SENSITIVITY-BASED DECISION SUPPORT FOR CRITICAL MEASURES USING THE EXAMPLE OF COVID-19 DYNAMICS

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Abstract. We parametrize public policies in the context of the COVID-19 pandemic to evaluate the effectiveness of policies through sensitivity-based methods in order to offer insights into understanding the contributions to critical measures in retrospective. The study utilizes a group-specific SEIR model with a tracing and isolation strategy and vaccination programs. Public policies are applied to minimize death tolls, mitigate the social and economic effects caused by infections and avoid the overburden of the health system. We propose derivative-based sensitivity analysis to evaluate the priorities of different strategies. As apposed to purely scenario-based approaches, the proposed method only uses qualitative properties of the underlying mathematical model. Combined with compartment models the strategy permits to assess the relative significance of policies under the typically large uncertainties. The study carries out experiments under past situations with Delta and Omicron variants in Germany. These experiments confirm a positive influence of tracing apps as earlier observed in simulation-based case studies as well as the importance of booster programs, especially for the elderly. Insights and methods gained from this study may provide support for decision-making processes in future public health crises and can be advanced to assessing criticality of measures for other societal challenges.

1. Introduction

In March 2020, the new coronavirus was declared a pandemic by WHO because of its rapid spread throughout the world. During the pandemic years, the German government has locked down to varying degrees several times, which came at huge social and economic costs. As the country's statistic agency reported, the German economy fell into a recession in 2020 after ten years of growth. Statistics showed a gross domestic product (GDP) reduction of 4,9% compared to 2019. Measured against GDP at current prices, the deficit ratio for 2020 is 4.2% [7]. The inflation rate increased still by 3.1% on average in 2021 compared to 2020 [13]. Furthermore, the social damage was non-negligible regarding the impacts of lower population growth, unemployment and a greater risk of psychological disorders due to fewer social connections and an increase in despair and anxiety[4].

The unprecedented global health crisis caused by the COVID-19 outbreak has prompted policymakers at that time to implement various strategies to control the spread of virus and reduce the social and economic costs. Making rational decisions entails incorporating the available scientific evidence, which is often derived from expert opinions and modelling studies[2]. Other than the contributions from medical aspects, mathematical models are increasingly being used to better understand illness transmission patterns and to simulate the effectiveness of public policies in mitigating the adverse impacts of the pandemic. Mathematical studies have the

ability to provide numerical insights and guidance for informed decision-making and play the role of a systematic and rigorous approach facing complex challenges. Quantifying and measuring various public policy factors can provide a preliminary understanding for policy formulation, improving the efficiency of public policies and reducing the expenditures in the implementation. However, many public issues, such as the COVID-19 problem, are facing large uncertainties and rapidly evolving changes[2]. Reviewing past pandemic, it is crucial to find an appropriate way with sufficient robustness in order to provide information in decision-making processes.

In the context of COVID-19, it was questionable how to strike a balance between the cost and effectiveness of different strategies, in a way that does not overburden the health system. Non-pharmaceutical intervention methods such as social distancing and putting infectious patients without complications into home quarantine are critical to control the pandemic. In the technological environment, it is reasonable to use the "tracing and isolation strategy" to decrease expenses. The use of tracing apps can effectively break the transmission chain when the contact persons can be put into quarantine quickly enough. The vaccination program is also an important strategy to prevent infections, while it has been implemented since 2021. Given the weakening of the vaccine efficacy over time, the government also considers the booster program as an addition. Facing the possible wave caused by new variants, it is interesting to investigate the impacts of each strategy in different situations influenced by Delta and Omicron variants, respectively.

Many mathematical modelling studies have been conducted, trying to simulate and analyze the situation, see e.g. [8, 12, 15, 17, 28, 34, 33, 16, 10, 5, 25, 27]. Most of the models extended from the classic SIR and SEIR models. Some of them lay the focus on the social distancing policy and found it to be an effective method to control the disease [28, 25]. Age-structure in the transmission of the disease is also an interesting topic in the studies [8, 15, 27]. Some studies have investigated the basic reproduction number R_0 and the sensitivity of fixed parameters regarding R_0 [12, 33]. Various researchers have taken advantage of control theory and studied the optimal control of the intervention. Some studies have utilized statistical and machine learning methods to predict the situation[10, 5]. Many studies have also extended the original SIR or SEIR models to include the implications of the vaccine policy [25, 22, 31, 29, 32, 27, 21].

This work combines the mathematical epidemiological models with a derivative-based sensitivity analysis to assess the impacts of different public strategies in controlling the epidemic under the situations of different variants. The model used in this work is based on the models in the work of Grimm et al. [8]. This model utilized different groups with various infection severity and risks, which leads to diverse parameters in asymptomatic and death rates. Also, the model parameterizes the compliance with tracing apps to share the private location information and includes a quarantine process, which makes the model more complex and more complete for the situation than many others. There are also various studies about contact tracing apps [14], most of which used empirical and stochastic methods along with individual-based message-passing models. Those models only cover part of the aspects which are mentioned in the model of Grimm et al.

For our study, the model by Grimm et al. is further extended in several aspects. First, the model allows discussions about the vaccination and booster programs. It

is obvious that people with and without antibodies differ in the possibilities to get infected or have severe complications. To fulfill these particular purposes, the model is extended into more detailed compartments. In addition, the model parameterizes different strategies into controllable arguments, including the *non-pharmaceutical interventions* (NPIs), the compliance of tracing apps, triage mechanisms in hospitalization and the acceptance of vaccinations and boosters. The group-specific model offers the opportunity to choose varying parameters in different groups. The controllability of parameters corresponds to the implementation of various policies to suppress the pandemic. Furthermore, a functional is defined to model the costs of the pandemic which needs to be minimized. The minimization of the cost functional aims to minimize the death toll, reduce spending on the treatment and testing of infections, lessen impacts of lock-downs and quarantines as well as avoid the overburden of the health system.

Such simple compartment models have been criticized for their limitations in quantitative predictions [30]. The accuracy has a high dependence on the preciseness of parameters. This leads to the unreliability of the results, since the predictions are not absolute. However, our work focuses on the qualitative effects of the controllable factors using derivative-based sensitivities, which is a replenishment of other studies. The combination of compartment models with sensitivity analysis offers an analytically supported evaluation of the strategies, which is precise enough for policy making. Local sensitivities are implemented using a fourth-order Runge-Kutta discretization method (RK4) and the techinques of *internal numerical differentiation* (IND). Globally, the model generates the so-called *derivative-based global sensitivity measures* (DGSM) in a certain threshold with the trapezoidal rule.

The model and methods are used in this paper to analyze past situations under the Delta and Omicron variants. They are applicable in different future scenarios to provide assessment of public policies in public health crises and they can be adapted to assess measures for other societal challenges as well.

The rest of the paper is organized as follows. Section 2 presents the model and methods used in the experiments. Section 3 discusses the parameterization of the experiments and illustrates the results of experiments. Section 4 presents the conclusion.

2. Models and Methods

The SEIR model is one of the most commonly employed models in the epidemiological studies [11]. This system uses different compartments to distinguish people in different infectious periods: susceptible (S), exposed (E), infectious (I) and recovered (R). Each compartment may be considered as a function of time $(S(t), E(t), I(t), R(t))$. The following nonlinear ordinary differential equations describe the dynamics of the transitions between compartments:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\beta IS \tag{2.1a}$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \beta IS - \varepsilon E \tag{2.1b}$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \varepsilon E - \gamma I \tag{2.1c}$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \gamma I \tag{2.1d}$$

along with the initial conditions

$$S(0) = s_0 \geq 0, \quad E(0) = e_0 \geq 0, \quad I(0) = i_0 \geq 0, \quad R(0) = r_0 \geq 0.$$

In this system, $S(t), E(t), I(t)$ and $R(t)$ are the susceptible, exposed, infectious and recovered fraction in the whole population at time t, respectively. Thus, it holds that $S(t)+E(t)+I(t)+R(t) = 1$ and $(S(t)+E(t)+I(t)+R(t))N = N$ with N being the total population size. β describes the average number of adequate contacts, which are sufficient for the infection of a person per unit time. Thus, βISN is the amount of new cases per unit time. $1/\varepsilon$ and $1/\gamma$ denote the average incubation and infectious period. The basic reproduction number R_0 is defined as the average number of secondary infections that may exist after one infection is introduced into the susceptible host group[11], which can be represented as $R_0 = \beta/\gamma$.

2.1. **Extension of the Model.** The model being used in the following is primarily based on the models presented in [8]. Since the outbreak of COVID-19 pandemic, there have been many researches discussing different measures including non-pharmaceutical intervention methods and the use of tracing apps. The model in [8] combined different strategies in a mathematical compartment model. It was based on the basic SEIR model and was extended to analyze the effects of group-specific interventions as well as the impact of tracing apps and quarantine policies. Furthermore, the model allows us to distinguish between different severities of infectiousness and takes the ICU capacity of the health system into consideration and provides the chance to study the development of pandemics in different scenarios.

Our model inherits the "tracing and isolation" strategy and the categories of different severity of infections from the model in [8]. The infection chains can be broken using tracing app and quarantine policy in the pre-infectious period. It is assumed that a fraction of individuals in the whole population uses the tracing app effectively[1]. When both of the infectious and susceptible sides have the tracing app, the infection chain can be broken, and the susceptible side can be quarantined throughout the latent period until he or she is recovered or dead. The traced individuals who are exposed are placed in home quarantine during the incubation period or, if they are extremely infectious, later in collective isolation, which is denoted as Group Q_k. All individuals who are not traced enter compartment I_k when they become infectious. The ratio of people in group k who uses the tracing app efficiently is denoted as ψ_k. The compartments of infectious individuals (both traced and not traced) are divided into three parts: asymptomatic, symptomatic and severely infectious people. The fraction of asymptomatic individuals among all infectious people in each group is denoted with $\eta_k = \frac{I_k^{asym}}{I_k} = \frac{Q_k^{asym}}{Q_k} \in (0,1)$. Hence, $1-\eta_k$ of the infectious people exhibit symptoms. Among those symptomatic infected, ν_k of them are severely infectious or even have complications. These individuals might have more symptoms and respiratory diseases such as pneumonia and bronchitis and need to be taken to the hospitals and receive intensive medical care. The fractions of each particular compartment also differ between groups k. The average infectious duration is denoted with $\frac{1}{\gamma}$. The model also divides the recovered compartment in classic SIR models into compartments R and D, which

[1]Group-specific rations of people using the tracing apps might be interesting to look at and easily realized by modifying the model.

Symbol	Description
K	Number of groups
S_k	Sesceptible individuals in group k
V_k	Individuals in group k with complete vaccinations
B_k	Individuals in group k with booster dose
$E_k^{nt,s}$	Exposed individuals without tracing apps in k - susceptible
$E_k^{nt,v}$	Exposed individuals without tracing apps in k - vaccinated
$E_k^{nt,b}$	Exposed individuals without tracing apps in k - boostered
$E_k^{tr,s}$	Exposed individuals with tracing apps in k - susceptible
$E_k^{tr,v}$	Exposed individuals with tracing apps in k - vaccinated
$E_k^{tr,b}$	Exposed individuals with tracing apps in k - boostered
I_k^{asym}	Asymptomatic infectious people in group k
I_k^{sym}	Symptomatic infectious people in group k
I_k^{sev}	Severely infectious people in group k
Q_k^{asym}	Asymptomatic infectious people in quarantine in group k
Q_k^{sym}	Symptomatic infectious people in quarantine in group k
Q_k^{sev}	Severely infectious people in quarantine in group k
R_k	Recovered individuals with immunity in group k
D_k	Dead individuals in group k

TABLE 1. Notations of compartments

describe the recovered individuals and dead individuals during the infection. (Note that the group of recovered people in SIR models also includes deaths.)

The model is then extended in different aspects: First, different groups $k \in \{1, \dots, K\}$ are considered in this model. The number of individuals is denoted with N_k. Then all the compartments S, E, I, R are split up into the compartments of different groups, e.g., S_k, E_k, I_k, R_k. The total sum of all the compartments is still 1. The various groups can distinguish for example people of different ages, people with less or more risk to be infected, people with different rates of vaccinations, or people who are less or more compliant with the NPIs. Transmissions can occur both within groups or across groups with different rates. It is assumed in our model that the basic infection rate is fixed as β. The actual transmission rates are modified by the intervention rate $u_k \in [0, 1]$. The intervention rates combine the effects of NPIs from the aspect of the policy and the willingness to stay at home of each group. Here, $u_k = 1$ means no intervention, and $u_k = 0$ corresponds to the extreme case of total isolation of the whole group k. The contact across two groups is mainly determined by the group with a lower intervention rate since this group is more inclined to stay at home and to reduce public contacts. In general, the actual infectious rates across groups are denoted with multiplications βu_{ij}, where $u_{ij} := \min\{u_i, u_j\}$.

Second, it is assumed that only severely infectious people may not survive at different lethality rates when they receive the hospitalization care or not. The fraction of deaths among critically ill patients is denoted with σ_k. This parameter is determined by the mortality rate of the virus both with ICU and without ICU, as well as the rationing mechanisms of people who receive intensive health cares and the ICU capacities in the hospitals. Denoting the lethality rate of a critical patient with ICU in group k as $\hat{\sigma}_k$, we assume that the mortality without ICU is

$2\hat{\sigma}_k$. The percentage of seriously infected patients who are admitted to hospitals is denoted with α_k in possibly different hospitalization triage methods. Thus, the total death rate of group k is defined as

$$\sigma_k = \begin{cases} \hat{\sigma}_k\alpha_k + 2\hat{\sigma}_k(1-\alpha_k) & \alpha_k(I_k+Q_k)^{sev}N_k < \frac{N_k}{N}B \\ \frac{\hat{\sigma}_k B_k + 2\hat{\sigma}_k((I_k^{sev}+Q_k^{sev})N_k - B_k)}{(I_k^{sev}+Q_k^{sev})N_k} & \alpha_k(I_k+Q_k)^{sev}N_k \geq \frac{N_k}{N}B \end{cases} \tag{2.2}$$

with $B_k := \frac{N_k}{N}B$. Denote the severely infectious people in group k with $C_k(t) := (I_k^{sev}(t) + Q_k^{sev}(t))N_k$ and the number of ICU beds for group k with $B_k := \frac{N_k}{N}B$, the death rate in (2.2) can be written as

$$\sigma_k(t) = \begin{cases} \hat{\sigma}_k\alpha_k + 2\hat{\sigma}_k(1-\alpha_k), & \alpha_k C_k(t) < B_k \\ \frac{\hat{\sigma}_k B_k + 2\hat{\sigma}_k(C_k(t)-B_k)}{C_k(t)}, & C_k(t) \geq B_k. \end{cases}$$

If $\alpha_k = 1$, then it can be further simplified to

$$\sigma_k(t) = \begin{cases} \hat{\sigma}_k, & C_k < B_k \\ 2\hat{\sigma}_k - \hat{\sigma}_k\frac{B_k}{C_k(t)}, & C_k \geq B_k. \end{cases}$$

Remark. *The death rate function is non-smooth when $C_k(t) = B_k$. In the model, the differentiations of R_k and D_k are linear with the death rate σ_k. Thus, the solutions of R_k and D_k also have the non-smoothness at the point $\alpha_k C_k(t) = B_k$, which might lead to inaccuracies in the numerical solution of the ODE system. To resolve this, we use the right derivative at the point which is non-zero. In practice, the non-smooth state is quite rare to be reached.*

Furthermore, the model introduces vaccination and booster programs $(V_k(t), B_k(t))$ according to past situations. A proportion a_k of susceptible people will choose to be vaccinated and build resilience in the body against the coronavirus after getting the complete vaccines. The time of immunization after complete vaccines is denoted as $\frac{1}{\gamma_v}$. However, vaccine protection can diminish over time when antibodies from vaccination are gradually metabolized in months. For the elderly, this situation is quite common, and the time when antibodies stay is much shorter. Hence, the booster vaccination program is carried out to enhance the immune system. In the extended model, people are encouraged to take the booster vaccination after at least three months ($\frac{1}{\gamma_b}$). The fraction of vaccinated individuals who are willing to take the booster dose is denoted as b_k. Hence, a vaccination campaign can reduce the risk of being infected and the severity of being infectious. Meanwhile, the protection may also protect infected people against severe, critical or even fatal diseases. The compartments E_k^{nt} and E_k^{tr} are then split into groups denoted with s-susceptible without vaccination, v-vaccinated and b-vaccinated with booster dose. The fractions of asymptomatic individuals η_k and severe infectious individuals ν_k also vary for people with or without vaccinations. It is assumed that vaccinated or recovered individuals lose the immunity during the time $\frac{1}{\tilde{\gamma}}$. Those individuals will enter the susceptible compartment after the stage of vaccine protection.

Moreover, we aim to model suppressing the pandemic by minimizing the death toll, avoid ICU overflow, and at the same time reduce the economic and social costs for (light-) lockdowns and quarantines. Thus, the motivated cost functional

Symbol	Description
N_k	Number of individuals in the group k
β_k^s	Infection rates of individuals without immunity
β_k^v	Infection rates of vaccinated individuals
β_k^b	Infection rates of boostered individuals
η_k^s	Fraction of asymptomatic individuals - susceptible
η_k^v	Fraction of asymptomatic individuals - vaccinated
η_k^b	Fraction of asymptomatic individuals - boostered
ν_k^s	Fraction of severely infectious individuals - susceptible
ν_k^v	Fraction of severely infectious individuals - vaccinated
ν_k^b	Fraction of severely infectious individuals - boostered
$\hat{\sigma}_k$	Lethality rate
$1/\varepsilon$	Average incubation time
$1/\gamma$	Infection duration
$1/\gamma_v$	Duration for immunization after normal vaccinations
$1/\gamma_b$	Duration between normal vaccines and booster
$1/\tilde{\gamma}$	Duration to lose the immunity
B	Number of ICU beds
	Controllable parameters
a_k	Fraction of susceptible individuals who get vaccinated
b_k	Fraction of vaccinated individuals who get boostered
u_k	1: no intervention in group k
	0: no public activities in group k
ψ_k	Rate of tracing app users in group k
α_k	Fraction of severely ill patiants sent to hospitals in group k

TABLE 2. Notations of all parameters in the model (2.4).

is defined as

$$J(z_0,p)=\sum_{k=1}^{K}D_k(T)+\frac{1}{T}\int_0^T\left(k_1\sum_{k=1}^{K}(I_k^{asym}+Q_k^{asym})\right.\\ \left.+k_2\sum_{k=1}^{K}(I_k^{sym}+Q_k^{sym}+I_k^{sev}+Q_k^{sev})\right)\mathrm{d}t. \tag{2.3}$$

The cost consists of two parts: the total death toll and the running cost for the infections, which include not only the medical expenses, but also the economic and societal impacts. It is assumed that the asymptomatic and symptomatic infections lead to various weights in the cost functional.

The full dynamics of the ODE system reads

$$\frac{\mathrm{d}S_k}{\mathrm{d}t}=\tilde{\gamma}(R_k+V_k+B_k)-a_k\gamma_v S_k-\beta_k^s\left(\sum_{j=1}^{K}u_{jk}I_j^{sum}\right)S_k \tag{2.4a}$$

$$\frac{\mathrm{d}V_k}{\mathrm{d}t}=a_k\gamma_v S_k-\beta_k^v\left(\sum_{j=1}^{K}u_{jk}I_j^{sum}\right)V_k-b_k\gamma_b V_k-\tilde{\gamma}V_k \tag{2.4b}$$

$$\frac{\mathrm{d}B_k}{\mathrm{d}t} = b_k\gamma_b V_k - \beta_k^b \left(\sum_{j=1}^{K} u_{jk} I_j^{sum}\right) B_k - \tilde{\gamma} B_k \tag{2.4c}$$

$$\begin{aligned}\frac{\mathrm{d}E_k^{nt,m}}{\mathrm{d}t} &= \beta_k^m \left(\sum_{j=1}^{K} u_{jk} I_j^{asym}\right) M_k - \varepsilon E_k^{nt,m} \\ &\quad + \beta_k^m \left(\sum_{j=1}^{K} u_{jk}(1-\psi_j\psi_k)(I_j^{sym} + I_j^{sev})\right) M_k, m = s, v, b\end{aligned} \tag{2.4d}$$

$$\frac{\mathrm{d}E_k^{tr,m}}{\mathrm{d}t} = \beta_k^m \left(\sum_{j=1}^{K} u_{jk}\psi_j\psi_k(I_j^{sym} + I_j^{sev})\right) M_k, m = s, v, b \tag{2.4e}$$

$$\frac{\mathrm{d}I_k^{asym}}{\mathrm{d}t} = \sum_{m=s,v,b} \eta_k^m \varepsilon_k E_k^{nt,m} - \gamma I_k^{asym} \tag{2.4f}$$

$$\frac{\mathrm{d}I_k^{sym}}{\mathrm{d}t} = \sum_{m=s,v,b} (1-\eta_k^m)(1-\nu_k^m)\varepsilon_k E_k^{nt,m} - \gamma I_k^{sym} \tag{2.4g}$$

$$\frac{\mathrm{d}I_k^{sev}}{\mathrm{d}t} = \sum_{m=s,v,b} (1-\eta_k^m)\nu_k^m \varepsilon_k E_k^{nt,m} - \gamma I_k^{sev} \tag{2.4h}$$

$$\frac{\mathrm{d}Q_k^{asym}}{\mathrm{d}t} = \sum_{m=s,v,b} \eta_k^m \varepsilon_k E_k^{tr,m} - \gamma Q_k^{asym} \tag{2.4i}$$

$$\frac{\mathrm{d}Q_k^{sym}}{\mathrm{d}t} = \sum_{m=s,v,b} (1-\eta_k^m)(1-\nu_k^m)\varepsilon_k E_k^{tr,m} - \gamma Q_k^{sym} \tag{2.4j}$$

$$\frac{\mathrm{d}Q_k^{sev}}{\mathrm{d}t} = \sum_{m=s,v,b} (1-\eta_k^m)\nu_k^m \varepsilon_k E_k^{tr,m} - \gamma Q_k^{sev} \tag{2.4k}$$

$$\begin{aligned}\frac{\mathrm{d}R_k}{\mathrm{d}t} &= \gamma(I_k^{asym} + I_k^{sym} + (1-\sigma_k(t)I_k^{sev}) - \tilde{\gamma} R_k \\ &\quad + \gamma(Q_k^{asym} + Q_k^{sym} + (1-\sigma_k(t)Q_k^{sev})\end{aligned} \tag{2.4l}$$

$$\frac{\mathrm{d}D_k}{\mathrm{d}t} = \sigma_k(t)\gamma(I_k^{sev} + Q_k^{sev}), \tag{2.4m}$$

and is illustrated in Figure 1. All notations used in this model are listed in Table 1 and Table 2.

2.2. **Methods.** To study how the impact of each controllable parameters in Table 2 can result in the uncertainty in the cost functional (2.3), we conducted a sensitivity analysis of the variables with respect to the cost. The sensitivity analysis simplifies the model and reduces it into a qualitative study of the robustness. It identifies significant factors which contribute the most to the objective function. As the classical SIR and SEIR models are often criticized for its inability to quantify the uncertainty in the predictions [30], the sensitivity analysis is compatible here for a qualitative statement. The combination of a compartment model with the sensitivity analysis is sufficiently precise to give useful information for the public policy.

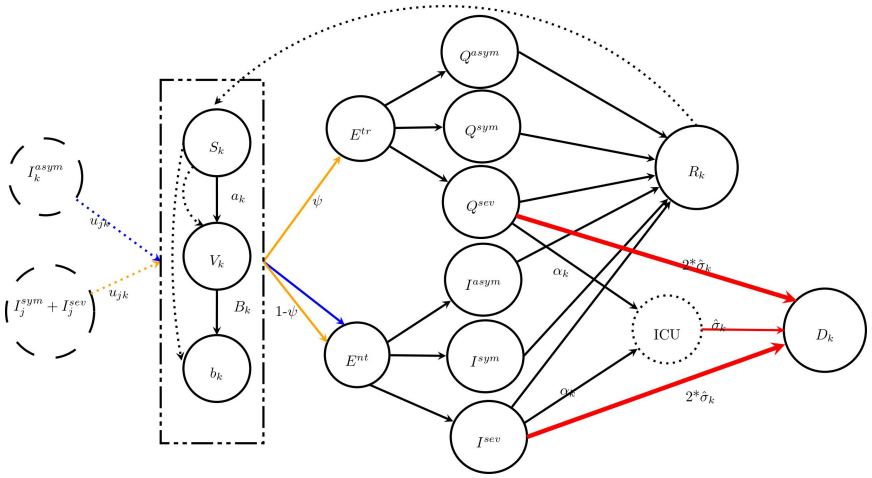

FIGURE 1. Illustration of the transmission model of group k in the model (2.4).

For the ordinary differential equation system (2.4), the solutions were approximated iteratively with the Runge Kutta 4th order method [9] and a suitable discretization in time. The principle of internal numerical differentiation [3], adds the calculation of derivatives into the approximation of solutions in differential equations. The derivatives then present sensitivities of each variable along with each parameter in the equation system. Compared to variational methods, this principle applies the idea of "first-integrate-then-differentiate" and compute the value of both the trajectory x_i and the sensitivity $S_i = \frac{\partial x_i}{\partial(x_0,p)}$ simultaneously with given temporal grids.

$$x'(t) = f(x(t),t) \overset{\text{integrate}}{\Longrightarrow} x_i = x_{i-1} + h_i f(x_{i-1}, t_{i-1})$$
$$\overset{\text{differentiate}}{\Longrightarrow} S_i = S_{i-1} + h_i \frac{\partial f}{\partial x} S_{i-1}.$$

Considering the discretizations x_i of the solution x as iteratively defined functions of the initial value x_0 and the parameters p, the sensitivity S_i w.r.t the arguments (x_0, p) can be derived with $\frac{\partial f}{\partial x}$. For (2.4), these derivatives were calculated explicitly with the analytical form of gradients of the right-hand-sides in (2.4).

The global sensitivity measure employed in this article averages local derivatives using Monte Carlo or Quasi Monte Carlo sampling methods [20]. Consider the cost function $J(x, \mathbf{p})$ with $\mathbf{p} = (p_1, \cdots, p_M) \in \Theta \subseteq \mathbb{R}^M$ a vector of the parameters in the model. Local sensitivity of the parameters is presented as an approximation of partial derivatives $\frac{\partial J}{\partial p_i}$. The local sensitivity varies when $\mathbf{p}$ changes. In the concept of derivative-based sensitivity measures, $p_1, \cdots, p_n$ are considered as independent random variables defined in each domain. The cumulative density functions are noted as $F_1(p_1), \cdots, F_n(p_n)$. The derivative-based global sensitivity measure used

in the sequel is defined as

$$\omega_i := \int_\Theta \left(\frac{\partial J(x,\mathbf{p})}{\partial p_i}\right)^2 \mathrm{d}F(\mathbf{p}) = \mathbb{E}_\Theta\left[\left(\frac{\partial J(x,\mathbf{p})}{\partial p_i}\right)^2\right] \tag{2.5}$$

where $F(\mathbf{p})$ is the joint distribution function (cdf) of $\mathbf{p}$.

The most applied methods for global sensitivity make use of the local information, but the result does not rely much on the choice of a nominal point. The local sensitivities can also be used to estimate the contribution of each parameter and highlight the unimportant ones. Then, using a "factors fixing setting", certain parameters may be fixed at a specific point and ignored in the global analysis. Direct realization of the global measures for models with high-dimensional $\mathbf{p}$ can be very time-consuming and inefficient. Thus, the different triage mechanisms in hospitals will not be considered in the analysis of global sensitivities, as they were not applied in retrospective. In addition, they have a relatively low impact on the cost and the sensitivities of other parameters.

3. Experiments and Results

The following case study is based on the German population of about 83 million people [6]. Our results function as an extension and a supplement to the study in [8] and provides a sensitivity analysis of the effectiveness of different policies and approaches to control the epidemic mainly caused by the Delta variant and Omicron variant now in retrospective.

3.1. **Parameterization.** In the case study, the group number K was set to 3. The entire population is divided into groups of people under the age of 18, people aged 19 to 60 and people over 60 years old. These categories also vary in the susceptibility against virus and proportions of getting vaccinated, which are represented by the numbers 1 (aged under 18), 2 (aged 19 to 60) and 3 (aged over 60). The initial values of the experiments about the infection cases and vaccination rates for Delta and Omicron variants are mainly estimated from the reports from Robert-Koch-Institute at the beginning of November and at the end of December in 2021, respectively [18, 19].

The parameters from the top part of Table 2 are fixed in the model since they are affected by the virus's basic characteristics. Note that some parameters for Omicrons are approximated from the Delta variant since the research for Omicron was still ongoing at that time being. The reproduction number R was estimated between 3.0 and 3.5 for Delta variant [24] and around 3.7 for Omicron variant [1]. The basic reproduction number R_0 was calculated between 3 and 8 for Delta variants [23, 24]. Thus, we assume a basic reproduction number of 5.08 for the Delta variant [23] and 7 for the Omicron variant. Based on the basic reproduction numbers, the infectious rates β's are fixed as 0.36 for Delta and 0.5 for Omicron for people without immunity. The basic infectious rates for vaccinated and boostered people are approximated from the case numbers and the attack rate, in reality, [26]. The asymptomatic rates and severe infectious rates are also estimated from the real case numbers [26, 18] as in Table 3.

The latent period is estimated to be four days for Delta [24] and three days for Omicron from the Centers for Disease Control and Prevention. The typical infection (from infectious and until recovered) lasts 14 days on average, which is

Variant	Delta			Omicron		
Age	<19	19–60	>60	<19	19–60	>60
β^s	0.36	0.36	0.36	0.5	0.5	0.5
β^v	0.022	0.087	0.095	0.14	0.14	0.142
β^b	0.005	0.02	0.025	0.01	0.0106	0.016
η^s	0.4	0.3	0.08	0.4	0.3	0.08
η^v	0.2	0.2	0.08	0.3	0.3	0.08
η^b	0.2	0.2	0.08	0.3	0.3	0.08
ν^s	0.0058	0.033	0.218	0.0041	0.026	0.1963
ν^v	0.0047	0.013	0.116	0.0051	0.0116	0.0887
ν^b	0.003	0.005	0.05	0	0.005	0.04
$\hat{\sigma}$	0.003	0.014	0.16	0.003	0.014	0.16
$1/\varepsilon$	4			3		
$1/\gamma$	14			14		
$1/\gamma_v$	14			14		
$1/\gamma_b$	90			90		
$1/\tilde{\gamma}$	180			180		
B	24000			24000		

TABLE 3. Parametriztion of fixed parameters in the model (2.4).

also estimated as the period for a vaccinated individual to get complete immunity ($1/\gamma_v$). Booster vaccination can be taken at least three months after the second dose of the vaccine. It is assumed that all the individuals who have taken the vaccines or recovered from the infection will lose their immunity against the virus after six months and might be infected again.

The bottom part of Table 2 lists the parameters whose sensitivities are studied. Since more variants may appear in three to six months, it is meaningful to analyze the situation during three months. The constants in the cost function are chosen as $k_1 = 0.1, k_2 = 1$, since more financial and material resources are needed for confirmed and critically ill patients.

All the experiments employ the fourth-order Runge Kutta method and IND technique. The local sensitivities are taken as their absolute value. The non-smoothness in the death rate is possible to influence the consistency and accuracy of the computational results. Thus, an adaptive step size is set to avoid the point of non-smoothness, when this point is exactly reached. However, this situation has never been seen in any experiment. The multidimensional integration in derivative-based global sensitivity measure is calculated with the trapezoidal rule implemented in *The MathWorks MATLAB*.

3.2. **Local Sensitivity Analysis.**

3.2.1. *Benchmark Scenarios.* In the scenario for local sensitivity, it is assumed that a uniform social distancing strategy is taken, and people who haven't taken the vaccine or the booster are reluctant to get it. However, a high amount of people use the tracing apps properly. Children and elder people have a higher possibility to get hospitalizations when they are severely infected. As kindergardens and schools are closed only when the pandemic hits Group 1, the children are more inclined to have a lower rate in the social activities and in the usage of tracing apps. The benchmark

Parameters	Value	Sensitivity Delta	Sensitivity Omicron
a_1	0.1	0.000874641	0.003850389
a_2	0.2	0.000390293	0.001678988
a_3	0.2	0.000345878	0.001673189
b_1	0.02	1.86213E-05	0.000104434
b_2	0.02	0.000181992	0.000734436
b_3	0.06	0.00184786	0.006424272
u_1	0.6	0.001916091	0.007131778
u_2	0.8	0.000636341	0.002862005
u_3	0.8	0.000458237	0.001211584
ψ_1	0.4	0.001207402	0.003999195
ψ_2	1	0.002978279	0.007682046
ψ_3	0.8	0.001855098	0.00380382
α_1	1	8.85904E-08	3.12262E-07
α_2	0.8	4.79465E-06	1.4748E-05
α_3	1	0.000137452	2.75204E-05

TABLE 4. Benchmark scenario

scenario simulates an actual realistic situation when few people have gotten the booster shot at the beginning of a booster program. The parameterization and local sensitivity of all parameters are given in Table 4.

It is obvious that all the parameters have a higher local sensitivity in the Omicron situation, except the proportion of elderly with fatal illness who can get hospitalizations. Besides, by both variants, the local sensitivities of parameters a_1, b_3, u_1, ψ_2 are larger than the others. It corresponds to a higher importance of the vaccination rate and social activity intervention in children and young teenagers, the booster rate in elder people and the usage of tracing apps in young adults.

3.2.2. *Scenarios with different parameterization.* This subsection shows the results under different policy strategies. Four different scenarios are considered: VAC, BOOST, NPI and TR. VAC represents the situation where people who haven't got the basic vaccination are more inclined to take it, while BOOST represents the case when the rate of people who take the booster dose is higher. In the NPI scenario, we assume fewer non-pharmaceutical interventions, while fewer individuals take advantage of the tracing apps and stay in quarantine when they are warned in the TR scenario. The values of parameters are listed in 5.

The local sensitivities of each parameter are shown in Figure 2 and Figure 3 respectively for Delta and Omicron cases.

In the VAC and BOOST scenarios, almost all parameters a, b, u, ψ have larger values of local sensitivities than in the benchmark scenario for both variants. The results are the opposite in the NPI and TR scenarios when less intervention participates. These lead to a conclusion that all the strategies a, b, u, ψ tend to be more important when the pandemic is more serious. As in the benchmark scenario, the application of tracing apps and quarantine strategy is still significant for all the three groups from the local sensitivity. Besides this strategy, the booster program in Group 3 and the intervention in the activities of children and young teenagers also play a crucial role in affecting the costs. Also, local sensitivities of parameters

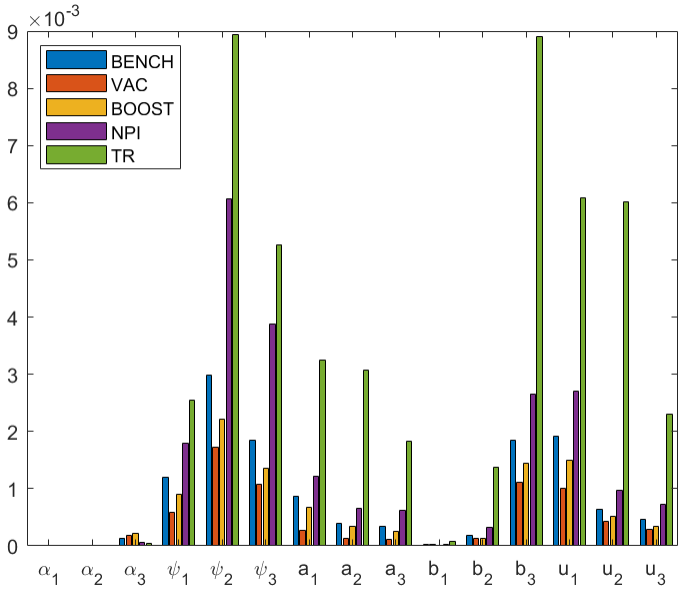

FIGURE 2. Local sensitivities by Delta-variant.

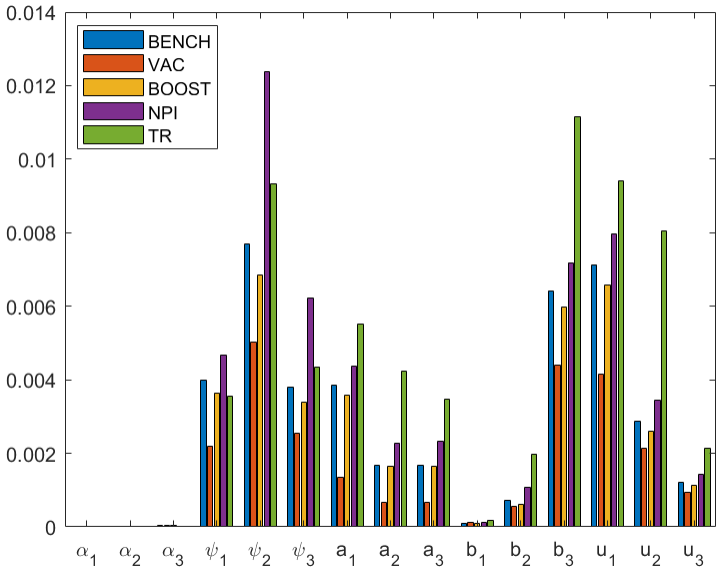

FIGURE 3. Local sensitivities by Omicron-variant.

Parameters	Bench	VAC	BOOST	NPI	TR
a_1	0.1	0.4	0.1	0.1	0.1
a_2	0.2	0.5	0.2	0.2	0.2
a_3	0.2	0.5	0.2	0.2	0.2
b_1	0.02	0.02	0.1	0.02	0.02
b_2	0.02	0.02	0.3	0.02	0.02
b_3	0.06	0.06	0.3	0.06	0.06
u_1	0.6	0.6	0.6	0.6	0.6
u_2	0.8	0.8	0.8	1	0.8
u_3	0.8	0.8	0.8	1	0.8
ψ_1	0.4	0.4	0.4	0.4	0.4
ψ_2	1	1	1	1	0.6
ψ_3	0.8	0.8	0.8	0.8	0.6
α_1	1	1	1	1	1
α_2	0.8	0.8	0.8	0.8	0.8
α_3	1	1	1	1	1

TABLE 5. Parametrization in different scenarios

a, b, u, ψ in all three groups are larger under the Omicron conditions than the Delta, which implies the influence of more actions facing the Omicron variant.

3.3. **Global Sensitivity Analysis.** In this subsection, we will take a look at the global sensitivity of the parameters. Table 6 presents the domains of the parameters. All parameters are uniformly distributed in the domain.

Parameters	Range
a_1	[0.2, 0.6]
a_2	[0.2, 0.8]
a_3	[0.2, 0.8]
b_1	[0.02, 0.1]
b_2	[0.02, 0.3]
b_3	[0.06, 0.5]
u_1	[0.6, 1]
u_2	[0.6, 1]
u_3	[0.6, 1]
ψ_1	[0.4, 0.6]
ψ_2	[0.6, 1]
ψ_3	[0.6, 1]

TABLE 6. Range of parameters for the global sensitivity anyalsis.

The vaccination rates vary from 0.2 to a value less than 1 since it is impossible to force all the people to get the vaccine. The booster rates have different domains due to the reason that the vaccination program in young teenagers begins much later than in older people. Thus the booster program has also been affected by the time difference. The admissible rates for NPI strategies are larger than 0.6 because complete isolation of the population is unrealistic.

Figure 4 shows the global sensitivity of the parameters.

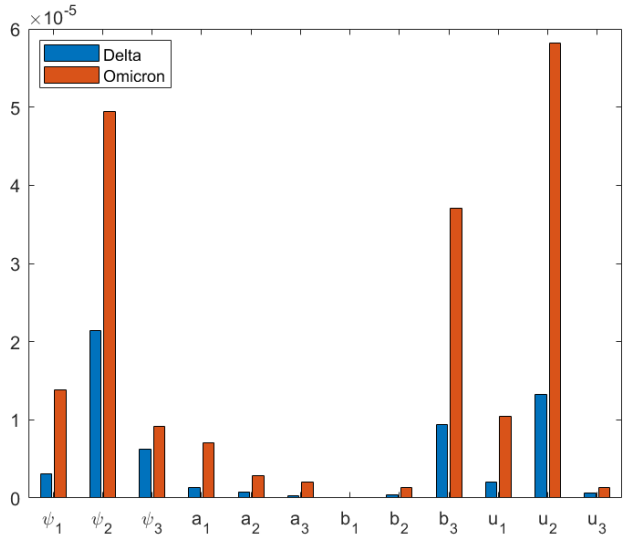

FIGURE 4. Derivative-based global sensitivity measures

The global sensitivity measures of ψ's can be seen obviously in the chart, especially for the young adults. This observation is consistent with the conclusion in [8] that contact tracing with a high compliance rate is effective to control the pandemic. Other than this, b_3 and u_2 (booster for elderly and NPI in the adults) also possess larger global sensitivities than other parameters by both variants.

When comparing the results in the contexts of different variants, it can be seen that the global sensitivities of all parameters in Table 6 are greater in the situation of the Omicron variant than the Delta. This again emphasizes the importance of the intervention strategies under Omicron conditions, not only in vaccination programs but also in social activities. Besides the factors that have been mentioned in both cases, a_1 and u_1 (vaccination and NPI in children) can cause impacts that are not to be neglected facing the Omicron variant.

4. Conclusion

The purpose of this study was to assess the sensitivity of controllable parameters regarding different strategies in a COVID-19 model based on [8] under the impacts of Delta and Omicron variants, respectively, and to get insights on how public policies can have an impact on mitigating the spread of the pandemic. It aims at evaluating the efficiency of anti-COVID policies in decision-making processes and providing inspiration for future policy formulation by incorporating scientific support.

The model was extended with vaccination and booster programs to simulate the situation. Individuals with antibodies are less inclined to have severe symptoms or need the intensive health care. Varying group-specific strategies were parameterized as controllable arguments, including the NPI interventions such as social distancing restriction to reduce close contacts, usage of tracing apps and isolation policy, compliance of taking vaccines and boosters as well as triage mechanism in

hospitalizations. We defined a cost functional to reduce the casualties and minimize the impact of infections from the economic and societal aspects.

Concerning the criticisms of compartment models like SIR and SEIR models on failing to provide absolute predictions, we used sensitivity analysis in this work to give a mathematically supported qualitative evaluation of public policies. The combination of compartment models with sensitivity analysis is sufficiently accurate to assess significance of the polices and strategies. We conducted experiments on past circumstances of Delta and Omicron variants using the given model and methods. The model and methods in this study may be of assistance to also analyze later variants or other highly contagious diseases.

The main results of the experiments under the situations of Delta and Omicron variants are as follows:

(1) Locally and globally, all the strategies a, b, ψ, u show larger sensitivities by Omicron variant than Delta.
(2) The compliance of tracing apps, the intervention in social activities in adults ψ_2, u_2 as well as the booster rate in older people b_3 show greater contributions in J than other parameters under conditions of both variants.
(3) Besides the factors in (2), the compliance of tracing apps in all 3 groups b's, the vaccination program and the NPI strategy in children and teenagers a_1, u_1 should not be neglected especially facing the Omicron variant.
(4) Effects of hospitalization rate in the young group α_1, α_2 are negligible. Contributions of α_3 are small but still detectable.

Results 2 and 3 support the conclusion in [8] that mild social distancing in combination with a "tracing and isolation" approach is a promising way to control the epidemic in the medium run. Furthermore, public policies play a more significant role in the context of the Omicron variant. To defeat COVID, it confirms significants to introduce boosters especially to the elderly.

Future work may be dedicated to investigate other sensitivity measures, to combine this with techniques from uncertainty quantification and to extend these techniques to more detailed models for example based on agent-type models and homogenization techniques.

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