A binary linear programming approach for supporting administrative territorial consolidation

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ARTICLE INFO

Keywords:
Spatial optimization
Districting problem
Binary linear programming
Heuristic

ABSTRACT

The objective of this paper is to develop a scalable binary linear programming model for finding the optimal aggregation of communes into spatially contiguous administrative territorial units (ATUs) constrained on certain balancing criteria. The requirement for the ATUs to be contiguous represents the main computational bottleneck and, therefore, it prevents one from using such models on a large state level. We propose a novel scalable model that bypasses the contiguity bottleneck by using a shortest path heuristic. In our approach, the contiguity of an ATU is viewed as the connectedness between any pair of its communes mediated through its center by means of shortest paths. We apply the developed model for obtaining optimal administrative territorial consolidation scenarios for the Republic of Moldova which totally corresponds to legal requirements and achieves a reasonable compromise between all balancing criteria.

1. Introduction

The territorial fragmentation of a state is associated with a limited capacity of local governments to mobilize local own source revenues. This situation significantly undermines the ability of local governments for effective, efficient and equitable public services provision [1], [2]. The main purpose of administrative territorial consolidation is to increase cost-efficiency in provision of the public services and also to make administrative units more competitive nationally and internationally.

Administrative territorial consolidation is closely related to the districting problem which consists in grouping small basic areas into larger districts subject to planning criteria. The districting problem is a well-studied combinatorial optimization problem that finds many practical applications: design of sales territories [3], political districts [4], police districting [5], districting for salt spreading operations [6], school redistricting [7], urban land use [8], home health care [9]. Multiple approaches to solving it such as exact methods based on integer linear and quadratic programming, construction methods, computational geometry methods and metaheuristics were proposed [10], [11], [12].

Distrticting problems related to administrative territorial consolidation involve merging a set of communes connected by roads into administrative territorial units (ATUs) that meet multiple criteria such as connectivity by roads (contiguity), compactness, ethnic balance, equitable population size and economic advancement. The contiguity of an ATU means that it is possible to travel by roads between any two communes within the ATU without having to visit communes from other ATUs. An ATU is geographically compact if it is somewhat round-shaped [13]. Generally, the districting problem in many of its formulations is NP-hard [14] and only very small instances of the districting problem can be solved by linear programming models in reasonable time [15].

In this article, we propose a binary linear programming (BLP) model for solving the administrative territorial consolidation problem in which the contiguity of ATUs is expressed by the shortest roads between the ATU centers and any other communes assigned to these centers. This formulation substantially reduces the number of binary variables and constraints and as a result, a balance between the quality of the solution and the computational effort is achieved. We applied the developed model to determine the optimal scenarios of administrative territorial organization of the Republic of Moldova.

The paper is organized as follows. Section 2 contains a description of the model. Section 3 describes the case study of the Republic of Moldova. Finally, the conclusions of this paper are presented in Section 4.

2. Problem description

Let $G = (V, E)$ be an undirected graph, where the set of nodes (communes) is denoted by $V = \{v_1, \dots, v_n\}$ and edge e_{ij} belongs to E if and only if communes v_i and v_j are connected by a direct road. Each node $v_i \in V$ is annotated with the following three attributes: population size, per capita fiscal capacity as economic indicator and ethnic minority rate. Each edge $e_{ij} \in E$ is weighted by the distance on the direct road between communes v_i and v_j.

The set of communes V has to be partitioned into p ATUs, $p \geq 2$, subject to the following constraints. First of all, each node has to be assigned to exactly one ATU. Each ATU must have at least a minimum allowable number of inhabitants. The administrative center of an ATU has to be the commune with the maximum population size and per capita fiscal capacity and it has to be located as close as possible to all other communes belonging to this ATU. The latter condition can be interpreted as a measure of compactness. Also, ATUs

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must be geographically contiguous, i.e. each ATU has to induce a connected subgraph of G. Another important constraint refers to the grouping of communes with similar ethnic structure (similar minority rate) into ATUs.

So, the problem is to partition V into a number of sets according to the specified planning requirements of balancing and connectivity such that the sum of distances between communes and their corresponding ATUs centers is minimized. We refer to this problem as the ATC problem (administrative territorial consolidation).

2.1. Binary Linear Programming approach

We formulate a binary linear programming model for solving the ATC problem. We introduce binary variables x_{ij}, $1 \le i \le n$, $1 \le j \le n$, having the following meaning:

$$x_{ij} = \begin{cases} 1, & \text{if commune } v_j \text{ is assigned to ATU with center in } v_i \\ 0, & \text{otherwise} \end{cases}$$

and, in particular, $x_{ii} = 1$ if commune v_i is chosen as a center of an ATU and $x_{ii} = 0$ otherwise. Additional binary variables y_i, $1 \le i \le n$, are used to enforce the ethnic constraints. In order to provide more flexibility in identification of administrative centers of ATUs, we introduce some tolerance parameters pertaining to the population size and per capita fiscal capacity. We use the following notations:

- N – number of ATUs;
- D – maximum allowable travel distance from the administrative center of ATU to any commune assigned to it;
- C – minimum allowable per capita fiscal capacity of the administrative center of ATU;
- P – minimum allowable population size of the administrative center of ATU;
- P^* – minimum allowable population size of ATU;
- M – a large enough number;
- p_i – population size of commune v_i;
- c_i – per capita fiscal capacity of commune v_i;
- γ_i – ethnic coefficient of commune v_i ($\gamma_i = 1$ if ethnic minority rate in commune v_i is greater than a given threshold, $\gamma_i = 0$ otherwise);
- d_{ij} – travel distance between communes v_i and v_j;
- α – population tolerance, $\alpha \in [0, 1]$ (population size of any commune of ATU has to be at most $(1 + \alpha) * 100\%$ of the population size of the administrative center of ATU);
- β – fiscal capacity tolerance, $\beta \in [0, 1]$ (per capita fiscal capacity of any commune of ATU has to be at most $(1 + \beta) * 100\%$ of the per capita fiscal capacity of the administrative center of ATU);
- $\langle v_i, v_j \rangle$ – set of communes lying on a shortest paths (roads) between communes v_i and v_j.

Depending on the objective function, we formulate two BLP models: one model that seeks to minimize the total number of ATUs (we will refer to it as Min-BLP model) and the other model that seeks to minimize the total distances between communes and their corresponding ATU centers given the target number of ATUs (Fixed-BLP).

The Min-BLP model is presented below.

$$\text{minimize} \quad \sum_{i=1}^{n} x_{ii} \tag{1}$$

subject to:

$$d_{ij} x_{ij} \le D, \qquad i, j = \overline{1, n} \tag{2}$$

$$\sum_{v_k \in \langle v_i, v_j \rangle} x_{ik} \ge |\langle v_i, v_j \rangle| x_{ij}, \qquad i, j = \overline{1, n} \tag{3}$$

$$\sum_{j=1}^{n} p_j x_{ij} \ge P^* x_{ii}, \qquad i = \overline{1, n} \tag{4}$$

$$p_i x_{ii} \ge P x_{ii}, \qquad i = \overline{1, n} \tag{5}$$

$$p_j x_{ij} \le (1 + \alpha) p_i x_{ii}, \qquad i, j = \overline{1, n} \tag{6}$$

$$c_i x_{ii} \ge C x_{ii}, \qquad i = \overline{1, n} \tag{7}$$

$$c_j x_{ij} \le (1 + \beta) c_i x_{ii}, \qquad i, j = \overline{1, n} \tag{8}$$

$$\sum_{j=1}^{n} (1 - \gamma_j) x_{ij} \le M y_i, \qquad i = \overline{1, n} \tag{9}$$

$$\sum_{j=1}^{n} \gamma_j x_{ij} \le M (1 - y_i), \qquad i = \overline{1, n} \tag{10}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \qquad j = \overline{1, n} \tag{11}$$

$$\sum_{i=1}^{n} x_{ii} \ge N \tag{12}$$

$$x_{ij}, y_i \in \{0, 1\}, \qquad i, j = \overline{1, n} \tag{13}$$

The objective function (1) minimizes the total number of ATUs. Conditions (2) check whether the distances between the administrative center and each commune assigned to it do not exceed the maximum allowable distance. Constraints (3) guarantee the connectivity of each ATU by means of shortest roads between the administrative center and the rest of the communes of ATU. Note that these constraints represent an efficient approach to satisfying the contiguity requirement without adding any extra computational burden to the model. Constraints (4)–(6) assure that the population size of each administrative center and each ATU is not less than the lowest admissible population size, while constraints (7)–(8) impose similar conditions on the per capita fiscal capacity. Constraints (9) and (10) assure that only communes with the similar ethnic structure can be merged into a single ATU. At most two national groups are considered to be distinct in the model (the ethnic majority as one group and an ethnic minority as another group). Constraints (11) ensure that each node is assigned to only one ATU. Constraint (12) implies the creation of at least N ATUs. Finally, constraints (13) define the type of the decision variables (i.e., binary).

By replacing objective function (1) with

$$\text{minimize} \quad \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij} \tag{14}$$

and restriction (12) with

$$\sum_{i=1}^{n} x_{ii} = N \tag{15}$$

we get the Fixed-BLP model. In this model the objective function minimizes the sum of the distances between each commune and the administrative center that it is allocated to. This objective function reflects our intuitive comprehension of compactness. Constraint (15) implies the creation of exactly N ATUs.

The solution process of ATC problem consists of the following steps:

Step 1. Determining, by using the Min-BLP model, the minimum number of ATUs for which the imposed distance, population and per capita fiscal capacity restrictions are satisfied;

Step 2. Determining, by using the Fixed-BLP model, the most compact and balanced scenario for the optimal number of ATUs established at the first step.

3. Case study of the Republic of Moldova

In this section, we applied our model to the case of the Republic of Moldova to determine the optimal scenarios of administrative territorial consolidation. We set the maximum allowable travel distance D as the main parameter of our model. Depending on the choice of this parameter, we found three distinct optimal scenarios.

3.1. Actual administrative territorial organization

Generally, the administrative territorial organization of Moldova is two-level (see Figure 1). Since the independence, the Republic of Moldova has undergone two administrative territorial reforms. In 1998, the Moldovan Parliament approved the division of the country into 10 second-level ATUs, but kept the existing large number of first-level ATUs. One of the main criticisms of this reform was that it increased the distance for the citizens to access public services and did not provide any mechanism to access them remotely. In 2001, Moldova reverted to the structure existing before 1998, with a total of 898 first-level ATUs (communes) and 35 second-level ATUs (32 rayons, Chisinau and Balti municipalities, and one autonomous region – Gagauzia) . A special status was assigned to the Transnistria region which actually is outside of government control. These back-and-forth reforms are an evidence that the administrative territorial structure of Moldova requires a balance between many criteria such as the travel distance from any commune to its ATU center, the number of inhabitants in the ATUs, the economic development, etc.

The existing administrative territorial structure of the Republic of Moldova is characterized by a significant degree of fragmentation. According to the National Bureau of Statistics, the population of Moldova is 3.35 million inhabitants (as of 2017) [16]. On average, first level ATUs (communes) have less than 3,000 inhabitants with almost a third of them having less than the legally defined minimum threshold of 1,500 inhabitants [17]. In addition, communes with at most 5,000 residents form about 90% of all ATUs at level I. The present administrative territorial organization is not economically efficient, particularly because of the high share and volume of administrative expenditures. The average ratio between the administrative costs and the own revenues of ATUs of level I is 239.4%, and only 10% of ATUs of level I can cover their administrative expenses from their own revenues.

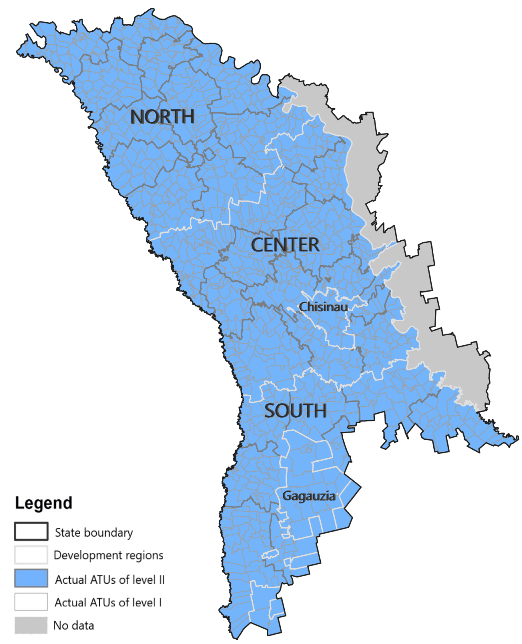

Figure 1: Actual administrative territorial organization of Moldova. The development regions do not have the status of administrative regions, but are designed to facilitate planning, evaluation and implementation of regional development policy.

In comparison with other South-Eastern European states, the Republic of Moldova has the smallest level I ATU population size (the average population size of actual communes is 2897 inhabitants), while the average population of such ATUs in Romania, Croatia, Albania and Slovenia is only a little less than 10,000 inhabitants [18]. The international experience shows that ATUs below 3,000-5,000 inhabitants cannot undertake significant public sector responsibilities and are not able to execute their functions efficiently. Moreover, the optimal size of ATUs for the purpose of delivering effective services and promoting economic development is 30,000 citizens [1].

The commune size is a significant factor associated with efficiency and effectiveness of local public administrations. Figure 2 shows that the share of administrative costs decreases with increasing size of communes. Also, there is a weak correlation between per capita fiscal capacity and size of commune (see Figure 3). The larger the commune size is, the more efficient and effective its governance is. Therefore, the administrative territorial consolidation would increase the economic capacity and the autonomy of local governments of the Republic of Moldova.

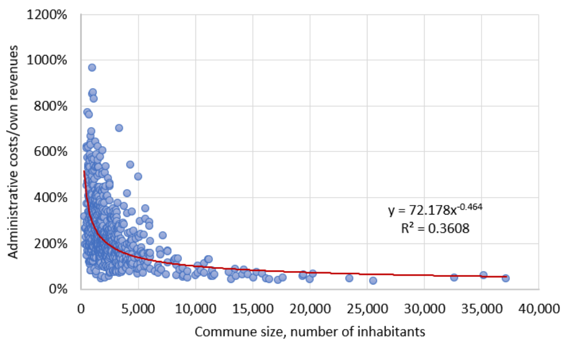

Figure 2: Relationship between ratio of administrative costs to own revenues and size of commune.

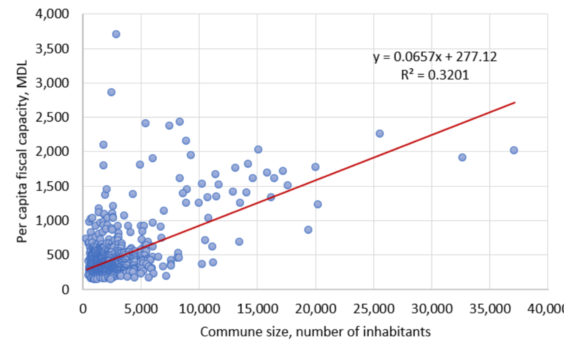

Figure 3: Relationship between per capita fiscal capacity and size of commune.

3.2. Optimal scenarios of administrative territorial consolidation

As we are aware, to date, only two studies conducting detailed analysis of the possible scenarios of the administrative territorial consolidation of the Republic of Moldova exist. The first study published in 2010 [19] proposed two consolidation scenarios: the first scenario implies the elimination of ATUs of level II and the consolidation of a number of ATUs of level I to 111; the second scenario involves the reorganization of second-level ATUs into regions of development and consolidation of the number of first level ATUs to 289. According to the other study conducted with the support of UNDP Moldova in 2015 [20], it was recommended to consolidate first-level ATUs to 111 as well as to transform ATUs of level II into three development regions. It should be noted that all these scenarios were developed manually without a serious mathematical justification. In contrast, we propose a BLP based methodology for determining the optimal consolidated administrative territorial organization of the state with regard to geographic, demographic and economic regional peculiarities.

We focus our research on optimal consolidation of ATUs of level I. We applied the BLP model to the whole territory of Moldova except for Transnistria region. In our approach, the second-level ATUs are associated with the actual regions of development: North, Center, South, Chisinau municipality and Autonomous Territorial Unit Gagauzia (see Figure 1). We slightly adjusted the limits of regions of development according to the following rule: if before the consolidation the center of a new ATU belongs to a region of development, then after consolidation the entire ATU belongs to the same region of development (ATU of level II). Subsequently, we will refer to ATUs of level I as ATUs. In our analysis, we used economic and socio-demographic data available from the National Bureau of Statistics [16], and we used Google Maps API in order to generate road network information.

Moldova is represented by multiple ethnicities that can be divided into two major groups based on the spoken language. The first group, majoritarian, is composed of the autochthonous Romanian-speaking population and the second group is a mixture of ethnic minorities (Russians, Ukrainians, Gagauz, Bulgarians, etc) for whom Russian is the main language of communication. In order to preserve actual ethnic minority representation, the communes with significant share of ethnic minorities (we consider that the threshold is 40%) can be merged only with communes having a similar ethnic composition. At the same time, the use of ethnic criteria can considerably reduce the number of potential spatial scenarios by maintaining a high degree of territorial fragmentation due to the uneven geographical location of communes. Therefore, this criterion is applied exclusively within administrative boundaries of autonomous region of Gagauzia.

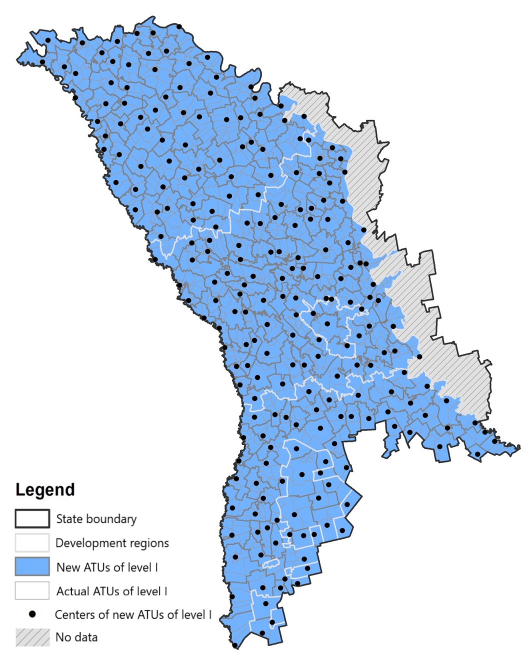

Figure 4: Moderate scenario with 231 ATUs of level I and 5 ATUs of level II.

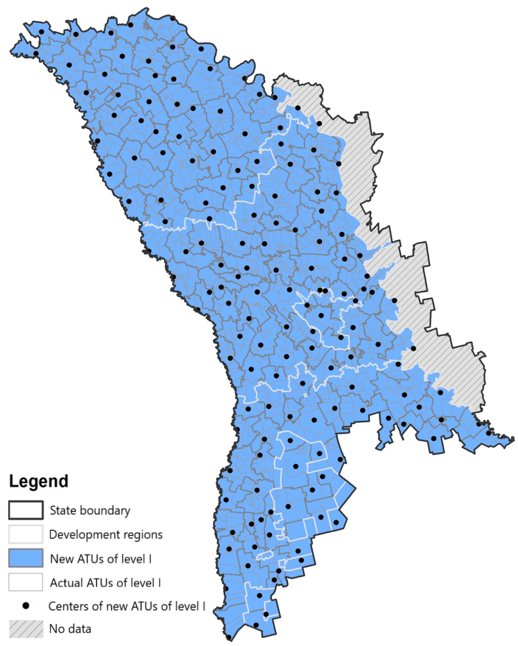

Figure 5: Intermediate scenario with 154 ATUs of level I and 5 ATUs of level II.

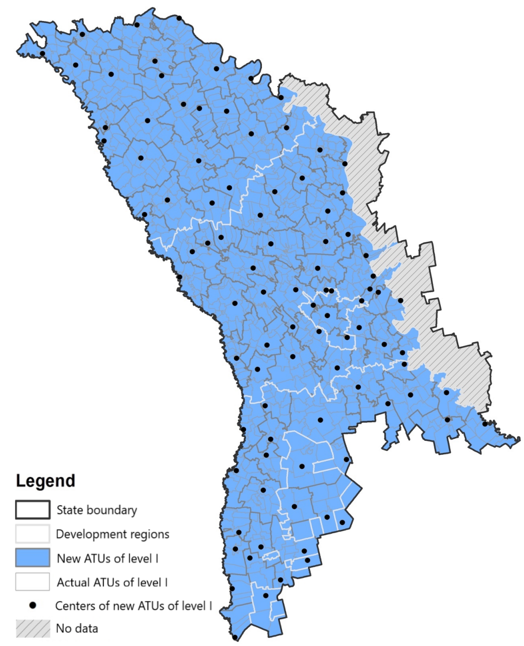

Figure 6: Strong scenario with 93 ATUs of level I and 5 ATUs of level II.

We considered the maximum allowable travel distance from the administrative center of any ATU to any commune assigned to it as the main criterion for establishing the optimal administrative territorial consolidation. We implemented the BLP model in MATLAB and applied it to the case of the Republic of Moldova. Our experiments showed that for values of maximum travel distance less than 12 km, can not be obtained solutions which would correspond to minimum legal requirements in sens of minimum allowable population size of ATU (1500 inhabitants). Also, the maximum travel distance greater than 25 km leads to a radical amalgamation, and, as a result, significantly decreases the efficiency of the services provided by local public administrations. Finally, we generated three optimal spatial scenarios: the moderate scenario with 231 ATUs (maximum allowable distance – 12 km, see Figure 4), the intermediate scenario with 154 ATUs (maximum allowable distance – 18 km, see Figure 5) and the strong scenario with 93 ATUs (maximum allowable distance – 25 km, see Figure 6). The input parameters used for these scenarios were obtained by manual calibration based on multiple preliminary runs (see Table 1).

Before consolidation, the proportion of ATUs with population less than 1500 inhabitants was 30% and less than 5000 inhabitants – 89%. After consolidation, a considerable reduction in the number of ATUs with up to 5000 inhabitants is observed. In all the scenarios, there were no longer ATUs with population less than 1500 inhabitants. ATUs with population up to 5000 accounted for 19% of the total number of ATUs in the moderate scenario, 7.8% – in the intermediate, and in the compact scenario there were no such ATUs. The average ATU population is 11.5, 17.4, and 28.2 thousand inhabitants in the moderate, intermediate, and compact scenarios correspondingly. The distribution of ATUs according to their population is presented in Figure 7. From the point of view of proximity to the administrative centers, the average distances between administrative centers and the communes within ATUs constitute 7.9 km in the moderate scenario, 10.1 km in the intermediate scenario and 12.5 km in the compact one.

Among these three spatial scenarios, the intermediate scenario is the most promising and allows one to achieve an

Table 1
Modeling parameters.

Moderate	Intermediate	Strong
D – 12	D – 18	D – 25
C – 150	C – 150	C – 170
P – 700	P – 900	P – 1500
P^* – 1500	P^* – 1500	P^* – 2000
α – 0.7	α – 0.6	α – 0.6
β – 0.6	β – 0.4	β – 0.4

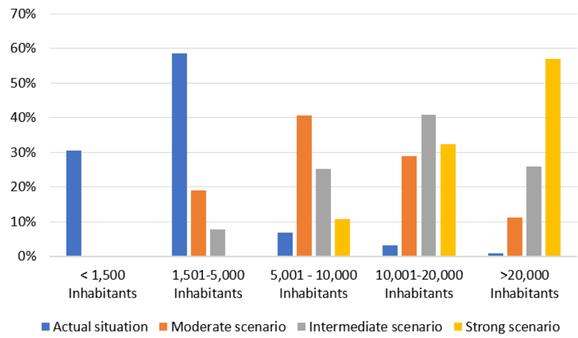

Figure 7: Distribution of ATUs by size (number of inhabitants).

optimal compromise between ensuring both an acceptable distance to the administrative centers and an efficient demographic and economic consolidation. Compared to the moderate scenario, the maximum allowable distance of 18 km exhibits a much more efficient choice of administrative centers, maintaining a balance between population size, fiscal capacity and the distance between the administrative centers of ATUs to communes assigned to them. At the same time, increasing the distance up to 25 km can be considered a serious weakness given that it can reduce the access of citizens to local public services. So, the intermediate scenario presents a great potential for increasing the efficiency of public institutions and can stimulate social and economic development of regions and of the entire state.

4. Conclusion

This paper introduces the BLP formulation for the problem of administrative territorial consolidation of the state.

Our model simultaneously decides the location of ATU centers and the assignment of communes to ATUs. Moreover, it generates connected, sufficiently compact and balanced ATUs in terms of population and economic development. Since the assurance of spatial contiguity is the crucial stumbling block that makes this problem challenging, our model incorporates contiguity constraints using the shortest paths between ATU centers and communes allocated to them on the basis of the existing road network, thereby leading to considerable reduction of problem size and computational effectiveness.

The practical usefulness of this approach is illustrated by applying it to the case of the Republic of Moldova for which the optimal administrative territorial consolidation scenarios were obtained. We identified that the most promising scenario of administrative territorial consolidation based on the maximum allowable travel distance between ATU centers and communes assigned to them equals 18 km. Obviously, the proposed model can be effectively applied for determining optimal consolidation of states whose size is much larger than the size of Moldova.

References

[1] Paweł Swianiewicz. *Territorial consolidation reforms in Europe*. Local Government and Public Service Reform Initiative, Open Society Institute, 2010.

[2] David Bartolini. Municipal Fragmentation and Economic Performance of OECD TL2 Regions. OECD, 2015.

[3] Bernhard Fleischmann and Jannis N Paraschis. Solving a large scale districting problem: a case report. *Computers & Operations Research*, 15(6):521–533, 1988.

[4] Justin C Williams Jr. Political redistricting: a review. *Papers in Regional Science*, 74(1):13–40, 1995.

[5] Steven J D'Amico, Shoou-Jiun Wang, Rajan Batta, and Christopher M Rump. A simulated annealing approach to police district design. *Computers & Operations Research*, 29(6):667–684, 2002.

[6] Luc Muyldermans, Dirk Cattrysse, Dirk Van Oudheusden, and Tsippy Lotan. Districting for salt spreading operations. *European Journal of Operational Research*, 139(3):521–532, 2002.

[7] Felipe Caro, Takeshi Shirabe, Monique Guignard, and Andrés Weintraub. School redistricting: Embedding GIS tools with integer programming. *Journal of the Operational Research Society*, 55(8):836–849, 2004.

[8] Wenwen Li, Richard L Church, and Michael F Goodchild. An extendable heuristic framework to solve the p-compact-regions problem for urban economic modeling. *Computers, Environment and Urban Systems*, 43:1–13, 2014.

[9] Emna Benzarti, Evren Sahin, and Yves Dallery. Operations management applied to home care services: Analysis of the districting problem. *Decision Support Systems*, 55(2):587–598, 2013.

[10] Jörg Kalcsics, Stefan Nickel, and Michael Schröder. Towards a unified territorial design approach—Applications, algorithms and GIS integration. *Top*, 13(1):1–56, 2005.

[11] Federica Ricca, Andrea Scozzari, and Bruno Simeone. Political districting: from classical models to recent approaches. *Annals of Operations Research*, 204(1):271–299, 2013.

[12] Roger Z Ríos-Mercado. *Optimal Districting and Territory Design*, volume 284. Springer, 2020.

[13] Alan M MacEachren. Compactness of geographic shape: Comparison and evaluation of measures. *Geografiska Annaler: Series B, Human Geography*, 67(1):53–67, 1985.

[14] Micah Altman and Michael McDonald. The promise and perils of computers in redistricting. *Duke J. Const. L. & Pub. Pol'y*, 5:69, 2010.

[15] Juan C Duque, Richard L Church, and Richard S Middleton. The p-Regions Problem. *Geographical Analysis*, 43(1):104–126, 2011.

[16] National Bureau of Statistics. https://statistica.gov.md/index.php?l=en.

[17] The Law on Territorial-Administrative Organization (10/20/2017 version). https://www.legis.md/cautare/getResults?doc_id=101662&lang=ro#, 2017.

[18] Anthony Levitas et al. *FISCAL decentralization indicators for South-East Europe: 2006-2014*. NALAS, 2016.

[19] Ion Osoianu, Valeriu Prohnițchi, Igor Sîrodoev, and Eugenia Veverița. Alytical Study on Optimal Administrative-Territorial Structure for Republic of Moldova. Expert-Grup, 2010.

[20] A Ionescu, S Drezgie, and I Rusu. Report on the Territorial-Administrative Structure Options for the Republic of Moldova. UNDP, 2015.