Water resources management: A bibliometric analysis and future research directions

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Abstract

The stochastic dual dynamic programming (SDDP) algorithm introduced by Pereira and Pinto in 1991 has sparked essential research in the context of water resources management, mainly due to its ability to address large-scale multistage stochastic problems. This paper aims to provide a tutorial-type review of 32 years of research since the publication of the SDDP algorithm. A systematic academic literature search identified 174 scientific papers on water resource management published in 96 different journals. A bibliometric analysis is conducted to identify the main methods used to tackle this type of problem and to determine recent and future research trends. Our analysis reveals that stochastic dynamic programming, which was initially the most used approach, has now been replaced by multistage stochastic programming. Risk-averse and robust approaches are also gaining strength in recent years due to uncertainty related to climate change. Water inflows have been the main source of uncertainty considered in the literature by far, followed by, e.g., electricity demand, electricity prices, fuel costs, and renewable energy availability. In addition, as computational capacity continues to increase, aspects of nonlinearities, disaggregated networks, and different water management strategies are increasingly considered to make modeling more realistic. This work suggests there is still a need for tractable stochastic optimization models for large-scale power and water systems that deal with multiple uncertainty sources and nonlinearity approximations.

Keywords: Water resources management, Hydrothermal scheduling, Optimization under uncertainty, Stochastic dual dynamic programming, Bibliometric approach.

1. Introduction

Managing energy production based on hydroelectric resources in a centralized system is a complex task carried out by a central dispatcher whose primary goal is to determine an optimal dispatch policy for thermal and hydroelectric power plants at the lowest possible operational cost (Labadie, 2004). This problem is typically known as hydrothermal scheduling. It is also named hydrothermal coordination or hydrothermal planning. The energy production process for a hydroelectric plant consists on releasing water stored in a reservoir that falls from a certain height, goes through a hydraulic turbine, and makes it rotates, thus generating electricity. Therefore, this gives rise to a sequential decision process based on when and how much water to store or release from reservoirs. It is typically approached with optimization models, particularly in optimization under uncertainty schemes due to the high presence of uncertainties like hydro inflows (de Queiroz, 2016).

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The hydrothermal scheduling problem generally seeks to meet the electricity demand while minimizing the expected operational cost. This problem is commonly solved in annual horizons on a monthly or weekly basis, and it depends directly on reservoirs' storage capacity and spatial interrelation. The water inflows for these reservoirs play a fundamental role since they are not known beforehand. Therefore, they are usually modeled as uncertain variables to consider different possible scenarios. Thus, for this problem's decisions, the main challenge is that using water resources inefficiently today can lead to high operational costs in the future. In an unforeseen future drought, using more water than it should, might result in load shedding and blackouts, among others. Additionally, storing too much water today could overflow reservoirs in unforeseen wet futures, resulting in possible water spills.

There is a more general problem called reservoir management. It involves minimizing the risk of floods for a multi-reservoir system over a horizon planning subject to operational constraints such as water balance, storage, spills, releases, and other physical and regulatory constraints. Both hydrothermal scheduling and reservoir management problems are related to water resources management and consist of finding an optimal resource (water) allocation policy. Their mathematical structures are very similar. In the following, we refer to these two problems as water resources management.

Since the seminal paper of Pereira & Pinto (1991), significant progress has been made in water resources management problems. In (Pereira & Pinto, 1991), the authors proposed the stochastic dual dynamic algorithm (SDDP), which emerged as an effective algorithm for successfully tackling multi-dimensional stochastic reservoir problems. This has allowed obtaining essential insights into the estimation of the value of water and how to manage it efficiently, which is more necessary in periods where hydrological uncertainty becomes more relevant as a consequence of climate change (Vicuña et al., 2008).

So far, a few literature reviews have been devoted to water resources management and its variants. In (Farhat & El-Hawary, 2009), the authors focus on optimization methods used to solve the short-term hydrothermal programming problem. The paper discusses Lagrangian relaxation and Benders decomposition-based methods, mixed-integer programming, dynamic programming, evolutionary computing, artificial intelligence techniques, interior-point methods, optimal control, and others. Kong et al. (2014) summarizes hydrothermal-related research progress and current status in China. The authors highlight the potential of hydrothermal resources in that country, the necessity of continuing research and technology development, and political support for sustainable energy sources. In (de Queiroz, 2016), the author presents an overview of model formulations, solution techniques, and new developments for hydrothermal scheduling and emphasizes the classical balance between computational effort and the granularity of power and water network representation. The review performed in (Hossain & Shiblee, 2017) exhibits a brief overview of various techniques and analytical strategies to solve the hydrothermal scheduling problem without elaborating much on them. Archibald & Marshall (2018) presents a literature review on optimization models for water resources management, but limited to the period between 2010 and 2017. Lastly, in (Lorca et al., 2020), the authors discuss the challenges of the hydrothermal scheduling problem more specifically on the following topics: flexibility in power systems with deep penetration of renewable energy sources, modeling of hydro inflow and other source uncertainties, and non-convexity handling.

The present paper attempts to provide a systematic and comprehensive tutorial-type review differing from previous works in the following aspects:

1. We carry out a bibliometric analysis to examine the research progress since the publication of the seminal work of Pereira & Pinto (1991) in the context of water resources management problems. We cover all relevant aspects published in leading journals rather than emphasizing a specific one or a determined country.
2. Based on the bibliometric analysis and the visualization software VOSviewer, we identify the relationship between journals, papers, authors, and keywords and which are the most influential on the topic.
3. A critical review of the previous works and discussions on current literature gaps and future research lines are provided. This discussion is divided into three main categories: modeling approaches, uncertainty sources and modeling, and water resources management strategies. In addition, we formally introduce some of the basics of the main modeling approaches considered in the literature.

The remainder of this paper is organized as follows. Section 2 presents the bibliometric study. Section 3 presents a critical review and discussion based on the literature characterization. Section 4 concludes the paper and offers some potential directions for further research.

2. Bibliometric Analysis

This section provides a general bibliometric analysis of water resources management research using quantitative methods to classify and summarize bibliometric data. It analyzes, in particular, the performance of journals, institutes, and authors, as well as the characteristics of research fields or topics. Using visualization techniques, bibliometric networks, such as the co-citation network, co-authorship network, and keyword co-occurrence network, can be constructed and presented visually. We use the VOSviewer software[1] (van Eck & Waltman, 2010) for this analysis, and we considered several bibliometric indicators such as the number of papers, total citations, and citations per paper (and year).

To collect the publication data, we use the well-recognized Scopus database, searching in the title, abstract, and keywords any of the following topics: "water resources management", "reservoir management", "hydrothermal scheduling", "hydrothermal coordination", "hydrothermal planning", "hydro-thermal scheduling", "hydro-thermal coordination", "hydro-thermal planning", "hydropower generation", or "hydroelectric generation". We also check all references citing the seminal work of Pereira & Pinto (1991), entitled "Multi-stage stochastic optimization applied to energy planning". After repeated sifting and checking, a total of 174 relevant papers were finally retrieved on the period of 1995–2023. Note that the data collection was conducted in January 2023.

Figure 1 depicts the evolution of the number of published papers from 1995 to 2023. It can be noted that little attention was paid to water resources management problems during the first 20 years (1995 to 2006), with only 23 papers. This may be related to the low computational power back then. However, from 2007 onward, the number of published papers increased significantly. In addition to computational advances, this increase is related to the growing interest in modeling the impacts of climate change, including possible water scarcity that directly affect water resource management. During the last 13 years, 135 papers were published, accounting for about 73% of the total number of papers in the past 29 years. Although the last 4-year period includes just one month of 2023, the yearly average number of publications does not seem to decrease. This continued growing trend clearly shows that water resources management issues are still critical, in particular, due to the increasingly devastating effects of climate change.

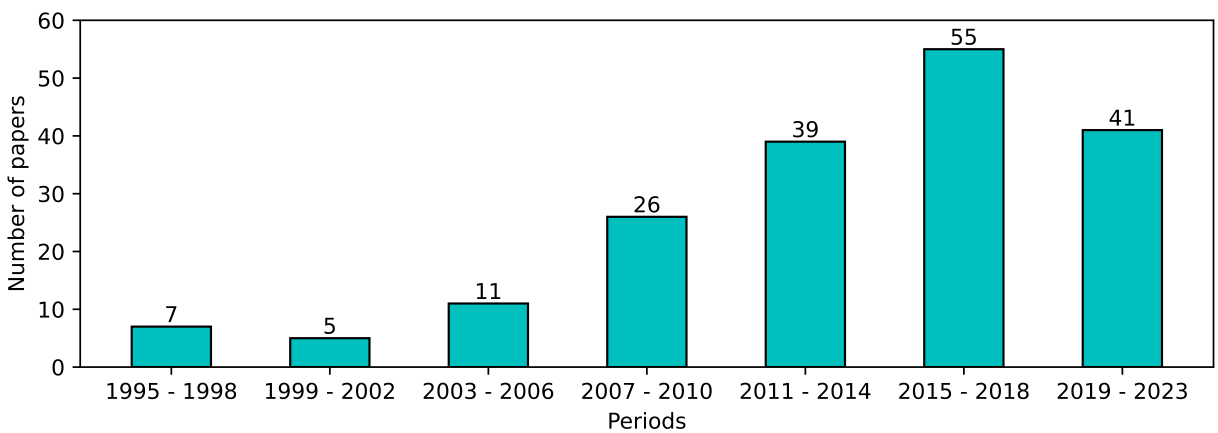

Figure 1: Number of papers per period (4 years)

Table 1 presents the top 24 (over 96) journals regarding the number of published papers. It can be noted that these journals mainly belong to "operations research", "engineering", "energy", and "environmental science" areas, based on the journal categories of SCImago Journal Rank (SJR). In particular, the European Journal of Operational Research (EJOR), a leading journal in the operations research field, tops the ranking with 12 papers. IEEE Transactions on Power Systems (IEEE-TPS), a leading journal in the energy field, follows with 8 papers. Additionally, Water Resources Research (WRR), a leading journal in the environmental

[1]VOSviewer is freely available, and further information can be found at http://www.vosviewer.com/.

science field, follows the latter with 7 publications. These three journals account for almost 15.5% of the total published papers.

Rank	Journal	Number of papers
1	European Journal of Operational Research	12
2	IEEE Transactions on Power Systems	8
3	Water Resources Research	7
4	Electric Power Systems Research	6
5	International Journal of Electrical Power & Energy Systems	6
6	Journal of Water Resources Planning and Management	6
7	Power Systems Computation Conference	6
8	Energy Systems	5
9	IEEE Power & Energy Society General Meeting	5
10	Stochastic Environmental Research and Risk Assessment	5
11	Water Resources Management	5
12	Annals of Operations Research	3
13	Energy Procedia	3
14	International Transactions in Operational Research	3
15	Journal of Environmental Management	3
16	Mathematical Programming	3
17	Optimization and Engineering	3
18	Advances in Water Resources	2
19	Computational Optimization and Applications	2
20	IEEE Grenoble Conference PowerTech	2
21	IEEE Manchester PowerTech	2
22	IET Generation, Transmission & Distribution	2
23	Renewable and Sustainable Energy Reviews	2
24	Journal of Irrigation and Drainage Engineering	2

Table 1: Top journals by the number of published papers (at least two)

Figure 2 shows the co-relation among journals through a bibliographic coupling. Note that to not overload the figure, we only considered journals with at least one citation among the selected papers. The size of a circle represents the number of citations of the corresponding journal papers. The width of lines between circles represents the co-citation between journals. It can be noted that papers published in Mathematical Programming (MP), where Pereira & Pinto (1991) is published, have a strong citation connection with papers published in the vast majority of journals, which of course results from the applied papers search methodology. In addition, the papers published in EJOR have highly been cited by MP, IEEE-TPS, WRR, International Transactions in Operational Research (ITOR), and Civil Engineering and Environmental Systems (CEES).

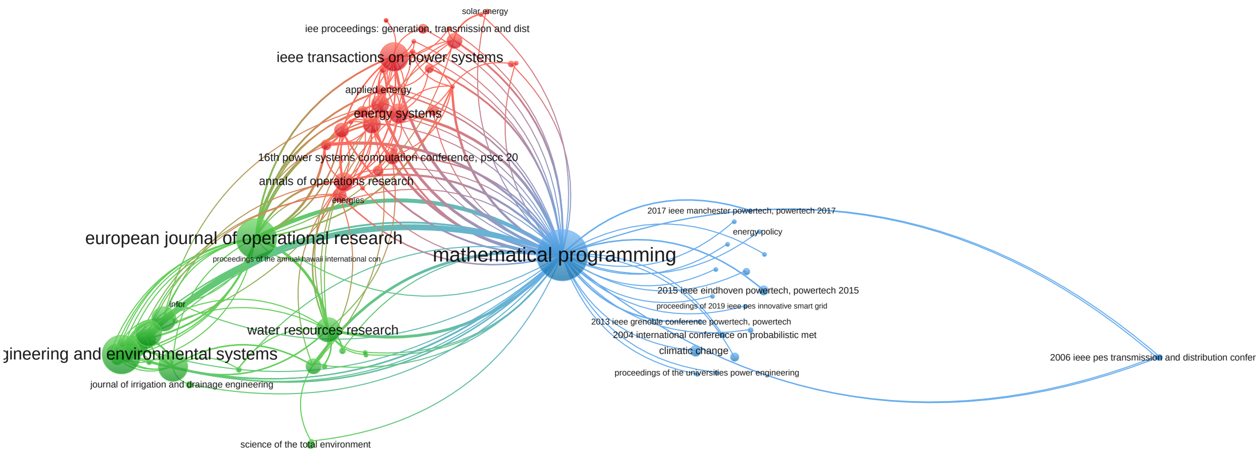

Figure 2: Bibliographic coupling of journals

We now emphasize the most influential papers based on total citations and average citations per year. Table 2 shows the top 30 most influential papers having at least 40 citations. The top three papers are Pereira & Pinto (1991), Huang & Loucks (2000) and Maqsood et al. (2005). In particular, the seminal work of Pereira & Pinto (1991) is the most influential paper with a total of 812 citations and the second most influential in terms of average citations per year (with 24.6 citations per year). In the top ten, two ranking positions (3 and 5) are occupied by EJOR papers, the well-recognized operations research journal. There are 15 papers with at least 70 citations and 7 with at least 7 citations per year. It is worth mentioning that IEEE-TPS is the journal with the highest number of papers in the ranking (four). Journals such as Advances in Water Resources (AWR), EJOR, ITOR, MP, Water Resources Management (WRM), and WRR have two papers in this rank, which demonstrates the multidisciplinary context of water resources management.

Rank	Reference	Title	Year	Journal	Cited by	Citations per year (CPY)	Rank by CPY
1	Pereira & Pinto (1991)	Multi-Stage Stochastic Optimization Applied to Energy Planning	1991	Mathematical Programming	812	24.6	1
2	Huang & Loucks (2000)	An inexact two-stage stochastic programming model for water resources management under uncertainty	2000	Civil Engineering and Environmental Systems	489	20.4	2
3	Maqsood et al. (2005)	An interval-parameter fuzzy two-stage stochastic program for water resources management under uncertainty	2005	European Journal of Operational Research	257	13.5	3
4	Li et al. (2006)	An interval-parameter multi-stage stochastic programming model for water resources management under uncertainty	2006	Advances in Water Resources	241	13.4	4
5	Philpott & De Matos (2012)	Dynamic sampling algorithms for multi-stage stochastic programs with risk aversion	2012	European Journal of Operational Research	131	10.9	5
6	Li et al. (2008a)	Inexact multistage stochastic integer programming for water resources management under uncertainty	2008	Journal of Environmental Management	103	6.4	10
7	Jacobs et al. (1995)	SOCRATES: A system for scheduling hydroelectric generation under uncertainty	1995	Annals of Operations Research	99	3.4	27
8	Scott & Read (1996)	Modelling hydro reservoir operation in a deregulated electricity market	1996	International Transactions in Operational Research	97	3.5	26
9	Homem-De-Mello et al. (2011)	Sampling strategies and stopping criteria for stochastic dual dynamic programming: A case study in long-term hydrothermal scheduling	2011	Energy Systems	88	6.8	8
10	Lu et al. (2008)	An inexact two-stage fuzzy-stochastic programming model for water resources management	2008	Water Resources Management	86	5.4	13
11	Watkins et al. (2000)	A scenario-based stochastic programming model for water supplies from the highland lakes	2000	International Transactions in Operational Research	85	3.5	25
12	Li & Huang (2009)	Fuzzy-stochastic-based violation analysis method for planning water resources management systems with uncertain information	2009	Information Sciences	85	5.7	11

Rank	Reference	Title	Year	Journal	Cited by	Citations per year (CPY)	Rank by CPY
13	Li et al. (2008b)	IFMP: Interval-fuzzy multistage programming for water resources management under uncertainty	2008	Resources, Conservation and Recycling	84	5.3	14
14	Tilmant et al. (2008)	Assessing marginal water values in multipurpose multireservoir systems via stochastic programming	2008	Water Resources Research	84	5.3	14
15	van Ackooij et al. (2014)	Joint chance constrained programming for hydro reservoir management	2014	Optimization and Engineering	70	7.0	7
16	Maceira et al. (2008)	Ten years of application of stochastic dual dynamic programming in official and agent studies in Brazil - Description of the NEWAVE program	2008	Power Systems Computation Conference	64	4.0	19
17	Rebennack et al. (2012)	Stochastic hydro-thermal scheduling under CO2 emissions constraints	2012	IEEE Transactions on Power Systems	60	5.0	16
18	de Queiroz et al. (2016)	Stochastic hydro-thermal scheduling optimization: An overview	2016	Renewable and Sustainable Energy Reviews	59	7.4	6
19	Li et al. (2009a)	Multistage scenario-based interval-stochastic programming for planning water resources allocation	2009	Stochastic Environmental Research and Risk Assessment	56	3.7	23
20	Raso et al. (2014)	Short-term optimal operation of water systems using ensemble forecasts	2014	Advances in Water Resources	56	5.6	12
21	Diaz (2009)	Optimal scheduling of a price-taker cascaded reservoir system in a pool-based electricity market	2011	IEEE Transactions on Power Systems	52	4.0	19
22	Rebennack (2016)	Combining sampling-based and scenario-based nested Benders decomposition methods: application to stochastic dual dynamic programming	2016	Mathematical Programming	52	6.5	9
23	Li et al. (2009b)	Water resources management and planning under uncertainty: An inexact multistage joint-probabilistic programming method	2009	Water Resources Management	51	3.4	28

Rank	Reference	Title	Year	Journal	Cited by	Citations per year (CPY)	Rank by CPY
24	Flach et al. (2010)	Long-term optimal allocation of hydro generation for a price-maker company in a competitive market: Latest developments and a stochastic dual dynamic programming approach	2010	IET Generation, Transmission & Distribution	51	3.6	24
25	Turgeon & Charbonneau (1998)	An aggregation-disaggregation approach to long-term reservoir management	1998	Water Resources Research	50	1.9	30
26	de Matos & Finardi (2012)	A computational study of a stochastic optimization model for long term hydrothermal scheduling	2012	International Journal of Electrical Power & Energy Systems	48	4.0	19
27	Zhou et al. (2013a)	Water resources management under multi-parameter interactions: A factorial multi-stage stochastic programming approach	2013	Omega	48	4.4	17
28	dos Santos & Diniz (2009)	A new multiperiod stage definition for the multistage benders decomposition approach applied to hydrothermal scheduling	2009	IEEE Transactions on Power Systems	44	2.9	29
29	Lima et al. (2013)	On the computational studies of deterministic global optimization of head dependent short-term hydro scheduling	2013	IEEE Transactions on Power Systems	43	3.9	22
30	Rebennack (2014)	Generation expansion planning under uncertainty with emissions quotas	2014	Electric Power Systems Research	43	4.3	18

Table 2: Top 30 most cited papers

We now investigate co-citation relationship. Figure 3 depicts the bibliographic coupling of authors. Here, circles and lines represent authors and co-citation degrees, respectively. The circle size is proportional to the number of links of a researcher, whereas the line width represents the co-citation degree between the papers of the corresponding two authors. It can be noted that regarding the total number of links, authors such as de Matos V., Rebennack S., Maceira M., Finardi E., and Favereau M. are the most influential authors.

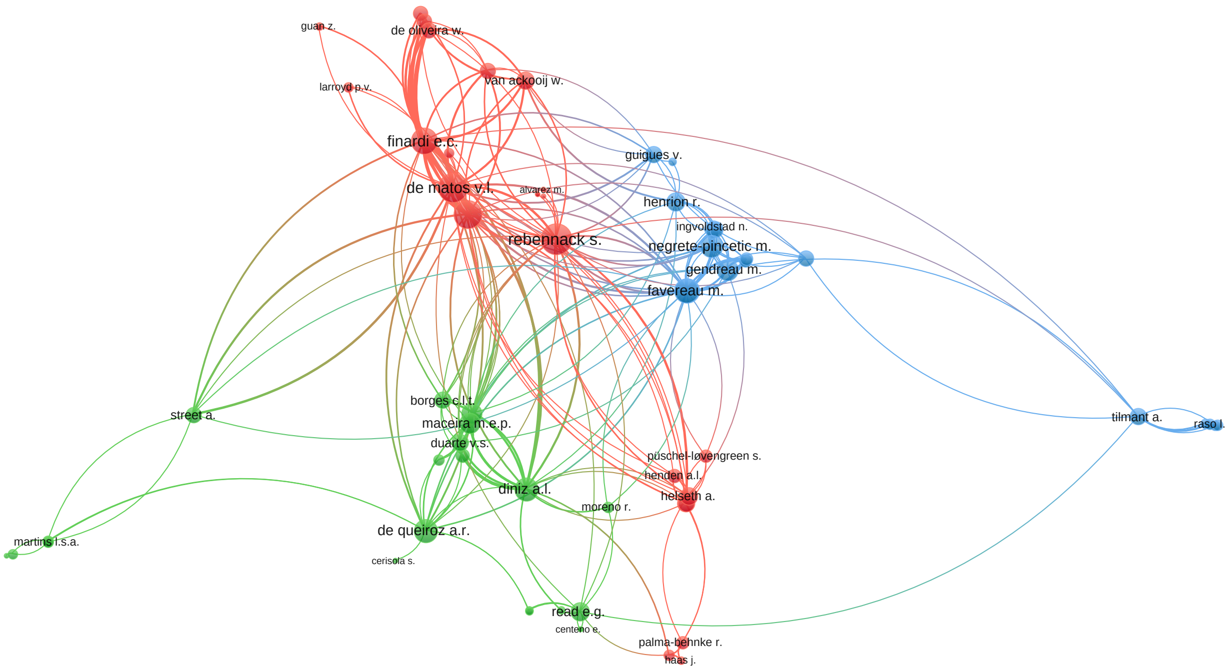

Figure 3: Bibliographic coupling of authors

Table 3 focuses on the top 15 most influential authors with respect to the number of published papers (at least 5) and to yearly average citations. Huang G.H. leads this ranking with a total number of 16 published papers. Interestingly, the top five authors (with more than 100 citations together) come from China and Brazil, two countries where hydropower generation is very critical for the generation matrice (IHA, 2022). Similarly, most of the other authors come from China, Brazil, Norway, and Denmark.

Rank by TP	Institution/Country	Author	Total papers (TP)	Total citations (TC)	TC/TP	Rank by TC/TP
1	Beijing Normal University, China	Huang G.H.	16	1640	102.5	1
2	Electrical Energy Research Center, Brazil	Diniz A.L.	12	125	10.4	13
3	Universidade Federal de Santa Catarina, Brazil	Finardi E.C.	11	228	20.7	6
4	Norus, Brazil	de Matos V.L.	9	304	33.8	3
5	Research Institute of Highway Ministry of Transport, China	Li Y.P.	8	615	76.9	2
6	SINTEF Energy Research, Norway	Mo B.	7	116	16.6	8
7	Technical University of Denmark, Denmark	Bauer-Gottwein P.	6	104	17.3	7
8	IBM, Brazil	Martins L.S.A.	6	46	7.7	14

Table 3 continued from previous page

Rank by TP	Institution/Country	Author	Total papers (TP)	Total citations (TC)	TC/TP	Rank by TC/TP
9	University of Porto, Portugal	Santos T.N.	6	43	7.2	15
10	Chinese Academy of Sciences, China	Liu S.	5	65	13.0	11
11	Rio de Janeiro State University, Brazil	Maceira M.E.P.	5	105	21.0	4
12	Chinese Academy of Sciences, China	Mo X.	5	65	13.0	11
13	Delft University of Technology, Netherlands	Raso L.	5	104	20.8	5
14	State University of Campinas, Brazil	Soares S.	5	73	14.6	9
15	SINTEF Energy Research, Norway	Warland G.	5	73	14.6	9

Table 3: Top leading authors with at least five published papers

Figure 4 depicts the bibliographic coupling of keywords where some research hotspots in water resources management problems can be identified, given that a keyword needs to appear at least seven times to be included in the network. Here, the larger the circle, the greater the number of occurrences of the keyword, and vice versa. In addition, the size of a line represents the occurrence degree between the two connected keywords. Some of the most prominent keywords in Figure 4 are "stochastic systems", "optimization", "dynamic programming", "reservoirs", "stochastic programming", "stochastic models", "stochastic dual dynamic programming", "scheduling", "water management", "water resources", "reservoirs management", "stochasticity", "hydroelectric power", "uncertainty", and "decision making".

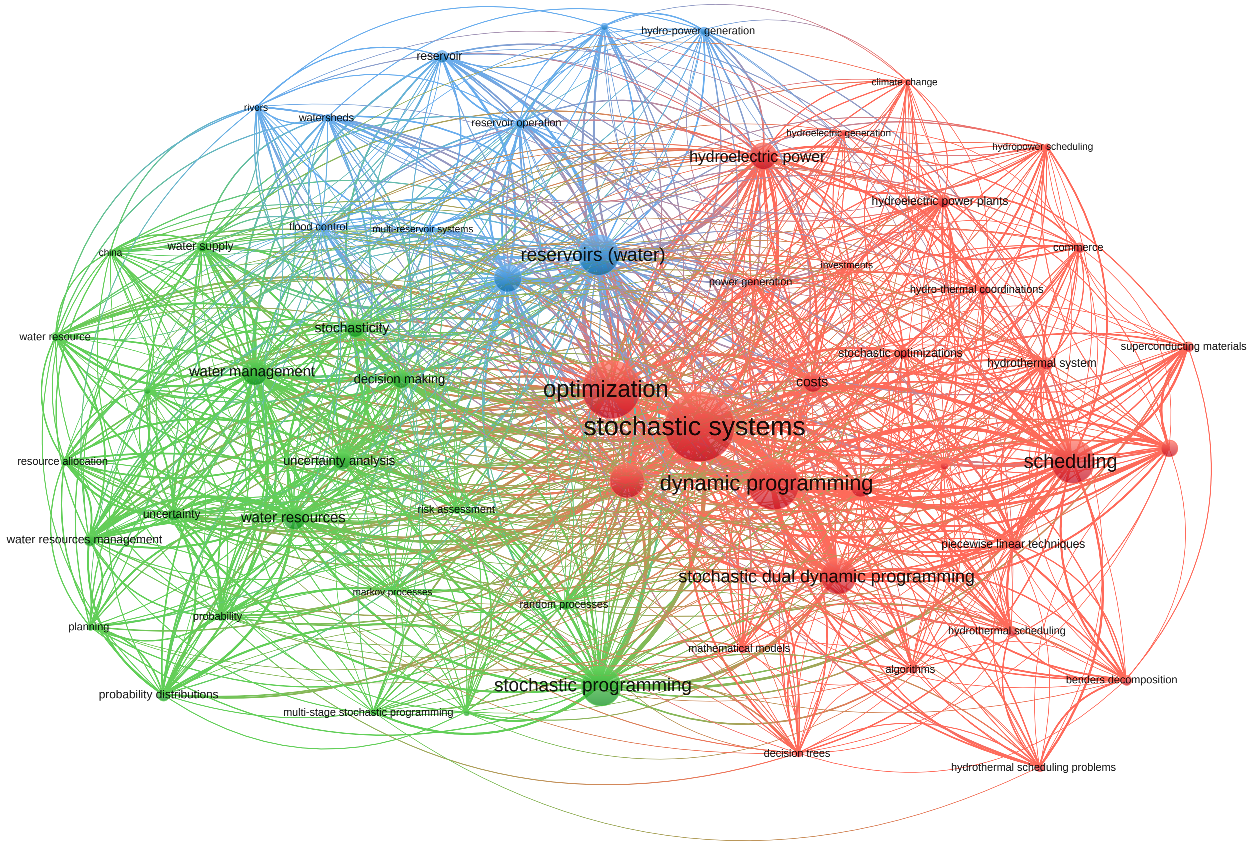

Figure 4: Bibliographic coupling of keywords

Figure 4 allows us to identify three main research areas studied in the literature, which will be discussed in more detail in Section 3. These three topics are *Modeling and solution approaches* whose keywords are highlighted in red on Figure 4, *Uncertainty sources and modeling* in green, and *Water resources management strategies* in blue.

3. Literature Review

This section presents a critical review and discussion of the collected papers around the three research areas identified in the previous section, e.g., modeling and solution approaches, uncertainty sources in models, and water resources management strategies.

3.1. Modeling and solution approaches

Water resources management problems typically consider stochastic hydro inflows in their formulation. Due to the random nature of hydro inflows and the intertemporal dependence of the decisions associated with the amount of water to store or release from reservoirs, this problem is commonly formulated as a multistage stochastic optimization problem (MSP) (de Queiroz et al., 2016). Typically, the solvability of large-scale MSPs is computationally intractable, so the design of efficient algorithms becomes crucial. An algorithm commonly used for this type of problem is SDDP, first proposed by Pereira & Pinto (1991). It is based on approximating the expected future cost (cost-to-go) function through the generation of scenario trees and the iterative use of the Benders decomposition (Benders, 1962). In particular, SDDP was proposed with an application in hydrothermal scheduling in Brazil at that time and is still used today (Lorca et al., 2020). Risk-neutral optimization models, such as the classic formulation of hydrothermal scheduling, do not allow to capture the variability of operational costs since it focuses on minimizing the expected costs, so some research has been focused on including risk-averse approaches and they become more relevant considering the increasing uncertainty of future hydro inflows. The following sections will review the most common modeling and solution approaches for water resources management employed in the literature. For the convenience of readers, Appendix A presents a brief technical discussion of the optimization under uncertainty foundations in the context of water resources management.

3.1.1. Dynamic programming

Since the origins of problems related to water resources management and before the publication of the SDDP algorithm, the most common approach was dynamic programming (DP) and its uncertain derivation, stochastic dynamic programming (SDP, or Markov decision processes, MDP) (Lindqvist, 1962; Sniedovich, 1979; Baker & Daellenbach, 1984; White, 1985). Even some years after, MDP was still an essential approach to stochastic sequential decision problems, especially in the context of hydro-power systems (Lamond & Boukhtouta, 1996).

In Turgeon & Charbonneau (1998), an optimal policy of a whole system (Hydro-Québec's 26 large reservoirs) was found with SDP, giving not only an optimal operation policy but also the expected marginal value of hydroelectric energy produced and stored. This was done using the so-called aggregation-disaggregation approach, which reduces computational efforts by aggregating reservoirs, but losing some essential details in the water network modeling. Consequently, evaluating the flexibility in complex hydroelectric power systems using aggregated elements of the water (or power) network becomes increasingly challenging.

In Lamond (2003), an algorithm based on splines was presented to compute the expectation of a piecewise polynomial approximation of the future value function for a single hydroelectric reservoir, avoiding the discretization of inflow distribution, thus deriving a continuous DP faster and more accurately than classical DP approaches. For a water resources planning problem, Zhao et al. (2010) proposed a finite-horizon MDP with continuous state and decision spaces and a sensitivity-based solution approach. The authors show that the proposed method improved the system's performance without affecting computational efficiency.

There is an application of SDP known as the water value method on which important research is based. This method aims to obtain reservoir operating rules given uncertain inflows. It consists of calculating the total cost (immediate cost plus expected future cost) for each stage and state and then taking the derivative with respect to the reservoir level (resulting in the water value, measured in currency over the amount of water or energy). Pereira-Cardenal et al. (2014) analyzed climate change impacts, based on the water value method, in operating rules for the Iberian power system based on hydrological uncertainty given by a hydrological model. Within the aspects that most affect the quality of the solutions, the authors highlight the uncertainty associated with the impacts of climate change, the spatial aggregation of the hydrological model, and the temporal representation of the power system (directly affecting power supply and demand). In addition to water quantity, water quality substantially affects optimal water allocation policies, as Davidsen et al. (2015a) showed using the water value method in a hydroeconomic optimization approach and Martinsen et al. (2019) in a hydroeconomic optimization modeling framework for joint water allocation and quality optimization. Also based on a hydroeconomic optimization approach, Davidsen et al. (2015b) derived a pricing policy to bring an overexploited groundwater aquifer to a sustainable state in the long term, quantifying potential savings of joint (surface and groundwater) water management of a river basin in China. A similar result was found in Davidsen et al. (2015c) using a hydroeconomic approach, showing that the South-North Water Transfer Project (an infrastructure mega-project) reduces the impacts of water scarcity and enhances optimal water management in a Northern China basin.

Although the accuracy of SDP is limited by the dimensionality of decision space, state space, or uncertainty space (the so-called "curse of dimensionality"), some research has been done to improve it and determine near-optimum decisions cooperating with other approaches. One example is the work of Saadat & Asghari (2018), where a nonlinear programming model is embodied in an SDP model, increasing the dimensionality of the uncertainty and state spaces without sacrificing computational times and solutions quality (in terms of electricity and water reliability).

Table 4 summarizes the different works reviewed around dynamic programming according to the particular approach, planning horizon, and uncertainty sources. Note that in the following tables, we indicate the horizon of the models. We define short-, medium-, and long-term planning horizons as 0 to 1, 1 to 5, and 5 and more years, respectively.

Modeling approach	Planning horizon	Uncertainty sources	References
DP	Long-term	All deterministic	Turgeon & Charbonneau (1998) Lamond (2003)
MDP/SDP	Long-term	Hydro inflows	Zhao et al. (2010) Pereira-Cardenal et al. (2014) Davidsen et al. (2015a,b,c) Martinsen et al. (2019) Saadat & Asghari (2018)

Table 4: A summary of modeling approaches of reviewed papers based on dynamic programming

3.1.2. Stochastic optimization

Although dynamic programming was one of the leading modeling approaches for water resources management problems, stochastic optimization has become the primary modeling approach in recent years (de Queiroz, 2016; Lorca et al., 2020). The principal decision-making structures in this approach are two-stage or multistage schemes.

A multistage framework typically approaches water resources management problems. However, there are exciting works based on two-stage stochastic optimization. In (Huang & Loucks, 2000), an inexact two-stage stochastic model is proposed for water resources management. It consists of a hybrid of inexact optimization and two-stage stochastic optimization, allowing it to work not only with probability distribution-based uncertainties but also with discrete intervals. Despite the computational advantages that this entails, this modeling approach has substantial limitations for large-scale systems, in addition to the over-simplification of uncertain intervals, reducing the system's reliability. An extension of this work is the one by Maqsood et al. (2005), where an interval-parameter fuzzy two-stage stochastic optimization model is proposed, introducing the concept of interval fuzzy membership function, which reflects the complexities of system uncertainties presented as fuzzy, stochastic, and interval parameters as well as their combinations. This work was extended by Wang & Huang (2013) and Khosrojerdi et al. (2019) mainly by adding the concept of type-2 fuzzy sets, which helps capture the uncertainty in membership functions. Additionally, Lu et al. (2008) present an inexact two-stage fuzzy stochastic optimization model differing from previous works in incorporating variable penalty policies (economic measures for authorities and water users) for water resources management systems based on fuzzy events. The previous works can hardly solve complex problems on the existence of parameters with dual-layer information, e.g., when the quantity of an unregulated water source is imprecise. Then, the estimation is made based on the decision maker's subjective experience and objectively recorded data. Lu et al. (2009) proposed the incorporation of rough intervals (i.e., upper and lower approximation intervals) to express such information for agricultural irrigation systems, defined as an inexact rough-interval two-stage stochastic optimization.

One of the main advantages of two-stage stochastic optimization models is related to the efficiency of reformulations, algorithms, and decomposition procedures to solve them. Guerra et al. (2019) proposed a two-stage stochastic optimization model for power system design and planning evaluating climate change impacts on a hydro-dominated power system, and it was solved through the deterministic equivalent MILP problem. In (Helseth et al., 2010), a sequence of two-stage stochastic optimization models was solved for a long-term hydrothermal scheduling problem using the Benders decomposition (Benders, 1962). The proposed model is actually a simulator of a sequence of the production schedule one week ahead, and it was shown that it is easier to implement and computationally more tractable than solving a multistage scheme via SDDP. In the case of (Helseth et al., 2018), the authors present a method for long-term hydrothermal scheduling, where weekly (dispatch) decisions are obtained by solving a two-stage stochastic optimization problem with uncertainty in weather and exogenous market prices, embedding such two-stage problems in a rolling horizon simulator. Hydrothermal scheduling models with long planning horizons (2-7 years) are essential because they can approximate the future cost function of stored water. With this approximation at hand, a short-term model with more operational detail can be solved to obtain the schedule of releases based on simpler optimization models, as done by Alvarez et al. (2018).

With respect to multistage stochastic optimization, we must first highlight the work done by Pereira & Pinto (1991), which is essential in this paper. This seminal work presented for the first time the SDDP algorithm, which is based on an approximation of the expected future cost function through the generation of scenario trees and the iterative use of the Benders decomposition to solve linear multistage stochastic optimization problems. We refer the reader to (Shapiro, 2011) for a thorough explanation and analysis of this algorithm's statistical properties and convergence, and to Füllner & Rebennack (2019) for a tutorial-type review on SDDP and its variants. Some other works also analyze computationally efficient heuristics and SDDP enhancements to solve multistage stochastic optimization models (Guigues, 2014; Street et al., 2020; Siddig et al., 2021; Beltrán et al., 2021; Dowson et al., 2022; Borges et al., 2022).

Since the publication of the SDDP algorithm, some important research has taken place. Based on multistage stochastic optimization, Jacobs et al. (1995) described a generation scheduling model developed by PG&E (Pacific Gas and Electric Company), called SOCRATES (Stochastic Optimal Coordination of River-basin And Thermal electric power systems), which was designed to schedule hydrothermal generation considering uncertain hydro inflows. A multistage stochastic optimization model was also developed in (Watkins et al., 2000) to support the Lower Colorado River Authority's decision regarding the amount of interruptible water to contract each year, based on two objectives (maximize recreational benefits and revenues from the sale of interruptible water). For the New Zealand power system, a hydrothermal scheduling model was also developed and satisfactorily solved with SDDP in the work of Halliburton (2004), considering all relevant physical features of the power system for medium- to long-term planning horizons. Although conventional optimization techniques such as SDDP and Lagrangian relaxation effectively solve large-scale water resources management problems, they have severe limitations regarding incorporating nonlinearities or stochasticity of hydro inflows, respectively. Therefore, Modarres et al. (2004) proposed a hybrid genetic algorithm to solve (almost optimally) the hydrothermal scheduling problem with quadratic costs for thermal power plants.

Although inexact optimization has been effectively combined with two-stage stochastic optimization, as mentioned earlier, it has also been combined with multistage schemes. This permits revised decisions in each stage based on the uncertainty revealed up to that stage. In (Li & Huang, 2007), an inexact multistage stochastic quadratic programming model for water resources management was proposed, which can directly tackle uncertainties presented as discrete intervals and probability distributions. In addition, it can accommodate real-time dynamics of system uncertainties based on a complete set of scenarios and deal with nonlinearities in the objective function (such as the effects of marginal utility on system benefits and costs). In a similar work, Li et al. (2008a) proposed an inexact multistage stochastic integer programming incorporating economies-of-scale effects in diversion costs based on fixed-charge cost functions. Factorial analysis, a multivariate inference method, was introduced into inexact multistage stochastic optimization in (Zhou et al., 2013a) to analyze the potential interrelationships between several uncertain parameters to support the water resources management problem. These previous works still cannot approach the problem of partially-known information, which has, as a practical consequence, the challenge of assuming some probability distributions. As an extension of the previous works, Chen & Wang (2021) established an inexact multi-stage interval-parameter partial information programming model for urban water resources management, considering the linear partial information theory to represent hydro inflows probabilities.

Generally, an essential issue in stochastic optimization is modeling uncertain parameters, especially when data is not available enough or is of poor quality. In (Li et al., 2006), an interval-parameter multistage stochastic optimization model for water resources management was proposed. This multistage version admits to incorporating more dynamic and uncertain information within the model compared to a two-stage version. However, it could not account for the risk of violating system constraints under several uncertainties as the multistage scenario-based interval-stochastic optimization model of Li et al. (2009a) did. The work of Li et al. (2006) was extended by Li et al. (2008b), where uncertainties were presented in terms of fuzzy sets that deal with the practical questionability of the solutions obtained. By coupling fuzzy mathematical programming with multistage stochastic optimization, a fuzzy-stochastic-based violation analysis was proposed by Li & Huang (2009), where violation variables for the objective and constraints were introduced into the modeling formulation, helping to generate a range of decision alternatives balancing (i) environmental and economic objectives, and (ii) system optimality and reliability. By incorporating an interactive fuzzy resolution method within an inexact multistage stochastic programming, Wang & Huang (2012) proposed a model capable of reflecting the uncertainty dynamics and the corresponding decision processes based on the use of representative scenarios, allowing to identify a desired compromise between the degree of satisfaction of goals and constraints feasibility. Other modeling approaches related to fuzzy-stochastic programming were proposed and analyzed, where interval-stochastic fractile optimization (Zhou et al., 2013b) and multistage stochastic fuzzy optimization (Li et al., 2018) stand out.

Interval-parameter multistage stochastic chance-constrained mixed-integer optimization, another extension of the work of Li et al. (2006) that incorporates chance-constrained techniques and mixed-integer variables, was proposed by Han et al. (2008). This extension allows the authors to address more complex needs such as sewage plant construction, water-saving technologies, municipality water requirements, and regional food supply security. By considering a chance-constrained scheme for minimum and maximum stored water level constraints, the work of van Ackooij et al. (2014) presents a joint chance-constrained optimization model for a hydro reservoir management problem, obtaining an interesting trade-off between costs and robustness and can be tractable for complex, realistic models. A similar balance between costs and solutions' robustness was obtained in (El Karfi et al., 2022) with a chance-constrained optimization model of an optimal investment into agricultural infrastructure. Other works incorporating chance constraints with inexact multistage stochastic optimization (Li et al., 2009b) and factorial multistage stochastic optimization (Liu et al., 2016) are also proposed.

Although medium-term water resources management problems are the most common in literature, long-term analyses are critical for water and power networks planning. See, for example, the tutorial on stochastic programming of Finardi et al. (2013) based on the long-term hydrothermal scheduling problem, De Matos et al. (2011), which also describes the long-term hydrothermal scheduling problem and some strategies to improve the performance of SDDP to solve it, the description of some modeling issues by de Matos & Finardi (2012), and de Matos et al. (2017) that asses the quality of an operational policy based on SDDP. An essential challenge for water resources planning is the determination of optimal bidding decisions for price-maker energy producers. In (Flach et al., 2010), a long-term multistage stochastic optimization model was proposed with that purpose, considering uncertain hydro inflows and solved with an SDDP-based approach. Solving long-term multistage problems can be challenging, so different techniques have been applied. Typically, bidding on flexible reservoir hydropower in the day-ahead market is part of a short-term problem. However, Fleten et al. (2011) presented a long-term multistage stochastic mixed-integer optimization model with fine time granularity and uncertain electricity prices and inflows, which is satisfactorily detailed for short-term bidding still solvable for long-term planning horizons. Another fundamental challenge for water resources planning is the incorporation of (i) other sources of uncertainty in addition to hydro inflows and (ii) a detailed formulation of nonlinear or nonconvex functions. Tong et al. (2013) propose a multistage (3-stage) stochastic mixed-integer programming model for long-term hydrothermal scheduling considering uncertain weather conditions (hydro inflows and water demand) and net load (grid load demand and wind power generation), in addition to a piecewise linearization of nonlinear and nonconvex thermal generation costs, hydropower production, water recession, reservoir evaporation.

Table 5 summarizes the different works reviewed around stochastic optimization according to the particular approach, planning horizon, and uncertainty sources.

Modeling approach	Planning horizon	Uncertainty sources	References
Two-stage stochastic optimization	Short-term	Hydro inflows	Huang & Loucks (2000) Maqsood et al. (2005) Lu et al. (2008) Wang & Huang (2013) Khosrojerdi et al. (2019)
	Long-term	Hydro inflows	Lu et al. (2009) Helseth et al. (2010)
		Hydro inflows, power prices	Helseth et al. (2018)
		Hydro inflows, power demand	Alvarez et al. (2018)
		Hydro generation, gas prices, climate policy	Guerra et al. (2019)

Table 5 continued from previous page

Modeling approach	Planning horizon	Uncertainty sources	References
Multistage stochastic optimization	Medium-term	Hydro inflows	Pereira & Pinto (1991) Beltrán et al. (2021) Dowson et al. (2022) Jacobs et al. (1995) Watkins et al. (2000) Halliburton (2004) van Ackooij et al. (2014)
		Hydro inflows, power prices	Borges et al. (2022)
		Demand of cereals, yield coefficient of soil	El Karfi et al. (2022)
	Long-term	Hydro inflows	Guigues (2014) Street et al. (2020) Siddig et al. (2021) Modarres et al. (2004) Finardi et al. (2013) De Matos et al. (2011) de Matos & Finardi (2012) de Matos et al. (2017) Flach et al. (2010)
		Hydro inflows, electricity prices	Fleten et al. (2011)
		Hydro inflows, water demand, load demand, wind generation	Tong et al. (2013)
		Water-allocation targets, cost, benefit, water deficit	Li et al. (2006) Li & Huang (2007) Han et al. (2008) Li et al. (2008a,b) Li & Huang (2009) Li et al. (2009a,b) Liu et al. (2016) Wang & Huang (2012) Zhou et al. (2013a,b) Li et al. (2018) Chen & Wang (2021)

Table 5: A summary of modeling approaches of reviewed papers based on stochastic programming

3.1.3. Risk-averse approaches and robust optimization

Risk-neutral optimization models, such as the classic formulation of water resources management problems, do not allow to capture the variability of operational costs since it typically focuses on minimizing the expected costs, so some research has been focused on how to include risk-averse (and robust) approaches. They are more relevant considering the increasing uncertainty of future hydro inflows due to the consequences of climate change.

A risk-averse approach can be straightforwardly incorporated into multistage stochastic optimization models, and given an adequate level of risk aversion, it can be possible to reduce the probability of poor outcomes only by slightly affecting the overall cost. A classical (and coherent) risk functional used for this purpose is the conditional value at risk (CVaR), and a convex combination between it and expectation remains a coherent risk measure. It was shown in (Philpott & De Matos, 2012) how to incorporate this convex combination into SDDP and implement it in an extensive numerical study. A modified version of the risk-averse SDDP algorithm is proposed for the medium-term hydrothermal scheduling problem by

Soares et al. (2014), and it aims to reduce the variability of marginal costs and decisions induced by the use of autoregressive models. Also, Maceira et al. (2015) applied a risk-averse approach based on the convex combination of expectation and CVaR in the expansion and operation planning and for setting the spot price in the Brazilian hydrothermal system, obtaining more conservative solutions but without sacrificing total costs. In addition to CVaR, other risk-measure strategies have been proposed for multistage optimization models. For instance, in (Larroyd et al., 2017), a risk-averse long-term hydrothermal scheduling problem was approached by (i) a convex combination between CVaR and the expected operating costs and (ii) a reservoir risk curve. The latter consists of checking periodically if reservoir levels are lower than the risk curve in each stage, and if some violation occurs, hydro generation is reduced. Although this method seems interesting, in the work of Larroyd et al. (2017), the CVaR-based approach behaves better than the reservoir-risk curve-based, providing more coherent operation policies. A more general class of convex risk measures embodied into SDDP can be seen in (Dowson et al., 2022) where it was analyzed the entropic risk measure which does not have the shortcoming of leading to inconsistencies that other risk measures do. Even though risk-averse SDDP approaches have been demonstrated to perform efficiently, the rolling horizon policy compares favorably with SDDP policies, involving shorter computational times. Guigues & Sagastizábal (2012) analyzed this in a rolling horizon model that iteratively solves and implements the solution to a chance-constrained single-stage problem.

Robust optimization is an emerging modeling technique of recent years, and exciting works have been proposed in the context of water resources management. One of the main differences between robust optimization and other stochastic approaches is that a sampling process is avoided, and all that it means, e.g., balancing the computational effort and statistical representativeness of stochastic processes. Mejia-Giraldo et al. (2014) presented an adjustable robust long-term hydrothermal scheduling model where the hydrological uncertainty was represented by ellipsoidal sets based on sample means and covariance of a Colombian hydro subsystem (four hydro plants). The chance-constrained optimization model of van Ackooij et al. (2014) for a hydro reservoir management problem is compared to a robust model with an ellipsoidal uncertainty set for hydro inflows, and the better performance of the chance-constrained model can be mainly due to the conservativeness of the uncertainty set.

Typically, multistage robust optimization models are computationally challenging to solve, mainly because the decision space is in an infinite-dimensional function space. Therefore, to handle this problem, an alternative is to restrict the decision variables to affine functions of the sequential realizations of the uncertainty, known as linear (or affine) decision rules. In the work of Gauvin et al. (2017), several numerical experiments in a robust optimization model for a reservoir management problem confirm the value of affine decision rules, providing efficient implementable solutions but maintaining the tractability of the model. In (Egging et al., 2017), linear decision rules were also applied for a model that takes the perspective of a hydropower producer aiming to maximize the expected market value of power production, considering symmetric polyhedral uncertainty sets for electricity prices and hydro inflows. Methodologically, Gauvin et al. (2018) extends the previous works by considering linear decision rules and time series for uncertainty representation, but for a reservoir management problem, providing reasonable quality solutions and illustrating the importance of considering the persistence of hydro inflows. In (Favereau et al., 2023), a multistage adaptive robust optimization model was proposed for the medium-term hydrothermal scheduling problem. In this case, a direct reformulation was obtained when considering using linear decision rules and symmetric polyhedral uncertainty sets (based on vector autoregressive models). This allows to solve the problem for a large-scale instance with hundreds of hydro nodes and connections, hundreds of buses, and almost a thousand generators.

Table 6 summarizes the different works reviewed around risk-aver stochastic and robust optimization according to the particular approach, planning horizon, and uncertainty sources.

Modeling approach	Planning horizon	Uncertainty sources	References
Risk-averse stochastic optimization	Short-term	Hydro inflows	van Ackooij et al. (2014)
	Medium-term	Hydro inflows	Soares et al. (2014) Dowson et al. (2022)

Table 6 continued from previous page

Modeling approach	Planning horizon	Uncertainty sources	References
	Long-term	Hydro inflows	Philpott & De Matos (2012) Maceira et al. (2015) Larroyd et al. (2017) Gauvin et al. (2017)
		Hydro inflows, power demand	Guigues & Sagastizábal (2012)
Robust optimization	Medium-term	Hydro inflows	Favereau et al. (2023)
		Hydro inflows, power prices	Egging et al. (2017)
	Long-term	Hydro inflows	Mejia-Giraldo et al. (2014) Gauvin et al. (2018)

Table 6: A summary of modeling approaches of reviewed papers based on risk-averse stochastic and robust optimization

3.1.4. Other formulation approaches

For long-term hydrothermal scheduling, Zambelli et al. (2009b) proposed a model predictive control (MPC) approach that outperforms an SDP model. Here, the nonlinear aspects of inflow correlation, hydropower generation, and thermal fuel costs are represented based on a neural fuzzy network forecasting model and a nonlinear optimization model to determine the discharge decisions. This is a similar result compared to Zambelli et al. (2009a), where an MPC approach outperforms an SDP model for long-term hydrothermal scheduling, mainly due to the benefits of implementing a fuzzy inference system for monthly and annual inflow forecasts. Although MPC has shown significant benefits in addressing long-term water resources management problems, it has also been effective for short-term problems. Raso & Malaterre (2017) proposed an infinite horizon MPC (short-term model with integrated long-term effects) based on basis functions to couple short- and long-term objectives.

Not only has the performance of an MPC model been analyzed against SDP, but also stochastic optimization models. In (Zambelli et al., 2011), it was shown that an MPC model (with a deterministic nonlinear optimization model for operational decisions) gives an 8.23% smaller expected operating cost in contrast to a stochastic optimization approach, based on a simulation over 75 inflow scenarios. Another case study with favorable results for MPC is the one of Zambelli et al. (2013) where a decrease of 3.5% of the operating costs was obtained versus a stochastic optimization model, highlighting that a more detailed system modeling should prevail over sophisticated uncertain inflow representation.

Due to the "curse of dimensionality" in dynamic programming, some research focuses on developing heuristics-based algorithms to solve water resources management problems. In (Cau & Kaye, 2002), an evolutionary method was proposed for minimizing the total operational cost of hydrothermal scheduling, combining dynamic programming and evolutionary programming, and evolving the piecewise linear cost-to-go functions (water values) instead of decision variables. Also based on evolutionary algorithm techniques, Hinojosa & Leyton (2012) applied a mixed-binary evolutionary particle swarm optimization to short-term hydrothermal scheduling (considering binary thermal states as decision variables), obtaining an enhancement in solutions quality and significantly improving the convergence (by working only with feasible solutions).

Although the hydrological component of water resources management problems is inherently stochastic, some deterministic approaches have been carried out to study different approximations and the impact of certain phenomena. Santos & Diniz (2009) explored two strategies to handle infeasible subproblems during the multistage Benders decomposition for a deterministic large-scale short-term hydrothermal scheduling problem, i.e., strategy 1 uses slack variables in each constraint. In contrast, strategy 2 considers feasibility cuts to try to vanish infeasibilities, being strategy 1 computationally slower but consistently delivering a solution. By comparing two models, namely EMPS (a deterministic heuristic-based approach) and SOVN (a stochastic optimization model), Warland & Mo (2016) indicate that replacing aggregate hydro description and heuristics with a more formal optimization may give a model better suited for power systems analysis.

The physics of hydroelectricity generation is commonly simplified in the literature to account for its fundamentally nonlinear nature, and its consideration and some approximations have been studied. A nonconvex mixed-integer nonlinear optimization model for the hydrothermal scheduling problem was proposed in (Lima et al., 2013), considering a detailed representation of the net head water and a nonlinear hydropower generation function. A simplified model was also formulated based on a relaxation framework, exhibiting better computational performance than commercial solvers. Vieira et al. (2015) analyzed the mathematical and physical properties of the hydrothermal scheduling problem. In particular, it studied the hydropower generation nonlinear function and established sufficient mathematical properties to guarantee optimality conditions. Paredes & Martins (2018) show how semidefinite programming relaxation is only exact for a quadratically constrained quadratic hydrothermal scheduling problem if a very strict condition regarding turbine efficiency is observed (concavity of the hydropower generation function).

Additional analyses have been carried out concerning estimating future cost functions or integrating other uncertainty sources. Alvarez et al. (2017) proposed a model for short-term hydrothermal scheduling linked to long-term conditions through a piecewise-linear future cost function to maximize the value of the electricity production from hydropower. With the integration of renewable sources like solar and wind, there is a growing need to approach hydrothermal scheduling problems considering this. For a long-term hydrothermal-solar scheduling problem, Singh & Banerjee (2017) proposes a two-stage algorithm based on dynamic programming and linear programming techniques. This allows for assessing the impact of large-scale rooftop solar photovoltaic integration in hydrothermal systems.

From the perspective of market equilibrium models, some interesting research has been carried out. Barquín et al. (2005) proposed a model to represent a medium-term operation of the electrical market that introduces hydrological uncertainty formulation naturally and practically, where utilities are explicitly considered to be maximizing their expected profits. Market equilibrium conditions are presented through the optimality conditions of the problem, which has a structure that strongly resembles the hydrothermal scheduling problem. Cruz et al. (2016) proposed an equilibrium model to determine price and quantity strategic bids for generation companies in a day-ahead market. Each company aims to maximize its profit by solving a bilevel optimization problem, where the upper-level corresponds to the producer revenue maximization. At the same time, the lower one represents the minimization of system operation faced by the independent system operator. Solutions to short-term water resources management problems usually consider boundary conditions provided by long-term models. In this case, the centralized dispatch is performed by solving single-level optimization problems. Nevertheless, Bernardinelli & Martins (2017) also presented a (deterministic) bilevel formulation with equilibrium constraints, in which the upper-level problem considers long-term objectives and constraints, and the lower-level problem represents short-term operational conditions. It is important to note that risk criteria in market participants' decisions have not been strongly considered. Consequently, Jovanovic et al. (2018) presented a multistage market equilibrium model of risk-averse agents in the context of fuel price volatility and renewable energy source penetration to analyze the impacts of risk-averse decisions of market participants.

Simulation is another approach that has been considered for water resources management problems. Perea et al. (2010) presented a mixed approach for the long-term hydrothermal scheduling problem based on a simulation model and a stochastic optimization model, enabling the assessment of different design options for new assets or considering the repowering of old ones. Huang et al. (2013) presented a multistage simulation-based optimization model for supporting water resources management, coupling a lumped rainfall-runoff model with an inexact multistage stochastic optimization model. Unlike other stochastic optimization works, this can obtain random hydro inflows through the statistical analysis of hydrological simulation outcomes and incorporate more dynamic and uncertain information within its modeling framework.

Table 7 summarizes the different works reviewed around other modeling approaches according to the considered planning horizon, particular modeling approach, and uncertainty sources.

Planning horizon	Modeling approach	Uncertainty sources	References
Short-term	Dynamic programming (heuristic-based)	All deterministic	Cau & Kaye (2002) Hinojosa & Leyton (2012)
	Linear programming	All deterministic	Santos & Diniz (2009)
	Nonlinear programming	All deterministic	Lima et al. (2013) Vieira et al. (2015) citeParedes2018
	Model predictive control	Hydro inflows	Raso & Malaterre (2017)

Table 7 continued from previous page

Planning horizon	Modeling approach	Uncertainty sources	References
	Stochastic optimization	Hydro inflows	Alvarez et al. (2017)
Medium-term	Market equilibrium model	All deterministic	Cruz et al. (2016) Bernardinelli & Martins (2017)
		Hydro inflows	Barquín et al. (2005)
		Hydro inflows, power demand, renewables availability, fuel costs	Jovanovic et al. (2018)
Long-term	Dynamic programming	All deterministic	Singh & Banerjee (2017)
	Model predictive control	Hydro inflows	Zambelli et al. (2009a,b) Zambelli et al. (2011) Zambelli et al. (2013)
	Stochastic optimization	Hydro inflows	Warland & Mo (2016)
	Simulation	Hydro inflows	Perea et al. (2010) Huang et al. (2013)

Table 7: A summary of modeling approaches of reviewed papers based on other approaches

3.2. Uncertainty sources and modeling

As mentioned in previous sections, and as expected, the main uncertain component in the reviewed papers corresponds to hydrological uncertainty. However, other uncertain parameters have been considered in the literature. This section presents the models representing the different uncertain parameters in water resources management problems.

In general, extensive literature addresses hydrological uncertainty in water resources management problems (as will be discussed later). Still, it may often be challenging to consider additional uncertain parameters (and together with hydro inflows). Besides, the quality of information may be poor to be presented as a probability distribution (in the stochastic case), and their reflection in large-scale models could be extremely hard to solve. Typically, works based on inexact and fuzzy optimization have been able to consider more than one source of uncertainty, such as hydro inflows, water allocation targets, economic data, or irrigation schedules. Instead of working with probability distributions, these works consider discrete intervals in which the uncertain parameters are supported, providing an effective way to consider different uncertainty sources but without affecting computational efficiency in their resolution (Huang & Loucks, 2000; Maqsood et al., 2005; Li et al., 2006; Li & Huang, 2007; Han et al., 2008; Li et al., 2009a,b; Wang & Huang, 2013; Zhou et al., 2013b,a; Guo et al., 2014; Liu et al., 2016; Li et al., 2018; Chen & Wang, 2021).

Fuzzy sets have also been considered in uncertainty modeling to represent different phenomena. In particular, a fuzzy set is a pair (Ω, m), where Ω is a (typically non-empty) set known as a universe of discourse, and $m : \Omega \to [0, 1]$ a membership function where for each $z \in \Omega$, $m(z)$ represents the grade of membership of z in (Ω, m). One of the limitations of interval-based models is their over-simplification of the fuzzy membership information into intervals. Based on fuzzy sets theory, works such as Lu et al. (2008); Li et al. (2008b); Zambelli & Soares (2009); Zambelli et al. (2009b); Wang & Huang (2012); Khosrojerdi et al. (2019) have been able to consider uncertain inflows, water allocation targets, costs, and benefits more realistically.

Multistage stochastic optimization models typically approximate a stochastic process $\boldsymbol{\xi} = \{\boldsymbol{\xi}_t\}_{t=1}^{T}$ by a process having finitely many scenarios exhibiting a tree structure and starting with a fixed element $\boldsymbol{\xi}_1 \in \mathbb{R}^d$. This structure allows the outcome of uncertainty to be gradually revealed, and the decisions are made dynamically without anticipating future uncertainty outcomes. In (Barquín et al., 2004), a scenario tree was considered to represent the hydrological uncertainty in a medium-term generation planning model, but only considering three scenarios (wet, medium, and dry). Tong et al. (2013) considered four sources of uncertainty (hydro inflows, water demand, grid load demand, and wind power generation) based on weather conditions and net load, and it was developed based on a scenario tree structure. Similarly, in (Jovanovic et al., 2018), a scenario tree was considered for modeling uncertain hydro inflows, power demand, and fuel costs for a medium-term market equilibrium hydroelectric operation model. Additionally, Borges et al. (2022) addressed the optimal management of cascade hydropower generation, considering uncertain hydro inflows and electricity prices, although it does not explicitly state which distributions were used. A significant challenge associated with scenario trees is the balance between the statistical representativeness (how many scenarios are considered) and the computational efficiency of solving the corresponding optimization model. Therefore, for large-scale problems, a reduction technique could benefit statistical representativeness. However, this should be done carefully to avoid modifying probability distribution moments, such as the expectation of the stochastic processes. With this purpose, Fan et al. (2016) presented a novel scenario tree generation (or reduction) using probabilistic ensemble hydro inflows forecasts for a short-term reservoir optimization problem. A similar alternative is using ensemble forecasts, a set of representative trajectories of the possible future outputs. Raso et al. (2014) used ensemble forecasts for a short-term water system operation model, proving to be effective compared to other real-time optimal control methods.

As a more general case, some optimization under uncertainty models considers traditional probability distributions to represent uncertain parameters (via scenario trees or simply a Monte Carlo analysis). This approach could be more limited since it does not allow the consideration of temporal or spatial correlations as other kinds of models like autoregressive (this will be discussed later). However, occasionally, it is enough to consider probability distributions to represent uncertain phenomena without correlations, in the absence of complex statistical structures, or simply as proof of concept. For instance, Andrieu et al. (2010) considered a bivariate normal distribution with independent components to represent two uncertain hydro inflows for a water reservoir management problem, discretizing the decision variables but keeping the probability distribution continuous. Although the distribution considered is not explicit, Fleten et al. (2011) generated a tree of 1000 scenarios for uncertain electricity prices in an integrated short- and long-term hydrothermal scheduling problem. For an agricultural investment problem, El Karfi et al. (2022) proposed a chance-constrained optimization model with (multivariate) Gaussian cereal demand and yield coefficient of soil. Even though distribution fitting is essential, the sampling strategy applied is just as important. Homem-De-Mello et al. (2011) studied applying two alternative sampling strategies (of variance reduction) for generating scenario trees in very large problems, namely, Latin hypercube and randomized quasi-Monte Carlo. Results showed that both strategies were more consistent, producing an operation policy with a much smaller variance than classical Monte Carlo. A similar result was obtained in de Matos et al. (2017), where Latin hypercube and randomized quasi-Monte Carlo sampling strategies provided better policies than Monte Carlo, yielding better confidence intervals on the optimality gap. Cassagnole et al. (2021) studied the impact of short-term (7 days) hydro inflow forecasts with different qualities on reservoir management and revenue based on a conceptual method that introduces different biases to log-normal hydro inflows. The results show that the quality of forecasts can lead to certain economic losses and unnecessary management problems.

A classical alternative to modeling uncertain parameters in optimization under uncertainty models is linear regression due to its linearity and properties of noise processes (typically independent and identically distributed, iid). With the purpose of including emissions quota schemes for medium- and long-term hydrothermal scheduling models, Belsnes et al. (2003) considered quota markets with uncertain prices via linear regression. An extension of linear regression is the autoregressive (AR) model, in which a stochastic process is defined by a linear combination of its past observations plus an iid noise process. In particular, the uncertain components $\{b_t\}_{t\in\mathcal{T}}$ can be modeled with an autoregressive process of order p, or AR(p), given by

$$b_t = \mu_t + \sum_{l=1}^{p} \phi_l b_{t-l} + \xi_t, \tag{1}$$

where μ_t is a deterministic component (trend) in the stage t, ϕ_l corresponds to the autoregressive coefficient of lag l (temporal correlation), and ξ_t is a white noise process centered at the origin with finite and known variance.

AR models have been commonly used to model uncertain hydro inflows since their ability to capture temporal correlations (Yang & Read, 1999; van Ackooij et al., 2014; Soares et al., 2014; Pritchard, 2015; Yildiran, 2019). Despite the application of AR models being straightforward and can capture interesting statistical features, some extensions have been considered. In (Gjerden et al., 2015), the dimensionality reduction technique of principal component analysis was applied to the (hydro inflows) noise process, capturing some spatial correlation and allowing better computational performance in the model's resolution without sacrificing the statistical representativeness of the inflows. Guan et al. (2018) obtained a similar result when using SDDP to develop water-value functions for a multi-reservoir system. In Gauvin et al. (2018), hydro inflows were represented by ARMA (autoregressive moving average) and GARCH (generalized autoregressive conditional heteroscedasticity) models to consider heteroscedasticity (nonconstant variability), providing better forecasts than traditional methods and without sacrificing computational efficiency.

Periodic autoregressive (PAR) models are seasonally varying autoregressive models considering dynamic autoregressive parameters. In particular, PAR models extend the AR models of (1), and they are given by

$$b_t = \sum_{s=1}^{S} \delta_{st}(\mu_{ts} + \sum_{l=1}^{p} \phi_{ls} b_{t-l}) + \xi_t, \tag{2}$$

where δ_{st} is a dummy variable which is equal to 1 if the stage t corresponds to the season s and 0 otherwise, μ_{ts} is a deterministic component for the time t in the season s, ϕ_{ls} is the autoregressive coefficient of lag l in the season s, and ξ_t is again the white noise process.

PAR models have been extensively evaluated for modeling hydro inflows Zambelli et al. (2009a); de Matos & Finardi (2012); Guigues & Sagastizábal (2012); De Matos et al. (2014). A classical challenge with AR models (and their extensions) is that they can yield negative values, which has no practical sense (at least in terms of hydro inflows). To avoid that problem, De Castro et al. (2015) considered the bootstrap technique to generate viable synthetic scenarios from a PAR model, an alternative to model the innovation process as log-normal. Similarly, Raso et al. (2017) proposed an innovative hydro inflow process model to be used in SDDP, including a log-normal multiplicative stochastic component and a nonuniform time step, which makes the process approximately homoscedastic. Some extensions for PAR models have been studied. Treistman et al. (2020) presented an extended memory approach for the PAR model to overcome the drawback of returning to the historical average roughly in some months, even when the current regime is very wet or dry. To account for spatial correlation, Lohmann et al. (2016) proposed a spatial-PAR and Espanmanesh & Tilmant (2022) a multisite PAR. The former takes spatial information, such as the distance between hydro plants, whereas the latter includes a climate variable governed by a hidden Markov model.

The multivariate version of AR models, known as the vector autoregressive (VAR) model, also allows for capturing spatial information. In particular, a vector autoregressive process of order p, or VAR(P), is given by

$$\boldsymbol{b}_t = \boldsymbol{\mu}_t + \sum_{l=1}^{p} \boldsymbol{\Phi}_l \boldsymbol{b}_{t-l} + \boldsymbol{\xi}_t, \tag{3}$$

which represents the d-dimensional version of an AR model. Here, it is important to note that $\boldsymbol{\Phi} \in \mathbb{R}^{d \times p}$ accounts not only for temporal correlation but also for spatial correlation.

Some interesting works have considered VAR processes. In Dashti et al. (2016), the performance of a two-stage robust optimization model is analyzed based on a classical polyhedral uncertainty set compared to a VAR-based uncertainty set for hydro inflows, where the latter performs better due to a more appropriate uncertainty modeling. In Favereau et al. (2022), a multivariate Student's t-mixture was proposed for modeling the error process of a VAR model for hydro inflows, outperforming a classical VAR with Gaussian white noise. Also considering a VAR model, Favereau et al. (2023) presented a multistage adaptive robust optimization model for medium-term hydrothermal scheduling. The uncertainty set for hydro inflows was built based on a VAR model, applying the dimensionality reduction technique of principal component analysis to obtain a tractable robust counterpart. Additionally, it is worth mentioning that some classical uncertainty sets could be ellipsoidal Mejia-Giraldo et al. (2014) and polyhedral Gronvik et al. (2014).

Although all the previously mentioned models are based on statistics, physics-based models are a powerful tool for modeling uncertain hydro inflows. de Queiroz et al. (2016) considered scenarios from a global climate model (GCM), the HadCM3, for a hydrothermal scheduling problem, allowing the identification of future behavior of different climate patterns. Guerra et al. (2019) considered the four representative concentration pathways (RCP) of greenhouse emissions, i.e., RCP2.6, RCP4.5, RCP6.0, and RCP8.5 as climate change scenarios in combination with 18 GCMs to forecast climatic variables for a medium-term hydrothermal scheduling problem. In addition to an RCP8.5 model, Zakeri et al. (2020) included an AR model for producing synthetic hydro inflows, yielding exciting results thanks to the ability to capture seasonal patterns. Lastly, Tucci et al. (2019) considered a rainfall-runoff model instead of classical stochastic models for short-term hydrothermal scheduling, proving that a benefit from ensemble forecasts is the ability to issue intermediate probabilities of occurrence.

Table 8 characterizes the different uncertainty modeling approaches. Note that some works proposed extensions of some classical models (as in the autoregressive or its derivations), but Table 8 characterized them according to their fundamental approach.

Uncertain parameter	Modeling approach	References
Hydro inflows	Scenario-tree	Barquín et al. (2004) Homem-De-Mello et al. (2011) Tong et al. (2013) Raso et al. (2014) Fan et al. (2016) de Matos et al. (2017) Jovanovic et al. (2018) Borges et al. (2022)
	Probability distributions	Andrieu et al. (2010) Cassagnole et al. (2021)
	Autoregressive models	Yang & Read (1999) van Ackooij et al. (2014) Soares et al. (2014) Pritchard (2015) Gjerden et al. (2015) Guan et al. (2018) Yildiran (2019) Gauvin et al. (2018)
	Periodic autoregressive models	Zambelli et al. (2009a) de Matos & Finardi (2012) Guigues & Sagastizábal (2012) De Matos et al. (2014) De Castro et al. (2015) Lohmann et al. (2016) Raso et al. (2017) Treistman et al. (2020) Espanmanesh & Tilmant (2022)
	Vector autoregressive models	Dashti et al. (2016) Favereau et al. (2022) Favereau et al. (2023)
	Classical uncertainty sets	Mejia-Giraldo et al. (2014) Gronvik et al. (2014)
	Physics-based models	de Queiroz et al. (2016) Guerra et al. (2019) Zakeri et al. (2020) Tucci et al. (2019)
Power demand	Scenario-tree	Tong et al. (2013) Jovanovic et al. (2018)
Renewables availability	Scenario-tree	Tong et al. (2013)
Water demand	Scenario-tree	Tong et al. (2013)
Fuel costs	Scenario-tree	Jovanovic et al. (2018)
Electricity prices	Scenario-tree	Fleten et al. (2011) Borges et al. (2022)

Table 8 continued from previous page

Uncertain parameter	Modeling approach	References
Cereal demand and yield coefficient of soil	Probability distributions	El Karfi et al. (2022)
Emission quotas	Linear regression	Belsnes et al. (2003)

Table 8: A summary of uncertainty modeling approaches of reviewed papers

3.3. Water resources management strategies and considerations

This section presents a practical analysis of the selected papers regarding the topography of the considered power and/or water networks, the approximation of nonlinear functions, and the operation strategies and considerations evaluated.

3.3.1. Network representation

As discussed, water resources management is a challenging problem often approached by large-scale optimization under uncertainty models. Typically, there are two representations of such a problem: disaggregated and aggregated networks (de Queiroz, 2016). The first is modeling the resolution of individual elements (reservoirs, hydro, and thermal units, among others) and stochastic hydro inflows. The second uses an energy-based equivalent reservoir representation with stochastic inflows of energy. As the representation clearly impacts the computational efficiency of the solution methods, it is relevant to analyze the different network representations considered in the literature. Many works have been done on electricity or water networks, and we focus here on the most significant ones.

According to IHA (2022), Brazil is the second world leader with respect to total hydropower installed capacity with over 109 GW, with hydro generation representing about 60% of its generation matrix for 2021 (considering that in 2021, Brazil suffered the worst drought in 91 years). Therefore, many research works are expected to be associated with this country and its electrical system. Costa et al. (2008) compare different representations of the electrical network: (i) single bus approach, (ii) multi-area approach, (iii) transportation model, (iv) DC model without losses, (v) DC model with line losses, (vi) AC model OPF. The study cases comprise real 24-hour scheduling for the complete Brazilian electrical system, with over 110 hydro plants, 40 thermal units, 3400 buses, and 4800 branches. The most significant differences were observed when including power transmission losses, concluding that these losses are as important (or even more critical) than representing each branch in the electrical network (instead of only interchange among areas). Diniz et al. (2011) proposed a mid- (2 months) and long-term (56 months) stochastic optimization model for hydrothermal scheduling that seeks to assess the penetration of liquified natural gas thermal plants in the whole Brazilian system, composed of more than 120 hydro plants and 40 thermal plants. To evaluate the impact of nonlinearities (given by an energy equivalent reservoir modeling and a PAR model) for a six months hydrothermal scheduling problem, de Matos & Finardi (2012) considered the whole Brazilian interconnected system (139 hydro and 146 thermal plants). A case study with the application of model predictive control to the Brazilian power system was performed by Zambelli et al. (2013), simulating the monthly operation schedule for 56 months for the whole Brazilian power system, comprising 149 hydro plants and 133 thermal plants. In (Martins et al., 2014), a nonlinear medium-term hydrothermal scheduling problem considered the entire Brazilian power system, including 127 hydro and 132 thermal plants.

Although previous works considered the whole Brazilian power system, others focused on more aggregated or specific subsystems. For instance, for a long-term hydrothermal scheduling problem, Zambelli & Soares (2009), considered a multi-reservoir system composed of 19 cascaded plants with ten reservoirs along a river basin (representing 38% of the Brazilian Southeast subsystem). De Matos et al. (2015) performed a computational study of the tuning strategies for SDDP based on a long-term hydrothermal scheduling problem. It is considered an aggregated representation of the Brazilian power system comprising 158 hydropower plants and 151 thermal power plants summarized in four energy equivalent reservoirs. To test a novel hydro inflow forecast method, Lohmann et al. (2016) focused the analysis on 3 of 15 basins described by the power system operator that does not contain any artificially constructed series: Southeast Atlantic (17 hydro plants), Sao Francisco (5 hydro plants), and Tocantins (7 hydro plants).

In addition to Brazil, other countries have been studied in the literature. For instance, for a medium-term (two years) stochastic hydrothermal scheduling, with nonlinear water head effects, Ramos et al. (2011) considered the Spanish electric power system with 118 thermal units, five main basins with 49 hydro reservoirs, 56 hydro plants, and two pumped storage hydro plants. In the case of Greece, Ourani et al. (2012) considered six river basins and four major blocks of multiple reservoirs in cascade with 13 hydropower units for a stochastic medium-term hydrothermal scheduling problem. In addition, the Chilean case has also been considered. In (Maluenda et al., 2018), existing generators and transmission lines were reduced to 68 aggregated generators and 23 lines connecting 20 buses. The generation plants were divided into 25 hydro, 11 wind, nine solar, 16 gas, and seven coal units. In (Favereau et al., 2023), a large-scale representation of the Chilean power system was considered for a multistage robust optimization hydrothermal scheduling problem, comprising up to 227 buses, 334 lines, 933 generators, ten reservoirs, 353 hydro connections, and 178 hydro nodes.

3.3.2. Nonlinearities and functions approximation

Another interesting and challenging problem in water resources management consists in capturing relevant nonlinearities and approximating nonlinear or nonconvex functions (Lorca et al., 2020). For instance, the relationship between hydropower generation and water discharge is typically nonconvex. Reservoir seepage, thermal generation costs, water-use regulatory constraints, or transmission losses are also generally nonlinear. Thus, some simplifications are commonly considered to preserve the convexity in optimization models to be handled by classic algorithms such as SDDP.

Hydropower production is one of the most approached nonlinear functions for water resources management problems, particularly hydrothermal scheduling. The physics of hydroelectric generation leads it to be typically a nonlinear function of its storage and turbined outflow. This function has been generally approximated with an efficiency constant to preserve linearity, although nonlinear formulations have been studied. Diniz et al. (2006); Tong et al. (2013); Martins et al. (2014); Diniz et al. (2016) approximated hydropower production using a piecewise linear function of discharge, storage, and spillage to consider water head effects accurately. Zambelli et al. (2009a,b); Diaz (2009); Díaz et al. (2011); Ramos et al. (2011); Zambelli et al. (2011); De Souza et al. (2013); Lima et al. (2013) represented hydropower generation by considering water head as a nonlinear function of storage, discharge, and spillage, typically giving rise to nonconvexities due to the bilinear terms (water head multiplied by discharged flow). Although linear approximations guarantee a global optimum (if it exists), they must balance accuracy with solution efficiency. Nonlinear representations are more accurate in modeling, which makes it difficult to obtain a global optimum. However, Vieira et al. (2015) established some mathematical properties that could ensure the global optimality of a solution to nonlinear hydrothermal scheduling by keeping the problem concave. In addition, Paredes & Martins (2018) showed how semidefinite programming relaxation is exact if a rigorous condition about turbine efficiency is observed, particularly if hydroelectric power generation is a concave function. An inaccurate representation of the nonlinear hydroelectric generation function could lead to a poor estimation of the marginal cost of a hydro plant, thus affecting market signals.

Another approximation in the literature corresponds to the value function in stochastic dynamic programming models. Drouin et al. (1996) considered replacing the value function with a piecewise linear approximation (two segments) to obtain near-optimal policies and easily computed bounds on the optimal value function based on a value iteration method. Lamond (2003) extends the previous work by considering (i) a piecewise polynomial function of higher degree and (ii) continuous distributions. A similar approach was proposed by Flach et al. (2010) with a piecewise linear approximation of the expected value function for a hydrothermal scheduling problem solved with SDDP, preserving the concavity of the problem as needed by SDDP.

Different cost functions have also been approximated. An example is the approximation of marginal costs of electricity production for a medium-term hydrothermal scheduling model done by Jacobs (1995), based on estimation methods such as crude Monte Carlo, control variate, method of moments, and indirect approximation. Nonlinear thermal generation costs have also been approached. A piecewise linear approximation was proposed in (Tong et al., 2013) to avoid nonlinearities, and a linearization was proposed in (Ennes & Diniz, 2014) based on the combination of two procedures: an exact piecewise quadratic equivalent function and a dynamic piecewise linear model. Although dynamic piecewise linear models are advantageous as compared to piecewise linear models, the fact that it is not possible to build a model for all nodes in SDDP challenges its applicability. Therefore, Cabral & Diniz (2016) proposed a state-dependent dynamic piecewise linear model for thermal generation costs, where a dynamic piecewise linear model is built for each system state.

The hydrothermal scheduling model is usually solved for different time scales. In particular, system constraints are more detailed in short-term models, where transmission losses are a classic example. Transmission constraints are typically built based on direct current (DC) equations, where DC model losses are naturally nonlinear. For a short-term hydrothermal scheduling problem, Diniz et al. (2006) consider quadratic transmission losses for each line, represented by a piecewise linear function, presenting very small deviations between real and considered losses. Olivares et al. (2015) represented transmission losses quadratically, but approximating it with n piecewise linear segments. Because a poor representation of transmission losses can lead to an insecure system dispatch, Costa et al. (2008) compared different electrical network representations: (i) single bus approach, (ii) multi-area approach, (iii) transportation model, (iv) DC model without losses, (v) DC model with line losses (expressed as a quadratic convex function of the phase angle difference), and (vi) alternating current model optimal power flow. The authors concluded that representing transmission losses is as important as representing each branch in the electrical network. In addition, a DC model with losses accurately represents the active flows (when the system load is low).

Other water-related functions, such as crop water production, water quality, and irrigation constraints, have been approximated or considered nonlinear. In (Guo et al., 2014), it was considered a nonlinear crop water production function that reflects the relationship between crop production and required water, that not only tackle water requirements during the entire crop growth period but also during planting stages when is hard to compensate with more irrigation during other planting stages. In (Davidsen et al., 2015a), nonlinearity arises from the water quality constraints, and it is handled with an effective hybrid method that combines genetic algorithms and linear programming. Because water stored in reservoirs can support several economic activities, such as agriculture and hydroelectricity generation, they must be appropriately coordinated. In that sense, Pereira-Bonvallet et al. (2016) considered nonconvex irrigation constraints in an SDDP-based approach, yielding near-optimal solution through an alternative mixed integer formulation that can capture these nonconvexities.

Table 9 summarizes the literature review according to the different approximation techniques used for nonlinear or nonconvex original functions or parameters.

Original function	Type of approximation	General approach	References
Hydropower generation	Piecewise linear function of discharge, storage, and spillage	Multistage stochastic Optimization	Diniz et al. (2006)
		Multistage stochastic mixed integer linear optimization	Tong et al. (2013)
		Nonlinear optimization	Martins et al. (2014)
		Linear programming	Diniz et al. (2016)
Water head (for hydropower generation)	Nonlinear function of storage, discharge, and spillage	Nonlinear optimization	Zambelli et al. (2009a,b) Diaz (2009) De Souza et al. (2013) Lima et al. (2013)
		Mixed integer nonlinear optimization	Díaz et al. (2011)
		Stochastic nonlinear optimization	Ramos et al. (2011)
		Stochastic optimization	Zambelli et al. (2011)
Revenue (cost-to-go function)	Piecewise linear approximation	Stochastic dynamic programming	Drouin et al. (1996)
	Piecewise linear approximation	Multistage stochastic Optimization	Flach et al. (2010)

Table 9 continued from previous page

Original function	Type of approximation	General approach	References
	Piecewise polynomial approximation	Stochastic dynamic programming	Lamond (2003)
Thermal generation costs	Piecewise linear approximation	Multistage stochastic mixed integer linear optimization	Tong et al. (2013)
	Piecewise quadratic approximation	Multistage stochastic optimization	Ennes & Diniz (2014)
	Dynamic piecewise linear approximation	Multistage stochastic optimization	Ennes & Diniz (2014) Cabral & Diniz (2016)
Transmission losses	Piecewise linear approximation	Dynamic programming	Diniz et al. (2006)
		Multistage stochastic optimization	Olivares et al. (2015)
	Quadratic convex approximation	Nonlinear optimization	Costa et al. (2008)
Crop water production	Nonlinear approximation	Inexact nonlinear optimization	Guo et al. (2014)
Water quality (pollutant concentration)	Nonlinear function of volume of surface water, reservoir release, and non-treated pollution release	Multistage stochastic optimization	Davidsen et al. (2015a)
Irrigation constraints	Nonconvex representation	Multistage stochastic optimization	Pereira-Bonvallet et al. (2016)

Table 9: A summary of functions approximation of reviewed papers

3.3.3. Climate change and operation strategies

Finally, another critical challenge in water resources management is related to climate change's current and expected impacts. To deal with this issue, power, and water systems must be prepared to minimize its consequences, and concepts like operation policies, emission quotas, and flexibility must be reanalyzed.

Reservoirs are commonly operated to reduce the natural variability of hydro inflows and comply with the water requirements of different activities such as hydroelectric generation, agriculture, and tourism, among others; therefore, efficient operation policies are essential (Macian-Sorribes & Pulido-Velazquez, 2020; Giuliani et al., 2021). Current phenomena like deep penetration of renewable energy sources, normative changes, water scarcity, and others imply a reanalysis of actual operation policies. For instance, and in the context of the growing installation of natural gas-fired power plants, Ojeda-Esteybar et al. (2017) presented an optimization approach to the operational planning of electric power and natural gas systems, taking into account different energy storage facilities and showing the benefits of integrated operational planning of both systems. Another important concept is reservoir reliability, comprehended as not running out of water nor suffering flooding, for which Ware (2018) proposed a pair of strategies (safety-first and revenue-maximizing) evaluated with a stochastic dynamic programming model that helped to quantify the costs of maintaining a certain level of reliability. Similarly, Raso et al. (2019) presented a novel SDDP-based approach that combines reservoir and hydraulic models for flood and drought protection, increasing flood protection and reducing the duration and intensity of droughts. Clearly, water resources management is typically limited to water quantity, neglecting water quality analysis. Martinsen et al. (2019) addressed joint water allocation and water quality management, showing that the latter strongly impact shadow prices on water availability.

Other analysis regarding operation policies have been considered. For example, Street et al. (2020) proposed a Decision-Hazard approach for long-term hydrothermal scheduling, unlike the traditional Hazard-

Decision approach where dispatch decisions are determined assuming perfect information about the current inflows. Here, the authors showed that Hazard-Decision inconsistency generates an optimistic assessment of the opportunity cost of water, leading to sub-optimal use of energy resources. Gonzalez et al. (2020) proposed co-optimizing water for irrigation and hydropower. The authors proposed a water allocation scheme that integrates monthly marginal benefits of water for irrigation and hydropower at the basin level, alleviating the conflicts between water uses. Although mathematically optimum policies are generally obtained, in practice, their implementation is sometimes tricky for political reasons. Cid et al. (2023) proposed a collaborative modeling approach to define hedging rules as an alternative to politically viable operation policy, considering stakeholders' consensus within the methodology.

In the context of climate change and adaptation strategies for the energy sector, there is a need for systematic tools that help support decision-making processes. New developments in the energy sector, i.e., transformation and expansion of power systems to low-carbon and more resilient designs, will be required in an effort to mitigate and adapt to the effects of climate change. Scianni et al. (2013) determined a hydrothermal scheduling optimal policy, calculating the power system assured energy–or the amount of energy available to supply the power demand at a deficit risk of 5%. The work considered the effects of climate change on the annual variation of precipitation and its seasonality, and it allowed to determine which new (run-of-river) hydro plants will enter the Brazilian power system. Pereira-Cardenal et al. (2014) coupled a hydrological model with a power market model to study the impacts of climate change on the Iberian power system, founding that changes in precipitation will reduce runoff, decrease hydropower production (consequently, increasing thermal generation) and increase irrigation water use, while higher temperatures will shift power demand from winter to summer months. In the work of Guerra et al. (2019), it was found that climate change will reduce the hydropower capacity factor by 5.5-17.1% between 2015 and 2029, based on an integrated analysis that considered general circulation models, global sensitivity analysis, and stochastic optimization.

Some previous efforts, such as emissions quota modeling to mitigate the effects of climate change, have been developed. Belsnes et al. (2003) proposed methods for medium- and long-term hydrothermal scheduling with two different emissions quota schemes: fixed and uncertain quotas. Also, Avellà Fluvià et al. (2005) analyzed the impact of CO_2 emissions, where the whole thermal units emissions were modeled as a unique annual reservoir, namely, the allowances stock. In a similar approach, Rebennack et al. (2012) presented a modeling approach for considering CO_2 emissions into an SDDP-based hydrothermal scheduling problem, proposing a reservoir model for CO_2 emissions, yielding valuable insights for policy-makers to determine socially acceptable CO_2 emissions quotas. Rebennack (2014) developed an optimal investment tool to set quota levels and quantify emissions savings and cost changes associated with the imposed quota levels. For the Panama case study, the authors showed that wind resources are economically attractive at moderate emission quota levels.

In some electrical power systems, such as the Brazilian, due to the randomness of hydro inflows, the hydroelectric plants receive an assured energy certificate, also known as physical guarantees. These assured energy certificates correspond to the energy each generation unit can produce at a certain percentage, typically 95%. In addition, the certificates also represent the maximum energy each hydro plant can contract. Lorey et al. (2015) proposed an SDDP-based approach to optimize the seasonalization of assured energy considering bilateral contracts previously assigned and the projection of spot prices, obtaining a substantial revenue increase of a generator utility from the Brazilian system. Lorey et al. (2017) extends the previous work by implementing a different heuristic technique, decreasing the computational times compared to the work of Lorey et al. (2015) based on genetic algorithms.

Ancillary services play a fundamental role in helping grid operators maintain a reliable electricity system. Street et al. (2017) proposed a multistage stochastic optimization model for planning hydrothermal coordination, co-optimizing the nominal energy dispatch and individual up and down reserve allocations. The main goal of this paper is to address a general $n - K$ security criterion, such that, for each inflow scenario, the system can withstand the loss of up to K components (transmission or generation assets). Concerning financial storage rights, Martins & Hochstetler (2019) discussed the applicability of financial storage rights for hydroelectric power plants as a mechanism to decouple ownership and operation of reservoirs, being able to be centrally dispatched by the system operator, making itself available to the energy and ancillary services markets to maximize its economic value to the system.

Lastly, it is important to consider that the water allocation from multipurpose reservoirs is typically strongly affected by the consequences of climate change. Thus, direct, detailed modeling of different uses of water becomes necessary. For instance, in González et al. (2016) assessed the economically optimal operation of a multipurpose reservoir in Chile for irrigated agriculture and hydropower, examining a dynamic water allocation scheme in which monthly marginal benefits of water in agriculture and hydropower are considered. In addition, Pereira-Bonvallet et al. (2016) proposed an SDDP-based sub-optimal hydrothermal scheduling approach incorporating nonconvex irrigation constraints for the Chilean power system. The authors showed that this approach determines feasible and near-optimal solutions with costs reasonably close to those optimally determined through alternative mixed-integer linear programming formulation (which can capture nonconvexities).

4. Conclusions and Future Research Directions

The main goal of this paper is to develop a tutorial-type review of water resources management problems over the last decades since the publication of the seminal work of Pereira & Pinto (1991). Two additional components were also included in this work. First, it provides a formal introduction to some of the basics of the modeling approaches considered in the literature. Second, a bibliometric analysis was carried out, in which 174 relevant papers were extracted from Scopus, the well-known journal database. This allowed to identify the distribution of published papers across time, by journal, and the leading topics, authors, and papers. It also allowed identifying where the literature has evolved. In particular, the literature was divided into three main groups: modeling and solution approaches, uncertainty sources and modeling, and water resources management considerations. According to the systematic review, future research directions have been identified, providing potential, exciting research opportunities and ideas for further studies. They are presented as follows.

4.1. Incorporation of operational effects in medium- and long-term models

With the deep penetration of renewable energy sources, hydroelectricity took a fundamental role in the flexibility capability of electric power systems. Therefore, it becomes essential to consider short-term operational effects in medium- and long-term hydrothermal scheduling problems.

In the case of hydroelectric operations planning, this can be related to incorporating operational constraints such as short-term storage, ramp requirements, reserve requirements, and operating costs (like turn-on and shut-down of generators). This typically assumes the incorporation of integer decision variables, which can harden the model solution's efficiency. Otherwise, to avoid incorporating this type of decision variables, some approximations could be studied to at least contemplate some effects of these operational constraints. Regarding more general water resources management problems, some operational or short-term effects could also be considered. For instance, reservoir seepage, evapotranspiration, maintenance, or irrigation constraints induce some impact that, when incorporated into the modeling, could eventually lead to more efficient operating policies.

4.2. Incorporation of uncertainty sources additional to hydro inflows

As presented in Section 3.2, there are several uncertain parameters considered in the reviewed literature. For example, power demand, renewables availability, water demand, fuel costs, electricity prices, emission quotas, and others. However, in general, the modeling approach considered for all of these uncertain parameters has been simple, without capturing essential statistical features. On the other hand, uncertain hydro inflows have been approached with sophisticated models to consider statistical characteristics such as trend, seasonality, and spatial and temporal correlations, among others. Thus, an exciting research area, still open in the literature, corresponds to incorporating other uncertainty sources in addition to hydro inflows but applying modeling tools as sophisticated as those used for the latter. As expected, this creates additional challenges concerning the balance between the distributional or statistical representation of these uncertainty sources and the efficiency of solving the corresponding optimization problems.

Here, an essential research direction is related to the impacts of climate change. As discussed in Section 3.3.3, some interesting approaches have been considered, providing important results and policy recommendations. Nevertheless, climate change not only impacts the availability and uncertainty of water but also exacerbates some risks, such as more elevated temperatures, loss of animal species, food scarcity, or health issues. Each affects water resources management and related activities to a greater or lesser extent. Therefore, future research should be aware of these risks and consider them in the corresponding analyses.

4.3. Design of efficient solution techniques to address more complex networks

As discussed in Section 4.1, modeling of operational effects usually leads to large-scale instances with detailed power and water networks. This implies an increase in the computational effort to solve the corresponding models.

As commented in Section 3.3.1, although some works considered large-scale power systems (such as the Brazilian one), the majority focused on aggregated or specific subsystems mainly for computational purposes. However, the aggregated value of considering more complex networks (and greater uncertainty spaces) is limited by the ability of algorithms such as SDDP to solve these instances. Therefore, this creates an exciting challenge in designing efficient solution techniques to solve large-scale instances, allowing a better assessment of flexibility requirements.

4.4. Approximation of nonlinearities

Another important characteristic related to flexibility issues concerns the incorporation (and approximation) of nonlinearities. Some examples are those reviewed in Section 3.3.2, such as thermal generation costs, transmission losses, crop water production, water quality, and irrigation, among others. However, the most approached nonlinearity corresponds to hydropower generation (and water head). This has been typically considered in nonlinear optimization models, showing the benefits of obtaining solutions that can be adequately implemented in practice. Some piecewise linear approximations have also been considered, delivering more effective results than a linear (constant) approximation.

The approximation of nonlinearities also has important consequences in computational efficiency due to the possible incorporation of integer variables that harden model resolution. In the case of stochastic optimization, some algorithms, such as SDDiP (Zou et al., 2019), allow solving multistage stochastic integer problems. However, its scalability is still not enough to solve for large-scale instances. Thus, again an interesting research direction is related to designing efficient solution techniques (for integer problems in this case) to be able to represent nonlinear functions such as irrigation constraints, hydropower generation, and generation costs, among others.

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Appendices for *Water resources management: A bibliometric analysis and future research directions*

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A Optimization Under Uncertainty Foundations

Stochastic dynamic programming. The *stochastic dynamic programming* (SDP) technique, also known as Markovian decision processes (MDP), was proposed by Bellman (1957) and extensively developed and standardized by Puterman (1994). SDP has also had important contributions, such as those of Ross (1983) and Bertsekas (2017). An MDP problem is essentially composed of the following elements:

1. Decision stages: a finite set of stages $t \in \mathcal{T} := \{1, \dots, T\}$.

2. State space: for each stage, a set of possible states $\mathcal{S}_t$, generally assumed as discrete.

3. Decision space: a compact (closed and bounded) space of possible actions (decisions) that can be taken and that depend on the state of the system at each stage. That is, $\mathcal{X}_t(\boldsymbol{s}_t)$ with $\boldsymbol{s}_t \in \mathcal{S}_t$ for $t \in \mathcal{T}$.

4. Stochastic transition function: as a function of the current state, $\boldsymbol{s}_t$, the decision taken, $\boldsymbol{x}_t$, and the random exogenous information, $\boldsymbol{\xi}_t \sim F_t$, the next state will be given by

$$\boldsymbol{s}_{t+1} = f_t(\boldsymbol{s}_t, \boldsymbol{x}_t, \boldsymbol{\xi}_t). \tag{1}$$

   This is equivalent to defining a transition probability that maps from the current state and decision space to the future state space, $p_t : \mathcal{S}_t \times \mathcal{X}_t \to \mathcal{S}_{t+1}$. That is,

$$p_t(\boldsymbol{s}_{t+1}|\boldsymbol{x}_t, \boldsymbol{s}_t) = \mathbb{P}(\boldsymbol{S}_{t+1} = \boldsymbol{s}_{t+1}|\boldsymbol{S}_t = \boldsymbol{s}_t, \boldsymbol{x}_t). \tag{2}$$

5. Immediate value: there exists an immediate benefit (value) in each stage t, $r_t : \mathcal{S}_t \times \mathcal{X}_t \to \mathbb{R}$. Thus, the objective will be

$$\max_{\boldsymbol{x} \in \mathcal{X}} \mathbb{E}\Bigg[\sum_{t \in \mathcal{T}} r_t(\boldsymbol{S}_t, \boldsymbol{x}_t|\boldsymbol{s}_1)\Bigg], \tag{3}$$

   where the state $\boldsymbol{s}_1$ is assumed as known and $|r_t(\boldsymbol{s}_t, \boldsymbol{x}_t)| < \infty$. As the uncertain parameters of the problem perturbate the future states, then (3) is given by

$$u_t^*(\boldsymbol{s}_t) = \sup_{\boldsymbol{x}_t \in \mathcal{X}_t(\boldsymbol{s}_t)} \Bigg\{ r_t(\boldsymbol{s}_t, \boldsymbol{x}_t) + \sum_{\boldsymbol{\xi} \in \mathcal{S}_{t+1}} p_t(\boldsymbol{\xi}|\boldsymbol{s}_t, \boldsymbol{x}_t) u_{t+1}^*(\boldsymbol{\xi}) \Bigg\}, \tag{4}$$

known as Bellman's equation. Then, if $\exists \boldsymbol{x}_t^* \in \mathcal{X}_t(\boldsymbol{s}_t)$ such that

$$u_t^*(\boldsymbol{s}_t) = \sup_{\boldsymbol{x}_t \in \mathcal{X}_t(\boldsymbol{s}_t)} \big\{ r_t(\boldsymbol{s}_t, \boldsymbol{x}_t) + \mathbb{E}_{\boldsymbol{S}_{t+1}}[u_{t+1}^*(\boldsymbol{S}_{t+1})|\boldsymbol{s}_t, \boldsymbol{x}_t] \big\}, \tag{5}$$

then, by Bellman's principle of recursion (Bellman, 1957), the decision policy $\boldsymbol{x}^* = (\boldsymbol{x}_1^*, \boldsymbol{x}_2^*, \ldots, \boldsymbol{x}_T^*)$ will be optimal.

SDP has been a widely used technique within the reservoir operation area because the structure of this problem allows it to be intuitively modeled as an MDP. The first work to use SDP to find the optimal operation rule of a water resources system with stochastic hydro inflows was Little (1955), where a simplified example based on the Columbia River were analyzed. Product of the simplicity of the previous example, in Butcher (1971), the use of SDP is extended by applying it to a more realistic case (Watasheamu dam, United States), considering returns from hydroelectricity, irrigation, and recreation.

During the following decades, several research works were carried out with the objective of extending the use of SDP for different water resource management applications such as reservoir operation, irrigation control, infrastructure development, and water quality, among others. A review of the evolution of these models can be found in Yakowitz (1982). On the other hand, Yeh (1985) presents a review of the state of the art of linear programming, dynamic programming, non-linear programming, and simulation models used in reservoir operation problems.

Until then (the mid-1980s), most research considered stationary policies using hydro inflows from strictly preceding periods, mainly based on Markov chains (or order 1 autoregressive models). This made it easier to solve the problems, but it also had certain disadvantages since considering greater hydro inflow lags allowed more precise forecasts (Bras et al., 1983; Alarcon and Marks, 1979). However, this caused the dynamic programming models to be non-stationary, and their resolution using SDP got complicated. Due to this, in (Stedinger et al., 1984), an alternative formulation of SDP was proposed based on the work of Curry and Bras (1980), which not only considers the immediately preceding hydro inflows for the forecasts of each period but also the best prediction of them, integrating control theory techniques and obtaining better reservoir operation policies.

In its classical formulation, SDP assumes that stochastic processes are stationary. Due to the effects of climate change, this assumption may not adequately represent the hydro inflows of water networks, so some methodological derivations of SDP were proposed. One of them is *sampling stochastic dynamic programming* (SSDP), initially proposed by Kelman et al. (1990). SSDP, unlike SDP, does not rely on the use of probability distributions, so it can incorporate non-stationary processes. In SSDP, the uncertainty vectors are represented by fixed scenarios, which makes it possible to ignore the stationarity assumption of SDP. As shown in equations (2) and (4), Bellman's equations assume a discrete intertemporal representation of the uncertain parameters, while in SSDP, the recursion equation corresponds to

$$u_t(\boldsymbol{s}_t, \boldsymbol{\delta}_t) = \sup_{\boldsymbol{x}_t \in \mathcal{X}_t(\boldsymbol{s}_t)} \big\{ r_t(\boldsymbol{s}_t, \boldsymbol{x}_t) + \mathbb{E}_{\boldsymbol{\theta}_{t+1}|\boldsymbol{\delta}_t}[u_{t+1}(\boldsymbol{s}_{t+1}, \boldsymbol{\theta}_{t+1})|\boldsymbol{s}_t, \boldsymbol{x}_t] \big\} \tag{6}$$

$$= \sup_{\boldsymbol{x}_t \in \mathcal{X}_t(\boldsymbol{s}_t)} \left\{ r_t(\boldsymbol{s}_t, \boldsymbol{x}_t) + \sum_{\boldsymbol{\theta}_{t+1} \in \Theta_{t+1}} p_t(\boldsymbol{\theta}_{t+1} | \boldsymbol{\delta}_t) u_{t+1}(\boldsymbol{s}_{t+1}, \boldsymbol{\theta}_{t+1}) \right\}, \tag{7}$$

where $p_t(\boldsymbol{\theta}_{t+1}|\boldsymbol{\delta}_t)$ represents the probability of going from the scenario $\boldsymbol{\delta}_t$ in stage t to the scenario $\boldsymbol{\theta}_{t+1}$ in stage $t+1$. Based on this definition, in Vicuña et al. (2010), an SSDP-based optimization algorithm was proposed, and it was capable of incorporating the nonstationarity of uncertain hydro inflow forecasts due to the effects of climate change.

Despite the benefits and potential of dynamic programming in general, in most real applications solving Bellman's equations becomes very challenging. The main reason for this is the so-called "curse of dimensionality". This concept refers to the following:

1. The size of the state space, $\mathcal{S}$, can be very large. Suppose a resource management problem with several (over 10, for instance) reservoirs. The possible number of states (level of each reservoir) is so large that it can be very difficult to find an optimal operating rule in a relatively short time.

2. Solving the model over the decision space $\mathcal{X}(s)$ can be very complex.

3. The number of possible future realizations of the uncertainty can be very large. In this way, computing the value function can be computationally intractable.

Due to the above, *approximate dynamic programming* (ADP) plays a fundamental role in solving dynamic problems, where Powell (2011) presents interesting and widely used techniques today. ADP consists of different heuristic methods to solve dynamic stochastic models by finding approximate policies sufficiently "close" to optimal ones. Among the most used are myopic decisions, lookahead, roll-out, and rolling-horizon. In general, these techniques seek to exploit the properties and benefits of future state simulation. In Guigues and Sagastizábal (2012), for the hydrothermal scheduling problem, the benefits delivered by rolling horizon policies are shown in comparison to those obtained with stochastic and robust optimization models. In particular, the challenge of the latter lies in the definition of uncertainty, which can often be very conservative (as will be discussed later).

Multistage stochastic programming. An alternative to SDP is multistage stochastic programming (MSP). Based on Section 3.1 of Shapiro et al. (2009), consider the nested (or dynamic programming) formulation of an MSP of the form

$$\min_{\boldsymbol{x}_1 \in \mathcal{X}_1} f_1(\boldsymbol{x}_1) + \mathbb{E}\left[\inf_{\boldsymbol{x}_2 \in \mathcal{X}_2(\boldsymbol{x}_1, \boldsymbol{\xi}_2)} f_2(\boldsymbol{x}_2, \boldsymbol{\xi}_2) + \mathbb{E}\left[\cdots + \mathbb{E}\left[\inf_{\boldsymbol{x}_T \in \mathcal{X}_T(\boldsymbol{x}_{T-1}, \boldsymbol{\xi}_T)} f_T(\boldsymbol{x}_T, \boldsymbol{\xi}_T) \right] \right] \right], \tag{8}$$

driven by the random data process $\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \ldots, \boldsymbol{\xi}_T$, where $f_t : \mathbb{R}^{n_t} \times \mathbb{R}^{d_t} \to \mathbb{R}$ and $\mathcal{X}_t \subset \mathbb{R}^{n_t}$. A possible way to rewrite the MSP problem of (8) is with the corresponding dynamic programming equations (value functions)

$$Q_t(\boldsymbol{x}_{t-1}, \boldsymbol{\xi}_{[t]}) = \inf_{\boldsymbol{x}_t \in \mathcal{X}_t(\boldsymbol{x}_{t-1}, \boldsymbol{\xi}_t)} \left\{ f_t(\boldsymbol{x}_t, \boldsymbol{\xi}_t) + \mathcal{Q}_{t+1}(\boldsymbol{x}_t, \boldsymbol{\xi}_{[t]}) \right\}, \tag{9}$$

where

$$\mathcal{Q}_{t+1}(\boldsymbol{x}_t, \boldsymbol{\xi}_{[t]}) := \mathbb{E}\left[Q_{t+1}(\boldsymbol{x}_t, \boldsymbol{\xi}_{[t+1]}) | \boldsymbol{\xi}_{[t]} \right], \tag{10}$$

for $t = 2, \ldots, T$, and $\mathcal{Q}_{T+1} \equiv 0$. $\mathcal{Q}_t(\cdot)$ is called expected value function or (expected) cost-to-go function. In addition, in the first stage, the problem to solve corresponds to

$$\min_{\boldsymbol{x}_1 \in \mathcal{X}_1} f_1(\boldsymbol{x}_1) + \mathbb{E}\big[Q_2(\boldsymbol{x}_1, \boldsymbol{\xi}_2)\big]. \tag{11}$$

It is important to note that two-stage stochastic programming corresponds to the case where $T = 2$.

For large-scale optimization problems, solving an MSP optimally can be very challenging, even more so if the number of possible future realizations is very large. Therefore, it becomes necessary to use methods that reduce said effort. One commonly used is *stochastic dual dynamic programming* (SDDP), proposed by Pereira and Pinto (1991). Assuming stage-wise independency of the random process $\boldsymbol{\xi}_t$ and that $f_t(\cdot, \boldsymbol{\xi}_t)$ is convex for $\boldsymbol{\xi}_t$ c.p.1, SDDP in its general form, seeks to approximate the functions $Q_t(\cdot)$ and $\mathcal{Q}_t(\cdot)$ of (9) and (10) through the generation of scenario trees and the iterative use of the Benders decomposition (Benders, 1962). That is, it is constructed a sample average approximation (SAA) of the "true" problem (8) by replacing the true distribution of $\{\boldsymbol{\xi}_t\}_{t=2}^{T}$ (recall that $\boldsymbol{\xi}_1$ is deterministic), P_t, by the empirical distribution P_{Ω_t} based on a random sample $\{\tilde{\boldsymbol{\xi}}_{\omega t}\}_{\omega=1}^{\Omega_t}$ from P_t of size Ω_t. The probability distribution of the SAA problem can be represented by a tree where at each stage $t-1$ every node has the same branches corresponding to $\{\tilde{\boldsymbol{\xi}}_{\omega t}\}_{\omega=1}^{\Omega_t}$, preserving the stagewise independence of the true problem. The value function equations for the SAA problem can be expressed as

$$\tilde{Q}_{\omega t}(\boldsymbol{x}_{t-1}) = \inf_{\boldsymbol{x}_t \in \mathcal{X}_t(\boldsymbol{x}_{t-1}, \tilde{\boldsymbol{\xi}}_{\omega t})} \big\{ f_t(\boldsymbol{x}_t, \tilde{\boldsymbol{\xi}}_{\omega t}) + \tilde{\mathcal{Q}}_{t+1}(\boldsymbol{x}_t) \big\}, \quad \forall \omega = 1, \ldots, \Omega_t$$

with

$$\tilde{\mathcal{Q}}_{t+1}(\boldsymbol{x}_t) = \frac{1}{\Omega_t} \sum_{\omega=1}^{\Omega_{t+1}} \tilde{Q}_{\omega,t+1}(\boldsymbol{x}_t)$$

for $t = 2, \ldots, T$ and $\tilde{\mathcal{Q}}_{T+1}(\cdot) \equiv 0$. Since $\mathcal{Q}_t(\cdot)$ and $\tilde{\mathcal{Q}}_t(\cdot)$ are convex functions and $\Omega_t < \infty$ for each t, then the functions $\tilde{\mathcal{Q}}_t(\cdot)$ are piecewise linear. A backward step applied to the SAA problem is described as follows. Let $\tilde{\boldsymbol{x}}_t \in \mathbb{R}^{n_t}$ at stage $t = 1, \ldots, T-1$ be a trial decision and $\mathfrak{Q}_t(\cdot)$ an approximation of $\tilde{\mathcal{Q}}_t(\cdot)$ at stage $t = 2, \ldots, T$. At the final stage $t = T$, for $\tilde{\boldsymbol{\xi}}_{\omega t} = (\tilde{\boldsymbol{c}}_{\omega t}, \tilde{\boldsymbol{A}}_{\omega t}, \tilde{\boldsymbol{B}}_{\omega t}, \tilde{\boldsymbol{b}}_{\omega t})$ for $\omega = 1, \ldots, \Omega_t$, it is solved the problem

$$\min_{\boldsymbol{x}_T \in \mathbb{R}^{n_T}} \quad \tilde{\boldsymbol{c}}_{\omega T}^{\top} \boldsymbol{x}_T \tag{12a}$$

$$\text{s.t.} \quad \tilde{\boldsymbol{B}}_{\omega T} \boldsymbol{x}_{T-1} + \tilde{\boldsymbol{A}}_{\omega T} \boldsymbol{x}_T = \tilde{\boldsymbol{b}}_{\omega T} \tag{12b}$$

$$\boldsymbol{x}_T \geq \boldsymbol{0} \tag{12c}$$

for $\boldsymbol{x}_{T-1} = \tilde{\boldsymbol{x}}_{T-1}$ and $\omega = 1, \ldots, \Omega_t$. It is important to note that $\tilde{Q}_{\omega T}(\boldsymbol{x}_{T-1})$ corresponds to the optimal value of problem (12). Let $\tilde{\boldsymbol{x}}_{\omega T}$ be an optimal solution of problem (12) and $\tilde{\boldsymbol{\pi}}_{\omega T}$ the optimal dual solution for $\boldsymbol{x}_{T-1} = \tilde{\boldsymbol{x}}_{T-1}$. Then, $\ell_T(\boldsymbol{x}_{T-1}) := \tilde{\mathcal{Q}}_T(\tilde{\boldsymbol{x}}_{T-1}) + \tilde{g}_T^{\top}(\boldsymbol{x}_{T-1} - \tilde{\boldsymbol{x}}_{T-1})$, with

$$\tilde{\mathcal{Q}}_T(\tilde{\boldsymbol{x}}_{T-1}) = \frac{1}{\Omega_T} \sum_{\omega=1}^{\Omega_T} \tilde{\boldsymbol{c}}_{\omega T}^{\top} \tilde{\boldsymbol{x}}_{\omega T}$$

and

$$\tilde{g}_T = -\frac{1}{\Omega_T}\sum_{\omega=1}^{\Omega_T} \tilde{\boldsymbol{B}}_{\omega T}^\top \tilde{\boldsymbol{\pi}}_{\omega T},$$

is a supporting plane for $\tilde{\mathcal{Q}}_T(\cdot)$ at $\tilde{\boldsymbol{x}}_{T-1}$. Then, $\mathfrak{Q}_T(\cdot)$ is replaced by $\max\{\mathfrak{Q}_T(\cdot), \ell_T(\cdot)\}$. With this updated approximation $\mathfrak{Q}_T(\cdot)$, for $t = T-1$ we solve the problem

$$\min_{\boldsymbol{x}_{T-1}\in\mathbb{R}^{n_{T-1}}} \quad \tilde{\boldsymbol{c}}_{\omega,T-1}^\top \boldsymbol{x}_{T-1} + \mathfrak{Q}_T(\boldsymbol{x}_{T-1}) \tag{13a}$$

$$\text{s.t.} \quad \tilde{\boldsymbol{B}}_{\omega,T-1}\boldsymbol{x}_{T-2} + \tilde{\boldsymbol{A}}_{\omega,T-1}\boldsymbol{x}_{T-1} = \tilde{\boldsymbol{b}}_{\omega,T-1} \tag{13b}$$

$$\boldsymbol{x}_{T-1} \geq \boldsymbol{0} \tag{13c}$$

which should be solved for $\boldsymbol{x}_{T-2} = \tilde{\boldsymbol{x}}_{T-2}$ and $\omega = 1,\ldots,\Omega_{T-1}$. Let $\tilde{Q}^\star_{\omega,T-1}(\boldsymbol{x}_{T-2})$ be the optimal value of the problem (13), and consider

$$\tilde{Q}^\star_{T-1}(\boldsymbol{x}_{T-2}) := \frac{1}{\Omega_{T-1}}\sum_{\omega=1}^{\Omega_{T-1}} \tilde{Q}^\star_{\omega,T-1}(\boldsymbol{x}_{T-2}).$$

Note that $\tilde{Q}^\star_{\omega,T-1}(\cdot) \leq \tilde{Q}_{\omega,T-1}(\cdot)$ for each $\omega = 1,\ldots,\Omega_{T-1}$ and, thus $\tilde{\mathfrak{Q}}^\star_{\omega,T-1}(\cdot) \leq \tilde{\mathfrak{Q}}_{\omega,T-1}(\cdot)$. Solving the problem (13) and the dual for each $\omega = 1,\ldots,\Omega_{T-1}$, $\tilde{Q}^\star_{T-1}(\tilde{\boldsymbol{x}}_{T-2})$ and a subgradient $\tilde{g}_{T-1}$ of $\tilde{Q}^\star_{T-1}(\boldsymbol{x}_{T-2})$ can be computed at $\boldsymbol{x}_{T-2} = \tilde{\boldsymbol{x}}_{T-2}$, constructing the supporting plane $\ell_{T-1}(\boldsymbol{x}_{T-2}) := \tilde{\mathcal{Q}}^\star_{T-1}(\tilde{\boldsymbol{x}}_{T-2}) + \tilde{g}_{T-1}^\top(\boldsymbol{x}_{T-2} - \tilde{\boldsymbol{x}}_{T-2})$ for $\tilde{Q}^\star_{T-1}(\boldsymbol{x}_{T-2})$ at $\boldsymbol{x}_{T-2} = \tilde{\boldsymbol{x}}_{T-2}$. As before, the approximation $\mathfrak{Q}_{T-1}(\cdot)$ is replaced with $\max\{\mathfrak{Q}_{T-1}(\cdot), \ell_{T-1}(\cdot)\}$, and this process continues until the first stage, where the following problem is solved

$$\min_{\boldsymbol{x}_1\in\mathbb{R}^{n_1}} \quad \boldsymbol{c}_1^\top \boldsymbol{x}_1 + \mathfrak{Q}_2(\boldsymbol{x}_1) \tag{14a}$$

$$\text{s.t.} \quad \boldsymbol{A}_1\boldsymbol{x}_1 = \boldsymbol{b}_1 \tag{14b}$$

$$\boldsymbol{x}_1 \geq \boldsymbol{0}. \tag{14c}$$

SDDP was mainly proposed to be applied in solving the problem of reservoir management in hydrothermal scheduling in Brazil at the beginning of the 1990s and is still used today (De Queiroz, 2016; Lorca et al., 2020). Risk-neutral MSPs, such as the classical formulation of the hydrothermal scheduling problem, do not allow capturing the variability of decisions, so some works that include risk-averse functionals as an objective have been proposed (Shapiro et al., 2013; de Mello and Pagnoncelli, 2016). Generally, this risk aversion is captured through the use of functionals such as *conditional value-at-risk* (CVaR), and they become even more relevant if the current and future scenarios of hydrological uncertainty are considered.

Although SDDP continues to be one of the most widely used techniques to solve reservoir operation problems, it becomes difficult to manage non-convexities such as hydropower generation, reservoir seepage, or certain regulatory constraints on the use of water (Lorca et al., 2020). An alternative technique to address these difficulties is *stochastic dual dynamic integer programming* (SDDiP), proposed by Zou et al. (2019), which seeks to solve multistage stochastic integer problems by approximating the state variables by means of binary variables. SDDiP extends the effort of SDDP by adding a reformulation of the subproblems of each stage and an approximation of the value function based on Lagrangian cuts. In particular, in Hjelmeland et al. (2019), this technique was applied to solve the hydrothermal scheduling problem considering non-convex functions in electricity generation and flow discharges. Through SDDiP, the authors showed the importance of considering these non-convexities in the operation of reservoirs since a linear approximation of these functions translates into an overestimation of the reserve capacity.

Robust optimization. Another interesting approach to addressing water management problems is robust optimization (RO). The essence of RO is that a probability distribution for the random parameter $\boldsymbol{\xi}_t$ is not known or considered, but only that $\boldsymbol{\xi}$ belongs to a support $\mathcal{U}$. In RO, $\mathcal{U}$ is known as the uncertainty set. Then, for a general problem of the form

$$f(\boldsymbol{\xi}) := \min_{\boldsymbol{x}\in\mathcal{X}} \left\{F(\boldsymbol{x},\boldsymbol{\xi}) : \boldsymbol{x}\in\mathcal{X}(\boldsymbol{\xi})\right\}, \tag{15}$$

$\boldsymbol{x}\in\mathcal{X}$ is sought such that $\boldsymbol{x}\in\mathcal{X}(\boldsymbol{\xi})$ for each $\boldsymbol{\xi}\in\mathcal{U}$ and minimizes the worst-case total cost. That is,

$$\min_{\boldsymbol{x}\in\mathcal{X}} \max_{\boldsymbol{\xi}\in\mathcal{U}} \left\{F(\boldsymbol{x},\boldsymbol{\xi}) : \boldsymbol{x}\in\mathcal{X}(\boldsymbol{\xi})\right\} \tag{16}$$

One of the main challenges of RO is to adequately represent random processes by the uncertainty set $\mathcal{U}$. For this, there are several classic definitions, such as the pioneering work of Soyster (1973) or those of Ben-Tal and Nemirovski (1999) and Bertsimas and Sim (2004). Some of the different classes of uncertainty sets are the following:

- Box: $\mathcal{U} = \{\boldsymbol{\xi}\in\mathbb{R}^d : ||\boldsymbol{\xi}||_\infty \leq \rho\}$, with $||\boldsymbol{\xi}||_\infty := \max_{i\in[d]}|\xi_i|$ and $\rho\in\mathbb{R}$.
- Ellipsoidal: $\mathcal{U} = \{\boldsymbol{\xi}\in\mathbb{R}^d : ||\boldsymbol{\xi}||_2 \leq \rho\}$, with $||\boldsymbol{\xi}||_2 := \sqrt{\xi_1^2 + \ldots \xi_d^2}$ and $\rho\in\mathbb{R}$.
- Polyhedral: $\mathcal{U} = \{\boldsymbol{\xi}\in\mathbb{R}^d : \boldsymbol{H}\boldsymbol{\xi} + \boldsymbol{d} \geq \boldsymbol{0}\}$, with $\boldsymbol{H}\in\mathbb{R}^{m\times d}$ and $\boldsymbol{d}\in\mathbb{R}^m$.

For a detailed survey on robust optimization, we refer the reader to the works of Ben-Tal et al. (2015); Yanıkoğlu et al. (2019). In summary, under RO, there is a willingness to accept suboptimal solutions as long as the solution continues to be feasible and "close" to optimal, even when the random parameters change. An interesting application of RO for the hydrothermal planning problem is the work of Dashti et al. (2016), where the uncertainty set associated with the hydro inflows was built from a vector autoregressive process. In this way, the authors demonstrated a clear advantage over the classic *budget uncertainty set* of Bertsimas and Sim (2004) because the hydro inflows forecast had a lower variance and was not as conservative.

Finally, another technique that has gained attention in recent years is distributionally robust optimization (DRO) (Ben-Tal et al., 2013). From stochastic optimization, it has been considered that the uncertain parameters follow a certain known distribution, while in robust optimization, only their support has been assumed to be known. DRO is positioned right in between both approaches: it provides probabilistic models as in the former, together with the incorporation of uncertainty sets as in the latter. However, here we do not assume a single probability distribution for random processes but rather a set of possible distributions constructed from historical observations, known as the ambiguity set. In this way, being $\mathcal{M}$ a set of possible probability distributions that explain the behavior of $\boldsymbol{\xi}$, a DRO problem has the following form:

$$\min_{\boldsymbol{x}\in\mathcal{X}} \max_{\mathbb{Q}\in\mathcal{M}} \{\mathbb{E}_{\mathbb{Q}}[F(\boldsymbol{x},\boldsymbol{\xi})]\}. \tag{17}$$

In this way, DRO seeks to optimize the expected value of the worst-case distribution of the ambiguity set. The work of Huang et al. (2017) shows the effectiveness of applying this technique to a risk-averse hydroelectric resource management problem solved through SDDP, compared to the classic SO modeling.

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