

# On the Relation Between Affinely Adjustable Robust Linear Complementarity and Mixed-Integer Linear Feasibility Problems

CHRISTIAN BIEFEL, MARTIN SCHMIDT

ABSTRACT. We consider adjustable robust linear complementarity problems and extend the results of Biefel et al. (2022) towards convex and compact uncertainty sets. Moreover, for the case of polyhedral uncertainty sets, we prove that computing an adjustable robust solution of a given linear complementarity problem is equivalent to solving a properly chosen mixed-integer linear feasibility problem.

## 1. INTRODUCTION

We consider affinely adjustable robust (AAR) linear complementarity problems (LCPs). The classic, i.e., deterministic, LCP is defined as follows. Given a matrix  $M \in \mathbb{R}^{n \times n}$  and a vector  $q \in \mathbb{R}^n$ , the LCP( $q, M$ ) is the problem to find a vector  $z \in \mathbb{R}^n$  that satisfies the conditions

$$z \geq 0, \quad Mz + q \geq 0, \quad z^\top (Mz + q) = 0 \quad (1)$$

or to show that no such vector exists. In the following, we use the standard  $\perp$ -notation and abbreviate (1) as

$$0 \leq z \perp Mz + q \geq 0. \quad (2)$$

LCPs are very important both in applications as well as in mathematical theory itself. For instance, they are used to model market equilibrium problems in many applied studies of gas or electricity markets [11] but also play an important role in mathematical optimization, game theory, or general matrix theory. We refer the interested reader to the seminal book [10] for an overview.

Although there is a very strong connection between LCPs and mathematical optimization and although the latter has been studied a lot in the recent decades under data uncertainty, the field of LCPs under uncertainty is still in its infancy. Stochastic approaches can be found in [7–9, 15] and are mainly based on minimizing the expected residual gap of the uncertain LCP. On the other hand, robust approaches for uncertain LCPs have been considered as well recently. The first rigorous analysis of robust LCPs can be found in [19, 20], where the authors apply the concept of strict robustness [18] to LCPs, which has been used later on in [16] in the context of Cournot–Bertrand equilibria in power networks. Moreover, in [13, 14], LCPs have been studied using  $\Gamma$ -robustness as introduced in [3, 4, 17]; see [6, 12] for some applications in power markets.

The most recent paper on robust LCPs, to the best of our knowledge, is [5], where robust LCPs are studied using the concept of adjustable robustness [2, 21]. In [5], the authors study adjustable robust LCPs in the most simplest setting, which is for affine decision rules and box uncertainties. In this short note, we stay with affine decision rules but generalize the results to general convex and compact uncertainty

---

*Date:* August 29, 2022.

*2010 Mathematics Subject Classification.* 90C33, 91B50, 90Cxx, 90C34.

*Key words and phrases.* Linear Complementarity Problems, Adjustable Robustness, Robust Optimization, Mixed-Integer Linear Optimization.

sets  $\mathcal{U}$ . In this context, our contribution is twofold. First, we characterize AAR solutions of robust LCPs and, second, use this characterization to prove that the AAR LCP is equivalent to a properly chosen mixed-integer linear problem (MILP).

Let us finally note that our study is related to [1], where the authors consider multi-parametric LCPs for sufficient matrices  $M$ . However, our robust approach as well as the studied relation to MILPs differ from the concepts and results of [1].

We introduce the problem under consideration in Section 2 and derive our main results in Section 3. Afterward, we comment on some special cases and extensions in Section 4.

## 2. PROBLEM STATEMENT

We now define the adjustable robust LCP with affine decision rules. To this end, let  $M \in \mathbb{R}^{n \times n}$  and  $q \in \mathbb{R}^n$  as before and let  $T \in \mathbb{R}^{n \times k}$  be given. We assume that  $q$  is perturbed by  $Tu$  with  $u \in \mathcal{U}$ . In what follows, we assume that  $\mathcal{U} \subset \mathbb{R}^k$  is a convex and compact uncertainty set with  $0 \in \text{relint}(\mathcal{U})$ . Then, the affinely adjustable robust LCP( $q, M, T, \mathcal{U}$ ) consists of finding an affine decision rule, i.e., we want to determine  $D \in \mathbb{R}^{n \times k}$  and  $r \in \mathbb{R}^n$  such that  $z(u) = Du + r$  satisfies

$$0 \leq z(u) \perp Mz(u) + q(u) \geq 0 \quad \text{for all } u \in \mathcal{U}. \quad (3)$$

Equivalently, we can state the problem more explicitly as

$$0 \leq Du + r \perp MDu + Mr + q + Tu \geq 0 \quad \text{for all } u \in \mathcal{U}. \quad (4)$$

Without loss of generality, we may assume that  $T \in \mathbb{R}^{n \times k}$  has full column rank; see [1]. To model  $h$  here-and-now variables, we simply require that the first  $h$  rows of  $D$  are zero. For more details, we refer to [5].

We close this section by briefly introducing the following notation. Let  $A \in \mathbb{R}^{m \times n}$ ,  $b \in \mathbb{R}^m$ , and index sets  $I \subseteq [m] := \{1, \dots, m\}$  as well as  $J \subseteq [n]$  be given. Then,  $A_{I,J} \in \mathbb{R}^{|I| \times |J|}$  denotes the submatrix of  $A$  consisting of the rows indexed by  $I$  and the columns indexed by  $J$ . Moreover,  $b_I$  denotes the subvector with components specified by entries in  $I$ . If  $I = J$ , we also write  $A_I$  instead of  $A_{I,I}$ .

## 3. MAIN RESULTS

We now state and prove our two main results and start with a full characterization of AAR solutions of robust LCPs.

**Theorem 1.** *Assume that  $\mathcal{U}$  is convex and compact with  $0 \in \text{relint}(\mathcal{U})$  and let  $\mathcal{B} = \{v^1, \dots, v^\ell\}$ ,  $\ell \in \mathbb{N}$ , be a basis of the linear hull  $\text{lin}(\mathcal{U})$  of  $\mathcal{U}$ . Moreover, let  $z(u) = Du + r$  such that  $z(u) \geq 0$  as well as  $Mz(u) + q + Tu \geq 0$  holds for all  $u \in \mathcal{U}$  and define  $I := \{i \in [n] : r_i > 0\}$ . Then,  $z(u) = Du + r$  is an AAR solution if and only if  $D$  and  $r$  satisfy the conditions*

$$M_{I,\cdot} r + q_I = 0, \quad (5)$$

$$(M_{I,\cdot} D + T_{I,\cdot}) v^j = 0, \quad j \in [\ell]. \quad (6)$$

*Proof.* First, let  $z(u) = Du + r$  be an AAR solution. Then,  $r$  is a nominal solution (as  $0 \in \mathcal{U}$ ) and therefore  $r$  satisfies  $M_{I,\cdot} r + q_I = 0$ , i.e., (5) is fulfilled. For every  $v^j$ ,  $j \in [\ell]$ , there exists a scalar  $\delta_j > 0$  such that  $\delta_j v^j \in \mathcal{U}$  and  $\delta_j D_{I,\cdot} v^j + r_I > 0$  holds. Thus, for every  $j \in [\ell]$  the AAR solution  $z$  satisfies

$$0 = M_{I,\cdot} z(\delta_j v^j) + q_I + \delta_j T_{I,\cdot} v^j = \delta_j M_{I,\cdot} D v^j + \delta_j T_{I,\cdot} v^j = \delta_j (M_{I,\cdot} D + T_{I,\cdot}) v^j,$$

where we used (5) for the second equality. Thus,  $z$  satisfies (6).

Let now  $D$  and  $r$  satisfy (5) and (6). From  $0 \in \text{relint}(\mathcal{U})$ , it follows that for all  $u \in \mathcal{U}$  there exists an  $\varepsilon > 0$  such that  $-\varepsilon u \in \mathcal{U}$ . Hence, nonnegativity of  $z(u) = Du + r$  yields

$$\{i \in [n]: \exists u \in \mathcal{U} : D_{i,\cdot}u + r_i > 0\} \subseteq I.$$

Thus, for  $\bar{I} = [n] \setminus I$ ,

$$z_{\bar{I}}(u)^\top (M_{\bar{I},\cdot}z(u) + q_{\bar{I}} + T_{\bar{I},\cdot}u) = 0$$

holds for all  $u \in \mathcal{U}$ . On the other hand, every  $u \in \mathcal{U}$  can be written as a linear combination  $u = \sum_{j=1}^{\ell} \lambda_j v^j$  with  $\lambda_j \in \mathbb{R}$ . Hence,

$$M_{\bar{I},\cdot}(Du + r) + q_{\bar{I}} + T_{\bar{I},\cdot}u = M_{\bar{I},\cdot}Du + T_{\bar{I},\cdot}u = (M_{\bar{I},\cdot}D + T_{\bar{I},\cdot}) \left( \sum_{j=1}^{\ell} \lambda_j v^j \right) = 0$$

holds, where we used (5) for the first and (6) for the last equality. Therefore,  $z(u) = Du + r$  fulfills complementarity and is an AAR solution due to the additional assumptions of the theorem.  $\square$

The second main result is about polyhedral uncertainty sets and uses the characterization of the last theorem to show that affinely adjustable robust solutions are the solutions of a properly chosen MILP.

**Theorem 2.** *Let  $\mathcal{U} = \{u \in \mathbb{R}^k : \Theta u \geq \zeta\}$  with  $\Theta \in \mathbb{R}^{g \times k}$  and  $\zeta \in \mathbb{R}^g$  and let  $\mathcal{B} = \{v^1, \dots, v^\ell\}$  be a basis of  $\text{lin}(\mathcal{U})$ . Furthermore, let  $B \in \mathbb{R}$  be sufficiently large and consider the mixed-integer linear feasibility problem*

$$\text{Find } x \in \{0, 1\}^n, D \in \mathbb{R}^{n \times k}, r \in \mathbb{R}_{\geq 0}^n, A, C \in \mathbb{R}_{\geq 0}^{g \times n} \quad (7a)$$

$$\text{s.t. } r_i \leq Bx_i, \quad i \in [n], \quad (7b)$$

$$B(1 - x_i) \geq M_{i,\cdot}r + q_i \geq 0, \quad i \in [n], \quad (7c)$$

$$B(1 - x_i) \geq (M_{i,\cdot}D + T_{i,\cdot})v^j \geq -B(1 - x_i), \quad i \in [n], j \in [\ell], \quad (7d)$$

$$D_{[h],\cdot} = 0, \quad (7e)$$

$$\zeta^\top A_{\cdot,i} + r_i \geq 0, \quad i \in [n], \quad (7f)$$

$$\Theta^\top A_{\cdot,i} = D_{i,\cdot}^\top, \quad i \in [n], \quad (7g)$$

$$\zeta^\top C_{\cdot,i} + M_{i,\cdot}r + q_i \geq 0, \quad i \in [n], \quad (7h)$$

$$\Theta^\top C_{\cdot,i} = (M_{i,\cdot}D + T_{i,\cdot})^\top, \quad i \in [n]. \quad (7i)$$

If (7) is feasible, it returns an AAR solution of the form  $z(u) = Du + r$  of (4). If it is infeasible, no AAR solution exists.

*Proof.* We show that  $z(u) = Du + r$  is an AAR solution if and only if there exist  $x, A, C$  such that  $x, D, r, A, C$  solve (7).

First, we prove nonnegativity of the solution, i.e.,  $Du + r \geq 0$  for all  $u \in \mathcal{U}$ , if and only if there exists a matrix  $A$  such that  $D, r, A$  satisfy (7f) and (7g). For all  $i \in [n]$ , we observe that  $D_{i,\cdot}u + r_i \geq 0$  holds for all  $u \in \mathcal{U}$  if and only if  $\min_{u \in \mathcal{U}} \{D_{i,\cdot}u + r_i\} \geq 0$ . We now employ duality and obtain that this is equivalent to the statement that there exists a vector  $a \in \mathbb{R}_{\geq 0}^g$  such that  $\zeta^\top a + r_i \geq 0$  and  $\Theta^\top a = D_{i,\cdot}^\top$ . The matrix  $A \in \mathbb{R}_{\geq 0}^{g \times n}$  then contains the vectors  $a$  as columns.

Next, we show that the  $Mz(u) + q + Tu \geq 0$  holds for all  $u \in \mathcal{U}$  if and only if there exists a matrix  $C$  such that  $D, r, C$  satisfy (7h) and (7i). This is analogous to the previous step and we observe that for every  $i \in [n]$ ,  $M_{i,\cdot}Du + M_{i,\cdot}r + q_i + T_{i,\cdot}u \geq 0$  being valid for all  $u \in \mathcal{U}$  is equivalent to  $\min_{u \in \mathcal{U}} \{M_{i,\cdot}Du + M_{i,\cdot}r + q_i + T_{i,\cdot}u\} \geq 0$ , which again is equivalent to that there exists a vector  $c \in \mathbb{R}_{\geq 0}^g$  such that  $\zeta^\top c + r_i + q_i \geq 0$ .

0 and  $\Theta^\top c = (M_{i,\cdot} D + T_{i,\cdot})^\top$  holds. The matrix  $C \in \mathbb{R}_{\geq 0}^{g \times n}$  then contains the vectors  $c$  as columns.

It remains to prove complementarity of the solutions. Let  $z(u) = Du + r$  be an AAR solution. We define  $I := \{i \in [n] : r_i > 0\}$  and  $x_i = 1$  for all  $i \in I$  and  $x_i = 0$  for all  $i \in [n] \setminus I$ . Then, Theorem 1 implies that  $x, D, r$  satisfy the constraints (7b)–(7d) for sufficiently large  $B$ . On the other hand, if  $x, D, r, A, C$  solve (7), all conditions of Theorem 1 are fulfilled and  $z(u) = Du + r$  is an AAR solution.

Finally, the remaining constraint (7e) enforces that the first  $h$  variables are nonadjustable.  $\square$

**Remark 1.** The linear hull  $\text{lin}(\mathcal{U})$  of the uncertainty set  $\mathcal{U}$  can be computed in polynomial time if  $\mathcal{U}$  is a polyhedron, i.e., if  $\mathcal{U} = \{u \in \mathbb{R}^k : \Theta u \geq \zeta\}$  as in Theorem 2. We can then maximize once in every direction  $\Theta_{j,\cdot}$ ,  $j \in [g]$ , and check if the optimal value is larger than  $\zeta_j$ . If it is equal to  $\zeta_j$ , we know  $\zeta_j = 0$  due to  $0 \in \text{relint}(\mathcal{U})$  and the inequality constraint can be replaced by an equality constraint. We obtain the representation  $\mathcal{U} = \{u \in \mathbb{R}^k : \Phi u = 0, \Theta' u \geq \zeta'\}$  with  $\Phi \in \mathbb{R}^{(g-f) \times k}$ ,  $f \leq g$ , and  $\Theta' \in \mathbb{R}^{f \times k}$ . The basis of  $\text{lin}(\mathcal{U})$  is then given by the basis of  $\ker(\Phi)$ .

We leave the study of other uncertainty sets for future work.

Let us also comment on a difference to the setting considered in [5]. There, the submatrix  $M_I$  has to be invertible for an AAR solution to exist if all entries of  $q$  are uncertain, cf. Theorem 4.5 in [5]. This is not the case in our setting as the following example shows.

**Example 1.** Consider the uncertain LCP given by

$$M = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}, \quad q(u) = \begin{pmatrix} -1 \\ -1 \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad \mathcal{U} = \{(u_1, u_2) : -2 \leq u_1 = u_2 \leq 2\}.$$

Then,

$$D = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}, \quad r = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

is an AAR solution with  $I = \{1, 2\}$ , but the matrix  $M$  is not invertible.

Finally note that if  $T$  is the identity matrix and  $\mathcal{U}$  is a box, the MILP (7) is equivalent to the MILP in Theorem 4.7 in [5].

#### 4. REMARKS AND EXTENSIONS

We now finally comment on a special case and two extensions.

**4.1. Positive Semidefinite  $M$ .** We first consider the case that the matrix  $M$  is positive semidefinite. In this case, we obtain a similar result as in [5] with the same arguments. The set  $I$  in Theorem 1 can be replaced by

$$P = \{i \in [n] : \exists z \in \text{SOL}(q, M) \text{ with } z_i > 0\},$$

where  $\text{SOL}(q, M)$  denotes the set of solutions of the LCP( $q, M$ ). This is due to the fact that every nominal solution  $r$  satisfies  $M_{P,\cdot} r + q_P = 0$ , which follows from Theorem 3.1.7 (a) in [10]. Therefore, every AAR solution has to satisfy

$$M_{P,\cdot} Du + T_{P,\cdot} u = 0$$

for all  $u \in \mathcal{U}$  and we obtain the statement of Theorem 1 with the set  $P$  instead of  $I$ . Hence, the MILP (7) can be simplified to an LP as we do not need the binary variables anymore. Since  $P$  can be computed in polynomial time using Theorem 3.1.7 (c) in [10], see also [5], this implies that we can find an AAR solution in polynomial time if  $M$  is positive semidefinite.

**4.2. Discrete Uncertainty Sets.** Next, we briefly discuss discrete uncertainty sets. In the following example, for any uncertainty realization in the discrete set, there exists a solution whereas there does not exist solutions for some realizations in the convex hull of the uncertainty set.

**Example 2.** Consider the LCP given by

$$M = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad q(u) = \begin{pmatrix} 1+u \\ u \end{pmatrix}$$

and  $\mathcal{U} = \{\pm 1\}$ . Then, for  $u = 1$ ,  $z = (0, 0)$  is a solution and for  $u = -1$ ,  $z = (1, 0)$  is a solution. If  $\mathcal{U}' = \text{conv}(\mathcal{U})$ , there is, however, no solution for  $u = -1/2$ .

This example is in contrast to results for classic robust linear optimization, where one can always replace the uncertainty set with its convex hull. The reason for this behavior can be explained with classic LCP theory. In the literature, the cone of vectors  $q$  for which the LCP( $q, M$ ) with a given matrix  $M$  has a solution is usually denoted by  $K(M)$ , i.e.,

$$K(M) = \{q \in \mathbb{R}^n : \text{SOL}(q, M) \neq \emptyset\}.$$

In general,  $K(M)$  is not convex, and hence the convex hull of some points that lie in  $K(M)$  is not necessarily contained in  $K(M)$ . However,  $K(M)$  is convex if and only if  $M$  is a so-called  $Q_0$ -matrix, cf. Proposition 3.2.1 in [10], and we obtain the following result.

**Proposition 1.** Suppose that  $M$  is a  $Q_0$ -matrix. Then, the uncertain LCP has a solution for all  $u \in \text{conv}(\mathcal{U})$  if it has a solution for all  $u \in \mathcal{U}$ .

**4.3. Decision-Dependent Uncertainty.** The MILP (7) can be also extended to cover simple decision-dependent uncertainty sets. To this end, consider the uncertainty set

$$\mathcal{U}(r) = \{u \in \mathbb{R}^k : \Theta u \geq \zeta + \Psi r\}, \quad \Psi \in \mathbb{R}^{g \times n},$$

that depends on the first-stage decision  $r$ . If the deviation caused by  $\Psi r$  is not too large, in some cases the linear hull does not change. Hence, in these cases we only have to replace the constraints (7f) and (7h) by their respective quadratic versions that include the terms  $(\Psi r)^\top A_{\cdot, i}$  and  $(\Psi r)^\top C_{\cdot, i}$ , respectively. We leave the detailed study of such situations for our future work.

## REFERENCES

- [1] N. Adeltgren and M. M. Wiecek. “A two-phase algorithm for the multiparametric linear complementarity problem.” In: *European Journal of Operational Research* 254.3 (2016), pp. 715–738. DOI: [10.1016/j.ejor.2016.04.043](https://doi.org/10.1016/j.ejor.2016.04.043).
- [2] A. Ben-Tal, A. Goryashko, E. Guslitzer, and A. Nemirovski. “Adjustable robust solutions of uncertain linear programs.” In: *Mathematical Programming* 99.2 (2004), pp. 351–376. DOI: [10.1007/s10107-003-0454-y](https://doi.org/10.1007/s10107-003-0454-y).
- [3] D. Bertsimas and M. Sim. “Robust discrete optimization and network flows.” In: *Mathematical Programming* 98.1 (2003), pp. 49–71. DOI: [10.1007/s10107-003-0396-4](https://doi.org/10.1007/s10107-003-0396-4).
- [4] D. Bertsimas and M. Sim. “The Price of Robustness.” In: *Operations Research* 52.1 (2004), pp. 35–53. DOI: [10.1287/opre.1030.0065](https://doi.org/10.1287/opre.1030.0065).
- [5] C. Biefel, F. Liers, J. Rolfes, and M. Schmidt. “Affinely Adjustable Robust Linear Complementarity Problems.” In: *SIAM Journal in Optimization* 32.1 (2022), pp. 152–172. DOI: [10.1137/20M1359778](https://doi.org/10.1137/20M1359778).
- [6] E. Çelebi, V. Krebs, and M. Schmidt. “T-Robust Electricity Market Equilibrium Models with Transmission and Generation Investments.” In: *Energy Systems* (2020). DOI: [10.1007/s12667-020-00411-x](https://doi.org/10.1007/s12667-020-00411-x). Forthcoming.

- [7] X. Chen and M. Fukushima. “Expected Residual Minimization Method for Stochastic Linear Complementarity Problems.” In: *Mathematics of Operations Research* 30.4 (2005), pp. 1022–1038. DOI: [10.1287/moor.1050.0160](https://doi.org/10.1287/moor.1050.0160).
- [8] X. Chen, R. J.-B. Wets, and Y. Zhang. “Stochastic Variational Inequalities: Residual Minimization Smoothing Sample Average Approximations.” In: *SIAM Journal on Optimization* 22.2 (2012), pp. 649–673. DOI: [10.1137/110825248](https://doi.org/10.1137/110825248).
- [9] X. Chen, C. Zhang, and M. Fukushima. “Robust solution of monotone stochastic linear complementarity problems.” In: *Mathematical Programming* 117.1 (2009), pp. 51–80. DOI: [10.1007/s10107-007-0163-z](https://doi.org/10.1007/s10107-007-0163-z).
- [10] R. W. Cottle, J.-S. Pang, and R. E. Stone. *The Linear Complementarity Problem*. Society for Industrial and Applied Mathematics, 2009. DOI: [10.1137/1.9780898719000](https://doi.org/10.1137/1.9780898719000).
- [11] S. A. Gabriel, A. J. Conejo, J. D. Fuller, B. F. Hobbs, and C. Ruiz. *Complementarity modeling in energy markets*. Vol. 180. Springer Science & Business Media, 2012.
- [12] A. Kramer, V. Krebs, and M. Schmidt. “Strictly and  $\Gamma$ -robust counterparts of electricity market models: Perfect competition and Nash–Cournot equilibria.” In: *Operations Research Perspectives* 8 (2021), p. 100197. DOI: [10.1016/j.orp.2021.100197](https://doi.org/10.1016/j.orp.2021.100197).
- [13] V. Krebs, M. Müller, and M. Schmidt. “ $\Gamma$ -robust linear complementarity problems with ellipsoidal uncertainty sets.” In: *International Transactions in Operational Research* 29.1 (2022), pp. 417–441. DOI: [10.1111/itor.12988](https://doi.org/10.1111/itor.12988).
- [14] V. Krebs and M. Schmidt. “ $\Gamma$ -Robust Linear Complementarity Problems.” In: *Optimization Methods and Software* (2020). DOI: [10.1080/10556788.2020.1825708](https://doi.org/10.1080/10556788.2020.1825708). Forthcoming.
- [15] G.-H. Lin and M. Fukushima. “New reformulations for stochastic nonlinear complementarity problems.” In: *Optimization Methods and Software* 21.4 (2006), pp. 551–564. DOI: [10.1080/10556780600627610](https://doi.org/10.1080/10556780600627610).
- [16] J. Mather and E. Munsing. “Robust Cournot-Bertrand equilibria on power networks.” In: *American Control Conference (ACC), 2017*. IEEE, 2017, pp. 2747–2754. DOI: [10.23919/ACC.2017.7963367](https://doi.org/10.23919/ACC.2017.7963367).
- [17] M. Sim. “Robust Optimization.” PhD thesis. Massachusetts Institute of Technology, 2004.
- [18] A. L. Soyster. “Technical Note—Convex Programming with Set-Inclusive Constraints and Applications to Inexact Linear Programming.” In: *Operations Research* 21.5 (1973), pp. 1154–1157. DOI: [10.1287/opre.21.5.1154](https://doi.org/10.1287/opre.21.5.1154).
- [19] Y. Xie and U. V. Shanbhag. “On Robust Solutions to Uncertain Linear Complementarity Problems and their Variants.” In: *SIAM Journal on Optimization* 26.4 (2016), pp. 2120–2159. DOI: [10.1137/15M1010427](https://doi.org/10.1137/15M1010427).
- [20] Y. Xie and U. V. Shanbhag. “On robust solutions to uncertain monotone linear complementarity problems (LCPs) and their variants.” In: *2014 IEEE 53rd Annual Conference on Decision and Control (CDC)*. IEEE, 2014, pp. 2834–2839. DOI: [10.1109/CDC.2014.7039824](https://doi.org/10.1109/CDC.2014.7039824).
- [21] İ. Yanıkoğlu, B. Gorissen, and D. den Hertog. “A Survey of Adjustable Robust Optimization.” In: *European Journal of Operational Research* 277 (2019), pp. 799–813. DOI: [10.1016/j.ejor.2018.08.031](https://doi.org/10.1016/j.ejor.2018.08.031).

(C. Biefel) FRIEDRICH-ALEXANDER-UNIVERSITÄT ERLANGEN-NÜRNBERG, DISCRETE OPTIMIZATION, CAUERSTR. 11, 91058 ERLANGEN, GERMANY  
*Email address:* [christian.biefel@fau.de](mailto:christian.biefel@fau.de)

(M. Schmidt) TRIER UNIVERSITY, DEPARTMENT OF MATHEMATICS, UNIVERSITÄTSRING 15, 54296 TRIER, GERMANY  
*Email address:* [martin.schmidt@uni-trier.de](mailto:martin.schmidt@uni-trier.de)