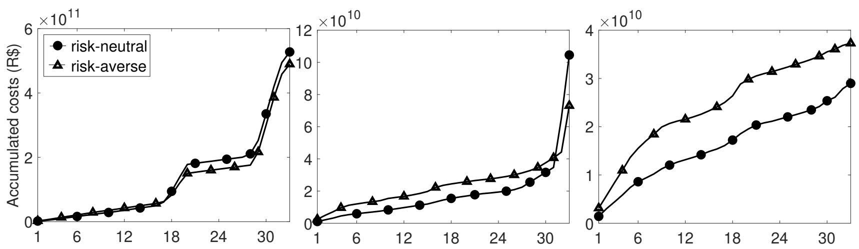
Algorithm 2. Simulation algorithm

Data: simulation horizon H; state variable $\bar{V}_0$;
    simulation scenario $\bar{A} = [\bar{A}_1, \bar{A}_2, ..., \bar{A}_H]$; T_h, for $h = 1, 2, \dots, H$
For $h = 1, 2, \dots, H$
    $V_0 = \bar{V}_{h-1}$
    Generate $[A^1, A^2, ..., A^L]$, $A^i \in \mathbb{R}^{R \times T_h}$, $A^i_1 = \bar{A}_h$, $i = 1, \dots, L$
    Compute the decision x_1 of the first stage and the state variable V_1.
    Using this decision as a target, compute $\bar{V}_h$ by an one-stage algorithm.

So, for each stage h we have a non-anticipative decision and consequently a non-anticipative policy with horizon H associated with the simulation scenario.

We presented simulations of the Brazilian system operation for three historical scenarios, started respectively in 1952 (very dry scenario), 1957 (medium scenario) and 1993 (wet scenario). The simulations start in 2020, and mean the following: "if from 2020 the given scenario occurs, what will be the optimal behavior of the system with or without risk aversion?". For that we considered 100 randomly generated inflow scenarios with $H = 33$ month horizon starting in April and ending in January, when empty reservoirs are allowed. To fix the same final planning stage we have taken $T_h = T_1 - h + 1$, for $h = 2, 3, \dots, H$. So, we have six simulations (three risk-neutral and three risk-averse) and each one requires the nonlinear non-anticipative problem to be solved 33 times, which takes many hours of machine.

Figures 2 and 3 synthesize the result of these six simulations, related to the dry, medium and wet scenarios with risk-neutral and risk-averse approaches. Figure 2 shows the accumulated costs along the months. We see that the costs for the dry simulation scenario are higher than those associated with the medium scenario, which in turn are higher than those obtained from the wet scenario. The total planning cost for the risk-averse approach is lower than the one for risk-neutral in drier scenarios. This occurs because the planning is done by foreseeing the future occurrence of bad scenarios, that is, with low inflows. However, the inclusion at risk does not bring advantages in the wet case shown in Figure 2c, since the inflows were high. Moreover, Figure 3 illustrates the differences between the results of the risk-neutral and the risk-averse approaches. These differences are shown for the deficit and thermal monthly costs. If the difference is negative, the risk-averse approach provides lower costs. We note that for the dry and medium scenarios, the inclusion of risk aversion avoided or reduced deficits, since more attention to critical events was given, by generating more thermoelectric energy at the beginning of the planning. On the other hand, for the wet scenario, we note that the case of including risk aversion is not advantageous. In fact, as we can see in Figure 3c, the risk-aversion approach resulted in higher thermal costs in the first stages by generating thermal energy to avoid deficit in the face of a drought. However, this thermal generation turned out to be unnecessary, since the inflows were higher than expected. As a result, the cost of planning in this case was higher than the one obtained by the risk-neutral approach. So, the inclusion of risk aversion in the planning problem is advantageous in critical cases, since the worst cases are optimized in the risk aversion problem (10).

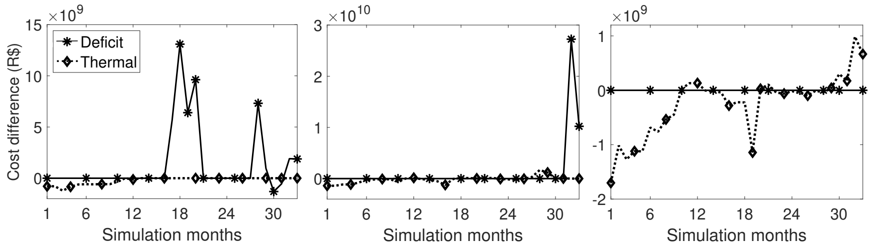


(a) Dry scenario (b) Medium scenario (c) Wet scenario

Fig. 2: Accumulated costs obtained from the simulations.

6 Conclusion

In this paper, we address long-term operation planning problems of hydrothermal power systems. The mathematical model considers individual hydro plants and non-



(a) Dry scenario (b) Medium scenario (c) Wet scenario

Fig. 3: Difference between monthly costs obtained by risk-neutral and risk-averse approaches, related to deficit and thermal generation.

linear model for the HPF. Non-anticipative solutions are obtained from scenario analysis with inclusion of a risk-averse measure. The concepts of *VaR* and *CVaR* are particularized to our case in which the random variables are discrete and the events are equiprobable. Two strategies to reduce the size of the non-anticipative risk-averse problem are proposed. Numerical experiments are presented by solving the LTHS problems applied to the Brazilian system composed by 141 hydroelectric and 105 thermoelectric plants, distributed in 5 interconnected subsystems. The experiments showed the effectiveness of the proposed approaches to reduce the number of scenarios and consequently the dimension of the non-anticipative problem. The *CVaR* minimization can prevent critical situations, particularly, in the occurrence of dry scenarios, since the worst events are optimized in this case. As the future is uncertain, we hope that our non-anticipative risk-averse approach can assist the operator in making a safer decision, minimizing the chances of energy deficit in the system.

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