Improving reliability with optimal allocation of maintenance resources: an application to power distribution networks

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Abstract

Power distribution networks should strive for reliable delivery of energy. In this paper, we support this endeavor by addressing the Maintenance Resources Allocation Problem (MRAP). This problem consists of scheduling preventive maintenance plans on the equipment of distribution networks for a planning horizon, seeking the best trade-offs between system reliability and maintenance budgets. We propose a novel integer linear programming (ILP) formulation to effectively model and solve the MRAP for a single distribution network. The formulation also enables flexibility to suit new developments, such as different reliability metrics and smart-grid innovations. Then we develop a straightforward ILP formulation to address the MRAP for several distribution networks which takes the advantages of exchanging maintenance information between local agents and upper management. Using a general-purpose ILP solver, we performed computational experiments to assess the performance of the proposed approaches. Optimal maintenance trade-offs were achieved with the new formulations for real-scale distribution networks within short running times.

Keywords: Maintenance, Reliability, Power distribution networks, Integer linear programming, Multicriteria optimization.

1. Introduction

Asset management takes a principal role in power systems since they are highly dependent on their capital-intensive assets to deliver energy. Unlike most non-electric systems, these systems operate continuously with resources spread over large geographic areas. Their resources deteriorate over time due to the natural aging process and weather disruptions. Most of the power supply interruptions are due to failures in the distribution networks since

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they are usually vulnerable to single fault events (Billinton & Billinton, 1989; Billinton & Allan, 1996). From a customer perspective, regulatory agencies penalty distribution utilities without satisfactory levels of system reliability (Assis et al., 2015). These issues motivate the development of optimizing analytical tools to support the decision-making process of distribution utilities seeking to improve their system reliability.

Maintenance actions are a way of increase the reliability of components and systems (Endrenyi et al., 2001). The purpose of maintenance is to extend the life cycle of the equipment or at least the average time to the next failure (Endrenyi et al., 1998), which can be costly to repair or replace. In this paper, we address the Maintenance Resources Allocation Problem (MRAP), which consists of scheduling preventive maintenance plans on the equipment of distribution networks for a planning horizon, seeking the best trade-offs between system reliability and maintenance budgets. In this sense, the main decision of this problem is to identify which equipment of the networks leads to the best impact on system reliability when receiving maintenance actions at a minimum cost. The MRAP can arise either as a problem for a single distribution network or as a more general problem for a distribution utility with several distribution networks. The MRAP for a single distribution network is known to be NP-hard (Usberti et al., 2015).

The optimization of preventive maintenance actions in distribution networks has been studied over the past two decades. We emphasize that related work in the literature approaches mainly the particularities of small or moderate-sized distribution networks by proposing optimized maintenance plans based on heuristics, in which the aging of equipment and impact of maintenance actions are estimated using a failure rate model (continuous model) or Markov diagrams with Monte Carlo simulation (discrete model with sampling). For instance, Sittithumwat et al. (2004) proposed an optimization model considering the failure rates of each maintenance action and equipment. Motivated by a real application in a Brazilian utility, Usberti et al. (2015) developed a multi-criteria approach for generating maintenance plans with distinct trade-offs of cost and reliability. They addressed the MRAP for a single distribution network by developing a genetic algorithm that searches for the lowest cost maintenance plan with a minimum level of reliability. They also developed an algorithm that optimally combines the solutions of a set of the distribution networks to generate maintenance plans for the whole utility. Assuming the optimal Pareto front is known *a priori* for each distribution network, the algorithm guarantees to find the optimal Pareto front for the entire set of networks. On the other hand, if only approximate or incomplete fronts are known for each distribution network, then the algorithm obtains an approximation of the Pareto front for the entire set of networks.

Reliability-centered maintenance aspects of power distribution networks were discussed in Moon et al. (2006), Arya et al. (2011), and Ardabili et al. (2021). Heuristics based on multiple criteria optimization approaches for generating optimized trade-offs between reliability and maintenance budgets are designed in Radmer et al. (2002) and Hilber et al. (2007). The literature also covers other facets of power systems maintenance. For example, the maintenance of overhead distribution lines can be distinguished from other components of the network given its correlation with vegetation growth, leading to customized methodologies like the ones proposed by Abiri-Jahromi et al. (2009), Radmer et al. (2002), Kuntz et al.

(2002), and Misari et al. (2020). Another example is the total maintenance, that is, the replacement of equipment (Mirsaeedi et al., 2018). More recently, Moradi et al. (2019) and Rafiei et al. (2020) focus on the new features arising from smartgrids and microgrids. Other papers explore the stochastic aspects of the contingencies to propose a scenario decomposition methodology (Basciftci et al., 2018; Shang et al., 2020). Recent surveys and approaches on maintenance scheduling of power systems or optimization of multi-asset systems are found in Pham & Wang (1996), Froger et al. (2016), Wang et al. (2019), Petchrompo & Parlikad (2019), Laksman et al. (2020), and Salari & Makis (2020).

In this paper we explore these concepts further by proposing an exact framework for the MRAP. Unlike related work based on specific applications, the proposed methodology attracts attention for tackling a diversity of equipment, different maintenance actions, and periodic planning cycles. Since it considers the power system features as an input, the methodology also enables flexibility to suit new reliability indices, network operation typologies, energy sources, and even non-electrical systems. The main contributions of this paper are two novel ILP formulations for the MRAP. The first formulation completely models the MRAP for a single distribution network problem, and the second one concerns the MRAP for several distribution networks with a hierarchical structure. Using a general-purpose ILP solver, the formulations provide optimal maintenance trade-offs for real-scale power distribution networks of cities in São Paulo (Brazil). Although the MRAP is NP-Hard, it showed amenable to the use of an exact bi-objective approach, as discussed in the computational experiments.

The remainder of this paper is organized as follows. In Section 2, we provide preliminary concepts for the MRAP. In Section 3, we state the MRAP for a single distribution network and propose a novel ILP formulation. In Section 4, we develop an ILP formulation to the MRAP for several distribution networks with a hierarchical structure. The computational experiments presented in Section 5 show the main advantages/difficulties of the approaches. To benchmark purposes, we develop a Biased Random-Key Genetic Algorithm (BRKGA) for the MRAP, which is presented in the Appendix, to play the role of a benchmark approach, motivated by the genetic algorithm proposed in Usberti et al. (2015). In Section 6, we present final remarks and perspectives of future research.

2. Preliminary concepts

Reliability assessment of power distribution is performed by regulatory agencies using performance indices. For instance, the customer-based *system average interruption frequency index* (SAIFI) measures the average number of interruptions observed by customers during a year (Billinton & Allan, 1996; Brown, 2009). The SAIFI will be the reference index throughout this paper since maintenance actions have prominent effects on the frequency of failure events. Furthermore, it is a cornerstone reliability index for most regulatory authorities, including the Brazilian regulatory agency ANEEL. The SAIFI is also the most widely available information concerning network reliability. Although most utilities are putting effort into updating their data concerning the reliability of equipment and systems, most databases are still primarily structured for accounting purposes, making engineer analysis difficult.

Nevertheless, additional reliability indices may be considered in the proposed methodology, as discussed in Section 3.3.

2.1. Network sections and distributed generation

Most distribution networks operate with a radial configuration. Using graph terminology, they operate as a tree with the substation at the root (Ahuja et al., 1993). Thus, there is a unique path from any component to the distribution substation. Given there are no redundancies, these networks are more susceptible to interruptions due to a single component failure (Billinton & Billinton, 1989; Billinton & Allan, 1996). These distribution networks can be partitioned by the protection devices and switches into sections (Usberti et al., 2015), as illustrated in Fig. 1. A section represents a set of electrically connected components that observe the same interruptions, and the edges connecting sections represent protection devices. After a failure, the protection device upstream of the fault triggers, interrupting its downstream network's power supply. That is why the equipment closer to the distribution substation (root node) are more relevant to the system reliability than the equipment further away.

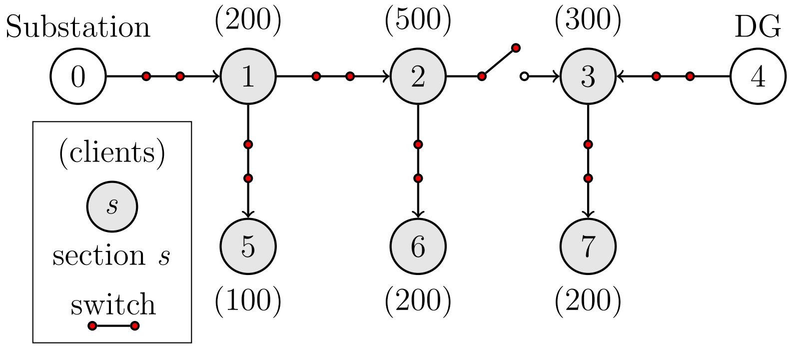


Figure 1: A radially operated network with distributed generation (DG).

After a failure, a path to an alternative power source can allow reconnecting part of the downstream network. For example, an interruption at node 2 in Fig. 1 disconnects all of its 700 downstream clients (nodes 2 and 6) given that switch $(1, 2)$ is opened. However, if an interruption occurs at node 1, and assuming the capacity of distribution generation (DG) is sufficient to provide power to nodes 2 and 6 only, a path can be formed by opening switch $(1, 2)$ and closing switch $(2, 3)$. This action would connect momentarily nodes 2 and 6 to the distributed generation located at node 4, preventing the interruption of these 700 clients.

Although the proposed framework is evaluated on distribution networks with radial configurations, all the concepts can be seamlessly developed for other operating topologies in the presence/absence of distributed generation, using only their respective methods to evaluate the number of customers affected by a failure. For a discussion of the impact of failures in networks with topologies other than radially operated, we refer to Billinton & Billinton (1989).

2.2. Failure rate model

The failure rate model seeks to describe the failure rate distribution during a given time interval. It should consider the natural aging of the equipment and the impacts of maintenance actions. It is represented in Eq. (1), where λ_e^t is the expected failure rate of equipment e in period t. A maintenance action k on equipment e improves the failure rate of the previous period by a factor given by multiplier m_{ek} (Sittithumwat et al., 2004; Usberti et al., 2015). Thus, $m_{ek} > 1$ indicates the absence of maintenance, otherwise $0 < m_{ek} \leq 1$. The vegetation adjacent to the distribution lines follows the same failure rate model as the equipment, that is, pruning the vegetation improves the reliability of the nearby lines.

$$\lambda_e^t = \lambda_e^{t-1} m_{ek}. \qquad \forall e, \forall t. \tag{1}$$

The lack of data on the initial failure rates for every piece of equipment in the network as well as the failure rate multipliers is not uncommon for most utilities. Nonetheless, mathematical models can approximate well these parameters based on historical values, as discussed in Sharifinia et al. (2020) and Doumpos et al. (2021).

3. MRAP for a single distribution network: an ILP model

The following assumptions give the environment of the MRAP for a single distribution network:

1. there are two conflicting objectives, namely the total maintenance costs (to be minimized) and the network reliability (to be maximized) measured by the SAIFI index;
2. the network operates with a radial configuration;
3. the initial failures rates for all the equipment and the maintenance multipliers for each maintenance action are input data.

Note that these objectives are conflicting because an increase in reliability usually requires an increase in investment. Furthermore, the minimization of the SAIFI index corresponds to the maximization of the network reliability. We define the following notation concerning sets, parameters, and decision variables to used when presenting the model:

Sets:

E	set of equipment.
S	set of sections.
E_s	set of equipment in section $s \in S$; $E_s \subseteq E$.
K	set of maintenance actions (which also includes the absence of maintenance).
T	set of periods in the planning horizon.

Parameters:

m_{ek} failure rate multiplier for maintenance action $k \in K$ on equipment $e \in E$.

p_{ek} preventive maintenance cost for action $k \in K$ on equipment $e \in E$.

c_e corrective maintenance cost of equipment $e \in E$.

N_s number of clients affected by an interruption in section $s \in S$.

NT number of clients served by the distribution network.

$\underline{\lambda_e^t}, \overline{\lambda_e^t}$ lower and upper bounds on failure rate of equipment $e \in E$ in period $t \in T$.

λ_e^0 initial failure rate of equipment $e \in E$.

r interest rate.

ϵ acceptable level for SAIFI index.

$\underline{S}, \overline{S}$ best and worst values for SAIFI index.

Binary variables:

x_{ek}^t binary variable which equals 1, if equipment $e \in E$ receives maintenance action $k \in K$ in period $t \in T$, and 0 otherwise.

Continuous variables:

λ_e^t failure rate of equipment $e \in E$ in period $t \in T$.

PC^t preventive maintenance costs in period $t \in T$.

CC^t corrective maintenance costs in period $t \in T$.

$SAIFI^t$ reliability index in period $t \in T$.

$SAIFI$ maximum reliability index in the planning horizon.

δ_{ek}^t auxiliary variable for representing the failure rate of equipment $e \in E$ in period $t-1 \in T$ when $x_{ek}^t = 1$.

3.1. Non-linear model

We present two main sets of variables for modeling the MRAP for a single distribution network. Continuous variables λ_e^t concerns the failure rate of equipment e in period t, and binary variables x_{ek}^t are equal to 1 if equipment e receives maintenance action k in period t, and 0 otherwise. Equations (2) and (3) define the preventive and corrective maintenance costs in period t, respectively. The preventive maintenance cost is directly associated with the actions arising from the maintenance plan. On the other hand, since the failures have not yet occurred, the corrective maintenance cost is only an estimate for the maintenance budget.

$$PC^t = \sum_{s \in S} \sum_{e \in E_s} \sum_{k \in K} p_{ek} x_{ek}^t. \tag{2}$$

$$CC^t = \sum_{s \in S} \sum_{e \in E_s} c_e \lambda_e^t. \tag{3}$$

Equation (4) evaluates the SAIFI index of period t.

$$SAIFI^t = \frac{\sum_{s \in S} (N_s \sum_{e \in E_s} \lambda_e^t)}{NT}, \qquad \forall t. \tag{4}$$

The following set of equations describes the MRAP as a bi-objective mixed-integer non-linear problem. This model generalizes the model of Usberti et al. (2015) to the case with multiple types of maintenance actions.

$$\mathbf{Min} \ \sum_{t \in T} \frac{1}{(1+r)^t} (PC^t + CC^t), \tag{5a}$$

$$\mathbf{Min} \ \max_{t \in T} \{SAIFI^t\}, \tag{5b}$$

s.t.

$$\sum_{k \in K} x_{ek}^t = 1, \qquad \forall e, \forall t, \tag{5c}$$

$$\lambda_e^t = \lambda_e^{t-1} \sum_{k \in K} x_{ek}^t m_{ek}, \qquad \forall e, \forall t, \tag{5d}$$

$$x_{ek}^t \in \{0, 1\}, \qquad \forall e, \forall k, \forall t, \tag{5e}$$

$$\lambda_e^t \geq 0, \qquad \forall e, \forall t, \tag{5f}$$

$$PC^t, CC^t, SAIFI^t \geq 0, \qquad \forall t. \tag{5g}$$

The objective function (5a) consists of minimizing the present value of the total maintenance cost. The objective function (5b) consists of minimizing the maximum $SAIFI^t$ along the planning horizon. Note that it is a conservative metric as it seeks to minimize the worst SAIFI over the planning horizon and not just its total sum or average. Constraints (5c) ensure that each equipment e receives one type of maintenance action in each period, which may be the absence of maintenance. Constraints (5d) represent the change in each equipment's failure rate with the maintenance actions in each period. Constraints (5e) to (5g) define the domain of the variables.

3.2. Reformulating the model

We formulate the MRAP for a single distribution network in model (5) as a non-convex model. This model can be reformulated into an equivalent mixed-integer linear model, which allows employing general-purpose ILP solvers, such as CPLEX and Gurobi. For instance, non-linear constraints (5b) can be reformulated by adding the auxiliary variable $SAIFI$ and constraints (6).

$$\textbf{Min} \quad SAIFI, \quad \textbf{s.t.} \; SAIFI \geq SAIFI^t, \qquad \forall t. \tag{6}$$

The non-linear constraints (5d) can be replaced by equivalent linear expressions. Let $\delta^t_{ek} = \lambda^{t-1}_e x^t_{ek}$, $\forall e, \forall t, \forall k$. Thus, variable δ^t_{ek} assumes the value of λ^{t-1}_e when $x^t_{ek} = 1$; otherwise $\delta^t_{ek} = 0$ as $x^t_{ek} = 0$. Let $\underline{\lambda^t_e}$ and $\overline{\lambda^t_e}$ be lower and upper bounds for variable λ^t_e, respectively. Constraints (7) are equivalent to non-linear constraints (5d) (Williams, 1999).

$$\underline{\lambda^{t-1}_e} x^t_{ek} \leq \delta^t_{ek} \leq \overline{\lambda^{t-1}_e} x^t_{ek}, \qquad \forall e, \forall t, \forall k, \tag{7a}$$

$$\lambda^{t-1}_e - \overline{\lambda^{t-1}_e}(1 - x^t_{ek}) \leq \delta^t_{ek} \leq \lambda^{t-1}_e, \qquad \forall e, \forall t, \forall k. \tag{7b}$$

Using these equivalences, the MRAP can be reformulated as a mono-objective integer linear programming model, given by Model (8), where the reliability objective (6) is replaced by an ϵ-constraint approach (Ehrgott, 2005).

$$\textbf{Min} \sum_{t \in T} \frac{1}{(1+r)^t}(PC^t + CC^t), \tag{8a}$$

s.t.

$$SAIFI \leq \epsilon, \tag{8b}$$

$$SAIFI \geq SAIFI^t, \qquad \forall t, \tag{8c}$$

$$\sum_{k \in K} x^t_{ek} = 1, \qquad \forall e, \forall t, \tag{8d}$$

$$\lambda^t_e = \sum_{k \in K} \delta^t_{ek} m_{ek}, \qquad \forall e, \forall t, \tag{8e}$$

$$\underline{\lambda^{t-1}_e} x^t_{ek} \leq \delta^t_{ek} \leq \overline{\lambda^{t-1}_e} x^t_{ek}, \qquad \forall e, \forall t, \forall k, \tag{8f}$$

$$\lambda^{t-1}_e - \overline{\lambda^{t-1}_e}(1 - x^t_{ek}) \leq \delta^t_{ek} \leq \lambda^{t-1}_e, \qquad \forall e, \forall t, \forall k, \tag{8g}$$

$$x^t_{ek} \in \{0, 1\}, \qquad \forall e, \forall t, \forall k, \tag{8h}$$

$$\lambda^t_e \geq 0, \delta^t_{ek} \geq 0, \qquad \forall e, \forall t, \forall k, \tag{8i}$$

$$SAIFI, PC^t, CC^t, SAIFI^t \geq 0, \qquad \forall t. \tag{8j}$$

The objective function (8a) consists of minimizing the total cost of maintenance in net present value. Constraints (8b) and (8c) determine the SAIFI while preventing it to become above scalar ϵ. Constraints (8d) ensure each equipment e receives one maintenance action in each period, which may be the absence of maintenance. Constraints (8e) express the failure rate fluctuation over the planning horizon. Constraints (8f) and (8g), as discussed previously, ensure the correct evaluation of the failure rates. Constraints (8h) to (8j) define the domain of the variables. The bounds $\underline{\lambda^t_e}$ and $\overline{\lambda^t_e}$ can be estimated as the best and the worst failure rates each equipment e can assume in period t, respectively. For example, $\overline{\lambda^t_e}$

can be obtained by applying t times the multiplier related to no maintenance on the initial failure rate λ_e^0.

Constraining the reliability metric in Model (8) is optional. The alternative of minimizing the SAIFI index while constraining the total maintenance costs could also be adopted (Aravinthan & Jewell, 2013). Using a general-purpose ILP solver, the efficient solutions of the Pareto frontier can be obtained by solving Model (8) in an iterative manner. Assuming $\mathcal{P}$ efficient solutions, each execution has to apply a different scalar ϵ according to Eq. (9), where the bounds $\underline{S}$ and $\overline{S}$ are the best and the worst values of SAIFI allowed for the network, respectively.

$$\epsilon = \underline{S} + \frac{(\overline{S} - \underline{S})(p-1)}{\mathcal{P} - 1}, \qquad p = 1, \ldots, \mathcal{P}. \tag{9}$$

3.3. Other reliability indices

The previous formulations for the MRAP assumed the SAIFI as the main reliability index. This decision was guided by the requirements of a research project with CPFL[1], since maintenance actions have prominent effects on the frequency of failure events. Still, these effects can also reverberate on the duration of failure events and even on the amount of energy not supplied. The assessment of these and other reliability measures can be accommodated by MRAP, provided the necessary information is at hand. For example, one can include constraint (10) in the model to determine the energy not supplied in period t, knowing the power demand p_s and average interruption time t_s for each section s. Another alternative is puttin constraint (11) in the objective function to consider the customer interruption cost, knowing the average customer cost for each interruption IC.

$$ENS^t = \sum_{s \in S} (p_s t_s \sum_{e \in E_s} \lambda_e^t), \qquad \forall t. \tag{10}$$

$$CIC^t = IC \sum_{s \in S} (N_s \sum_{e \in E_s} \lambda_e^t), \qquad \forall t. \tag{11}$$

3.4. An illustrative example

In this section, we present a small network to illustrate the MRAP for a single distribution network. It is the reference network, originally considered in Usberti et al. (2015), which is composed of 7 sections (3 primary sections and 4 secondary sections), 34 pieces of equipment divided into 10 types of equipment, and 5,200 clients. For this scenario, we assumed two types of maintenance actions (absence and presence of maintenance). Using CPLEX as ILP solver, Model (8) provided 50 maintenance plans for the reference network considering a planning horizon of three periods/years with scalar ϵ varying in the interval [1.493208, 3.957748] according to Eq. (9). Each of these maintenance plans corresponds to a solution for the MRAP. Fig. 2 shows the Pareto Optimal frontier for this network, as the optimality was proven for all these solutions.

[1]Acronym from Power and Light Company of São Paulo (in Portuguese), the second-largest non-state-owned electric energy holding company in Brazil.

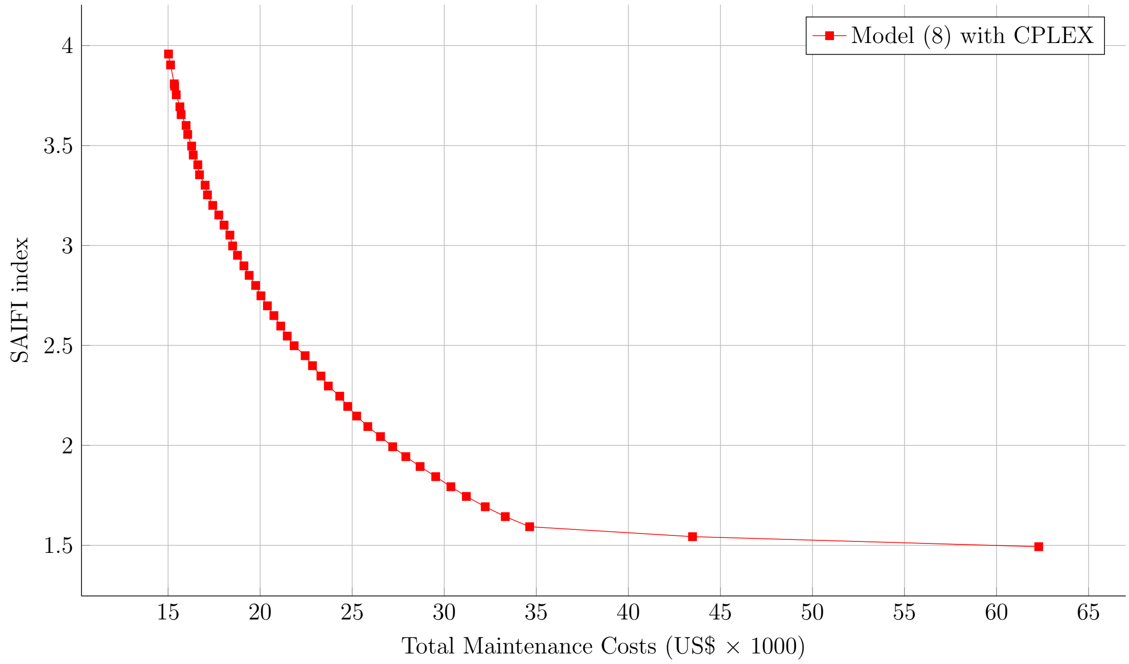


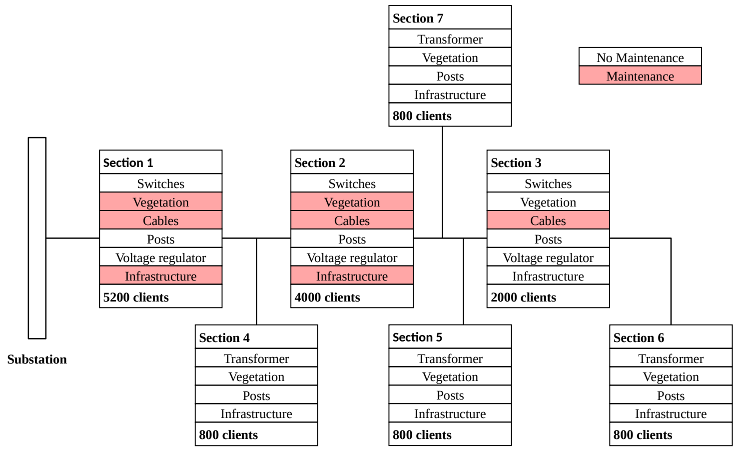
Figure 2: Pareto curve with 50 maintenance plans for the reference network.

We note that total maintenance costs are reported in Fig. 2, while Usberti et al. (2015) reported only preventive maintenance costs for the reference network. As expected, since optimality was achieved in all these solutions, the Pareto frontier of Fig. 2 dominates the Pareto frontier of Usberti et al. (2015). In Fig. 3, we report the maintenance plan corresponding to the solution of $z = (28.688366, 1.89558)$. As in Usberti et al. (2015), these figures display the sections with their equipment and the number of clients served.

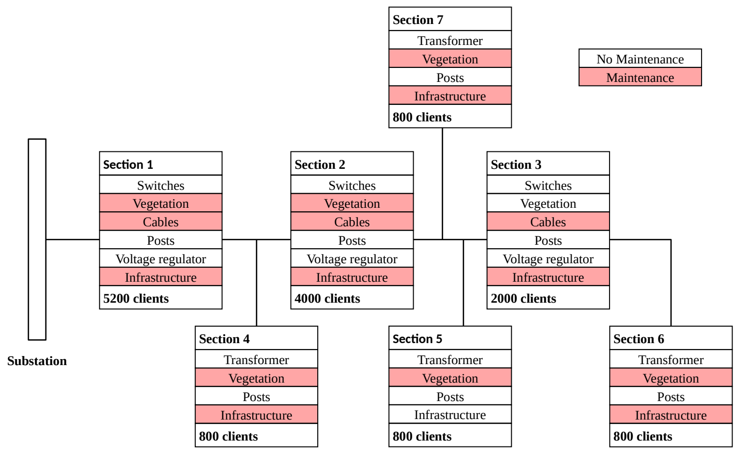
4. MRAP for several distribution networks: a heuristic approach

The methodology discussed in the previous section solves the MRAP of each distribution network locally circumscribed within the utility's concession area, i.e., the local-level solutions provide the respective optimal trade-off curves. Now consider the MRAP for several distribution networks, that is, the upper-level problem that examines the optimal allocation of maintenance resources for an electric group embodying several networks, or even the utility concession area as a whole. Model (8) can be employed to optimally solve the upper-level problem by setting the model's input as the union of the inputs for all individual networks. However, this process can result in a sizable instance possibly intractable for most exact approaches.

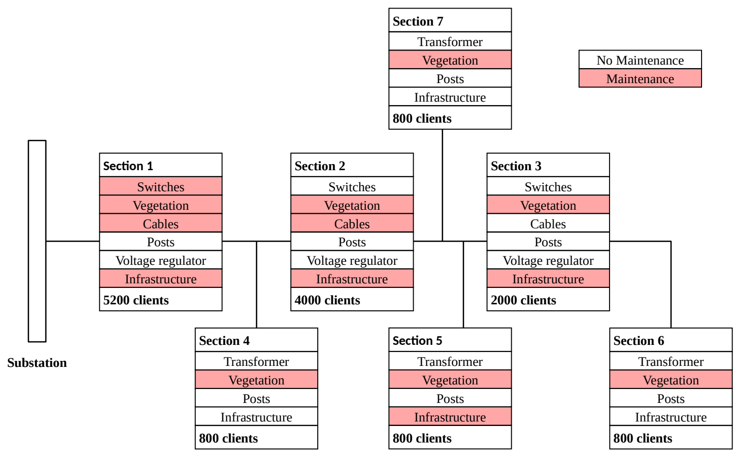
An alternative heuristic strategy is based on a two-level decomposition hierarchy, as illustrated in Fig. 4. This framework, originated in Usberti et al. (2015), was built upon the property that upper-level solutions have an optimal substructure: each optimal upper-level solution is composed of optimal local-level solutions. The decomposition approach is to solve heuristically or exactly the local problems and aggregate in a single shot their



(a) First year.



(b) Second year.



(c) Third year.

Figure 3: A maintenance plan of three years for the reference network.

solutions to obtain the upper-level decisions. The hierarchy of maintenance decisions has a managerial and a computational rationale. The managerial rationale is that the approach mirrors the organization and the flow of information and decisions in distribution utilities. The local teams in different geographical areas define and execute the best trade-offs between investments and benefits for their jurisdictions. Officers at a higher level decide on the allocation of budgets for each network to achieve the best decisions for the whole group of networks. The computational rationale comes from the implicit *divide and conquer* strategy by which the effort to solve the MRAP with an approach that allows dividing it into a set of smaller problems can keep the computational requirements within tractable limits.

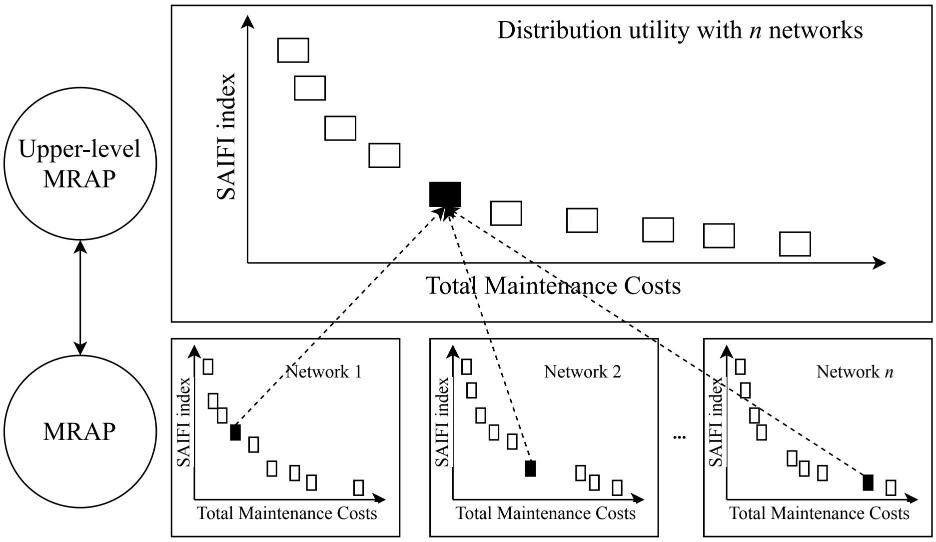


Figure 4: Hierarchical relationship between the two levels of the MRAP.

4.1. An ILP formulation for upper-level trade-offs

The following assumptions set the environment of the MRAP for several distribution networks:

1. there are two conflicting objectives, namely the total maintenance costs (to be minimized) and the network reliability (to be maximized) measured by the SAIFI index;

2. a set of maintenance plans is provided as input for each distribution network.

We define the following notation concerning sets, parameters, and decision variables to used when presenting the model:

Sets:

R set of distribution networks in the electric group.

P_r set of maintenance plans for distribution network $r \in R$.

Parameters:

MC_r^p cost of maintenance plan $p \in P_r$ for distribution network $r \in R$.

$SAIFI_r^p$ reliability index of distribution network $r \in R$ when maintenance plan $p \in P_r$ is applied.

NT_r number of clients served by distribution network $r \in R$.

Binary variables:

y_r^p distribution network $r \in R$ receives maintenance plan $p \in P_r$.

The inputs of the the upper-level MRAP are obtained after solving the local-level MRAP for each distribution network of the distribution utility. A maintenance plan, i.e., an efficient solution $p \in P_r$ of distribution network $r \in R$ is represented by its values of maintenance cost MC_r^p and system reliability metric $SAIFI_r^p$. The binary variables y_r^p are used to aggregate local-level solutions into upper-level decisions, in which y_r^p is equal to 1 if maintenance plan p is selected for distribution network r, and 0 otherwise. We formulate the upper-level MRAP as an ILP formulation, given by Model (12). As before, the reliability objective is replaced by an ϵ-constraint approach Ehrgott (2005) – of course, any of the objectives could be ϵ-constrained. Solving this model for a meaningful set of ϵ values according to Eq. (9) unveils the Pareto frontier; the bounds $\underline{S}$ and $\overline{S}$ are, respectively, now the best and worst allowed values for the SAIFI of the electric group.

$$\textbf{Min} \sum_{r\in R}\sum_{p\in P_r} MC_r^p \, y_r^p, \tag{12a}$$

$$\textbf{s.t.}$$

$$\sum_{p\in P_r} y_r^p = 1, \qquad \forall r, \tag{12b}$$

$$\frac{\sum_{r\in R} NT_r \sum_{p\in P_r} SAIFI_r^p}{\sum_{r\in R} NT_r} y_r^p \leq \epsilon, \tag{12c}$$

$$y_r^p \in \{0,1\}, \qquad \forall p, \forall r. \tag{12d}$$

The objective function (12a) consists of minimizing the total cost of maintenance. Constraints (12b) ensure a maintenance plan for each local distribution network. Constraint (12c) determines the SAIFI of the utility while preventing it from becoming above a scalar ϵ. Constraints (12d) define the domain of the variables.

4.2. On the Accuracy of the Decomposition Approach

A word of caution concerns the precision of the upper-level trade-off curve. Given that the decomposition approach considers a subset of non-dominated solutions for each network, the Pareto optimal solutions' accuracy depends on the number of non-dominated solutions provided for the local networks. Notice that accuracy improves and processing times increase as the number of non-dominated solutions per network increases. The computational experiments in Sec. 5 provide information to elucidate this issue.

5. Computational experiments

The computational experiments consider two sets of problem instances to evaluate the proposed ILP models in terms of the quality of solution and processing time. All data comes from a maintenance budget allocation project with a Brazilian distribution utility. The first set of instances consider three distribution networks from cities in São Paulo (Brazil). The second set of instances consider three even larger instances. Models (8) and (12) were coded in C++ using the Concert Studio library on top of IBM CPLEX Optimization Studio v20.1. For benchmarking purposes, we developed a BRKGA algorithm for the MRAP for a single distribution network (Appendix), inspired by the genetic algorithm proposed in Usberti et al. (2015). The BRKGA metaheuristic brings the latest developments in evolutionary algorithms and has successfully addressed other complex problems, such as cutting and packing, vehicle routing, and network design (Gonçalves & Resende, 2011). Detailed information on the algorithm, its source code in C++, and benchmark instances are available at `http://www.ic.unicamp.br/~fusberti/problems/mrap/`.

All experiments were carried out on a PC with Intel Xeon E5-2680v2 (2.8 GHz), 16 GB of RAM, under CentOS Linux 7.2.1511 Operating System. They considered two maintenance actions (absence of maintenance and a standard preventive maintenance), a planning horizon of three periods, and an interest rate of $r = 12\%$. All runs had a time limit of 3,600 seconds, including the experiments with the BRKGA; the acronym "tl" in Table 4 indicates the processing interruption due to the time limit.

5.1. Results for the MRAP of a single distribution network

We report in Table 1 the main characteristics of the first set of instances. Both Model (8) with CPLEX and BRKGA provided 50 maintenance plans for each of these networks, with scalar ϵ varying in the interval [Worst SAIFI, Best SAIFI], according to Eq. (9). We illustrate in Fig. 5 these maintenance plans. Notice that the maintenance plans' costs significantly increase as the scalar ϵ gets closer to the best SAIFI value.

The optimality was proven by the solver with Model (8) in all 150 ($= 3 \times 50$) maintenance plans. The results of the BRKGA metaheuristic are also competitive concerning the quality of the solutions. However, the average improvement of the exact approach over the BRKGA approach was 3.20%; the average processing time for each plan was 27.62 seconds for Model (8) with CPLEX, and 150.92 seconds for the BRKGA metaheuristic. Table 2 details some of these results for different values of the scalar ϵ, highlighting the optimal solutions in bold. For instance, it shows that the exact approach's improvement upon the BRKGA is 17.63% in the maintenance plan with $\epsilon = 0.4555$ for Network 2.

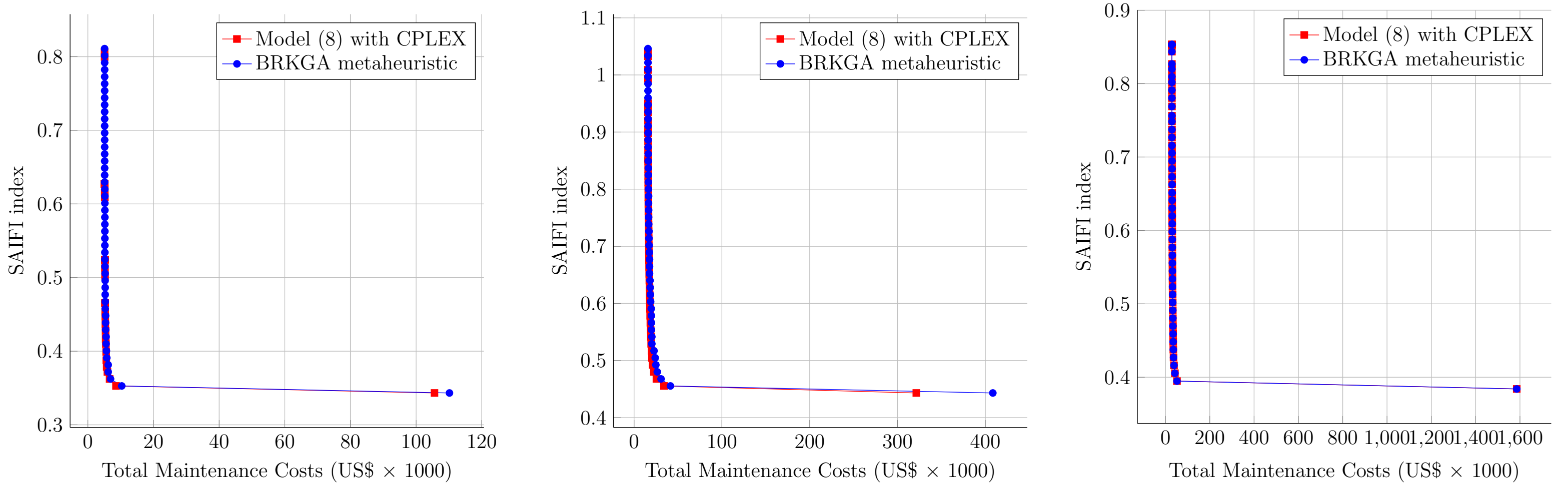
Table 1: Distribution Networks 1, 2 and 3.

	Network 1	Network 2	Network 3
Sections	166	447	820
Equipment	716	2,061	3,488
Equipment types	10	10	10
Served clients	4,513	18,268	25,466
Best SAIFI	0.3432	0.4432	0.3840
Worst SAIFI	0.8109	1.0461	0.9091

Table 2: Comparison of the proposed and benchmark approaches for distribution Networks 1, 2, and 3.

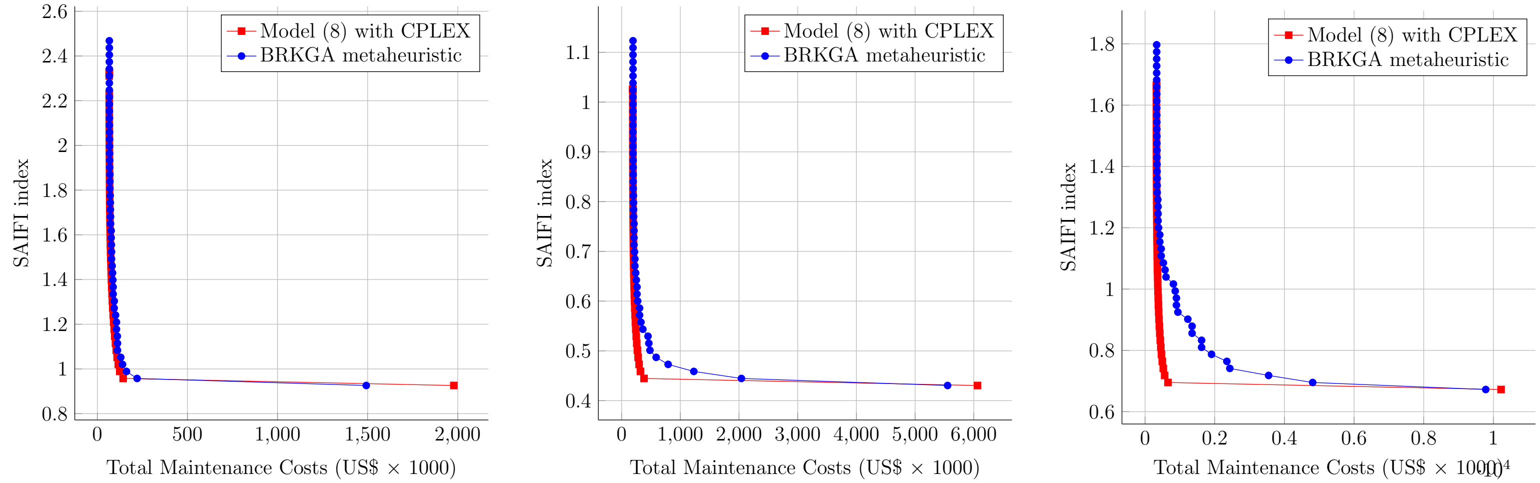
	Model (8)		BRKGA		
Scalar ϵ	Costs*	Time[s]	Costs*	Time[s]	Improv.[%]
Network 1					
0.3527	**8.58**	0.77	10.32	50.07	16.88
0.4864	**5.26**	0.46	5.27	0.66	0.25
0.6295	**5.12**	0.30	**5.12**	4.85	0.00
0.8014	**5.11**	0.38	5.12	0.35	0.23
Network 2					
0.4555	**35.09**	13.90	41.39	837.10	17.63
0.6277	**17.55**	4.62	18.35	34.55	4.35
0.8123	**16.13**	2.30	16.26	12.49	0.79
1.0337	**15.82**	20.07	15.83	0.70	0.06
Network 3					
0.3947	**51.93**	21.57	84.00	2524.06	38.18
0.5447	**31.61**	81.06	32.97	163.10	4.11
0.7055	**29.52**	81.38	30.10	61.89	1.90
0.8983	**28.99**	3.95	29.29	0.95	1.02

*in US$ $\times$ 1000



(a) Network 1 (b) Network 2 (c) Network 3

Figure 5: Pareto curves with 50 maintenance plans for distribution networks 1, 2, and 3.



(a) Network 4 (b) Network 5 (c) Network 6

Figure 6: Pareto curves with 50 maintenance plans for large-scale distribution networks 4, 5, and 6.

5.2. *Robustness analysis*

This section describes the experiments designed to stress the methodologies with solutions of the MRAP for very large-scale distribution networks while assessing their robustness in obtaining high-quality solutions. It evaluates solutions for three instances, ranging from 1000 sections and 4499 equipment up to 5000 sections and 22449 equipment. Table 3 shows their main characteristics. The data for each section (cable lengths, number of clients, number of equipment, and the failure rate of each equipment) come from distribution networks in São Paulo (Brazil).

Table 3: Large-scale distribution networks 4, 5, and 6.

	Network 4	Network 5	Network 6
Sections	1000	3000	5000
Equipment	4,499	13,483	22,449
Equipment types	10	10	10
Clients	7,124	48,247	48,247
Best SAIFI	0.9259	0.4304	0.6723
Worst SAIFI	2.4680	1.1231	1.7967

Model (8) with CPLEX and BRKGA unveiled 50 maintenance plans for each of these networks with scalar ϵ varying in the range [Worst SAIFI, Best SAIFI], according to Eq. (9). These maintenance plans are depicted in Fig. 6 to illustrate the Pareto-optimal trade-off solutions. Model (8) reached optimality with CPLEX in 31 maintenance plans out of 50 for Network 4, in 19 out of 50 for Network 5, and in 10 maintenance plans out of 50 or Network 6. The results with BRKGA were not competitive for these cases, especially when the scalar ϵ gets closer to the best SAIFI value. The average improvement of the exact solutions with CPLEX over the solutions with BRKGA was 15.84%; the average processing time for each plan with CPLEX was 2,293.59 seconds, and 2,382.67 with the BRKGA. Table 4 details some of these results for different values of the scalar ϵ; taking, for instance, the maintenance plans for Network 6 with $\epsilon = 0.6953$, the improvement of solution with CPLEX over the solution with BRKGA is 81.29%.

The results for Model (8) with CPLEX outperform the results with BRKGA, both in the quality of solution and processing time. Fis. 5 and 6 also show that there is a small variation in the costs of the maintenance plans for a significant part of the SAIFI interval. It is worth noting that solutions of the MRAP are likely to disclose different equally reliable maintenance strategies; indeed, similar maintenance actions at different periods can minimize the objective function, which measures the maximum SAIFI.

Another aspect that calls for attention concerning solutions for the MRAP is that most of the maintenance actions are on the equipment closer to the substations. This result should be expected for networks with a radial configuration because faults closer to the substations interrupt the energy supply to a larger number of clients.

Table 4: Comparison of the proposed approach for large-scale distribution networks 4, 5 and 6.

	Model (8)		BRKGA		
Scalar ϵ	Costs*	Time[s]	Costs*	Time[s]	Improv.[%]
Network 4					
0.9573	143.18	tl	220.55	tl	35.08
1.3979	77.99	tl	85.76	1,038.49	9.06
1.8700	**67.30**	276.35	69.53	481.70	3.20
2.4365	**65.82**	150.80	66.47	146.79	0.98
Network 5					
0.4446	381.74	tl	2,040.97	tl	81.29
0.6425	209.76	tl	250.16	tl	16.14
0.8545	192.79	tl	197.46	1,895.31	2.36
1.1090	**190.76**	92.50	193.91	750.04	1.62
Network 6					
0.6953	658.29	tl	4,811.67	tl	86.31
1.0165	364.50	tl	812.93	tl	55.16
1.3607	329.92	tl	347.40	tl	5.03
1.7738	**325.23**	137.72	332.59	2,214.39	2.21

*in US$ $\times$ 1000

5.3. Appraising the hierarchical approach of the MRAP for several distribution networks

In this section, we aim at evaluating the accuracy and processing time of the two-level heuristic approach presented in Section 4. We assumed that optimal trade-offs curves for maintenance budget decisions are required for an area comprising Networks 1, 2, and 3. In other words, Model (8) provides the local optimal trade-offs curves for each of the Networks 1, 2 and 3, which are inputs for Model (12) to generate upper-level trade-offs.

As observed in Section 4.2, the upper-level Pareto optimal solutions' accuracy depends on the number of non-dominated solutions provided for the local networks. As mentioned before, accuracy improves and processing time increase as the number of non-dominated solutions per network increases. The first experiment considers an extreme scenario where only 2 non-dominated solutions are given by Model (8) with CPLEX. Thus, 8 ($= 2^3$) possible compositions of local maintenance plans are evaluated with Model (12) using CPLEX to unveil the upper-level maintenance trade-offs curves. The experiment is repeated other three times with 4, 8 and 16 non-dominated solutions for each one of the Networks 1, 2 and 3, given by Model (8) with CPLEX. Therefore, Model (12) evaluates, respectively, 64 ($= 4^3$), 512 ($= 8^3$) and 4096 ($= 16^3$) compositions of local maintenance plans to unveil the upper-level maintenance trade-offs curves, as represented in Fig. 7. The average processing time to obtain each local solution for Network 1, 2, and 3 is, of course, the same $27,62$ seconds already mentioned for the results of Fig. 5. The average processing time to obtain each upper-level non-dominated solution (i.e., each point in the trade-off curve) is just about 1 second.

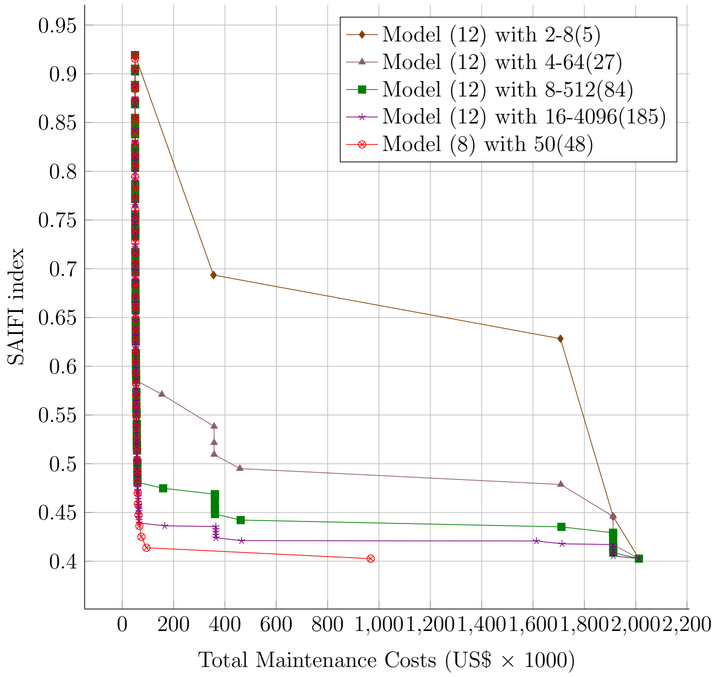


Figure 7: Illustration of the benefits of the hierarchical approach.

An additional experiment provides a yardstick for evaluating the accuracy of the upper-level maintenance trade-offs curves. This experiment unveils the upper-level optimal trade-offs curves for maintenance actions with Model (8), considering the whole network comprised by the Networks 1, 2 and 3, as discussed in the first paragraph of Secton 4. Model (8) runs 50 times with CPLEX, one for each different ϵ value, to obtain 50 efficient solutions and the associated Pareto upper-level trade-offs. The average processing time to achieve each solution is 172.66 seconds. Fig. 7 also depicts these solutions.

The analysis of the results in Fig. 7 shows that the accuracy of the upper-level optimal trade-offs curves for maintenance actions provided by the hierarchical approach significantly improves as the number of local non-dominated solutions given by Model (8) for each network increases. In contrast, the total processing time to obtain accurate upper-level optimal trade-offs curves with the hierarchical approach is negligible, compared to the processing times required by Model (8) considering the whole network comprised by the local networks. Therefore, the hierarchical strategy with Model (8) and Model (12) gives a flexible alternative to address the MRAP for a set of large networks.

6. Conclusions

We addressed the Maintenance Resources Allocation Problem (MRAP) that schedules preventive maintenance plans on the equipment of distribution networks for a planning horizon, seeking the best trade-offs between system reliability and maintenance budgets. We first proposed an integer linear programming (ILP) formulation capable of completely modeling the MRAP for a single distribution network. The methodology uses the recent developments in the estimation of parameters based on historical data (Sharifinia et al., 2020). The approach unveils optimal trade-offs between investments in maintenance actions and improvements in reliability for each distribution network locally circumscribed within the utility's concession area. Then we developed an additional ILP formulation to heuristically solve the MRAP for several distributions networks with a hierarchical structure, giving the (near-)optimal trade-off for the utility as a whole. Using a general-purpose ILP solver, the computational experiments with real large-scale networks showed that the approaches can provide optimal budget versus reliability trade-off curves within short processing times.

Future research should explore the flexibility of the proposed methodology with additional decisions, such as other reliability indices, network operation typologies, and energy sources. Other related approach could examine the consideration of uncertainty in the parameters, for example, in the impact of each maintenance action on the failure rate of the equipment. It is worth noticing that the methodology can be generalized to non-electrical systems, especially on systems with spatially distributed equipment, such as water supply systems and irrigation equipment for farming.

Acknowledgments

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References

Abiri-Jahromi, A., Fotuhi-Firuzabad, M., & Abbasi, E. (2009). An efficient mixed-integer linear formulation for long-term overhead lines maintenance scheduling in power distribution systems. *IEEE Transactions on Power Delivery*, *24*, 2043–2053. doi:10.1109/TPWRD.2009.2028785.

Ahuja, R. K., Magnanti, T. L., & Orlin, J. B. (1993). *Network Flows: Theory, Algorithms, and Applications*. Upper Saddle River, NJ, USA: Prentice-Hall, Inc.

Aravinthan, V., & Jewell, W. (2013). Optimized maintenance scheduling for budget-constrained distribution utility. *IEEE Transactions on Smart Grid*, *4*, 2328–2338. doi:10.1109/TSG.2013.2271616.

Ardabili, H. A. R., Haghifam, M.-R., & Abedi, S. M. (2021). A probabilistic reliability-centred maintenance approach for electrical distribution networks. *IET Generation, Transmission & Distribution*, *15*, 1070–1080. doi:https://doi.org/10.1049/gtd2.12081.

Arya, L. D., Choube, S. C., & Arya, R. (2011). Differential evolution applied for reliability optimization of radial distribution systems. *Electrical Power and Energy Systems*, *33*, 271–277.

Assis, L. S., González, J. F. V., Usberti, F. L., Lyra, C., Cavellucci, C., & Zuben, F. J. V. (2015). Switch allocation problems in power distribution systems. *IEEE Transactions on Power Systems*, *30*, 246–253. doi:10.1109/TPWRS.2014.2322811.

Basciftci, B., Ahmed, S., Gebraeel, N. Z., & Yildirim, M. (2018). Stochastic optimization of maintenance and operations schedules under unexpected failures. *IEEE Transactions on Power Systems*, *33*, 6755–6765. doi:10.1109/TPWRS.2018.2829175.

Billinton, R., & Allan, R. N. (1996). *Reliability evaluation of power systems*. New York: Plenum Press.

Billinton, R., & Billinton, J. E. (1989). Distribution system reliability indices. *IEEE Transactions on Power Delivery*, *4*, 1670–1676.

Brown, R. E. (2009). *Electric power distribution reliability*. New York: CRC Press.

Doumpos, M., Papastamos, D., Andritsos, D., & Zopounidis, C. (2021). Developing automated valuation models for estimating property values: a comparison of global and locally weighted approaches. *Annals of Operations Research*, *306*, 415–433. doi:10.1007/s10479-020-03556-1.

Ehrgott, M. (2005). *Multicriteria optimization* volume 491. Springer Science & Business Media.

Endrenyi, J., Aboresheid, S., Allan, R. N., Anders, G. J., Asgarpoor, S., Billinton, R., Chowdhury, N., Dialynas, E. N., Fipper, M., Fletcher, R. H., Grigg, C., McCalley, J., Meliopoulos, S., Mielnik, T. C., Nitu, P., Rau, N., Reppen, N. D., Salvaderi, L., Schneider, A., & Singh, C. (2001). The present status of maintenance strategies and the impact of maintenance on reliability. *IEEE Transactions on Power Systems*, *16*, 638–646. doi:10.1109/59.962408.

Endrenyi, J., Anders, G. J., & da Silva, A. M. L. (1998). Probabilistic evaluation of the effect of maintenance on reliability: an application. *IEEE Transactions on Power Systems*, *13*, 576–583.

Froger, A., Gendreau, M., Mendoza, J. E., ric Pinson, & Rousseau, L.-M. (2016). Maintenance scheduling in the electricity industry: A literature review. *European Journal of Operational Research*, *251*, 695 – 706. doi:https://doi.org/10.1016/j.ejor.2015.08.045.

Gonçalves, J. F., & Resende, M. G. C. (2011). Biased random-key genetic algorithms for combinatorial optimization. *Journal of Heuristics*, *17*, 487–525.

Hilber, P., Miranda, V., Mator, M. A., & Bertling, L. (2007). Multiobjective optimization applied to maintenance policy for electrical networks. *IEEE Transactions on Power Systems*, *22*, 1675–1682.

Kuntz, P. A., Christie, R. D., & Venkata, S. S. (2002). Optimal vegetation maintenance scheduling of overhead electric power distribution systems. *IEEE Transactions on Power Delivery*, *17*, 1164–1169.

Laksman, E., Strömberg, A.-B., & Patriksson, M. (2020). The stochastic opportunistic replacement problem, part III: improved bounding procedures. *Annals of Operations Research*, *292*, 711–733. doi:10.1007/s10479-019-03278-z.

Mirsaeedi, H., Fereidunian, A., Mohammadi-Hosseininejad, S. M., Dehghanian, P., & Lesani, H. (2018). Long-term maintenance scheduling and budgeting in electricity distribution systems equipped with automatic switches. *IEEE Transactions on Industrial Informatics*, *14*, 1909–1919. doi:10.1109/TII.2017.2772090.

Misari, A. R., Leite, J. B., Piasson, D., & Mantovani, J. R. S. (2020). Reliability-centered maintenance task planning for overhead electric power distribution networks. *Journal of Control, Automation and Electrical Systems*, *31*, 12781287.

Moon, J. F., Yoon, Y. T., Lee, S. S., Kim, J. C., Lee, H. T., & Park, G. P. (2006). Reliability-centered maintenance model to managing power distribution system equipment. In *Power Engineering Society General Meeting, 2006. IEEE*.

Moradi, S., Vahidinasab, V., Kia, M., & Dehghanian, P. (2019). A mathematical framework for reliability-centered maintenance in microgrids. *International Transactions on Electrical Energy Systems*, *29*, e2691.

Petchrompo, S., & Parlikad, A. K. (2019). A review of asset management literature on multi-asset systems. *Reliability Engineering & System Safety*, *181*, 181 – 201. doi:https://doi.org/10.1016/j.ress.2018.09.009.

Pham, H., & Wang, H. (1996). Imperfect maintenance. *European Journal of Operational Research*, *94*, 425–438. doi:10.1016/S0377-2217(96)00099-9.

Radmer, D. T., Kuntz, P. A., Christie, R. D., Venkata, S. S., & Fletcher, R. H. (2002). Predicting vegetation-related failure rates for overhead distribution feeders. *IEEE Transactions on Power Delivery*, *17*, 1170–1175.

Rafiei, M., Khooban, M., Igder, M. A., & Boudjadar, J. (2020). A novel approach to overcome the limitations of reliability centered maintenance implementation on the smart grid distance protection system. *IEEE Transactions on Circuits and Systems II: Express Briefs*, *67*, 320–324.

Salari, N., & Makis, V. (2020). Joint maintenance and just-in-time spare parts provisioning policy

for a multi-unit production system. *Annals of Operations Research*, *287*, 351–377. doi:10.1007/s10479-019-03371-3.
Shang, Y., Wu, W., Liao, J., Guo, J., Su, J., Liu, W., & Huang, Y. (2020). Stochastic maintenance schedules of active distribution networks based on monte-carlo tree search. *IEEE Transactions on Power Systems*, *35*, 3940–3952. doi:10.1109/TPWRS.2020.2973761.
Sharifinia, S., Rastegar, M., Allahbakhshi, M., & Fotuhi-Firuzabad, M. (2020). Inverse reliability evaluation in power distribution systems. *IEEE Transactions on Power Systems*, *35*, 818–820.
Sittithumwat, A., Soudi, F., & Tomsovic, K. (2004). Optimal allocation of ditribution maintenance resources with limited information. *Eletric Power Systems Research*, *68*, 208–220.
Usberti, F. L., Lyra, C., Cavellucci, C., & González, J. F. V. (2015). Hierarchical multiple criteria optimization of maintenance activities on power distribution networks. *Annals of Operations Research*, *224*, 171–192. doi:10.1007/s10479-012-1182-6.
Wang, J., Ye, J., & Wang, L. (2019). Extended age maintenance models and its optimization for series and parallel systems. *Annals of Operations Research*, . doi:10.1007/s10479-019-03355-3.
Williams, H. P. (1999). *Model building in mathematical programming*. (4th ed.). Chichester, England: John Wiley & Sons.

Appendix. A Biased Random-Key Genetic Algorithm for the MRAP

The BRKGA proposed in Gonçalves & Resende (2011) is an evolutionary metaheuristic in which a population of individuals evolves through the Darwinian principle of the survival of the fittest. Each individual of the population is represented by a chromosome Q, encoded as a vector with m alleles. An allele is a random key uniformly drawn over the interval $[0, 1]$. A chromosome Q is mapped into a solution R through a decoder algorithm, which is the problem-specific component of the metaheuristic.

The BRKGA starts by generating an initial population with p random chromosomes. Throughout g generations, the population goes through a selective pressure environment in which the fittest individuals are more likely to endure and produce offspring. A distinguished feature of the BRKGA evolutionary strategy is that it partitions the population into *elite* and *non-elite* sets. As the name suggests, the elite set comprises the fittest individuals; the non-elite set contains all the remaining individuals, including the so-called *mutants*, which are randomly generated chromosomes to promote diversification in the search process.

In a nutshell, at each generation the BRKGA performs the following steps:

1. Decode each chromosome Q into its corresponding solution R;
2. Evaluate the fitness of each solution R;
3. Identify the elite set (*i.e.*, the best p_e individuals);
4. Copy the elite set to the next generation;
5. Include p_m new random chromosomes (mutants) in the next generation;
6. Produce $p - (p_e + p_m)$ offspring using crossover operators, and insert them in the next generation.

The crossover generates a new individual by sampling each allele from one of its parents. Both parents are from the current generation, one from the elite set and the other from the non-elite set. An allele comes from the elite parent with probability ρ_e. After g generations, the BRKGA returns the best chromosome Q^* and its decoded solution R^*. The following items describe the problem-specific aspects of using BRKGA to tackle the local-level MRAP.

Chromosome. Each allele of a chromosome Q corresponds to a pair (e, t) of equipment e and period t with key $Q(e, t)$.

Decoder. Algorithm 1 details the BRKGA decoder for solving the MRAP of a single distribution network. It receives as input a chromosome Q and returns the fitness of its corresponding solution R. The decoding process starts with an initial empty solution, meaning no maintenance on any equipment in any period. Then, in decreasing order of the chromosome key values $Q(e^*, t^*)$, the maintenance of equipment e^* in period t^* is included in the solution until it becomes feasible with respect to the SAIFI target.

Algorithm 1: BRKGA decoder for the MRAP of a single distribution network.

Input: chromosome Q
Output: fitness of a corresponding solution R;
$R \leftarrow \emptyset$;
while $\max_{t \in T}\{SAIFI^t(R)\} > \epsilon$ **do**
  $(e^*, t^*) \leftarrow \arg\max_{e \in E, t \in T} \{Q(e, t)\}$;
  $R \leftarrow R \cup (e^*, t^*)$;
  $Q(e^*, t^*) \leftarrow -1$;
return $fitness(R)$

Fitness. The fitness of a solution is given by the sum of preventive and corrective costs, in present value, calculated according to Eq. (8a).

Table A.5 gives the BRKGA parameters for the experiments in Section 5.

Table A.5: BRKGA parameters.

Parameter	Value
Population size	75
Elite set size	15
Mutant set size	7
Prob. of an offspring inheriting from elite	70 %