Democratization of Complex-Problem Solving: Toward Privacy-Aware, Transparent and Inclusive Optimization

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Abstract

Critical operations often involve stakeholders with diverse perspectives, yet centralized optimization assumes participation or private information, neither of which is *a priori* guaranteed. Additionally, decision-making involves discrete decisions, making optimization computationally challenging. Centralized formulations use approximations to manage complexity, often overlooking stakeholder perspectives, leading to bias. To resolve these challenges, we adopt a privacy-aware participatory-distributed (PAPD) optimization approach that reduces the combinatorial complexity upon decomposition, efficiently coordinates stakeholder subproblems without requiring private information, and allows stakeholders to actively and flexibly participate. Using challenging instances of large-scale Generalized Assignment Problems (GAPs) from the OR library and electrification of transportation scheduling instances, we demonstrate that the quality of our PAPD-obtained solutions decreases marginally compared to non-privacy-aware centralized solutions. However, this minor reduction in solution quality is outweighed by significantly lower make-whole payments (MWP), namely, compensations provided to participants to ensure they cover their costs and do not incur losses following the coordinated solution, than those associated with centralized methods. Reducing MWP enhances market transparency by ensuring stakeholder compensation is based on their decisions rather than centralized or out-of-market settlements. This minimizes the need for additional compensation and promotes a more fair and transparent market. The main takeaway is that PAPD decision-making aligns closely with minimizing system costs while preserving privacy, improving participation, and enhancing transparency compared to both traditional centralized and non-privacy-aware decomposition methods.

Keywords: Discrete Programming, Complex-Problem Solving, Democratization, Participatory Decision Making, Distributed Decision-Making, Privacy-Aware Optimization

1 Introduction

Operations within critical social systems such as healthcare [1–3], transportation and logistics [4–6], humanitarian applications [7–9], and power and energy systems [10–14] involve optimization-based decision-making to optimize an objective, e.g., minimize the total cost, maximize coverage or maximize total social welfare. To accurately model optimization problems, access to private stakeholder information is often required. Moreover, traditional centralized models and optimization methods typically assume that stakeholders will passively participate, i.e., comply with the solutions determined by the centralized decision-makers. However, neither the participation of stakeholders nor the availability of their private data can be guaranteed *a priori* since the lack of the former can lead to non-negative make-whole payments (MWP) to keep stakeholders engaged, and the lack of the latter leads to privacy concerns. In addition, within centralized methods, opportunities for stakeholders to actively participate in decision-making are generally limited, if provided at all. These considerations are compounded by the intrinsically discrete nature of the associated mathematical optimization problems, frequently modeled as mixed-integer programming (MIP) problems. As the MIP problems increase in size, the problem-solving difficulty (problem combinatorial complexity) increases drastically as the number of combinations of possible solutions grows. Since solutions to these problems impact populations significantly large in number and diversity, privacy-aware, inclusive, participatory, and computationally efficient methodologies are required.

Traditionally, to mitigate the problem's complexity, centralized modeling approaches often resort to modeling simplifications. Without private data, modeling approximations are used, potentially misrepresenting or completely overlooking various aspects and needs of diverse stakeholders. Even when solutions are optimal from a centralized perspective, the limited representation of stakeholders can hinder achieving maximum cost-efficiency. Such approaches may also be perceived as biased and undemocratic, as they exclude individuals from contributing their unique insights, skills, knowledge, expertise, and perspectives for the collective benefit. Many centralized modeling and solution methodologies thus face trade-offs between computational efficiency, cost-efficiency, and democracy. Computational efficiency often requires simplifications and exclusions, while cost-efficiency may come at the expense of privacy and increased computational demands. Furthermore, many centralized approaches rely on static data and are not designed to react to changing circumstances. Granted, many applications' intrinsic nature requires offline solving, such as unit commitment problems, which are solved a day ahead [12–14], or manufacturing scheduling problems [15, 16], which are frequently solved before a shift. Within these problems, centralized methods may be appropriate, and privacy preservation may not be critical if decision-making is confined to one factory with one owner. However, this paradigm would be problematic in dynamic operations like ambulance relocation [17], large-scale system failure detection [18], post-disaster blood supply [7, 9], urgent surgery scheduling [19], and disaster relief operations [6].

The inflexibility of traditional centralized approaches can hinder efficient and secure decision-making. Centralized approaches can be subdivided into two broad classes: non-decomposition-based (NDB) and decomposition-based (DB). The former class is exemplified by the Branch-and-Cut method and other methods widely implemented in optimization packages such as CPLEX and GUROBI. All private information must be known in these packages before the optimization begins, violating privacy. Moreover, without decomposition, the participation of stakeholders is problematic, and the issue of combinatorial complexity may remain unresolved from the computational standpoint, depending on the problem under consideration. The latter class (i.e., DB methods) is exemplified by methods such as Lagrangian Relaxation (LR). When implemented centrally, LR retains many of the abovementioned issues, such as requiring high computational resources as the number of stakeholders increases. Although LR alleviates some combinatorial complexities, traditional implementations still face significant computational effort and slow convergence due to the zigzagging behavior of multipliers. Our approach mitigates these issues using a participatory distributed framework, enhancing scalability and efficiency while preserving stakeholder privacy.

This paper addresses the democratization of complex-problem solving by developing a participatory distributed version of Surrogate Lagrangian Relaxation that enhances efficiency, stakeholder engagement, and privacy. We focus on the limitations of traditional centralized Mixed-Integer Linear Programming (MILP) in Subsection 1.1. Given the multifaceted nature of democratization, we propose a participatory distributed (PAPD) approach that actively involves stakeholders in both problem formulation and solution processes, along with reviewing the conceptual and methodological aspects supporting democratized problem-solving in Section 1.2.

1.1 Centralized Mixed-Integer Linear Programming

A centralized Mixed-Integer Linear Programming (MILP) problem can be represented as:

$$\min_{x:=\{x_i\}_{i=1}^{I^c}} \left\{ \sum_{i=1}^{I^c} (c_i)^T \cdot x_i \right\}. \tag{1}$$

Here, c_i is the cost of the subsystem/stakeholder i, and x_i is a decision variable vector that typically includes integer and continuous variables. Additionally, individual subsystems are frequently coupled through the following constraints:

$$s.t. \sum_{i=1}^{I^c} A_i \cdot x_i = b^c, \{x_i\} \in \mathcal{F}_i^c, i = 1, \ldots, I^c, \tag{2}$$

where $\mathcal{F}_i^c$ is a feasible set that corresponds to stakeholder i subsystem. In this centralized paradigm, decision-making authority and control are vested in a single central entity to manage the diverse aspects of the entire system. The centralized decision-making paradigm faces several challenges and limitations, including the following:

1. **Privacy Violation.** Sharing private data (e.g., c_i and $\mathcal{F}_i^c$) with the central decision-maker can lead to privacy violations.
2. **Inaccurate Approximations.** In the absence of private information, central decision-makers might resort to approximations, which can misrepresent stakeholders' actual needs and preferences.
3. **Stakeholder Exclusion.** Stakeholders can be completely overlooked as the number of stakeholders considered, I^c, can fall short of the actual number of affected parties, resulting in exclusion and possible oversight of vital interests.
4. **Cost Inefficiency.** When stakeholders are not actively involved in decision-making, the centralized approach may fail to utilize their knowledge and resources. As a result, the exclusion of stakeholders leads to a suboptimal allocation of resources, resulting in higher overall costs.
5. **Expert Limitations.** Given 1., 2. and 3., central decision makers, while often experts, may not fully address the diverse needs and interests of all stakeholders. In some sense, expertise may not fully compensate for the lack of diverse stakeholder perspectives.
6. **Violation of Fairness; Bias:** Centralized decision-making can lead to unintentional biases due to familiarity with traditional stakeholders, resulting in inequitable solutions. This perpetuates existing power imbalances and overlooks the needs of marginalized groups, ultimately leading to suboptimal solutions that fail to incorporate diverse perspectives.
7. **Computational Challenges.** The centralized decision-making is further burdened by the $\mathcal{NP}$-hard nature of the problem due to discrete decision variables within x. Including more stakeholders exponentially increases the combinatorial complexity, making it challenging to find optimal (or even near-optimal) solutions computationally efficiently. This may lead to solutions perceived as unfair and non-transparent by the stakeholders.

Addressing these challenges requires a shift toward a more participatory and democratic problem-solving paradigm to leverage collective intelligence and ensure all stakeholders' preferences are represented. Stakeholders must have equal opportunities to participate flexibly, benefit from their participation, and remain protected from discrimination based on sensitive private information. Instead of relying on centralized decision-makers, participants should actively contribute to decision-making at the system level while making informed choices for their respective subsystems. Democratizing complex problem-solving offers the potential to address diverse stakeholder needs and achieve more equitable outcomes. Overcoming the limitations of centralized approaches requires expanding participation beyond a limited expert group to account for the increasing scale, complexity, and diversity of stakeholders in modern contexts. The next section explores the multifaceted nature of democratized problem-solving, including research efforts from diverse fields.

1.2 The Multi-Faceted Nature of Democratizing Problem-Solving

The democratization of problem-solving is increasingly becoming a crucial aspect of modern decision-making, emphasizing privacy, diversity, participation, collaboration, and inclusion. This paradigm shift is rooted in the philosophy of intellectual openness advocating incorporating diverse viewpoints to ensure democratic decision-making [20, 21]. Democracy has been viewed as a practical means to actively engage citizens in solving complex social issues, extending well beyond traditional electoral processes [22], with citizen participation and collaboration playing a critical role in collective decision-making and showcasing how participatory democracy effectively addresses complex social challenges. Effective collective decision-making hinges on four essential requirements: diversity of opinion, independence, decentralization, and aggregation [23] with aggregation being a mechanism to turn a collection of private decisions into a collective decision. Decentralization, as explored in [24–28], has transformative potential in democratizing problem solving through open-source, peer production, and commons-based approaches that disrupt traditional power structures, favoring more participatory and equitable decision-making processes. Aggregation, as emphasized by [29], enhances decision processes by combining diverse stakeholder understandings, ensuring inclusion and fairness. From a technological standpoint, collaboration and collective intelligence have been studied in the context of management science [30–32]. The concept of the "supermind" aims to enhance collective problem-solving and improve democratic decision-making through a synergy of human abilities and computational power. Emerging technologies, however, introduce concerns around public engagement, algorithmic fairness, and moral responsibility [33–36].

Privacy preservation is a critical concern in collective decision-making processes. Research has explored privacy at the intersection of technology and society [37, 38], and computer science perspectives on privacy and security in distributed systems have also been investigated [39–41]. Techniques such as data obfuscation have been employed to protect privacy, but these methods can compromise the utility of the data [42]. For continuous optimization, privacy preservation has been achieved [43] through the use of "consensus variables" within the alternate direction method of multipliers (ADMM). The consensus is essentially the way to "aggregate" information from agents/subsystems through coordination;[1] The distributed ADMM method, a variant of Lagrangian Relaxation, holds promise for enhancing privacy in optimization processes. However, challenges persist in its application to discrete optimization problems, where the non-smooth nature of dual functions poses convergence difficulties for ADMM, as discussed in the recent survey by [44]. Given these limitations, further exploration of other Lagrangian Relaxation-based methods is warranted to develop a "privacy-aware participatory distributed" version that overcomes the difficulties of traditional Lagrangian Relaxation.

Traditionally, to optimize the Lagrangian dual function

$$\max_{\lambda}\{q(\lambda) : \lambda \in \Omega \subset \mathbb{R}^m\}, \tag{3}$$

with

$$q(\lambda) = \min_{x:=\{x_i\}_{i=1}^{I}, \{x_i\} \in \mathcal{F}_i} \Big\{ L(x, \lambda) \equiv \sum_{i=}^{I} (c_i)^T \cdot x_i + (\lambda)^T \cdot \Big(\sum_{i=1}^{I} A_i \cdot x_i - b \Big) \Big\}, \tag{4}$$

a series of steps s^k are iteratively taken along subgradient directions $g(\lambda^k)$

$$\lambda^{k+1} = \lambda^k + s^k \cdot g(\lambda^k), \tag{5}$$

both stepsizes s^k and multiplier-updating directions (e.g., subgradients $g(\lambda^k)$) are critical for achieving overall convergence and efficiently solving associated problems. Optimal stepsizes, now commonly referred to as "Polyak stepsizes," which ensure a linear convergence rate, were first introduced by [45]:

$$s^k = \gamma \cdot \frac{q(\lambda^*) - q(\lambda^k)}{\left\| g(x^k) \right\|^2}, 0 < \varepsilon_1 \leq \gamma \leq 2 - \varepsilon_2, \varepsilon_2 > 0. \tag{6}$$

Building upon this foundation, Surrogate Subgradient Methods (e.g., [46]) utilize Polyak stepsizes

$$s^k = \gamma \cdot \frac{q(\lambda^*) - L(\tilde{x}^k, \lambda^k)}{\|g(\tilde{x}^k)\|^2}, \quad \gamma < 1. \tag{7}$$

which, together with smoother "surrogate" subgradient directions $g(\tilde{x}^k)$, significantly reduce the computational effort required per iteration as well as alleviate zigzagging of multipliers through the requirement $L(\tilde{x}^k, \lambda^k) < L(\tilde{x}^{k-1}, \lambda^k)$. The practical implementability of Polyak step sizes for subgradient methods was further established by [47] based on heuristic adjustments of the "level" $\overline{q}^k$ to approximate $q(\lambda^*)$. Recently, advances in the Surrogate "level-based" Lagrangian Relaxation (SLBLR) method [48, 49] led to smooth directions, reduced "step-by-step" computational requirements, as well as decision-based and user-friendly "level" adjustments (adjustments of $\overline{q}^k$ - the estimates of $q(\lambda^*)$) to exploit linear convergence of Polyak stepsizes thereby overcoming limitations

[1] Aggregation is used in a sense introduced by [23].

of previous methods

$$s^k = \zeta \cdot \gamma \cdot \frac{\overline{q}^k - L(\tilde{x}^k, \lambda^k)}{\left\|g(\tilde{x}^k)\right\|^2}, \ \ \gamma < 1, \zeta < 1. \tag{8}$$

The numerical results of [48] highlight that the SLBLR method solves generalized assignment problems to optimality and outperforms traditional branch-and-cut methods in the job-shop and pharmaceutical scheduling, demonstrating high scalability. The SLBLR method, although originally implemented in a centralized manner with one coordinator, requires all private information. It can be viewed as distributed due to its decomposition-based nature and can be used as a foundation for inclusive and privacy-aware decision-making. This aspect facilitates participation, supports privacy preservation, and reduces the central coordinator's role in decision-making, effectively transferring more decision-making power to the individual stakeholder subproblems.

2 Democratization of Complex-Problem Solving

In this section, we introduce a privacy-aware participatory-distributed Surrogate Lagrangian Relaxation (PAPD-SLR), which enables stakeholders to formulate and solve subproblems independently without sharing private information. In doing so, stakeholders collectively formulate the overall optimization problem. A centralized coordinator ensures coordination and aggregation of subproblem solutions without accessing private information. This approach, efficiently reduces combinatorial complexity, transitions from centralized to participatory-distributed and privacy-aware decision-making, leverages human and computational capabilities for more inclusive, fair, and efficient decision-making, and produces solutions aligned with diverse stakeholder needs, leading to more equitable outcomes.

Lagrangian relaxation methods offer an operationalizable path toward democratizing problem-solving by enabling distributed participatory decision-making through **decomposition**, with stakeholders formulating and solving their subproblems while aligning with collective goals through **coordination**. This method allows stakeholders to tailor subproblems to their unique needs, contributing to more effective and inclusive decision-making.

In our proposed formulation, we diverge from the conventional approach that begins with defining an objective since the overall objective function in the privacy-aware scenario is not defined from the system-wide perspective, and only "local" stakeholder objectives are locally known. The coordinator only knows the general form of the "global" system-wide coupling constraints, which need to be satisfied:

$$\sum_{i=1}^{I} A_i \cdot x_i = b. \tag{9}$$

where A_i and b are assumed to be known while the number of stakeholders I or their decisions x_i are unknown. The primary goal is to satisfy (9) collectively. The variables x_i denote the contributions of each stakeholder to satisfying (9). The matrices A_i define how these contributions are incorporated into (9). Crucially, the number of stakeholders, I, is not predetermined, offering a dynamic element that requires flexible coordination and integration of varying stakeholder inputs over time. Each stakeholder keeps their costs private and can compute their total cost as:

$$(c_i)^T \cdot x_i, \tag{10}$$

as well as formulate their local feasible sets $\{x_i\} \in \mathcal{F}_i$, which are also kept private. Notably, after summing the above cost functions and subjecting to constraints (9), the resulting formulation becomes

$$\min_{x:=\{x_i\}_{i=1}^{I}} \left\{ \sum_{i=1}^{I} (c_i)^T \cdot x_i \right\}, \ \ \text{s.t. } (9), \{x_i\} \in \mathcal{F}_i, \forall i = 1, \ldots, I, \tag{11}$$

which is mathematically similar to that of (1)-(2). However, conceptually, all the data are known to the centralized decision maker within (1)-(2). In contrast, problem (11) is collectively formulated by participating stakeholders, none of whom is assumed to know information about other stakeholders, and the goal is to solve the problem collectively.

To operationalize the solution process through democratized collective decision-making, we employ a participatory distributed framework where stakeholders collaborate to solve the optimization problem without revealing their private information. This approach leverages Lagrangian Relaxation to decompose the problem into manageable subproblems, allowing each stakeholder to independently optimize their contribution while coordinating with others to satisfy the global constraints. By fostering a democratic environment, this method ensures that all stakeholders have equal opportunity to participate actively in the decision-making process, improving the system's transparency, fairness, and overall efficiency.

After relaxing constraints (9) and collecting terms associated with each stakeholder i while ignoring constant terms, Lagrangian Relaxation distributes decision-making by decomposing the relaxed problem into the following subproblems

$$\min_{x_i}\Big\{(c_i)^T \cdot x_i + \lambda^T \cdot (A_i \cdot x_i)\Big\}. \tag{12}$$

This enables stakeholders to control their subproblems fully, aligning with privacy-aware, participatory, and democratized decision-making. From an economic point of view, the terms $(c_i)^T \cdot x_i + \lambda^T \cdot (A_i \cdot x_i)$ represent the total cost minus revenue, which corresponds to the net loss of the stakeholder. Consequently, minimizing (12) is effectively equivalent to maximizing stakeholder profit. In the economy of the distributed system, λ acts as a market price and stakeholders collectively work to determine the optimal price for the efficient allocation and utilization of resources.

To obtain system-wide feasible solutions, violations of system-wide constraints are penalized and are included in the stakeholder subproblem formulation as

$$\underbrace{\min_{x_i}}_{\text{Independence}} \left\{ \overbrace{(c_i)^T}^{\text{Private cost}} \cdot x_i + \underbrace{\underbrace{\overbrace{\lambda^T}^{\text{Price signal}} \cdot (A_i \cdot x_i)}_{\text{Coordination}} + \rho \cdot \left\| \sum_{i'=1:i'\neq i}^{I} A_{i'}^{k-1} \cdot x_{i'}^{k-1} + A_i \cdot x_i - b \right\|}_{\text{Aggregation}} \right\}, \tag{13}$$

where the penalty norm can be either l_1 or l_2 depending on applications. Here λ^T, ρ and $\sum_{i'=1:i'\neq i}^{I} A_{i'}^{k-1} \cdot x_{i'}^{k-1}$ are quantities provided by the coordinator.

A concern might arise from adding the penalty term, as it modifies the stakeholders' objective function, potentially leading to the perception that stakeholders are not directly maximizing their revenue. However, if all stakeholders adopt this logic and opt out of participation, decision-making power reverts to a centralized coordinator. This contradicts the premise of this paper. Moreover, the centralized coordinator cannot optimize the overall cost without access to private stakeholders' private cost information. Constraint violations tend to be larger without penalties, requiring the coordinator to make extensive adjustments. This may not only increase stakeholder costs, but also reduce their revenue. By incorporating penalties, stakeholders collectively reduce constraint violations, allowing the coordinator to make minimal adjustments, which keeps costs lower and aligns with the principles of participatory, privacy-aware decision-making.

Within this decentralized framework, two situations can occur. First, if the constraint violation is zero, stakeholders achieve a globally feasible solution without the intervention of the coordinator. Second, if the constraint violation is small (owing the penalties) but non-zero, the coordinator intervenes by slightly perturbing stakeholder solutions to satisfy (9). This approach also ensures privacy, as the coordinator does not need access to detailed cost data when making adjustments to satisfy (9). The cost-effectiveness and robustness of this method will be empirically demonstrated in the "Numerical Testing" subsection, where the practical benefits of the model are highlighted.

Broader Perspectives and Implications. The proposed approach allows the inclusion of stakeholders to actively participate in the decision-making process (**participation**), fostering more community-based and tailored solutions that align with the unique needs of each stakeholder. This approach also enables stakeholders to contribute to the overall solving process by formulating and solving their subproblems rather than accepting centralized decision-maker solutions. In the latter case, participating stakeholders may find themselves in financial loss. In the former case, the overall solution is collectively obtained by all participants to maximize their income (or minimize the loss), thereby reducing the possibility of wholesale out-of-market payments, leading to improved **transparency**.

By allowing stakeholders to formulate and solve their subproblems, our PAPD method promotes fairness, accountability, and transparency, addressing potential power imbalances in centralized decision-making (**fairness**). **Accountability** is ensured because stakeholders independently solve their subproblems while explicitly aiming to satisfy system-wide constraints, while the coordinator enforces compliance through penalties for constraint violations.

Transparency, explainability, and interpretability are crucial yet distinct concepts. Although transparency focuses on accessibility and understanding the decision-making process, **explainability** involves stakeholders' ability to articulate how pricing signals (multipliers) influence decisions and the final solution. This allows stakeholders to explain their decisions to local communities, fostering trust and credibility in the decision-making process. **Interpretability** entails understanding the relationships between pricing signals (multipliers) and resource supply and demand, helping stakeholders make informed decisions.

Coordinating and integrating local solutions into a global solution ensures feasible and **near-optimal** global solution. The PAPD-SLR approach, grounded in economic theory, employs Lagrangian multipliers as "shadow prices" to discourage less economically viable decisions, ensuring fairness, equity, and **economic viability and efficiency**.

The **sensitivity** of Lagrangian multipliers within our PAPD approach reflects the rate of change in optimal costs relative to constraint changes [50] and provides high-level insights into adapting resource allocation in response to demand or supply shifts, thereby providing valuable insights for both local stakeholders and governments. This information helps stakeholders understand the effects of global constraints on local resource allocation and associated costs and supports coordinated decision-making in dynamic circumstances.

The PAPD approach offers **flexibility** and **adaptability** by enabling participants to join/leave the optimization process flexibly based on their needs and continuous multiplier adaptation to demand/supply changes. Distributed approaches distribute communication and coordination, enhancing decision-making efficiency.

3 Numerical Testing

In this section, we use two mixed-integer linear programming (MILP) problems to demonstrate the efficiency of the proposed approach. Example 3.1 considers the joint routing and charging (JRC) problem for heavy-duty electric vehicles. This example illustrates the increase in the participation truck revenue and the decrease in the make-whole payment (MWP). Example 3.2 considers large-scale instances of generalized assignment problems. Aspects such as the advantages over noncoordination-based and non-privacy-preserving methods are demonstrated, and our method generally achieves much smaller duality gaps, demonstrating higher solution quality. Both examples demonstrate that reducing participating trucks and machines significantly increases overall system cost. This underlines that creating an inclusive optimization platform is mutually beneficial for both participants and the system, with only marginal degradation of the solution quality compared to the best available results obtained by decomposition and coordination non-privacy preserving Lagrangian Relaxation methods.

3.1 EV Truck Joint Routing-Charging Problems

In this section, we consider the Electric Vehicle (EV) Heavy Duty Truck Joint Routing-Charging (JRC) problem [51] without considering tardiness as a test case for our proposed privacy-aware participatory-distributed optimization method. The JRC problem involves optimizing the routes and charging schedules for a fleet of heavy-duty EV trucks, considering constraints such as the limited range of EVs, the availability of charging stations, and the need to minimize total operational costs. The problem is combinatorially complex due to the interplay between routing and charging decisions, which must be coordinated to ensure efficient and timely delivery of goods while maintaining the charging levels of EV trucks. This problem is representative of real-world logistics and transportation applications, where global constraints (e.g., delivery deadlines and overall cost minimization) and local constraints (e.g., individual truck range and charging requirements) must be considered simultaneously. This approach allows us to demonstrate the effectiveness of our privacy-aware solutions, highlighting the benefits of stakeholder participation and the flexibility of the proposed method in integrating new participants during the optimization process. As demonstrated in [51],

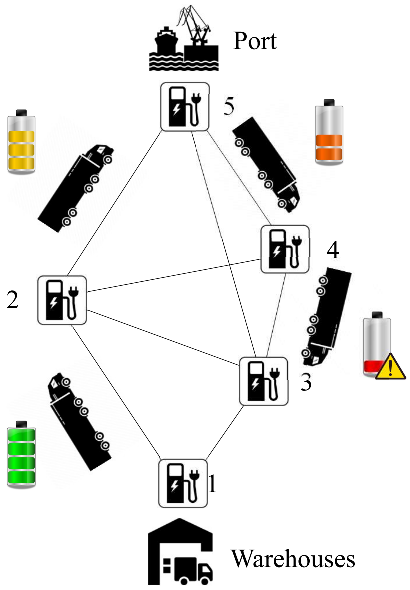


Fig. 1: A network schematic for Example 3.1.

non-decomposition-based methods such as Branch-and-Cut fail to obtain feasible solutions within a sufficiently long CPU time. The results will be compared with the decomposition-based centralized SLBLR method [51] to showcase the advantages of the proposed participatory-distributed approach. The network used is shown in Figure 1, and the results are presented in Table 1.

		PAPD-SLR					SLBLR				
Case/Number of Trucks	Lower Bound	Total Feas. Cost	Gap (%)	Time (s)	Total Revenue	Total MWP	Total Feas. Cost	Gap (%)	Time (s)	Total Revenue	Total MWP
9	26.07	26.48	1.55	631.7	13.23	0.00	26.43	1.39	444.5	9.50	0.25
10	24.26	24.51	1.03	1416.2	5.95	0.16	24.55	1.17	1410.8	5.84	0.22
11	23.15	23.75	2.53	788.8	9.15	0.00	23.52	1.56	557.7	2.92	0.25
12	22.20	22.67	2.08	888.3	7.58	0.00	22.67	2.08	948.3	7.60	0.00
13	22.40	22.75	1.57	1427.3	4.95	0.13	22.75	1.54	1480.5	4.90	0.13
14	22.06	22.78	3.15	1350.8	8.42	0.08	22.91	3.68	1286.0	1.72	1.31
	Average:	23.82	1.98	1083.8	8.21	0.06	23.80	1.90%	1021.3	5.41	0.36
	Improvement:	-0.09%	-0.08%	-6.12%	51.70%	82.78%					

Table 1: Comparison of PAPD-SLR and SLBLR

According to Table 1, as the number of participants/trucks increases from 9 to 10, the cost reduces from 26.48 to 24.51 (by 7.44%); as the number of participants increases from 10 to 11, the cost reduces from 24.51 to 23.75 (by 3.1%); and as the number of participants increases from 11 to 12, the cost reduces from 23.75 to 22.2 (by 4.55%). This underscores that creating an environment conducive to participation benefits the overall system. Further increases in the number of trucks do not lead to a reduction in cost. Another interesting feature is that while the non-privacy-aware SLBLR is generally faster, two issues occur: 1) feasible solutions are found before the multipliers have sufficiently converged, thereby impairing the revenue, and 2) a local search heuristic is needed to recover feasible solutions, which does not take into account whether each stakeholder will need to be compensated for the loss of profit, resulting in higher make-whole payments. In contrast, the PAPD-SLR is specifically designed for stakeholders to co-maximize their revenue while penalizing constraint violations, making it empirically a better option than local search heuristics.

The setup in the original paper [51] is suitable for a single company, where privacy is not a concern, and a central coordinator (the company) requires access to demand and supply data at depots (realistic), as well as data from ports and charging stations (less realistic). In contrast, our approach accommodates the coordination of multiple companies, including single-truck owners. Independent coordinators can operate at depots, ports, and charging stations, transmitting only Lagrangian multiplier values to participants, thereby preserving privacy and enabling broader participation.

3.2 Generalized Assignment Problems

In this Section, we use the Generalized Assignment Problem (GAP) to demonstrate the effectiveness of our privacy-aware method. This problem has the following desirable properties for empirical validation. First, the constraints of "global" job assignments and "local" machine capacity capture the high-level essence of many problems where global constraints couple local subsystems. Second, Lagrangian multipliers relax assignment constraints, which have interpretable economic meaning and price/reward for selecting a job, and each machine needs to decide what set of jobs to accept to maximize its profit. Third, after relaxing the global constraints, the problem can be distributed into smaller, independent subproblems. Fourth, it allows for reproducibility since GAP standard instances are well-studied and are readily available in OR libraries. The existing literature has extensively studied GAP, providing a solid baseline for comparison with existing methods. Many instances are well-solved to optimality using centralized methods, facilitating the comparison. Indeed, despite their simplicity, the GAP instances considered are combinatorially complex and can help demonstrate the advantages of the privacy-aware participatory distributed method compared to those of purely centralized ones (decomposition- as well as non-decomposition-based).

The generalized assignment problem assigns a set of jobs $\mathcal{J}$ to a set of machines $\mathcal{M}$ to minimize

$$\sum_{j \in \mathcal{J}} \sum_{m \in \mathcal{M}} c_{j,m} y_{j,m},$$

subject to:

1. Assignment constraints:

$$\sum_{m \in \mathcal{M}} y_{j,m} = 1, \quad \forall j \in \mathcal{J},$$

2. Capacity constraints:

$$\sum_{j \in \mathcal{J}} t_{j,m} y_{j,m} \leq T_m, \quad \forall m \in \mathcal{M}.$$

Here, $y_{j,m} \in \{0, 1\}$ are binary decision variables, $c_{j,m}$ is the cost of assigning job j to machine m, $t_{j,m}$ is the processing time of job j on machine m, and T_m is the capacity of machine m. In our method, the assignment constraints are relaxed and machine subproblems resulting from the Lagrangian decomposition exemplify the stakeholders' problems.

Example GAP d201600: Privacy-Aware Solutions and Flexible Participation. Consider a GAP instance with 20 machines and 1600 jobs—the third-largest GAP instance with the largest number of jobs (1600). Figure 2 shows that the PAPD-SLR method obtains results statistically similar in quality to those obtained by CPLEX, which requires machine cost and processing time information. Our privacy-aware method generates solutions without private cost information, assuming these costs *post factum* for evaluation and presentation purposes only. The privacy-aware feature of the method thus introduces variability in feasible costs, shown by the purple dots.

Examples GAP d401600 and d801600: Scalability and Effects of Drop Outs/Exclusion. Consider GAP instances with 40 and 80 machines— the largest GAP instances with 1600 jobs. In this example, optimal costs are used as a baseline [48], and both instances are solved to within 0.003% of the optimal costs by using a centralized implementation of SLBLR. The PAPD-SLR method is then compared with SLBLR. The CPLEX is then compared with the PAPD-SLR method. Finally, since participation is not guaranteed, we assume that 2.5% of participants (1 and 2, respectively) do not share information and do not participate. The resulting problems are solved using CPLEX. The results are shown in Figure 3.

The results presented in 3a for the case with 40 machines indicate that the degradation of the solution due to the privacy requirement is only 0.042%. In perspective, the quality of the non-privacy-aware CPLEX solution assuming complete participation is 0.129% lower (the corresponding gap is 0.174%) - the associated feasible cost is roughly four times farther from the optimal. The non-privacy-aware CPLEX's solution, assuming that one machine does not participate, is further degraded by 1.686% (the corresponding gap is 1.86%), which is more than ten times higher than the CPLEX's-obtained gap under the assumption of full participation. The results presented in 3b for the case with 80 machines indicate that the degradation of the solution due to the requirement of privacy is only 0.024%. The quality of the non-privacy-aware CPLEX solution, which requires complete participation, is 0.242% lower (the corresponding gap is 0.269%) - the associated feasible cost is roughly ten times farther from the optimal. The non-privacy-aware CPLEX's solution, assuming that two machines do not participate, is further degraded by 1.697% (the corresponding gap is 1.966%),

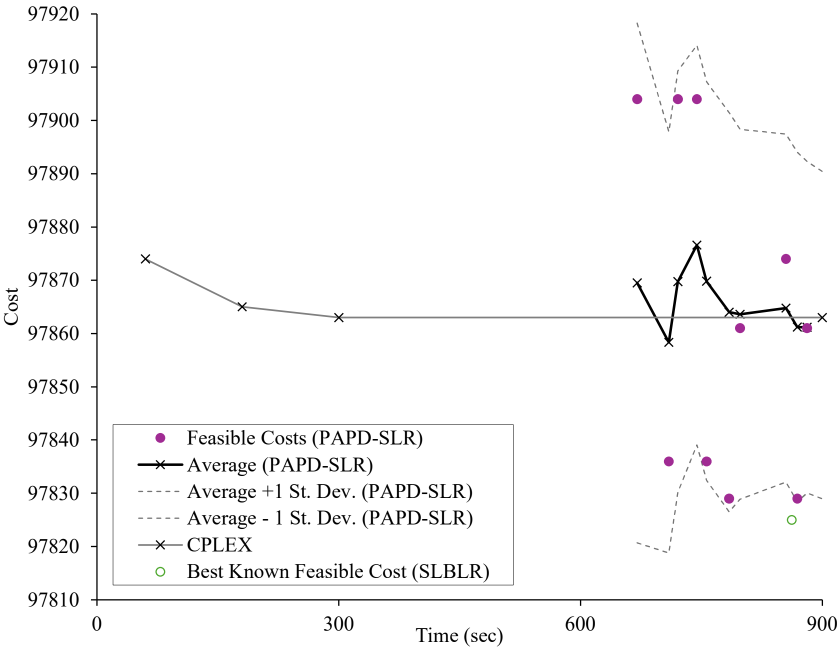


Fig. 2: Comparison of the PAPD-SLR with non-privacy-aware SLBLR and CPLEX.

which is roughly seven times higher than the CPLEX's-obtained gap assuming full participation. Following the results in Example 3.1, the consequences of the lack of participation are costly for the system, emphasizing the need and benefits of the participatory approach.

4 Conclusions

Democratization is an important concept with a long history. It encompasses more than passive acceptance of decisions made by a centralized authority and advocates the active participation and inclusion of all stakeholders in the decision-making process. This paper has explored democratization within the context of complex problem solving through a privacy-aware, participatory, and distributed optimization approach.

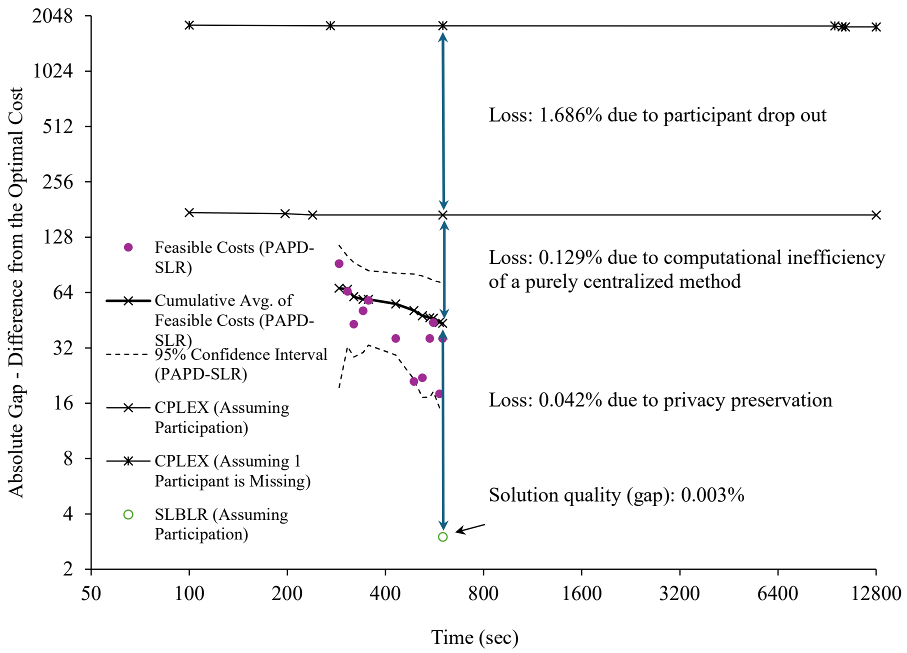
Traditional centralized methods often fail to adequately represent the diverse perspectives of stakeholders, leading to inefficiencies and potential biases. Our approach, leveraging Lagrangian Relaxation, addresses these limitations by enabling stakeholders to independently solve their sub-problems without disclosing private information. This ensures the preservation of privacy and efficient coordination, fostering active stakeholder participation.

Our numerical results demonstrate that privacy-aware solutions achieve quality comparable to traditional centralized methods, with only a minor decrease in solution quality. The substantial benefits of privacy protection and stakeholder inclusion outweigh this trade-off. The method has shown robustness and scalability, effectively handling large-scale problems and participant variability while maintaining solution quality.

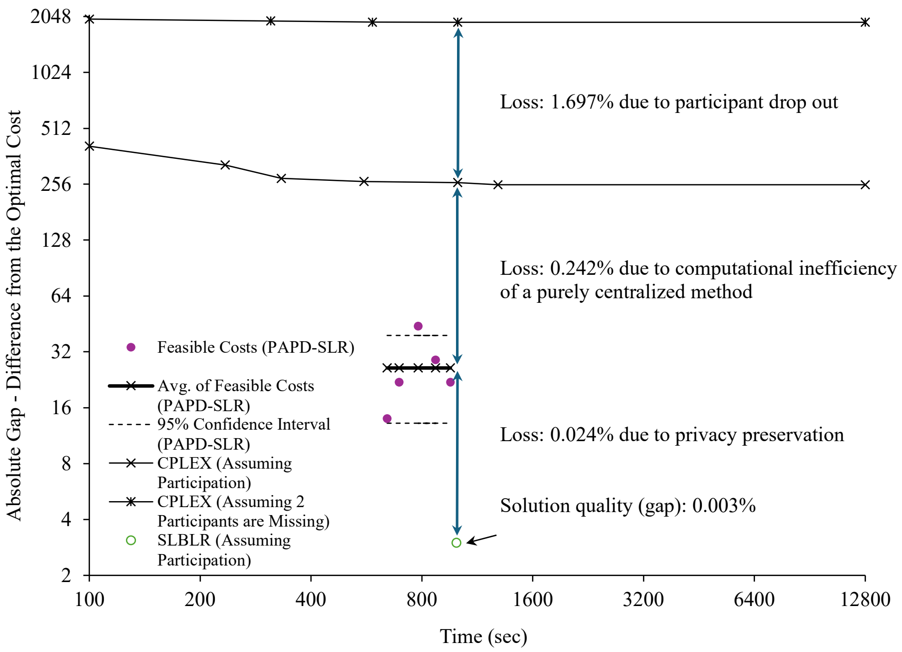
The findings highlight the transformative potential of democratizing problem-solving. By facilitating participatory, decentralized, privacy-preserving methodologies, we can achieve more equitable, efficient, and transparent outcomes that reflect all participants' diverse needs and contributions. This approach improves computational efficiency and ensures fairness and transparency, addressing potential power imbalances in decision-making.

While this study provides a foundational framework for enabling participation, future research must focus on further enhancing inclusivity and active engagement. In particular, developing mechanisms to incorporate incentives for participation is crucial. Incentives encourage stakeholders to contribute meaningfully and help sustain engagement over time, ensuring the long-term viability of participatory methodologies.

Additionally, the interplay of diverse participant strategies necessitates deeper investigation through the lens of game theory. Understanding how stakeholders might strategize, cooperate, or



(a) Results for the GAP instance d401600



(b) Results for the GAP instance d801600

Fig. 3: Comparison of PAPD-SLR with non-privacy aware SLBLR and Branch-and-Cut.

compete under varying conditions is critical to designing systems that are robust against potential manipulations or conflicts of interest. Game-theoretical analyses within Lagrangian coordination can provide insights into equilibrium behaviors, potential coalition formations, and the impact of incentive structures, ultimately guiding the design of fairer and more effective participatory frameworks.

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5 Appendix A: Formulation of EV Truck Joint Routing-Charging Problems

Description. Consider a transportation network with

1. A set of nodes $n \in \mathcal{N}$;
2. A set of road segments $r \in \mathcal{R}$ with each segment characterized by a "starting node" $s(r)$ and "ending node" $e(r)$;[2]
3. A set of heavy-duty electric vehicles (trucks) $v \in \mathcal{V}$;
4. A set of trips $t \in \mathcal{T} = \{1, \dots, T\}$ with each trip being a one-way trip either from a depot at node $n^{depot} \in \mathcal{N}^{depot} \subset \mathcal{N}$ to a port $n^{port} \in \mathcal{N}^{port}$ $(\subset \mathcal{N})$ or from n^{port} to n^{depot};
5. A set of nodes $n^{chrg} \in \mathcal{N}^{chrg}$ $(\subset \mathcal{N})$ containing charging stations;
6. A lookahead horizon $p \in \mathcal{P} = \{1, 2, \dots, P\}$ with P being the total number of time periods, and
7. Products $pr_n \in \mathcal{PR}$ that needs to be delivered either from depot to port $n = n^{port}$ or from port to depot $n = n^{depot}$.

The time required to travel through a road segment r is $T^{trvl}_{r,p}$, which may depend on the time of the day p as well as the direction traveled (e.g., for any two adjacent nodes n_1 and n_2, generally, $T^{trvl}_{(n_1,n_2),p} \neq T^{trvl}_{(n_2,n_1),p}$); the charging cost is $C_{p,n^{chrg}}$ per every period p at node n^{chrg}; and the labor driving cost per period is C^{lbr}.

The following assumptions are made:

1. Each truck v is designated to a certain depot n^{depot}_v and a certain port n^{port}_v.[3] Whether cargo needs to be delivered from a depot to a port or vice versa is determined through optimization as explained ahead;
2. Each truck is expected to be fully charged overnight, i.e., the initial charge is 100%. This assumption is not strict, since the methodology developed further will be able to handle any level of the initial charge, if feasible;
3. Each port is a node $n^{port} \in \mathcal{N}^{port}$ whereby cargo is loaded, and each port is equipped with a charging station;
4. Each depot is a node $n^{depot} \in \mathcal{N}^{depot}$ whereby the cargo is unloaded, and each depot is also equipped with a charging station;
5. Other nodes of the network $n \in \mathcal{N} \setminus \{\mathcal{N}^{port} \cup \mathcal{N}^{depot}\}$ may or may not contain a charging station;
6. Driving along a road segment r is *non-preemptive*, that is, once started driving at time p, the truck will arrive at the end of the road segment after $T^{trvl}_{r,p}$ periods;
7. Each truck v can take several trips (not necessarily T) during a scheduling horizon as long as the last trip ends at n^{depot}_v;
8. The increase in the truck battery's state of charge is a linear function of charging time;

The goal is to transport all goods between the depot and port nodes in both directions while minimizing total charging and labor costs.[4] To achieve this, the truck must determine the timing and selection of road segments, appropriate charging stations, and charging duration. Cargo collected from the port must be delivered to the depot. This process is repeated until the demand at both the depot and the port is met.

5.1 Constraints for Individual Trips

To represent trucks traveling and charging during a single-direction journey, multiple sets of binary decision variables are introduced:

1. To capture the beginning of a trip, let $x^{trvl,dprtr}_{v,n,p,t} = 1$ if truck v chooses to depart from node n at time p at trip t and $x^{trvl,dprtr}_{v,n,p,t} = 0$ otherwise;
2. The "arrival" binary decision variable $x^{trvl,arrvl}_{v,n,p,t}$ is similarly defined;
3. To capture the status of charging, let $x^{chrg}_{v,n^{chrg},p,t} = 1$ if truck v chooses to charge at node n^{chrg} during time period p and $x^{chrg}_{v,n^{chrg},p,t} = 0$ otherwise;

[2] Note that starting and ending nodes are not unique for a road segment since the road segments are generally bi-directional.
[3] The difference between n^{depot} and n^{depot}_v is that n^{depot} is a dummy index from a set $\mathcal{N}^{depot}$ whereas n^{depot}_v is a specific depot node where truck v is located.
[4] Unlike the original paper [51], tardiness is not considered in this study.

4. To capture beginning and completion of charging, let $x^{chrg,bgn}_{v,n^{chrg},p_1,t} = 1$ and $x^{chrg,cmplt}_{v,n^{chrg},p_2,t} = 1$ if truck v began charging during trip t at node n^{chrg} at time p_1 and ended charging at time p_2, respectively.

Truck Availability. For every truck v, the first trip $t = 1$ starts at depot $n = n^{depot}v$ after the truck becomes available at time av_v. To capture this, the binary variable $x^{trip}v, t$ is introduced:

$$x^{trip}v, 1 = 1 \Rightarrow dv, n^{depot}v, 1 \geq av_v, \quad \forall \left(v \in \mathcal{V}\right), \tag{14}$$

where $dv, n^{depot}_v, 1$ is the departure time.

Departure-Arrival Relationship. To ensure truck v can only depart from at most one node during each trip t, the following constraint is introduced:

$$\sum_{n \in \mathcal{N}} x^{trvl,dprtr}v, n, p, t \leq 1, \quad \forall \left(v \in \mathcal{V}, p \in \mathcal{P}, t \in \mathcal{T}\right). \tag{15}$$

Furthermore, the truck v can only depart once, requiring summation over periods:

$$\sum p \in \mathcal{P} x^{trvl,dprtr}v, n, p, t \leq 1, \quad \forall \left(v \in \mathcal{V}, n \in \mathcal{N}, t \in \mathcal{T}\right). \tag{16}$$

The corresponding departure time dv, n, t is determined by the following constraints:

$$\sum_{p \in \mathcal{P}} p \cdot x^{trvl,dprtr}_{v,n,p,t} = d_{v,n,t}, \forall \left(v \in \mathcal{V}, n \in \mathcal{N}, t \in \mathcal{T}\right). \tag{17}$$

The constraints (15)-(17) also apply to the binary arrival indicator $x^{trvl,arrvl}v, n, p, t$ and integer arrival time av, n, t decision variables, but are omitted for brevity.

Once truck v departs from node $s(r)$ (starting node of road segment r), it must arrive at one of the nodes $e(r)$ (ending node of road segment r) $T^{trvl}_{r,p}$ units of time later (due to non-preemptiveness), as captured by the following constraints:

$$x^{trvl,dprtr}_{v,n,p,t} = 1 \Rightarrow \sum_{r \in \mathcal{R}: s(r)=n} x^{trvl,arrvl}_{v,e(r),p+T^{trvl}_{r,p}-1,t} = 1, \forall \left(v \in \mathcal{V}, n \in \mathcal{N}, t \in \mathcal{T}, p \in \mathcal{P}\right). \tag{18}$$

This relationship is bidirectional: if truck v arrives at node n, it must have departed $T^{trvl}_{r,p}$ units of time ago from one of the nodes directly connected by road segments to node n:

$$x^{trvl,arrvl}_{v,n,p,t} = 1 \Rightarrow \sum_{r \in \mathcal{R}: e(r)=n} x^{trvl,dprtr}_{v,s(r),p-T^{trvl}_{r,p}+1,t} = 1, \forall \left(v \in \mathcal{V}, n \in \mathcal{N}, t \in \mathcal{T}, p \in \mathcal{P}\right). \tag{19}$$

In the above constraints, one unit of time is added or subtracted from the travel time due to the discrete nature of periods: departure is at the start of the period, while arrival is at the end. For example, if a truck departs at the beginning of time 3 and travels for 3 units of time, it arrives at 3 + 3 - 1 = 5, traveling during periods 3, 4, and 5.

Additionally, once truck v arrives at node n, it must depart from n in the future unless it reaches its destination $\in n^{port}_v \cup n^{depot}_v$.[5] The following constraints capture this relationship:

$$x^{trvl,arrvl}_{v,n,p,t} = 1 \Rightarrow \sum_{p' \in \mathcal{P}: p' \geq p+1} x^{trvl,dprtr}_{v,n,p,t} = 1, \forall \left(v \in \mathcal{V}, n \in \mathcal{N} \setminus \{n^{port}_v \cup n^{depot}_v\}, t \in \mathcal{T}, p \in \mathcal{P}\right). \tag{20}$$

Charging and Discharging. While traveling, truck v discharges at a rate of $\Delta s^{dchrg,ldd}\%$ per time period if loaded, and $\Delta s^{dchrg,empty}\%$ if empty. To capture these discharge rates, the binary decision variables $x^{ld}v, t$ indicate whether the truck v is loaded ($x^{ld}v, t = 1$) or empty ($x^{ld}_{v,t} = 0$) during the trip t. Using the 'departure' and 'arrival' decision variables, the state of charge becomes:

$$x^{trvl,dprtr}_{v,n,p,t} = 1 \ \wedge \ x^{trvl,arrvl}_{v,e(r),p+T^{trvl}_{r,p}-1,t} = 1 \ \wedge x^{ld}_{v,t} = 1 \Rightarrow$$

[5] After arriving at the destination, the truck may still depart, but this will occur during the next trip $t + 1$, discussed in subsection 5.2.

$$
\begin{aligned}
&s^{chrg}_{v,e(r),t} = s^{chrg}_{v,n,t} - \frac{\Delta s^{dchrg,ldd}_r}{100} \cdot T^{trvl}_{r,p} \cdot x^{trvl,arrvl}_{v,e(r),p+T^{trvl}_{r,p}-1,t}, \\
&\forall \left(v \in \mathcal{V}, n \in \mathcal{N}, r \in \mathcal{R} : s(r) = n, t \in \mathcal{T}, p \in \mathcal{P} : p + T^{trvl}_{r,p} - 1 \leq P\right),
\end{aligned}
\tag{21}
$$

where $s^{chrg}_{v,n,t} \in [0, 1]$ is a continuous decision variable representing the battery's state of charge for truck v at node n during trip t; $\wedge$ denotes the conjunction (logical AND). If truck v is empty during trip t, the constraint becomes:

$$
\begin{aligned}
&x^{trvl,dprtr}_{v,n,p,t} = 1 \ \wedge \ x^{trvl,arrvl}_{v,e(r),p+T^{trvl}_{r,p}-1,t} = 1 \ \wedge x^{ld}_{v,t} = 0 \Rightarrow \\
&s^{chrg}_{v,e(r),t} = s^{chrg}_{v,n,t} - \frac{\Delta s^{dchrg,empty}_r}{100} \cdot T^{trvl}_{r,p} \cdot x^{trvl,arrvl}_{v,e(r),p+T^{trvl}_{r,p}-1,t}, \\
&\forall \left(v \in \mathcal{V}, n \in \mathcal{N}, r \in \mathcal{R} : s(r) = n, t \in \mathcal{T}, p \in \mathcal{P} : p + T^{trvl}_{r,p} - 1 \leq P\right).
\end{aligned}
\tag{22}
$$

If the truck v arrives at the node n^{chrg} equipped with chargers, it can decide to charge. This decision is captured by the binary variable $x^{chrg}_{v,n^{chrg},p,t}$. Assuming $\Delta s^{chrg}\%$ is the charge rate per time period, (21) is modified as:

$$
\begin{aligned}
&x^{trvl,dprtr}_{v,n,p,t} = 1 \wedge x^{trvl,arrvl}_{v,e(r),p+T^{trvl}_{r,p}-1,t} = 1 \wedge x^{ld}_{v,t} = 1 \Rightarrow \\
&s^{chrg}_{v,e(r),t} = s^{chrg}_{v,n,t} - \frac{\Delta s^{dchrg,ldd}_r}{100} \cdot T^{trvl}_{r,p} \cdot x^{trvl,arrvl}_{v,e(r),p+T^{trvl}_{r,p}-1,t} + \sum_{p' \in \mathcal{P}} \frac{\Delta s^{chrg}_r}{100} \cdot x^{chrg}_{v,e(r),p',t}, \\
&\forall \left(v \in \mathcal{V}, n \in \mathcal{N}, r \in \mathcal{R} : s(r) = n, t \in \mathcal{T}, p \in \mathcal{P} : p + T^{trvl}_{r,p} - 1 \leq P\right).
\end{aligned}
\tag{23}
$$

In the above equation, $e(r)$ must be a member of the set $\mathcal{N}^{chrg}$. Equation (22) is modified similarly.

In (23), the amount of energy charged is determined by the number of periods used for charging. However, the appropriate charging time must be determined to ensure that the electric truck v can charge only if it has arrived and left the node n^{chrg}. Therefore, the following constraints are introduced:

$$
\sum_{p'=1}^{p-1} x^{trvl,arrvl}_{v,n^{chrg},p',t} \geq x^{chrg}_{v,n^{chrg},p,t}, \forall \left(v \in \mathcal{V}, n^{chrg} \in \mathcal{N}^{chrg}, t \in \mathcal{T}, p \in \mathcal{P}\right).
\tag{24}
$$

In the equation above, the upper limit of summation $p - 1$ is used due to the discrete nature of periods; if truck v arrives at the end of period $p - 1$, it can start charging at time p.

Charging will not be possible after departure, as captured by the following constraints:

$$
1 - \sum_{p'=1}^{p} x^{trvl,drptr}_{v,n^{chrg},p',t} \geq x^{chrg}_{v,n^{chrg},p,t}, \forall \left(v \in \mathcal{V}, n^{chrg} \in \mathcal{N}^{chrg}, t \in \mathcal{T}, p \in \mathcal{P}\right).
\tag{25}
$$

If truck v departs at time p, it is no longer eligible for charging starting at time p.

The beginning $b_{v,n^{chrg},t}$ and completion $c_{v,n^{chrg},t}$ times of charging are captured by introducing binary variables $x^{chrg,bgn}v, n^{chrg}, p, t$ and $x^{chrg,cmplt}v, n^{chrg}, p, t$, similar to how departure times are captured in (17). The same relations as in (15) and (16) apply to $x^{chrg,bgn}v, n^{chrg}, p, t$ and $x^{chrg,cmplt}v, n^{chrg}, p, t$. These binary variables are linked to $x^{chrg}_{v,n^{chrg},p,t}$ as follows:

$$
x^{chrg,bgn}_{v,n^{chrg},p,t} \geq x^{chrg}_{v,n^{chrg},p,t} - x^{chrg}_{v,n^{chrg},p-1,t}, \forall \left(v \in \mathcal{V}, n^{chrg} \in \mathcal{N}^{chrg}, t \in \mathcal{T}, p \in \mathcal{P} \setminus \{1\}\right),
\tag{26}
$$

$$
x^{chrg,cmplt}_{v,n^{chrg},p-1,t} \geq x^{chrg}_{v,n^{chrg},p-1,t} - x^{chrg}_{v,c,p,t}, \forall \left(v \in \mathcal{V}, n^{chrg} \in \mathcal{N}^{chrg}, t \in \mathcal{T}, p \in \mathcal{P} \setminus \{1\}\right).
\tag{27}
$$

5.2 Trip-Connecting Constraints

The number of trips is determined through optimization. To indicate whether a trip is taken, binary variables are introduced: $x^{trip}v, t = 1$ if truck v takes trip t, and $x^{trip}v, t = 0$ otherwise. If truck v is to take the next trip $t+1$ (decided at either a depot n_v^{depot} or a port n_v^{port}), the start time of trip $t+1$ will relate to the arrival time and charging completion time of the previous trip t as follows:

$$x_{v,t+1}^{trip} = 1 \ \wedge \ \sum_{p\in\mathcal{P}} x_{v,n,p,t}^{chrg} = 0 \Rightarrow d_{v,n,t+1} \geq a_{v,n,t} + 1, \forall \left(v \in \mathcal{V}, n \in \{n^{depot}, n^{port}\}, t \in \mathcal{T} \setminus \{T\}\right), \tag{28}$$

$$x_{v,t+1}^{trip} = 1 \ \wedge \ \sum_{p\in\mathcal{P}} x_{v,n,p,t}^{chrg} = 1 \Rightarrow d_{v,n,t+1} \geq c_{v,n,t} + 1, \forall \left(v \in \mathcal{V}, n \in \{n^{depot}, n^{port}\}, t \in \mathcal{T} \setminus \{T\}\right). \tag{29}$$

The above constraints ensure that if truck v is not charging (per (28)), the next trip $t+1$ can start immediately after the previous trip t ends. If the truck needs to charge (per (29)), the next trip $t+1$ can start immediately after charging is completed within the previous trip t.

Additionally, at a port ($n = n_v^{port}$), truck v must depart to return to a depot, regardless of whether it is loaded, as captured by the following constraints:

$$x_{v,t}^{trip} = 1 \ \wedge \ \sum_{p\in\mathcal{P}} x_{v,n,p,t}^{chrg} = 0 \Rightarrow d_{v,n,t+1} \geq a_{v,n,t} + 1, \forall \left(v \in \mathcal{V}, n = n_v^{port}, t \in \mathcal{T}\right), \tag{30}$$

$$x_{v,t}^{trip} = 1 \ \wedge \ \sum_{p\in\mathcal{P}} x_{v,n,p,t}^{chrg} = 1 \Rightarrow d_{v,n,t+1} \geq c_{v,n,t} + 1, \forall \left(v \in \mathcal{V}, n = n_v^{port}, t \in \mathcal{T}\right). \tag{31}$$

Since ports and depots are equipped with chargers, truck v can decide to charge. In the next trip, the charging levels are determined as:

$$x_{v,t+1}^{trip} = 1 \Rightarrow s_{v,n,t+1}^{chrg} = s_{v,n,t}^{chrg}, \forall \left(v \in \mathcal{V}, n \in \{n_v^{depot}, n_v^{port}\}, t \in \mathcal{T} \setminus \{T\}\right), \tag{32}$$

$$x_{v,t}^{trip} = 1 \Rightarrow s_{v,n,t+1}^{chrg} = s_{v,n,t}^{chrg}, \forall \left(v \in \mathcal{V}, n \in n_v^{port}, t \in \mathcal{T}\right). \tag{33}$$

Charging at nodes n^{depot}, n^{port} is already accounted for during trip t within (21)-(23).

To ensure a truck v is not loaded without being scheduled for a trip, we use the binary variables $x_{v,t}^{ld}$, as introduced in (21)-(23). To maintain logical consistency, the following constraint is introduced:

$$x^{ld}v,t \leq x^{trip}v,t, \quad \forall \left(v \in \mathcal{V}, t \in \mathcal{T}, pr \in \mathcal{PR}\right). \tag{34}$$

In addition to (21)-(23) and (34), $x_{v,t}^{ld}$ will be used to capture demand as explained in subsection 5.3.

To ensure the contiguity of trips, the following constraint is introduced:

$$x^{trip}v,t+1 \leq x^{trip}v,t, \quad \forall \left(v \in \mathcal{V}, t \in \mathcal{T} \setminus T\right). \tag{35}$$

5.3 Truck-Coupling Constraints

Demand Constraints. Loaded trucks unload cargo at the end of the trip. For inbound trips to the depot, the unloaded cargo $pr_{n^{depot}}$ satisfies the demand $D_{n^{depot},pr_{n^{depot}}}$ at depot n^{depot}. For outbound trips from the depot, the unloaded cargo $pr_{n^{port}}$ satisfies the demand $D_{n^{port},pr_{n^{port}}}$ at port n^{port}.

$$\sum_{t\in\mathcal{T}^{inbnd}\subset\mathcal{T}, v\in\Omega_{\left(pr_{n^{port}}, n_v^{depot}\right)}} x_{v,t}^{ld} = D_{n^{port}}, \forall (n^{port} \in \mathcal{N}^{port}, pr_{n^{port}} \in \mathcal{PR}), \tag{36}$$

$$\sum_{t\in\mathcal{T}^{otbnd}\subset\mathcal{T}, v\in\Omega_{\left(pr_{n^{depot}}, n^{port}\right)}} x_{v,t}^{ld} = D_{n^{depot}}, \forall (n^{depot} \in \mathcal{N}^{depot}, pr_{n^{depot}} \in \mathcal{PR}). \tag{37}$$

Here, $\mathcal{T}^{inbnd}$ represents inbound trips (e.g., trips with odd numbers), and $\mathcal{T}^{otbnd}$ represents outbound trips.

In (36), the summation is over trucks $\Omega_{\left(pr_{n^{port}}, n^{depot}v\right)}$ eligible to deliver product prn^{port} from $n^{depot}v$ to port n^{port} to meet demand Dn^{port}, provided truck v is loaded ($x^{ld}_{v,t} = 1$).

In (37), the summation is over trucks $\Omega_{\left(pr_{n^{depot}}, n^{port}\right)}$ eligible to deliver product $pr_{n^{depot}}$ from n^{port} to depot n^{depot}[6] to meet demand Dn^{depot}, provided truck v is loaded ($x^{ld}_{v,t} = 1$).

Charging Station Capacity Constraints. To prevent more trucks from charging simultaneously at node n^{chrg} than there are chargers $C_{n^{chrg}}$, the following 'charging station capacity' constraint is introduced:

$$\sum_{v \in \mathcal{V}, t \in \mathcal{T}} x^{chrg} v, n^{chrg}, p, t \leq Cn^{chrg}, \quad \forall (n^{chrg} \in \mathcal{N}^{chrg}, p \in \mathcal{P}). \tag{38}$$

In (38), the summation is over trips and trucks, as a given time p may correspond to different trips for different trucks.

5.4 Objective Function.

The objective function is to minimize the total labor and charging costs as follows:

$$\min \left\{ \sum_{v \in \mathcal{V}} O_v(d, \overline{a}, x) \right\} = \min \left\{ \sum_{v \in \mathcal{V}} C^{lbr} \cdot \left(\overline{a}_{v, n_v^{depot}} - d_{v, n_v^{depot}, 1} \right) + \sum_{v \in \mathcal{V}, n^{chrg} \in \mathcal{N}^{chrg}, t \in \mathcal{T}, p \in \mathcal{P}} C^{chrg}_{p, n^{chrg}} \cdot x^{ch}_{v, n^{chrg}, p, t} \right\}. \tag{39}$$

Here $\overline{a}_{v, n_v^{depot}}$ is introduced to capture the latest arrival time of vehicle v at the depot n_v^{depot} as:

$$\overline{a}_{v, n_v^{depot}} \geq a_{v, n_v^{depot}, t}, \forall (t \in \mathcal{T}). \tag{40}$$

The term $\left(\overline{a} v, nv^{depot} - d_{v, n_v^{depot}, 1} \right)$ represents the total time a driver spends on the road, calculated as the difference between the completion of the last trip and the departure of the first trip. The total labor and charge costs are expressed as the sum $\sum_{v \in \mathcal{V}} O_v(d, \overline{a}, x)$ for all trucks v.

[6] Here, n^{depot} must equal n_v^{depot} by assumption 1 stated at the beginning of the section.