Optimal Planning for the Electrification of Bus Fleets in Public Transit Systems

Filipe Cabral
School of Industrial and Systems Engineering, Georgia Institute of Technology, fcabral3@gatech.edu

Andy Sun
Sloan School of Management, Massachusetts Institute of Technology, sunx@mit.edu

Electric vehicles (EV) pave a promising way towards low-carbon transportation, but the transition to all EV fleets creates new challenges for the public transportation sector. Despite increasing adoption of electric buses, the main challenges presented by the battery electric bus technology include the lack of charging facilities, the reduced operating capacity per battery charge compared to fossil-fuel vehicles, and weather-induced degradation. Thus, the joint planning of electric bus fleets and charging infrastructure are essential to guarantee energy security of the transport service and the parsimony of investment. In this paper, we propose a multi-period investment model in which the transition to a 100% electric bus fleet and the expansion of the depot and on-route charging facilities are carried out jointly and gradually through bus retirement targets or annual budget constraints. An important feature of our model is the representation of two optimization time scales, one referring to yearly investment and the other to hourly operation; moreover, the hourly operation model captures the cyclic nature of the bus schedules as well as various EV charging strategies. We characterize the computational complexity of the proposed model and identify polynomially solvable problem subclasses. A primal heuristic algorithm is proposed that can significantly speed up Gurobi. Extensive computational experiments on public transit systems in major cities in the US and the world are carried out, using real data. Insights gained from real-world case studies are also explained through theoretical analysis.

Key words: Battery Electric Bus Fleet, Integer Programming, Complexity Analysis, Primal Heuristic

1. Introduction

Modern transportation relies heavily on fossil fuels. However, fossil fuel consumption endangers our world. According to the Intergovernmental Panel on Climate Change, IPCC (2021), carbon dioxide emission is the predominant cause of global warming and has already increased the global average temperature by one degree Celsius above the pre-industrial revolution level. An increase above two degrees Celsius can cause extreme weather events, a shortage of food supply, and higher sea levels. The United States, among many other countries, aims to reduce its net greenhouse gases (GHG) emissions by 50% below the 2005 levels in the coming decade, according to US Department of State (2021).

To curb the GHG emission, the world is seeking alternative energy sources. Worldwide, the transportation sector is the second largest contributor to greenhouse gas emissions EPA (2014) after the electricity industry, while in the US, it is the largest contributor EPA (2019). Although buses represent a fraction of the transportation segment, their effect on public health is significant, due to the fact that buses operate in densely populated urban areas and emissions such as nitrogen oxide and particulate matter adversely affects cardiovascular and respiratory health, see Bourdrel et al. (2017) and Ritz et al. (2019).

One attractive solution in the public transit segment is the Battery Electric Bus (BEB). A BEB produces zero tailpipe GHG emissions, its fuel cost is around 40% cheaper than a similar-sized fossil-fuel bus, its noise level is significantly lower, and it has less maintenance need. There has been an ever-growing number of transportation agencies all over the world switching to BEBs as a more sustainable option for public transportation Bus Sustainable (2020).

However, BEB technology poses new challenges to the bus operation planning, fleet sizing, and the charging placement due to the limited travel range and long charging hours. The optimal charging infrastructure plan may vary with the spatial distribution of the routes. The minimum fleet size to maintain the same service level may increase to compensate for the charging time. Indeed, when planning the transition to an entirely electric bus fleet, one should consider the fleet sizing, charging placement, and the impact on bus operation altogether.

There are two typical charging technologies that are adopted on a commercial scale, namely depot and on-route charging. Depot charging refers to charging BEBs with a low-voltage alternating current (AC) system in a bus depot or garage, which has lower deployment and usage costs but requires several hours to fully charge a BEB. On-route charging, in contrast, uses a direct current

(DC) fast charging system. It has a higher cost than a depot charger, but can be used in bus terminals for fast charging during the layover time of a bus to provide the energy needed for a round trip on a bus route.

In this paper, we propose a novel integer linear optimization model for the joint optimal planning and operation of depot and on-route charging facilities and BEB fleets. This model plans the transition to an entirely BEB fleet through annual investment targets which consider the agency's budget, the conventional bus retirement targets, and the operation of the mixed fleet of BEBs and conventional buses in transition. The model is realistic in capturing periodic bus schedules, BEB charging dynamics, various investment and operational costs, with depot locations, bus routes, and bus demand extracted from real transit agency data.

1.1. Literature Review

The scientific literature on electric vehicles (EVs) has investigated a wide variety of modeling techniques and applications. In this section, we mention some recent papers that are related to electrical bus fleet planning and operations.

Kleindorfer et al. (2012) presents a model for EV fleet renewal from the French national postal service. Mak et al. (2013) use distributionally robust optimization to plan the battery swapping infrastructure. The work of Schneider et al. (2014) evaluates the implementation of battery swapping for EVs in freight logistics, while Avci et al. (2015) consider the environmental impact of the adoption of electric passenger vehicles. Montoya et al. (2017) consider an EV routing model with a nonlinear charging function. Shen et al. (2019) present a literature review of other models related to EV infrastructure planning, EV charging operations, and public policy in the EV industry.

The work of Li (2014) considers battery charging scheduling, battery swapping, and the bus scheduling of a mixed bus fleet including battery-electric, compressed natural gas, diesel, and hybrid-diesel buses. Abdelwahed et al. (2020) concentrate on fast charging scheduling of battery electric buses to minimize charging costs and power grid impact. Wang et al. (2020) focus on battery capacity fade and propose an optimization model, which considers the reduction in the storage capacity of batteries, to schedule the electric bus charge. The works of Kunith et al. (2017), Rogge et al. (2018), and Houbbadi et al. (2019) consider a planning model for the composition of an electrical bus fleet using depot charge BEBs. With regards to technological factors, the paper of Yildirim and Yildiz (2021) assesses the impact of wireless chargers on battery-electric bus scheduling. Panah et al. (2021) consider a hybrid solution of hydrogen and electric buses with the concept of multi-product charging stations.

With regards to modeling specific features, the papers by He et al. (2019) and Kunith et al. (2017) propose models for installing fast chargers focused on the electric demand charge, which is the cost associated with the variations in power demand. Trocker et al. (2020) assess the potential reduction of the peak demand charge by installing energy storage units for on-route fast chargers. The works of Wu et al. (2021) and He et al. (2020) propose a fast charging location model for battery-electric buses, considering the bus operation and the power distribution. Liu and Liang (2020) propose a stochastic model for managing the electric bus charge, the photovoltaic energy production, and energy storage systems. Csonka (2021) and Dirks et al. (2021) present a long-term multi-period model for the electric bus integration into urban bus networks.

It is also worth commenting on the diversity of modeling techniques associated with battery-electric buses. Zhang et al. (2021b) use bi-level programming to formulate an electrical transit route planning problem, where the upper level determines the route structure and charging station location while the lower level calculates the user cost. Uslu and Kaya (2021) propose a charger location model that describes the bus charging using queuing theory. Zhang et al. (2021a)'s work is based on a stochastic model for the interaction between bus charge and battery swap stations for taxi and bus fleets. Lin et al. (2019) propose a mixed-integer second-order cone programming model for the charging planning of battery-electric buses.

1.2. Contributions

1. **Modeling**: This paper proposes an Optimal Planning of Charging Facilities and Electric Bus Fleet (OPCF-EBF) model for public transit systems to optimally plan the transition to an entirely electric bus fleet with several innovative features.

(a) The two-time-scale structure of the OPCF-EBF model brings together long-term planning and short-term operation, with annual investment decisions of charging infrastructure and fleet composition over a decade-long transition horizon and hourly operation decisions of bus charging and scheduling over 24 hours in each planning year.

(b) Modular arithmetic is used to model the repeating of daily 24-hour bus demand in each planning year. Various charging strategies, such as early charging (charging before battery full depletion), idling (neither working nor charging), non-preemptive charging (charging until full), are modeled, together with practical planning strategies such as utilization of existing bus depots and route terminals as potential charging facilities, respecting of retirement schedules of conventional buses, and various realistic investment and operational costs.

2. **Characterizations**: We characterize the computational complexity of the proposed model and identify special structures in a subclass of the proposed model that can be polynomially solved through an interesting transformation.

(a) The proposed OPCF-EBF model is shown to be NP-hard through reduction from the uncapacitated facility location problem. Even with one planning period and two charging states for depot-charged BEBs, the OPCF-EBF model is still NP-hard, as the numbers of bus routes and charging depots grow.

(b) We show that an important class of OPCF-EBF problems, which has one bus route with arbitrary numbers of investment periods and charging states for depot-charged BEBs under a simple charging policy, has rich structure and is polynomially solvable.

3. **Algorithm**: We propose an effective and computationally scalable primal heuristic called "Policy-Restriction" that significantly outperforms and improves Gurobi.

4. **Real-world case studies**: We conduct extensive computational studies using real-world data from public transit systems in major cities in the U.S. and around the world, which reveal insights into the optimal investment and operational strategies. For example, an optimal investment decision tends to invest in depot chargers before on-route chargers; an optimal operational solution tends to use on-route BEBs to meet base-load bus demand and to use depot BEBs to meet peaking bus demand. These empirical insights are also explained through theoretical analysis.

1.3. Organization of the Paper

The rest of the paper is structured as follows. We introduce the OPCF-EBF model in Section 2. In Section 3, we analyze the complexity of the proposed OPCF-EBF model. Section 4 proposes a primal heuristic to solve the challenging large-scale integer optimization model. Section 5 reports real-world case studies with observed insights and theoretical analysis. Section 6 concludes the paper. The electronic companion contains all the proofs.

2. An optimal planning model for charging facility and battery electric bus fleet

The OPCF-EBF model is formulated as a two-stage problem in Section 2.1, where investment problem is in the first stage and the operational problem is the second-stage recourse. The detailed operational problem is formulated in Section 2.2.

2.1. The OPCF-EBF model

We first define all the investment parameters and investment decision variables before laying out the overall two-stage OPCF-EBF model.

2.1.1. Investment parameters Let Θ denote the set of yearly investment periods, (i.e., $\Theta = \{1, 2 \ldots, N\}$ for some $N \in \mathbb{Z}_+$). Let $\mathcal{I}$ be the set of depot sites, $\mathcal{J}$ be the set of bus routes, $\mathcal{K}$ be the set of relevant depot chargers, and $\mathcal{R}$ be the set of terminal stations available for on-route charging. In the model, we allow BEBs from different manufacturers, since different BEB models can have different charging times, battery capacities, unit costs, and ability to perform on-route charging. In particular, denote $\mathcal{B}_{depot}$ and $\mathcal{B}_{route}$ as the set of BEB models that can be charged by depot and on-route chargers, respectively.

2.1.2. Investment decision variables Let $x \in \{0,1\}^{|\mathcal{I}|\times|\Theta|}$ be the vector of binary variables such that $x_i^\theta = 1$ implies that the depot site $i \in \mathcal{I}$ can install depot chargers during the investment period $\theta \in \Theta$, and $x_i^\theta = 0$ otherwise. Let $y \in \mathbb{Z}_+^{|\mathcal{I}|\times|\mathcal{K}|\times|\Theta|}$ be the vector of integer variables whose entry y_{ik}^θ represents the number of depot chargers of a given plug type $k \in \mathcal{K}$ (e.g. levels 1 and 2 chargers) for a given site $i \in \mathcal{I}$ and investment period $\theta \in \Theta$. Let $\chi \in \mathbb{Z}_+^{|\mathcal{R}|\times|\Theta|}$ represent the number of on-route chargers at each terminal station $r \in \mathcal{R}$ and investment period $\theta \in \Theta$. Let $\eta \in \mathbb{Z}_+^{|\mathcal{B}_{depot}|\times|\mathcal{J}|\times|\Theta|}$ and $\widetilde{\eta} \in \mathbb{Z}_+^{|\mathcal{B}_{route}|\times|\mathcal{J}|\times|\Theta|}$ be the numbers of depot and on-route BEBs respectively along each route $j \in \mathcal{J}$, given the BEB type, and investment period. Let $\xi \in \mathbb{Z}_+^{|\mathcal{J}|\times|\Theta|}$ be the vector of conventional buses in each route $j \in \mathcal{J}$ and investment period $\theta \in \Theta$.

2.1.3. Two-stage OPCF-EBF model The OPCF-EBF model is formulated as a two-stage integer optimization model in the investment variables below.

$$
\begin{aligned}
\min \quad & \sum_{\theta\in\Theta} \gamma_\theta \cdot \Big(I_\theta(x,y,\chi,\eta,\widetilde{\eta},\xi) + F_\theta(x,y,\chi,\eta,\widetilde{\eta},\xi) \Big) && && \text{(1a)}\\
\text{s.t.} \quad & I_\theta(x,y,\chi,\eta,\widetilde{\eta},\xi) \le C_\theta, && \theta\in\Theta, && \text{(1b)}\\
& x_i^\theta \ge x_i^{\theta-1}, \quad y_{ik}^\theta \ge y_{ik}^{\theta-1}, \quad \chi_r^\theta \ge \chi_r^{\theta-1}, && i\in\mathcal{I},\ k\in\mathcal{K}\ r\in\mathcal{R},\ \theta\in\Theta, && \text{(1c)}\\
& \eta_{bj}^\theta \ge \eta_{bj}^{\theta-1}, \quad \widetilde{\eta}_{\bar{b}j}^\theta \ge \widetilde{\eta}_{\bar{b}j}^{\theta-1}, \quad \xi_j^\theta \le \xi_j^{\theta-1}, && b\in\mathcal{B}_{depot},\ \bar{b}\in\mathcal{B}_{route},\ j\in\mathcal{J},\ \theta\in\Theta, && \text{(1d)}\\
& \underline{Q}_{ik}^\theta x_i^\theta \le y_{ik}^\theta \le \overline{Q}_{ik}^\theta x_i^\theta, \quad 0 \le \chi_r^\theta \le \chi_{UB,r}^\theta, && i\in\mathcal{I},\ k\in\mathcal{K},\ r\in\mathcal{R},\ \theta\in\Theta, && \text{(1e)}\\
& \xi_{LB,j}^\theta \le \xi_j^\theta \le \xi_{UB,j}^\theta, && j\in\mathcal{J},\ \theta\in\Theta, && \text{(1f)}\\
& x_i^\theta \in \{0,1\},\ y_{ik}^\theta \in \mathbb{Z}_+,\ \chi_r^\theta \in \mathbb{Z}_+, && i\in\mathcal{I},\ k\in\mathcal{K},\ r\in\mathcal{R},\ \theta\in\Theta, && \\
& \eta_{bj}^\theta \in \mathbb{Z}_+,\ \widetilde{\eta}_{\bar{b}j}^\theta \in \mathbb{Z}_+,\ \xi_j^\theta \in \mathbb{Z}_+, && b\in\mathcal{B}_{depot},\ \bar{b}\in\mathcal{B}_{route},\ j\in\mathcal{J},\ \theta\in\Theta. && \text{(1g)}
\end{aligned}
$$

The objective function (1a) has two parts I_θ and F_θ for each investment period θ, where I_θ represents the investment related cost, F_θ is the operational cost associated with the infrastructure decision $(x,y,\chi,\eta,\widetilde{\eta},\xi)$, and γ_θ is a discount factor. The investment cost I_θ is defined as

$$
\begin{aligned}
I_\theta(x,\chi,y,\eta,\widetilde{\eta},\xi) = & \sum_{i\in\mathcal{I}} c_{x,i}^\theta (x_i^\theta - x_i^{\theta-1}) + \sum_{(i,k)\in\mathcal{I}\times\mathcal{K}} c_{y,ik}^\theta (y_{ik}^\theta - y_{ik}^{\theta-1}) + \sum_{r\in\mathcal{R}} c_{\chi,r}^\theta (\chi_r^\theta - \chi_r^{\theta-1}) \\
& + \sum_{(b,j)\in\mathcal{B}_{depot}\times\mathcal{J}} c_{\eta,bj}^\theta (\eta_{bj}^\theta - \eta_{bj}^{\theta-1}) + \sum_{(b,j)\in\mathcal{B}_{depot}\times\mathcal{J}} c_{\widetilde{\eta},bj}^\theta (\widetilde{\eta}_{bj}^\theta - \widetilde{\eta}_{bj}^{\theta-1}) \\
& + \sum_{j\in\mathcal{J}} c_{\xi,j}^\theta \cdot (\xi_j^\theta - \xi_j^{\theta-1}),
\end{aligned} \tag{2}
$$

which is incurred on the incremental change of charging facilities and bus fleet in year θ. The vectors c_x, c_y, c_χ, c_η, $c_{\widetilde{\eta}}$, and c_ξ in (2) represent the unit cost of the corresponding decisions x, y,

χ, η, $\widetilde{\eta}$, and ξ. The cost c_ξ signifies the financial benefit of retiring a conventional bus, which can be also interpreted as a penalty for using fossil fuel-based buses. The operational cost F_θ is given by a recourse problem in operational decisions and will be presented in Section 2.2.

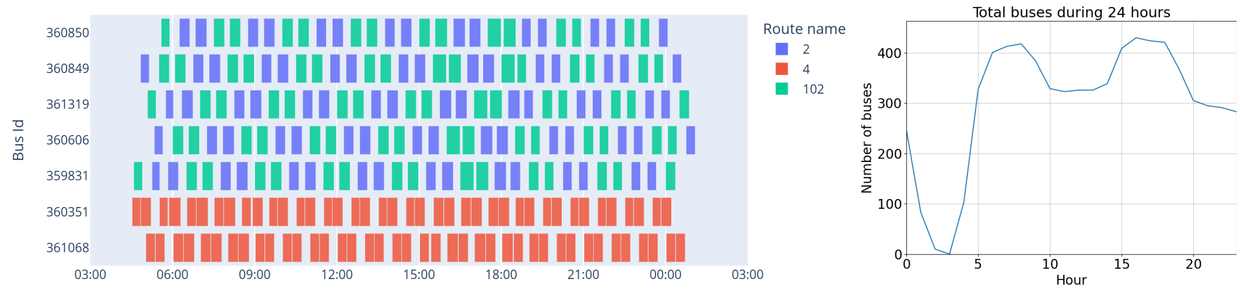
In terms of constraints, we have the budget constraint (1b) on the investment costs, where C_θ is the investment budget in period θ. Constraints (1c) and (1d) describe the monotone expansion of the charging infrastructure and the BEB fleet, and the monotone reduction of the conventional bus fleet. We have the upper and lower bounds (1e) on the numbers of depot and on-route chargers, where $\underline{Q}_{ik}^{\theta}$ and $\overline{Q}_{ik}^{\theta}$ are the upper and lower bounds on the number of depot plugs y_{ik}^{θ} given that the depot site i is open (i.e., $x_i^\theta = 1$), while $\chi_{UB,r}^{\theta}$ is the upper bound on the number of on-route chargers χ_r^θ. The bus retirement target constraint (1f) has upper and lower bounds $\xi_{LB,j}^{\theta}$ and $\xi_{UB,j}^{\theta}$ for the number of conventional buses ξ_j^θ in each year θ.

2.2. The operational problem as recourse

The goal of the operational problem is to find an optimal daily schedule for the charging and operation of a mixed fleet of BEBs and conventional buses in an investment period with a given investment decision. We use the information on existing bus routes and schedules published by public transit agencies, see MobilityData IO (2021), and assume that the mixed fleet should operate on the same routes and satisfy the same bus demand as in the current system. To obtain the bus demand for each hour and route, we count the number of operating buses from the published bus schedules. See Figure 1a as an illustration of the bus schedules for routes 2, 4, and 102 on a weekday in Atlanta's MARTA system and Figure 1b for the total bus demand in Atlanta on a weekday in August 2019.

In the following sections, we describe the operational problem as a recourse to the investment decisions. Sections 2.2.1, 2.2.2, and 2.2.3 describe the parameters, decisions, and constraints, which are put together in Section 2.2.4 to formulate the operational problem, whose optimal objective value is the operational cost F_θ in the overall model (1a).

Figure 1 Atlanta MARTA bus schedule for routes 2,4, and 102, and the total bus demand in August 2019.



(a) Weekday bus schedule for routes 2, 4, and 102.

(b) Weekday total bus demand.

2.2.1. Parameters in the operational problem.

1. **Time related parameters:** Let $[0:T-1]$ be the set of time intervals $\{0,1,\ldots,T-1\}$ of the operational horizon, which is treated as the cyclic group $\mathbb{Z}/T\mathbb{Z}$ of integers modulo T. In this way, the operation becomes cyclic, i.e., the operation at time $t=T-1$ loops back to time $t=0$ as the next time step. This construction models stationary, periodic operation rather than transient operation with fixed initial ($t=0$) and final conditions ($t=T-1$). If each $t\in[0:T-1]$ is an hourly interval and $T=24$, then the operational problem models the repeated daily bus operation.

2. **Charging related parameters:** Different types of BEBs may have different battery capacities, thus we define $[1:W_b+1]:=\{1,\ldots,W_b+1\}$ as the set of battery states of a depot BEB of type $b\in\mathcal{B}_{depot}$, where $s=1$ and $s=W_b+1$ denote the fully charged and the fully discharged states, respectively. The value W_b is the lowest battery state in which it is still safe to operate a depot BEB. The battery state index increases with time, that is, if a depot BEB is in a state s when it operates in time interval t, then it must be in the battery state $s+1$ at time $t+1$ to model the battery discharging in one interval of operation. Let $\mathcal{P}$ be the set of all pairs of depot BEB types and battery states, i.e., $\mathcal{P}:=\{(b,s):b\in\mathcal{B}_{depot},\, s\in[1:W_b+1]\}$. Let L_{bks} be the number of time intervals needed to fully charge a depot BEB of type $b\in\mathcal{B}_{depot}$ from state $s\in[1:W_b+1]$ to $s=1$ using a depot charger of type $k\in\mathcal{K}$. For instance, suppose an hourly interval operational model with $T=24$ hours. Consider a depot BEB b with battery capacity $W_b=12$ hours. If a depot charger k takes 6 hours to fully charge b, then we have $L_{bk,13}=6$. One can use a linear interpolation

rule to define the charging time L_{bks} for other states of charges $s \in [1:W_b]$. This means that if the BEB b starts to charge at time t with initial state $s=13$, then it will be fully charged at time $t+6$.

3. **Routes related parameters:** Let $\mathcal{J}(r)$ be the set of bus routes associated with terminal station r. Let $\mathcal{R}(j)$ be the set of terminal stations that are connected to the bus route j. We assume it is possible to accommodate up to CH_r on-route charging activities within one operational time interval at the terminal station r. Denote by $\mathcal{Q}$ the set of all pairs of routes and terminal stations, i.e., $\mathcal{Q} := \{(j,r) : j \in \mathcal{J},\, r \in \mathcal{R}(j)\}$.

2.2.2. Decision variables in the operational problem. Given an investment period $\theta \in \Theta$, consider the decisions $w^\theta \in \mathbb{Z}_+^{\mathcal{N}_{d,op}}$ and $v^\theta \in \mathbb{Z}_+^{\mathcal{N}_{d,op}}$ as the number of depot BEBs that are working and idling (i.e. neither working nor charging) respectively, where the set $\mathcal{N}_{d,op}$ is the Cartesian product $\mathcal{P} \times \mathcal{J} \times [0:T-1]$ and represents all the indices for w^θ and v^θ, including the depot BEB model, battery state, bus route, and time interval. Moreover, let $z \in \mathbb{Z}_+^{\mathcal{N}_{d,ch}}$ represent the battery state at which a group of depot BEBs starts charging, where $\mathcal{N}_{d,ch}$ is defined as $\mathcal{P} \times \mathcal{I} \times \mathcal{J} \times \mathcal{K} \times [0:T-1]$. Finally, the decision $\beta^\theta \in \mathbb{Z}_+^{\mathcal{N}_{d,beta}}$ contains the number of depot BEBs that are currently charging regardless of the charging state, where $\mathcal{N}_{d,beta}$ is defined as $\mathcal{B} \times \mathcal{J} \times [0:T-1]$.

Let $\widetilde{w}^\theta \in \mathbb{Z}_+^{\mathcal{N}_{route}}$ and $\widetilde{v}^\theta \in \mathbb{Z}_+^{\mathcal{N}_{route}}$ be the decisions that represent the number of on-route BEBs that are working and idling respectively, for each terminal station, route, and time interval. The set $\mathcal{N}_{route}$ is defined as $\mathcal{Q} \times [0:T-1]$. Let $\phi^\theta \in \mathbb{Z}_+^{\mathcal{N}_{conv}}$ and $\sigma^\theta \in \mathbb{Z}_+^{\mathcal{N}_{conv}}$ be the numbers of conventional buses working and idling for each route and time interval, where $\mathcal{N}_{conv}$ is $\mathcal{J} \times [0:T-1]$. Finally, let $u^\theta \in \mathbb{Z}_+^{\mathcal{N}_{slack}}$ be the slack variable of the demand constraint, where $\mathcal{N}_{slack}$ is $\mathcal{J} \times [0:T-1]$.

2.2.3. Constraints of the operational problem.

1. **Bus demand satisfaction:** Given $(j,t,\theta) \in \mathcal{J} \times [0:T-1] \times \Theta$, we have the demand constraint

$$\sum_{(b,s)\in\mathcal{P}} w_{bjs}^{t,\theta} + \sum_{(b,r)\in\mathcal{B}_{route}\times\mathcal{R}} \widetilde{w}_{bjr}^{t,\theta} + \phi_j^{t,\theta} + u_j^{t,\theta} \geq d_j^{t,\theta}, \tag{3}$$

where the bus demand $d_j^{t,\theta}$ must be satisfied by the total number of working depot BEBs (the first term on the left), working on-route BEBs (the second term), working conventional buses (the third term), and the slack variable $u_j^{t,\theta}$ (the fourth term).

2. **Depot BEB dynamics:** For all $t \in [0:T-1]$ and $(b,j,\theta) \in \mathcal{B}_{depot} \times \mathcal{J} \times \Theta$, we have the depot BEB charging dynamic equations

$$w_{bj1}^{t,\theta} + v_{bj1}^{t,\theta} = \sum_{s=2}^{W_b+1} \sum_{(i,k)\in\mathcal{I}\times\mathcal{K}} z_{bijks}^{(t-L_{bks}),\theta} + v_{bj1}^{(t-1),\theta}, \tag{4a}$$

$$w_{bjs}^{t,\theta} + \sum_{(i,k)\in\mathcal{I}\times\mathcal{K}} z_{bijks}^{t,\theta} + v_{bjs}^{t,\theta} = w_{bj(s-1)}^{(t-1),\theta} + v_{bjs}^{(t-1),\theta}, \qquad s \in [2:(W_b+1)], \tag{4b}$$

$$w_{bj(W_b+1)}^{t,\theta} = 0, \quad z_{bijk1}^{t,\theta} = 0. \tag{4c}$$

Equation 4a states that the total number of fully charged ($s=1$), working and idling depot BEBs at time t (the two terms on the left) must equal to the total number of depot BEBs that just finished charging at time t (the first term on the right) plus the fully charged idle depot BEBs at time $t-1$ (the second term on the right). The same dynamics applies to the partially charged state $s \in [2:(W_b+1)]$ in (4b). Equation (4c) enforces that the depot BEBs cannot work if fully depleted (the first equation) and cannot charge if fully charged (the second equation). Recall that all time indices are cyclic modulo T. Also note that these equations represent a non-preemptive charging policy (i.e. charging must continue until fully charged), which is practical for depot BEB charging and assumed throughout the paper.

3. **On-route BEB and conventional bus dynamics:** The dynamic equations for on-route BEBs and conventional buses are

$$\widetilde{w}_{bjr}^{t,\theta} + \widetilde{v}_{bjr}^{t,\theta} = \widetilde{w}_{bjr}^{(t-1),\theta} + \widetilde{v}_{bjr}^{(t-1),\theta}, \tag{5a}$$

$$\phi_j^{t,\theta} + \sigma_j^{t,\theta} = \phi_j^{(t-1),\theta} + \sigma_j^{(t-1),\theta}, \tag{5b}$$

for all $(j,\theta) \in \mathcal{J} \times \Theta$, $r \in \mathcal{R}(j)$, $b \in \mathcal{B}_{route}$, and $t \in [0:T-1]$. Equations (5a) and (5b) are conservation of on-route and conventional buses over each time interval, respectively. Because the on-route charging is accommodated within an operational time interval no state-of-charge index is needed.

4. **Bounds on depot BEBs simultaneously being charged:** The number of depot BEBs being charged and the upper bound by the number of depot chargers are given below

$$\beta_{bijk}^{t,\theta} - \sum_{s=2}^{W_b+1} \sum_{l=0}^{L_{bks}-1} z_{bijks}^{(t-l),\theta} = 0, \tag{6a}$$

$$\sum_{b\in\mathcal{B}_{depot}}\sum_{j\in\mathcal{J}}\beta^{t,\theta}_{bijk}\leq y^{\theta}_{ik}, \tag{6b}$$

for all $(i,j,k,\theta)\in\mathcal{I}\times\mathcal{J}\times\mathcal{K}\times\Theta$, $b\in\mathcal{B}_{depot}$, and $t\in[0:T-1]$. Here, equation (6a) has the total number of type b depot BEBs that are being charged at time t, depot i, route j, using charging plug type k. Equation (6b) is an upper bound on $\beta^{t,\theta}_{bijk}$ by the total number of depot chargers y^{θ}_{ik} that are invested.

5. **On-route charging capacity:** The number of working on-route BEBs that can charge at a given terminal is bounded by the following constraint

$$\sum_{b\in\mathcal{B}_{route}}\sum_{j\in\mathcal{J}(r)}\widetilde{w}^{t,\theta}_{bjr}\leq CH_r\cdot\chi^{\theta}_{r}, \tag{7}$$

for all $(r,\theta)\in\mathcal{R}\times\Theta$, and $t\in[0:T-1]$. Recall that CH_r is the charging capacity of an on-route charger over a time interval and χ^{θ}_{r} is the number of on-route chargers on route r, investment period θ.

6. **Total numbers of BEBs and conventional buses:** The link between the operational variables and the total number of BEBs and conventional buses is given below

$$\sum_{s=1}^{W_b}w^{0,\theta}_{bjs}+\sum_{s=1}^{W_b+1}v^{0,\theta}_{bjs}+\sum_{i\in I}\sum_{k\in K}\beta^{0,\theta}_{bijk}=\eta^{\theta}_{bj},\quad b\in\mathcal{B}_{depot}, \tag{8a}$$

$$\sum_{r\in\mathcal{R}(j)}\widetilde{w}^{0,\theta}_{bjr}+\widetilde{v}^{0,\theta}_{bjr}=\widetilde{\eta}^{\theta}_{bj},\quad b\in\mathcal{B}_{route}, \tag{8b}$$

$$\phi^{0,\theta}_{j}+\sigma^{0,\theta}_{j}=\xi^{\theta}_{j}, \tag{8c}$$

for all $(j,\theta)\in\mathcal{J}\times\Theta$. It is enough to relate the total number of buses to the operational variables at time $t=0$, because the dynamic equations (4)-(5) imply bus conservation, see Section 2.3.1.

2.2.4. The model of the operational problem Finally, we can formulate the operational problem using the constraints defined above:

$$\begin{aligned} F_\theta(x,y,\chi,\eta,\widetilde{\eta},\xi):=\min\quad & p_z^\top z^\theta+p_w^\top w^\theta+p_{\tilde{w}}^\top\widetilde{w}^\theta+p_\phi^\top\phi^\theta+p_u^\top u^\theta\\ \text{s.t.}\quad & (3)-(8),\\ & w^\theta,v^\theta\in\mathbb{Z}_+^{\mathcal{N}_{d,op}},\ z^\theta\in\mathbb{Z}_+^{\mathcal{N}_{d,ch}},\ \beta^\theta\in\mathbb{Z}_+^{\mathcal{N}_{d,beta}},\\ & \widetilde{w}^\theta,\widetilde{v}^\theta\in\mathbb{Z}_+^{\mathcal{N}_{route}},\ \phi^\theta,\sigma^\theta\in\mathbb{Z}_+^{\mathcal{N}_{conv}},u^\theta\in\mathbb{Z}_+^{\mathcal{N}_{slack}}. \end{aligned} \tag{9}$$

Some observations are instructive regarding the operational costs. The cost p_z contains the unit electricity cost for charging a depot BEB plus the deadhead cost of a trip between a route and a depot charging site. The p_w and p_ϕ represent the unit costs of operating depot BEBs and conventional buses, respectively, which are essentially the bus driver costs. The $p_{\tilde{w}}$ contains the unit electricity cost associated with the incremental charge at on-route stations plus the bus driver cost. The p_u represents the penalty for the demand constraint violation.

2.3. Properties of the optimal planning of model

2.3.1. Conservation of the total number of buses As our model does not track individual buses, but rather only tracks the total number of buses in different states, it would be assuring to verify that the total number of each type of buses in the fleet is conserved over operating times within each investment period. Indeed, Eq. (5a) and (8b) imply that the total number of on-route BEBs counted in interval t is equal to the total number of invested on-route BEBs $\widetilde{\eta}_{bj}^{\theta}$ as

$$\sum_{r\in\mathcal{R}(j)} \widetilde{w}_{bjr}^{t,\theta} + \widetilde{v}_{bjr}^{t,\theta} = \widetilde{\eta}_{bj}^{\theta}, \qquad b \in \mathcal{B}_{route}, \tag{10}$$

for all $(j,\theta)\in\mathcal{J}\times\Theta$ and $t\in[0:T-1]$. The conservation of the total number of conventional buses follows analogously as $\phi_j^{t,\theta}+\sigma_j^{t,\theta}=\xi_j^{\theta}$ for all $j\in\mathcal{J}$, $t\in[0:T-1]$, and $\theta\in\Theta$. For depot BEBs, the conservation is stated in Lemma 1 and the proof is referred to the Electronic Companion EC.1.

Lemma 1. *The total number of depot BEBs is constant through the operational horizon. That is, the following equality holds*

$$\sum_{s=1}^{W_b} w_{bjs}^{t,\theta} + \sum_{s=1}^{W_b+1} v_{bjs}^{t,\theta} + \sum_{i\in I}\sum_{k\in K} \beta_{bijk}^{t,\theta} = \eta_{bj}^{\theta}, \quad t\in[0:T-1],\ (b,j,\theta)\in\mathcal{B}_{depot}\times\mathcal{J}\times\Theta. \tag{11}$$

3. Computational complexity

3.1. Complexity of the OPCF-EBF model

The OPCF-EBF problem defined in (1)-(9) is NP-hard. The idea of the proof is to create a mapping between the charging depot and bus terminals in the OPCP-EBF and the facilities in an uncapacitated facility location problem. Moreover, some special classes of the OPCF-EBF problem are already NP-hard as shown in the following theorem. The proof is given in EC.2.1.

Theorem 1. *The OPCF-EBF problem is NP-hard. Even an OPCF-EBF problem with a single investment period and only depot BEBs of two battery states or only on-route BEBs is NP-hard.*

3.2. A polynomial time solvable class: the fleet sizing problem

In this subsection, we explore another dimension of the model with only one bus route and only depot BEBs, but allow an arbitrary number of battery states and investment periods. We call this a fleet-sizing problem. We show that, under a simple non-preemptive charging strategy with no early charging and idling, the following fleet-sizing problem is polynomially solvable.

$$\min_{\eta} \quad \sum_{\theta\in\Theta} c_\eta^\theta \cdot \eta^\theta + \widetilde{F}_\theta(\eta^\theta) \tag{12a}$$

$$\text{s.t.} \quad \eta^{\theta-1} \le \eta^\theta, \qquad \eta_{LB}^\theta \le \eta^\theta \le \eta_{UB}^\theta, \qquad \eta^\theta \in \mathbb{Z}_+, \qquad \theta \in \Theta. \tag{12b}$$

Here, $\widetilde{F}_\theta(\eta^\theta)$ is the operational cost of a depot BEB fleet of size η^θ during the investment period θ, which is given by the following operational problem

$$\widetilde{F}_\theta(\eta^\theta) = \min_{w,z} \quad \sum_{t\in[0:T-1]} \left(\sum_{s=1}^{W} p_{w,s}^{t,\theta} \cdot w_s^{t,\theta} + p_z^{t,\theta} \cdot z^{t,\theta} \right) \tag{13a}$$

$$\text{s.t.} \quad \sum_{s=1}^{W} w_s^{0,\theta} + \sum_{l=0}^{L-1} z^{-l,\theta} = \eta^\theta, \tag{13b}$$

$$w_1^{t,\theta} = z^{t-L,\theta}, \quad w_s^{t,\theta} = w_{s-1}^{t-1,\theta}, \quad z^{t,\theta} = w_W^{t-1,\theta}, \quad t \in [0:T-1], \tag{13c}$$

$$\sum_{s=1}^{W} w_s^{t,\theta} \ge d^{t,\theta}, \qquad t \in [0:T-1], \tag{13d}$$

$$w_s^{t,\theta}, \ z^{t,\theta} \in \mathbb{Z}_+, \qquad t \in [0:T-1], \ s \in [1:W], \tag{13e}$$

which minimizes the total working and charging cost in (13a), subject to the total number of depot BEBs equal to η^θ in (13b), the non-preemptive policy (13c) with non-stop working buses that start charging once it reaches the depleted battery state W and resume operation immediately after fully charged, and the bus demand constraint (13d).

The strategy to prove the polynomial solvability of (12)-(13) has three steps. First, we show that the operational problem (13) has a tight LP relaxation, thus a tight convex lower envelope

(Theorem 2). Then, using this, (12) can be viewed as a nonlinear integer program with a convex piecewise linear objective. Exploiting such a view allows us to reformulate and solve (12)-(13) through a small number of LPs (Theorem 3). Lastly, we do a detailed complexity analysis to show it is polynomial time to solve all the LPs involved (Theorem 4).

To carry out the first step, note that the constraints of (13) may *not* be totally unimodular (TU). However, interestingly, by exploiting the rich symmetry imposed by the non-preemptive charging policy and the modular arithmetic in (13), we can reformulate and unimodularly transform (13) to an equivalent formulation that does have the TU property (see Lemmas EC.1 and EC.2). Based on this, we can reach the following conclusion. The proof is given in EC.2.2.1.

THEOREM 2. *The domain of $\widetilde{F}_\theta$ is a set of integer multiples of $(W+L)/k$, where k is the greatest common divisor of $W+L$ and T, that is,* $\operatorname{dom}(\widetilde{F}_\theta) \subseteq \left\{ \frac{i(W+L)}{k} \in \mathbb{Z} \;\middle|\; i \in \mathbb{Z} \right\}$. *Moreover, the LP relaxation of* (13) *gives a tight lower convex envelope of $\widetilde{F}_\theta$, i.e., it is equal to $\widetilde{F}_\theta(\eta^\theta), \forall \eta^\theta \in \operatorname{dom}(\widetilde{F}_\theta)$.*

To carry out the second step, we first rescale the fleet variables η^θ of the fleet sizing problem (12) to the appropriate domain of $\widetilde{F}_\theta$ using the change of variables $\overline{\eta}^\theta = \frac{k}{W+L} \cdot \eta^\theta$. Define the new objective cost$c^\theta_{\overline{\eta}} = \frac{W+L}{k} \cdot c^\theta_\eta$, the new lower and upper bounds $\overline{\eta}^\theta_{LB} = \left\lceil \frac{k}{W+L} \cdot \eta^\theta_{LB} \right\rceil$ and $\overline{\eta}^\theta_{UB} = \left\lfloor \frac{k}{W+L} \cdot \eta^\theta_{UB} \right\rfloor$, and the new value function $\overline{F}_\theta(\overline{\eta}^\theta) = \widetilde{F}_\theta\left(\frac{W+L}{k} \cdot \overline{\eta}^\theta\right)$. By Theorem 2, $\overline{F}_\theta(\cdot)$ can be extended to a extended-real-valued convex piecewise linear function. Thus, (12) is essentially a separable convex integer program (SCIP), separable over θ. Now the key result, Theorem 3, shows that (12), viewed as an SCIP, can be further reformulated as a new integer linear program (15), which has an exact LP relaxation. Underlying this result is a proximity result proved for general SCIP that an optimal integer solution of SCIP is close to its LP relaxation's optimal solution (Theorem EC.2).

The new IP (15) takes as parameters an optimal solution, denoted as $\overline{\eta}_{*,LR}$, of the linear relaxation of (12)-(13), as well as a discretization of the convex function $\overline{F}_\theta(\cdot)$, which can be obtained by solving some LPs. In particular, let h^{θ}_{LB} and h^{θ}_{UB} be the smallest and largest integer $h \in [0:2|\Theta|]$ such that $\overline{F}_{\theta}(\lfloor \overline{\eta}^{\theta}_{*,LR} \rfloor - |\Theta| + h) < \infty$, and let $q^{\theta}_{\delta,h}$ be the cost vector defined as

$$q^{\theta}_{\delta,h} = \begin{cases} \overline{F}_{\theta}(\lfloor \overline{\eta}^{\theta}_{*,LR} \rfloor - |\Theta| + h^{\theta}_{LB}), & \text{if } h = h^{\theta}_{LB}, \\ \overline{F}_{\theta}(\lfloor \overline{\eta}^{\theta}_{*,LR} \rfloor - |\Theta| + h) - \overline{F}_{\theta}(\lfloor \overline{\eta}^{\theta}_{*,LR} \rfloor - |\Theta| + h - 1), & \text{if } h \in [h^{\theta}_{LB} + 1 : h^{\theta}_{UB}], \\ 0, & \text{if } h \notin [h^{\theta}_{LB} : h^{\theta}_{UB}], \end{cases} \tag{14}$$

for all $h = 0, \dots, 2|\Theta|$ and $\theta \in \Theta$. Thus, we have Theorem 3, which is proved in EC.2.2.2.

THEOREM 3. *The fleet sizing problem* (12) *can be reformulated as the following integer program:*

$$\min_{\overline{\eta},\delta} \sum_{\theta \in \Theta} \left(c^{\theta}_{\overline{\eta}} \cdot \overline{\eta}^{\theta} + \sum_{h=0}^{2|\Theta|} q^{\theta}_{\delta,h} \cdot \delta^{\theta}_{h} \right) \tag{15a}$$

$$s.t.\ \overline{\eta}^{\theta-1} \le \overline{\eta}^{\theta}, \qquad \overline{\eta}^{\theta}_{LB} \le \overline{\eta}^{\theta} \le \overline{\eta}^{\theta}_{UB}, \qquad \theta \in \Theta, \tag{15b}$$

$$\overline{\eta}^{\theta} - \sum_{h=0}^{2|\Theta|} \delta^{\theta}_{h} = \lfloor \overline{\eta}^{\theta}_{*,LR} \rfloor - |\Theta| - 1, \qquad \theta \in \Theta, \tag{15c}$$

$$\delta^{\theta}_{h} = 1, \qquad h \in [0 : h^{\theta}_{LB}],\ \theta \in \Theta, \tag{15d}$$

$$\delta^{\theta}_{h} = 0, \qquad h \in [h^{\theta}_{UB} + 1 : 2|\Theta|],\ \theta \in \Theta, \tag{15e}$$

$$\overline{\eta}^{\theta} \in \mathbb{Z}_{+},\ \delta^{\theta}_{h} \in \{0,1\}, \qquad h = 0, \dots, 2|\Theta|,\ \theta \in \Theta. \tag{15f}$$

An optimal solution $(\overline{\eta}_{*}, \delta_{*})$ *to* (15) *exists if and only if an optimal solution to* (12) *exists. The objective function requires* $O(|\Theta|)$ *evaluations of the value function* $\overline{F}_{\theta}(\cdot)$*, for each* $\theta \in \Theta$*. The constraint matrix induced by* (15b)-(15f) *is TU. Thus,* (15) *can be solved exactly by its LP relaxation.*

Finally in the last step, we show that all the LPs involved, i.e. the LP relaxation of (12)-(13), the evaluations of $\overline{F}_{\theta}(\cdot)$ in (14), and the LP relaxation of (15), can be solved in polynomial time by an algorithm of Vaidya (1990). Let k be defined as in Theorem 2 and $\mathbb{L}$ be the size of the integer program (12) (see (EC.38)). The theorem below is proved in EC.2.2.3.

THEOREM 4. *An optimal integral solution of* (12) *can be obtained in* $O\Big((k^3|\Theta|^3 + |\Theta|^6)\mathbb{L}\Big)$ *arithmetic operations.*

4. Primal Heuristic Method

The OPCF-EBF model (1)-(9) is an extremely challenging large-scale integer linear program. The state-of-the-art commercial solver such as Gurobi cannot obtain a good feasible solution within a reasonable computation time as will be shown in the computation part. After explorations of various computation methods, it became evident the need for primal heuristics to warm-start Gurobi. In this section, we describe a primal heuristic called the *Policy Restriction*.

Policy Restriction heuristic. This heuristic restricts the operation dynamics of depot BEBs to reduce the primal solution search. Indeed, we denote the set of positive demand time intervals as $T^{service}_{j,\theta} = \{t \in [0:T-1] \mid d^{t,\theta}_j > 0\}$ and refer to it as the service times. Analogously, we define the set of zero-demand time intervals as $T^{off}_{j,\theta} = [0:T-1] \backslash T^{service}_{j,\theta}$ and refer to it as off-service times.

The Policy Restriction heuristic prevents the depot BEBs from charging at any state s different from the depleted state $W_b + 1$ during the service times, that is,

$$z^{t,\theta}_{bijks} = 0, \qquad \forall t \in T^{service}_{j,\theta},\ s \in [2:W_b],\ (b,i,j,k,\theta) \in \mathcal{B}_{depot} \times \mathcal{I} \times \mathcal{J} \times \mathcal{K} \times \Theta. \tag{16}$$

It also prevents the depot and on-route BEBs from being idle during service times, with the exception of depot BEBs when fully charged ($s = 1$):

$$v^{t,\theta}_{bjs} = 0, \qquad \forall t \in T^{service}_{j,\theta},\ s \in [2:(W_b+1)],\ (b,j,\theta) \in \mathcal{B}_{depot} \times \mathcal{J} \times \Theta, \tag{17}$$

$$\widetilde{v}^{t,\theta}_{bjr} = 0, \qquad \forall t \in T^{service}_{j,\theta},\ r \in \mathcal{R}(j),\ (b,j,\theta) \in \mathcal{B}_{route} \times \mathcal{J} \times \Theta. \tag{18}$$

The idea of the above restriction is to use fully charged depot BEBs when it is most convenient in terms of cost. Lastly, the number of working depot and on-route BEBs must be zero during off-service times $t \in T^{off}_{j,\theta}$:

$$w^{t,\theta}_{bjs} = 0, \qquad \forall t \in T^{off}_{j,\theta},\ s \in [1:W_b],\ (b,j,\theta) \in \mathcal{B}_{depot} \times \mathcal{J} \times \Theta, \tag{19}$$

$$\widetilde{w}^{t,\theta}_{bjr} = 0, \qquad \forall t \in T^{off}_{j,\theta},\ r \in \mathcal{R}(j),\ (b,j,\theta) \in \mathcal{B}_{route} \times \mathcal{J} \times \Theta. \tag{20}$$

One advantage of the Policy Restriction heuristic is that it always leads to a feasible solution.

Proposition 1. *The OPCF-EBF problem with the Policy Restriction constraints is feasible.*

5. Case studies and analysis

We present in Section 5.1 a bus electrification plan for the Metropolitan Atlanta Rapid Transit Authority (MARTA) of Atlanta and a battery sensitivity analysis in Section 5.2 for bus electrification of the Massachusetts Bay Transportation Authority (MBTA) of Boston using depot BEBs. Section 5.3 highlights the performance of the primal heuristic compared to Gurobi over 11 US and 2 non-US cities using real data. Finally, Section 5.4 provides an analytical explanation for the patterns of investment and operation decisions observed in the computation studies.

5.1. Atlanta MARTA case study: Bus electrification plan

The data used in this case study corresponds to a weekday bus schedule and it is based on the MARTA GTFS file available at MobilityData IO (2021) from August 2019, before the COVID-19 pandemic. In 2019, Atlanta had 110 bus routes, from which there were 115 terminal stops that could serve as possible locations to install on-route chargers. We assume an installation capacity of 2 on-route chargers per terminal station, where each can serve up to 8 on-route BEBs each hour.

The bus depots operated by MARTA are taken as potential depot charging sites, with a total of five depots identified through CPTDB (2021), see the "D" marks in Figure 2. We use geospatial images to estimate the maximum installation capacity of depot chargers in each depot. The only depot charger considered in this study is a 70kW AC charger that costs \$60.05k. The on-route 325kW DC charger costs \$877.59k and both values comprise purchase, installation, and maintenance over 10 years Johnson et al. (2020).

We consider two models of BEBs in our studies. The first model is the New Flyer 40-foot BEB with a 160 kWh battery capacity, 6 hours of operational capacity when fully charged, and it requires 3 hours to fully charge using a depot charger. The New Flyer BEB has the on-route charging capability and costs \$943k each. The second model is the BYD 40-foot BEB with a 351 kWh battery capacity. We assume the BYD BEB has a 12-hour operational capacity and it requires 6 hours to fully charge. However, the BYD model does not have the on-route charging capability and it costs \$1,093k per unit.

Atlanta bus fleet electrification plan. The solution of our model gives an annual investment plan in depot and on-route chargers and BEBs over 10 years, summarized in Table 1. The investment plan is guided by the conventional bus retirement targets based on MARTA (2021), which is column '# Conv. buses" (e.g. -5 means retiring 5 conventional buses). All other columns are obtained from our numerical solution.

Table 1 Investment plan for MARTA on charging facilities and bus fleet units.

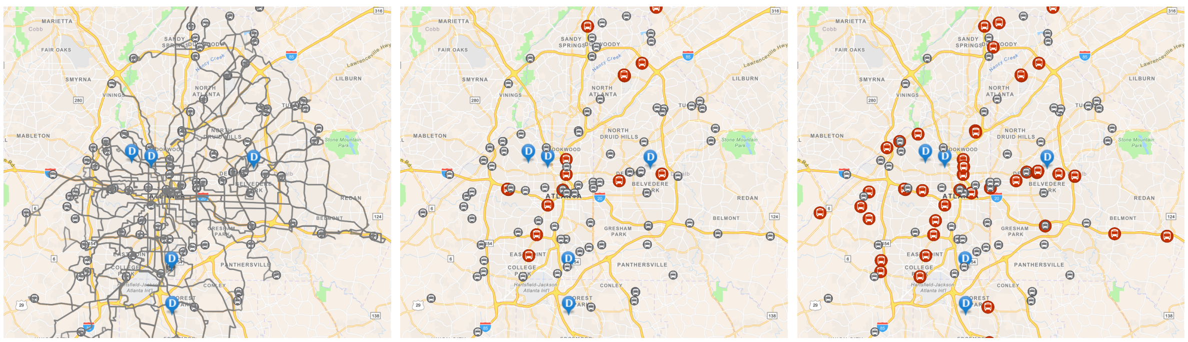
Year	# Depot BEBs	# On-route BEBs	# Conv. buses	# Depot chargers	# On-route chargers	Invest. cost ($ Million)	Op. cost ($ Million)
0	2	3	-5	1	2	$6.53	$38.14
1	11	0	-11	5	0	$10.26	$36.76
2	66	6	-72	33	2	$67.18	$36.87
3	46	27	-73	35	13	$72.92	$37.08
4	1	72	-72	2	14	$69.06	$35.71
5	1	27	-28	2	6	$25.94	$34.32
6	0	23	-23	0	3	$19.05	$32.94
7	1	46	-45	0	4	$35.98	$31.72
8	0	50	-49	0	5	$37.21	$30.43
9	19	40	-63	0	11	$45.25	$29.98
Total	147	294	-441	78	60	$389.39	$343.95

One interesting observation from Table 1 is that, during the first four years (years 0-3), investment is primarily on depot BEBs and chargers, but from year 4 onwards, the investment shifts towards on-route BEBs and chargers. A similar investment pattern is also observed in other cities, see Section 5.3.

Also from Table 1, the replacement factor of the conventional bus fleet is 1, that is, the total number of retired conventional buses is equal to the total number of the added depot and on-route BEBs. The yearly investment cost of such an investment plan remains below $70 million, except in year 3, with the total investment cost equal to $390 million. The total operational cost over 10 years is comparable to the investment cost.

The spatial distribution of on-route chargers from our numerical solution is depicted in Figure 2. The D markers represent the five depots used as charging depots, the small grey circles represent potential on-route charging locations, while the larger red circles are the installed on-route chargers. Generally, the model suggests the installation of on-route chargers from the area of the greatest confluence of bus routes in the downtown area towards the periphery of the city as shown in years 3 and 9 in Figures 2b and 2c.

Figure 2 Spatial distribution of on-route charge stations for Atlanta.

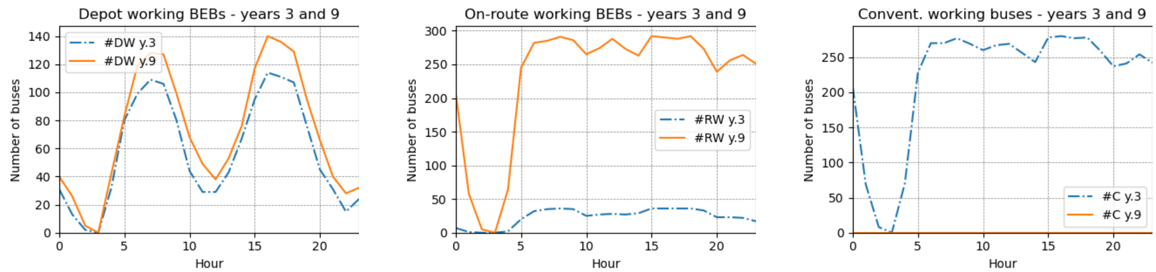


(a) Bus transportation network. (b) # Open stations: 16 – year 3. (c) # Open stations: 49 – year 9.

Operation of the BEB fleet. To understand how the mixed fleet of BEBs and conventional buses is operated by our model, we present the operational schedule of working depot BEBs, on-route BEBs, and conventional buses over 24 hours during investment years 3 and 9 in Figure 3. From Figure 3c, we note that the conventional buses meet the part of the demand that is essentially constant throughout the day, named base demand, during year 3, and diminish to zero in year 9.

Meanwhile, as seen in Figure 3a, the depot BEBs accommodate the rush-hour demand fluctuation for both years 3 and 9. The most likely explanation is that the depot BEB New Flyer 40ft (160 KWh) is the cheapest option, its 6 hours battery performance is sufficient to cover each rush wave, and the 3 hours charging time is less than the in-between rush hour times. We observed that the number of the other type of BEBs, namely the BYD BEBs, obtained in the solution is almost zero, which is possibly due to its purchase cost being slightly higher than the New Flyer (about 16% higher per BEB). As Figure 3b shows, the number of on-route BEBs from year 3 is insignificant

Figure 3 Fleet operational dynamics over 24 hours per bus type.



(a) Depot working (DW) BEBs. (b) On-route working (RW) BEBs. (c) Conventional (C) buses.

in comparison with the total bus demand but in year 9 the on-route BEBs essentially replaced the conventional bus fleet from year 3, see also Figure 3c. In summary, we observe that depot BEBs accommodate the variation in demand during rush hour waves, while on-route BEBs are responsible for handling the base demand. This pattern is partially explained in Section 5.4.

5.2. Boston MBTA case study: Battery sensitivity analysis

In this section, we present a case study on the Massachusetts Bay Transportation Authority (MBTA) of Boston, Massachusetts. The report MBTA (2021) points out that their electrification strategy considers only depot BEBs and a type of diesel-electric hybrid bus. The justification for this strategy instead of an entirely electric fleet is that during the winter season the efficiency of a depot BEB drops to 4 hours of operation due to the use of heaters. MBTA's plan is to use hybrid buses to retire most of the old conventional diesel buses in order to meet the GHG reduction target set for 2030, US Department of State (2021).

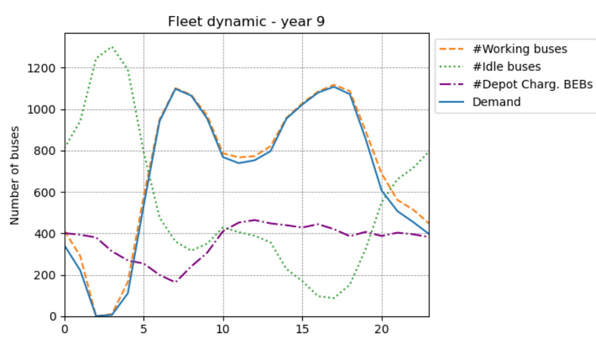
Based on this scenario, we carry out a sensitivity analysis for the MBTA's 10-year investment plan, assuming only depot BEBs with battery performance values of 4, 6, 8, 10, and 12 hours, and a charging time of 4 hours. These numbers are chosen based on the assumption that the insulation system and the battery capacity of electric buses may improve in the near future. The maximum demand for buses in this case study is 1108 buses and the result is summarized in Table 2. The column "Battery (h)" contains the battery performance in hours of operations for the depot BEBs; the column "# Depot BEBs" contains the number of depot BEBs needed to replace the conventional bus fleet, while maintaining the same level of service; the column "Ratio" is the ratio between the number of depot BEBs and the number of retired conventional buses. Note that

Table 2 Sensitivity analysis of the operating capacity depot BEBs for the MBTA case study.

Battery (h)	# Depot BEBs	Ratio
4	1902	1.72
6	1638	1.48
8	1625	1.47
10	1610	1.45
12	1518	1.37

the number of depot BEBs needed to replace the conventional bus fleet decreases as the battery capacity increases. To illustrate the need of extra depot BEBs, we present in Figure 4 the mixed fleet operating dynamics in year 9, in particular, the total numbers of working, charging, and idling depot BEBs with 8 hours of battery capacity. The curve of working depot BEBs is very close to the bus demand which indicates the same bus service level. But to compensate for the charging time, the depot BEBs require a coordinated operation that involves around 27% of the fleet constantly charging and the idle BEBs to start working at the specific times of day, as can be seen by the first and second rush waves.

Figure 4 Depot BEBs' operational dynamics over 24 hours with a battery capacity of 8 hours.



Thus, one cannot expect a replacement ratio equal to 1 using exclusively depot BEBs if their battery capacity is not enough to operate through the entire service day. In comparison, a mixed fleet solution in the Atlanta study involves a large proportion of on-route BEBs. If the deployment of on-route chargers may delay the replacement of conventional buses and negatively impact the GHG reduction goal set for 2030 (see MBTA (2021)), then the use of the diesel-electric hybrid buses is a reasonable solution.

5.3. Multi-city study: commonalities and differences

We benchmark the efficiency of the Policy Restriction heuristic with respect to Gurobi's internal heuristic over a total execution time of four hours for 17 public transit systems with 11 US cities and 2 non-US cities. All computation is performed on a cluster with 86 processors Intel Xeon Skylake and 317 Gb of shared RAM memory.

Table 3 Primal heuristics optimality gap after 4 hours of computation.

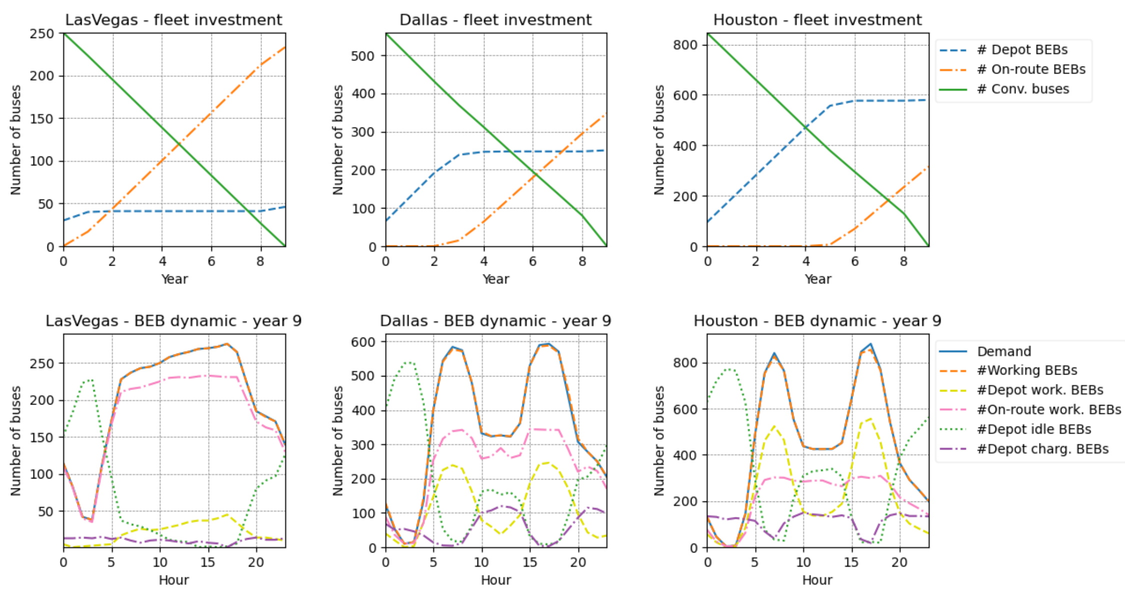
City	Gurobi gap	Policy-R gap	# Depot BEBs	# On-route BEBs	# Depot chargers	# On-route chargers	Invest. cost ($ Million)
Chicago	99.71%	7.62%	874	598	380	96	1256.15
Dallas	100.00%	11.23%	251	348	125	52	519.07
Houston	70.55%	8.78%	580	316	156	50	753.65
LasVegas	3.64%	1.82%	46	233	15	36	249.94
LosAngeles	99.21%	3.00%	1064	706	536	108	1507.30
NY (Bronx)	23.59%	8.23%	655	570	354	80	1046.20
NY (Brooklyn)	59.15%	8.42%	1481	1461	795	193	2512.65
NY (Manhatt.)	26.73%	4.32%	525	551	194	80	921.03
NY (Queens)	1.95%	6.08%	595	303	178	45	753.42
NY (St. Island)	0.32%	0.29%	565	52	105	6	498.63
San Francisco	3.95%	0.47%	745	572	169	82	1114.21
Seattle	3.61%	2.12%	169	26	29	4	159.01
Philadelphia	99.74%	7.56%	637	387	299	72	914.03
San Jose	7.15%	2.61%	209	198	48	42	406.33
Sydney	99.70%	3.84%	2249	726	838	115	2640.40
Toronto	99.75%	3.35%	982	808	456	154	1638.11
Washington DC	99.69%	3.24%	869	352	293	93	1058.05
Average Gap	52.85%	4.75%					
Std. Gap	43.33%	3.01%					

Table 3 shows the total number of depot BEBs and chargers, on-route BEBs and chargers, and total investment cost to preserve the same bus service level in those major cities. We assume for all instances a constant budget in every investment year and a target of 0 conventional bus in the last year. The following are some comments on the computational results.

Original: The solutions by Gurobi alone have an optimality gap of greater than 99% for 7 of the 17 instances and an average gap of 52.85%, even after four hours of computation.

Policy-R: Gurobi warm started by the Policy Restriction heuristic has a much lower optimality gap with quite stable results overall. All instances of the Policy-R heuristic have a gap smaller than 12% and the average gap is only 4.75%. Thus, the Policy-R heuristic is shown to be more reliable in producing a high-quality solution than using Gurobi alone. Below we provide more details for some important features of the Policy-R solution.

Figure 5 Multicity case study - fleet investment in 10 years (top row) and 24-hour operation (bottom row).



The proportion of depot and on-route BEBs from Table 3 can be primarily explained by the bus demand shape of each instance. For this, Figure 5 shows the fleet investment evolution over 10 years and the BEB operation in year 9 for Las Vegas, Dallas, and Houston. Indeed, we observe that the number of working on-route BEBs are relatively constant over the day to cover the base bus demand, while the depot BEBs supply the difference between the total bus demand and the working on-route BEBs, thus covering the peak waves. As a secondary influence, a larger spatial distribution of the bus routes (e.g. in Houston) may hinder the use of on-route chargers and increase the gap between on-route and depot BEBs, as reflected in the difference between fleet investment in Houston versus in Las Vegas and Dallas in the top row of Figure 5.

In the BEB fleet evolution from Figure 5, we see a preference for depot BEBs in the early investment years until a saturation point and then the investment in on-route BEBs. This observation is consistent with the intuition that depot BEBs are cheaper to deploy than on-route BEBs and, because of the discount factor $\gamma_\theta \in (0,1)$, depot BEBs should be invested first. See Proposition EC.1 for a mathematical explanation of the role of the discount factor in ordering decisions.

5.4. Analysis of the mixed depot and on-route fleet strategy

Now we provide an analytical explanation for the interesting pattern observed in the above computational studies. Namely, in a mixed fleet, depot BEBs tend to cover peak demand and on-route BEBs tend to supply base demand. Consider a simplified fleet sizing problem with one route and one investment period, where the chargers and other infrastructure costs are aggregated into the BEB unit costs. We consider depot and on-route BEBs only, i.e., no conventional buses in the fleet sizing problem. Suppose that each time interval covers several hours and a depot BEB can only work for *one* time interval and need to fully charge in the next interval. For example, 24 hours can be partitioned to 4 time intervals, each of 6 hours, in a way to model peak and off-peak hours.

Let η and $\widetilde{\eta}$ be the total number of the depot and on-route BEBs with unit costs c_d, c_r, respectively, and let w_t, v_t, and z_t be the number of working, idling, and charging depot BEBs at time t. Let $\widetilde{w}_t$ and $\widetilde{v}_t$ be the number of working and idling on-route BEBs, and let d_t be the bus demand at time t. Assume there are no charging or working costs. Then, our simplified fleet sizing model is

$$\begin{aligned}
&\min\ c_d\eta + c_r\widetilde{\eta} \\
&\text{s.t. } \eta = w_0 + v_0 + z_0, && \widetilde{\eta} = \widetilde{w}_0 + \widetilde{v}_0, \\
&\quad w_t + v_t = z_{t-1} + v_{t-1}, && z_t = w_{t-1}, && \forall t \in [0:T-1], \\
&\quad \widetilde{w}_t + \widetilde{v}_t = \widetilde{w}_{t-1} + \widetilde{v}_{t-1}, && w_t + \widetilde{w}_t \geq d_t, && \forall t \in [0:T-1], \\
&\quad \eta, \widetilde{\eta}, w_t, \widetilde{w}_t, v_t, \widetilde{v}_t, z_t \in \mathbb{Z}_+, && && \forall t \in [0:T-1].
\end{aligned} \tag{21}$$

It turns out the optimal solution of (21) can be obtained in closed form.

PROPOSITION 2. *For every scenario of unit costs c_d and c_r, the optimal number of the depot and on-route BEBs to* (21) *and the corresponding numbers of working BEBs for each time interval $t \in [0:T-1]$ is obtained in Table 4, where $D_1 := \max_{t\in[0:T-1]} d_t$ and $D_2 := \max_{t\in[0:T-1]}(d_t + d_{t-1})$. Moreover, the optimal solution of the variables z_t, v_t, and $\widetilde{v}_t$ is given by the relations $z_t = w_{t-1}$, $v_t = \eta - w_t - z_t$, and $\widetilde{v}_t = \widetilde{\eta} - \widetilde{w}_t$.*

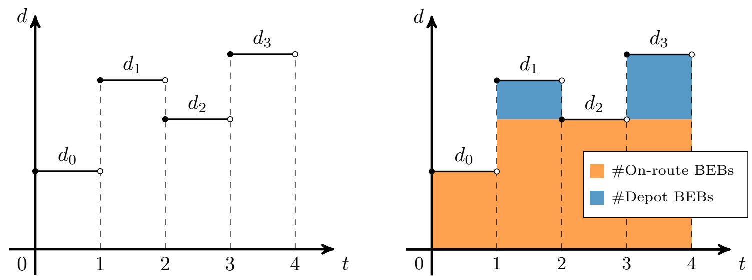
Table 4 Optimal solution table of (21) for each objective coefficients c_r and c_d.

Coeff.	η	$\widetilde{\eta}$	w_t	$\widetilde{w}_t$
$c_r \leq c_d$	0	D_1	0	d_t
$c_r \geq 2c_d$	D_2	0	d_t	0
$c_d < c_r < 2c_d$	$2D_1 - D_2$	$D_2 - D_1$	$\max\{d_t + D_1 - D_2, 0\}$	$\min\{D_2 - D_1, d_t\}$

The following example provides the intuition behind the optimal solution in Proposition 2 and how it matches the patterns observed in computation. Consider a 24-hour horizon partitioned to 4 time intervals: early morning $t = 0$, morning rush $t = 1$, off-peak $t = 2$, and evening rush $t = 3$. On this timescale, it is reasonable to assume that a depot BEB can work during only one time interval and needs to fully charge in a consecutive interval. Suppose the demand $\{d_t\}_{t=0}^3$ is such that $d_0 < d_2 < d_1 < d_3$ and $d_3 - d_2 < d_1 - d_0$, which mimics the buses' rush waves, see Figure 6a. Then, peak demand is $D_1 = d_3$ and the highest two-period demand $D_2 = d_2 + d_3$.

It is a reasonable approximation to assume that the deployment of an on-route BEB is more expensive than that of a depot BEB but less expensive than that of two depot BEBs, i.e., $c_d < c_r <$

Figure 6 Bus demand and the optimal number of working BEBs if $c_d < c_r < 2c_d$.



(a) Simplified demand. (b) Optimal working BEBs.

$2c_d$, since the cost of an on-route charger can be divided equally among the on-route BEBs. This implies that the optimal fleet is $\eta = d_3 - d_2$ and $\widetilde{\eta} = d_2$ and the optimal working BEBs are given by $w_t = \max\{d_t - d_2, 0\}$ and $\widetilde{w}_t = \min\{d_2, d_t\}$, for each $t \in [0:3]$, see the illustration of Figure 6b. Thus, the optimal operation is to use on-route BEBs for the base demand and depot BEBs for the rush wave fluctuations. In reality, the OPCF-EBF solution may suggest more depot BEBs since there may exist many routes without common terminals, which increases the unit cost c_r.

6. Conclusions

In this paper, we propose a novel investment planning model for the electrification of bus fleets and the building up of charging infrastructure for public transit systems. We carry out a detailed complexity analysis of the proposed model and develop an effective primal heuristic, which significantly speeds up Gurobi. We present two detailed case studies and a multi-city analysis. In the Atlanta case study, we present an investment plan that achieves a 1:1 replacement ratio of conventional buses and sheds light on the operation of the bus fleet in transition. In the Boston case study, we assess the sensitivity of the bus electrification plan with regard to BEB charging times, motivated by the significant weather-induced battery performance change in Boston winters. In the multi-city analysis, we observe that the proportion of depot and on-route BEBs is primarily dictated by the shape of the total bus demand curve. These patterns are also corroborated by an analytical study. Overall, the proposed model, algorithm, and analysis provide a valuable tool to facilitate public transit systems to carry out one of the most important and challenging tasks facing modern society, namely to electrify transportation in a timely and efficient manner.

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Proofs of Statements

EC.1. Conservation of the total number of depot BEBs

The goal of this section is to prove the Lemma 1 about the conservation of the total number of depot BEBs in the system. In other words, the number of depot BEBs either working, charging, or idling is constant over the entire operational horizon $t \in [0:T-1]$.

LEMMA 1. *The total number of depot BEBs is constant through the operational horizon. In other words, the following equality holds*

$$\sum_{s=1}^{W_b} w_{bjs}^{t,\theta} + \sum_{s=1}^{W_b+1} v_{bjs}^{t,\theta} + \sum_{i\in I}\sum_{k\in K} \beta_{bijk}^{t,\theta} = \eta_{bj}^{\theta}, \tag{EC.1}$$

for all $t \in [0:T-1]$ *and* $(b,j,\theta) \in \mathcal{B}_{depot} \times \mathcal{J} \times \Theta$.

Proof of Lemma 1. We prove equation (EC.1) by induction. The base case $t=0$ follows from the constraint (8a), so assume the cases $0,1,\ldots,t-1$. Denote by C_t the term $\sum_{s=1}^{W_b} w_{bjs}^{t,\theta} + \sum_{s=1}^{W_b+1} v_{bjs}^{t,\theta}$. Note that on the left-hand side of (EC.1) we have

$$\begin{aligned}
\sum_{s=1}^{W_b} w_{bjs}^{t,\theta} + \sum_{s=1}^{W_b+1} v_{bjs}^{t,\theta} + \sum_{i\in I}\sum_{k\in K} \beta_{bijk}^{t,\theta} &= C_t + \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}}\sum_{s=2}^{W_b+1}\sum_{l=0}^{L_{bks}-1} z_{ijks}^{(t-l),\theta} \\
&= C_t + \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}}\sum_{s=2}^{W_b+1}\left(\sum_{l=0}^{L_{bks}-2} z_{bijks}^{(t-l-1),\theta} + z_{bijks}^{t,\theta}\right),
\end{aligned} \tag{EC.2}$$

where we get the equation (EC.2) by splitting the sum $\sum_{l=0}^{L_{ks}-1} z_{bijks}^{(t-l),\theta}$ into the term $\sum_{l=1}^{L_{ks}-1} z_{ijks}^{(t-l),\theta}$ plus $z_{ijkst}^{t,\theta}$ and by re-indexing l to range from 0 to $L_{bks}-2$. We sum the depot transition equations (4a) and (4b) over the states of charge $s \in [1:W_b+1]$ to get

$$C_t + \sum_{i\in I}\sum_{k\in K}\sum_{s=2}^{W_b+1} z_{ijks}^{t,\theta} = C_{(t-1)} + \sum_{i\in I}\sum_{k\in K}\sum_{s=2}^{W_b+1} z_{ijks}^{(t-L_{ks}),\theta}. \tag{EC.3}$$

By replacing (EC.3) into (EC.2) and using the induction hypothesis, we conclude our proof:

$$\begin{aligned}
&\sum_{s\in[1:W]} w_{bjs}^{t,\theta} + \sum_{s=1}^{W_b+1} v_{bst}^{t,\theta} + \sum_{i\in I}\sum_{k\in K} \beta_{bijk}^{t,\theta} \\
&\overset{((\text{EC.2})+(\text{EC.3}))}{=} C_{(t-1)} + \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}}\sum_{s=2}^{W_b+1}\left(\sum_{l=0}^{L_{ks}-2} z_{ijks}^{(t-l-1),\theta} + z_{ijks}^{(t-L_{ks}),\theta}\right) \\
&= C_{(t-1)} + \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}}\sum_{s=2}^{W_b+1}\sum_{l=0}^{L_{bks}-1} z_{ijks}^{(t-1-l),\theta} \overset{(\beta\ \text{def.})}{=} C_{(t-1)} + \sum_{i\in I}\sum_{k\in K} \beta_{bijk}^{(t-1),\theta} = \eta_{bj}^{\theta}.
\end{aligned}$$

$\square$

EC.2. Complexity Analysis

EC.2.1. Complexity of the OPCF-EBF model

In this section, we prove that the Uncapacitated Facility Location (UFL) problem can be polynomially reduced to the single period *depot only* OPCF-EBF instance. In particular, this implies that the OPCF-EBF problem is NP-hard.

THEOREM 1 *The UFL problem can be polynomially reduced to the single period depot-only OPCF-EBF problem. In particular, the OPCF-EBF problem is NP-Hard.*

Proof of Theorem 1. Let us denote by $\lambda_i \in \{0,1\}$ the binary variable that corresponds to decision to open or not the facility $i \in [n] := \{1,\ldots,n\}$ and by $\pi_{ij} \in \{0,1\}$ the binary variable that corresponds to meet the demand of j-th client using the i-th installation. Consider the facility setup cost f_i associated with variable λ_i and the supply cost g_{ij} associated with λ_{ij}. Below, we present an instance of the UFL problem:

$$\begin{aligned}
\min\ & \textstyle\sum_{i=1}^{n} f_i\lambda_i + \sum_{i=1}^{n}\sum_{j=1}^{m} g_{ij}\pi_{ij} & \\
\text{s.t.}\ & \textstyle\sum_{i=1}^{n} \pi_{ij} = 1, & \forall j \in [m], \\
& \textstyle\sum_{j=1}^{m} \pi_{ij} \le m \cdot \lambda_i, & \forall i \in [n], \\
& \lambda_i \in \{0,1\},\ \pi_{ij} \in \{0,1\}, & \forall i \in [n],\ \forall j \in [m].
\end{aligned}$$

We now define the reduction to an instance of the OPCF-EBF problem starting with the set of indices. Consider just one type of depot BEB, $\mathcal{B}_{depot} = \{1\}$, but not a single on-route BEB, $\mathcal{B}_{route} = \emptyset$, n potential charging sites, $\mathcal{I} = \{1,\ldots,n\}$, m routes, $\mathcal{J} = \{1,\ldots,m\}$, only one plug type, $\mathcal{K} = \{1\}$, not a single on-route charging facility, $\mathcal{R} = \emptyset$, battery performance of one time-interval, $W_1 = 1$, operational horizon $T = 2$, and single investment period $\Theta = \{1\}$. To improve the presentation of this instance of the OPCF-EBF problem, we omit the sub-indices that have only one possible value such as the depot BEB type b, the plug type k, and the investment period θ.

Consider the lower $\underline{Q}_i$ and upper $\overline{Q}_i$ bounds of plugs as being equal to 0 and m, respectively, for all site $i \in \mathcal{I}$. Let d_{jt} be the demand for buses and let L_s be the charging time as defined below

$$d_{jt} = \begin{cases} 1, & \text{if } t = 0, \\ \\ 0, & \text{if } t = 1, \end{cases} \qquad , \qquad L_s = \begin{cases} 0, & \text{if } s = 1, \\ \\ 1, & \text{if } s = 2, \end{cases}$$

for every route $j \in \mathcal{J}$. Note that $s = 1$ is the fully charged state, and $s = 2$ is the fully depleted state, since $W = 1$. The idea of our construction is to have the depot BEBs working at time $t = 0$ and charging at time $t = 1$.

We assume that the initial infrastructure condition is zero, that is, $x_{i0} = 0$, $y_{i0} = 0$, $\eta_{j0} = 0$, and $\xi_{j0} = 0$, for all depots $i \in [n]$ and routes $j \in [m]$. If the initial condition of conventional buses is zero, $\xi_{j0} = 0$, then the number of conventional buses in the period of investment $\theta = 1$ is also zero, that is, $\xi_{j1} = 0$. This implies that the number of working $\phi_{jt\theta}$ and idling $\sigma_{jt\theta}$ conventional buses are zero for all time intervals $t \in [0:1]$, route $j \in [m]$, and investment period $\theta = 1$.

Let H be the constant $\sum_{i=1}^n f_i + \sum_{i=1}^n \sum_{j=1}^m g_{ij}$, and consider the unit cost of a depot BEB c_{beb} as $H + 1$. Let the investment budget C be equal to $\sum_{i=1}^n f_i + m \cdot c_{beb}$, which is essentially a large enough constant so all possible investments are feasible. Then the investment part of this OPCF-EBF instance is given below:

$$\begin{aligned}
\min \ & \textstyle\sum_{i=1}^n f_i x_i + \sum_{j=1}^m c_{beb} \cdot \eta_j + F(x, y, \eta) \\
\text{s.t. } & \textstyle\sum_{i=1}^n f_i x_i + \sum_{j=1}^m c_{beb} \cdot \eta_j \leq C, \\
& 0 \leq y_i \leq m \cdot x_i, && \forall i \in [n], \\
& x_i \in \{0, 1\}, \ y_i, \ \eta_j \in \mathbb{Z}_+, && \forall i \in [n], \ \forall j \in [m],
\end{aligned}$$

and there is no on-route and conventional bus variables since those are zero.

For the operational problem, we have two remarks regarding the depot working and charging variables w and z, respectively. We omit the state of charge $s = 1$ of the depot working variable w, since this is the only possible state for a working depot BEB given that $W = 1$. Similarly, we omit the state of charge $s = 2$ for z, since this is also the only possible state of charge for a depot

BEB to start charging in our instance. For the depot idling BEBs v, we keep the state of charge index $s \in [1:2]$ because it is possible for a depot BEB to be idle in both fully charged ($s=1$) and fully discharged ($s=2$) states. Let the demand constraint violation cost c_u be equal to $(m+1)\cdot c_{beb}$. Below, we present the operational part of our OPCF-EBF instance:

$$
\begin{aligned}
F(x,y,\eta) = \min\ & \textstyle\sum_{i=1}^{n}\sum_{j=1}^{m}\left[0\cdot z_{ij}^{0} + g_{ij}\cdot z_{ij}^{1}\right] + \sum_{j=1}^{m}\sum_{t=0}^{1} c_u \cdot u_{jt} \\
\text{s.t. }\ & w_j^t + u_j^t \geq d_j^t, && \forall j \in [m],\ \forall t \in [0:1] \\
& w_j^t + v_{j1}^t = \textstyle\sum_{i=1}^{n} z_{ij}^{(t-1)} + v_{j1}^{(t-1)}, && \forall j \in [m],\ \forall t \in [0:1], \\
& \textstyle\sum_{i=1}^{n} z_{ij}^t + v_{j2}^t = w_j^{(t-1)} + v_{j2}^{(t-1)}, && \forall j \in [m],\ \forall t \in [0:1], \\
& \beta_{ij}^t = z_{ij}^t, && \forall i \in [n],\ \forall j \in [m],\ \forall t \in [0:1], \\
& \textstyle\sum_{j\in[m]} \beta_{ij}^t \leq y_i, && \forall i \in [n],\ \forall t \in [0:1], \\
& \eta_j = \left(v_{j1}^t + v_{j2}^t + w_j^t + \textstyle\sum_{i=1}^{n}\beta_{ij}^t\right), && \forall j \in [m],\ \forall t = 0, \\
& w_j^t,\ v_{js}^t,\ z_{ij}^t,\ \beta_{ij}^t,\ u_j^t \in \mathbb{Z}_+, && \forall i \in [n],\ \forall j \in [m],\ \forall t \in [0:1], \\
& \forall s \in [1:2].
\end{aligned}
$$

Now that we have defined the instance of the OPCF-EBF, we focus on the reduction of the UFL problem. Let (λ, π) be a feasible solution of the UFL problem with an objective value less than or to K. Note that K is less than or equal to $H = \sum_{i=1}^{n} f_i + \sum_{i=1}^{n}\sum_{j=1}^{m} g_{ij}$, because H is an upper bound for the UFL objective cost. Consider the following OPCF-EBF-induced solution:

$$x_i = \xi_i, \quad y_i = m\cdot\lambda_i, \quad \eta_j = 1, \tag{EC.4a}$$

$$w_j^t = \begin{cases} 1, & \text{if } t = 0, \\ 0, & \text{if } t = 1, \end{cases} \qquad z_{ij}^t = \begin{cases} 0, & \text{if } t = 0, \\ \pi_{ij}, & \text{if } t = 1, \end{cases}, \tag{EC.4b}$$

$$v_{js}^t = 0, \quad \beta_{ij}^t = z_{ij}^t, \quad u_j^t = 0, \tag{EC.4c}$$

for every site $i \in [n]$, route $j \in [m]$, charge state $s \in [1:2]$, and time interval $t \in [0:1]$. The solution defined by the equations (EC.4a), (EC.4b), and (EC.4c) is feasible for the OPCF-EBF instance, and it has objective value equal to $\sum_{i=1}^{n} f_i\lambda_i + \sum_{i=1}^{n}\sum_{j=1}^{m} g_{ij}\pi_{ij} + c_{beb}\cdot m$, which is equal to the objective value of the UFL problem plus the constant $c_{beb} \cdot m$. Therefore, the objective value of the OPCF-EBF instance is less or equal to $K + c_{beb} \cdot m$, which is also less than or equal to $H + c_{beb} \cdot m$.

We now check the other side of the reduction. Consider a feasible solution of the OPCF-EBF instance $(x, y, \eta, w, v, z, \beta, u)$ with objective value $K + c_{beb} \cdot m$, where K is less than or equal to H. Such solution exist, since we can take $(x, y, \eta, w, v, z, \beta, u)$ as defined by (EC.4a), (EC.4b), and (EC.4c), and the following feasible solution (λ, π) for the UFL problem:

$$\lambda_i = \begin{cases} 1, & \text{if } i = 1, \\ 0, & \text{otherwise,} \end{cases} \qquad \pi_{ij} = \begin{cases} 1, & \text{if } i = 1, \\ 0, & \text{otherwise,} \end{cases}$$

for each site $i \in [n]$ and route $j \in [m]$.

The first observation regarding the feasible solution of the OPCF-EBF is that the the demand constraint violation u_{jt} is 0, and that the number of depot BEBs η_j equals 1, for every route $j \in [m]$ and interval $t \in [0:1]$. Indeed, the objective function

$$Obj := \sum_{i=1}^{n} f_i x_i + \sum_{j=1}^{m} c_{beb} \cdot \eta_j + \sum_{i=1}^{n} \sum_{j=1}^{m} g_{ij} z_{ij}^1 + \sum_{j=1}^{m} \sum_{t=0}^{1} c_u \cdot u_j^t$$

evaluated at the OPCF-EBF solution is such that $Obj \leq H + c_{beb} \cdot m$, by hypothesis, and from the choice of c_{beb} we have that $H < c_{beb}$. Thus, $Obj < c_{beb} \cdot (m+1)$. Since c_u equals $c_{beb} \cdot (m+1)$ this implies that u_{jt} is 0, for every route $j \in [m]$ and every time interval $t \in [0:1]$. The demand constraint $w_j^t + u_j^t \geq d_j^t$ at $t = 0$ implies that

$$\eta_j \geq w_j^0 \geq 1 - u_j^0 = 1, \qquad \forall j \in \mathcal{J}.$$

With this lower bound on η_j, we have that Obj satisfies $c_{beb} \cdot m \leq Obj < c_{beb} \cdot (m+1)$, which implies that both η_j and w_j^0 must be equal to 1 for every route $j \in [m]$.

The second observation is that the number of working w_j^t and idling v_{js}^t depot BEBs satisfy

$$w_j^t = \begin{cases} 1, & \text{if } t = 0, \\ 0, & \text{if } t = 1, \end{cases} \qquad \text{and} \qquad v_{js}^t = 0,$$

for every route $j \in [m]$, state of charge $s \in [1:2]$, and time interval $t \in [0:1]$. Indeed, because the depot BEBs have enough charge for only one time interval, all the buses must be charging at time $t=1$ to be able to work again at time $t=0$. This implies that $w_{j1}=0$, for all $j \in [m]$. Consequently, the number of idling depot BEBs v_{js}^t is equal to 0, for all $j \in [m],\ s \in [1:2],\ t \in [0:1]$.

The third observation is that the solution (λ, π) defined by

$$\lambda_i = x_i, \quad \pi_{ij} = z_{ij}^1,$$

is feasible for the UFL problem with objective value K less than or equal to H. From the state transition dynamics $w_j^t + v_{j1}^t = \sum_{i=1}^n z_{ij}^{(t-1)} + v_{j1}^{(t-1)}$ at time $t=1$, we conclude the identity $\sum_{i=1}^n z_{ij}^1 = 1$, for every route $j \in [m]$. It follows from $0 \le y_i \le m \cdot x_i$, $\beta_{ij}^t = z_{ij}^t$, and $\sum_{j\in[m]} \beta_{ij}^t \le y_i$ the constraint $\sum_{j\in[m]} z_{ij}^1 \le m \cdot x_i$, for every site $i \in I$. Note that z_{ij}^1 is a binary variable as there is only one BEB in each route. In particular,

$$K := \sum_{i=1}^n f_i \lambda_i + \sum_{i=1}^n \sum_{j=1}^m g_{ij} \pi_{ij} = Obj - c_{beb} \cdot m \le H,$$

and this concludes the reduction proof. $\square$

We note that one can prove a similar reduction from the UFL to the single period *on-route BEB* only OPCF-EBF.

EC.2.2. Complexity of the depot BEB fleet sizing with simple charging policy

In this section, we analyze the complexity of a simplified OPCF-EBF with a single route, depot BEB fleet only, unlimited depot charging capacity, and arbitrary numbers of battery states and investment periods:

$$\min_{\eta} \quad \sum_{\theta\in\Theta} c_\eta^\theta \cdot \eta^\theta + \widetilde{F}_\theta(\eta^\theta) \tag{EC.5a}$$

$$\text{s.t.} \quad \eta^{\theta-1} \le \eta^\theta, \qquad \eta_{LB}^\theta \le \eta^\theta \le \eta_{UB}^\theta, \qquad \eta^\theta \in \mathbb{Z}_+, \qquad \theta \in \Theta. \tag{EC.5b}$$

The value function $\widetilde{F}_\theta(\eta^\theta)$ represents the operational cost impact of the fleet η^θ at the investment period $\theta \in \Theta$. We omit the index θ to improve the exposition.

$$\widetilde{F}(\eta) = \min_{w,z} \quad \sum_{t\in[0:T-1]} \left(\sum_{s=1}^{W} p_{w,s}^t \cdot w_s^t + p_z^t \cdot z^t \right) \tag{EC.6a}$$

$$\text{s.t.} \quad \sum_{s=1}^{W} w_s^0 + \sum_{l=0}^{L-1} z^{-l} = \eta, \tag{EC.6b}$$

$$w_1^t = z^{t-L}, \qquad t \in [0:T-1], \tag{EC.6c}$$

$$w_s^t = w_{s-1}^{t-1}, \qquad t \in [0:T-1],\ s \in [2:W], \tag{EC.6d}$$

$$z^t = w_W^{t-1}, \qquad t \in [0:T-1], \tag{EC.6e}$$

$$\sum_{s=1}^{W} w_s^t \geq d^t, \qquad t \in [0:T-1], \tag{EC.6f}$$

$$w_s^t,\ z^t \in \mathbb{Z}_+, \qquad t \in [0:T-1],\ s \in [1:W]. \tag{EC.6g}$$

EC.2.2.1. Properties of the operational problem with simple charging policy Let $P(\eta)$ be the feasible polyhedron defined by the constraints (EC.6a)-(EC.6g). The goal of this section is to understand the properties of the polyhedron $P(\eta)$ and the value function $\widetilde{F}(\eta)$ for every integral coefficients $p_{w,s}^t, p_z^t, \eta \in \mathbb{Z}$.

Lemma EC.1 **(Variable Reduction)**. *Let k be the greatest common divisor of $(W+L)$ and T. The problem* (EC.6) *is equivalent to the following model in ζ variables only*

$$\widetilde{F}(\eta) = \min_{\zeta} \quad \sum_{i=0}^{k-1} p_{\zeta,i} \cdot \zeta_i \tag{EC.7a}$$

$$s.t. \quad \sum_{i=0}^{k-1} \zeta_i = \frac{k}{W+L} \cdot \eta, \tag{EC.7b}$$

$$\sum_{l=0}^{W-1} \zeta_{i-l} \geq \widetilde{d}_i, \qquad i \in [0:k-1], \tag{EC.7c}$$

$$\zeta_i \in \mathbb{Z}_+, \qquad i \in [0:k-1], \tag{EC.7d}$$

where the ζ indexes i's are equivalence classes modulo k. The coefficients $p_{\zeta,i}$ and $\widetilde{d}_i$ are defined as

$$p_{\zeta,i} = \sum_{\substack{t\in[0:T-1] \\ t\%k=i}} \left[p_z^t + \sum_{\substack{\tau\in[0:T-1],\ s\in[1:W], \\ s.t.\ \tau-s-L+1=t.}} p_{w,s}^\tau \right], \quad \textit{and} \quad \widetilde{d}_i = \max_{\substack{t\in[0:T-1] \\ t\%k=i}} d^{t+L}, \tag{EC.8}$$

for all $i \in [0:k-1]$, *and* $t\%k$ *represents the remainder of* t *divided by* k. *The map between the feasible solutions from problem* (EC.7) *and the original operational problem* (EC.6) *is given by the relation*

$$z^t = \zeta_{t\%k}, \quad \text{and} \quad w_s^t = z^{t-s-L+1}, \tag{EC.9}$$

for all $s \in [1:W]$ *and* $t \in [0:T-1]$. *In particular, the fleet size* η *must be a multiple of* $(W+L)/k$, *otherwise the original operational problem* (EC.6) *is infeasible.*

Proof of Lemma EC.1. First, it follows from (EC.6c)-(EC.6e) that $w_s^t = z^{t-s-L+1}$ for all charge states $s \in [1:W]$ and time intervals $t \in [0:T-1]$. This reduces the original operational problem (EC.6a) to the following:

$$\widetilde{F}(\eta) = \min_{z} \sum_{t\in[0:T-1]} \widetilde{p}_z^t \cdot z^t \tag{EC.10a}$$

$$\text{s.t.} \sum_{l=0}^{L+W-1} z^{-l} = \eta \tag{EC.10b}$$

$$z^t = z^{t-W-L}, \qquad t \in [0:T-1], \tag{EC.10c}$$

$$\sum_{l=0}^{W-1} z^{t-l-L} \geq d^t, \quad t \in [0:T-1], \tag{EC.10d}$$

$$z_t \in \mathbb{Z}_+, \qquad t \in [0:T-1], \tag{EC.10e}$$

where the cost vector $\widetilde{p}_z$ is defined as

$$\widetilde{p}_z^t = p_z^t + \sum_{\substack{\tau\in[0:T-1],\ s\in[1:W], \\ \text{s.t. } \tau-s-L+1=t.}} p_{w,s}^{\tau}, \tag{EC.11}$$

for all $t \in [0:T-1]$.

The equality constraint (EC.10c) creates a symmetry, i.e. a periodicity of $W+L$, on the z-variable space. Moreover, recall that t is an equivalence class modulo T, so $t+yT$ is equal to t for all $y \in \mathbb{Z}$. This fact together with the constraint (EC.10c) implies that

$$z^t = z^{t+x\cdot(W+L)+y\cdot T}, \tag{EC.12}$$

for every $x, y \in \mathbb{Z}$, and every $t \in [0:T-1]$. By the Bezout's identity, there are integers $\overline{x}$ and $\overline{y}$ such that $k = \overline{x} \cdot (W+L) + y \cdot T$, where k is the greatest common divisor of $(W+L)$ and T, and k is also the smallest positive integer given by any integral combination of $(W+L)$ and T. Thus, the number of distinct z variables is k, and the constraint (EC.12) can be equivalently represented as $z^t = z^{t+a\cdot k}$, for every $a \in \mathbb{Z}$ and every $t \in [0:T-1]$. Let ζ_i be defined as $\zeta_i = z^i$ for all $i \in [0:k-1]$. Because of the identity $z^t = z^{t+a\cdot k}$, we have that

$$z^t = \zeta_{t\%k}, \tag{EC.13}$$

so the total fleet constraint (EC.10b) can be described in terms ζ as

$$\eta = \sum_{l=0}^{W+L-1} z^{-l} = \sum_{l=0}^{\frac{(W+L)}{k}\cdot k-1} \zeta_{(-l)\%k} = \frac{(W+L)}{k} \cdot \sum_{i=0}^{k-1} \zeta_l. \tag{EC.14}$$

The demand constraint (EC.10d) in terms of the variables ζ becomes $\sum_{l=L}^{W+L-1} \zeta_{(t-l)\%k} \geq d_t$ for all $t \in [0:T-1]$. So, by the change of variable $t := t+L$, and by taking a maximum of the right-hand side demand d^{t+L} over $t \in [0:T-1]$ such that $t\%k = i$ we obtain the following expression:

$$\sum_{l=0}^{W-1} \zeta_{i-l} \geq \max_{\substack{t\in[0:T-1] \\ t\%k=i}} d^{t+L}, \tag{EC.15}$$

for all $t \in [0:T-1]$. Finally, the cost $p_{\zeta,i}$ follows from (EC.11) similarly by adding $\widetilde{p}_z^t$ over $t \in [0:T-1]$ such that $t\%k = i$, that is, $p_{\zeta,i} = \sum_{\substack{t\in[0:T-1] \\ t\%k=i}} \widetilde{p}_z^t$, for all $i \in [0:k-1]$. $\square$

Although the reduced problem (EC.7a) has a simpler structure compared to the original operational model (EC.6a), the new demand constraint (EC.7c) is inconvenient to analyze. Indeed, the wrap-around property of the indexes i's leads to a complicated expression for the summation $\sum_{l=0}^{W-1} \zeta_{i-l}$ in terms of $\zeta_0, \zeta_1, \ldots, \zeta_{k-1}$ with coefficients that may be greater than 1. In order to improve the analysis we perform a symmetry-breaking transformation, this time with a unimodular linear transformation $R : \mathbb{R}^k \to \mathbb{R}^k$ defined as

$$(R\zeta)_i = \sum_{l=0}^{i} \zeta_l, \quad \forall 0 \leq i \leq k-1. \tag{EC.16}$$

Its inverse R^{-1} is given by

$$(R^{-1}\overline{\zeta})_i = \begin{cases} \overline{\zeta}_0, & \text{if } i = 0, \\ \overline{\zeta}_i - \overline{\zeta}_{i-1}, & \text{if } 1 \le i \le k-1. \end{cases} \tag{EC.17}$$

Recall that a matrix M is called *unimodular* if M is a square integral matrix with determinant $+1$ or -1. The unimodularity property holds for R since it is an integral lower triangular matrix with ones in its main diagonal.

LEMMA EC.2 **(Unimodular transformation)**. *The change of variables* $\overline{\zeta} := R\zeta$ *applied to the reduced operational problem* (EC.7) *results in the following problem:*

$$\widetilde{F}(\eta) = \min_{\overline{\zeta}} \quad \sum_{i=0}^{k-1} \overline{p}_{\zeta,i} \cdot \overline{\zeta}_i \tag{EC.18a}$$

$$s.t. \quad \overline{\zeta}_{k-1} = \frac{k}{W+L} \cdot \eta, \tag{EC.18b}$$

$$\overline{\zeta}_i - \overline{\zeta}_{i-W} + \left(\left\lfloor \frac{W}{k} \right\rfloor + \mathbb{I}_{[i+1,\infty)}(W\%k) \right) \overline{\zeta}_{k-1} \ge \widetilde{d}_i, \quad i \in [0:k-1], \tag{EC.18c}$$

$$\overline{\zeta}_0 \ge 0, \tag{EC.18d}$$

$$\overline{\zeta}_i - \overline{\zeta}_{i-1} \ge 0, \quad i \in [1:k-1], \tag{EC.18e}$$

$$\overline{\zeta}_i \in \mathbb{Z}, \quad i \in [0:k-1], \tag{EC.18f}$$

where $\mathbb{I}_{[i+1,\infty)}(x)$ *is an indicator function that is* 1 *if* x *is greater than or equal to* $i+1$, *and* 0 *otherwise, and the cost coefficient* $\overline{p}_{\zeta,i}$ *is defined as*

$$\overline{p}_{\zeta,i} = \begin{cases} p_{\zeta,i} - p_{\zeta,i+1}, & \text{if } 0 \le i \le k-2, \\ p_{\zeta,k-1}, & \text{if } i = k-1, \end{cases} \tag{EC.19}$$

for all $i \in [0:k-1]$. *In particular, the polyhedron defined by the linear relaxation of* (EC.18) *is integral whenever* η *is a multiple of* $(W+L)/k$ *and thus* $\widetilde{F}(\eta)$ *is an extended real-valued convex piecewise linear function for continuous values of* η.

Proof of Lemma EC.2 Because $\zeta_i = \overline{\zeta}_i - \overline{\zeta}_{i-1}$, for every $i \in [1:k-1]$, and $\zeta_0 = \overline{\zeta}_0$. we can describe the left-hand side of the constraint (EC.7b) as $\sum_{i=0}^{k-1} \zeta_i = \overline{\zeta}_0 + \sum_{i=1}^{k-1} (\overline{\zeta}_i - \overline{\zeta}_{i-1}) = \overline{\zeta}_{k-1}$. Similarly for the left-hand side of (EC.7c). Indeed,

$$\sum_{l=0}^{W-1} \zeta_{i-l} = \sum_{l=0}^{k\left\lfloor \frac{W}{k} \right\rfloor - 1} \zeta_{i-l} + \sum_{l=k\left\lfloor \frac{W}{k} \right\rfloor}^{W-1} \zeta_{i-l} \tag{EC.20a}$$

$$= \left\lfloor \frac{W}{k} \right\rfloor \left(\zeta_i + \zeta_{i-1} + \cdots + \zeta_0 + \zeta_{k-1} + \zeta_{k-2} + \cdots + \zeta_{i+1} \right) + \sum_{l=k\left\lfloor \frac{W}{k} \right\rfloor}^{W-1} \zeta_{i-l} \tag{EC.20b}$$

$$= \left\lfloor \frac{W}{k} \right\rfloor \overline{\zeta}_{k-1} + \sum_{l=k\left\lfloor \frac{W}{k} \right\rfloor}^{W-1} \zeta_{i-l} \tag{EC.20c}$$

Since any integer W can be described as $W = k\left\lfloor \frac{W}{k} \right\rfloor + W\%k$, we have the following equalities for $\sum_{l=k\left\lfloor \frac{W}{k} \right\rfloor}^{W-1} \zeta_{i-l}$:

$$\sum_{l=k\left\lfloor \frac{W}{k} \right\rfloor}^{W-1} \zeta_{i-l} = \sum_{l=0}^{W\%k-1} \zeta_{i-l} = \begin{cases} \overline{\zeta}_i - \overline{\zeta}_{i-W\%k}, & \text{if } W\%k \leq i, \\ \overline{\zeta}_i - \overline{\zeta}_{i-W\%k} + \overline{\zeta}_{k-1}, & \text{if } W\%k \geq i+1, \end{cases} \tag{EC.21}$$

where the last equality follows from noting that the term $\overline{\zeta}_{k-1}$ is added to the final expression whenever ζ_0 appears in a consecutive summation. The expression (EC.18c) follows from (EC.21) because of the identity $i - W\%k = (i-W)\%k$ and that we can drop the remainder operator % since the indexes of ζ and $\overline{\zeta}$ are equivalence classes modulo k. The expression (EC.19) for the objective costs is straightforward.

Finally, we prove that the linear relaxation polyhedron induced by (EC.18b)-(EC.18f) is integral whenever η is a multiple of $(W+L)/k$. Indeed, the last variable $\overline{\zeta}_{k-1}$ is fixed and equal to $\frac{k}{W+L} \cdot \eta$, so we can replace it in every occurrence of $\overline{\zeta}_{k-1}$, which leads to an integral right-hand side vector. We conclude the integrality of linear relaxation polyhedron by noting that the constraint matrix associated to the variables $\overline{\zeta}_0, \ldots, \overline{\zeta}_{k-2}$ is totally unimodular since it has at most one $+1$ and -1 at each row. $\square$

We can now prove Theorem 2 using the properties of the reduced models.

THEOREM 2 *The domain of $\widetilde{F}_\theta$ is a set of integer multiples of $(W+L)/k$, where k is the greatest common divisor of $W+L$ and T, that is,* $\mathrm{dom}(\widetilde{F}_\theta) \subseteq \left\{ \frac{i(W+L)}{k} \in \mathbb{Z} \;\middle|\; i \in \mathbb{Z} \right\}$. *Moreover, the LP relaxation of* (13) *gives a tight lower convex envelope of $\widetilde{F}_\theta$, i.e., it is equal to* $\widetilde{F}_\theta(\eta^\theta), \forall \eta^\theta \in \mathrm{dom}(\widetilde{F}_\theta)$.

Proof of Theorem 2 From Lemmas EC.1 and EC.2, we know that the feasible solutions of the original operational problem (13) have a one-to-one correspondence with the feasible solutions of the reformulated model (EC.18). So, it is straightforward to note that the original operational problem is infeasible when η^θ is not a multiple of $(W+L)/k$.

Denote the polyhedron defined by the linear relaxation of the constraints (13b)-(13e) as $P_\theta(\eta^\theta)$. Then $P_\theta(\eta^\theta)$ is integral if and only if the minimum of

$$\min_{(w,z)\in P_\theta(\eta^\theta)} p_w^\top w + p_z^\top z \tag{EC.22}$$

is either integral or $-\infty$, for every $p_w \in \mathbb{Z}^{W\times T}$ and $p_z \in \mathbb{Z}^T$. Using the variable reduction map from Lemma EC.1 and the unimodular change of variables from Lemma EC.2, we have that (EC.22) can be the reduced to the following problem:

$$\min_{\overline{\zeta}} \quad \sum_{i=0}^{k-1} \overline{p}_{\zeta,i} \cdot \overline{\zeta}_i \tag{EC.23a}$$

$$\text{s.t.} \quad \text{(EC.18b)} - \text{(EC.18c)} \tag{EC.23b}$$

$$\overline{\zeta}_i \geq 0, \qquad i \in [0:k-1]. \tag{EC.23c}$$

Since $\overline{p}_{\zeta,i}$ is integral whenever $p_{w,s}^t$ and p_z^t are integral, and the constraints (EC.18b)-(EC.18c) induce an integral polyhedron by Lemma EC.2, we conclude that the optimal value of (EC.22) is integral or $-\infty$. $\square$

Theorem 2 provides important insights on how to solve the fleet sizing problem (EC.5). First, it states that the relevant fleet sizes η^θ are multiple of $(W+L)/k$, and second, we can evaluate the function $\widetilde{F}_\theta(\eta^\theta)$ by solving a simple linear program. Below we present the fleet sizing problem (EC.5) after a change of variables $\eta^\theta = \frac{W+L}{k}\overline{\eta}^\theta$:

$$\min_{\overline{\eta}} \quad \sum_{\theta\in\Theta} c_{\overline{\eta}}^\theta \cdot \overline{\eta}^\theta + \widetilde{F}_\theta\Big(\frac{W+L}{k} \cdot \overline{\eta}^\theta\Big) \tag{EC.24a}$$

$$\text{s.t.} \quad \overline{\eta}^{\theta-1} \leq \overline{\eta}^\theta, \qquad \overline{\eta}_{LB}^\theta \leq \overline{\eta}^\theta \leq \overline{\eta}_{UB}^\theta, \qquad \overline{\eta}^\theta \in \mathbb{Z}_+, \qquad \theta \in \Theta, \tag{EC.24b}$$

where $c_{\overline{\eta}}^{\theta} = \frac{W+L}{k} c_{\eta}^{\theta}$, $\overline{\eta}_{LB}^{\theta} = \left\lceil \eta_{LB}^{\theta} \cdot \frac{k}{W+L} \right\rceil$, and $\overline{\eta}_{UB}^{\theta} = \left\lfloor \eta_{UB}^{\theta} \cdot \frac{k}{W+L} \right\rfloor$. Thus, the overall problem structure remains the same after this variable rescaling with the additional benefit that the value function $\overline{F}_{\theta}(\overline{\eta}^{\theta}) := \widetilde{F}_{\theta}\left(\frac{W+L}{k} \cdot \overline{\eta}^{\theta} \right)$ is convex piecewise linear for continuous values of $\overline{\eta}^{\theta}$ by using the linear relaxation of (EC.6).

Thus, we have reduced the main fleet sizing program (EC.5) to the solution of a separable convex integral program (EC.24) with the objective function separable over investment periods and subject to total unimodular constraints (EC.24b). In the next section, we review the solution proximity results for this class of problems and propose a polynomial time algorithm to solve (EC.24) if the number of investment periods $\theta \in \Theta$ is fixed.

EC.2.2.2. The Proximity Theorem Let $\{f_i\}_{i=1}^{n}$ be univariate real-valued convex functions, and consider the following separable convex integer programming problem:

$$\min \sum_{i=1}^{n} f_i(y_i) \tag{EC.25a}$$

$$\text{s.t. } Ay \geq b, \tag{EC.25b}$$

$$y \in \mathbb{Z}^n, \tag{EC.25c}$$

where A is a totally unimodular (TU) matrix, and b is an integer vector. The goal of this section is to prove that we can use the linear relaxation to perform an efficient local search for an optimal integral solution. We assume the minimum of the linear relaxation of the integer program (EC.25) exists and it is attainable. The feasibility of the integer program (EC.25) is implied by the feasibility of the corresponding linear relaxation and the fact that A is TU and b is integral.

Even extended real-valued functions fit the scope of the program (EC.25). Let f_i be an *extended* real-valued proper convex lower semi-continuous function of the form:

$$f_i(x) = \begin{cases} g_i(x), & \text{if } x \in [a_i, b_i], \\ +\infty, & \text{otherwise,} \end{cases} \tag{EC.26}$$

where $g_i(x)$ is a univariate real-valued convex function. A relevant example of such a function is the polyhedral function $\overline{F}_\theta(\cdot)$. Indeed, if $\{f_i\}_{i=1}^n$ are extended real-valued convex functions such as (EC.26) we can reformulate (EC.25) as follows:

$$\min \sum_{i=1}^n g_i(y_i) \tag{EC.27a}$$

$$\text{s.t. } Ay \geq b, \tag{EC.27b}$$

$$\lceil a \rceil \leq y \leq \lfloor b \rfloor, \tag{EC.27c}$$

$$y \in \mathbb{Z}^n. \tag{EC.27d}$$

Note that the constraint matrix induced by (EC.27b)-(EC.27c) is still TU, and the right-hand side vectors are still integral.

Theorem EC.1 **(Proximity Theorem for Separable Convex Integer Programs)**. *Suppose $\{f_i\}_{i=1}^n$ are convex proper real-valued functions and let y^* and w^* be optimal integral and continuous linear relaxation (LR) solutions to* (EC.25)*, respectively. Then,*

1. *there exists an optimal* integral *solution $\widehat{y}$ to* (EC.25) *such that $\|\widehat{y} - w^*\|_\infty \leq n$.*
2. *there exists an optimal LR solution $\widehat{w}$ to* (EC.25) *such that $\|y^* - \widehat{w}\|_\infty \leq n$.*

Proof to Theorem EC.1 This proof can be found in Hochbaum and Shanthikumar (1990).

□

The next result provides a method to solve separable convex integer programs assuming that the summation terms $\{f_i\}_{i=1}^n$ are cheap to evaluate. Let $h_{i,LB}$ and $h_{i,UB}$ be the minimum and maximum index $h \in \{0,1\dots,2n\}$ such that $f_i(\lfloor w_i^* \rfloor - n + h) < +\infty$, respectively, and let $q_{i,h}$ be the following objective cost:

$$q_{i,h} = \begin{cases} f_i(\lfloor w_i^* \rfloor - n + h_{i,LB}), & \text{if } h = h_{i,LB}, \\ f_i(\lfloor w_i^* \rfloor - n + h) - f_i(\lfloor w_i^* \rfloor - n + h - 1), & \text{if } h \in [h_{i,LB}+1, h_{i,UB}], \\ 0, & \text{if } h \notin [h_{i,LB}, h_{i,UB}], \end{cases} \tag{EC.28}$$

for every $i \in [1:n]$ and $h \in [0:2n]$. The cost vector q defined in (EC.28) provides a linearization of the objective function at integral points y such that $\|y - w^*\|_\infty \leq n$.

THEOREM EC.2 **(Solution of separable convex integer program)**. *Suppose that* $\{f_i\}_{i=1}^n$ *are extended real-valued proper convex functions. Let* w^* *be an optimal solution to the linear relaxation of* (EC.25), *and let* $h_{i,LB}$, $h_{i,UB}$, *and* $q_{i,h}$ *be the constants defined previously. Then, the separable convex integer program* (EC.25) *can be reformulated as follows:*

$$\min_{y,\delta} \sum_{i=1}^{n}\sum_{h=0}^{2n} q_{i,h}\cdot\delta_{i,h} \tag{EC.29a}$$

$$s.t.\ Ay \geq b, \tag{EC.29b}$$

$$y_i - \sum_{h=0}^{2n}\delta_{i,h} = \lfloor w_i^* \rfloor - n - 1, \qquad i\in[1:n], \tag{EC.29c}$$

$$\delta_{i,h} = 1, \qquad h\in[0:h_{i,LB}],\ i\in[1:n], \tag{EC.29d}$$

$$\delta_{i,h} = 0, \qquad h\in[h_{i,UB}+1:2n],\ i\in[1:n], \tag{EC.29e}$$

$$y_i\in\mathbb{Z}_+,\ \delta_{i,h}\in\{0,1\}, \qquad h\in[0:2n],\ i\in[1:n]. \tag{EC.29f}$$

In particular, an optimal solution (y^*,δ^*) *to* (EC.29) *exists if and only if an optimal solution to* (EC.25) *exists. The constraint matrix of the integer program* (EC.29) *is totally unimodular, therefore, it is sufficient to solve the linear relaxation of* (EC.29).

Proof of Theorem EC.2 First, note that constraints (EC.29c) and (EC.29f) imply that any solution $y\in\mathbb{Z}^n$ is such that $\|y-w^*\|_\infty \leq n$, where the infinite norm is defined as $\|a\|_\infty = \max_{i\in[1:n]}|a_i|$. By definition of the $q_{i,h}$, we note that

$$f_i\left(\lfloor w_i^*\rfloor - n + h\right) = \sum_{h=0}^{k} q_{i,h}, \tag{EC.30}$$

for each $h\in[h_{i,LB},h_{i,UB}]$ and $i\in[1:n]$. Because f_i is convex and univariate, the slopes of f_i are non-decreasing functions, so the sequence $\{q_{i,h}\}$ is non-decreasing over $h\in[h_{i,LB}+1:h_{i,UB}]$, for every $i\in[1:n]$. This proves that among all possible representations of $f_i\left(\lfloor w_i^*\rfloor - n + h\right)$ as the binary variable encoding $\sum_{h=0}^{2n} q_{i,h}\cdot\delta_{i,h}$ the one with the least objective cost is the right-hand side of (EC.30). Thus, the formulation (EC.29) is equivalent to

$$\min_{y} \quad \sum_{i=1}^{n} f_i(y_i) \tag{EC.31a}$$

$$\text{s.t.} \quad \|y - w^*\|_\infty \leq n, \tag{EC.31b}$$

$$\text{(EC.25b)} - \text{(EC.25c)}, \tag{EC.31c}$$

and we know from Theorem EC.1 that an optimal solution to (EC.31a) is also optimal to (EC.25).

Recall that the constraint matrix A defined by the constraint (EC.25b) is totally unimodular (TU), and by appending any canonical vector to columns or rows of a TU matrix, we preserve the TU property. Since the constraint matrix formed by (EC.29b) and (EC.29c) can be represented as

$$\widetilde{A} = \begin{array}{c} \begin{array}{cccc} \overline{\eta} & \delta_0 & \cdots & \delta_{2n} \end{array} \\ \begin{bmatrix} A & 0 & \cdots & 0 \\ I & -I & \cdots & -I \end{bmatrix} \end{array}, \tag{EC.32}$$

where $\delta_h := (\delta_{i,h})_{i=1}^n$, for all $h \in [0:2n]$, we conclude that $\widetilde{A}$ is also TU. It is straightforward to see that all the other constraints coefficients when appended to $\widetilde{A}$ preserves the TU property. $\square$

Note that Theorem EC.2 is a more general statement of Theorem 3.

THEOREM 3 *The fleet sizing problem* (12) *can be reformulated as the following integer program:*

$$\min_{\overline{\eta},\delta} \sum_{\theta\in\Theta} \left(c^\theta_{\overline{\eta}} \cdot \overline{\eta}^\theta + \sum_{h=0}^{2|\Theta|} q^\theta_{\delta,h} \cdot \delta^\theta_h \right) \tag{EC.33a}$$

$$s.t.\ \overline{\eta}^{\theta-1} \leq \overline{\eta}^\theta, \qquad \overline{\eta}^\theta_{LB} \leq \overline{\eta}^\theta \leq \overline{\eta}^\theta_{UB}, \qquad \theta\in\Theta, \tag{EC.33b}$$

$$\overline{\eta}^\theta - \sum_{h=0}^{2|\Theta|} \delta^\theta_h = \lfloor \overline{\eta}^\theta_{*,LR} \rfloor - |\Theta| - 1, \qquad \theta\in\Theta, \tag{EC.33c}$$

$$\delta^\theta_h = 1, \qquad h\in[0:h^\theta_{LB}],\ \theta\in\Theta, \tag{EC.33d}$$

$$\delta^\theta_h = 0, \qquad h\in[h^\theta_{UB}+1:2|\Theta|],\ \theta\in\Theta, \tag{EC.33e}$$

$$\overline{\eta}^\theta \in \mathbb{Z}_+,\ \delta^\theta_h \in \{0,1\}, \qquad h = 0,\ldots,2|\Theta|,\ \theta\in\Theta. \tag{EC.33f}$$

An optimal solution $(\overline{\eta}_*, \delta_*)$ *to* (15) *exists if and only if an optimal solution to* (12) *exists. The objective function requires* $O(|\Theta|)$ *evaluations of the value function* $\overline{F}_\theta(\cdot)$*, for each* $\theta\in\Theta$*. The constraint matrix induced by* (15b)-(15f) *is TU. Thus,* (15) *can be solved exactly by its LP relaxation.*

Proof of Theorem 3 The proof is a direct application of Theorem EC.2. $\square$

EC.2.2.3. Polynomial solvability Consider any linear program with integral coefficients:

$$\begin{aligned} &\min\ c^\top x \\ &\text{s.t. } Ax \le b, \\ &x \in \mathbb{R}^n, \end{aligned} \tag{EC.34}$$

where $c \in \mathbb{Z}^n$, $A \in \mathbb{Z}^{m\times n}$, and $b \in \mathbb{Z}^m$. In all our integer programs the coefficient matrices and the right-hand side vectors are integral. The objective coefficients can be converted to integral numbers if one multiplies the denominator of each rational coefficient by the least common multiple among all denominators.

Assume the feasible set $P = \{x \in \mathbb{R}^n \mid Ax \le b\}$ is a non-empty polytope. Let Δ be the largest absolute value of the determinant of a submatrix of A. We define the size of (EC.34) as

$$\mathbb{L} = \log_2(\Delta+1) + \log_2\left(\max_{j\in[n]} |c_j| + 1\right) + \log_2\left(\max_{i\in[m]} |b_i| + 1\right) + \log_2(m+n). \tag{EC.35}$$

We note that any basic feasible solution x^* to (EC.34) and the associated objective costs $c^\top x^*$ have the following upper bound:

$$\|x^*\|_\infty \le 2^{\mathbb{L}}, \qquad |c^\top x^*| \le 2^{2\mathbb{L}}, \tag{EC.36}$$

see (Vaidya 1990, page 191).

After the change of variables to the new fleet sizing variable $\widetilde{\eta}$, see (EC.24), and the reformulation of the operational variables as described in Lemma EC.2, the fleet-sizing problem (EC.5) becomes equivalent to

$$\min_{\overline{\eta},\overline{\zeta}} \quad \sum_{\theta\in\Theta} \left[c^\theta_{\overline{\eta}} \cdot \overline{\eta}^\theta + \sum_{i=0}^{k-1} \overline{p}^\theta_{\zeta,i} \cdot \overline{\zeta}^\theta_i \right] \tag{EC.37a}$$

$$\text{s.t.} \quad \overline{\eta}^{\theta-1} \le \overline{\eta}^\theta, \qquad \overline{\eta}^\theta_{LB} \le \overline{\eta}^\theta \le \overline{\eta}^\theta_{UB}, \qquad \overline{\eta}^\theta \ge 0, \qquad \theta \in \Theta, \tag{EC.37b}$$

$$\overline{\zeta}^\theta_{k-1} - \overline{\eta}^\theta = 0, \qquad \theta \in \Theta, \tag{EC.37c}$$

$$\overline{\zeta}^\theta_i - \overline{\zeta}^\theta_{i-W} + \left(\left\lfloor \frac{W}{k} \right\rfloor + \mathbb{I}_{[i+1,\infty)}(W\%k) \right) \overline{\zeta}^\theta_{k-1} \ge \widetilde{d}^\theta_i, \qquad i \in [0:k-1],\ \theta \in \Theta, \tag{EC.37d}$$

$$\overline{\zeta}^\theta_0 \ge 0, \qquad \theta \in \Theta, \tag{EC.37e}$$

$$\overline{\zeta}^\theta_i - \overline{\zeta}^\theta_{i-1} \ge 0, \qquad i \in [1:k-1],\ \theta \in \Theta, \tag{EC.37f}$$

$$\overline{\eta}^\theta,\ \overline{\zeta}^\theta_i \in \mathbb{Z}, \qquad i \in [0:k-1],\ \theta \in \Theta. \tag{EC.37g}$$

Note that the integer program (EC.37) has $n=(k+1)|\Theta|$ variables and $m=(2k+5)|\Theta|$ linear constraints. Denote by Δ the maximum absolute value of the determinant of a submatrix of the constraint matrix in (EC.37). Let $c=[c_{\overline{\eta}}, \overline{p}_{\zeta}]$ and $b=[\overline{\eta}_{LB}, \overline{\eta}_{UB}, \widetilde{d}]$. The size of the integer program (EC.37) is well-defined by the formula (EC.35):

$$\begin{aligned}\mathbb{L} = &\log_2(\Delta+1)+\log_2\left(\max\left\{\|c_{\overline{\eta}}\|_\infty,\ \|\overline{p}_\zeta\|_\infty\right\}+1\right)\\ &+\log_2\left(\max\left\{\|\overline{\eta}_{LB}\|_\infty, \|\overline{\eta}_{UB}\|_\infty, \|\widetilde{d}\|_\infty\right\}+1\right)\\ &+\log_2(m+n).\end{aligned} \tag{EC.38}$$

We note that the feasible region of the linear relaxation of (EC.37) is a polytope since all the variables are bounded. Indeed, it follows from (EC.37e),(EC.37f), (EC.37c), and (EC.37b) the inequalities

$$0\le \overline{\zeta}_0^\theta \le \overline{\zeta}_1^\theta \le \cdots \le \overline{\zeta}_{k-1}^\theta = \overline{\eta}^\theta \le \overline{\eta}_{UB}^\theta.$$

To recap, in order to obtain an optimal integral solution to (EC.37) using the Proximity result from Theorem 3, one needs to:

1. Find an optimal basic feasible solution $(\overline{\eta}_{*,LR}, \overline{\zeta}_{*,LR})$ to the linear relaxation of (EC.37).

2. Compute the optimal value $\widetilde{F}_\theta(\eta)$ of the operational subproblem (EC.18) with fleet sizes $\eta = \frac{(W+L)}{k}\left(\lfloor\overline{\eta}_{*,LR}^\theta\rfloor - |\Theta| - 1 + h\right)$, for all $h=0,1,\ldots,2|\Theta|$, and all $\theta\in\Theta$. Then, define $q_{\delta,h}^\theta$ according to the expression (14).

3. Solve the proximity problem (EC.33) using $q_{\delta,h}^\theta$ and $\lfloor\overline{\eta}_{*,LR}^\theta\rfloor$ as inputs.

In our complexity analysis proof, we must guarantee that the sizes of all the intermediate linear programs are polynomially bounded by the size of the integer program (EC.37). Indeed, denote by $\mathbb{L}_{\theta,h}$ the size of the linear relaxation of (EC.18), and let $n_{\theta,h}$ and $m_{\theta,h}$ be the associated numbers of variables and constraints. Denote by $\widetilde{\Delta}$ the maximum absolute value of the determinant of a submatrix of the constraint matrix in (EC.18) Note that $\widetilde{\Delta}$ is less than or equal to Δ. Then, the size of the linear relaxation of (EC.18) is given by

$$\begin{aligned}\mathbb{L}_{\theta,h} =& \log_2(\widetilde{\Delta}+1)+\log_2\left(\|\overline{p}^{\theta}_{\zeta}\|_\infty+1\right)\\ &+\log_2\left(\max\left\{\|\widetilde{d}^{\theta}\|_\infty,\ \left|\lfloor\overline{\eta}^{\theta}_{*,LR}\rfloor-|\Theta|-1+h\right|\right\}+1\right)\\ &+\log_2\left(m_{\theta,h}+n_{\theta,h}\right),\end{aligned}\tag{EC.39}$$

where the number of variables is $n_{\theta,h}=k$ and the number of constraints is $m_{\theta,h}=2k+1$. Similarly, denote by $\mathbb{L}_P$ the size of the proximity problem (EC.33), and let n_P and m_P be the corresponding number of variables and constraints. Recall that the coefficient matrix of (EC.33) is also TU. Then, note that

$$\begin{aligned}\mathbb{L}_P =& 1+\log_2\left(\max\left\{\|c_{\overline{\eta}}\|_\infty,\ \|q_\delta\|_\infty\right\}+1\right)\\ &+\log_2\left(\max\left\{\|\overline{\eta}_{LB}\|_\infty,\ \|\overline{\eta}_{UB}\|_\infty,\ \|\lfloor\overline{\eta}_{*,LR}\rfloor-(|\Theta|+1)\cdot e\|_\infty\right\}+1\right)\\ &+\log_2\left(m_P+n_P\right).\end{aligned}\tag{EC.40}$$

where e is a vector of 1's, the number of variables of (EC.33) is $n_P=2|\Theta|^2+|\Theta|$, and the number of constraints is $m_P=2|\Theta|^2+4|\Theta|+1$.

Lemma EC.3. *The linear program sizes $\mathbb{L}_{\theta,h}$ and $\mathbb{L}_P$ are linearly bounded by the size $\mathbb{L}$:*

$$\mathbb{L}_{\theta,h}\le 2\mathbb{L}+2,\qquad \mathbb{L}_P\le 8\mathbb{L}+13,\tag{EC.41}$$

for all $\theta\in\Theta$ and $h=0,1,\ldots,2|\Theta|$.

Proof of Lemma EC.3 From the definition of $\mathbb{L}$, we obtain the upper bound:

$$\mathbb{L}_{\theta,h}\le\mathbb{L}+\log_2\left(\max\left\{\|\widetilde{d}^{\theta}\|_\infty,\ \left|\lfloor\overline{\eta}^{\theta}_{*,LR}\rfloor-|\Theta|-1+h\right|\right\}+1\right).\tag{EC.42}$$

Because $(\overline{\eta}_{*,LR},\overline{\zeta}_{*,LR})$ is an optimal basic feasible solution, we know that $0\le\lfloor\overline{\eta}^{\theta}_{*,LR}\rfloor\le\overline{\eta}^{\theta}_{*,LR}\le 2^{\mathbb{L}}$. This implies that

$$\left|\lfloor\overline{\eta}^{\theta}_{*,LR}\rfloor-|\Theta|-1+h\right|\le 2^{\mathbb{L}}+\left|h-|\Theta|-1\right|\le 2^{\mathbb{L}+1}\tag{EC.43a}$$

$$\Longrightarrow\ \mathbb{L}_{\theta,h}\le\mathbb{L}+\log_2(2^{\mathbb{L}+1}+1)\le 2\mathbb{L}+2,\tag{EC.43b}$$

for all $\theta \in \Theta$ and $h = 0, 1, \ldots, 2|\Theta|$.

It follows from (EC.38) the upper bound $|\widetilde{F}_\theta(\eta)| \leq 2^{2\mathbb{L}_{\theta,h}}$. Then, we use this inequality and (EC.43b) to get $|q^\theta_{\delta,h}| \leq 2^{2\mathbb{L}_{\theta,h}+1} \leq 2^{4\mathbb{L}+5}$. Hence, we have the following upper bound for $\mathbb{L}_P$:

$$\mathbb{L}_P \leq 1 + \log_2\left(2^{4\mathbb{L}+5} + 1\right) + \log_2\left(2^{2\mathbb{L}+1} + 1\right) + \log_2(m_P + n_P) \tag{EC.44a}$$

$$\leq (6\mathbb{L} + 9) + \log_2(m_P + n_P) \tag{EC.44b}$$

$$= (6\mathbb{L} + 9) + \log_2(4|\Theta|^2 + 5|\Theta| + 1) \tag{EC.44c}$$

$$\leq (6\mathbb{L} + 9) + \log_2\left(16 \cdot |\Theta|^2\right) \tag{EC.44d}$$

$$= (6\mathbb{L} + 13) + 2\log_2(|\Theta|) \tag{EC.44e}$$

$$\leq 8\mathbb{L} + 13. \tag{EC.44f}$$

This completes the proof. □

We can finally prove the polynomial solvability of the fleet sizing problem (12).We use the algorithm of Vaidya (1990) that finds an optimal basic feasible solution of a linear program in $O(((m+n)n^2 + (m+n)^{1.5}n)\mathbb{L})$ arithmetic operations, where $\mathbb{L}$ is the size of the linear program, m is the number of constraints, and n is the number of variables. Note that if $m = O(n)$ this arithmetic complexity becomes $O(n^3\mathbb{L})$.

Theorem 4 *An optimal integral solution of* (12) *can be obtained in* $O\Big((k^3|\Theta|^3 + |\Theta|^6)\mathbb{L}\Big)$ *arithmetic operations, where* $\mathbb{L}$ *is the size of the integer program* (12) *defined in* (EC.38).

Proof The number of variables n and constraints m of the linear relaxation of (EC.37) is $O(k|\Theta|)$. So, the arithmetic complexity to compute an optimal basic feasible solution $\overline{\eta}_{*,LR}$ is $O\Big((k|\Theta|)^3\mathbb{L}\Big)$.

One needs to solve the linear relaxation of (EC.18) to compute the optimal value $\widetilde{F}_\theta(\eta)$ with $\eta = \frac{(W+L)}{k}\Big(\lfloor\overline{\eta}^\theta_{*,LR}\rfloor - |\Theta| - 1 + h\Big)$, for all $h = 0, 1, \ldots, 2|\Theta|$ and all $\theta \in \Theta$. Since the number of variables $n_{\theta,h}$ and constraints $m_{\theta,h}$ of each subproblem is $O(k)$, it takes $O(k^3\mathbb{L}_{\theta,h})$ arithmetic operations to solve each of them. By Lemma EC.3, we have that $\mathbb{L}_{\theta,h} = O(\mathbb{L})$ and this implies that the arithmetic complexity to solve (EC.18) is indeed $O(k^3\mathbb{L})$. Hence, it takes $O(k^3|\Theta|^2\mathbb{L})$ arithmetic operations to find the coefficient q_δ of the proximity problem (EC.33).

Finally, one needs to solve the proximity problem (EC.33) which has $O(|\Theta|^2)$ variables and constraints. This implies a arithmetic complexity of $O(|\Theta|^6\mathbb{L}_P)$, and again by Lemma EC.3, we can replace the size $\mathbb{L}_p$ by $\mathbb{L}$. Therefore, the arithmetic complexity of the whole algorithm is

$$O(k^3|\Theta|^3\mathbb{L}) + O(k^3|\Theta|^2\mathbb{L}) + O(|\Theta|^6\mathbb{L}) = O\Big((k^3|\Theta|^3 + |\Theta|^6)\mathbb{L}\Big). \tag{EC.45}$$

This completes the proof. $\square$

EC.3. Feasibility of the Policy Restriction heuristic

The goal of this section is to prove that the Policy Restriction (PR) heuristic is always feasible. First, we recall the constraints of the PR heuristic. Indeed, the PR heuristic prevents the depot BEBs from charging at any state s different from the depleted state $W_b + 1$ during the service times $T^{service}_{j,\theta}$, that is,

$$z^{t,\theta}_{bijk} = 0, \qquad \forall t \in T^{service}_{j,\theta},\ s \in [2 : W_b],\ (b,i,j,k,\theta) \in \mathcal{B}_{depot} \times \mathcal{I} \times \mathcal{J} \times \mathcal{K} \times \Theta.$$

It also prevents the depot and on-route BEBs from being idle during service times, with the exception of depot BEBs when fully charged ($s = 1$):

$$v^{t,\theta}_{bjs} = 0, \qquad \forall t \in T^{service}_{j,\theta},\ s \in [2 : (W_b + 1)],\ (b,j,\theta) \in \mathcal{B}_{depot} \times \mathcal{J} \times \Theta,$$

$$\widetilde{v}^{t,\theta}_{bjr} = 0, \qquad \forall t \in T^{service}_{j,\theta},\ r \in \mathcal{R}(j),\ (b,j,\theta) \in \mathcal{B}_{route} \times \mathcal{J} \times \Theta.$$

Lastly, the number of working depot and on-route BEBs must be zero during off-service times $t \in T^{off}_{j,\theta}$:

$$w^{t,\theta}_{bjs} = 0, \qquad \forall t \in T^{off}_{j,\theta},\ s \in [1 : W_b],\ (b,j,\theta) \in \mathcal{B}_{depot} \times \mathcal{J} \times \Theta,$$

$$\widetilde{w}^{t,\theta}_{bjr} = 0, \qquad \forall t \in T^{off}_{j,\theta},\ r \in \mathcal{R}(j),\ (b,j,\theta) \in \mathcal{B}_{route} \times \mathcal{J} \times \Theta.$$

Proposition 1. *The OPCF-EBF problem with the Policy Restriction constraints is feasible.*

Proof of Proposition 1 Consider a solution defined as follows:

- Define all the depot infrastructure x and y, depot BEBs η, and the associated operational variables w, v, and z as zero vectors.

- Define all the on-route infrastructure χ, on-route BEBs $\widetilde{\eta}$, and the associated operational variables $\widetilde{w}$ and $\widetilde{v}$ as zero vectors as well.

- Let the conventional buses ξ be such that it satisfies the retirement targets $\xi^\theta_{LB,j} \le \xi^\theta_j \le \xi^\theta_{UB,j}$ and the monotonicity constraints $\xi^\theta_j \le \xi^{\theta-1}_j$ for all routes $j \in J$, and all investment periods $\theta \in \Theta$. Define the working conventional buses $\phi^{t,\theta}_j$ as zero and the idle conventional buses $\sigma^{t,\theta}_j$ as ξ^θ_j for all time intervals $t \in [0:T-1]$, routes $j \in J$, and investment periods $\theta \in \Theta$.

- Let the demand slack variable $u^{t,\theta}_j$ be equal to $d^{t,\theta}_j$ for all time intervals $t \in [0:T-1]$, routes $j \in J$, and investment periods $\theta \in \Theta$.

It is straightforward to check that this solution is feasible. Hence, the OPCF-EBF problem with Policy Restriction constraints is feasible. $\square$

EC.4. Optimal solution of the simplified model

The purpose of this section is to prove Proposition 2. Recall the simplified fleet sizing model (21):

$$
\begin{aligned}
\min\ & c_d\eta + c_r\widetilde{\eta} \\
\text{s.t. } & \eta = w^0 + v^0 + z^0, && \widetilde{\eta} = \widetilde{w}^0 + \widetilde{v}^0, \\
& w^t + v^t = z^{t-1} + v^{t-1}, && z^t = w^{t-1}, && \forall t \in [0:T-1], \\
& \widetilde{w}^t + \widetilde{v}^t = \widetilde{w}_{t-1} + \widetilde{v}_{t-1}, && w^t + \widetilde{w}^t \ge d^t, && \forall t \in [0:T-1], \\
& \eta, \widetilde{\eta}, w^t, \widetilde{w}^t, v^t, \widetilde{v}^t, z^t \in \mathbb{Z}_+, && && \forall t \in [0:T-1],
\end{aligned}
\tag{EC.46}
$$

where c_d and c_r are non-negative unit costs, the demand $\{d^t\}_{t=0}^{T-1}$ is a scalar sequence taking non-negative values, and the quantities D_1 and D_2 are defined by:

$$
D_1 = \max_{t\in[0:T-1]} d^t, \qquad D_2 = \max_{t\in[0:T-1]} d^t + d^{t-1}.
$$

Proposition 2. *For every scenario of unit costs c_d and c_r, the optimal number of the depot and on-route BEBs to* (21) *and the corresponding numbers of working BEBs for each time interval $t \in$*

$[0:T-1]$ is obtained in Table 4, where $D_1 := \max_{t\in[0:T-1]} d_t$ and $D_2 := \max_{t\in[0:T-1]}(d_t + d_{t-1})$. Moreover, the optimal solution of the variables z_t, v_t, and $\widetilde{v}_t$ is given by the relations $z_t = w_{t-1}$, $v_t = \eta - w_t - z_t$, and $\widetilde{v}_t = \widetilde{\eta} - \widetilde{w}_t$.

Table EC.1 Optimal solution table of (21) for each objective coefficients c_r and c_d.

Coeff.	η	$\widetilde{\eta}$	w_t	$\widetilde{w}_t$
$c_r \le c_d$	0	D_1	0	d_t
$c_r \ge 2c_d$	D_2	0	d_t	0
$c_d < c_r < 2c_d$	$2D_1 - D_2$	$D_2 - D_1$	$\max\{d_t + D_1 - D_2, 0\}$	$\min\{D_2 - D_1, d^t\}$

Proof of Proposition 2. We first simplify (EC.46) by eliminating the charging variable z^t, and the idling variables v^t and $\widetilde{v}^t$. Indeed, we can replace z_t by w_{t-1} everywhere in (EC.46) and this leads to the following model:

$$\begin{aligned}
&\min\ c_d\eta + c_r\widetilde{\eta} \\
&\text{s.t. } \eta = w^0 + v^0 + w^{T-1}, \quad \widetilde{\eta} = \widetilde{w}^0 + \widetilde{v}^0, \\
&\quad w^t + v^t = w^{t-2} + v^{t-1}, && \forall t \in [0:T-1], \\
&\quad \widetilde{w}^t + \widetilde{v}^t = \widetilde{w}^{t-1} + \widetilde{v}^{t-1},\ w^t + \widetilde{w}^t \ge d^t, && \forall t \in [0:T-1], \\
&\quad \eta, \widetilde{\eta}, w^t, \widetilde{w}^t, v^t, \widetilde{v}^t \in \mathbb{Z}_+, && \forall t \in [0:T-1].
\end{aligned} \tag{EC.47}$$

Note that $\eta = w^t + v^t + w^{t-1}$ is equivalent to $w^t + v^t = w^{t-2} + v^{t-1}$, for all $t \in [0:T-1]$, and that $\widetilde{\eta} = \widetilde{w}^t + \widetilde{v}^t$ is equivalent to $\widetilde{w}^t + \widetilde{v}^t = \widetilde{w}^{t-1} + \widetilde{v}^{t-1}$, for all $t \in [0:T-1]$. This leads to the following equivalent formulation:

$$\begin{aligned}
&\min\ c_d\eta + c_r\widetilde{\eta} \\
&\text{s.t. } \eta = w^t + v^t + w^{t-1}, \quad \widetilde{\eta} = \widetilde{w}^t + \widetilde{v}^t, && \forall t \in [0:T-1] \\
&\quad w^t + \widetilde{w}^t \ge d^t, && \forall t \in [0:T-1], \\
&\quad \eta, \widetilde{\eta}, w^t, \widetilde{w}^t, v^t, \widetilde{v}^t \in \mathbb{Z}_+, && \forall t \in [0:T-1].
\end{aligned} \tag{EC.48}$$

From (EC.48), it is straightforward to eliminate the idling variables v^t and $\widetilde{v}^t$. Let $v^t = \eta - w^t - w^{t-1}$ and $\widetilde{v}^t = \widetilde{\eta} - \widetilde{w}^t$, and because both variables are non-negative, we have the formulation below:

$$\begin{aligned}
\min\ & c_d\eta + c_r\widetilde{\eta} \\
\text{s.t. } & \eta \geq w^t + w^{t-1}, \quad \widetilde{\eta} \geq \widetilde{w}^t, && \forall t \in [0:T-1] \\
& w^t + \widetilde{w}^t \geq d^t, && \forall t \in [0:T-1], \\
& \eta, \widetilde{\eta}, w^t, \widetilde{w}^t \in \mathbb{Z}_+, && \forall t \in [0:T-1].
\end{aligned} \tag{EC.49}$$

The lines of Table EC.1 induce feasible solutions to (EC.49) with objectives c_rD_1, c_dD_2, and $c_r(2D_1 - D_2) + c_d(D_2 - D_1)$. Consider the dual of the linear relaxation of (EC.49):

$$\begin{aligned}
\max\ & \textstyle\sum_{t\in[0:T-1]} d^t\phi^t \\
\text{s.t. } & \textstyle\sum_{t\in[0:T-1]} \pi_t \leq c_d, \quad \sum_{t\in[0:T-1]} \widetilde{\pi}^t \leq c_r, \\
& -\pi^t - \pi^{t+1} + \phi^t \leq 0, \quad -\widetilde{\pi}^t + \phi^t \leq 0, && t \in [0:T-1], \\
& \pi^t, \widetilde{\pi}^t, \phi^t \geq 0, && t \in [0:T-1],
\end{aligned} \tag{EC.50}$$

and let $a, b \in [0 : T-1]$ be such that $D_1 = d^a$ and $D_2 = d^b + d^{b-1}$. We use the Kronecker delta vectors δ^a and δ^b to define the dual feasible solutions, where

$$(\delta^a)^t := \begin{cases} 1, & \text{if } t = a, \\ 0, & \text{otherwise.} \end{cases}$$

One can check that the lines of Table EC.2 induce feasible solutions to the dual problem (EC.50) with the same objective values c_rD_1, c_dD_2, and $c_r(2D_1 - D_2) + c_d(D_2 - D_1)$. Therefore, the solutions of Table EC.1 are optimal to (EC.49). $\square$

Table EC.2 Optimal solutions of the dual problem (EC.50) for each objective coefficients c_r and c_d.

Coeff.	ϕ^t	π^t	$\widetilde{\pi}^t$
$c_r \leq c_d$	$c_r\delta^a$	$c_r\delta^a$	$c_r\delta^a$
$c_r \geq 2c_d$	$c_d(\delta^b + \delta^{b-1})$	$c_d\delta^b$	$c_d(\delta^b + \delta^{b-1})$
$c_d < c_r < 2c_d$	$(2c_d - c_r)\delta^a + (c_r - c_d)(\delta^b + \delta^{b-1})$	$(2c_d - c_r)\delta^a + (c_r - c_d)\delta^b$	$(2c_d - c_r)\delta^a + (c_r - c_d)(\delta^b + \delta^{b-1})$

The discount factor and decision order Below we show the role of the discount factor in ordering decisions.

PROPOSITION EC.1. *Let* $c_1 < c_2 < \cdots < c_n$ *and* $0 < \beta < 1$ *be given. Let* $\pi : [n] \to [n]$ *denote a permutation, i.e. a bijection, where* $[n] := \{1, 2, \ldots, n\}$*. Then, the following minimization problem*

$$\min_{\substack{\pi:[n]\to[n] \\ \pi \;\; permutation}} \sum_{\theta=1}^{n} \beta^{\theta-1} c_{\pi(\theta)}$$

has a unique optimal solution given by the identity permutation $\pi^*(\theta) = \theta$ *for all* $\theta \in [n]$*. That is, the minimum sum of a sequence of distinct numbers discounted by* β *is achieved by the increasing ordering of the numbers.*

Proof of Proposition EC.1. The solution to this problem can be find by induction. Indeed, the case $n = 1$ and $n = 2$ are trivial. Given a permutation π, we create another permutation $\widehat{\pi}$ by swapping two numbers:

$$\widehat{\pi}(i) = \begin{cases} n, & \text{if } i = n, \\ \pi(n), & \text{if } i = \pi^{-1}(n), \\ \pi(i), & \text{if } i \neq n, \pi^{-1}(n). \end{cases}$$

Note that the new permutation $\hat{\pi}$ is identical to the original permutation π except at two places: $\hat{\pi}(n) = n$, whereas $\pi(n) = i$, and $\hat{\pi}(i) = \pi(n)$, whereas $\pi(i) = n$.

Let $r = \pi^{-1}(n)$, and note that

$$\begin{aligned} & c_{\widehat{\pi}(r)} \cdot \beta^{r-1} + c_{\widehat{\pi}(n)} \cdot \beta^{n-1} < c_{\pi(r)} \cdot \beta^{r-1} + c_{\pi(n)} \cdot \beta^{n-1}, \\ \iff \quad & c_{\pi(n)} \beta^{r-1} + c_n \cdot \beta^{n-1} < c_n \cdot \beta^{r-1} + c_{\pi(n)} \cdot \beta^{n-1}, \\ \iff \quad & \beta^{n-1}(c_n - c_{\pi(n)}) < \beta^{r-1}(c_n - c_{\pi(n)}), \end{aligned}$$

where the last inequality holds since n is greater than r. Therefore,

$$\sum_{i=1}^{n-1} c_{\widehat{\pi}(i)} \beta^{i-1} + c_n \beta^{n-1} < \sum_{i=1}^{n} c_{\pi(i)} \beta^{i-1},$$

and because $\widehat{\pi}$ restricted to $[n-1]$ defines a permutation in $[n-1]$, we conclude the result by the induction hypothesis:

$$\sum_{i=1}^{n-1} c_i \beta^{i-1} + c_n \beta^{n-1} < \sum_{i=1}^{n} c_{\pi(i)} \beta^{i-1}.$$

$\square$