Optimal Reconfiguration with Variant Transmission Times on Network

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Abstract: Contraflow means lane reversals on networks. In lane reversal reconfiguration, the capacity of arc increases by reorienting arcs towards demand nodes, which maximizes the flow value and reduces the travel time. In this work, we survey the existing pieces of literature on single and multi-commodity contraflow problems with symmetric and asymmetric travel times on parallel but oppositely oriented edges. A number of illustrations are included to support the main results.

Keywords: Network flow, contraflow, asymmetric transit times, time-expanded network, Δ-condensed graph.

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1 Introduction

Hurricanes, floods, earthquakes, industrial catastrophes, nuclear mishaps, terrorist attacks, and other disastrous situations endanger people's lives. Evacuation under these situations is one method of increasing safety and preventing damage from escalating. Over the previous two decades, there has been a greater focus on evacuation issues. The approaches can be generally classified as optimization or simulation methods (see Kotsireas et al. [1]).

In either scenario, the evacuation problem is addressed by a network flow model in which arcs or edges serve as roadways connecting two locations or network nodes. The dangerous zones represent source nodes, while the safe sites where evacuees relocated are called sink nodes. Every arc has a certain capacity. Furthermore, each edge is assigned a travel time or a cost. The transshipment of evacuees or vehicles or commodities via the lanes of the network is modeled as flow.

One of the most critical aspects of disaster preparation is evacuating individuals from dangerous regions to safe areas. Network flow approaches are the most efficient among the several disciplines of mathematical studies, such as fluid mechanics, differential equations, control theory, traffic simulations, and variational inequalities in evacuation planning. An emergency optimizer manages an evacuation network plan to ensure that the maximum number of evacuees are transferred from risky zones (sources) to safer places (sinks) as rapidly as feasible. Selecting the most secure venues and providing humanitarian logistics are difficult in these situations. The survey studies of Lovetskii and Melamed [2], Aronson [3], Hamacher and Tjandra [4], Cova and Johnson [5], Kotnyek [6], Yusoff et al. [7], Skutella [8], and Dhamala et al. [9] provide a detailed discussion of many theories and applications. The main concern of this study is to examine contraflow transportation plannings that has applications not only in emergency evacuations but also during peak traffic hours in a metropolitan city.

Ford and Fulkerson [10, 11] introduced a dynamic flow problem by including the time component in the conventional network flow problem. Gale [12] wonders if it is feasible to transship the maximum amount of flow from a source to a sink at each time point. He introduces a more general problem, known as earliest arrival flow problem, in which flow is maximized at each time point. However, he is unable to provide a method to address this problem. The strategies to tackle this problem in a two-terminal network were devised by Wilkinson [13] and Minieka [14]. The earliest arrival flow problem does not exist in a multi-source and multi-sink network. However, for a multi-source single-sink network with known supply and demands, it always exists [15]. In an emergency, it may be important to provide priority to specific terminals. The lexicographic maximum flow occurs when the flow from the origin node(s) to the destination node(s) is maximized with a certain priority on terminals. The multi-source multi-sink network's lexicographic maximum static flow problem is solved in polynomial time [14]. Hoppe and Tardos [16] and Hoppe [17] provide a dynamic variant of this problem with specified priority ordering on terminals. They also provide a polynomial-time solution for this problem, which is useful in some evacuation planning situations.

Using Fleischer and Tardos [18] natural transformation, discrete dynamic flow solutions are extended to a continuous-time environment with the same time complexity. The first exponential-time augmenting path methods for a single-source single-sink generalized static maximum flow problem were proposed by Jewell [19] and Onaga [20, 21]. The authors in [22] created the first polynomial-time combinatorial approach for the problem. A maximum generalized dynamic flow model in which each arc comprises both gain factors and travel times, was presented by Gross and Skutella [23] and Gross [24]. The issue of generalized dynamic flow is $\mathcal{NP}$-hard in general. They presented a pseudo-polynomial time algorithm to the problem with a single source-sink for a single commodity on a lossy network, where the loss rate per time unit is the same on all arcs.

In both regular and emergency disastrous situations, the location of facilities such as hospitals, warehouses, supermarkets, fire brigades, security offices, and so on is critical. Jia et al. provided the location model of a large-scale emergency medical service center in [25]. Rupp [26], Heller and Hamacher [27], and Hamacher et al. [28] integrate location and flow decisions in a network flow problem, finding that placing a facility on a lane of a network can reduce the maximum flow value. Based on a set of facilities and a set of lanes wherein the facilities are to be put strategy is to assign the facilities to the lanes in such a way that it minimizes the reduction in the maximum flow value.

The best lane reversal approach makes traffic more organized and smooth by alleviating traffic congestion caused by natural and human-induced large-scale disasters, busy office hours, special events, and public demonstrations. Contraflow reconfiguration reverts the usual orientation of unoccupied lanes towards sinks, fulfills the given constraints, increasing the value of the flow and decreasing the average duration of evacuation, using various operations research models, heuristics, optimization algorithms, and simulation.

The goal is to provide a survey that categorizes the approaches available in the literature. Based on the methodologies utilized in the evacuation models, a first categorization was formed, namely optimization-based or simulation-based approaches. Furthermore, a subclassification was created based on the model's consideration of key critical properties. These are elements that will have a substantial influence on evacuation efficiency. The solution approaches for each model in these categories were examined in-depth and evaluated based on their computing performance and realizability. In the event of unanticipated circumstances, the computational efficiency of the model is critical. The models run as efficiently as possible to establish alternate strategies and prepare for the dynamic scenarios.

The remainder of the paper is summarized below. All network flow theory parameters and flow models are described in Section 2. Section 3 reviews the literature on single-commodity contraflow problems with symmetric and asymmetric transmission times on anti-parallel arcs.

Different solution strategies for contraflow problems such as heuristics, simulation and analytical are discussed in Subsections 4.1 and 4.2, respectively, whereas the $\mathcal{NP}$-hardness of the problem is presented in Subsection 4.3. Section 5 surveys multi-commodity contraflow problems with solution strategies, and Section 6 concludes the paper.

2 Preliminaries

To keep this article self-contained, we provide some fundamental notations and definitions alongside the flow models in this section.

2.1 Auxiliary Network

For a network $\mathcal{Q}$, the corresponding auxiliary network is denoted by $\mathcal{Q}^a = (N, E_a, K, b_a, \tau_a, d_i, S_+, S_-, T)$, with undirected edges in $E_a = \{(x, y) : (x, y) \text{ or } (y, x) \in E\}$, where $e^r = (y, x)$ is the backward edge of $e = (x, y)$.

Capacity. The capacity of the auxiliary lane is given by

$$b_a = \begin{cases} b_e & \text{if } e^r \notin E \\ b_{e^r} & \text{if } e \notin E \\ b_e + b_{e^r} & \text{otherwise.} \end{cases} \tag{1}$$

Transit times. On network $\mathcal{Q}$, the arcs are associated with a non-negative travel time taken by flow (commodities) to travel through an arc from the initial point to the final point. The transit time may be constant or it may be flow-dependent. We consider constant travel times throughout this work.

(i) Symmetric. Arulselvan [29] and Rebennack et al. [30] considered symmetric travel times on anti-parallel arcs. The travel time of the auxiliary arc is

$$\tau_a = \begin{cases} \tau_e & \text{if } e \in E \\ \tau_{e^r} & \text{otherwise.} \end{cases} \tag{2}$$

(ii) Non-symmetric. To model, the scenario of uneven road network topology, authors in [31, 32, 33, 34, 35] consider the non-symmetric transit times on anti-parallel arcs and modify the idea of Rebennack et al. [30] (cf.Figure 1). The travel time of the auxiliary arc is

$$\tau_a = \begin{cases} \tau_e & \text{if } e^r \text{ is oriented towards } e \\ \tau_{e^r} & \text{if } e \text{ is oriented towards } e^r \\ \tau_e = \tau_{e^r} & \text{for one way arc } e \text{ or } e^r. \end{cases} \tag{3}$$

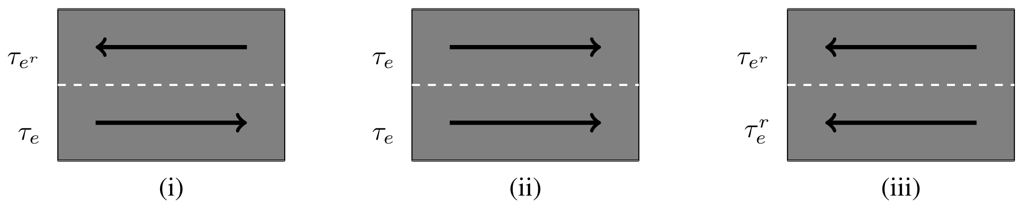


Figure 1: (i) Represents a two-way lane, (ii) represents the network, if lane e^r is reverted towards lane e, and (iii) represents the network, if lane e is reverted towards lane e^r.

2.2 Flow models and notations

Consider the network $\mathcal{Q} = (N, E, K, b, \tau, S_+, S_-, T)$, where N represents sets of vertices, E is the set of edges (arcs), and $K = \{1, 2, \ldots, k\}$ be the set of commodities with $|N| = n$ and $|E| = m$. Each commodity $i \in K$ is routed through a unique source-sink pair (s_i, t_i). The sets S_+ and $S_- \subset V$ denote origin nodes and destination nodes of all commodities respectively. On each arc $e = (x, y)$, the capacity function $b : E \rightarrow \mathbb{R}_{\geq 0}$ limits the flow of commodities, and a non-negative travel time function $\tau : E \rightarrow \mathbb{R}_{\geq 0}$ measures the time to transship the flow from the intial point x to the terminal point y of edge e. The sets $\delta_v^{in} = \{e = (w, v) : \forall w \in N\}$ and $\delta_v^{out} = \{e = (w, v) : \forall w \in N\}$ designate the sets of edges leaving from vertex v and entering to vertex v, respectively. The sets $\mathbb{T} = \{0, 1, 2, \ldots, T\}$ and $\mathbb{T} = [0, T + 1)$ denote the time frame in discrete and continuous-time settings. A network $\mathcal{Q} = (N, E, K, b, S_+, S_-)$ without time component is a static network.

Generalized dynamic multi-commodity flow. For continuous-time GDMCF ξ_c is a sum of flows described by a Lebesque measurable function $\xi_c^i : E \times \mathbb{T} \rightarrow \mathbb{R}^+$ satisfying the constraints (4 - 6).

$$\sum_{e \in \delta^{in}(v)} \int_0^{T-\tau_e} \lambda_e \xi_c^i(e, \rho) d\rho - \sum_{e \in \delta^{out}(v)} \int_0^{T} \xi_c^i(e, \rho) d\rho \quad = \quad 0, \;\; v \notin \{S_+, S_-\}, \tag{4}$$

$$\sum_{e \in \delta^{in}(v)} \int_0^{\theta-\tau_e} \lambda_e \xi_c^i(e, \rho) d\rho - \sum_{e \in \delta^{out}(v)} \int_0^{\theta} \xi_c^i(e, \rho) d\rho \quad \geq \quad 0, \; \forall \; \theta \in \mathbb{T}, \; v \neq S_+, \tag{5}$$

$$0 \leq \sum_{i=1}^{k} \xi_c^i(e, \theta) \quad \leq \quad b_e + b_{e^r}, \; \forall \; e \in E, \; \theta \in \mathbb{T}. \tag{6}$$

The MDMCF problem is to maximize the multi-commodity flow over time $\sum |\xi^i|$ in (7)

$$\max \sum_{i \in K} |\xi^i| = \max \sum_{e \in \delta^{in}(t)} \int_0^{T-\tau_e} \lambda_e \xi^i(e, \rho) d\rho. \tag{7}$$

In this case, the constraints in (4) represent flow conservation constraints at the intermediate vertex in time T. The inequality in (5) indicates moderate flow conservation restrictions that enable the flow to be stored at intermediate vertices, while the equality in (5) depicts flow conservation at intermediate vertices at all times with no storage. Furthermore, constraints in (6) represent capacity constraints on the arcs.

This mathematical formulation is reduced to single-commodity maximum generalized dynamic flow if $i = 1, \forall i \in K$ and $S_+ = \{s\}, S_- = \{t\}$. If we replace the integral sign by summation and remove $d\rho$ in constraints (4, 5, and 7) it reduces to discrete-time maximum generalized dynamic flow, whereas if $\lambda_e = 1, \forall e \in E$ then it reduces to maximum dynamic flow. The maximum static flow model has an analogous formulation by reducing temporal dimension from the above constraints and objective function.

2.3 FlowLoc

Let $L \subseteq E$ represent the set of all feasible sites, F represent the set of all facilities, $r : F \rightarrow \mathcal{Q}$ represent the size of the facilities, and $nol : L \rightarrow \mathcal{Q}$ represent the number of facilities that may be installed on the conceivable locations. The FlowLoc issue involves allocating facilities to arcs in such a way that the $s - t$ flow value in the network $\mathcal{Q}^{loc} = (N, E, b'_e, \tau_e, s, t)$ is maximized, with $b'_e = u_e - \max\{r_f : loc(f) = e\}$. If more than one facility is installed on position l, the capacity on the arc is reduced solely by the size of the largest facility. Other modeling options for putting many facilities on an edge were described in [28]. The multi-facility FlowLoc problems (q-FlowLoc) locate q facilities $f \in F$ of size r_f in such a way that the maximum flow value is reduced as little as feasible and no more than $nol(l)$ facilities are put on each arc $l \in L$. The Single-FlowLoc issue, in instance, allocates one facility from the provided set of facilities to $q = 1$.

2.4 Flow model with intermediate storage

In network flow models with intermediate storage, the inflow into intermediate vertices can be higher than the outflow, and the extra flow can be kept in that vertex as long as the node capacity is not exceeded. As a result, the outflow from a source does not have to be the same as the inflow into a sink it might be more. From this standpoint, authors in [36] suggest a change to the present maximum flow models. One goal is to employ maximal arc capacity to push as much flow out of the source as feasible. Only if the total capacity of arcs leaving the source exceeds the network's minimum cut capacity can only newly suggested model be employed. In this model, I denotes the set of intermediate nodes and $u\colon V \to \mathbb{R}_{\geq 0}$ represents node capacity.

$$\sum_{e\in\delta^{in}(v)}\sum_{\rho=0}^{T-\tau_e}\xi^i(e,\rho) - \sum_{e\in\delta^{out}(v)}\sum_{\rho=0}^{T}\xi^i(e,\rho) \quad \geq \quad 0,\ \ v \notin \{S_+, S_-\}, \tag{8}$$

$$\sum_{e\in\delta^{in}(v)}\sum_{\rho=0}^{\theta-\tau_e}\xi^i(e,\rho) - \sum_{e\in\delta^{out}(v)}\sum_{\rho=0}^{\theta}\xi^i(e,\rho) \quad \geq \quad 0,\ \forall\ \theta\in\mathbb{T},\ v\neq S_+, \tag{9}$$

$$0 \leq \sum_{i=1}^{k}\xi^i(e,\theta) \quad \leq \quad b_e + b_{e^r},\ \forall\, e\in E,\ \theta\in\mathbb{T}. \tag{10}$$

$$0 \leq \sum_{i=1}^{k}\xi^i_v(\theta) \quad \leq \quad u_v,\ \forall\, v\in I,\ \theta\in\mathbb{T}. \tag{11}$$

The MDMCF with intermediate storage problem is to maximize the multi-commodity flow over time with intermediate storage $\sum|\xi^i|$ in (12) (For details see in [37]).

$$\max\sum_{i\in K}|\xi^i| = \max\sum_{e\in\delta^{out}(S_+)}\sum_{\rho=0}^{T}\xi^i(e,\rho) = \max\sum_{e\in\delta^{in}(S_-)}\sum_{\rho=0}^{T-\tau_e}\xi^i(e,\rho) + \sum_{v\in I:u_v\geq 0}\xi^i_v(T). \tag{12}$$

If $i=1, \forall i\in K$ and $S_+=\{s\}, S_-=\{t\}$, this model reduces to single-commodity maximum dynamic flow model with intermediate storage.

3 Contraflow approach

People are discouraged from going to risky regions from safer locations in an emergency. As a consequence, the roads leading to the safe zones grow overcrowded, while those leading to the danger areas become unoccupied. In such instances, turning a two-way lane to a one-way in the proper direction becomes desirable to increase traffic flow and decrease evacuation time. This is called contraflow configuration, and it involves reversing the direction of traffic on unoccupied road segments towards demand points to improve the capacity of the road sections. Contraflow arrangement boosts flow value while reducing road congestion and smoothing vehicle flow. However, determining the best orientations for a network's arcs to optimize flow is a challenging optimization problem. The average evacuation time will be shortened, and certain routes with surplus capacity will be freed up for the use of emergency vehicles and logistical assistance to get to the sources. The contraflow arrangement may be dealt with using a variety of operational research models, heuristics, optimization, and simulation approaches.

Example 1. *Consider the network $\mathcal{Q}=(N,E,b,\tau,s,t,T)$, as given in the Figure 2(i) with asymmetric capacity and symmetric (or asymmetric) travel times on edges. For a time frame of 6 units, before contraflow, a maximum* *of 20 units of flow is transshipped from the origin to the destination, and a maximum of 24 units of flow is sent after contraflow with symmetric travel times, whereas 26 units of flow are sent with asymmetric transit times (cf. Figure 2(ii),(iii), Figure 3(i),(ii), and Table 1).*

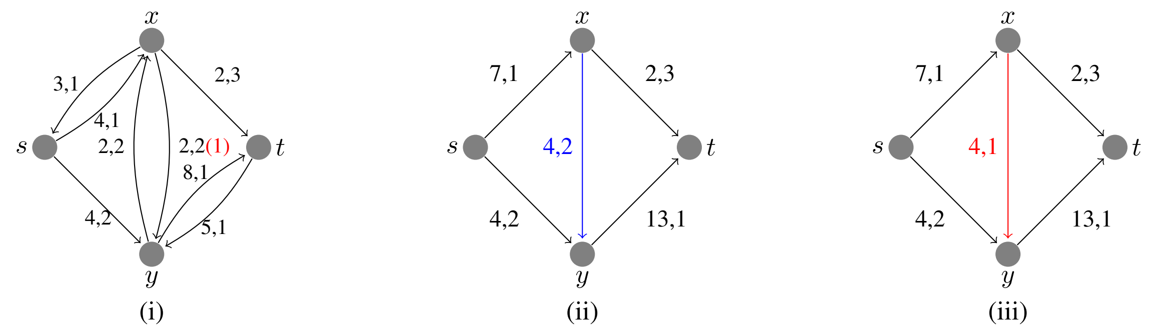


Figure 2: (i) Given network (ii) network after contraflow with symmetric transit times (iii) network after contraflow with asymmetric transit times.

Table 1: Maximum Dynamic Flow before and after LR with ST and AST

Paths	Time	F. before LR	T. F.	F. after LR(ST)	T.F.	F. after LR(AST)	T. F.
$s-y-t$	3	4	12	4	12	4	12
$s-x-y-t$	4(3)	2	4	4	8	4	12
$s-x-t$	4	2	4	2	4	2	4
Total			20		24		26

T = 5, T.F. = Total Flow, LR = Lane Reversals, ST = Symmetric Transit Times, AST = Asymmetric Transit Times

However, if we flip the orientation of arc (x, y) in the direction of the lane (y, x), transit times in both the cases symmetric and asymmetric are the same, and the flow is 16 units (cf. Figure 2(iv)), as shown in Table 2.

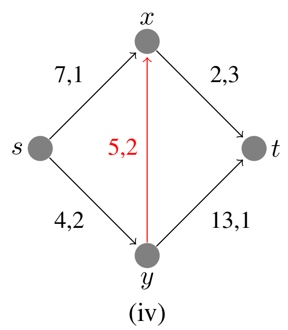


Table 2: Maximum Flow with ST and AST

Paths	Time	Flow with ST and AST	T. F.
$s-y-t$	3	4	12
$s-y-x-t$	7	-	-
$s-x-t$	4	2	4
Total			16

T = 5, T.F.=Total Flow

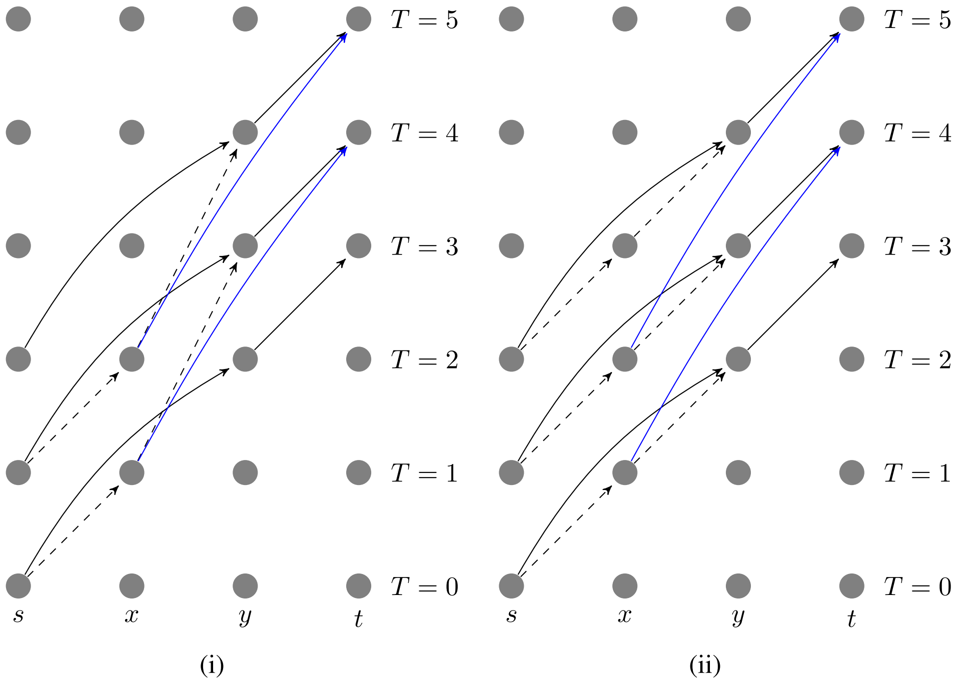


Figure 3: (i) Time-expanded network of Figure 1(ii) and (ii) Time-expanded network of Figure 1(iii).

4 Solution Approaches

4.1 Heuristic and simulation

Contraflows, also known as lane reversals, are considered in simulation models,as they are in optimization models. Lane reversals have been demonstrated to reduce overall evacuation time in recent trends and studies. The authors of [38] conducted a study to investigate the impact of contraflow installations in the New Orleans evacuation.

The studies were carried out to illustrate the advantages of contraflow. The effectiveness measurements, particularly travel duration and average speed, improved significantly for the designs that allowed contraflow. In most circumstances, the contraflows take complete lane reversals, which means that the whole capacity of the route is moved to the destinations. There are a few reasons why you should keep a capacity either way. A network link in each direction might be used to establish a route to the destination in the case of a large network. The reversed links can no longer be utilized to reach the target if a connection breaks and a different path is required.

There is existing literature on heuristic ways to lane reversal strategies and their applications. Authors from a variety of professions have documented considerable time savings and the necessity for good contraflow strategies in many tragic events. Kim and shekhar [39] proposed a simulated annealing procedure for this problem together with emperical results. Kim et al. [40] prove the $\mathcal{NP}$-hardness of the contraflow issue by modeling it as an integer programming problem. They propose two strategies for possible numerical approximation solutions to the quickest contraflow problem: greedy and bottleneck relief heuristics. To obtain precise mathematical solutions for general contraflow strategies is expensive.

It has been demonstrated through computational research that reversing at most 30% of arcs can save at least 40% of the evacuation time. Vogiatzis et al. [41] describe a heuristic approach to address the problem of transporting vehicles from dangerous vertices to safe vertices, reverting at most a specific number of lanes to minimize the number of vehicles that must spend time on the most dangerous vertices. To tackle a large-scale problem effectively, they apply smart clustering of similar vertices to construct subgraphs.

Hamza-Lup et al. [42] constructed the first contraflow algorithms, known as all-links and fastest-links, to assist an intelligent transportation evacuation system formed to create dynamic evacuation plans focused on the accident site, scope, and current traffic circumstances, with the goal of providing a quick and reliable humanitarian relief.

The all-links algorithm reduces traffic congestion by traversing all possible streets just once, beginning at the source. The faster-links method directs traffic to the shortest pathways between the source and exit locations, which are generated using an ideal multicast tree. However, because these algorithms are unconcerned with the total capacity of the road, they are ineffective if the number of evacuees, lane capacity, specific safe regions, or evacuees are dispersed over many sites. Contraflow has been commonly used to evacuate hurricane-prone areas in the southeastern United States for numerous years. Litman [43] not only noted storm Katrina and Rita's planning flaws but also condemned the unscheduled contraflow instructions and refusal to employ contraflow lanes. Wolshon [44] claims that a considerable increase in flow and time was obtained instantly without the effort or cost of planning, designing, and constructing new lanes.

In order to establish an ideal contraflow scheme, many algorithms and simulation approaches have been utilized to analyze the consequences of various contraflow schemes. For the calculation of traffic volume and journey time under various contraflow systems, software packages such as CORSIM [45], DYNASMART [46], and DynusT [47], were created. To find the optimal contraflow methods heuristic [48], genetic [49], greedy [40], and Tabu search [50], algorithms have been utilized. Major advancements in the utilization of traffic contraflow for mass evacuation have been accomplished through modeling and simulations done with the assistance of such algorithms.

Wang et al. [51] proposed a multi-model evacuation issue in which the lane reversal model and road segment repair are tackled at the same time. The result demonstrates that by creating one new road and replanning the resource, the evacuation time on the damaged transportation network was decreased by more than 50% and by 20%, respectively. Wang et al. [52] examined a relaxed lane reversal model incorporating setup time for contraflow operation, taking into account the priority ordering of evacuees' flow. Furthermore, Lv et al. [53] provided the root choice opportunity for evacuees in a contraflow network model by disregarding background traffic and conducting complete contraflow reconfiguration. It increases the efficiency of evacuation and reduces the evacuation time by 30 to 60%. In execution, the Monticello, Minnesota area was evacuated by employing both the lane-based contraflow and crossing-elimination tactics at the same time. According to Xie and Turnquist [54], the experiment was done with a fixed number of terminals and a complete lane reversal of the transportation network. Xie et al. [55] employed a bi-level model to tackle the Monticello nuclear facility evacuation problem in the same location, including contraflow at road segments and crossing removal at intersection. The lane-based network optimization and simulation models are included in the bi-level. Hua et al. [56] conducted a case study for a super typhoon on an evacuation network utilizing the integrated contraflow technique.

The effects of shifting bottlenecks created by coaches were investigated using the contraflow technique by the authors in [57]. In a contraflow method, the empirical data was used to build a Vissim simulation model to explore the influence of shifting bottlenecks caused by trucks. The contraflow problem was defined by Bagloee et al. [58] as a bilevel, non-linear, and discrete problem that had to be solved to solve a traffic assignment problem. Wollenstein-Betech et al. [59] use a piecewise affine approximation of the travel latency function to reformulate the lane reversal problem, allowing us to use integer linear programming's total uni-modularity. They relax the integer variables to convert an integer linear program to a linear program. Their approach can solve the problem of any number of lane reversals. Darvinshan and Lim [60] propose a rerouting strategy for an evacuation network disrupted by road closures. To make the model adaptable to large evacuation networks, they presented a path-based dynamic flow optimization.

4.2 Analytical

The development of analytical solution approaches for contraflow setups has recently sparked an interest. It does not have a rich history. The contraflow strategies were implemented based on previous evacuation experiences, and the analytical results were insufficient. Arulselvan [29] and Rebennack et al. [30] present analytical models and solution strategies for the contraflow arrangement. They present an algorithm to solve the maximum static contraflow problem by using graph transformation. To obtain the solution they use the maximum flow algorithm and provide the following theorem.

Theorem 1. *[30] A single-source single-sink maximum static contraflow problem can be solved optimally in strongly polynomial-time.*

Authors in [29, 30] introduce the maximum dynamic contraflow (MDCF) problem in discrete-time parameters. They use Ford and Fulkerson's [11] maximum dynamic flow model to model this problem. They also present polynomial-time algorithm to solve the problem in the s-t network that enabled arc reversal at time zero, which means that if we chose to reverse an arc, it will remain reversed for the duration of the time period. The capacities of two-way arcs are combined to provide new capacity, but the travel time remains the same as it was before contraflow and uses the temporally repeated flow technique. The resulting flow is decomposed into pathways and removes cycles. If $\psi_e > b_e$, or if flow $\psi_e \geq 0$ through lane $e \notin E$, lane $e^r \in E$ is reversed. The cost of a contraflow setup is assumed to be zero. The general contraflow evacuation problem via arc reversals, on the other hand, is $\mathcal{NP}$-hard (Kim et al. [40], Rebennack et al. [30]).

Algorithm 1: [30] The MDCF Algorithm with Symmetric Travel Times

Input : A network $\mathcal{Q} = (N, E, b, \tau, s, t, T)$ with symmetric travel time on arcs

Output: A maximum dynamic contraflow on $\mathcal{Q}$

1. The network $\mathcal{Q}$ is transformed into $\mathcal{Q}^a = (V, E^a, b_a, \tau_a, s, t, T)$ as
$$b_a = b_e + b_{e^r},$$
$$\tau_a := \begin{cases} \tau_e & e \in E, \\ \tau_{e^r} & \text{otherwise.} \end{cases}$$
2. To compute MDCF use a temporally repeated flow algorithm on $\mathcal{Q}^a$.
3. Reverse $e^r \in E$ up to the capacity $\psi_e - b_e$ iff $\psi_e > b_e$, b_e replaced by 0 whenever $e \notin E$.
4. For any $e \in E$, if e^r is reverted, $s_c(e^r) = b_a - \psi_e$ and $s_c(e) = 0$. If neither e nor e^r is reverted, $s_c(e) = b_e - \psi_e > 0$, where $s_c(e)$ is the saved capacity of e.

Theorem 2. *[30] Algorithm 1 solves a single-source single-sink maximum dynamic contraflow problem optimally in $\mathcal{O}(nm + nm \log n)$ strongly polynomial-time.*

4.3 $\mathcal{NP}$-completeness of multi-source multi-sink MDCF

Multi-source and multi-sink static contraflow problems are managed by including super-terminals. They are then linked to the sources and sink accordingly. However, this solution approach is no longer relevant in a dynamic scenario. For the same reason, the dynamic contraflow issue with numerous sources and sinks is $\mathcal{NP}$-complete. Authors in [30] present an example demonstrating this and a demonstration of its $\mathcal{NP}$-completeness. Kim et al. [40] provided a sketch of the proof of $\mathcal{NP}$-completeness. However, rigorous proof of the problem has been developed by Rebennack et al. [30] using reduction by the same problem, 3-SAT. From the 3-SAT problem author constructed a graph $G_{3SAT} = (N, E)$ for multi-source multi-sink MDCF with the help of clauses and variables. Garey and Johnson [61] proved that 3-SAT is $\mathcal{NP}$-complete in the strong sense. The equivalence of 3-SAT and $G_{3SAT} = (N, E)$ for multi-source multi-sink MDCF network problems show that multi-source multi-sink MDCF problem is at least as hard as the 3-SAT problem. Hence it is $\mathcal{NP}$-complete in the strong sense.

In a brief, the methods maximum static and dynamic contraflow revert lanes on the fly and are blind to whether or not they revert a lane. In the case of static flows or s-t dynamic flows, this is unproblematic since, in a conventional chain decomposition, an optimal solution can always be found by utilizing only one of the lanes throughout the whole time frame. Nevertheless, when there are several sources and sinks, the ability to use both lanes creates the challenge of determining whether an arc has been reversed or not. Therefore, the task is $\mathcal{NP}$-complete due to this memory and the choice of reverting the lane now or afterward. Examine two sources and a single-single network as depicted in Figure 4. When the concept of procedures maximum dynamic contraflow is used in this situation, the following conclusion is obtained: We would flip lane (x, y) to enhance capacity at time 1, and then switch it back at time 3, producing a flow that required less time than it is reverted for whole time frame [29, 30].

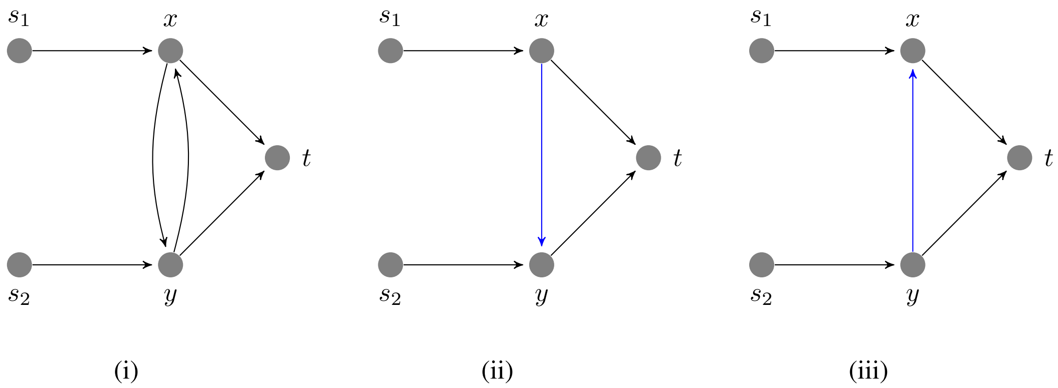


Figure 4: (i) Given network (ii) contraflow network with (x, y) (iii) contraflow network with (y, x).

Assume $\mathcal{Q}$ is a single source-sink network with a supply d at the source. The quickest contraflow (QCF) is a flow over time of value d with the shortest time horizon that reverses the required arcs in E at time zero. The inverse problem of the MDCF problem is the QCF problem. To solve this problem in the same way as the MDCF problem is addressed the quickest flow in the temporally repeated form must be found. Some approaches to locating such a flow are discussed further below.

Assume ξ is a maximum dynamic single source-sink flow with a time frame T. The value of ξ then grows as T increases. Burkard et al. [62] use this fact to create multiple methods for determining the quickest flow. The basic concept is, to begin with, an interval $[T_l, T_u]$ such that $v_{T_l}(\xi) \leq d \leq v_{T_u}(\xi)$, and then seek for the minimum T^* such that $v_{T^*}(\xi) \geq d$. Their methods need several calls to solve a minimum-cost circulation problem. The running time of their highly polynomial approach is $\mathcal{O}(m^2 log^3 n(m + nlogn))$.

Using the concept that the temporally repeated maximum flow over time with a time frame T may be achieved by obtaining the static flow that maximizes $Tv(\psi) - \sum_{e \in E} \tau_e \psi_e$ (Ford & Fulkerson [10], Fleischer and Tardos [18]).

Lin and Jaillet [63] introduced the QFP as the fractional programming problem for the network $\mathcal{Q} = (N, E, b, s, t)$ and a supply d at s as given below

$$\min \ \frac{d + \sum_{e \in E} \tau_e \psi_e}{v} \tag{13}$$

subject to

$$\sum_{e \in \delta^{in}(v)} \psi_{ij} - \sum_{e \in \delta^{out}(v)} \psi_{ji} \quad = \quad \begin{cases} -v & \text{if } i = s \\ 0 & \text{if } i \in V \setminus \{s \cup t\} \\ v & \text{if } i = t \end{cases} \tag{14}$$

$$0 \leq \psi_{ij} \leq u_{ij}, \ \forall \ (i, j) \in E \tag{15}$$

Authors in [63] applied a cost scaling algorithm to compute the solution of the problem in running time $\mathcal{O}(n^3 log(nC))$. Saho and Shigeno [64] improved this bound by using the cancel and tighten algorithm to $\mathcal{O}(nm^2 log^2 n)$. The quickest contraflow problem is solved within the same complexity in [65].

Dhamala and Pyakurel [66] and Pyakurel [67] use time as a discrete parameter to address the earliest arrival and maximum contraflow issues. Pyakurel and Dhamala [68, 69] discuss how to solve such issues in a continuous-time scenario. The same authors in [70] devise pseudo-polynomial time methods to solve the earliest arrival contraflow on single-source single-sink networks. They additionally present the lexicographic maximum dynamic contraflow issue, wherein the flow is maximized in a given priority sequence, and develop polynomial-time solution techniques. The earliest arrival transshipment contraflow (EATCF) issue is described in discrete-time and addressed on a multi-source network using a polynomial-time approach in Pyakurel and Dhamala [69] with the provided supply and demands. The issue can also be addressed on a multi-sink network with a polynomial-time efficiency if each edge has a zero travel time.

They provide approximation strategies to tackle the EATCF problem for the multi-terminal network. In Pyakurel and Dhamala [69] and Pyakurel et al. [71], discrete-time approaches are extended to continuous-time strategies. The maximum generalized dynamic contraflow problems are investigated in Pyakurel et al. [72]. The network flow method, wherein a network is depicted as a group of vertices and edges, is used in the analytical approaches outlined above. A formulation of a similar problem using abstract flow on abstract networks in which a network is assumed to be made up of elements and pathways has recently garnered attention.

Pyakurel et al. [71, 73] implement the lane reversals technique in network with elements and paths instead of nodes and arcs known as an abstract network, and present algorithms for solving the maximum static and maximum dynamic contraflow problems in continuous-time settings. In Dhungana et al. [74], researchers examined models and solutions for abstract contraflow issues with discrete-time settings.

Dhungana and Dhamala [75] introduced maximum dynamic FlowLoc problem with lane reversals and provided polynomial time solution. Same authors in [76] look at the challenge of optimizing flow within a budget while taking into account the cost of arc reversal. Nath et al. [77] presented polynomial time solution to the quickest FlowLoc with lane reversals. Adhikari and Dhamala [78] used the quickest contra-transshipment approach and solved the prioritized integrated evacuation network in minimum clearance time.

4.4 Single-FlowLoc Maximum Dynamic Contraflow

Authors in [75] introduced the single FlowLoc maximum dynamic contraflow problem by incorporating contraflow approach in the Single-FlowLoc maximum dynamic flow problem of [28]. They also present efficient algorithm to solve the Problem 1.

Problem 1. *Given a network $\mathcal{Q} = (N, E, b_e, \tau_e, s, t, T)$, locations L and size r_p of facility p, the Single-FlowLoc MDCF problem on $\mathcal{Q}$ is to find the maximum dynamic flow on updated network $\mathcal{Q}^{loc} = (N, E, b_e, \tau_e, s, t, T)$ with arc reversals allowed initially.*

Algorithm 2: [75] Single-FlowLoc Maximum Dynamic Contraflow

Input : A dynamic network $\mathcal{Q} = (N, E, b_e, \tau_e, s, t, T)$, locations L, size r_p of facility p

Output: MDCF value max_dyna_cont, location $loc(p)$ of facility p in $\mathcal{Q} = (N, E, b_e, \tau_e, s, t, T)$

1. Construct auxiliary $\mathcal{Q}^a = (N, E^a, b_a, \tau_a, s, t, T)$ with new capacity $b_a = b_e + b_{e^r}$ and symmetric transit time
2. Apply single facility maximum dynamic FlowLoc algorithm of [28] on $\mathcal{Q}^a$
3. Decompose the flow resulting from Step 2 into chain and cycle flows then remove the cycle flow
4. A lane $e^r \in E$ is reverted iff the flow along $e \in E$ is greater than b_e or there is a non-negative flow along the path $e \notin E$

Theorem 3. *[75]Algorithm 2 solves Problem 1 optimally in polynomial time, i.e.,* $\mathcal{O}(|L|(m \log n)(m + n \log n))$.

Example 2. *Consider the network given in Figure 2(i). The numbers on each lane show the lanes capacity and travel time. Let* $F = f, r(f) = 1, L = \{(x, y), (x, t), (s, y)\}$, *implying that a facility* f *of size* $r(f) = 1$ *will be installed on one of the lanes in* L. *When the time frame* $T = 3$ *before contraflow, only one path from* s *to* t *exists, i.e.,* $s - y - t$. *It makes no difference whether the facility is located on lane* (x, y) *or lane* (x, t) *because these lanes are not on the route. However, if we place the facility on lane* (s, y), *only three units are transferred from* s *to* t, *and one unit capacity is occupied. As a result, the locations* (x, y) *and* (x, t) *are ideal. The pathways from* s *to* t *are* $s - y - t$, $s - x - t$, *and* $s - x - y - t$ *when* $T = 4$. *When we place the facility on* (x, y), *one unit of the arcs capacity is impeded. As a result, flow units of 4, 2, and 1 are transmitted down the pathways* $s - y - t$, $s - x - t$, *and* $s - x - y - t$, *respectively. The sink receives a total of 11 units of flow. Similarly, if the facility set to arc* (x, t), *11 units of flow arrive at the sink. However, if facility set to arc* (s, y), *only 10 units arrive at the sink. As a result,* (x, y) *and* (x, t) *are the best sites. After the contraflow, we can compute the flow, as in Table 3.*

Table 3: Maximum Dynamic FlowLoc

Location	Time Horizon	Paths	Length	Flow	Total Flow
(x, y)	3	$s - y - t$	3	4	
	4	$s - y - t$	3	4	
		$s - x - t$	4	2	
		$s - x - y - t$	4	3	13
(x, t)	3	$s - y - t$	3	4	
	4	$s - y - t$	3	4	
		$s - x - t$	4	1	
		$s - x - y - t$	4	4	13
(s, y)	3	$s - y - t$	3	3	
	4	$s - y - t$	3	3	
		$s - x - t$	4	2	
		$s - x - y - t$	4	4	12

When a facility is placed on an arc, the flow is obstracted, which means that less flow may be shipped from the source to the sink than in the usual scenario, i.e., the network without the facility. Because to obstraction, the time it takes to transport a specific amount of flow has increased. As a result, rather than maximizing flow in a single FlowLoc, the authors in [77] looked at time minimization for a given supply and demand, i.e., the quickest contra FlowLoc problem. They proposed an algorithm to solve the problem in strongly polynomial-time, i.e., $\mathcal{O}(|L|nm^2 \log^2 n)$, where $|L| \leq m$.

4.5 Maximum dynamic contraflow with intermediate storage

Authors in [36] investigated dynamic contraflow problems with intermediate storage. The network's contraflow arrangement, in particular, has been evaluated from an emergency standpoint. This reconfiguration flow model with intermediate storage can be employed if the intermediate nodes were created to fulfill the needs of emergency scenarios due to various large-scale disasters. The MDCF problem with intermediate storage optimizes flow departing from the source and sends flow as far as feasible towards the sink in the specified time frame T by reverting the orientation of lanes from the begining. They presented an algorithm to solve the problem in a polynomial-time.

Algorithm 3: [36] The MDCF Algorithm with Intermediate Storage

Input : A network $\mathcal{Q} = (N, E, b, u, \tau, s, t, T)$ with symmetric travel time on arcs

Output: A MDCF with intermediate storage on $\mathcal{Q}$

1. The network $\mathcal{Q}$ is transformed into $\mathcal{Q}^a = (V, E^a, b_a, u, \tau_a, s, S_-, T)$ as

$$b_a = b_e + b_{e^r},$$

$$\tau_a := \begin{cases} \tau_e & e \in E, \\ \tau_{e^r} & \text{otherwise.} \end{cases}$$

2. Construct the modified auxiliary network.
   - Compute the minimum distance $d(s, v), \forall v \in I$ with $\sum_{e \in \delta^-(v)} b_e \leq u_v$.
   - Assign first priority to the sink, second priority to the farthest intermediate vertex v and so on.
   - Transform the single-source multi-sink network by creating dummy locations.
3. Compute the prioritized maximum dynamic flow, without intermediate storage on modified auxiliary network.
4. Decompose the flow into path and cycles and remove all cycle flows.
5. Revert $e^r \in E$ to the capacity $\psi_e - b_e$ iff $\psi_e > b_e$, b_e replaced by 0 whenever $e \notin E$.
6. For any $e \in E$, if e^r is reverted, $s_c(e^r) = b_a - \psi_e$ and $s_c(e) = 0$. If neither e nor e^r is reverted, $s_c(e) = b_e - \psi_e > 0$, where $s_c(e)$ is the saved capacity of e.

Theorem 4. *[36] Algorithm 3 computes two terminal MDCF problem with intermediate storage optimally by using in polynomial time.*

In all the problems discussed above, travel time is symmetric on parallel but oppositely oriented arcs. However, in real-life scenarios it may not be symmetrical. Bhandari and Khadka [79] consider the two-terminal maximum dynamic contraflow issue on anti-parallel arcs with non-symmetric travel times so that the reversals utilize the same arc travel time as previously. Using the technique of [30], this interprets the situation of parallel arcs on a network. Nath et al. [31] addressed the contraflow problem on lanes with non-symmetric capacity and travel time and offered a new approach to handle the problem in which a reverted edge takes the same transmission time as its unreverted counterpart. As a result, it modifies the algorithm of [30] by using the fact of asymmetric arc trip durations. All the steps of the algorithm are same only transit time on the auxiliary arc is defined by the Equation 3. Hence the complexity is also same. Authors in [32] proposed the multi-source single-sink EATCF and lexicographic maximum dynamic partial contraflow problems and provided polynomial-time solution utilizing the method of Nath et al. [31]. They also extended this approach to generalized dynamic partial contraflow in [80]. However, for easy reference, we summarize the currently known complexities for single-commodity flow problems with symmetric and asymmetric transit times on anti-parallel arcs in Table 4.

Table 4: Complexity of Contraflow Problems

Single Commodity with $\tau_e = \tau_{e^r}$	Date	Complexity	Approximation	References
Maximum Dynamic Flow				
& Quickest Flow	2010	Strongly polynomial	-	[30]
Earliest Arrival Problem on SP-graph	2013	Polynomial	-	[66]
Generalized Maximum Dynamic Flow				
on Lossy Network	2014	Pseudo polynomial	-	[72]
Continuous Time Dynamic	2016, 17	Polynomial time	-	[68, 69]
Abstract Flow	2017,18	Polynomial time	-	[71, 73]
Maximum FlowLoc	2019	Polynomial time	-	[75]
Inflow Dependent Transit Times	2019	Polynomial time	-	[65]
Quickest FlowLoc	2020	Polynomial time	-	[77]
MDF with Arc Switching Cost	2020	Polynomial time	-	[76]
Prioritized Integrated Evacuation Network	2020	Polynomial time	-	[78]
Dynamic Flow with Intermediate Storage	2020	Polynomial time	-	[89]
Single-commodity with $\tau_e \neq \tau_{e^r}$				
Maximum Dynamic Flow				
& Quickest Flow	2020	Strongly polynomial	-	[79]
	2021	Strongly polynomial	-	[31]
Maximum Lexicographic Flow				
Earliest Arrival Transshipment	2021	Polynomial, pseudo polynomial	-	[32]
Generalized Flow	2021	Pseudo polynomial	-	[33]
Dynamic Flow with Intermediate Storage	2021	Polynomial time	-	[35]

5 Multi-commodity Contraflow

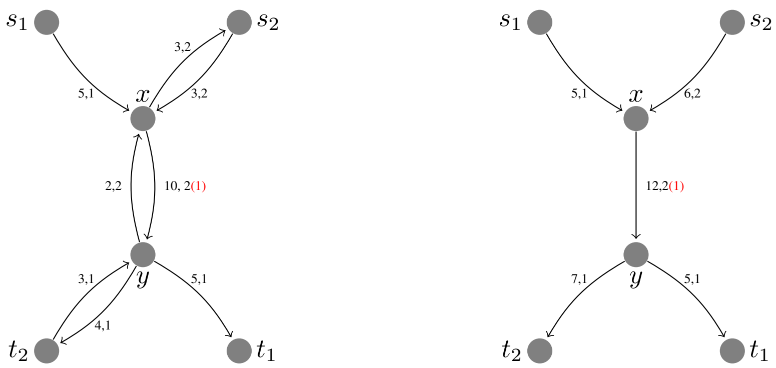
The multi-commodity network flow problem entails sending multiple commodities from distinct sources to different sinks with the best flow assignment possible while staying within the arcs' capacity restrictions. It expands the single-commodity network flow issue in the sense that, if the bundle restrictions and arc capacity constraints that connect flows of various commodities traveling through the same arc are ignored, an MCNF problem may be seen as multiple independent single commodity flow problems. MCNF problems are classified into static and dynamic MCNF problems. Many researchers have extended the models and algorithms by adding different aspects of the problem such as maximum flow, maximum concurrent flow, quickest flow, and minimum cost flow. For more details we refers to [81, 82, 83, 84, 85, 86, 87] and references therein.

This problem was first introduced by Ford and Fulkerson [10]. Because they carry more than one commodity, multi-commodity flow issues in bundle arcs differ significantly from single-commodity flow problems. Unlike multi-commodity models, single-commodity models cancel flows, preventing cycles in opposing directions. The goal of the maximum MCNF issue is to maximize the total of all commodity flows between their origins and destinations. The inverse of this problem wherein, instead of maximizing the flow, the delivery time to satisfy the demands of commodities is minimized is known as the QMCF problem. The static version of the MCNF problem is solved polynomially, whereas the dynamic version is $\mathcal{NP}$-hard [88]. The authors in [34, 89, 90, 91] introduced MDMCF and QMCF with lane reversals and presented approximation algorithms.

5.1 Solution approach to MDMCF with lane reversals

Ford and Fulkerson [11] proposed the notion of time expansion to solve the problem of maximum flow over time. In the scenario of a dynamic MCNF problem this well-known technique can be used. The equivalence of static MCNF on a time-expanded graph and MCNF over time on the original network has been demonstrated by Kappmeier [92]. It can be addressed in pseudo-polynomial running time because the dynamic MCNF issue on network $\mathcal{Q}$ is reduced to the static MCNF problem on a time-expanded network $\mathcal{Q}_T$.

Example 3. *Suppose a two-commodity network having capacity and transmission time on lanes, (cf. Figure 5(i)) and T=6. The maximum flow from the sources s_i to the corresponding sinks t_i is 21 units. By using partial contraflow approach the maximum flow with symmetric transit time is 27 units, whereas with asymmetric transit time is 38 units and preserves the unoccupied arc capacity (cf. Figure 5(ii), Figure 6, and Table 5).*



(i) (ii)

Figure 5: (i) Given two-commodity network (ii) network after contraflow.

Table 5: Maximum Flow before and after LR with ST and AST

Paths	Time	F. before LR	T. F.	F. after LR(ST)	T.F.	F. after LR(AST)	T. F.
$s_1 - x - y - t_1$	4 (3)	5	15	5	15	5	20
$s_2 - x - y - t_2$	5 (4)	3	6	6	12	6	18
Total			21		27		38

T = 6, T.F.=Total Flow, LR = Lane Reversals, ST = Symmetric Transit Times, AST = Asymmetric Transit Times

To compute the solution of the maximum dynamic MCNF problem with partial contraflow, authors in [76] presented Algorithm 4.

Theorem 5. *[89]Algorithm 4 provides a pseudo-polynomial-time solution to the MDMCF problem with partial lane reversals.*

To solve the difficulties of time expansion, a scaling method might be used, in which every vertex and arc is replaced by T/Δ copies instead of T, greatly lowering the problem size and establishing a compromise between

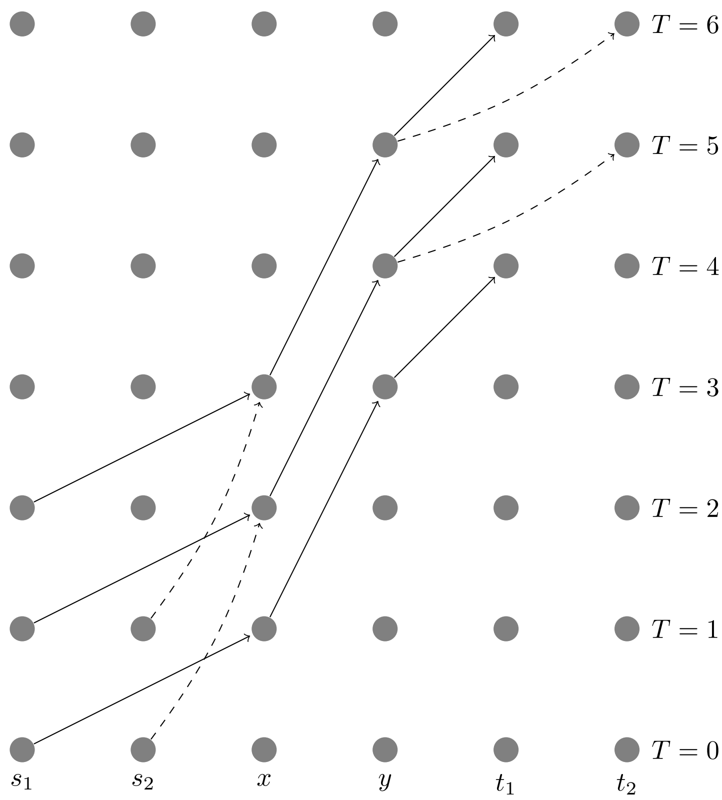


Figure 6: Time-expanded network of Figure 5 (ii)

the precision of the solution and the algorithm's running time [61]. By using the scaling approach an FPTAS is presented to solve the MDMCF problem in [89].

Authors in [34] modified Algorithm 4 to the case of non-symmetric transit times on anti-parallel arcs by using relation 3, and the same approach is applied to compute the solution of EAMCT in pseudo-polynomial time complexity.

5.2 Approximation approach to QMCCF

The quickest multi-commodity flow problem contains the transshipment of various commodities from their respective supply points to corresponding destination points through a network system so the total demand of every commodity is met within the shortest computation time for given supplies and demands. Even in the case of a series-parallel network or only two commodities, Hall et al. [88] have proved that the dynamic MCNF issue is $\mathcal{NP}$-hard. They also established that the QMCF issue is $\mathcal{NP}$-hard with or without intermediate storage and simple pathways. To address the QMCF issue with polynomial-time complexity, Fleischer and Skutella [93] proposed a length-bounded approximation and a condensed time expanded graph. If the flow on each route $P \in \mathbf{P}_i$ can be decomposed into the sum of flows ψ^i_P, i.e., $\psi^i = \sum_{P \in \mathbf{P}_i} \psi^i_P$ with $\psi^i_P > 0$, the multi-commodity path flow ψ^i meeting demands and supplies d_i at terminals $S_+ \cup S_+$ is a T-length bounded flow if $\tau_P = \sum_{e \in P} \tau_e \leq T$. $\mathbf{P}^T_i = \{P \in \mathbf{P}_i : \tau_P \leq T\} \subseteq \mathbf{P}_i$ denotes the collection of all T-length bounded pathways. Because the T-length bounded static flow problem meeting multi-commodity needs is $\mathcal{NP}$-hard, [93] presents an approximation solution with polynomial-time complexity.

Authors in [76, 91] incorporated the lane reversals approach in the QMCF and introduced the QMCCF problem in discrete and continuous-time settings. Based on the approach of [93], the authors of [76] presented a length-bound approximation and an FPTAS by using the condensed network. Furthermore, with the help of the natural

Algorithm 4: [89]Algorithm for MDMCF with Partial Contraflow

Input : A network $\mathcal{Q} = (N, E, u, \tau, K, S_+, S_-, T)$

Output: The MDMCF with contraflow

1. A network $\mathcal{Q}$ is transformed into $\mathcal{Q}_T = (N_T, E_T = E_M \cup E_H \cup E_+ \cup E_-, K, b, \tau, S'_+, S'_-, T)$
2. Construct a modified network $\mathcal{Q}_T^a = (N_T, E_T^a, b^a, \tau^a K, S'_+, S'_-, T)$ by combining two-way arc capacities.
3. Find a MSMCF solution on $\mathcal{Q}_T^a$.
4. Decompose the flow along the $s_i - t_i, \forall i \in K$ paths and cycles and remove cycle flows .
5. Revert $e^r(\theta) \in E_T$ up to the capacity $\psi_a(\theta) - u_e(\theta)$ iff $\psi_a(\theta) > u_e(\theta)$, u_e replaced by 0 whenever $e(\theta) \notin A_T$ where $\psi_e = \sum_{i=1}^k \psi_e^i$ and $u_e = \sum_{i=1}^k u_e^i$.
6. For every $e(\theta) \in A_T$, if $e^r(\theta)$ is reverted, $s_c(e^r(\theta)) = u_e^a(\theta) - \psi_e(\theta)$ and $s_c(e(\theta)) = 0$. If neither e nor e^r is reverted, $s_c(e(\theta)) = u_e(\theta) - \psi_e(\theta) > 0$, where $s_c(e(\theta))$ is the saved capacity of e.

transformation of [18], it can be extended to a continuous-time parameter in [91].

Algorithm 5: [90] FPTAS for QMCCF Problem

Input : A network $\mathcal{Q} = (N, E, K, b, \tau, d_i, S_+, S_-, T)$

Output: The continuous-time QMCCF

1. Transform network $\mathcal{Q}$ into $\mathcal{Q}^{\Delta a} = (N, E^a, K, b^a, \tau_a, d_i, S_+, S_-, T)$ as
$$b_e^a = \Delta(b_e + b_{e^r})$$
$$\tau_a := \begin{cases} \lceil \tau_e/\Delta \rceil \Delta & \text{if } e \in E \\ \lceil \tau_{\overleftarrow{e}}/\Delta \rceil \Delta & \text{otherwise.} \end{cases}$$
2. Compute the QMCF on $\mathcal{Q}^{\Delta a}$ by using FPTAS Core of [93].
3. Decompose the flow along the $s_i - t_i, \forall i$ paths and cycles and remove cycle flows.
4. Revert $e^r \in E$ up to the capacity $\psi_e - b_e$ iff $\psi_e > b_e$, b_e replaced by 0 whenever $e \notin E, \forall i$, where $\psi_e = \sum_{i=1}^k \psi_e^i$.
5. For each $e \in E$, if e^r is reverted, $s_c(e^r) = b_a - \psi_e$ and $s_c(e) = 0$. If neither e nor e^r is reverted, $s_c(e) = b_e - \psi_e > 0$, where $s_c(e)$ is the saved capacity of e.

Theorem 6. *[90] An FPTAS for the QMCF problem with bounded cost can be computed on an auxiliary network in a fully polynomial-time.*

Example 4. *Assume the networks as depicted in Figure 4(i) with demands $d_1 = 10, d_2 = 12$. To compute the quickest time without lane reversals by using length bound approximation (cf. Figure 5(i)), 4-length bound is essential and repeated two times. So, the quickest time to satisfy the demand for the Commodity-1 is $T = 5$. Similarly, 5-length bounded is essential and repeated four times. So, the quickest time to fulfill the demand for Commodity-2 is $T = 8$. Hence the minimum time to satisfy both the demands is $T = 8$. On the other hand, if contraflow is applied (cf. Figure 5(ii)), then it takes $T = 6$ units of time to fulfill both the demands.*

Next, construct a condensed time expended network by taking $\Delta = 2$ and scaling the capacity and transit time on arcs of the network depicted in Figure 5(ii). The quickest time to satisfy both the demands after contraflow is $T = 6$. Since the size of the time-expanded graph is reduced by the factor of Δ in the Δ-condensed time expanded graph it provides a fully polynomial-time solution (cf. Figure 7).

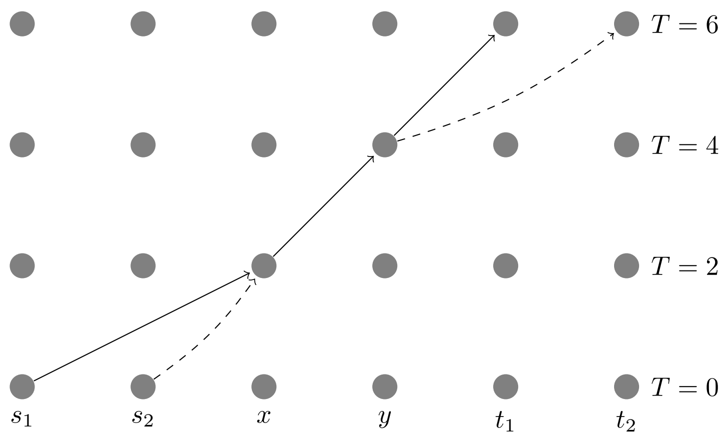


Figure 7: Condensed time-expanded graph of Figure 5 (ii)

Table 6 summarizes the currently known complexities for multi-commodity flow problems with symmetric and asymmetric transit times on anti-parallel arcs for quick reference.

Table 6: Complexity of Contraflow Problems

Multi-Commodity with $\tau_e = \tau_{e^r}$	Date	Complexity	Approximation	References
Quickest Transshipment	2020	$\mathcal{NP}$-hard	PTAS, FPTAS	[90]
Maximum Flow Over Time	2020	Pseudo polynomial	FPTAS	[89]
Continuous Time Quickest Transshipment	2020	$\mathcal{NP}$-hard	PTAS, FPTAS	[91]
Multi-Commodity with $\tau_e \neq \tau_{e^r}$				
Maximum Multicommodity Flow with Intermediate Storage	2021	pseudo polynomial	-	[37]
Maximum Dynamic Flow	2022	Pseudo polynomial	-	[34]

6 Conclusions

Due to the increasingly wide applications and important impacts of the contraflows, this paper surveys the mathematical models, solution approaches, and applications of the single and multi-commodity contraflow problems. We illustrated the single-commodity and multi-commodity flow models with partial lane reversal strategies that are very relevant in saving the capacity of unoccupied lanes that can be used for the placement of facilities or logistics support in emergencies maximizing the flow value and reducing the transmission time. This work includes heuristics, simulations, and analytical approaches for solving contraflow problems. The main focus is on analytical approaches of maximum static, maximum dynamic, lexicographic maximum static, lexicographic maximum dynamic, quickest, earliest arrival, generalized static, and generalized dynamic contraflow problems with symmetric and non-symmetric tranmission times on anti-parallel arcs appeared in the literature in the last two decades for single-commodity.

The maximum MCNF problem aims to optimize the aggregate of all commodities in a particular period by shipping various commodities (goods) on an underlying network architecture while adhering to capacity limits on the lanes. The study also includes approximation approaches to maximum static, maximum dynamic, and quickest multi-commodity contraflow problems.

The applicability of FlowLoc problems in evacuation planning is widespread. Choosing the appropriate arcs for facility location with a given set of objectives from a provided subset of edges in a directed network is critical in emergency flow optimization. Because a facility reduces an arc's capacity, the maximum flow may be reduced, or the quickest time may rise. The quickest ContraFlowLoc problem is crucial in evacuation planning to organize traffic flow with facility location while reducing evacuation time. Another issue where contraflow approach is incorporated is the maximum flow problems with intermediate storage. It is important in the case of emergency management.

The applications of these problems vary from emergency evacuation, rush hour traffic management, routing in logistics and transportation networks, and message routing in telecommunication. The insights into the offered models and solution techniques cover multiple challenges for the operations research community in dealing with even more complicated models and alternative approaches to solving that might tackle complex real-life situations. The study objectives are theoretical as well as practical interests. We believe that this research will lead to a variety of new directions for model development and investigation of novel solution methodologies.

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