EVALUATION OF POLITICAL REDISTRICTING IN JAPAN BY OPTIMIZATION AND ENUMERATION

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Abstract The political/electoral districting problem for the single-seat constituency system is a problem of decomposing a graph into connected components of a given number of seats under several conditions and objectives. We evaluate and analyze the current division of single-seat constituencies for the House of Representatives using optimization and enumeration. The objective function is to minimize the maximum one-vote difference of disparities. Showing the limit value by optimization, the room for improvement of the current district becomes clear. We also support and evaluate the work of zoning revision by enumerating solutions and calculating various indices. As a result, there is a lot of room for improvement in the current district, and it is possible to reconfirm that better electoral districts can be presented.

Keywords: optimization, enumeration, OR practice, political districting/electoral districting, apportionment

1. Introduction

The Japanese Diet consists of two houses, the House of Representatives and the House of Councilors. Members of the House of Representatives are elected by a system that combines single-seat constituencies and proportional-seat representation, while the House of Councilors elects members by a constituency system and a proportional representation system. The electoral system is reviewed every five years in line with the national census, which is conducted in years that are multiples of 5 in the Western calendar. The House of Representatives single-seat constituency system, which began in 1994, held nine elections by 2022, all of which have been contested in court about equality of vote value. In the four elections from 1996 to 2005, the maximum difference per vote was 2.1 to 2.3 times, but the Supreme Court ruled that it was constitutional. Since then, the situation has not improved, and the 5th time in 2009 to the 7th time in 2014, when the rate was 2.1 to 2.3 times higher, was declared unconstitutional. The 8th in 2017 was 1.980 times, which was the first time it was less than 2 times, and it was considered constitutional. The ratio of the latest 9th in 2021 was 2.079. As of January 2023, it is still in dispute. Thus, in Japan's single-seat constituency system, it is strongly desired to improve the equality of voting value.

In Japan, Sakaguchi and Wada[37] were the first to point out the political necessity of finding an optimal solution rather than an approximate one for the political districting problem. This is because if we can obtain the limit values from the optimal solution, we will be able to determine whether it is unavoidable that there is a difference between the districts. In response to this, Nemoto and Hotta[28] were the first to obtain optimal solutions for all 300 single-seat constituencies of the House of Representatives based on 2000 population data, and showed the limit values. Since then, results have been shown every five years[30, 31, 14, 18, 22]. They also use their models to perform various analysis (cf. [29, 30, 31, 13, 21]). Similarly, for the evaluation of joint constituencies in the House of Councilors constituency system and the local assembly system, various optimization models were presented and results of analysis were shown[15, 17, 19, 23].

The political/electoral districting problem is a kind of territory design or region segmentation problem[36]. Since the 1960s, various methods have been proposed according to the needs of each country (cf.[6, 35, 41]). There are few exact algorithms. And most methods are heuristics (cf. Table1[35], [2, 3, 4, 5, 9, 10, 11, 25, 46]). One of the reasons is that the concept of element points when constructing an adjacency graph is different. Although the details and objectives differ from country to country, the following five major constraints are common to some extent.

1. population balance
2. contiguity
3. integrity / conformity of administrative boundary
4. compactness
5. community connection

The procedure for demarcating single-seat constituencies is as follows.

1. Allocate the total number to each administrative district in proportion to the population
2. Create electoral districts for the number of members assigned to each administrative district

Administrative districts here are 47 prefectures in the Japanese House of Representatives single-seat constituency system, 50 states in the U.S. House of Representatives, and 13 states (Land) + 3 cities (Stadtstaat) in the German Bundestag single-seat constituency system. From the point of view of the disparity of one vote, the disparity in fixed allocation occurs due to the distribution to administrative districts, and the maximum disparity is determined by partitioning each administrative district.

Each country has different requirements for conditions. Regarding condition 1, Japan is less than 2 times the maximum difference in all constituencies[1]. In the United States, after distributing the fixed number of 435 legislators to 50 states by the geometric mean divisor method (GMD), each state is made within an average of $\pm 1\%$[2]. Therefore, the maximum difference of one vote is about 1.9 times when allocating seats. Germany has all 299 constituencies within $\pm 15\%$ on average.

In Europe and the United States, there are many proposals of heuristics that emphasize the avoidance of gerrymandering and quickly search for partitions with condition 4 better than the current situation. In Japan, the boundaries of each region are complicated, and there are circumstances such as being divided by mountains and valleys. So, the requirement for the shape of the electoral district is low, and condition 4 is not important.

Japan tends to emphasize condition 5, and the constituent elements of single-seat districts are municipalities(Shi/Ku/Cho/Son) in principle[3]. Therefore, compared to the United States, Germany, etc., the components as the vertex of the graph are larger[4]. For example, the regions of Bavaria, Germany (46 constituencies) are RB(7), Kr(96), VB(1426), and Gem(2099). They want to solve in a more detailed area (cf,Table1[11]). On the other hand, for example, Kanagawa prefecture (20 electoral districts) in Japan has 50 municipalities (Shi/Ku/Gun), 58 municipalities (Shi/Ku/Cho/Son), 2133 towns (Oaza/machi), 5097 towns

[1]In the creation policy from 1994 to 2010, all constituencies were within $\pm 33.3\%$ on average.

[2]Some states are exceptions.

[3]There are some exceptions, since in practice this does not achieve the condition for population equity.

[4]Due to the severe demographic conditions in Germany, single-seat constituencies are constructed with more detailed elements.

(Aza/Chome). We have to find the solution with the 58 municipalities (Shi/Ku/Cho/Son) as the node of the graph according to Japanese law[5].

The constituent elements are relatively large, the number of members per capita is small compared to European parliaments, and the population disparity tends to be large, so it is natural to use the disparity ratio as the objective function for optimization. Therefore, we construct and solve an optimization problem that minimizes condition 1 under conditions 2, 3 and 5[28].

Since the limit value is derived without considering condition 4 and without considering connections other than administrative boundaries with condition 5, the limit value of population disparity is specified under the minimum necessary constraints. There is a political significance here that requires an optimal solution rather than an approximate one. This is because the room for improvement is clarified numerically.

As already mentioned, in Japan, Nemoto and Hotta were the first to succeed in finding the optimal solution for all constituencies in the House of Representatives single-seat constituency system[28]. They used two types of models, the set m partition type and the network flow type, and used the MIP solver. However, the solver cannot solve it as it is. Preprocessing is very important to obtain the optimal solution, and Nemoto and Hotta [28] proposed the idea of the 'second valid constituency' and created an algorithm using the solver as a subroutine. For example, in Shimane prefecture (3 electoral districts), which is the simplest example, it takes 37 hours to find the optimal solution by using the network flow type model as it is with PC around 2002 and MIP solver (cplex ver5), but if you solve it after a few seconds of preprocessing can be solved in seconds[28]. Also, in Japan, condition 1 is evaluated as a ratio, so the objective function is the ratio of the maximum population to the minimum population, which is a fractional programming problem. However, instead of binary search or Newton's method[34] using parameters, they propose a simpler method that makes good use of the structure of the problem and incorporate it into the algorithm to solve it[28]. Please refer to Hotta[20] for details of the method.

Although the optimal solution is politically significant in terms of specifying the gap limit, conditions 4 and 5 should be considered when considering the electoral districts to be used in practice, and the trade-off with condition 1 is becomes. Therefore, it is better to enumerate the solutions sorted by condition 1 and calculate and present an index that expresses the degree of conditions 4 and 5. Kawahara et al.[24] developed a frontier method-based high-speed enumeration algorithm that divides a graph into a given number, and established a method to solve a huge amount of enumeration solutions while keeping them compact in a data structure, called ZDD[27]. Hotta [16] has proposed some indices after enumeration. There are various indices for condition 4, but the diameter of the graph is appropriate in Japan, where the topography is complicated. The index of condition 5 can also be considered variously, such as economic zone, commercial zone, topography, and historical background. In this study, we use the degree of intimacy proposed by Hotta[16], which uses the traffic data of people between municipalities.

In Japan, a method was established to evaluate and support the creation of electoral districts through optimization and enumeration. Based on population data and administrative boundaries in 2020, this paper uses the optimization method of Nemoto and Hotta [28] and the results of the enumeration algorithm of Kawahara et al.[24], and evaluates the current electoral district, which was recommended by the Council on the House of Representatives

[5]According to Japanese law, we had to solve the problem with 50 municipalities (Shi/Ku/Gun) as the node of the graph from 1994 to 2010.

Electoral District in June 2022 and passed by the Diet in November of the same year.

2. Results and Evaluation

2.1. Evaluation and Analysis of Seat Allocation

On June 16th, 2022 (R4), the Council on the House of Representatives Electoral District recommended revisions to the House of Representatives single-seat constituencies (289 seats). Of the 289 single-seat constituencies, it is a large-scale revision proposal that changes 140 electoral districts in 25 prefectures, which is the largest ever. But the maximum difference per vote is 1.999, which is insufficient in terms of population balance. The revised Public Offices Election Law, including this bill, was passed and enacted on November 18th, 2022(R4), promulgated on November 28th, and put into effect on December 28th.

In seat allocation, the remainder method and the divisor method are well known. See [1] for each property. Here we use 1 remainder method and 8 divisor methods, LR; Largest Remainder method, LD; Largest Divisor method, HMD; Harmonic Mean Divisor method, GMD; Geometric Mean Divisor method, LMD; Logarithmic Mean Divisor method, IMD; Identric Mean Divisor method, AMD; Arithmetic Mean Divisor method and SD; Smallest Divisor method. The current electoral division uses LD (known as Adams method), which is one of the divisor methods. It also uses a seat allocation optimization model [18] with the goal of minimizing the ratio.

Table 1 shows the results of allocating 289 seats to 47 prefectures based on the 2020 population.

Table 1: Apportionment: Maximum Disparity by Seat Allocation

<table>
<tr><th colspan="2">method</th><th>max</th><th>min</th><th>ratio</th></tr>
<tr><td colspan="2">current electoral district(=LD)</td><td>465,829</td><td>274,549</td><td>1.697</td></tr>
<tr><td>remainder method</td><td>LR</td><td>549,097</td><td>331,448</td><td>1.657</td></tr>
<tr><td rowspan="3">divisor methods</td><td>LD</td><td>465,829</td><td>274,549</td><td>1.697</td></tr>
<tr><td>HMD,GMD,LMD,IMD,AMD</td><td>549,097</td><td>331,448</td><td>1.657</td></tr>
<tr><td>SD</td><td>753,067</td><td>394,555</td><td>1.909</td></tr>
<tr><td colspan="2">optimal allocation</td><td>549,097</td><td>331,448</td><td>1.657</td></tr>
</table>

The items ‘max’, ‘min’, and ‘ratio’ in Table 1 mean the maximum and minimum values of the average of the population divided by the allocated seats for each prefecture, and their ratio. The population of min 274,549 in the current district is the average population with 2 seats allocated to Tottori Prefecture, and the 549,097 appearing in max is the population of Tottori Prefecture (1 seat allocation). The details of seat allocation to 47 prefectures by each method are shown in Figure 1.

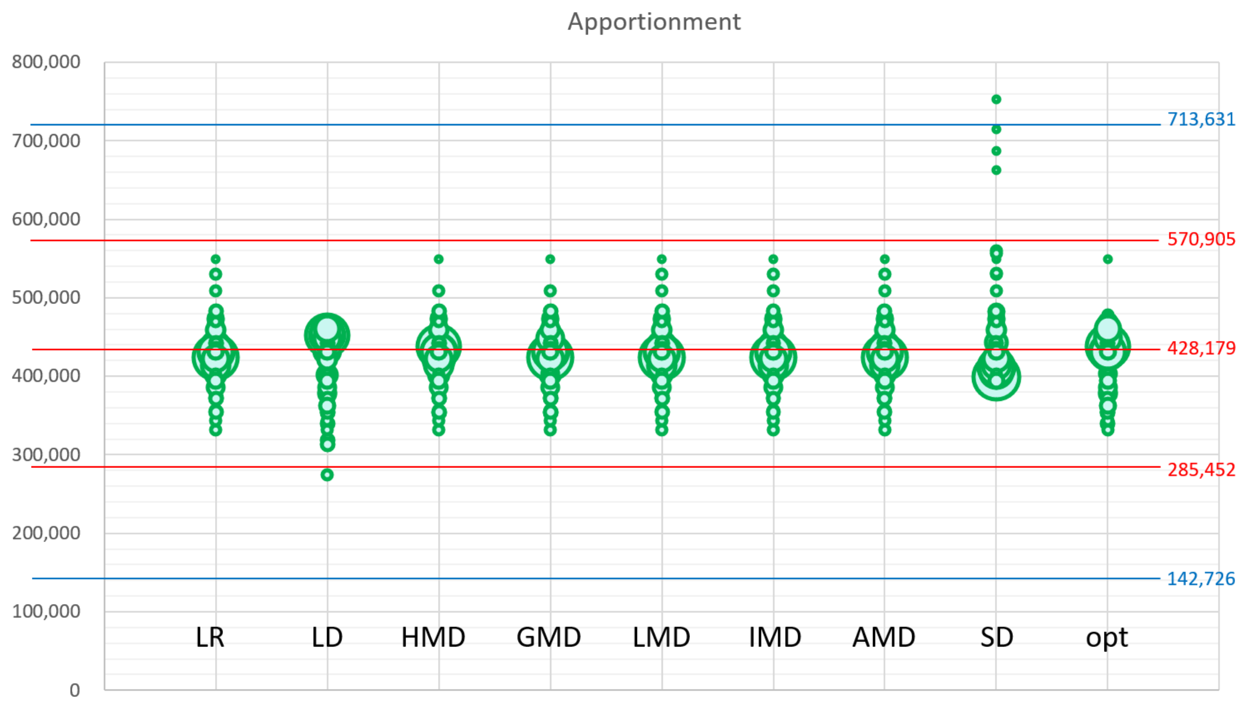

Figure 1: Apportionment: Allocated Seats and Distribution of Average Population

Figure 1 plots the average population of one constituency in 47 prefectures for each method. The size of the circle represents the relative value of the number of allocated seats. The smallest circle is the prefecture with one seat, and the largest circle is Tokyo (30-34 seats). The national average of 428,178.7, the average of ±33.3% (285,453-570,904), and the average of ±66.7% (142,726-713,631) are indicated by horizontal lines.

LD is more distributed below the mean, while SD is more distributed above the mean. It can be seen that the constant distribution of the current district by LD distorts the overall distribution. The LR and the 5 mean divisor methods (XMD) are distributed around the national average population. The result by the seat allocation optimization model[22] is relatively suppressed to the lower side. This is because after the maximum difference was determined, the maximum and minimum values were fixed, and the problem of minimizing the maximum values of the remaining 45 prefectures was repeatedly solved.

2.2. Evaluation and Analysis of Electoral Districting

For the optimization model of the political/electoral districting problem, the algorithm of Nemoto and Hotta [28] is used to find the optimal solution for each prefecture. Two models, the set m partition type and the network flow type, are used, but they cannot be solved by the MIP solver as they are. The set m partition type implements restriction by 'the second valid constituency', and the network flow type implements preprocessing of fixing 0-1 variables using the information of 'the second valid constituency'. After that, the MIP solver is used as a subroutine to solve with an algorithm aiming at ratio minimization[28, 20]. The MIP solver used is cplex 20.10.0, and the PC is Windows11 64bit Ryzen7 4800H (2.90Ghz) 16GB and Windows10 64bit Core i3-7100 (3.90GHz) 8GB.

The constituency is defined as 'the second valid constituency' if all the subgraphs (connected components) of the adjacency graph excluding the constituency are 'the first valid constituency'[28]. In addition, at this time, cities with an average population exceeding

+33.3% are considered overpopulated and subject to division, and the average population is divided and excluded as one electoral district. In terms of 2020 population and administrative boundaries, there are 9 target municipalities; One in Tochigi (Utsunomiya city), one in Kagoshima (Kagoshima city), six in Tokyo (Ota-ku/Setagaya-ku/Suginami-ku/Nerima-ku/Adachi-ku/Edogawa-ku), one in Chiba (Funabashi city). Table 2 shows the preprocessing results for 'the second valid constituencies' for each prefecture.

Table 2: preprocessing results by 'the second valid constituencies'[22]

pref	pop	m	$\|V\|$	$\|E\|$	div	all const	1st const	2nd const	fix	all	%	mod
Niigata	2,186,244	5	38	81	-	1,179,281	1,175,295	129,336	104	380	27.4%	B
Okayama	1,863,316	4	30	66	-	1,747,498	1,619,557	25,347	82	240	34.2%	B
Yamanashi	795,981	2	27	61	-	11,179,924	10,849,175	24,574	30	108	27.8%	B
Miyagi	2,282,543	5	39	86	-	327,773	315,958	18,502	118	390	30.3%	B
Hiroshima	2,751,969	6	31	65	-	120,633	119,631	14,805	142	372	38.2%	B
Gunma	1,885,678	5	37	80	-	293,801	280,985	9,641	221	370	59.7%	B
Tokushima	714,526	2	25	56	-	2,780,385	2,385,538	8,428	28	100	28.0%	B
Miyazaki	1,063,102	3	26	55	-	1,023,368	903,502	5,947	64	156	41.0%	B
Tochigi	1,895,738	5	25	56	1	39,980	37,776	5,258	129	250	51.6%	B
Kyoto	2,525,645	6	36	78	-	13,896	12,616	1,918	126	432	29.2%	B
Shiga	1,384,906	3	19	39	-	12,740	11,963	1,750	46	114	40.4%	B
Okinawa	1,449,323	4	41	67	-	969,905	965,862	1,730	126	328	38.4%	B
Akita	955,851	3	25	52	-	161,158	151,902	1,448	67	150	44.7%	B
Nagasaki	1,304,001	3	21	33	-	7,916	7,580	216	92	126	73.0%	B
Wakayama	916,555	2	30	57	-	17,113,425	12,452,214	212	43	120	35.8%	B
Kochi	687,307	2	34	66	-	30,770,291	22,776,275	132	53	136	39.0%	B
Oita	1,113,684	3	18	33	-	6,781	5,738	120	60	108	55.6%	B
Tottori	549,097	2	19	33	-	9,981	9,233	114	48	76	63.2%	B
Ehime	1,323,682	3	20	33	-	9,675	7,551	67	60	120	50.0%	B
Fukui	753,067	2	17	26	-	1,457	1,193	62	20	68	29.4%	B
Yamaguchi	1,327,681	3	20	34	-	3,627	3,175	48	67	120	55.8%	B
Saga	805,502	2	20	33	-	9,476	8,215	48	52	80	65.0%	B
Shimane	662,896	2	19	33	-	6,708	6,103	32	23	76	30.3%	B
Nara	1,312,968	3	39	94	-	9,499,514,939	9,422,984,690	48,446,761	44	234	18.8%	NF
Yamagata	1,060,878	3	35	84	-	681,335,604	668,005,855	16,557,200	40	210	19.0%	NF
Gifu	1,929,763	5	45	105	-	512,911,379	512,790,338	11,271,768	126	450	28.0%	NF
Aomori	1,232,575	3	43	90	-	2,629,996,051	2,627,850,997	1,409,206	104	258	40.3%	NF
Kagoshima	1,578,219	4	45	86	1	2,626,106,794	2,624,332,714	1,135,058	54	270	20.0%	NF
Iwate	1,203,597	3	33	74	-	84,257,462	83,743,008	438,514	38	198	19.2%	NF
Kumamoto	1,723,710	4	49	106	-	7,742,232,822	7,734,699,115	356,264	70	392	17.9%	NF
Hyougo	5,377,722	12	49	107	-	1,250,118	1,227,518	63,447	665	1,176	56.5%	SP
Ibaraki	2,809,190	7	44	95	-	258,877	256,863	49,452	126	616	20.5%	SP
Aichi	7,311,046	16	69	173	-	33,688	32,907	20,284	1,054	2,208	47.7%	SP
Osaka	8,629,004	19	72	166	-	13,957	13,434	10,044	1,624	2,736	59.4%	SP
Kanagawa	9,041,802	20	58	135	-	33,757	31,515	8,824	1,573	2,320	67.8%	SP
Shizuoka	3,547,156	8	43	92	-	31,473	29,873	5,231	235	688	34.2%	SP
Tokyo	13,564,222	30	62	145	6	6,261	5,768	2,293	2,131	2,976	71.6%	SP
Chiba	6,142,303	14	59	133	2	19,495,534	19,455,388	313,201	916	1,560	58.7%	NF
Kagawa	939,390	3	17	30	1	12,495	11,946	80	20	68	29.4%	B
Mie	1,725,533	4	29	51	1	7,604	4,200	75	159	240	66.3%	B
Toyama	1,018,488	3	15	27	1	3,029	2,533	22	20	60	33.3%	B
Ishikawa	1,118,841	3	19	30	1	4,113	3,417	8	22	76	28.9%	B
Fukuoka	5,068,515	11	72	169	-	3,400,414,087	3,399,229,548	693,762,217	564	1,584	35.6%	-
Saitama	7,183,326	16	72	185	-	30,379,090	30,173,191	9,191,491	986	2,304	42.8%	-
Hokkaido	5,190,293	12	68	161	-	-	-	-	-	-	-	-
Fukushima	1,820,284	4	58	142	-	-	-	-	-	-	-	-
Nagano	2,016,520	5	77	185	-	-	-	-	-	-	-	-

In Table 2, 'pref' and 'pop' indicate the name of prefectures and their populations, and $m, |V|, |E|$ indicate the number of seats allocated by the LD, the number of nodes, and the number of edges of the graph. 'div' is the number of municipal divisions, 'all const', '1st const', and '2nd const' are the number of all constituency candidates, the number of 'first valid constituencies', and the number of 'second valid constituencies', and 'fix', 'all' and '%' are the number of fixed 0-1 variables for the network flow type model by preprocessing, all variables and the fixed rate. 'mod' represents which model was used to solve after preprocessing. 'B' indicates that both models can be solved, 'NF' was solved only by the network flow type, and 'SP' was solved only by the set m partition type. '$-$' indicates that the problem cannot be solved by both models as it is. The difficulty of finding an optimal solution depends on the shape of the graph, $|V|, |E|$, the degree of connectivity, population distribution, the number of electoral districts, and combinations of these.

Obtaining results from optimization reveals the existence of overpopulated and underpopulated constituencies or proves to be infeasible. In that case as well, appropriate divisions of municipalities will be made. The targets are 5 cities: one in Chiba prefecture (Ichikawa city), one in Kagawa prefecture (Takamatsu city), one in Mie prefecture (Yokkaichi city), one in Toyama prefecture (Toyama city), and one in Ishikawa prefecture (Kanazawa city). Of these, only in Kanazawa city, Ishikawa prefecture, the necessary division is not made in the current district. Thus, the intra-prefectural disparity is correspondingly large (See Table 4).

Optimization shows that total 14 divisions (9 pre-cities and 5 post-cities) turned out to be sufficient. On the other hand, there are 32 divisions in the current district, and we can see that unnecessary divisions are being made. This is because all cities and electoral districts that are more than twice the size of the national minimum constituency (Tottori 2nd district) are divided.

Table 3 shows the maximum difference in all prefectures between one vote for the current district and that for the optimal district.

Table 3: maximum vote disparity[22]

method	max	min	ratio
current	547,664	273,973	1.999
optimal 1	567,565	273,973	2.072
optimal 2	567,565	315,157	1.801

'optimal 1' is calculated with the same seat allocation as the current district. The reason that optimal 1 has a larger gap than the current district is that the creation policy is different. When calculating the optimal district, the current creation policy based on the Tottori 2nd electoral district, which has the smallest population in all prefectures, is not used, and the conventional creation policy (created with an average of $\pm 33.3\%$) is used. Therefore, we did not divide cities and constituencies that are below the smaller of 4/3 times the national average (570,904) and 4/3 times the prefectural average. And since the seat allocation is the same as the current district, the Tottori constituency with the smallest population is too small, resulting in a larger disparity than the current district.

As we have already seen, 14 or more city divisions are essentially unnecessary. However, the current district first fixes the smallest electoral district (Tottori 2nd district) and divides all the districts that are more than twice as large as that, so many unnecessary divisions occur. This is an unnecessary division caused by the poor system, especially the biased quota allocation. The result is 1.999 times, which looks like a joke.

'optimal 2' is the result of using seats allocated by the seat allocation optimization model[18]. It can be seen that the limit value of 1.801 times can be achieved under appropriate seat allocation.

Figures 2, 3, 4 are obtained by plotting all 289 constituencies for the current district and the two optimal districts (optimal 1 and optimal 2). The vertical axis is the population, and the horizontal axis is the average descending order of the population divided by the seats allocated to the 47 prefectures from right to left. Also note that all of Japan is included, and there are 48 in total on the horizontal axis. It means that prefectures on the right side of Japan have a higher average population than the national average (the value of one vote is relatively low), and that on the left side is smaller (the value of one vote is relatively high). The center line and upper and lower lines in the white frame represent the prefectural average population and its ±33.3%, respectively. The difference between the rightmost center line and the leftmost center line is the maximum difference caused by seat allocation. Each short bar represents one constituency, and there are 289 in total. The difference between the highest short bar and the lowest short bar is the maximum difference of one vote determined by electoral districting. Since the seat allocation of the current district and the 'optimal 1' is the same, the order of prefectures on the horizontal axis in Figures 2 and 3 is the same. However, since the apportioned seats in 'optimal 2' are based on the seat allocation optimization model, the order of prefectures on the horizontal axis of Figure 4 is different from those of Figures 2 and 3.

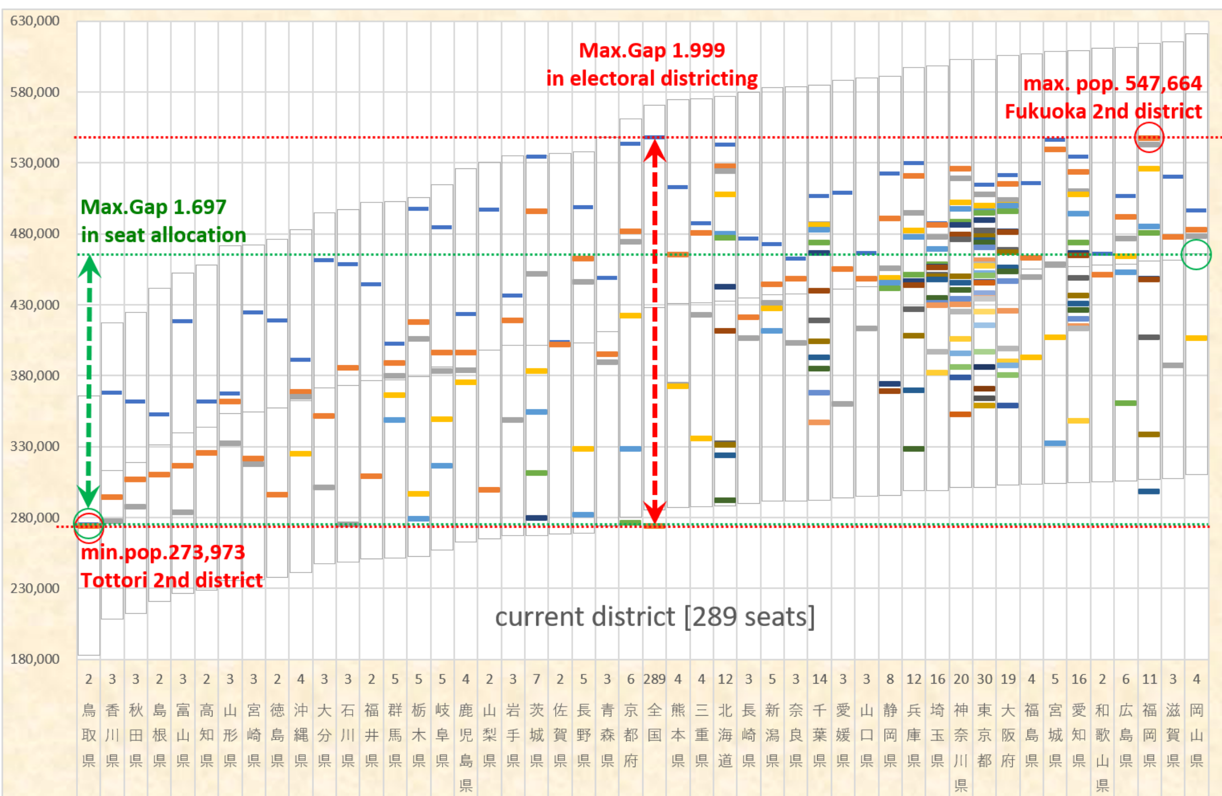

Figure 2: current districting

Comparing the distributions of the horizontal bars for the current district and the optimal districts (optimal 1 and 2) shows that the current distrcit has a fairly large disparity between electoral districts within prefectures. Not only the maximum disparity, but also the disparities in each electoral district are quite large.

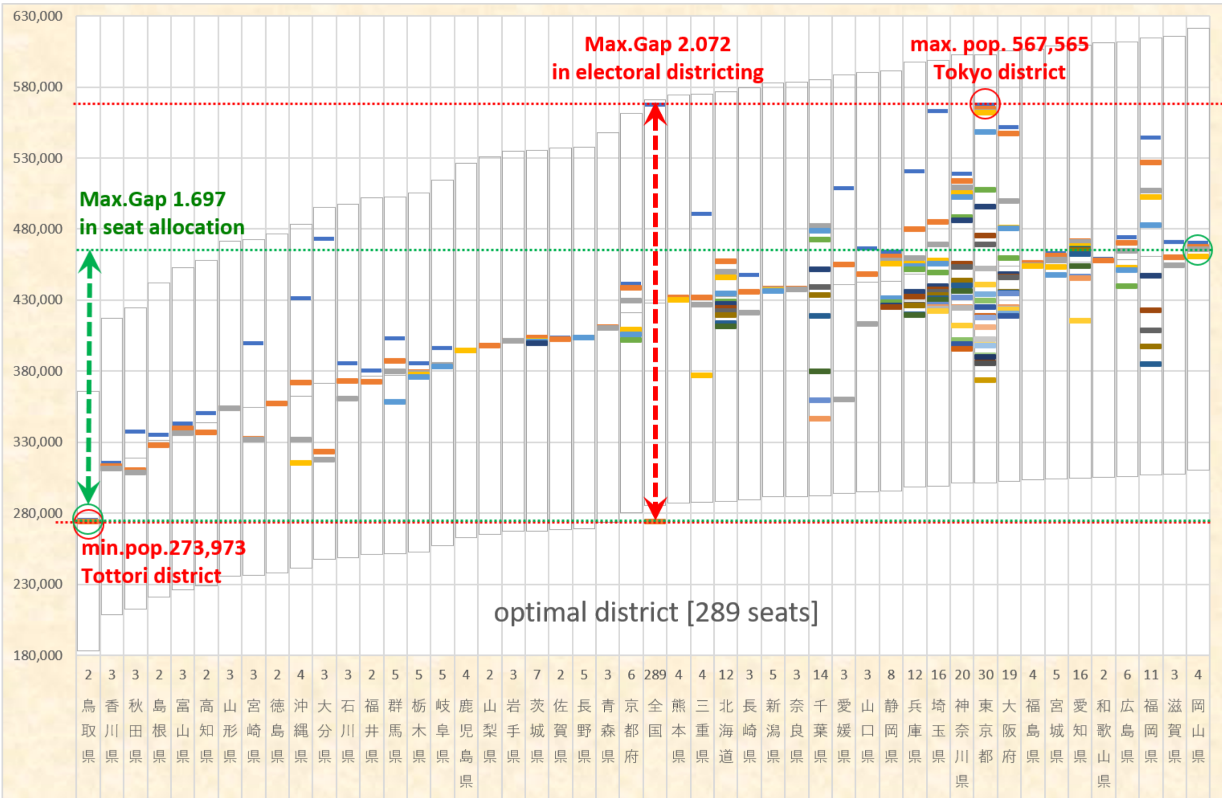

Figure 3: optimal district 1

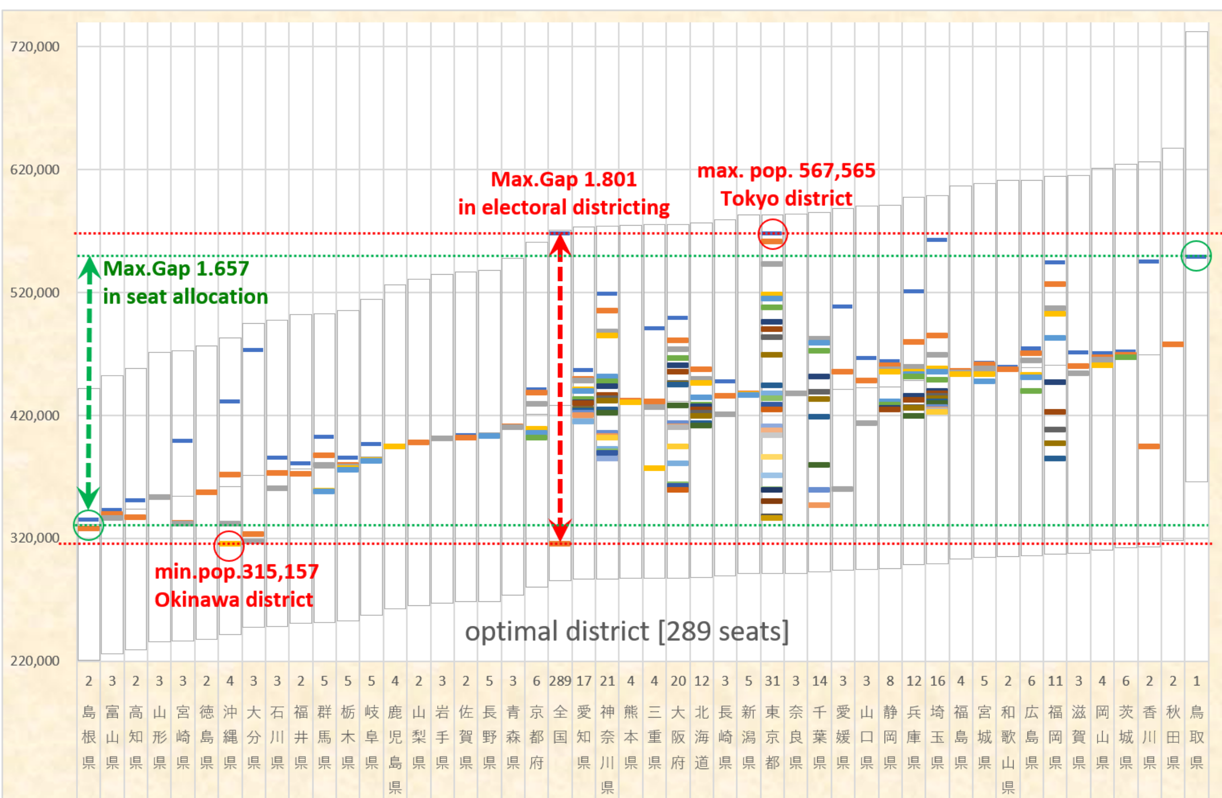

Figure 4: optimal district 2

Table 4 shows a comparison of current district and optimal district for each prefecture.

Table 4: comparative table

Japan	123,743,639	289	32	547664	273973	1.999		2.072	567565	273973	14
				current					optimal		
prefecture	pop	m	div	max	min	ratio	diff	ratio	max	min	div
Ibaraki	2809190	7	0	534093	279586	1.910	0.900	1.011	403754	399540	0
Kyoto	2525645	6	0	543462	276235	1.967	0.869	1.098	441230	401763	0
Nagano	2016520	5	1	498447	281735	1.769	0.769	1.000	403368	403214	0
Tochigi	1895738	5	1	497454	278740	1.785	0.759	1.026	385398	375623	1
Hokkido	5190293	12	3	542597	292062	1.858	0.746	1.111	457008	411214	0
Yamanashi	795981	2	0	496915	299066	1.662	0.662	1.000	398001	397980	0
Miyagi	2282543	5	0	546107	332408	1.643	0.609	1.034	462786	447451	0
Ishikawa	1118841	3	0	458456	274903	1.668	0.598	1.070	385482	360412	1
Gifu	1929763	5	0	484467	316550	1.530	0.495	1.035	396195	382807	0
Toyama	1018488	3	1	418341	283638	1.475	0.456	1.019	342746	336246	1
Fukuoka	5068515	11	3	547664	298262	1.836	0.422	1.414	544188	384834	0
Tokushima	714526	2	0	418829	295697	1.416	0.416	1.000	357292	357234	0
Fukui	753067	2	0	443998	309069	1.437	0.415	1.021	380488	372579	0
Aichi	7311046	16	0	533904	348078	1.534	0.399	1.135	471098	414983	0
Kumamoto	1723710	4	0	512777	372579	1.376	0.373	1.003	431670	430200	0
Hyougo	5377722	12	3	529619	328292	1.613	0.372	1.241	520450	419233	0
Hiroshima	2751969	6	0	506501	360306	1.406	0.328	1.078	474204	439811	0
Shizuoka	3547156	8	1	522187	368912	1.415	0.323	1.092	463787	424596	0
Kagawa	939390	3	2	367982	277036	1.328	0.315	1.014	315254	311006	1
Fukushima	1820284	4	0	515644	392425	1.314	0.308	1.006	456379	453651	0
Shiga	1384906	3	0	520167	386950	1.344	0.308	1.036	470733	454238	0
Iwate	1203597	3	0	436323	348423	1.252	0.252	1.000	401255	401164	0
Okayama	1863316	4	0	496175	406199	1.222	0.200	1.021	470231	460547	0
Kanagawa	9041802	20	0	526038	352447	1.493	0.180	1.312	518722	395245	0
Akita	955851	3	0	361839	287527	1.258	0.165	1.093	337318	308527	0
Mie	1725533	4	1	487465	335408	1.453	0.152	1.301	490694	377101	1
Aomori	1232575	3	0	448486	389245	1.152	0.149	1.003	411505	410129	0
Nara	1312968	3	1	462282	402650	1.148	0.147	1.001	437797	437501	0
Niigata	2186244	5	0	472383	411574	1.148	0.143	1.005	438318	436078	0
Osaka	8629004	19	0	521441	358897	1.453	0.135	1.318	551736	418572	0
Miyazaki	1063102	3	0	424415	317502	1.337	0.133	1.204	399230	331600	0
Kagoshima	1578219	4	1	423040	375274	1.127	0.127	1.000	394556	394554	1
Shimane	662896	2	0	352723	310173	1.137	0.115	1.022	335078	327818	0
Nagasaki	1304001	3	0	476719	406517	1.173	0.109	1.064	447581	420765	0
Yamagata	1060878	3	0	367188	332317	1.105	0.105	1.000	353707	353582	0
Kouchi	687307	2	1	361825	325482	1.112	0.071	1.040	350436	336871	0
Chiba	6142303	14	2	506548	346579	1.462	0.069	1.393	482719	346579	2
Oita	1113684	3	1	461379	301044	1.533	0.042	1.490	472953	317374	0
Gunma	1885678	5	1	402370	348518	1.155	0.030	1.125	402656	357981	0
Wakayama	916555	2	0	465687	450868	1.033	0.030	1.003	459003	457552	0
Ehime	1323682	3	0	508520	360071	1.412	0.000	1.412	508520	360071	0
Yamaguchi	1327681	3	0	466111	413267	1.128	0.000	1.128	466111	413267	0
Tottori	549097	2	0	275124	273973	1.004	0.000	1.004	275124	273973	0
Saga	805502	2	0	403528	401974	1.004	0.000	1.004	403528	401974	0
Saitama	7183326	16	1	487352	381857	1.276	-0.056	1.332	562682	422298	0
Tokyo	13564222	30	8	514227	358963	1.433	-0.087	1.520	567565	373426	6
Okinawa	1449323	4	0	391249	324660	1.205	-0.162	1.367	430915	315157	0

'm' means the allocated seat, 'div' means the number of divided municipalities, and 'diff' means the difference between the current division and the optimal division for the maximum prefectural disparity. The optimal in this table is 'optimal 1', which is the same number of seats as the current district. This table is sorted by prefectures in descending order of 'diff'. In other words, the prefectures are arranged in order of the room for improvement in terms of the difference of one vote. Above the item are values for all of Japan. For Fukushima, Saitama, and Fukuoka prefectures, the optimal solution for each municipality(Shi/Ku/Cho/Son) has not been obtained, and the result is obtained for each municipality(Shi/Ku/Gun). Saitama prefecture and Tokyo-to are better than optimal because they are divided into unnecessary city divisions. Okinawa prefecture is better than optimal because it is an island-only constituency and includes an enclave via Naha. Four prefectures, Ehime, Yamaguchi, Tottori, and Saga, have the same district as the optimal solution. Since there were two prefectures, Saga and Wakayama where the current district corresponds with the optimal one, in 2015, the number of such prefectures increased. The difference from the optimum is less than 0.1pt in 5 prefectures, less than 0.2pt in 12 prefectures, less than 0.3pt in 2 prefectures, less than 0.4pt in 8 prefectures, and less than 0.5pt in 5 prefectures. Ishikawa prefecture, where the difference is less than 0.6pt, is the only prefecture where the number of city divisions is smaller in the current district than in the optimal district. However, according to the old rule that the average ±33.3% in the prefecture is the condition, it is subject to division, and it is better to divide from the point of view of the difference.

On the other hand, the 6 prefectures with a difference of 0.6pt or more are shown in the table, and these did not improve the intra-prefectural disparity. These 6 prefectures are the same even in the comparison five years ago. In addition, although Nagano prefecture and Hokkaido are divided into city divisions, the difference within the prefecture is very large. Since the maximum difference of the optimal solution is approximately 1, a reasonable explanation is required to cause this difference despite the city division. Fukuoka, Hyogo and Shizuoka prefectures are similar.

When the limit value was unknown, no one could clearly point out the inadequacies, so it was left unchecked. However, by finding the optimal solution, the severity of the disparities in districts in each prefecture came to light. For prefectures with large disparities, appropriate explanations are required, and prompt improvements are desired.

2.3. Evaluation and Analysis by Enumeration of Electoral Districts

Optimal district is politically marginal, but may not be sufficient. In extreme cases, it is possible to say that there is no feasible solution between the optimal district and the current one, so the current district must be used. Thus, it is better to clarify how many solutions exist between the optimal district and the current one, and we want to count the number.

In addition, the optimal district is often difficult to use as an actual district. As the Supreme Court stated in its decision in the disparity suit, it is within the discretion of the Diet to consider conditions other than disparity. It is natural that there is a desire to search for district that can be acutually used even if the population equity is somewhat sacrificed. From the point of view of finding practical districts, it is tempting to enumerate feasible solutions that exist between the optimal district and the current one.

By enumerating the solutions and calculating various feature values for each, it becomes possible to find better districts, and the content of improvements to the current district becomes clearer. Using the frontier method-based fast enumeration algorithm by Kawahara et. al.[24, 42], we enumerate all feasible solutions and solutions below a specified ratio for 47 prefectures. Table 5 shows the result.

Table 5: Enumerated all feasible solutions and the # of solutions below the specified ratio

pref	pop	m	$\|V\|$	$\|E\|$	div	# of all sols	time	ratio	# of ≦ r. sols
Kagoshima	1,578,219	4	45	86	1	3,687,291,359	0.05	≦ 2.0	4,505,935
Aomori	1,232,575	3	43	90	-	16,650,869	0.02	≦ 2.0	2,868,391
Iwate	1,203,597	3	33	74	-	16,145,645	0.01	≦ 2.0	1,508,321
Kyoto	2,525,645	6	36	78	-	44,063,998,540	0.02	≦ 2.0	1,436,215
Hiroshima	2,751,969	6	31	65	-	2,585,257,864	0.02	≦ 2.0	856,063
Okayama	1,863,316	4	30	66	-	77,799,080	0.02	≦ 2.0	329,934
Gunma	1,885,678	5	37	80	-	2,722,606,272	0.02	≦ 2.0	25,251
Miyazaki	1,063,102	3	26	55	-	326,617	0.02	≦ 2.0	13,706
Yamanashi	795,981	2	27	61	-	18,740	0.02	≦ 2.0	12,287
Tochigi	1,895,738	5	25	56	1	192,572,103	0.02	≦ 2.0	11,944
Tokushima	714,526	2	25	56	-	12,442	0.02	≦ 2.0	4,214
Akita	955,851	3	25	52	-	140,653	0.01	≦ 2.0	2,109
Shiga	1,384,906	3	19	39	-	61,811	0.02	≦ 2.0	1,966
Okinawa	1,449,323	4	41	67	-	4,066,535	0.02	≦ 2.0	1,548
Nagasaki	1,304,001	3	21	33	-	5,071	0.03	≦ 2.0	447
Mie	1,725,533	4	29	51	1	264,430	0.02	≦ 2.0	108
Wakayama	916,555	2	30	57	-	364	0.02	≦ 2.0	106
Kochi	687,307	2	34	66	-	745	0.02	≦ 2.0	66
Tottori	549,097	2	19	33	-	121	0.02	≦ 2.0	57
Ehime	1,323,682	3	20	33	-	2,887	0.02	≦ 2.0	42
Kagawa	939,390	3	17	30	1	1,995	0.02	≦ 2.0	40
Oita	1,113,684	3	18	33	-	5,187	0.02	≦ 2.0	37
Yamaguchi	1,327,681	3	20	34	-	2,348	0.02	≦ 2.0	33
Fukui	753,067	2	17	26	-	87	0.02	≦ 2.0	31
Saga	805,502	2	20	33	-	216	0.02	≦ 2.0	24
Shimane	662,896	2	19	33	-	103	0.02	≦ 2.0	16
Toyama	1,018,488	3	15	27	1	1,189	0.02	≦ 2.0	11
Ishikawa	1,118,841	3	19	30	1	882	0.01	≦ 2.0	4
Fukushima	1,820,284	4	58	142	-	768,984,516	0.05	≦ 1.6	3,077,753
Miyagi	2,282,543	5	39	86	-	22,734,283,810	0.01	≦ 1.6	538,375
Tokyo	13,564,222	30	62	145	6	4.106295159E+27	0.10	≦ 1.6	9,927
Chiba	6,142,303	14	59	133	2	6.691778220E+22	3.40	≦ 1.5	822,181,618
Nagano	2,016,520	5	77	185	-	2.770350502E+17	0.97	≦ 1.5	78,651,461
Nara	1,312,968	3	39	94	-	2,540,276,610	0.03	≦ 1.5	43,962,650
Kanagawa	9,041,802	20	58	135	-	1.517836967E+25	0.04	≦ 1.5	25,053,125
Niigata	2,186,244	5	38	81	-	39,223,935,720	0.02	≦ 1.5	15,609,574
Yamagata	1,060,878	3	35	84	-	465,018,021	0.01	≦ 1.5	10,627,970
Shizuoka	3,547,156	8	43	92	-	1.992806409E+13	0.02	≦ 1.5	3,963,748
Kumamoto	1,723,710	4	49	106	-	15,048,399,700	0.03	≦ 1.5	2,888,921
Hyougo	5,377,722	12	49	107	-	1.765013542E+18	0.10	≦ 1.5	642,860
Ibaraki	2,809,190	7	44	95	-	6.349717438E+13	0.02	≦ 1.4	85,736,324
Gifu	1,929,763	5	45	105	-	3.227673666E+12	0.05	≦ 1.2	4,158,586
Osaka	8,629,004	19	72	166	-	8.448840915E+29	0.10		T/O
Aichi	7,311,046	16	69	173	-	3.015592395E+29	1.86		T/O
Saitama	7,183,326	16	72	185	-	2.411084997E+27	0.57		T/O
Hokkaido	5,190,293	12	68	161	-	6.002053108E+24	1.43		T/O
Fukuoka	5,068,515	11	72	169	-	5.207847643E+24	0.44		T/O

In Table 5, 'time' represents CPU time (seconds). Calculations were performed on a PC with Windows 11, Intel Core(TM) i9-10900K CPU (3.60 GHz), and 32GB memory. Municipalities(Shi/Ku/Cho/Son) are used as elements of the graph, but only Fukushima, Saitama and the right side of Nagano prefecture (feasible solution less than 1.5 times) are calculated by municipalities(Shi/Ku/Gun). 'T/O' means Timeout in 1000 seconds or 'Out of memory'.

In this paper, among the six prefectures with large gaps between the current district and the optimal one in Table 4, four prefectures, Ibaraki, Miyagi, Kyoto, and Yamanashi, which have not been divided into municipalities, are selected, and a more detailed analysis is performed. Table 6 shows the number of solutions from current district to optimal one. The PC used is Intel Xeon(TM) Gold 5215L CPU (2.50 GHz), and 6TB memory.

Table 6: enumerative solutions from optimal to current

prefecture	m	$\|V\|$	$\|E\|$	diff	# of solutions	cpu time	memory
Ibaraki	7	44	95	0.900	5,435,462,667	761.86 sec.	2.05 TB
Miyagi	5	39	86	0.609	805,189	32.25 sec.	5.05 GB
Kyoto	6	36	78	0.869	1,152,414	8.33 sec.	1.36 GB
Yamanashi	2	27	61	0.662	9,547	0.01 sec.	7.00 MB

It can be seen that there are a considerable number of feasible solutions between the current and optimal districts. The fact that there are so many feasible solutions and that the current district is still used requires a proper explanation. Listing the districts in ascending order of maximum difference is a very effective means of providing that information.

For your reference, Nagano without municipal division has an optimal solution of 1.000 times and more than 2.63 billion solutions with a difference of 1.08 times or less. It took 40 hours and consumed 5.8TB of memory to find them. Hokkaido without municipal division has an optimal of 1.111 times and more than 660 million solutions with a gap of 1.18 times or less. The calculation took 13 hours and consumed 4.7TB of memory.

Table 7 lists the number of solutions below each gap for the four prefectures in Table 6.

Table 7: enumerative solutions from optimal to specified ratio

	Ibaraki	Miyagi	Kyoto	Yamanashi
current ratio	1.910	1.643	1.967	1.662
# of sols $\leq$cur.ratio	5,435,462,667	805,189	1,152,414	9,547
# of sols ≤ 1.400	85,736,324	50,822	13,329	6,502
# of sols ≤ 1.300	16,237,666	11,267	4,850	5,050
# of sols ≤ 1.200	1,472,833	2,072	682	3,512
# of sols ≤ 1.100	16,856	164	3	1,815
optimal ratio	1.011	1.034	1.098	1.000

For Yamanashi prefecture, all enumerated solutions (9,546 without current) are targeted, and for Ibaraki, Miyagi, and Kyoto prefectures, a necessary and sufficient number of feasible solutions are targeted for analysis, and various feature values are calculated. As various features, we calculate the standard deviation of the electoral district population, intimacy and diameter for each district. Standard deviation is one of the indicators for condition 1, intimacy for condition 5, and diameter for condition 4.

The scatter plot of the standard deviation and the maximum difference within the prefecture is as follows.

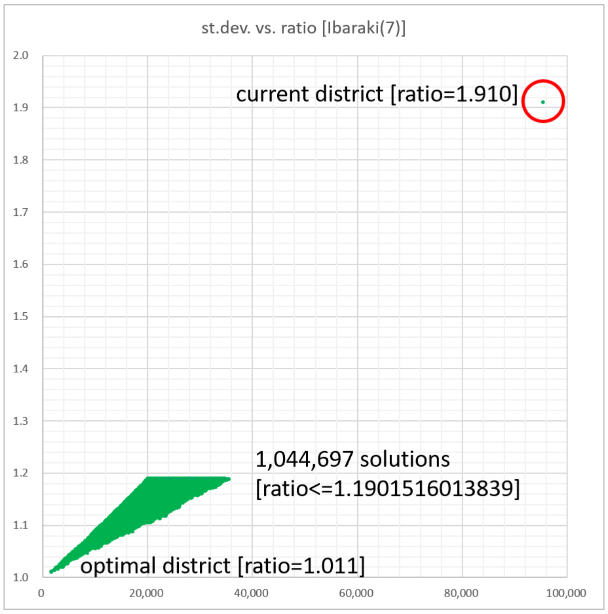

Figure 5: stdev vs ratio: Ibaraki(7)

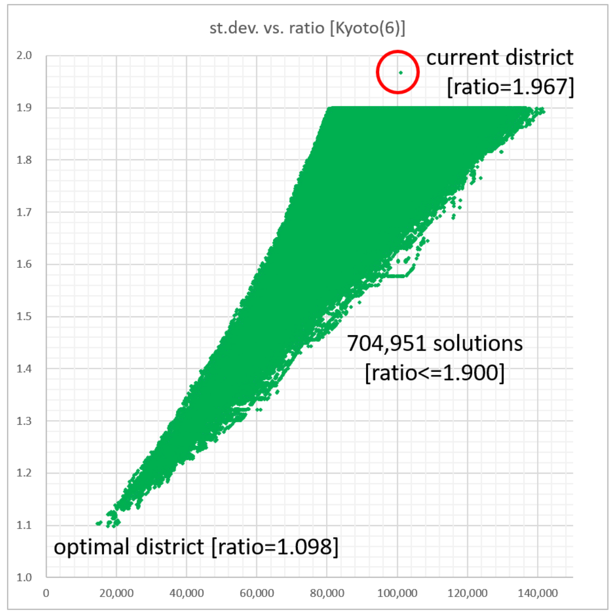

Figure 6: stdev vs ratio: Kyoto(6)

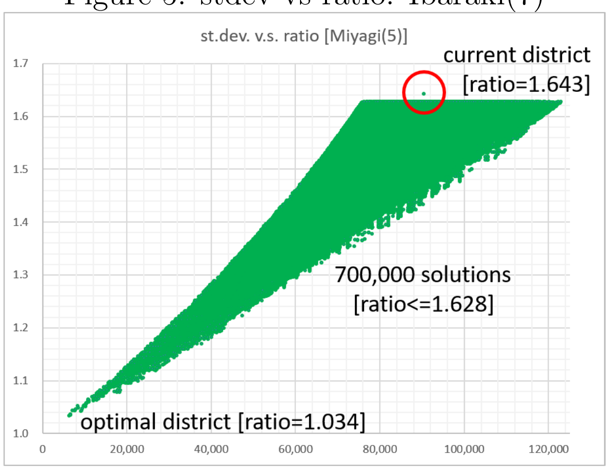

Figure 7: stdev vs ratio: Miyagi(5)

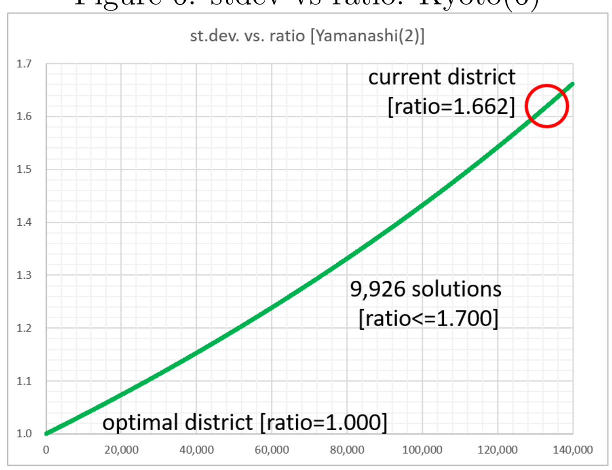

Figure 8: stdev vs ratio: Yamanashi(2)

Each point represents a feasible solution, and the circled point is the current district. The larger the maximum difference, the larger the standard deviation, which is a natural result. Since Yamanashi prefecture has only two electoral districts, there are few solutions with the same maximum difference, and most of them lie on one curve. For Ibaraki, Miyagi, and Kyoto prefectures, there are multiple solutions with the same difference, and the standard deviations are largely different as the maximum disparity increases. From this result, the current district is very bad for the standard deviation.

Next, we calculate the degree of intimacy and the diameter, and draw a scatterplot with the maximum difference, respectively. First, we show the results for Ibaraki prefecture, which has the largest 'diff' and the greatest room for improvement. In Figure 9, the horizontal axis is the maximum gap, and the left side is the better value. The vertical axis is the intimacy, and the upper side is the better value. The figure on the left shows the average value of intimacy, and the figure on the right shows the minimum value of intimacy. The points corresponding to 1,044,697 feasible solutions(ratio$\leq$ 1.19015) are plotted, including the optimal and current district, and the current district is circled.

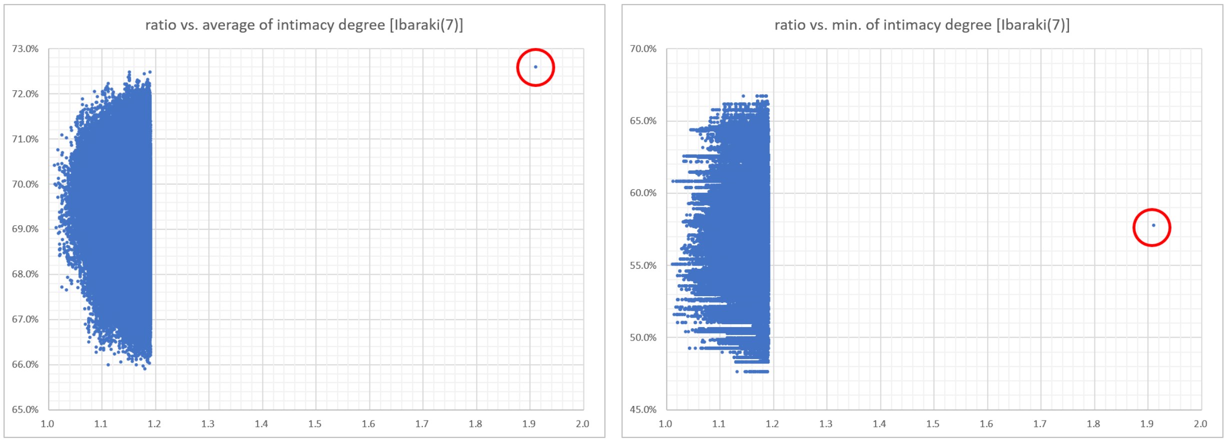

Figure 9: ratio vs intimacy degree: Ibaraki(7) 1,044,697 sols.

In Figure 10, the horizontal axis is the maximum disparity, the vertical axis is the diameter of the graph, and the lower one is the better value. The left figure is the average diameter, and the right figure is the maximum diameter. The points corresponding to 1,044,697 feasible solutions are plotted, including the optimal and current district, and the current district is circled.

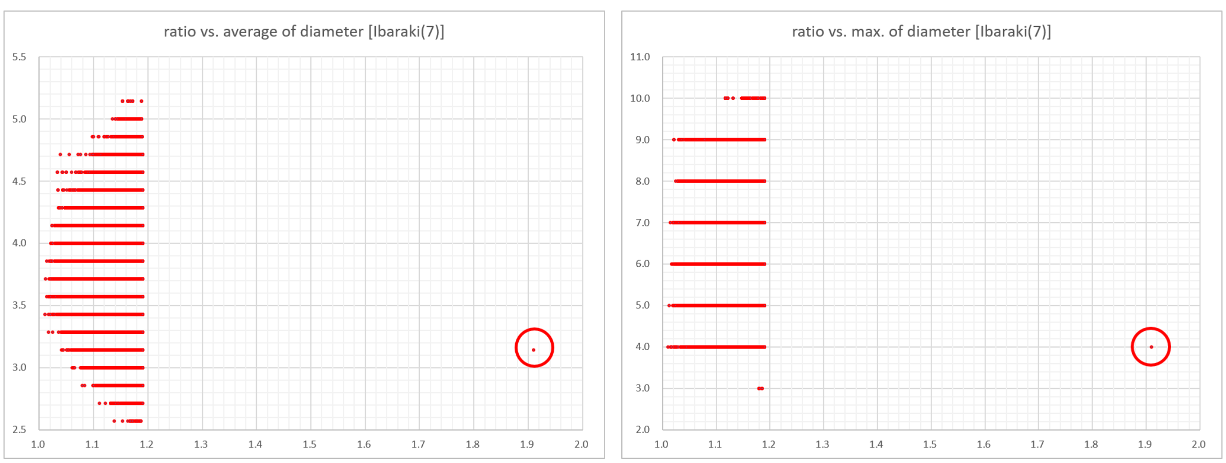

Figure 10: ratio vs diameter: Ibaraki(7) 1,044,697 sols.

Table 8 shows the number of solutions with better intimacy and diameter than the current district for Ibaraki prefecture (7 constituencies) for each maximum difference. The first row of the table gives information about the current district. The maximum difference (1.910 times), the average (72.6%) and minimum (57.8%) of intimacy degree, and the average (3.143) and maximum (4) diameters of the subgraphs representing the seven electoral districts (connected components).

Table 8: intimacy degree & diameter: Ibaraki(7)

1.910	current	72.6%		57.8%		3.143	4		
		intimacy degree				diameter			
ratio	# of sols	ave.	72.0%	min.	both	ave.	max.	both	all
≦ 1.10	16,856	0	1	5,491	1	487	1,580	314	0
≦ 1.11	36,883	0	8	12,836	8	1,481	2,803	692	0
≦ 1.12	62,146	0	13	22,162	13	2,562	4,241	1,153	0
≦ 1.13	99,343	0	14	36,800	14	4,116	6,963	1,864	0
≦ 1.14	154,681	0	14	57,423	14	6,759	11,038	3,046	0
≦ 1.15	234,378	0	28	88,041	28	9,966	16,213	4,498	1
≦ 1.16	351,402	0	44	132,490	40	15,016	23,614	6,446	2
≦ 1.17	521,361	0	69	194,944	63	22,057	34,403	9,382	5
≦ 1.18	747,496	0	100	282,545	89	32,862	48,343	13,435	7
≦ 1.19	1,039,639	0	156	402,759	137	43,808	67,822	18,313	11

The left half of Table 8 represents the number of solutions with 'ave' better than the average of intimacy degree of the current district for the number of solutions less than or equal to the specified ratio. But, there are 0 solutions for which the difference is less than or equal to 1.19 times. Therefore, '72.0%' in the next column shows the number of solutions that are greater than or equal to this value. 'min' represents the number of solutions with a value better than the minimum intimacy degree of the current district. 'both' represents the number of solutions with an average value of 72.0% or more and a better 'min' value. The right half of the table shows the number of solutions with 'ave' better than the average diameter of the current district and the number of solutions with 'max' better than the maximum diameter of the current district for the number of solutions less than or equal to the same ratio. 'all' is the number of solutions with better values for all four above. It can be seen that there is one solution at 1.15 times or less and one at 1.16 times or less.

The average intimacy degree of the current district is good, but the minimum value is not so good. As a result, some electoral districts are suffering. It can be seen that the diameter of the current division is good both in average and maximum, and is compact. However, the maximum disparity in the prefecture is 1.910 times, which is very large[6]. By enumerating the solutions and comparing the features, we can see that there exists a solution that greatly reduces the gap (condition 1), has a good diameter (condition 4), and has a good intimacy degree (condition 5). Immediate improvement is desired.

Table 9 and Figure 11 show numerical values and graphs for the optimal district, the current district and the solution with better 4 indices. In both Table 9 and Figure 11, the left side is the optimal solution, the right side is the current division, and the center is the solution with similar or better intimacy degree and diameter than the current one. Table 9 is arranged in descending order of population for each of the seven constituencies. The population of each constituency is shown on the left, and 'ratio', 'inti.', and 'dia.' on the right are the difference from the minimum population of each constituency, intimacy degree and diameter. The last row of the table is the average intimacy and the average diameter. You can see the maximum difference(ratio), the minimum intimacy degree and the maximum diameter value in the table.

From Table 9, the optimal district with the best maximum disparity is inferior to the current one in terms of intimacy and diameter, but the solution in the middle appears to

[6]Ibaraki prefecture continues to adopt the district with large gap, not only this time[14, 18, 21, 22].

be a good district with a much better ratio compared to the current one, a similar degree of intimacy and diameter. From Figure 11, it is difficult to say that the shape of the optimal district is compact, but the solution in the center looks not bad in terms of compactness compared to the current district.

Table 9: optimal, better solution, current district: Ibaraki(7)

optimal	ratio	inti.	dia.	better	ratio	inti.	dia.	current	ratio	inti.	dia.
403,754	1.011	55.1%	4	423,710	1.145	59.6%	4	534,093	1.910	74.5%	3
403,408	1.010	70.2%	4	418,382	1.131	67.8%	3	495,619	1.773	75.3%	3
401,153	1.004	70.0%	2	413,911	1.119	73.4%	4	451,442	1.615	57.8%	4
400,988	1.004	67.2%	3	400,870	1.083	71.0%	2	382,793	1.369	71.2%	3
400,723	1.003	80.8%	4	399,624	1.080	75.3%	4	354,299	1.267	78.4%	3
399,624	1.000	75.3%	4	382,692	1.034	82.3%	3	311,358	1.114	66.5%	3
399,540	1.000	74.4%	3	370,001	1.000	75.4%	2	279,586	1.000	84.5%	3
	ave	70.4%	3.43		ave	72.1%	3.14		ave	72.6%	3.14

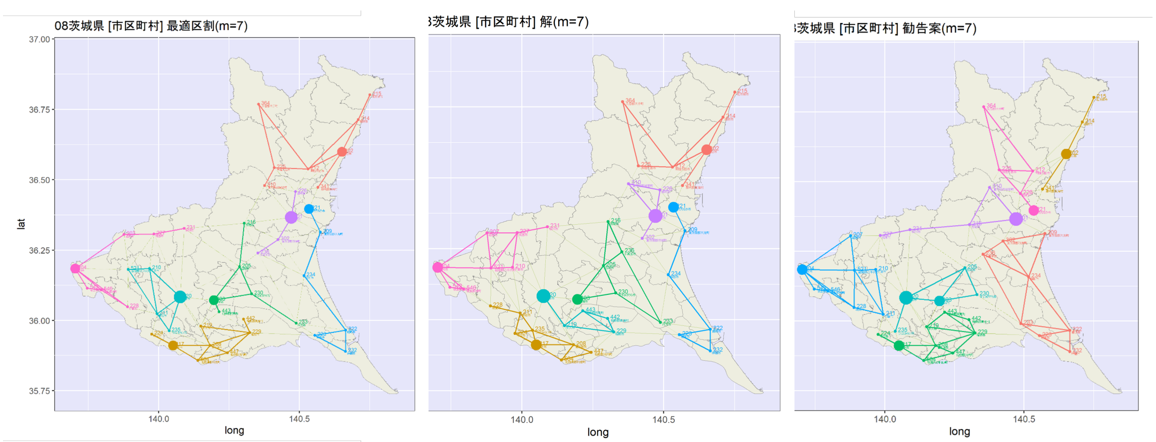

Figure 11: optimal, better solution, current district: Ibaraki(7)

The remaining 3 prefectures, Kyoto, Miyagi and Yamanashi, are shown in the same way.

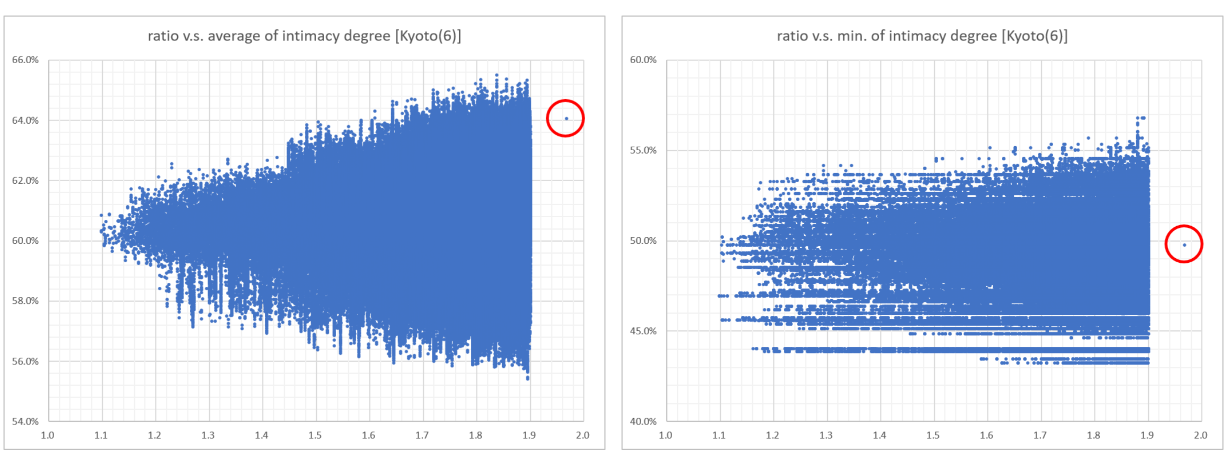

Figure 12: ratio vs intimacy degree: Kyoto(6) 704,951 sols.

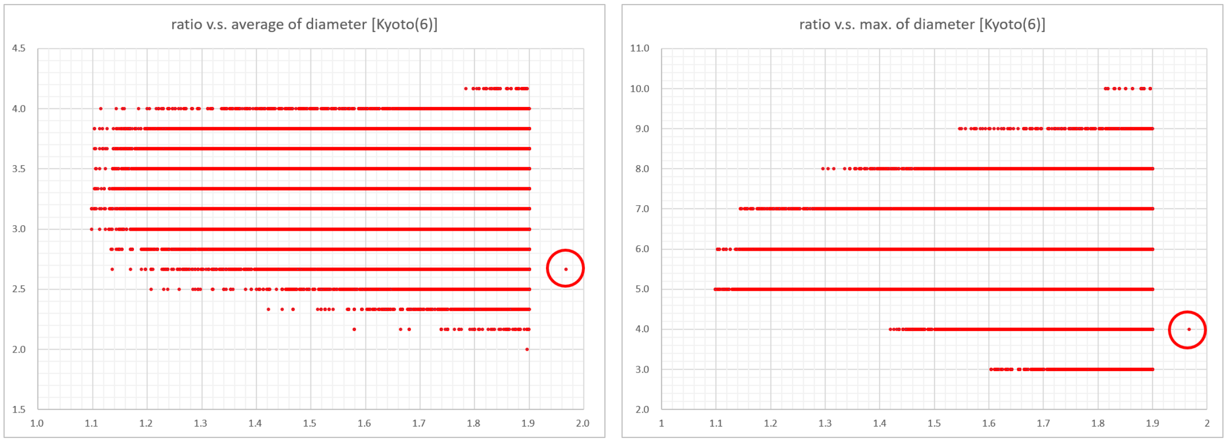

Figure 13: ratio vs diameter: Kyoto(6) 704,951 sols.

Table 10: intimacy degree & diameter: Kyoto(6)

1.967	current	64.1%	49.8%		2.667	4		
		intimacy degree			diameter			
ratio	# of sols	ave.	min.	both	ave.	max.	both	all
≦ 1.10	3	0	0	0	3	0	0	0
≦ 1.20	682	0	238	0	682	0	0	0
≦ 1.30	4,850	0	1,959	0	4,841	0	0	0
≦ 1.40	13,329	0	5,903	0	13,309	0	0	0
≦ 1.50	28,536	0	12,247	0	28,440	795	767	0
≦ 1.60	67,359	0	29,635	0	66,971	6,221	5,955	0
≦ 1.70	145,857	109	61,973	100	144,481	21,295	20,144	51
≦ 1.80	325,963	798	142,138	618	322,215	56,597	53,417	335
≦ 1.90	704,951	3,439	310,153	2,594	694,253	139,417	129,878	1,163

Kyoto prefecture has a solution with a maximum disparity 1.610 times better than the current district for the average and minimum intimacy values and the average and maximum diameter values. The solution is shown in the center of Table 11 and Figure 14.

Table 11: optimal, better solution, current district: Kyoto(6)

optimal	ratio	inti.	dia.	better	ratio	inti.	dia.	current	ratio	inti.	dia.
441,230	1.098	47.0%	4	465,267	1.610	52.3%	4	543,462	1.967	58.6%	4
438,552	1.092	54.2%	1	463,678	1.604	69.0%	1	481,286	1.742	67.6%	3
429,377	1.069	66.4%	2	459,864	1.591	54.5%	3	474,147	1.716	57.3%	3
409,089	1.018	54.8%	2	453,289	1.569	55.6%	2	422,154	1.528	49.8%	2
405,634	1.010	87.8%	5	394,555	1.365	59.9%	2	328,361	1.189	55.7%	1
401,763	1.000	54.9%	4	288,992	1.000	94.5%	3	276,235	1.000	95.4%	3
	ave	60.9%	3.00	0	ave	64.3%	2.50	0	ave	64.1%	2.67

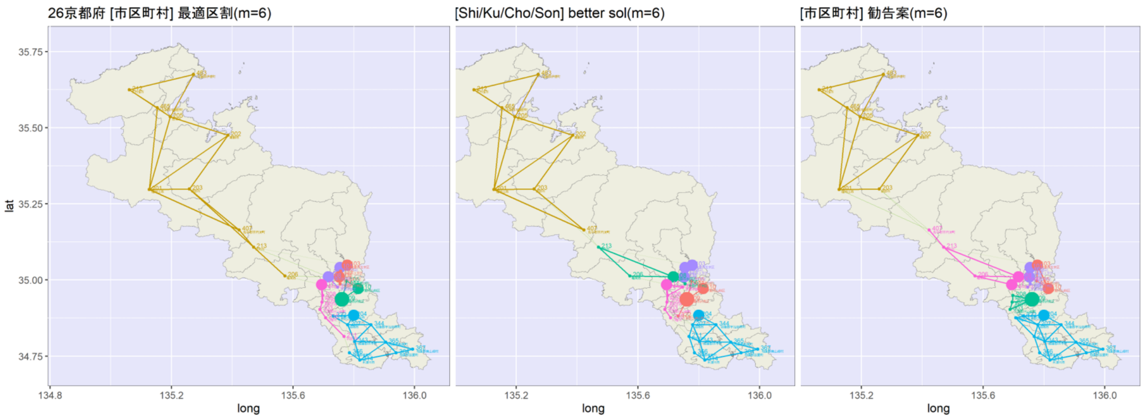

Figure 14: opt, sol, cur : Kyoto(6)

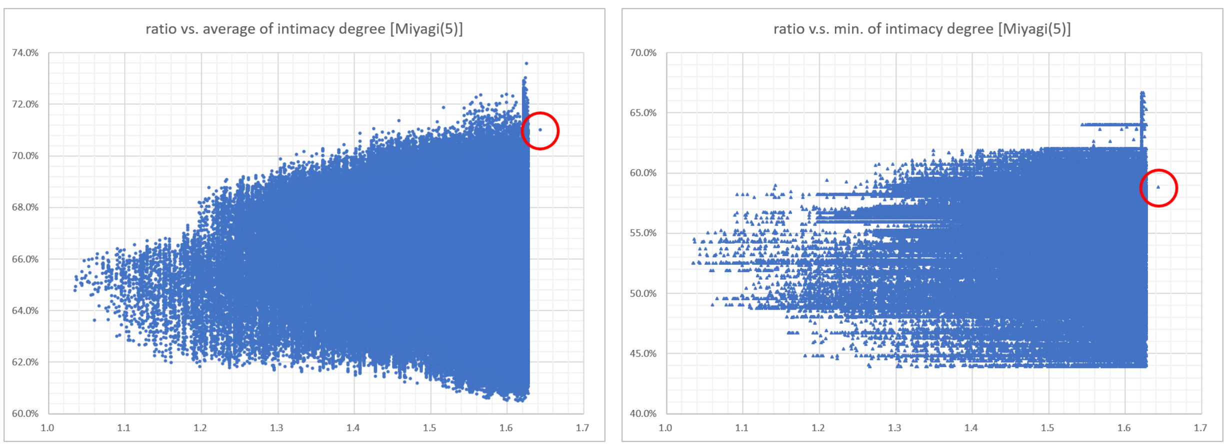

Figure 15: ratio vs intimacy degree: Miyagi(5) 700,000 sols.

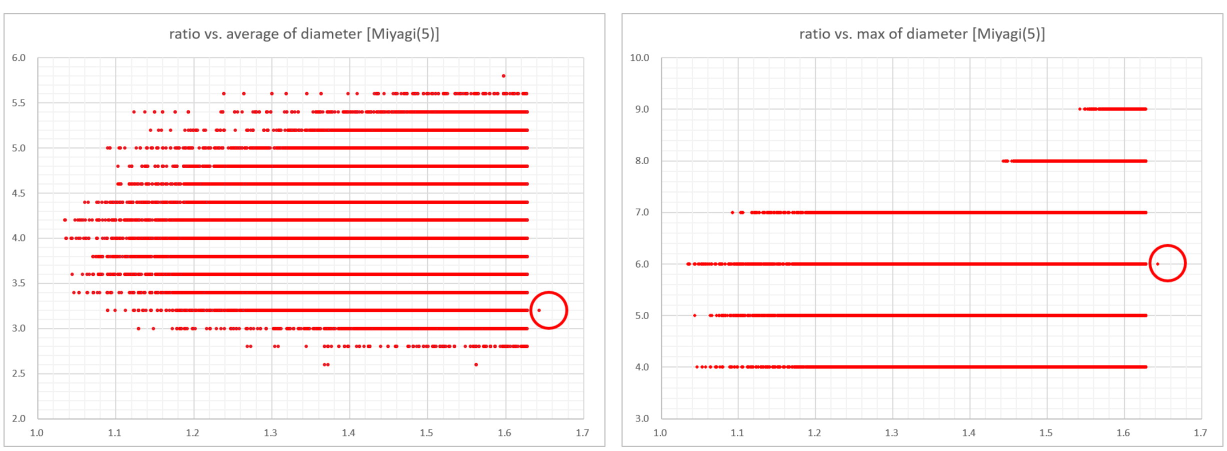

Figure 16: ratio vs diameter: Miyagi(5) 700,000 sols.

Table 12: intimacy degree & diameter: Miyagi(5)

1.643	current	71.0%	58.9%		3.200	6		
		intimacy degree			diameter			
ratio	# sols	ave.	min.	both	ave.	max.	both	all
≦ 1.10	164	0	0	0	3	161	3	0
≦ 1.20	2,072	0	1	0	96	1,747	96	0
≦ 1.30	11,267	0	55	0	350	9,635	350	0
≦ 1.40	50,822	0	1,167	0	1,498	42,679	1,498	0
≦ 1.50	184,526	4	5,479	4	4,569	152,468	4,558	4
≦ 1.60	538,375	181	17,088	165	9,796	417,777	9,721	53
≦ 1.643	804,812	698	25,586	645	13,663	602,841	13,537	256

Miyagi prefecture also has a solution with a maximum disparity 1.459 times better than the current district for the average and minimum intimacy values and the average and maximum diameter values. The solution is shown in the center of Table 13 and Figure 17.

Table 13: optimal, better solution, current district: Miyagi(5)

optimal	ratio	inti.	dia.	better	ratio	inti.	dia.	current	ratio	inti.	dia.
462,786	1.034	84.8%	5	565,489	1.459	67.1%	4	546,107	1.643	58.9%	2
461,058	1.030	72.7%	2	517,082	1.334	73.7%	1	539,090	1.622	69.9%	1
458,176	1.024	52.5%	4	415,531	1.072	60.4%	2	458,142	1.378	67.9%	6
453,072	1.013	57.1%	6	396,740	1.023	88.8%	3	406,796	1.224	85.2%	3
447,451	1.000	56.7%	4	387,701	1.000	66.8%	4	332,408	1.000	73.2%	4
	ave	64.8%	4.20		ave	71.4%	2.80		ave	71.0%	3.20

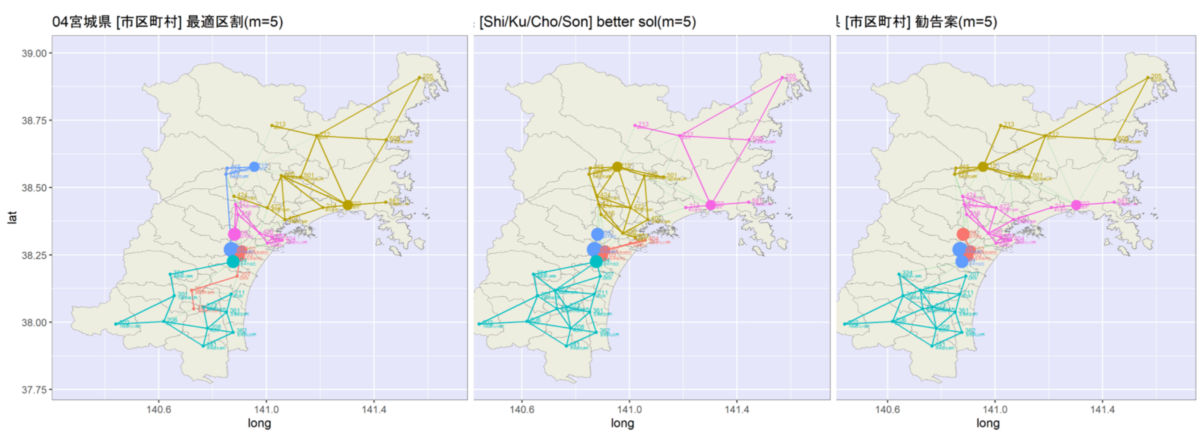

Figure 17: optimal, better solution, current district: Miyagi(5)

From Table 13 the solution in the center is better than the current district at all points, and it appears to be more compact from Figure 17.

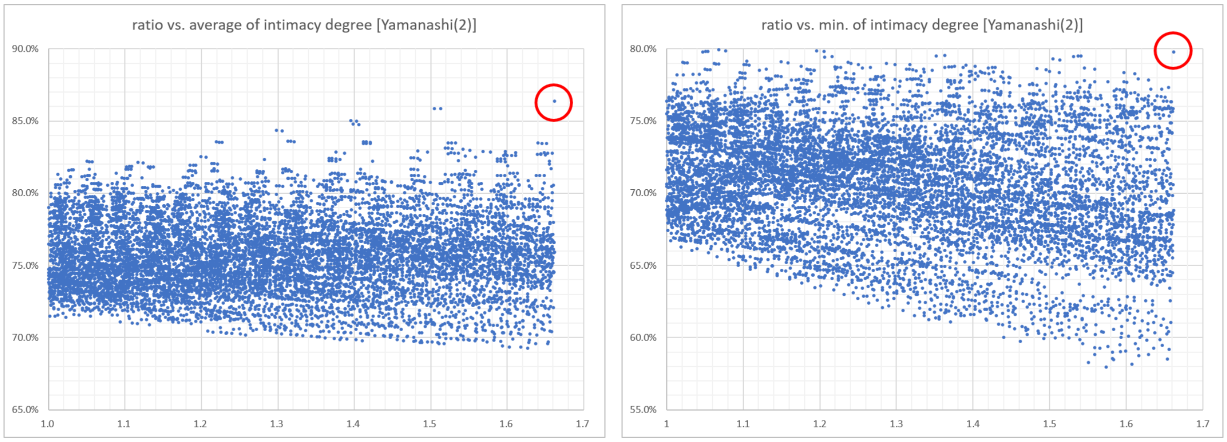

Figure 18: ratio vs intimacy degree: Yamanashi(2) 9,546 sols.

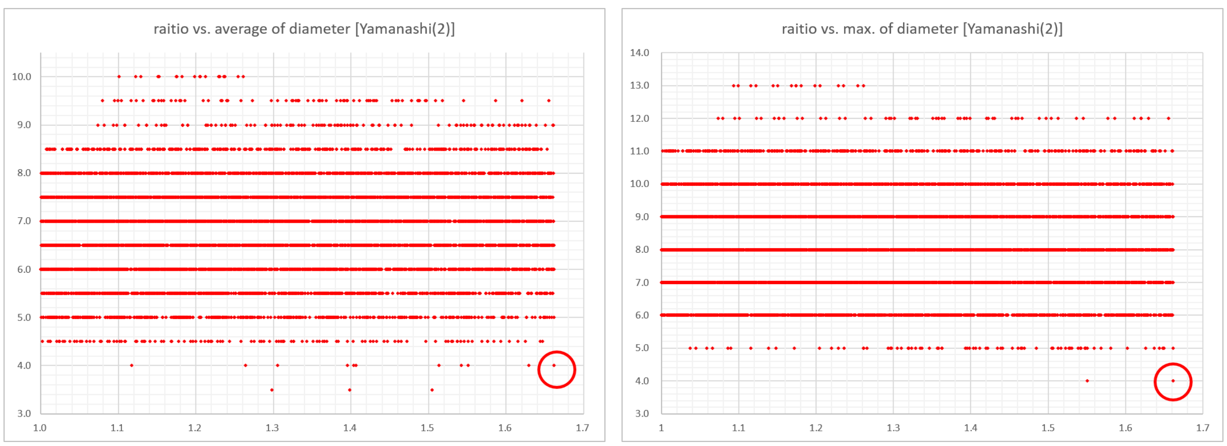

Figure 19: ratio vs diameter: Yamanashi(2) 9,546 sols.

Table 14: intimacy degree & diameter: Yamanashi(2)

1.662	current	86.4%		79.8%		4.00	4		
		intimacy degree				diameter			
0	# sols	ave.	80.0%	min.	both	ave.	max.	both	all
≦ 1.10	1,815	0	69	5	5	0	0	0	0
≦ 1.20	3,512	0	132	6	6	1	0	0	0
≦ 1.30	5,050	0	209	7	7	3	0	0	0
≦ 1.40	6,502	0	296	7	7	6	0	0	0
≦ 1.50	7,798	0	381	7	7	8	0	0	0
≦ 1.60	8,926	0	465	7	7	12	1	1	0
≦ 1.662	9,546	0	508	7	7	13	1	1	0

For Yamanashi refecture, all feasible solutions from the current district to the optimal one were enumerated, and the intimacy degree and diameter were calculated. As a result, there was no better solution than the current district for the average value of intimacy degree. Therefore, as in Ibaraki prefecture, the solutions with an average intimacy value of 80% or more are added in Table 14. Therefore, the 'both' column in the table means the number of solutions with an average intimacy degree of 80% or more and a minimum intimacy degree better than the current district, and the 'all' column means the number of solutions whose mean and maximum diameters are better than the current district with those 2 criteria. In addition, there is no solution with the average intimacy degree of 80% or more and the other three indices better than the current district (See 'all' in Table 14). It can be seen that the current district is not good in terms of gap in the value of individual votes, but it is good in terms of diameter (condition 4) and intimacy degree (condition 5).

In the middle of Table 15 and Figure 20, we show the solutions with a maximum difference of 1.551 times. This solution has the same average and maximum diameters as the current district, the average intimacy degree is more than 80%, and the minimum intimacy degree is worse than the current district.

Table 15: optimal, better solution, current district: Yamanashi(2)

optimal	ratio	inti.	dia.	better	ratio	inti.	dia.	current	ratio	inti.	dia.
398,001	1.000	78.5%	6	483,914	1.551	86.5%	4	496,915	1.662	92.9%	4
397,980	1.000	70.5%	6	312,067	1.000	74.5%	4	299,066	1.000	79.8%	4
	ave	74.5%	6.00		ave	80.5%	4.00		ave	86.4%	4.00

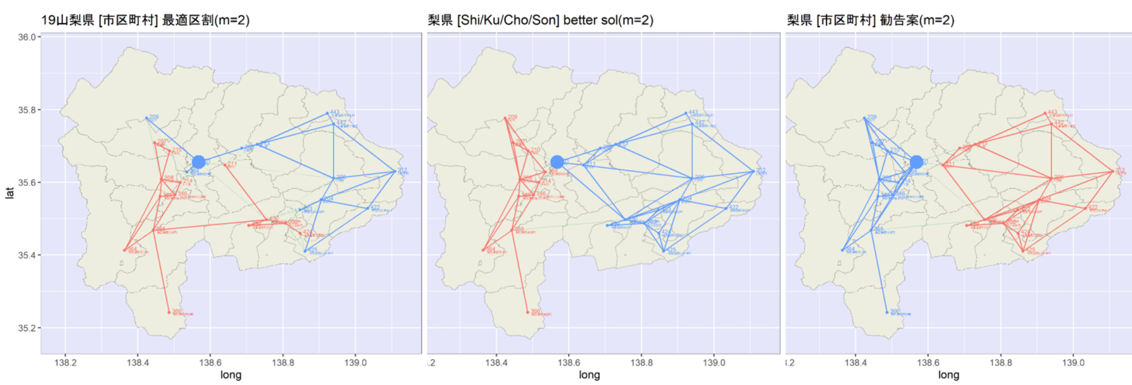

Figure 20: optimal, better solution, current district: Yamanashi(2)

There is a weak correlation between intimacy degree and diameter; Ibaraki(-0.394), Kyoto(-0.323), Miyagi(-0.375) and Yamanashi(-0.558). See Figure 21.

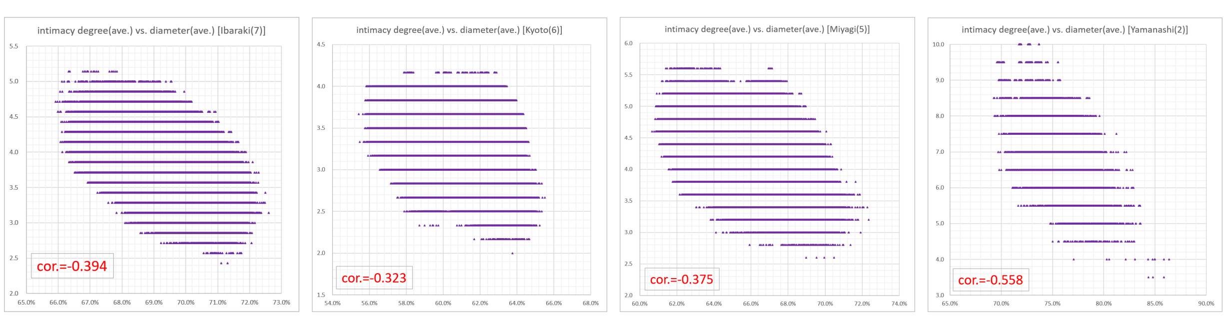

Figure 21: intimacy degree(ave) vs. diameter(ave)

3. Concluding Remarks

We evaluated and analyzed the single-seat constituencies of the House of Representatives, which were recommended in June 2022 and passed in November, through optimization and enumeration. By showing the limit value by optimization, we were able to highlight the problems of the work policy and system of electoral districting. In addition, we quantitatively showed the room for improvement in the maximum disparity of one vote for each prefecture in the current district, and also clarified the disparity in electoral districts. By enumerating all electoral districts and calculating some feature values, we clarified the bad points and good points of the current district. For example, the current districts of Ibaraki, Kyoto and Miyagi prefectures have a large difference from the limit value by the optimal solution, and it can be seen that there is a lot of room for improvement. Furthermore, by looking at enumerated solutions and their feature values, we were able to quantitatively clarify the existence of better districts. On the other hand, in Yamanashi prefecture, which has at most two electoral districts, the difference exceeds 1.6 times, and the difference from the optimum limit value is also large. However, it turned out to be a good division from the viewpoint of compactness (Condition 4) and regional connection (Condition 5). These will become clear only when all solutions are actually enumerated. It turns out that enumerating is very meaningful. In Europe and the United States, a better partitioning is found by heuristics for the purpose of various feature values, but in the Japanese single-seat electoral districting problem, it is possible to enumerate the solutions, so it is better to enumerate all the solutions and calculate the feature values.

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