Optimal configurations for modular systems at the example of crane bridges

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Abstract

The aim of this paper is to optimize modular systems which cover the construction of products that can be assembled on a modular basis. Increasing the number of different variants of individual components on the one hand decreases the cost of oversizing the assembled product, while on the other hand the cost for maintaining the modular system increases. For the minimization of the overall cost a mixed-integer problem is derived. However, this problem cannot simply be passed to a solver for mixed-integer optimization, since certain dependency structures of the variables occur. We propose a solution approach for this complicating structure using binary variables to transform the problem into a mixed-integer optimization problem, which can be solved deterministically. In a numerical study, this formulation is investigated using the example of a modular system for crane bridges.

Keywords: Modular system, nonconvex mixed-integer model, crane bridge.

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1 Introduction

In this paper, modular system problems are considered from an optimization point of view. Modular systems prove to be very useful in practice for products that can be assembled modularly from different components. Each of these components can consist of different variants or size which can be combined with each other. Modular systems are a common concept in many fields of design and manufacturing, where the goal is to decompose a complex system into simpler modules in order to decrease complexity and increase cost-efficiency [16].

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An optimization model is derived that determines an optimal configuration of the modular system. That is, on the one hand, the optimal configuration shows how many types of a component should be manufactured and, on the other hand, exactly what these components look like, for example the geometric dimensions of the variants.

The goal is that the modular system is on the one hand cheap to maintain, so that we have, for example, few different variants per component. On the other hand, meeting product properties with such a 'coarse' modular system leads to high oversizing costs, which can be reduced by sufficient variability and flexibility in the modular system.

In Section 2 a short introduction to crane bridges and a literature review on optimization models for modular systems are given, before we introduce a general model for treating the trade-off between maintenance and oversizing costs of modular systems. However, the resulting optimization problem cannot simply be passed to a solver for mixed-integer optimization problems, since certain dependency structures of the variables occur by which in the beginning it is not even clear how many decision variables the problem has. The problem could be reformulated into a two-level problem that takes the dependency structure into account. For such problems, especially also in the mixed-integer case, decomposition methods or methods from bilevel optimization are well-known, see [12] for decomposition methods and [13, 3, 11] for bilevel optimization. In Section 3 of this work, however, a more straightforward solution approach is proposed, which uses binary variables to transform the problem into a mixed-integer single-level problem, for which standard solvers can be used. In Section 4 this formulation is substantiated using the example of a modular system for crane bridges, and a numerical study indicates that the problem formulation as a single-level problem possesses potential also for the optimization of other modular systems. Section 5 closes the article with some final remarks.

2 An optimization model for modular systems

In this paper we consider an optimization approach for modular systems. The aim of a modular system is to have a minimal set of different parts for various tasks or a set of different sizes of the same part which can be combined into a product that fulfills a given requirement of properties [15]. One of the main challenges in the design of modular system is to balance the number of variants and the product performance [16].

For a better understanding, we look at the example of crane bridges at this point, which will be considered in detail later on. After presenting the example, we will discuss the existing literature.

2.1 Crane bridges

Overhead cranes are mostly used to transport objects in production halls and warehouses. They consist of the crane bridge, the crab with hoist and trolley, and two end carriages at the ends of the crane bridge. The end carriages travel on the crane runway. End carriages, hoists and trolleys are already offered by the respective manufacturers as modular systems. Crane bridges are usually built in one part in the shape of a box or I-profile girder. These

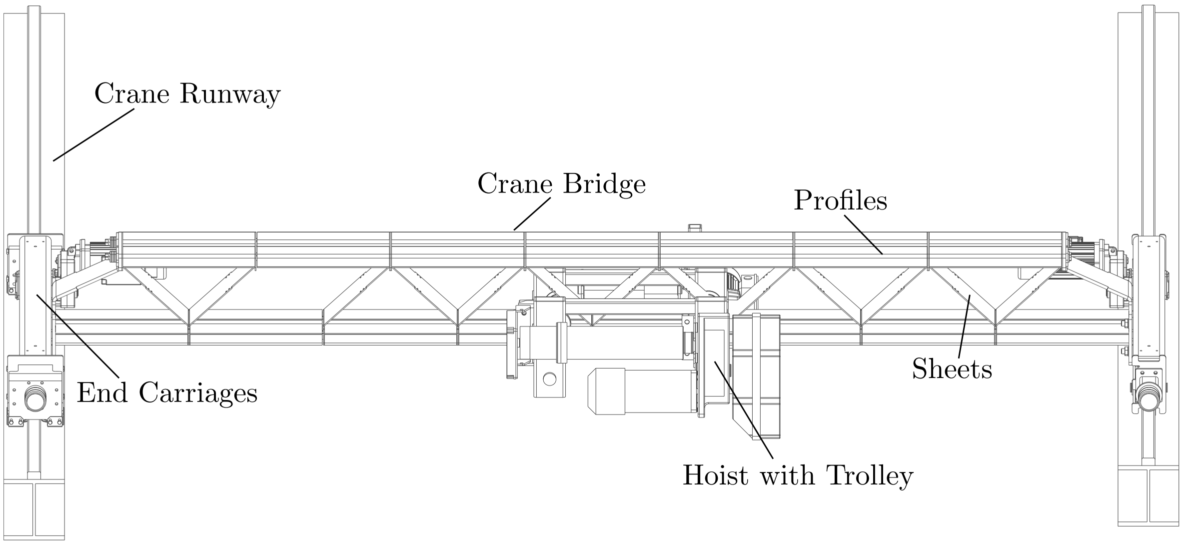

Figure 1: Crane concept

are manufactured in a series of various different sizes. In order to make the advantages of a modular system usable for crane bridges as well, a concept was developed in [2, 14] on how crane bridges can also be constructed modulary. This segmented crane bridge is designed as a truss profile made of hollow profiles and diagonal connecting plates. The components can be mass-produced, easily transported and assembled at the crane's place of use. The structure of the crane bridge is shown in Figure 1 and is described in detail in [2, 14].

The crane bridge modular system we consider consists of the two components profile and sheet, each of which is designed as a series in different sizes, as well as matching end sheets, connecting elements and an individually manufactured compensating element. By combining a sheet size and a profile size as well as varying the number of these elements, various requirements and properties of the crane bridge can be covered while using only a small number of different part sizes. In our model in Section 4 we will consider profiles that differ in the three geometry parameters height, thickness and width, and sheets with four geometry parameters.

Each crane bridge is built for a given span and load capacity based on the site and the use case for the crane. It has to fulfill the requirements for the safety and stability of overhead cranes. We have oriented ourselves on the German series of standards for crane design EN 13001 [8, 9, 10] and the international standard for the stiffness of overhead cranes ISO 22986 [4]. These mainly include the maximum stress in the material and the deflection under load. For the model, it is assumed that every combination of profiles and sheets can be used for a number of span and load capacity combinations while meeting the given safety requirements.

The properties that should be fulfilled by the modular system, as mentioned above, are therefore the load capacity and the span of a crane bridge. We are interested in a modular system that is cost-effective in some sense. On the one hand we wish to avoid a large number of different variants of a component and, on the other hand, we want to remain flexible in fulfilling the required properties of span and load capacity. In detail, this means that on the one hand we want as few profile and sheet variants as possible, but still want to cover span and load capacity as well as possible and avoid building oversized crane bridges.

In Section 4 we will consider the example of crane bridges in more detail. Before that, we will briefly discuss the existing literature and deduce a flexible model for the optimization of general modular systems.

2.2 Existing literature

The literature on modular systems in combination with mathematical optimization is very sparse.

In [6] a formal description of modular systems is presented, possible system boundaries, a cost model for the optimization of modular systems and a possible procedure for the mathematical modeling of modular systems is discussed.

The paper [5] divides modular system optimization into three classes: Class I describes the optimization of individual components, e.g. the design of one component, under a fixed combination of the components for each product. Class II describes the optimization of components combinations with explicitly given variants of the components. Class III describes the simultaneous optimization of the combination of the components and of the design of the component.

In [7] the optimization of a modular system is modeled as an integer optimization problem (with only binary variables) and solved using simulated annealing. They use components with existing parameters and solve an allocation problem with a simple cost model. They classify it as an Class II problem, since the component parameters are fixed. There is no optimization of the different parameters (Class III). In [5] it is only mentioned that the optimization problem resulting from the class III problem is a nonlinear mixed-integer optimization problem, and that heuristics can play an important role here.

The paper [17] describes the optimization of modular products in reconfigurable production lines using the example of a powertrain. The optimization problem is described as a subset selection problem. However, it is based on modules with predefined parameters and there is no adjustment or optimization of the module parameters here. According to [5], this is again a Class II problem.

The problem we consider in this paper fits best into the framework of class III problems. In our model, we want to keep the combination of the individual components variable, as well as the exact design of the components. In addition, we do not want to fix the number of variants per component of the modular system. Whether the last-mentioned aspect is considered in the class III problems remains unclear, as no exact problem is specified.

2.3 An optimization model for general modular systems

In some underlying market we assume a demand of N products which can be characterized by the same s properties. Therefore we have products $p^\ell \in \mathbb{R}^s$, $\ell = 1, \ldots, N$. A product can be built of R different components. In the example of the crane bridges we have a demand of N crane bridges that are built out of $R = 2$ components, profiles and sheets.

Each component has specific parameters and through them the components in a modular systems will differ. In our example, these are different geometry parameters such as length and widths of the profiles and sheets. For each component r, $r = 1, \dots, R$, there are $k_r \in \mathbb{N}$ different variants in the modular system. In our model the number of components R will be a fixed known number and the vector of number of variants $\kappa = (k_1, \dots, k_R)$ will be variable, but bounded. Hence we have $\kappa \in \mathbb{N}^R \cap [l, u]$ with a vector $l \in \mathbb{N}^R$ for the lower bounds and $u \in \mathbb{N}^R$ for the upper bounds of k_r, $r = 1, \dots, R$.

Besides the variables for the number of variants κ we have a second type of variables, the variables for the different geometry parameters. With $x^{r,k} \subseteq \mathbb{R}^{n_r}$ we denote the vector for variant k, $k = 1, \dots, k_r$ of component r, $r = 1, \dots, R$, where n_r is the number of geometry parameters of component r. For the crane bridges we have for instance for component 1, the profiles, $n_1 = 3$ geometry parameters: length, thickness and width. Each of these vectors $x^{r,k}$ can additionally be box-constrained for each of the components, which we collect for simplicity in the set X^r. For instance the length of each profile variant should be nonnegative and bounded from above. Therefore we have $x^{r,k} \in X^r \subseteq \mathbb{R}^{n_r}$. We summarize all $x^{r,k}$ in the vector $\xi \in X(\kappa) \subseteq \mathbb{R}^{\sum_{r=1}^{R} k_r n_r}$, which is the second variable for our model. The description of the set $X(\kappa)$ collects the restrictions for each $x^{r,k}$ and depends on the number of variants κ. Through this formulation we obtain a dependence of the length of the vector ξ on the variable κ, which complicates our model.

Besides the specification of the components by the entries of ξ we are interested in the number of pieces in the modular system. We assume that, to fulfil the product properties, we are allowed to choose pieces of only one variant for each component. For our example, this means that we use one profile variant and one sheet variant for each crane bridge in the required number, which will be specified later. With $z^{\ell}_{r,k} \in \mathbb{N}_0$ we denote the number of pieces we choose from variant k of component r for product ℓ, $k = 1, \dots, k_r$, $r = 1, \dots, R$, $\ell = 1, \dots, N$. Since we are only allowed to choose one variant we have for each component r, $r = 1, \dots, R$,

$$|\{k \in \{1, \dots, r\} | z^{\ell}_{r,k} > 0\}| = 1.$$

With z^{ℓ} we denote the vector of all variables $z^{\ell}_{r,k}$ for one product p^{ℓ}. To satisfy the required product properties, additional constraints on the variables z^{ℓ} have to be expected which also depend on the variable ξ. So we have $z^{\ell} \in Z^{\ell}(\kappa, \xi)$ with the set $Z^{\ell}(\kappa, \xi)$ of all feasible configurations for product p^{ℓ}. By z we denote the vector of all z^{ℓ}, $\ell = 1, \dots, N$.

With given κ and ξ, for each product p^{ℓ} we choose among all feasible configurations $z^{\ell} \in Z^{\ell}(\kappa, \xi)$ the cheapest one. Therefore we are looking for the minimal point of a cost function $c^{\ell}(\kappa, \xi, z^{\ell})$ over $Z^{\ell}(\kappa, \xi)$. In addition, there exist maintenance costs $C(\kappa, \xi)$ which depend on the size of the modular system. In many applications these costs develop in opposite directions under changes in the configuration of the modular system.

We obtain the total costs $C(\kappa, \xi) + \sum_{\ell=1}^{N} c^{\ell}(\kappa, \xi, z^{\ell})$ and, therefore, altogether the optimization problem

$$P: \quad \min_{\kappa, \xi, z} \; C(\kappa, \xi) + \sum_{\ell=1}^{N} c^{\ell}(\kappa, \xi, z^{\ell}) \quad \text{s.t.} \quad \xi \in X(\kappa), \tag{1}$$

$$z \in Z(\kappa, \xi), \tag{2}$$

$$\kappa \in \mathbb{N}^R \cap [l, u] \tag{3}$$

with $Z(\kappa, \xi) = Z^1(\kappa, \xi) \times \ldots Z^N(\kappa, \xi)$. Since ξ can be a continuous variable while κ and z are integer variables, P is in general formulated with mixed-integer variables. As mentioned before, the constraints yield a dependency structure between the variables that complicates the solution of the optimization problem. Indeed, the length of the vector ξ is determined by the entries of the vector κ. In the following we will suggest a reformulation of P which allows to deal with this fact.

3 Solution approach

One idea to deal with the dependency of ξ on κ is to fix a number of maximum variants for each component and link each possible number of variants with a binary variable. In applications the possible maximum number of variants of a component is often known since the warehouse capacity is bounded. Since the optimization problem P is already mixed-integer, additional binary variables and corresponding constraints do not change the problem structure, but the number of variables and restrictions just increase. Although the class of mixed-integer problems is NP-hard, in many real-world problems standard solvers can treat instances with several thousands of integer variables in reasonable time.

The solver Gurobi [1] can also be used if the objective function and the constraints are quadratic (convex or nonconvex). For this reason we will reformulate the problem P into a linear or quadratic mixed-integer problem. In particular, in Section 4 we will reformulate the modular system for crane bridges as a nonconvex multiquadratic problem.

Indeed, in the following we assume that for every component r we know a maximum number of variants, denoted by $\overline{k}_r \in \mathbb{N}$. While with the previous notation this yields $k_r \leqslant \overline{k}_r$, from now on we will no longer consider the vector κ of variables k_r but the known constants $\overline{k}_r$ with associated binary variables $v_{r,k} \in \mathbb{B}$, $k = 1, \ldots, \overline{k}_r$, $r = 1, \ldots, R$, which indicate whether a variant is available in the modular system or not. More precisely, we have

$$v_{r,k} = \begin{cases} 1, & \text{if for component } r \text{ variant } k \text{ is available,} \\ 0, & \text{else.} \end{cases}$$

We need to link the new variables $v_{r,k}$ with the variables $z^\ell_{r,k}$ since, of course, there can only be a positive number of a variant k of component r in the modular system if the variant actually exists. In other words the value of $z^\ell_{r,k}$ has to be zero if variant k of component r is not available, which we denote as

$$v_{r,k} = 0 \;\Rightarrow\; z^\ell_{r,k} = 0, \qquad \ell = 1, \ldots, N. \tag{4}$$

This type of restriction is called indicator constraint and can be modeled directly in Gurobi. It should be noted at this point that the implication arrow is based on Gurobi notation. We add condition (4) to the set Z and thus obtain the additional dependence of Z on v.

On the other hand, under our assumption that the maximum number of variants is given, the length of the vector ξ is known to be $\overline{k} = \sum_{r=1}^{R} \overline{k}_r$. The sets $X(\kappa)$, $Z(\kappa, v, \xi)$ thus simplify to

X and $Z(v,\xi)$. Furthermore, we can omit the dependency on κ everywhere else in P. Every function (in objective and constraints) which depends on a variant k of component r must be supplemented by the binary variable $v_{r,k}$ since the variables $x^{\ell}_{r,k}$ and $z^{\ell}_{r,k}$ may of course only enter there if they are available. In summary we obtain the reformulated problem

$$\overline{P}: \quad \min_{v,\xi,z} \ C(v,\xi) + \sum_{\ell=1}^{N} c^{\ell}(v,\xi,z^{\ell}) \quad \text{s.t.} \quad \begin{aligned} &\xi \in X, \\ &z \in Z(v,\xi), \\ &v \in \mathbb{B}^{\overline{k}}. \end{aligned}$$

Like in problem P, since ξ can be a continuous variable while v and z are integer variables, $\overline{P}$ is in general a mixed-integer optimization problem. In contrast to P, however, the length of ξ is fixed in $\overline{P}$.

4 A specification of the general optimization model to the modular system for crane bridges

In Section 2 a modular system for crane bridges was briefly introduced. Then a general optimization model for the cost-minimal configuration of a modular system has been derived. However, explicit forms of cost functions and constraints have not yet been discussed. In the present section we use a concrete example of a modular system to derive an explicit optimization problem. We choose the modeling approach described in Section 3 to formulate a problem without dependency structures. After some reformulations, we obtain a mixed-integer optimization problem with quadratic objective function and quadratic constraints. These functions do not have to be convex, when the resulting problem is solved by Gurobi, since this solver does not require convexity for quadratic problems. We test our modeling approach for some example problems.

4.1 Derivation of the optimization model

A rough sketch of a crane bridge is shown in Figure 1. As decribed above, we have a demand of N crane bridges, the products p^{ℓ}, $\ell = 1,\dots,N$, and we take into account the two properties span width L^{ℓ} and load capacity M^{ℓ}.

We assume that the modular system contains at most $\overline{k}_1 = n$ profile variants and $\overline{k}_2 = m$ sheet variants. Profiles are characterized by the three geometry parameters height, thickness and width, i.e. $x^{1,i} = P^i = \left(h_P^i, t_P^i, w_P^i\right) \in X^1 \subseteq \mathbb{R}^3$, $i = 1\dots,n$. Sheets, on the other hand, are described by the four geometry parameters height, segment length, thickness and width i.e. $x^{2,j} = S^j = \left(h_S^j, l_S^j, t_S^j, w_S^j\right) \in X^2 \subseteq \mathbb{R}^4$, $j = 1,\dots,m$. The description of the sets X^1 and X^2 also contains box constraints for the individual geometry parameters which we will specify later. It should be noted that the length of the profiles is not explicitly included in the model. This is characterized by the double segment length of the corresponding sheet, see Figure 2 or Figure 3. With the above notation we obtain the variable $\xi = \left(P^1,\dots,P^n,S^1,\dots,S^m\right)$. The modular concept and the geometry parameters are shown in Figure 2 in detail.

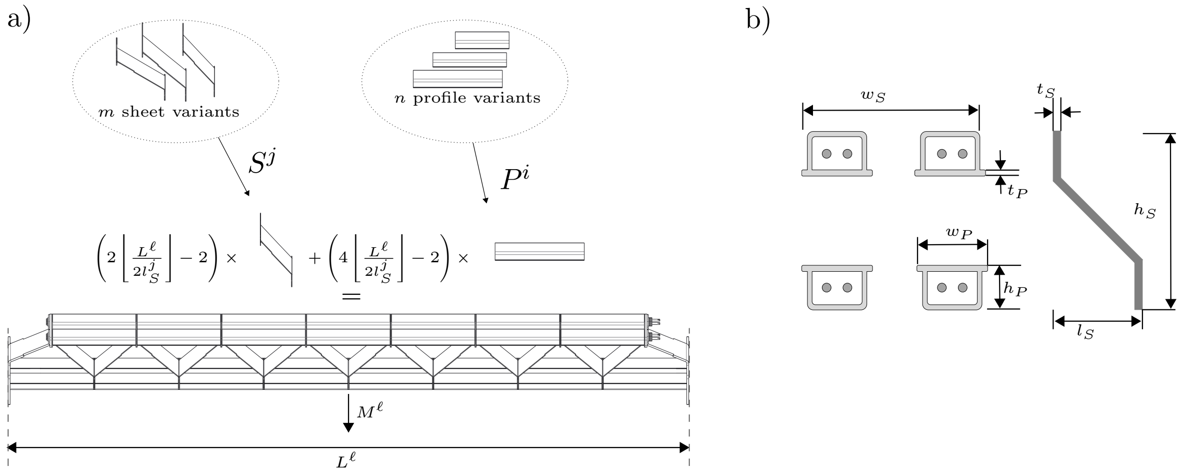

Figure 2: a) Modular concept of the crane's parts; b) Geometry parameters

Next we describe the functions and sets from problem $\overline{P}$ in more detail. The main effort will be to describe the set of feasible configurations of the modular system, that is, the description of the set $Z(v, \xi)$.

Set of feasible configurations

A crane bridge (here explicitly crane bridge ℓ) must fulfill two properties, as described above. Profiles of one variant and sheets of one variant must be taken from the modular system in such a way that the crane bridge results in the span L^ℓ and that the crane bridge carries at least a load of M^ℓ (in tons). Figure 2a shows how a crane bridge can be built from profiles and sheets. For the chosen profile-sheet combination (i, j) we can calculate the total number $z_{1,i}^\ell$ of profiles and the total number of sheets $z_{2,j}^\ell$ by

$$z_{1,i}^\ell = 4\left\lfloor\frac{L^\ell}{2l_S^j}\right\rfloor - 2, \quad z_{2,j}^\ell = 2\left\lfloor\frac{L^\ell}{2l_S^j}\right\rfloor - 2.$$

Let M be the load capacity function for a combination of profile P^i and a sheet S^j. Since this function is generally difficult to determine, we approximate it by the (rough) estimate

$$M(P^i, S^j) = \frac{c_1}{L^\ell}\left(c_2 h_S^j + c_3 h_P^i + c_4 w_P^i + c_5 w_S^j - c_6\left(\frac{h_S^j - 2h_P^i}{l_S^j} - \sqrt{3}\right)^2\right) \tag{5}$$

with $c \in \mathbb{R}_+^6$. To motivate the shape of this estimate, note that, the larger the span of the crane bridge, the lower is the load capacity. Moreover, the height of the profiles and sheets is more important than their widths, so that we choose c_2 and c_3 greater than c_4 and c_5, respectively. Parameter studies for a truss model have shown that the best distributions of forces in the truss are achieved at an angle of around 60° between the sheets and the profiles. With Figure 3 it can be seen that this exists when $\frac{h_S^j - 2h_P^i}{l_S^j} = \sqrt{3} = \tan(60°)$ holds.

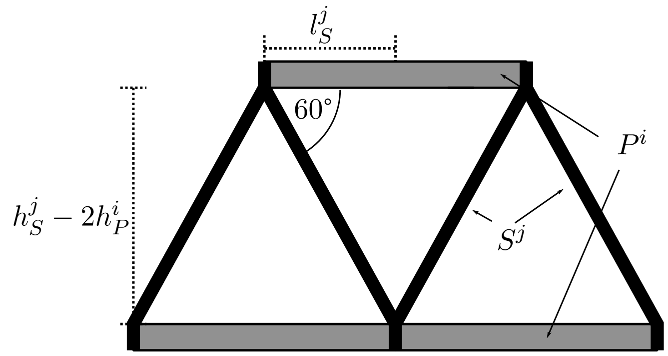

Figure 3: Geometry parameters in detail

With the load capacity function we can claim the second important property for crane bridge ℓ, $\ell = 1, \ldots, N$, namely the load capacity M^ℓ that has to be achieved at least by the chosen profile-sheet combination. This yields the constraint

$$M(P^i, S^j) \geqslant M^\ell.$$

In addition, the set of permissible configurations must take into account that not every profile-sheet combination is possible due to possible instabilities. Indeed, the four additional restrictions

$$w_S^j \geqslant 2w_P^i + t_S^j,\ h_S^j \geqslant 3h_P^i,\ 2l_S^j \geqslant h_S^j,\ 2l_S^j \leqslant 3h_S^j \tag{6}$$

must also apply.

By the first condition in (6) we achieve that there is a minimum distance between the profiles, see Figure 2b. The other three conditions achieve a boundary of the angle of the sheet, see Figures 2 and 3. We summarize these inequalities into a set F, which can be described with linear functions. Therefore we require $(P^i, S^j) \in F$ for a chosen combination.

Under our assumption that only one variant may be chosen, we obtain for crane bridge ℓ, $\ell = 1, \ldots, N$, the set of feasible configurations

$$\begin{aligned} Z^\ell(v, \xi) = \{z_{1,1}^\ell, \ldots, z_{1,n}^\ell, z_{2,1}^\ell, \ldots, z_{2,m}^\ell \in \mathbb{N}_0 |& \\ & |\{i \in \{1, \ldots, n\}|\ z_{1,i}^\ell > 0\}| = 1, && (7)\\ & |\{j \in \{1, \ldots, m\}|\ z_{2,j}^\ell > 0\}| = 1, && (8)\\ & z_{1,i}^\ell, z_{2,j}^\ell > 0 \ \Rightarrow\ z_{1,i}^\ell = 4\left\lfloor \frac{L^\ell}{2l_S^j} \right\rfloor - 2, && (9)\\ & z_{2,j}^\ell > 0 \ \Rightarrow\ z_{2,j}^\ell = 2\left\lfloor \frac{L^\ell}{2l_S^j} \right\rfloor - 2 && (10)\\ & z_{1,i}^\ell, z_{2,j}^\ell > 0 \ \Rightarrow\ M(P^i, S^j) \geqslant M^\ell && (11)\\ & z_{1,i}^\ell, z_{2,j}^\ell > 0 \ \Rightarrow\ (P^i, S^j) \in F && (12)\\ & v_{1,i} = 0 \ \Rightarrow\ z_{1,i}^\ell = 0, && (13)\\ & v_{2,j} = 0 \ \Rightarrow\ z_{2,j}^\ell = 0\}. && (14) \end{aligned}$$

From this definition of $Z^\ell(v,\xi)$ it is clear how the set of feasible configurations explicitly depends on the variable ξ. In Section 4.2 we will derive reformulations for the constraints in $Z^\ell(v,\xi)$ as well as for the appearing fractions, since this problem cannot be passed directly to a solver like Gurobi.

Cost functions

As described in Section 2, the objective function of the optimization problem is composed of two parts. On the one hand, there is the cost C of the size of the modular system, which leads to increasing costs for an increasing number of variants. On the other hand, we have the cost component c^ℓ which decreases for increasing numbers of variants. A significant part of this term is the cost of oversizing the crane bridges. Indeed, if profiles and sheets can be chosen from a large number of variants, we can expect that the desired load capacities will hardly be exceeded. If however, when there is little choice, crane bridges will be oversized, and respective penalizing costs occur.

In our model we assume that the cost for maintaining the modular system amounts to

$$C^P \sum_{i=1}^{n} v_{1,i} + C^S \sum_{j=1}^{m} v_{2,j}$$

with costs C^P per profile variant and C^S per sheet variant. The costs for oversizing the modular system are modelled as

$$C^O \sum_{\ell=1}^{N} \sum_{i=1}^{n} \sum_{j=1}^{m} c_{i,j}^\ell \left(M(P^i, S^j) - M^\ell \right)$$

with cost C^O per ton discrepancy in load capacity and binary variables $c_{i,j}^\ell$, $i = 1,\ldots,n, j = 1,\ldots,m, \ell = 1,\ldots,N$ with

$$c_{i,j}^\ell = \begin{cases} 1, & \text{if profile-sheet combination } (i,j) \text{ is chosen for crane bridge } \ell, \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

Through reformulations and renaming variables in the function M, we can find a quadratic reformulation of the objective function.

In addition to the costs mentioned above, it is also interesting to investigate the weight of the bridges. The goal is to build the crane bridges as light as possible. In penalizing the overdimensioning of bridges, it is certainly already included that the bridges are not too heavy. Nevertheless, we will explicitly consider the weight of the bridges in the numerical studies in Section 4.2 and investigate whether it makes a difference in the solution whether the total weight is part of the objective function or not. In this cost, we then also include for the first time the number variables $z_{1,i}^\ell$ and $z_{2,j}^\ell$ of the chosen variant of profiles and sheets from the modular system. We denote by w the weight function for a crane bridge, which depends on the geometry parameters of the profiles and sheets and the corresponding number variables. Thus, we additionally obtain the weight cost

$$C^W \sum_{\ell=1}^{N} \sum_{i=1}^{n} \sum_{j=1}^{m} c_{i,j}^{\ell} w(P^i, S^j, z^{\ell}) \tag{16}$$

with costs C^W per ton. The weight functions can be easily determined with the geometry of the profiles and sheets, see Figure 2 and 3. Through renaming variables we can find again a quadratic reformulation of the total weight costs from (16).

4.2 Reformulation of the set of feasible configurations

The set $Z^{\ell}(v, \xi)$ contains fractions and constraints that we cannot directly pass to a solver like Gurobi, but we need reformulations for them. We start with the fractions. We define new integer variables $y_j^{\ell} \in \mathbb{Z}$, $j = 1, \ldots, m$, $\ell = 1, \ldots, N$, which correspond to $\left\lfloor \frac{L^{\ell}}{2l_S^j} \right\rfloor$. Therefore we have for $j = 1, \ldots, m$, $\ell = 1, \ldots, N$,

$$\frac{L^{\ell}}{2l_S^j} + \varepsilon_{floor} - 1 \leqslant y_j^{\ell} \leqslant \frac{L^{\ell}}{2l_S^j} \tag{17}$$

while ε_{floor} is small enough (a little bit larger than the feasibily tolerance and integer feasibily tolerance of the solver Gurobi). The constraints in (17) can be reformulated to quadratic constraints through multiplication by $2l_S^j$. We recall at this point that, for passing them to Gurobi, quadratic constraints do not have to be convex.

Next we consider the indicator constraints. Gurobi can handle indicator constraints of the type

$$y = f \;\Rightarrow\; a^{\top} x \leqslant b,$$

with variables $y \in \mathbb{B}$ and $x \in \mathbb{R}^n$. This means that, if the binary variable y is equal to $f \in \{0, 1\}$, the linear constraint $a^{\top} x \leqslant b$ with $a \in \mathbb{R}^n$, $b \in \mathbb{R}$, has to be satisfied. In the other case ($y = 1 - f$) the constraint may be violated [1].

Through this form of the constraints, we introduce binary variables $b_{1,i}^{\ell} \in \mathbb{B}$, $i = 1, \ldots, n$, and $b_{2,j}^{\ell} \in \mathbb{B}$ which specify if variant i of profiles and variant j of sheets is chosen for crane bridge ℓ. Therefore we have

$$b_{1,i}^{\ell} = \begin{cases} 1, & \text{if profile variant } i \text{ is chosen for crane bridge } \ell, \\ 0, & \text{else,} \end{cases}$$

$$b_{2,j}^{\ell} = \begin{cases} 1, & \text{if sheet variant } j \text{ is chosen for crane bridge } \ell, \\ 0, & \text{else,} \end{cases}$$

$i = 1, \ldots, n$, $j = 1, \ldots, m$, $\ell = 1, \ldots, N$. The conditions (7) and (8) can be summarized to

$$\sum_{i=1}^{n} b_{1,i}^{\ell} = 1, \;\sum_{j=1}^{m} b_{2,j}^{\ell} = 1.$$

Further we use $b^\ell_{1,i} = 1$ instead of $z_{1,i} > 0$ in (9) and for the sheets $b^\ell_{2,j} = 1$ instead of $z^\ell_{2,j} > 0$, so that we can use the Gurobi indicator constraints. The condition $b^\ell_{1,i} = b^\ell_{2,j} = 1$ can be easily reformulated with the binary variable $c^\ell_{i,j}$ from (15) since we have

$$c^\ell_{i,j} = \begin{cases} 1, & \text{if } b^\ell_{1,i} = b^\ell_{2,j} = 1, \\ 0, & \text{else} \end{cases}, \quad i = 1,\dots,n, j = 1,\dots,m, \ell = 1,\dots,N.$$

We may formulate this relation by the linear constraints

$$c^\ell_{i,j} \in \mathbb{B}, \quad c^\ell_{i,j} \leqslant b^\ell_{1,i}, \quad c^\ell_{i,j} \leqslant b^\ell_{2,j}, \quad c^\ell_{i,j} \geqslant b^\ell_{1,i} + b^\ell_{2,j} - 1. \tag{18}$$

We summarize all the binary variables $b^\ell_{1,i}$ and $b^\ell_{2,j}$ for crane bridge ℓ into the vector b^ℓ, all binary variables $c^\ell_{i,j}$ into c^ℓ, all y^ℓ_i into y^ℓ and all the variables for the number of the variants into a vector z^ℓ (as before). Hence we obtain the set of feasible configurations

$$\begin{aligned} Z^\ell(v,\xi) = \{\ b^\ell \in \mathbb{B}^{n+m}, c^\ell \in \mathbb{B}^{nm}, y^\ell \in \mathbb{Z}^{mN}, z^\ell \in \mathbb{N}^{n+m} |\ & \sum_{i=1}^{n} b^\ell_{1,i} = 1,\ \sum_{j=1}^{m} b^\ell_{2,j} = 1, \\ & c^\ell_{i,j} = 1 \ \Rightarrow\ z^\ell_{1,i} = 4y^\ell_j - 2, \\ & b^\ell_{2,j} = 1 \ \Rightarrow\ z^\ell_{2,j} = 2y^\ell_j - 2, \\ & c^\ell_{i,j} = 1 \ \Rightarrow\ M(P^i, S^j) \geqslant M^\ell, \\ & c^\ell_{i,j} = 1 \ \Rightarrow\ (P^i, S^j) \in F, \\ & b^\ell_{1,i} = 0 \ \Rightarrow\ z^\ell_{1,i} = 0, \\ & b^\ell_{2,j} = 0 \ \Rightarrow\ z^\ell_{2,j} = 0, \\ & \text{reformulation (17)}, \\ & \text{coupling(18)}, \\ & v_{1,i} = 0 \ \Rightarrow\ b^\ell_{1,i} = 0, \\ & v_{2,j} = 0 \ \Rightarrow\ b^\ell_{2,j} = 0\}. \end{aligned} \tag{19}$$

We also reformulate (4) in terms of the new binary variables, which explains the last two constraints. All constraints in (19) can be passed to Gurobi, as long as we find a linear formulation of the function M.

4.3 A mixed-integer quadratic optimization problem for the modular system for crane bridges

With the results from Sections 4.1 and 4.2 we can specify the optimization problem $\overline{P}$ for the example of crane bridges

$$
\begin{aligned}
P^{\text{crane}}: \quad \min_{b,c,v,\xi,y,z} \quad & C^P \sum_{i=1}^{n} v_{1,i} + C^S \sum_{j=1}^{m} v_{2,j} + C^O \sum_{\ell=1}^{N} \sum_{i=1}^{n} \sum_{j=1}^{m} c_{i,j}^{\ell} \left(M(P^i, S^j) - M^{\ell} \right) \\
\text{s.t.} \quad & P^i \in X^1, \ i = 1, \dots, n, \\
& S^j \in X^2, \ j = 1, \dots, m, \\
& (b^{\ell}, c^{\ell}, y^{\ell}, z^{\ell}) \in Z^{\ell}(v, \xi), \ \ell = 1, \dots, N, \\
& v_{1,i}, v_{2,j} \in \mathbb{B}, \ i = 1, \dots, n, j = 1, \dots, m,
\end{aligned} \tag{20}
$$

with $b = (b^1, \dots, b^L)$, y and z respectively and $\xi = \left(P^i, \dots, P^n, S^1, \dots, S^m\right)$ and $Z^{\ell}(v, \xi)$ from (19). In P^{crane} we do not consider the total weight costs in particular.

The following problem considers the total weight cost of the crane bridges in the objective function

$$
\begin{aligned}
P^{\text{crane,w}}: \quad \min_{b,c,v,\xi,y,z} \quad & C^P \sum_{i=1}^{n} v_{1,i} + C^S \sum_{j=1}^{m} v_{2,j} + C^O \sum_{\ell=1}^{N} \sum_{i=1}^{n} \sum_{j=1}^{m} c_{i,j}^{\ell} \left(M(P^i, S^j) - M^{\ell} \right) \\
& + C^W \sum_{\ell=1}^{N} \sum_{i=1}^{n} \sum_{j=1}^{m} c_{i,j}^{\ell} w(P^i, S^j, z^{\ell}) \\
\text{s.t.} \quad & P^i \in X^1, \ i = 1, \dots, n, \\
& S^j \in X^2, \ j = 1, \dots, m, \\
& (b^{\ell}, c^{\ell}, y^{\ell}, z^{\ell}) \in Z^{\ell}(v, \xi), \ \ell = 1, \dots, N, \\
& v_{1,i}, v_{2,j} \in \mathbb{B}, \ i = 1, \dots, n, j = 1, \dots, m,
\end{aligned} \tag{21}
$$

with the same designations as above.

The problems P^{crane} and $P^{\text{crane,w}}$ have clearly more variables than the problem $\overline{P}$. However, we need these for the necessary reformulations so that the problem can be passed to the solver Gurobi.

Overall, we obtain mixed-integer optimization problems in (20) and (21). The load capacity function M and the weight function w can greatly complicate the problems. In particular the load capacity function enters both in objective function and the constraints. If we find a linear formulation of these functions, the resulting objective functions are quadratic and the constraint $c_{i,j}^{\ell} = 1 \Rightarrow M(P^i, S^j) \geqslant M^{\ell}$ can be modelled and solved globally with Gurobi. In the following subsection, we examine some example problems.

4.4 Numerical results

As mentioned in Section 4.3, we can solve the problems P^{crane} and $P^{\text{crane,w}}$ globally with Gurobi if we find linear representations for the load function M and the weight functions w^P

and w^S. Using the structure of M from (5), a linear representation can be found by renaming variables. The same can be done for the weight functions. By adding quadratic equality constraints, which can be handled with Gurobi, any polynomially representable function can be linearized.

For simplicity, we fix the profile and sheet thickness, so we have $t_P^i = t_S^j = 6$, $i = 1, \dots, n$, $j = 1, \dots, m$. All lengths are measured in millimeters. For the profiles we set $h_P^i \in [40, 100]$, $w_P^i \in [100, 200]$ and for the sheets $h_S^j \in [400, 1000]$, $l_S^j \in [150, 600]$, and $w_S^j \in [300, 400]$.

We choose $c = \left(50, 1, 3, \frac{2}{5}, \frac{1}{5}, 100\right)$. With this choice of c, we obtain approximately suitable load capacities. A more accurate representation of the load capacity function with suitable validated values for c remains an open question and is worth further investigation in the future. The input data given in the following were randomly generated and can be found in the tables below as well as the optimization results and the optimal configuration of the modular systems. By formulating the problems and reformulating them into a quadratic problem, we need to solve optimization problems that are nonconvex in both the objective function and the constraints.

Example 1

In **Ex1** and **Ex1W** we consider a small example with 5 demanded cranes and 5 possible profile and sheet variants. In **Ex1W** also the weight costs are considered in the objective function.

n	5
m	5
L	5
C^O	10.0000
C^P	10.0000
C^S	5.0000
runtime [s]	1.79
total costs	42.1999
number of binary variables	185
number of continuous variables	85
number of integer variables (with binary)	260
number of quadratic constraints	300
number of indicator constraints	925
total weight [t]	2.06

Table 1: General input data and optimization stats for Ex1

i	w^P	h^P
1	100.00	68.89
4	100.00	95.90

j	w^S	h^S	l^S
1	300.00	400.00	375.00
2	391.64	532.81	464.75
3	394.53	815.97	496.55
4	400.00	1,000.00	555.55

Table 2: Optimal configuration of the modular system for Ex1: profiles (left) and sheets (right)

ℓ	i	j	$M[t]$	$M^\ell[t]$	$L^\ell[mm]$	weight $[t]$
0	4	4	14.00	14.00	5,000	0.27
1	1	1	10.00	10.00	3,000	0.16
2	4	2	8.39	8.00	5,000	0.29
3	1	2	3.00	3.00	13,000	0.69
4	4	3	6.00	6.00	10,000	0.65

costs variants	40.00
costs oversizing	3.92

Table 3: Mapping crane bridges to profile and sheet variant with requested load capacity and actual load capacity and span for Ex1 (left); overview of the costs (right)

The following are the tables for **Ex1W**

n	5
m	5
L	5
C^O	10.0000
C^P	10.0000
C^S	5.0000
C^W	100.0000
runtime [s]	1.96
total costs	247.7879
number of binary variables	185
number of continuous variables	315
number of integer variables (with binary)	260
number of quadratic constraints	520
number of indicator constraints	925
total weight $[t]$	2.39

Table 4: General input data and optimization stats for Ex1W

i	w^P	h^P
1	153.31	66.89
3	154.65	90.00
4	100.00	55.00

j	w^S	h^S	l_S
0	400.00	449.21	225.705368
3	400.00	1,000.00	590.909628

Table 5: Optimal configuration of the modular system for Ex1W: profiles (left) and sheets (right)

ℓ	i	j	$M[t]$	$M^\ell[t]$	$L^\ell[mm]$	weight $[t]$
0	3	3	14.00	14.00	5,000	0.36
1	3	0	13.87	10.00	3,000	0.22
2	3	3	14.00	8.00	5,000	0.36
3	1	0	3.00	3.00	13,000	0.97
4	4	3	6.40	6.00	10,000	0.48

costs variants	40.00
costs oversizing	102.66
costs total weight	239.43

Table 6: Mapping crane bridges to profile and sheet variant with requested load capacity and actual load capacity and span for Ex1W (left); overview of the costs (right)

As expected, Ex1W is more complex to solve because the number of variables and constraints is larger. This has an effect on the runtime. Nevertheless, the solver quickly finds a global optimum point for both problems. We can also see that it makes a difference whether we take the weight into account in the optimization or not. However, it is difficult to say in general terms which problem should be solved. Depending on the application, it must be decided how much the weight costs should be taken into account or whether they should be included in the objective function.

Example 2 (Ex2)

In the second example, we consider a larger data set with 50 crane bridges. The solver takes considerably more time, but terminates with a global optimal point. The problem **Ex2** is the variant without weight costs, **Ex2W** considers the total weight of the crane bridges.

n	10
m	5
L	20
C^O	1.0000
C^P	20.0000
C^S	10.0000
runtime [s]	85.96
total costs	105.1276
number of binary variables	1,315
number of continuous variables	150
number of integer variables (with binary)	1,715
number of quadratic constraints	2,100
number of indicator constraints	7,300
total weight [t]	8.68

Table 7: General input data and optimization stats for Ex2

i	w^P	h^P
5	146.52	87.35

j	w^S	h^S	l^S
1	400.00	517.25	261.73
2	400.00	1,000.00	500.00
3	300.00	400.00	422.43

Table 8: Optimal configuration of the modular system for Ex2: profiles (left) and sheets (right)

ℓ	i	j	$M[t]$	$M^\ell[t]$	$L^\ell[mm]$	weight $[t]$
0	5	2	14.00	14.00	5,000	0.29
1	5	3	10.62	10.00	3,000	0.18
2	5	1	9.00	8.00	5,000	0.37
3	5	2	5.38	3.00	13,000	0.96
4	5	2	7.00	6.00	10,000	0.71
5	5	1	11.25	5.00	4,000	0.28
6	5	3	9.10	5.00	3,500	0.25
7	5	2	7.78	6.00	9,000	0.63
8	5	2	8.75	7.00	8,000	0.54
9	5	3	15.92	8.00	2,000	0.11
10	5	3	6.37	6.00	5,000	0.32
11	5	1	15.00	7.00	3,000	0.20
12	5	1	9.00	9.00	5,000	0.37
13	5	2	5.38	4.00	13,000	0.96
14	5	2	7.00	7.00	10,000	0.71
15	5	1	11.25	6.00	4,000	0.28
16	5	1	12.86	7.00	3,500	0.24
17	5	2	7.78	7.00	9,000	0.63
18	5	2	8.75	8.00	8,000	0.54
19	5	3	15.92	10.00	2,000	0.11

costs variants	50.00
costs oversizing	55.12

Table 9: Mapping crane bridges to profile and sheet variant with requested load capacity and actual load capacity and span for Ex2 (left); overview of the costs (right)

The following are the tables for **Ex2W**:

n	10
m	5
L	20
C^O	1.0000
C^P	20.0000
C^S	10.0000
C^W	10.0000
runtime [s]	140.45
total costs	165.6109
number of binary variables	1,315
number of continuous variables	1,360
number of integer variables (with binary)	1,715
number of quadratic constraints	3,290
number of indicator constraints	7,300
total weight [t]	7.68

Table 10: General input data and optimization stats for Ex2W

i	w^P	h^P
9	100.00	93.71

j	w^S	h^S	l^S
0	300.00	409.65	416.67
2	400.00	1,000.00	500.00
4	300.00	400.00	250.28

Table 11: Optimal configuration of the modular system for Ex2W: profiles (left) and sheets (right)

ℓ	i	j	$M[t]$	$M^\ell[t]$	$L^\ell[mm]$	weight $[t]$
0	9	2	14.00	14.00	5,000	0.31
1	9	0	10.79	10.00	3,000	0.14
2	9	2	14.00	8.00	5,000	0.31
3	9	2	5.38	3.00	13,000	0.86
4	9	2	7.00	6.00	10,000	0.65
5	9	0	8.09	5.00	4,000	0.20
6	9	0	9.24	5.00	3,500	0.20
7	9	2	7.78	6.00	9,000	0.58
8	9	2	8.75	7.00	8,000	0.51
9	9	4	17.58	8.00	2,000	0.09
10	9	4	7.03	6.00	5,000	0.29
11	9	0	10.79	7.00	3,000	0.14
12	9	2	14.00	9.00	5,000	0.31
13	9	2	5.38	4.00	13,000	0.86
14	9	2	7.00	7.00	10,000	0.65
15	9	0	8.09	6.00	4,000	0.20
16	9	4	10.05	7.00	3,500	0.19
17	9	2	7.78	7.00	9,000	0.58
18	9	2	8.75	8.00	8,000	0.51
19	9	2	35.00	10.00	2,000	0.10

costs variants	50.00
costs oversizing	73.48
costs total weight	76.81

Table 12: Mapping crane bridges to profile and sheet variant with requested load capacity and actual load capacity and span for Ex2W (left); overview of the costs (right)

All experiments were run on an Intel i7 processor with 8 cores with 3.60 GHz and 32 GB of RAM and with version 9.5.1 of GUROBI.

We observe that the optimization model finds a global solution even for a larger data set. This indicates that the modeling approach for modular systems problems from Section 3 may be considered suitable. However, the computational cost becomes very large due to the large increase in binary variables, despite the use of an efficient solver.

It should also be mentioned that the example is simplified and may not yet have a relevant dimension for practice, but that for the first time a model and a deterministic solution method were presented for modular system problems in which neither component combinations, numbers of variants nor component design were fixed.

5 Final remarks

As described in Section 4.4, the high problem dimension, especially the many binary variables that arise when we reformulate the original optimization problem, makes it very time consuming to solve the problem P^{crane} and $P^{\text{crane,w}}$ globally. Therefore, there is an interest in solving the problem faster.

One possibility to treat larger applications is to solve the problem locally instead of globally. So it must be weighed whether only local optimal points would be sufficient in place of global ones. Another difficulty is the nonconvexity of the optimization problem, which is largely due to the product of a binary variable with the load function M. Another idea would be to find a convex reformulation of the problem by a more suitable load capacity function or to work with a convex relaxation. Another idea for future research is to try to exploit the dependency structure of the original problem P in (1) by techniques from bilevel optimization and decomposition methods.

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