

On the Regulatory and Economic Incentives for Renewable Hybrid Power Plants in Brazil

Pedro Prescott, Alexandre Street, *Senior Member, IEEE*

Abstract—The complementarity between renewable generation profiles has been widely explored in the literature. Notwithstanding, the regulatory and economic frameworks for hybrid power plants add interesting challenges and opportunities for investors, regulators, and planners. Focusing on the Brazilian power market, this paper proposes a unified and isonomic firm energy certificate (FEC) calculation for non-controllable renewable generators, which allows us to 1) generalize the FEC concept for hybrid units and 2) capture the regulatory and economic synergies between sources. Based on the non-discriminatory FEC proposed for hybrid power plants, the co-optimization of both forward-market and network-access contracting strategies is studied, and its economic incentives are demonstrated. The optimal share of renewable sources composing the hybrid power plant is also considered in the model and analyzed in our case studies. Based on real data from the Brazilian power market, we quantify the benefits of the proposed market structures and model for a typical wind–solar hybrid unit.

Index Terms—Hybrid power plants, renewable generation, firm energy certificate, network-access contract, risk management, stochastic programming.

I. NOMENCLATURE

Sets and Indices

\mathcal{M}	Set of months of the optimization horizon. We use m to denote its elements.
$\mathcal{H}, \mathcal{H}_m$	Set of hours in the historical data (past) and subset of hours of a given month m of the year, respectively.
$\mathcal{T}, \mathcal{T}_m$	Set of hours defining the optimization horizon and subset of hours of a given month m of the year. We use $t \in \mathcal{T}$ to denote the hours in the study horizon (future).
Ω	Set of scenarios (sample space). We use $\omega \in \Omega$ to denote a given scenario.

Random variables

$\tilde{\pi}_t$	Spot price of a given hour $t \in \mathcal{T}$ (\$/MWh). We assume $\pi_{t,\omega}$ as its realization for a given scenario $\omega \in \Omega$.
\tilde{g}_t^S	Solar available generation of a given hour t (MWh). We assume $g_{t,\omega}^S$ as its realization for a given scenario $\omega \in \Omega$.
\tilde{g}_t^W	Wind available power generation of a given hour $t \in \mathcal{T}$ (MWh). We assume $g_{t,\omega}^W$ as its realization for a given scenario $\omega \in \Omega$.

Constants and parameters

c	Network-access contract tariff (\$/MW).
C^S	Investment expenditure CAPEX on solar installed capacity (\$/MW).

C^W	Investment expenditure CAPEX on wind installed capacity (\$/MW).
p	Forward contract price (\$/MWh).
η	Penalty for network-access contract violations.
ϵ	Upper tolerance for network-access contract violation.
α	CVaR confidence level (risk-aversion parameter).
λ	CVaR weight (risk-aversion parameter).
G_t^S	Historical generation data of the solar generator for hour $t \in \mathcal{H}$ (MWh).
G_t^W	Historical generation data of the wind generator for hour $t \in \mathcal{H}$ (MWh).
\mathbf{G}	Generic vector stacking the whole historical generation data (MWh). In the text, we consider variants of it for solar generation data (\mathbf{G}^S), wind generation data (\mathbf{G}^W), and the generation data of the hybrid unit (\mathbf{G}^H).

Decision Variables

M	Network-access contract quantity amount (MW).
Q	Forward contract quantity amount (MWh).
x	Percentage of solar generation on installed capacity composing the 1–MW hybrid power plant.
$\hat{g}_{t,\omega}^H, \hat{G}_t^H$	Auxiliary decision variables used to implement the truncated generation (after curtailment) for both simulated and historical data, respectively.

Functions

$\mathbb{F}(\mathbf{G}, M)$	Firm energy certificate (FEC) of a non-controllable renewable generator as a function of \mathbf{G} and M .
$G_t^H(x)$	Historical available generation data of the hybrid power plant composed of x –MW of solar and $(1-x)$ –MW of wind for an hour $t \in \mathcal{H}$.
$\hat{g}_t^H(x, M)$	Generation output truncated on M for the hybrid power plant composed of x –MW of solar and $(1-x)$ –MW of wind for an hour $t \in \mathcal{T}$, defined as $\hat{g}_t^H = \min\{x\tilde{g}_t^S + (1-x)\tilde{g}_t^W, M\}$. We assume $\hat{g}_{t,\omega}^H$ as its realization for a given scenario $\omega \in \Omega$.

II. INTRODUCTION

Brazil has one of the most renewable generation fleets [1] in the world. However, it is a latecomer in contrast to the US and

EU in the installation of wind, solar, and hybrid generation. Interestingly, hybrid generation units have shown to be an effective way to reduce the intermittence of non-controllable renewable generation (see [2]) and, consequently, have the potential to increase renewables' firm energy production. The complementarity between solar and wind generation in Texas (US) has been studied in [3], wherein relevant gains in the firm supply capacity were found from paring both sources. In the energy transition context, renewable sources, not only in Brazil, have been demanding a great effort from both the academy and industry sides to address the increasing grid congestion and detrimental effects on reliability due to intermittence on the operational, planning, and market sides (see [4], and [5] for further reference). Although the complementarity between renewable sources has been demonstrated to be beneficial for both the renewable generation owners and system sides, as will be further demonstrated, the regulatory and market design rules for hybrid units may interact with these benefits.

The core of the Brazilian power market and the regulatory system relies on long-term forward contracts and firm energy certificates (FEC). While the former allows for a more stable net revenue (see [6]), the latter assigns to each unit a regulatory stamp regarding its capability to supply energy under adverse conditions (see [7]) for a detailed description of the concept and allocation methods). Based on these two instruments, as described in [6], a link between demand growth and supply adequacy is created by a regulatory system that requires: 1) loads need to be 100% contracted, and 2) every contract needs to be 100% covered by physical generation capacity, which in Brazil is certified by FECs. So, in the current regulatory framework, no generation can sell more than its FEC amount. However, renewable generators face an additional risk due to their uncertain generation. The so-called price-and-quantity risk poses a severe risk when renewables fall short in generation during high spot price periods (see [8] and [9] for relevant publications on the subject). So, in the case of contracted renewables, a less uncertain (or more firm) generation is key to achieving profitable contracting positions.

The economic viability of hybrid renewable generation in this country mainly relies on two key and opposite forces, namely, 1) the risk-adjusted maximization of forward and spot market revenues and 2) the minimization of regulatory expenses with network-access contract (NAC) payments, which applies to all agents connected to the transmission system and, in the case of generators, is charged per maximum MW injected per month in the network. NAC's are mainly used to share transmission costs among generation and consumers while signaling through locational tariffs expansion needs [10]. For the interested reader, we refer to [11] and [12] for a detailed discussion on the network access charges in Brazil for the demand side. Thus, the challenge is that both forces are directly affected by the firmness resulting from the complementarity between the sources being combined to form the hybrid power plant.

Within this context, the advantages of combining renewables in a portfolio of complementary sources to back long-term forward contract sales have been demonstrated in several

papers for the Brazilian power system. In [8], for the first time, complementary renewable sources, namely, a biomass unit and run-of-riven small hydro, were combined through a risk-constrained optimized portfolio to mitigate the price-and-quantity risk and back a forward contract sell in the free trading forward market. In [13], the complementarity and risk-mitigation benefit of a large set of renewables coordinated in a single renewable pool to sell contracts in the market with lower risk was studied. More specifically, an allocation rule to distribute the shares among the renewables participating in the pool was devised based on the nucleolus of a stochastic cooperative game. In [14], the complementarity of renewables was also studied in the presence of different markets and contract formats. Finally, more recently in [9], forward contracts and call options were used to mitigate the price-and-quantity risk in complementary portfolios of renewables considering ambiguity on the probability distributions. Therefore, the complementarity of renewables in the various time scales and market structures is a key factor in fostering the sustainable integration of large shares of renewables.

Notwithstanding, the regulatory framework for hybrid generation in Brazil, as well as in other developing countries, is still under development (see the preliminary discussions in [15], [16] and [17]). None of the previously reported work deals with the regulatory benefits of the hybridization of complementary generation, such as wind and solar, considering the interactions between forward and network-access contracts as happens in Brazil. In this case, hybridization allows us to define a single regulatory FEC and a single NAC for the two complementary sources composing the hybrid power plant based on the final generation output. As the FEC limits the maximum amount of forward contracting and the NAC limits the FEC (as further explained in the next sections), hybrid units may benefit from jointly defining the two contracting strategies to maximize profits. Additionally, the recent update in the Brazilian regulatory framework allowed hybrid power plants to reduce NAC amounts. This regulatory act potentialized the benefits of co-optimizing network-access and forward contracts while creating an interesting link between transmission and renewable generation expansion.

The Brazilian concept of FEC for non-controllable renewable generation (wind, solar, biomass, and small run-of-the-river hydros) lacks uniformity. Yet, the generation constrained-off because of grid congestion has not been properly addressed¹. The situation is especially relevant for hybrid power plants, which can adjust the maximum NAC amount (M) according to generation characteristics (we refer to [15] and [19]). The amount M defines the maximum power that a generating unit can inject into the network (measured as the average injected energy within short intervals of time). Thus, all the generation above this value is curtailed by the generator, or a penalty of three times the NAC tariff is applied. However, the absence of uniformity on the definition of FEC among renewables and on how this relevant regulatory stamp is affected by M in the case of hybrid units challenges the regulatory framework and weakens the economic incentives

¹For the interested reader, we refer to the local regulation [16] and [18]

for the development of hybrid units in Brazil [20].

A. Objective and contributions

The objectives of this paper are threefold: 1) to define a non-discriminatory FEC calculation methodology for hybrid power plants in Brazil, 2) to propose a co-optimization model to define the optimal forward and network-access contracting strategies for a renewable risk-averse hybrid power plant, and 3) to study the optimal share of renewable sources composing the hybrid power plant. In this context, the contributions of this paper are the following:

- 1) A unified methodology to calculate the FEC of non-controllable renewable units that is consistent with the more general hybrid power plant FEC proposed in this work.
- 2) An analytic tool for generation companies and system entities (planner, operator, and regulator) to simulate the complementarity benefits that hybrid power plants may provide on i) the price-and-quantity risk mitigation, ii) the reduction of network costs, and iii) the potential and typical shares of renewable sources on future hybrid units.

The above contributions are studied and corroborated with real data from the Brazilian power system.

The remainder of this paper is organized as follows. In Section III, we describe the general data framework used in this work. In Section IV, we describe the current regulatory framework in Brazil regarding NAC and FEC, whereas in Section V we present and study the proposed unified FEC concept for non-controllable renewables and hybrid power plants. In Section VI, we introduce the network-access and forward contract co-optimization model and in Section VII we present a series of three case studies to illustrate its usage and benefits with realistic data from the Brazilian power system. Finally, in Section VIII we summarize the main conclusions of this work and highlight future extensions of this work.

III. DATA FRAMEWORK AND UNCERTAINTY REPRESENTATION

In this work, two temporal horizons are considered, namely, 1) the historical horizon, which is characterized by the set of hours in the past (\mathcal{H}) for which generation data is assumed available, and 2) the future horizon, characterized by the set of hours (\mathcal{T}) for which the both the NAC and forward contract amounts are optimized. While the FEC is defined using historical generation availability data ($\{G_t^S, G_t^W\}_{t \in \mathcal{H}}$), thereby defined based on \mathcal{H} , the stochastic revenue of the hybrid power plant devising its optimal joint contracting strategy is defined based on simulated scenarios of the future generation availability and spot prices ($\{g_{t,\omega}^H, \pi_{t,\omega}\}_{t \in \mathcal{T}, \omega \in \Omega}$) within the contracting horizon \mathcal{T} . We refer the reader to the nomenclature section for further details.

It is relevant to mention that the historical generation availability, defined as the generation given the resource availability disregarding curtailments, might not be observable in some cases. For instance, new projects with no historical generation data or existing units that are subject to systematic curtailments

constitute relevant examples. In this case, methods to create synthetic historical generation [21] data are often used by certification companies and generation companies. In this work, the generation availability is assumed as an input, and the methods used to generate synthetic historical data (if needed), despite impacting the final results, are out of the scope of this paper. However, as the inputs of any data-driven regulatory metric may have a significant impact on agents, standards and procedures should be determined to ensure isonomy.

The uncertainty of the future generation of the hybrid power plant and spot prices is modeled, in this work, through a non-parametric Bayesian network model and a sample of scenarios is generated through a Monte Carlo simulation [22]. To account for the cross-dependency between renewable generation and spot prices, we simulated 200 coupled scenarios for the three-dimensional time series comprising the spot prices, wind generation, and solar generation for every hour in \mathcal{T} . To do that, we used the commercial hydrothermal dispatch model SDDP (from PSR Consulting) to simulate Brazilian spot prices based on the scenarios for the main renewable spots of the Brazilian system, including the solar and wind generation studied in this paper. We selected the year 2025 as the target year for the contracting horizon \mathcal{T} . Finally, the simulated scenarios, represented by the sample-space set Ω , will be used as inputs in the risk-adjusted two-stage stochastic model presented in Section VI.

IV. REGULATORY FRAMEWORK

A. Network-access contract (NAC)

The network cost is shared through consumers and generators connected to the Brazilian high-voltage transmission network (230+ kV). In the general case, as per the current Brazilian regulation, a minimum amount of NAC M must be equal to the total installed capacity of the power plant, assuming that at least in one 15-minute interval the power plant will inject its full capacity. However, recently a new understanding was applied to non-controllable renewable hybrid power plants. As it combines two or more (generally) complementary sources, the probability of the two or more sources achieving the maximum at the same time would be very small. Therefore, the benefit of complementarity between sources composing hybrid power plants could also be captured at the network level. In this case, the current regulatory framework allowed a reduced amount of NAC lower than the sum of the individual sources' installed capacities within hybrid power plants. This feature was considered in the proposed regulation [19] regarding the flexibilization of NAC whereas the hybrid may contract M between the highest source installed capacity and the total installed capacity of the plant. This regulatory innovation allows for relevant NAC cost savings while incentivizing the hybridization of complementary sources that relieve network expansion needs. Yet, annually the hybrid generation is allowed by regulation to reduce or increase NAC. We shall remark that in October of 2022, it was published the first flexibilization for a solar-wind hybrid power plant actually receiving permission to establish M in a range between 471 MW and 590 MW.

Nevertheless, when generation injection surpasses the network-access contract M , a penalty must be paid. According to regulation [23], generators are subject to a penalty, monthly calculated when the injection is higher than the contracted amount M plus a tolerance ϵ . For each month m , the penalty can be expressed as the maximum violation of the tolerance within all $t \in \mathcal{T}_m$. This can be translated in the following expression:

$$\tilde{\mu}_m(\tilde{g}_t, M) = \max_{t \in \mathcal{T}_m} \{\max(\tilde{g}_t - (1 + \epsilon)M, 0)\}. \quad (1)$$

In practical cases, depending on the generation technology, there are means to prevent these penalties by curtailing generation in real-time operation.

B. Firm Energy Certificates in Brazil

The concept of FEC is an energetic reliability stamp, measured in average MW (avgMW), which aims to quantify the amount of energy that can be sustainably supplied by a generator. Therefore, the FEC is used as a relevant generation expansion index in planning studies [24]. This general concept is currently implemented in Brazil as an allocation of shares of a global long-term energetic supply capacity calculated based on dispatch simulations by the system planner (EPE – *Empresa de Pesquisa Energética*) and is issued by the Ministry of Mines and Energy. This happens because the total energy that can be supplied by a coordinated system with many hydros operating under different inflow regimes and interconnected by complex cascades is higher than the sum of the energies that would be achieved if operated on an individual basis. Therefore, the FEC of dispatchable units (units operating under the coordination of the national system operator) is calculated by EPE as a share of the system-global energetic supply capacity. We refer to [7] and [24] for further discussions and references.

Nevertheless, for non-controllable renewables (which are considered non-dispatchable units), such as wind and solar sources, the FEC is issued based on the expected value or quantiles of the annual average historical generation availability, depending on the source. Historical variables and other parameters are calculated by certification companies according to each technology. For instance, for wind power plants, the 10th percentile of the annual average generation is used, whereas for solar units, the 50th percentile is adopted. Similarly, the average production considering historical inflows is used for run-of-river small hydro plants. Interestingly, and in addition to the previous differences between sources, the assessment and revision procedures are mainly based on the historical average. Therefore, although the regulation aims at assigning and revising FECs of non-controllable renewables that are intrinsically associated with their expected annual generation capacity, for historical reasons, the current regulation uses a different methodology for each type of source. Furthermore, besides its relevance to planning studies, the FEC also has a significant impact on market decisions, as it defines the maximum regulatory amount that a generator can sell in forward contracts (see [24] for further details). Thus, the discrepancies among the FEC calculations constitute a regulatory distortion that can discriminate sources in both long-term

studies, auctions, and market competitiveness. It is interesting to mention that none of the methodologies acknowledges the strong generation seasonality to which most of these sources are subject, and only wind generators have their FEC calculated based on a reliability index (low percentile).

In addition to the above regulatory incompatibilities, the recent interest in hybrid power plants has triggered further issues on the subject. First, how to calculate a single FEC for a hybrid power plant composed of two different sources, e.g., wind and solar, each of which with its own FEC calculation methodology? Second, due to the high complementarity between wind and solar sources, the NAC amount, M , of the hybrid power plant can be, in general, reduced to a value lower than the sum of the installed capacity of the two sources. This can reduce NAC payments and network expansion costs without significantly compromising the revenue of the hybrid plant, as will be further demonstrated. However, in case of reducing the NAC amount, the generator would curtail the exceeding generation, forgoing some potential revenue from the spot market, to avoid heavy penalty charges for violating the NAC. This introduces a second layer of complexity, as the FEC calculation would be affected by a reduced (truncated on the NAC amount) generation of the hybrid power plant. *Hence, the coherence between the main transmission and generation expansion indexes (NAC and FEC) with a non-discriminatory regulatory contracting limit for non-controllable renewables and hybrid power plants relies on a unified FEC calculation methodology for these sources.*

V. THE PROPOSED UNIFIED FEC FOR RENEWABLES AND HYBRID POWER PLANTS

Based on the coherence requirement described at the end of the previous Section, we propose a unified FEC for non-controllable sources that can coherently accommodate 1) the hybridization of different sources, 2) capture the complementarity synergy among them, and 3) consider the effect of NAC. For didactic purposes, hereinafter, we will consider only the case of two sources, wind and solar. Notwithstanding, the whole developments and proofs in the sequel can be easily extended to the case of more than two generic non-controllable sources.

Additionally, bearing in mind the parsimony principle, in this Section, we will present the simplest version of our proposal, the *truncated expected-value-based FEC* (TEV-FEC), which constitutes the minimal change in the current regulation needed to make a coherent unified and non-discriminatory framework for renewables. Additionally, it is important to highlight that if the aim of FECs in the current regulation relies on energetic contributions, expectations (or averages) play a better role in quantifying the integral of the energy supplied within a given period than quantiles. Nonetheless, we provide in the Appendix an extension for the proposed FEC considering another interesting aspect left behind in the current regulatory framework, such as seasonality.

A. The unified truncated expected-value-based FEC (TEV-FEC) for renewables and hybrid power plants

Considering a vector \mathbf{G} with historical generation availability for a given non-controllable source, i.e., $\mathbf{G} = [G_1, \dots, G_{|\mathcal{H}|}]'$, and a network-access contract amount M , the FEC of the power plant can be defined as follows:

$$\mathbb{F}(\mathbf{G}, M) = \frac{1}{|\mathcal{H}|} \sum_{t \in \mathcal{H}} \min\{G_t, M\}. \quad (2)$$

Function (2) is concave on M and \mathbf{G} as it is a weighted sum of the minimum between linear functions. Figure 1 illustrates this function on the M dimension.

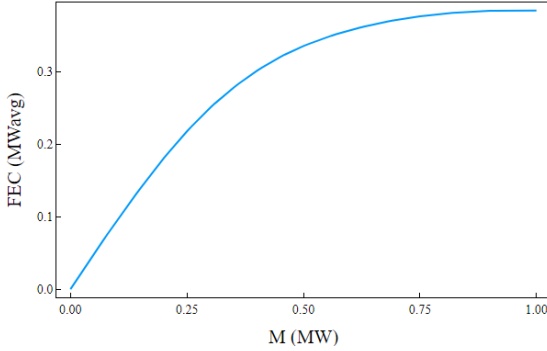


Figure 1. FEC as a function of NAC amount M .

Based on the general definition of the TEV-FEC, provided in (2), we can define the FEC of a hybrid power plant with 1 MW of total installed capacity, composed of a x -MW solar generator and a $(1-x)$ -MW wind generator, as $\mathbb{F}(\mathbf{G}^H(x), M)$, where $\mathbf{G}^H(x)$ is defined as follows:

$$\mathbf{G}^H(x) = x\mathbf{G}^S + (1-x)\mathbf{G}^W. \quad (3)$$

Thus, the TEV-FEC of non-hybrid units rests on the specific cases of $x = 0$ and $x = 1$. Note that in (3), we need to consider paired historical generation for the wind and solar sources, i.e., $G_t^H(x) = xG_t^S + (1-x)G_t^W$ for $t \in \mathcal{H}$, and that both wind and solar generation vectors consider normalized generation availability in percentages of their maximum installed capacity to meet the one-MW definition above. Based on (2) and (3), we can state the following theorem:

Theorem 1: *The TEV-FEC of the hybrid power plant composed of x 100% of solar and $(1-x)$ 100% of wind and with a NAC amount M is super-additive, i.e., is greater or equal to the sum of the FEC of its parts splitting M in proportion to x and $1-x$, for any $x \in [0, 1]$. In mathematical terms, it means that*

$$\mathbb{F}(\mathbf{G}^H(x), M) \geq \mathbb{F}(x\mathbf{G}^S, xM) + \mathbb{F}((1-x)\mathbf{G}^W, (1-x)M). \quad (4)$$

Proof: As we know that the minimum between two linear functions is a concave function [25], the following inequality

holds $\forall t \in \mathcal{H}$ and $x \in [0, 1]$:

$$\begin{aligned} & \min\{xG_t^S + (1-x)G_t^W, xM + (1-x)M\} \geq \\ & x \min\{G_t^S, M\} + (1-x) \min\{G_t^W, M\} = \\ & \min\{xG_t^S, xM\} + \min\{(1-x)G_t^W, (1-x)M\}. \end{aligned} \quad (5)$$

By averaging the first and the last term of (5) we have:

$$\begin{aligned} & \frac{1}{|\mathcal{H}|} \sum_{t \in \mathcal{H}} \min\{xG_t^S + (1-x)G_t^W, xM + (1-x)M\} \geq \\ & \frac{1}{|\mathcal{H}|} \sum_{t \in \mathcal{H}} \min\{xG_t^S, xM\} + \\ & \frac{1}{|\mathcal{H}|} \sum_{t \in \mathcal{H}} \min\{(1-x)G_t^W, (1-x)M\}. \end{aligned} \quad (6)$$

It happens that the left-hand-side of (6) meets the definition of the FEC for a hybrid power plant, whereas the right-hand-side meets the sum of the FECs of its parts when M is split in proportion to x and $1-x$, i.e.,

$$\mathbb{F}(xG_t^S + (1-x)G_t^W, M) \geq \mathbb{F}(xG_t^S, xM) + \mathbb{F}((1-x)G_t^W, (1-x)M). \blacksquare \quad (7)$$

Remark 1: *Theorem 1 tells us that there is a potential gain of synergy in terms of FEC associated with hybrid units, which constitutes an additional incentive for hybridization. We will further see in our empirical studies that this gain, for a hybrid power plant with $x \in (0, 1)$, is maximum on values of NAC lower than the sum of the total installed capacities, i.e., $M < 1$. This provides evidence that the proposed TEV-FEC connects the incentives for hybridization with the benefit of transmission expansion cost savings.*

Finally, it is relevant to mention that, because the TEV-FEC is based on the expected value of an annual average, intervals typically considered in annual revision processes can be based on the critical intervals of a hypothesis test for the mean. Although the definition of such a process is out of the scope of this paper, typical error type I and II analysis can be used to define the significance level of the test and define revision intervals that are tailored made for each power plant.

B. Empirical studies

The key aspect behind the inequality (and possible gain) in **Theorem 1** is the fact that the combination of two complementary sources may reduce the risk of curtailments, which happen whenever $xG_t^S + (1-x)G_t^W > M$. It is easy to see this fact, as the cap (truncation) on M is the only part that, if removed, would turn (4) into equality, which is due to the linearity property of the expected value operator. To empirically quantify this gain, we can define it as the difference between the TEV-FEC for the hybrid power plant and the sum of the TEV-FEC for the individual sources composing it, i.e.,

$$\begin{aligned} & \text{gain}(x, M) = \mathbb{F}(\mathbf{G}^H(x), M) \\ & - [\mathbb{F}(x\mathbf{G}_t^S, xM) + \mathbb{F}((1-x)\mathbf{G}_t^W, (1-x)M)] \end{aligned} \quad (8)$$

Let's consider the hybrid power plant composed of solar and wind sources with a total installed capacity of 1 MW.

To illustrate the gain for different values of M , we arbitrarily select $x = 0.5$ (hybrid half wind/solar) and plot the $gain(0.5, M)$ in Figure 2. The summation of wind (green line) and solar (yellow line) is represented by the segmented black line, while the hybrid is the continuous black line. The gap between these two black lines represents the gain of a hybrid power plant in terms of FEC which is highlighted in the bar graph below. It must be noted that the $gain$ is zero at the borders of M , especially important where the network-access contract amount reaches the total installed capacity. Below that, particularly in the range $M \in [0.3, 0.6]$ we see a considerable gain of hybrid plants in terms of FEC.

Additionally, applying the same calculus in Figure 3, for x 's ranging from 0.0 to 1.0, it is possible to see the highest gain of hybrid behaviour according to different share of sources. We observe that the maximum $gain$ happens for a particular M which depends on each x .

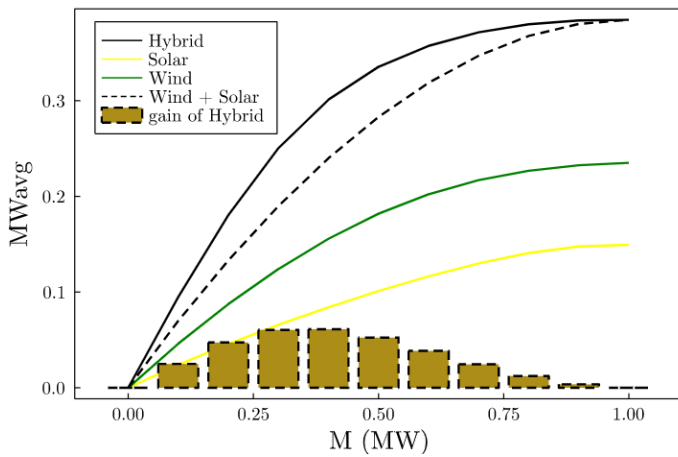


Figure 2. FEC Comparison of Hybrid and spitted sources for $x = 0.5$.

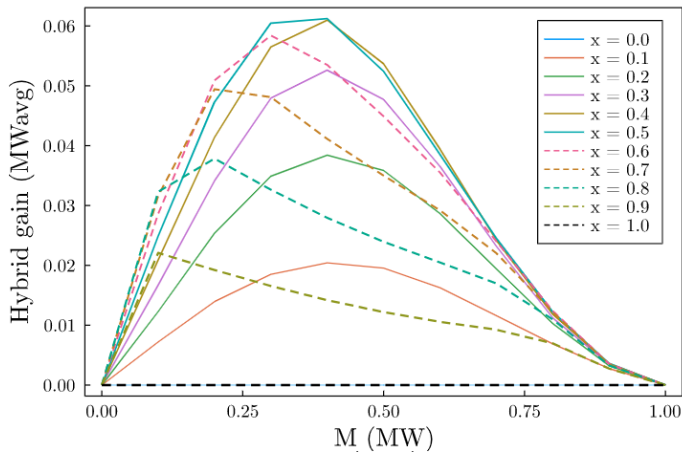


Figure 3. Hybrid's gain in FEC compared to separated sources, according to x .

The relationship between higher revenue and higher M is not direct, and the definition of M should consider three non-trivial relations: 1) the higher the value of M , the greater the value of FEC is, thereby the greater one can sell in forward contracts; 2) the complementarity gain in terms of

FEC for the hybrid power plant intrinsically and non-trivially depends on M and x ; 3) the higher M , the greater are the cost of transmission. Then, in the next Section, the interaction between M , x , and the risk-adjusted revenue of a hybrid power plant is characterized through a coherent risk measure and the optimal joint contracting strategy and is defined by a two-stage stochastic optimization model.

VI. CO-OPTIMIZATION OF NETWORK-ACCESS AND FORWARD CONTRACTS

A. Generators revenue and costs

The generator's net revenue can be expressed as the difference between the revenue (9) and costs (10). So, the net revenue is a function of the network-access contract M , forward contract Q , and share of solar x . The annual revenue \tilde{R} , which is a random variable, comprises the contract income plus the revenue in the spot market regarding the differences between the generation and the contract as follows:

$$\tilde{R}(Q, M, x) = \sum_{t \in \mathcal{T}} [pQ + (\hat{g}_t^H(M, x) - Q)\tilde{\pi}_t]. \quad (9)$$

The cost is a deterministic function of the NAC amount and the proportion of renewable energy sources multiplied by their annualized capital expenditures (CAPEX) C^S and C^W . It is represented as follows:

$$C(M, x) = \sum_{m \in \mathcal{M}} M c_m + x C^S + (1 - x) C^W. \quad (10)$$

Given the previous equations, it is clear that the optimal point regarding network-access amount M is not necessarily the one with the maximum energy. In fact, as the cost of NAC increases linearly with M , and the effective generation output of the hybrid power plant $\hat{g}_t^H(M, x)$ is a random non-decreasing concave function of M , there should be a value of M for which the incremental marginal cost with NAC tariff of increasing the NAC amount is equal to the incremental marginal certainty equivalent (utility) with additional spot revenues. The certainty equivalent concept used in this work will be defined in the next subsection.

B. Risk profile

To assess the value of random variables, in this paper, we make use of a coherent risk measure, namely, the conditional value at risk, to generate a certainty equivalent as per [26]. To do that, let us consider a risk profile characterized by a certainty equivalent (ρ) based on the Conditional Value at Risk (CVaR). So, in this setting, given a random revenue \tilde{R} , the certainty equivalent is defined as follows:

$$\rho_{\alpha, \lambda}(\tilde{R}) = \lambda CVaR_{\alpha}(\tilde{R}) + (1 - \lambda)E(\tilde{R}). \quad (11)$$

This certainty equivalent metric can be recast as a linear optimization problem by means of a linear programming representation for the CVaR in $\rho_{\alpha, \lambda}$ according to expressions (7) and (8) in [27], firstly proposed in [27]. For a discrete

distribution of \tilde{R} , with scenarios and probabilities given by $\{(R_\omega, 1/|\Omega|)\}_{\omega \in \Omega}$, $\rho_{\alpha,\lambda}$ can be represented as follows:

$$\rho_{\alpha,\lambda}(\tilde{R}) = \lambda \max_z \left\{ z - \sum_{\omega \in \Omega} \frac{(z - R_\omega)^+}{(1-\alpha)|\Omega|} \right\} + (1-\lambda) \frac{1}{|\Omega|} \sum_{\omega \in \Omega} R_\omega. \quad (12)$$

The above formulation is suitable for linear programming problems as widely used in the related literature (e.g., see [8] and [9]).

C. Co-optimization model

In this Section, we present the proposed risk-adjusted two-stage stochastic model to define 1) the optimal joint contracting strategy for both the forward market and the network-access contracts (Q^*, M^*), and 2) the optimal share (x^*) of sources for a one-MW hybrid power plant. The mathematical formulation of the model is as follows:

$$\max_{M, Q, x, \hat{g}_{t,\omega}^H, \hat{G}_t^H} \rho_{\alpha,\lambda} \left(\sum_{t \in \mathcal{T}} [pQ + (\hat{g}_t^H - Q)\tilde{\pi}_t] \right) - \sum_{m \in \mathcal{M}} M c_m - x C^S - (1-x) C^W \quad (13)$$

s.t.:

$$\hat{g}_{t,\omega}^H \leq M \quad \forall t \in \mathcal{T}, \omega \in \Omega \quad (14)$$

$$\hat{g}_{t,\omega}^H \leq (1-x)g_{t,\omega}^W + xg_{t,\omega}^S \quad \forall t \in \mathcal{T}, \omega \in \Omega \quad (15)$$

$$\hat{G}_t^H \leq M \quad \forall t \in \mathcal{H} \quad (16)$$

$$\hat{G}_t^H \leq (1-x)G_t^W + xG_t^S \quad \forall t \in \mathcal{H} \quad (17)$$

$$Q \leq \frac{1}{|\mathcal{H}|} \sum_{t \in \mathcal{H}} \hat{G}_t^H \quad (18)$$

$$M, Q, \hat{g}_{t,\omega}^H, \hat{G}_t^H \geq 0, \text{ and } x \in [0, 1] \quad (19)$$

In (13)–(19), the objective function comprises the maximization of the CVaR-based certainty equivalent, (11), applied to the net revenue, i.e., the difference between expression (9) and (10). As costs are deterministic, and the $\rho_{\alpha,\lambda}$ is shift additive (see [26]), the cost can be considered out of the certainty equivalent. Additionally, by considering $\hat{g}_{t,\omega}^H$ as a decision variable, expressions (14) and (15), impose that this variable lies in the hypograph of the hybrid power plant truncated generation, i.e., that $\hat{g}_{t,\omega}^H \leq \min\{(1-x)g_{t,\omega}^W + xg_{t,\omega}^S, M\}$. Actually, as 1) $\hat{g}_{t,\omega}^H$ is multiplied by positive coefficients (as spot prices are positive, i.e., $\tilde{\pi}_t \geq 0$) and 2) $\rho_{\alpha,\lambda}$, for $\lambda > 0$, is a non-decreasing function (see [26]), the optimal solution will always be attained on the equality, i.e., $\hat{g}_{t,\omega}^{H*} = \min\{(1-x)g_{t,\omega}^W + xg_{t,\omega}^S, M\}$. The same rationale applies to \hat{G}_t^H and expressions (16) and (17). Finally, expression (18) defines the regulatory limit for the forward involvement based on the FEC, and (19) defines the limits of each variable. Model (13)–(19) can be recast as a linear optimization problem by replacing $\rho_{\alpha,\lambda}$ in (13), with its linear counterpart (12). For the sake of brevity and to avoid redundancy, we omit this reformulation.

VII. CASE STUDIES

In this section, we present three case studies, each of which aims to demonstrate one relevant feature of the proposed

methodology. First, we test the co-optimization of forward and network-access contracts idea for a given hybrid power plant composed of 50-50 % (arbitrarily selected) wind and solar. So, in this case study, model (13)–(19) is used disregarding the CAPEX parcel in the objective function and with an additional constraint imposing $x = 0.5$. In the second study, we run sensitivity analyses on the shares (x) to study the effect of the share in the optimal contracting strategies, i.e., on the optimal responses $M^*(x)$ and $Q^*(x)$. In this sense, the same constrained model of the first study is used to run these analyses, but varying the constraint on x for different values. Finally, in the third study, we let x to be co-optimized with M and Q . In this case, we study the optimal solution as a function of the annualized CAPEX of two sources composing the hybrid power plant, thereby considering the complete objective function (13). In all case studies, we considered $\lambda = \alpha = 0.95$.

The model is implemented in Julia Language (JuMP) and solved by Gurobi. The data set used in this paper is available at [28]. We used a Notebook Intel(R) Core(TM) i7-8565U with 4 cores (1.99 GHz each) and 8 GB of RAM.

A. Case Study 1: Co-optimizing network-access and forward contracts

In this first case study, we assume a hybrid power plant half solar, half wind, i.e., $x = 0.5$. First, we assume a neutral forward market with a forward price p equal to the expected value of the average annual spot price, i.e., 83 R\$/MWh, a constant network-access tariff $c_m = 7$ R\$/kW/month and null CAPEX, $C^W = C^S = 0$. The effect of different forward prices will be further studied.

First, we showcase the relevance of optimizing both forward and network-access contracts. To do that, we benchmark the results of our model with the base case, compatible with the current regulatory framework, where the network-access contract should be equal to the total installed capacity. Thus, the benchmark model also comprises another constraint, i.e., $M = 1.0$.

For the benchmark, we find $Q^* = 0.3294$ and an objective value equal to 150,796\$. Then, by co-optimizing both variables (M and Q), we find $M^* = 0.5548$ (44.52% lower than the benchmark), $Q = 0.3177$ (3.57% lower than the optimal forward contracting level in the benchmark) and an objective value of 178,159\$ (18.15% higher than that obtained in the benchmark). In other words, the co-optimization significantly reduces the network-access contracting amount to a value that is only 10.96% higher than the installed capacity of each source (recall that in this case study, each source has 0.5 of the total installed capacity). On the forward contracting side, it is worth mentioning that the new joint contracting strategy, which is responsible for reducing 44.52% of the network-access annual costs, is made without changing too much the forward involvement strategy (only 3.57% of reduction in comparison to the benchmark). Hence, the significant improvement in the risk-adjusted revenue metric is mostly related to the reduction of network-access expenditures. Notwithstanding, the 3.57% adjustment in the forward market is necessary to compensate for the price and quantity risk due to the reduction

in the generation profile. For instance, by keeping the forward involvement equal to the benchmark, i.e., also making $Q^* = 0.3294$ and optimizing only M , the optimal solution would be $M^* = 0.5756$, and the certainty equivalent would be equal to 176,985\$ (0.66% lower than the co-optimized strategy, yet still 17.37% higher than the benchmark).

It is worth mentioning that, in contrast to the significant reduction in the NAC amount and related fixed cost savings (44.52%), the expected energy curtailment implemented to avoid exceeding the reduced NAC amount M^* is only 7.63% of the total expected value of the available generation of the hybrid power plant (with an expected frequency of 18.12% of the hours in the year). In terms of expected revenue in the spot market, this curtailed energy represents only 6.28% of the total expected revenue of the co-optimized case.

To further analyze the co-optimized strategy, we can vary the forward contract price to simulate different market conditions. To do that, we run the model (13)–(19) with $p = p_0\beta$, where $p_0 = 83$ R\$/MWh (neutral market) and $\beta \in \{0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0\}$. In Figure 4, we can see that the network-access contract M^* and the forward contract Q^* increase, especially the first, as the contract price grows, which is somewhat expected. However, it is interesting to see that the NAC grows faster than forward involvement, whereas M^* does not surpass 0.8102 and Q^* does not surpass 0.3805. This brings relevant evidence that hybrid power plants should be allowed to reduce their NAC amount. Additionally, the benefits of this reduction are intrinsically linked to the forward contracting strategy, thereby requiring a regulatory framework that acknowledges this link on both the FEC and NAC payments.

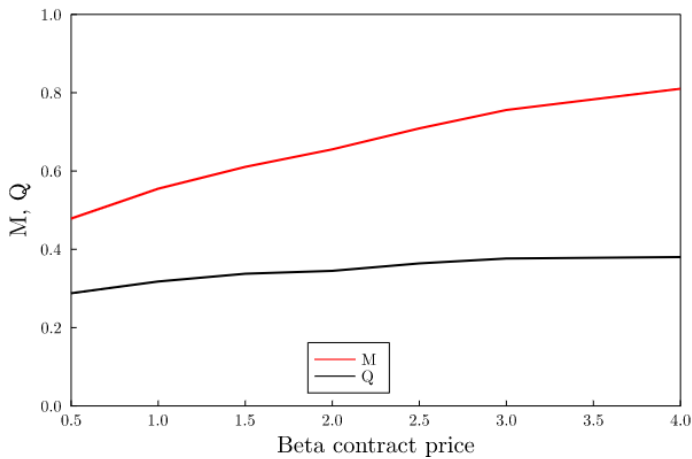


Figure 4. Network-access contract and Forward Contract Q according to β contract price.

B. Case Study 2: Sensitivity on the share of solar and wind

In this subsection, we study the sensitivity of the share of sources (x) in the hybrid power plant. Thus, in this investigation, we run the same two models from the previous subsection, namely, the benchmark, where only the Q is optimized and $M = 1$, and the co-optimized model, where both Q and M are jointly optimized, for different values

of x . Figure 5 shows results for different values of x from 0.0 to 1.0. Note that the forward contract Q is lower in the co-optimization case. As expected, co-optimizing M and Q increases the objective value. However, it is worth highlighting that, according to Figure 6, the gain in terms of certainty equivalent (difference between the objective function in the two cases – red and blue lines), is higher for intermediate values of x , i.e., hybrid plants benefit more from co-optimizing the NAC when developing their contracting strategy. The reason for that becomes clear in Figure 7, where the optimal co-optimized NAC amount M^* is depicted in red. Note that the value of M^* exhibits lower values in the intermediate values of x (with a minimum for a hybrid power plant with $x = 0.46$), where the interpretations and insights discussed in the previous section for $x = 0.5$ also apply. Notwithstanding, the boundary cases of single units, i.e., $x = 0$ (pure wind) and $x = 1$ (pure solar), also exhibit non-negligible gains and optimal contracting strategies with $M^* < 1$.

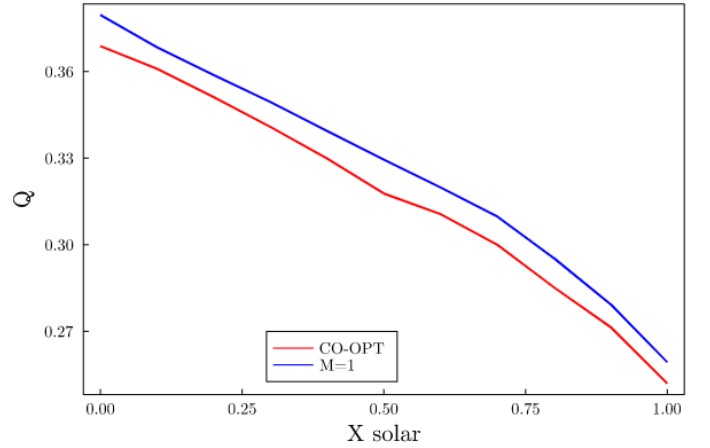


Figure 5. Forward Contract Q comparison co-optimization and maximum M , according to x .

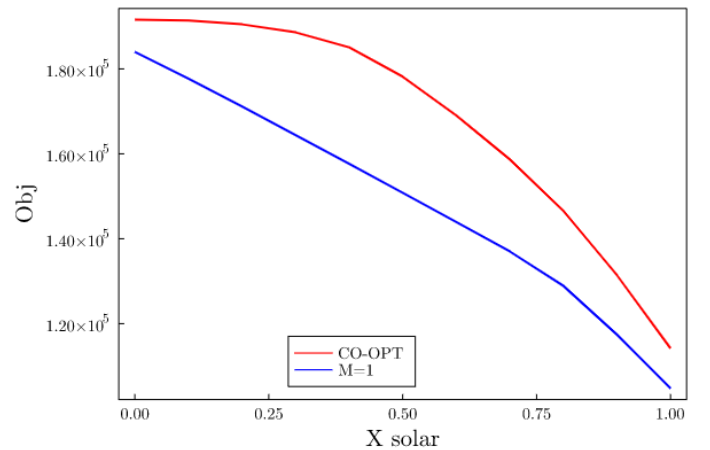


Figure 6. Objective value comparison co-optimization and maximum M , according to x .

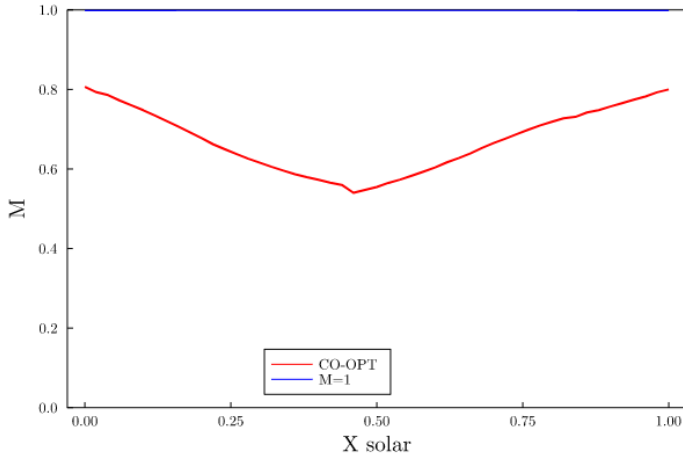


Figure 7. Optimum values of M according solar partition x .

C. Case Study 3: Sensitivity on the CAPEX of solar and wind

From Figure 6, the best configuration when ignoring CAPEX costs (recall that all the previously reported results ignore this term) is $x = 0$; in other words, the higher capacity factor (generation per unit of installed capacity) of the wind power producer selected in this study makes this source the most economical. However, a more comprehensive analysis should account for the annual CAPEX of the different sources composing the hybrid power plant. To study the optimal joint contracting strategy and the optimal share of the hybrid power plant, in this subsection we run model (13)–(19) with all its term and no additional constraints to define the optimal vector $[M^*, Q^*, x^*]$.

As all previously reported studies are related to the case where $C^S = C^W$, and as we know that for this case, the best solution is $x = 0$, here we run our model for different combinations of $C^S < C^W$. To facilitate this sensitivity, we parameterize $C^S = \gamma \cdot C^W$. As per [29], in Brazil, the CAPEX for wind energy ranges from 3,200 to 5,500 R\$/installed-kW while solar ranges from 2,500 to 5,000 R\$/installed-kW. So, we arbitrarily select $C^W = 4,000$ R\$/installed-kW within the range for wind power and run a sensitivity analysis on $\gamma = \frac{C^S}{C^W} \in [0, 1]$.

The consideration of CAPEX shows a clear advantage of source combination. Figure 8 depicts the co-optimized strategy $[M^*, Q^*, x^*]$ for each value of γ . It is clear that for reasonable values of γ , e.g., $\gamma = 0.8$ ($C^W = 4,000$ and $C^S = 3,200$), hybridization becomes the best option. For instance, for $\gamma = 0.8$, the best strategy is $[M^* = 0.5968, Q^* = 0.3367, x^* = 0.3388]$. The maximum NAC reduction is obtained with $\gamma = 0.5$ ($C^W = 4,000$ and $C^S = 2,000$), which, although out of the range of CAPEX for solar units, would produce an optimal co-optimized strategy with $[M^* = 0.5410, Q^* = 0.3210, x^* = 0.4589]$.

VIII. CONCLUSION

This paper proposed a unified formulation for the firm energy certificates of renewables and hybrid power plants in Brazil. Based on that, a co-optimization tool for jointly

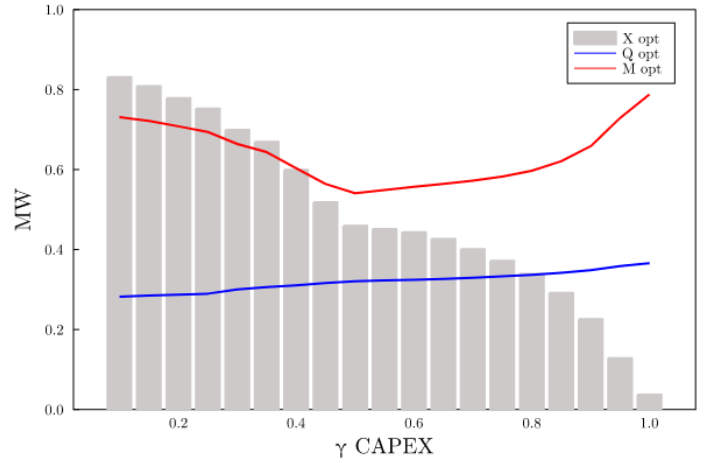


Figure 8. Optimum x solar, M and Q according to CAPEX defined by γ .

defining the optimal network-access and forward contracting strategy was proposed and tested with realistic data from typical profiles of wind and solar generation. The proposed truncated expected-value-based firm energy certificate (TEV-FEC) constitutes the minimal change in the current regulation needed to make a coherent, unified, and non-discriminatory framework for renewables and hybrid power plants. Additionally, according to our Theorem 1, it also enjoys the relevant property of super-additivity, which creates the link between investment incentives in hybrid power plants and reducing transmission expansion costs.

Within the limitations of the presented case study, which include all assumptions of the proposed model and the specific data, the results and analyses carried out in this work allow us to convey the following concluding remarks:

- The co-optimization of NAC and forward contracting strategies is capable of providing relevant gains for the hybrid power producers.
- The reduction in the NAC with respect to the benchmark value (total installed capacity) is responsible for most of the monetary benefit, whereas the curtailed excess of energy is relatively low, thereby justifying the reduction in NAC fix expenditures.
- When disregarding CAPEX, the optimal joint strategy of forward and network-access contracting results in the minimum network usage for a hybrid plant that is composed of 54% wind and 46% solar generation (results valid only for the analyzed data).
- Based on typical values of CAPEX ($C^W = 4,000$ and $C^S = 3,200$) and generation profiles of wind and solar power plants in the Northeastern region of Brazil, hybridization can be not only the best economical option for investors but also the one that provides the highest benefit for consumers in terms of reducing transmission costs. For instance, for the aforementioned CAPEX, the best strategy is $[M^* = 0.5968, Q^* = 0.3367, x^* = 0.3388]$.

We highlight as a relevant topic for future research the study of the regulatory incentives and the optimal trading strategy of hybrid power plants considering more sophisticated

FECs, e.g., the seasonal FEC proposed in the appendix. Additionally, the hybridization idea can be expanded for a pool of renewable plants within a given bus of the system or within a "transmission-free" zone. In this case, the ideas proposed in this paper can be expanded for $n > 2$ units, and a cooperative game theory such as that proposed in [13] can be used to share the total FEC of the pool. Finally, the study of revision processes for the proposed TEV-FEC based on hypothesis tests can also be an interesting topic of research.

APPENDIX

In this Appendix, we discuss the extension of the proposed TEV-FEC $\mathbb{F}_m(\mathbf{G}^H(x), M)$ for a hybrid power plant to consider seasonality and reliability.

The seasonality extension is straightforward. We just need to consider interannual, such as monthly or quarterly, calculations of (2). For didactic purposes, hereinafter, we use a monthly discretization. Thus, for each month $m \in \{1, \dots, 12\}$, one just needs to filter from the complete historical data \mathcal{H} used to calculate (3), the generation associated with hours in month m , i.e., \mathcal{H}_m , as follows:

$$\mathbb{F}_m(\mathbf{G}^H(x), M) = \frac{1}{|\mathcal{H}_m|} \sum_{t \in \mathcal{H}_m} \min\{G_t^H(x), M\}. \quad (20)$$

It is easy to see that **Theorem 1** is valid for (20), as it has the same structure, but considering a subset of data in the summation in comparison to (2). Although the optimal contracting strategy based on extensions of the TEV-FEC was not covered in this paper, the model (13)–(19) can be easily extended to consider any concave FEC as (20). The consideration of a seasonal FEC, notwithstanding, may trigger further discussions on more sophisticated seasonal or multistage trading strategies. This is left as an interesting topic for future research.

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