A Robust Approach to Food Aid Supply Chains

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One of the great challenges in reaching zero hunger is to secure the availability of sufficient nourishment in the worst of times such as humanitarian emergencies. Food aid operations during a humanitarian emergency are typically subject to a high level of uncertainty. In this paper, we develop a novel robust optimization model for food aid operations during a humanitarian emergency, where we include uncertainty in the procurement prices, which is one of the primary sources of uncertainty in practice. Due to the multi-period and dynamic nature of food aid operations, we extend this robust optimization model to an adaptive robust optimization model, in which part of the decisions is taken after some of the uncertainty has been revealed. Moreover, we analyse a folding horizon approach for the nominal, robust, and adaptive robust optimization models in which decisions can be altered in later time periods. We compare the different approaches based on a food operations case in Syria. We show that the (adaptive) robust optimization approach outperforms the nominal approach in the non-folding horizon case, while the nominal approach performs best in the folding horizon case. Consequently, in case decisions have to be made early on, we show that applying robust optimization to food aid operations can make a difference. However, in case small adaptations can be made to the decisions taken in later time periods, then food aid operations can use a relatively simple approach in practice and apply a folding horizon approach each month to optimize decisions.

Key words: OR in developing countries, Supply chain, humanitarian logistics, optimization, adjustable robust optimization

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1. Introduction

According to FAO et al. (2022) between 702 and 828 million people in the world suffered from hunger in 2021. In addition, they state that about 3.1 billion people in the world experience nutrient deficiencies. This means that more than one in three people in the world do not have access to adequate food and healthy diets on a regular basis. Consequently, the United Nations (UN) adopted zero hunger as its second Sustainable Development Goal, making an urgent call for action by all countries in a global partnership to fight hunger.

One of the great challenges in reaching zero hunger is securing the availability of sufficient nourishment in the worst of times, namely during humanitarian emergencies. It is during these humanitarian crises, like for instance the civil war in Syria, where food shortages are very common. Unfortunately, these humanitarian emergencies are often long-lasting, ensuring protracted food aid is necessary. To guarantee adequate food and healthy diets for beneficiaries caught in humanitarian emergencies, humanitarian organizations like the United Nations' World Food Programme (WFP) are providing food aid and food assistance to those in need. Food aid consists of procuring, transporting and delivering food to the beneficiaries, whereas food assistance takes food aid a step further, by helping to (re)build and support communities to become self-sufficient in the future.

Humanitarian organizations providing food aid and assistance, often operate on restricted budgets. Consequently, these organizations are not always able to secure and distribute adequate food to all in need. Saving only one dollar a day already provides an adequate daily meal for two more people, see Peters et al. (2021), emphasizing efficiency in food operations is key.

Literature survey

The focus of this paper is on food aid operations within the Humanitarian Supply Chain (HSC). Generally, within HSCs the decisions to be made include the transfer modality selection, the food basket design, the sourcing and procurement plan, and the routing and delivery plan. The *transfer modality selection* details what type of modality to use; food in kind or direct food assistance. With food in kind assistance, the organization procures, transports and distributes all commodities, while direct food assistance is done through cash-based or voucher-based assistance, i.e., through cash or vouchers, which can be redeemed at retailers, for fixed quantities of specific commodities. The search for *food basket designs* started during World War 2, in which Cornfield formulated the "Diet Problem" in order to find a diet or food basket satisfying the nutritional needs of a soldier at minimum cost. It was only from 2000 onward, with the rise of calculating capacities of computers and consequently the development of linear programming tools, solutions for both linear as nonlinear large-scale diet problems where extensively published (see e.g., Briend et al. (2003), Chastre et al. (2007), Ryan et al. (2014), Seljak (2006)). The *sourcing and procurement plan* details what and how much of a commodity is procured at which supplier. According to Falasca and Zobel (2011), the sourcing and procurement plan constitutes approximately 65% of the expenses of humanitarian operations. Despite an increase in studies on this topic in recent years (see e.g., Ozpolat et al. (2015), Schiffling and Hughes (2017)), this rise is limited considering the eminent role of the sourcing and procurement plan in HSC (Moshtari et al., 2021). The *routing and delivery* *plan* specifies by what means (e.g., trucks, boats, planes) and via which route the food is delivered to the beneficiaries. Routing and delivery plans have been studied extensively in humanitarian operations (see e.g., Balcik et al. (2008), Ozdamar and Demir (2012), Rancourt et al. (2015), Anuar et al. (2021), Hu and Dong (2019)).

Peters et al. (2021) and Peters et al. (2022) are the first to integrate all these decisions in food aid operations into one mathematical model. They construct a model that simultaneously optimizes a capacitated, multi-commodity, multi-period network flow problem, accounting for the sourcing, procurement, routing and delivery plan, and a diet problem, in which the transfer modality selection is integrated. The inclusion of food basket selection adds a substantial amount of flexibility to the model in comparison to previous models within this subject. By not specifying a subset of the available commodities that make up a food basket, but only the nutrients required for a sufficient daily ration, more beneficiaries receive food aid than with traditional supply chain models. This innovative approach saw great results when implemented in real-life humanitarian supply chains of the WFP, saving millions in operating costs.

Humanitarian organizations often operate in chaotic environments, which are subject to unpredictability and uncertainty (see Sigala et al. (2020)). The lack of knowledge of demand for food aid, procurement prices (especially regionally), and delays in harbors, lead to high uncertainties during humanitarian emergencies and thus make the HSC a complex problem. There are various approaches dealing with optimization under uncertainty, including stochastic programming and robust optimization. In stochastic programming (see for example Ruszczyński and Shapiro (2003)) it is assumed that the underlying probability distribution is known. However, due to the lack of data quality and quantity in humanitarian supply chains, full knowledge about the probability distribution is hard to obtain. Robust optimization (e.g. Ben-Tal et al. (2009)) does not require any knowledge of the underlying probability distribution. Instead, it assumes the uncertain parameters reside within a so-called uncertainty set and requires the constraints to be hard constraints, i.e., constraints should hold for all possible realizations of the uncertain parameters lying within this uncertainty set. Even with incomplete or limited information on the uncertain parameters, these uncertainty sets are relatively easy to determine. Moreover, robust optimization models often remain computationally tractable. Robust optimization therefore seems the appropriate approach in HSC problems and has already been applied to HSC problems before (e.g. Ben-Tal et al. (2011), Balcik and Yanıkoğlu (2020), Stienen et al. (2021)).

Robust optimization originates from the seventies, by the work of Soyster (1973). However, it was not until the late nineties (Ben-Tal and Nemirovski, 1998, 1999; El Ghaoui and Lebret, 1997; El Ghaoui et al., 1998) that the interest in the field of robust optimization was sparked. In robust optimization, the decision variables have to be determined before any of the uncertainty is realized, i.e., the decision variables are "here-and-now". However, humanitarian supply chains are often multi-period and dynamic, hence it may be better to consider also "wait-and-see" variables, i.e., variables that can be decided on after part of the uncertainty has been realized. An extension to robust optimization is adaptive robust optimization, first introduced by Ben-Tal et al. (2004), in which part of the decisions can be made in a later stage, after more information on the uncertain parameter values is known.

The model of Peters et al. (2021) does not capture the uncertainty present in HSCs. However, they argue that their current solutions are not robust against uncertainties in the procurement prices. Hence, in this paper we expand a nominal version of the model of Peters et al. (2021) to two robust optimization models and two adaptive robust optimization models to capture the uncertainty in procurement prices. Moreover, when decisions can be updated periodically, we analyse a Folding Horizon approach for the nominal, robust, and adaptive robust optimization models. This means that at the beginning of the planning period, HSCs estimate the future amount of commodities needed to fulfill the demand for a given number of future periods. The supplier of the commodities then develops an agreement where a reservation is placed for future periods against the costs that apply in those periods and the HSCs can only deviate a certain percentage from the reservation in future periods. For example, in the first time-period a reservation is made for 1 metric ton of beans to be used in the third time-period where we can at most deviate 10% from this reservation. In the third time-period the costs for beans are much higher than expected, and we thus alter our reservation to the minimum possible (0.9 metric ton). In this way, the supplier can make the necessary preparations for supplying the commodities to the HSCs and at the same time it enables the HSCs to deviate somewhat from the commitments in case of changes in prices. Ben-Tal et al. (2005) were the first to demonstrate the benefits of implementing robust optimization to solve flexible commitment contracts. As more knowledge about the uncertain parameters becomes available over time, decisions can be revised in subsequent time periods based on the agreement made in the first time period.

The approaches are tested on a simplified real-life food aid operation in Syria, for which we have historical data available on the procurement prices.

Contributions

The main contributions of this paper are threefold.

First of all, we develop two novel robust optimization models for food aid operations in order to include uncertainties in procurement prices, which is generally present within humanitarian supply chains. We extend these robust optimization models to two adaptive robust optimization models. A part of the decisions within food aid can be taken after some of the uncertainty has been revealed, and to this end, an adaptive robust optimization approach closely resembles how real-life operations work.

Secondly, we compare the nominal, robust and adaptive robust approaches on a simplified real-life case of a food aid operation in Syria. As WFP is currently using the nominal model of Peters et al. (2022) in all their operations, it is important for WFP to know if using (adaptive) robust optimization can be beneficial to their operations. We show that the adaptive robust optimization approach does not differ much from the nominal approach applied in a folding horizon way. Consequently, in this paper we show that HSCs could use a relatively simple approach in practice and apply a folding horizon approach each month to optimize decisions. However, in case reservations of commodities cannot be altered in subsequent time periods, we show that applying robust optimization to humanitarian supply chains can make a difference.

Finally, we offer this simplified real-life case of a food aid operation in Syria and the corresponding data to be used for educational and/or research purposes.

2. Humanitarian supply chain model

We consider a basic version of the humanitarian supply chain model as described in Peters et al. (2021) as our nominal model without any of the uncertainties included. The objective of the model is fulfilling the *demand* of food aid, which can be seen as the nutritional needs of all beneficiaries together, at minimum cost. This demand is thus based on the number of beneficiaries in the operation, the number of days the aid is needed, and the daily nutritional requirements an average beneficiary needs. The model simultaneously optimizes the sourcing and procurement plan, routing plan, food basket design, and transfer modality selection over a pre-defined period of time.

The food in kind consists of commodities which can be procured at various *suppliers*. The set of suppliers are divided into *international suppliers* and *regional suppliers*. International suppliers are suppliers of food commodities from outside the country facing the emergency, whereas regional suppliers are suppliers within the borders of the country at risk. The flow of commodities is from suppliers, possibly via different transshipment points, to beneficiaries at the final delivery points. The *food basket design* entails the combination of food commodities satisfying the nutritional needs of an average beneficiary per day. This part of the problem is integrated into the network flow model by defining a ration variable that governs the commodities flowing into a delivery point. In contrast to the manufacturing industry, where the fulfilled demand is variable and the end-product is fixed, in this model all beneficiaries receive a food basket, i.e., the fulfilled demand is fixed, and the end-product is variable. This means that in case there is a funding shortfall, instead of supplying the full food basket to fewer beneficiaries, it supplies a less nutritious food basket to all beneficiaries. The model developed in this paper includes both transfer modalities (food in kind and voucher-based assistance) by introducing *local markets*, which are modeled as suppliers that are linked directly to delivery points, i.e., there are no transportation costs involved on this link. Beneficiaries can then receive some or all of their commodities from these local markets through vouchers.

The sets used in the humanitarian supply chain model are described in Table 1. Here, the time periods correspond to the different months for which a planning is required in order to aid the beneficiaries. A detailed description of all parameters and decision variables used is given in Table 2 and Table 3 respectively.

Table 1 *Set notation*

Set	Description	Definition	Cardinality
$\mathcal{N}$	All nodes		N
$\mathcal{N}_\mathcal{S}$	Suppliers	$\subset \mathcal{N}$	N_S
$\mathcal{N}_{\mathcal{SI}}$	International suppliers	$\subset \mathcal{N}_\mathcal{S}$	N_{SI}
$\mathcal{N}_{\mathcal{SR}}$	Regional suppliers	$\subset \mathcal{N}_\mathcal{S}$	N_{SR}
$\mathcal{N}_{\mathcal{SL}}$	Local markets	$\subset \mathcal{N}_\mathcal{S}$	N_{SL}
$\mathcal{N}_\mathcal{T}$	Transshipment points	$\subset \mathcal{N}$	N_T
$\mathcal{N}_\mathcal{D}$	Delivery points	$\subset \mathcal{N}$	N_D
$\mathcal{N}_{\mathcal{ST}}$	$\mathcal{N}_\mathcal{S} \cup \mathcal{N}_\mathcal{T}$	$\subset \mathcal{N}$	N_{ST}
$\mathcal{N}_{\mathcal{TD}}$	$\mathcal{N}_\mathcal{T} \cup \mathcal{N}_\mathcal{D}$	$\subset \mathcal{N}$	N_{TD}
$\mathcal{K}$	Commodities		K
$\mathcal{L}$	Nutrients		L
$\mathcal{T}$	Time periods		T

The objective of the optimization model is to minimize the total operational costs. The different costs considered in this model are procurement costs (PC), transportation costs (TC), handling costs (HC) and storage costs (SC) described as follows:

$$\mathrm{PC} = \sum_{i \in \mathcal{N}_\mathcal{S}} \sum_{j \in \mathcal{N}_{\mathcal{TD}}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} p^P_{ikt} F_{ijkt}, \tag{1}$$

$$\mathrm{TC} = \sum_{i \in \mathcal{N}_{\mathcal{ST}}} \sum_{j \in \mathcal{N}_{\mathcal{TD}}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} p^T_{ijkt} F_{ijkt}, \tag{2}$$

$$\mathrm{HC} = \sum_{i \in \mathcal{N}_{\mathcal{ST}}} \sum_{j \in \mathcal{N}_{\mathcal{TD}}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} p^H_{j} F_{ijkt}, \tag{3}$$

$$\mathrm{SC} = \sum_{i \in \mathcal{N}_\mathcal{T}} \sum_{j \in \mathcal{N}_{\mathcal{TD}}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} p^S_{i} F_{ijkt}. \tag{4}$$

Using all these definitions, the mathematical model is given by:

$$\min_{\boldsymbol{F},\boldsymbol{R},\boldsymbol{S}} \quad \mathrm{PC} + \mathrm{TC} + \mathrm{HC} + \mathrm{SC} \tag{5a}$$

$$\text{s.t.} \quad \sum_{j \in \mathcal{N}_{\mathcal{TD}}} F_{ijkt} = \sum_{j \in \mathcal{N}_{\mathcal{ST}}} F_{jik,t-\tau(j,i)} \qquad i \in \mathcal{N}_\mathcal{T}, k \in \mathcal{K}, t \in \mathcal{T} \tag{5b}$$

Table 2 *Parameter notation*

Parameter	Description
c^H_{it}	Handling capacity (in mt) at node $i \in \mathcal{N}_{\mathcal{TD}}$ in time period $t \in \mathcal{T}$.
c^T_{ijt}	Transportation capacity (in mt) from node $i \in \mathcal{N}_{\mathcal{ST}}$ to node $j \in \mathcal{N}_{\mathcal{TD}}$ in time period $t \in \mathcal{T}$.
c^P_{ikt}	Procurement capacity (in mt) for commodity $k \in \mathcal{K}$ at node $i \in \mathcal{N}_{\mathcal{S}}$ in time period $t \in \mathcal{T}$.
p^P_{ikt}	Costs (in \$/mt) of procuring 1 mt of commodity $k \in \mathcal{K}$ at supply node $i \in \mathcal{N}_{\mathcal{S}}$ in time period $t \in \mathcal{T}$.
p^T_{ijkt}	Costs (in \$/mt) of moving 1 mt commodity $k \in \mathcal{K}$ from node $i \in \mathcal{N}_{\mathcal{ST}}$ to node $j \in \mathcal{N}_{\mathcal{ST}}$ in time period $t \in \mathcal{T}$.
p^H_i	Costs (in \$/mt) of handling 1 mt commodity $k \in \mathcal{K}$ at node $i \in \mathcal{N}_{\mathcal{TD}}$.
p^S_{it}	Costs (in \$/mt) of storing 1 mt at node $i \in \mathcal{N}_{\mathcal{T}}$ in time period $t \in \mathcal{T}$.
d_{it}	Number of beneficiaries at delivery point $i \in \mathcal{N}_{\mathcal{D}}$ in time period $t \in \mathcal{T}$.
β_{kl}	Nutritional value for nutrient $l \in \mathcal{L}$ per 100 gram of commodity $k \in \mathcal{K}$.
η_l	Nutritional requirement for nutrient $l \in \mathcal{L}$ (grams/average beneficiary/day).
γ_{ij}	Duration (days) of shipping from node $i \in \mathcal{N}_{\mathcal{ST}}$ to $j \in \mathcal{N}_{\mathcal{TD}}$.
δ_t	Number of days in time period $t \in \mathcal{T}$.
$\tau_{i,j}$	$\lfloor \frac{\gamma_{ij} + \frac{\delta_t}{2}}{\delta_t} \rfloor$. Used to rescale the shipping duration from days to time periods.
α	Used to convert from metric tons to 100 grams ($= 10,000$).
sf_l	Maximum shortfall in nutrient $l \in \mathcal{L}$ as a fraction of the total nutrition required.

Table 3 *Variable notation*

Variable	Description
F_{ijkt}	Amount of commodity $k \in \mathcal{K}$ transported between node $i \in \mathcal{N}_{\mathcal{ST}}$ and node $j \in \mathcal{N}_{\mathcal{TD}}$ in time period $t \in \mathcal{T}$ in metric tons.
R_{kt}	Daily ration of commodity $k \in \mathcal{K}$ provided in time period $t \in \mathcal{T}$ in 100 grams to a single average beneficiary.
S_{lt}	Realized shortfall of nutrient $l \in \mathcal{L}$ in time period $t \in \mathcal{T}$ to all beneficiaries as a fraction of the total amount of nutrients needed.

$$\sum_{i \in \mathcal{N}_{\mathcal{ST}}} \alpha F_{ijkt} = d_{jt} \delta_t R_{kt} \qquad j \in \mathcal{N}_{\mathcal{D}}, k \in \mathcal{K}, t \in \mathcal{T} \tag{5c}$$

$$\sum_{j \in \mathcal{N}_{\mathcal{TD}}} F_{ijkt} \leq c^P_{ikt} \qquad i \in \mathcal{N}_{\mathcal{S}}, k \in \mathcal{K}, t \in \mathcal{T} \tag{5d}$$

$$\sum_{k \in \mathcal{K}} F_{ijkt} \leq c^T_{ijt} \qquad i \in \mathcal{N}_{\mathcal{ST}}, j \in \mathcal{N}_{\mathcal{TD}}, t \in \mathcal{T} \tag{5e}$$

$$\sum_{i \in \mathcal{N}_{\mathcal{ST}}} \sum_{k \in \mathcal{K}} F_{ijkt} \leq c^H_{jt} \qquad j \in \mathcal{N}_{\mathcal{TD}}, t \in \mathcal{T} \tag{5f}$$

$$\sum_{k \in \mathcal{K}} \beta_{kl} R_{kt} \geq \eta_l (1 - S_{lt}) \qquad l \in \mathcal{L}, t \in \mathcal{T} \tag{5g}$$

$$S_{lt} \leq sf_l \qquad l \in \mathcal{L}, t \in \mathcal{T} \tag{5h}$$

$$\boldsymbol{F} \geq \boldsymbol{0} \tag{5i}$$

$$\boldsymbol{R} \geq \boldsymbol{0}. \tag{5j}$$

Constraints (5b) make sure all transshipment points are balanced, i.e., the incoming flow of commodities in a transshipment point must equal the outgoing flow. Constraints (5c) ensure the incoming flow at delivery points equal the demanded rations per commodity for a given time period. Constraints (5d)-(5f) are capacity constraints, i.e., they set a capacity restriction on respectively the amount procured, transported and handled. Constraints (5g) ensure the total nutritional value of a certain nutrient l within a given time period t going to a single beneficiary is greater or equal than the required nutritional value for that nutrient, or there is a shortage of S_{lt}. For every nutrient, this shortage may never be larger than the maximum shortfall for that nutrient, given by sf_l as modeled in constraints (5h). Constraints (5i) and (5j) are the nonnegativity constraints for the flow variables $\boldsymbol{F}$ and ration variables $\boldsymbol{R}$ respectively.

3. Robust optimization formulation

In reality, the biggest uncertainty occurs in the procurement prices of commodities at regional suppliers and local markets, since those are located in the country at risk. There is much less uncertainty in the procurement prices at international suppliers. This is observed by WFP (2022) and Al-Saidi (2023), both stating that a global food crisis fuelled by climate shocks, the COVID-19 pandemic, and the war in Ukraine pushes local food prices up especially in countries at risk. Consequently, we assume that uncertainty is present in the procurement prices for regional suppliers and local markets and that the procurement prices for commodities at international suppliers do not face uncertainties. However, from a mathematical perspective, our formulation can easily be adapted to take uncertainties in international markers into account as well. Moreover, we assume uncertainty in procurement prices for each commodity k and time t to be the same for all suppliers in the same market. This is because local markets are located within the same region and regional suppliers are positioned within the same country. Hence, we capture the uncertainty per market $m \in \mathcal{M} = \{\mathcal{N}_{\mathcal{SR}}, \mathcal{N}_{\mathcal{SL}}\}$. Observe that our formulation can easily be adapted to take different levels of uncertainties per supplier into account by stating that each supplier is its own market.

In order to formulate the robust optimization model, we make a distinction in procurement prices between international suppliers and regional and local suppliers:

$$p^P_{ikt} = \begin{cases} \theta_{ik} & \text{if } i \in \mathcal{N}_{\mathcal{SI}} \\ \mu_{ikt} & \text{if } i \in \mathcal{N}_{\mathcal{SR}} \cup \mathcal{N}_{\mathcal{SL}}, t = 1 \\ \mu_{ikt} + \zeta_{m_i kt} & \text{if } i \in \mathcal{N}_{\mathcal{SR}} \cup \mathcal{N}_{\mathcal{SL}}, t \geq 2. \end{cases} \tag{6}$$

Here $\boldsymbol{\theta} \in \mathbb{R}^{N_{SI}K}$ is the nominal value of the procurement prices at international suppliers, $\boldsymbol{\mu} \in \mathbb{R}^{(N-N_{SI})KT}$ is the nominal value of the procurement prices at regional and local suppliers, m_i denotes the market $m \in \mathcal{M}$ to which supplier i belongs, and $\boldsymbol{\zeta} \in \mathbb{R}^{|\mathcal{M}|KT}$ denotes the uncertainty in procurement prices at regional and local markets. Note that the prices for $t=1$ are known, but the prices for $t\geq 2$ are uncertain.

We assume that the uncertain parameters lie within an ellipsoidal uncertainty set given by

$$\mathcal{U}=\{\boldsymbol{\zeta}: \boldsymbol{\zeta}^\top \boldsymbol{\Sigma}^{-1}\boldsymbol{\zeta}\leq \Omega^2\}, \tag{7}$$

where $\boldsymbol{\Sigma}$ represents the covariance matrix of the procurement prices per market, which is positive semi-definite, and Ω represents the safety parameter, limiting the amount of uncertainty to be covered by the robust approach. By using an ellipsoidal uncertainty set, we reduce the conservative approach of box-uncertainty, by ensuring that the uncertain parameters do not take on their worst-case values simultaneously, while still maintaining a large probability of constraint satisfaction (Ben-Tal et al. (2009)).

3.1. Robust formulation of the HSC model

Since the uncertain procurement prices appear in the objective, we can use a similar method as Ben-Tal et al. (2009), by defining an epigraph variable q for the procurement costs at all regional and local suppliers at $t\geq 2$ to obtain the following robust optimization problem:

$$\begin{aligned}
\min_{\boldsymbol{F},\boldsymbol{R},\boldsymbol{S},q} \quad & q+\sum_{i\in\mathcal{N}_{\mathcal{SR}}\cup\mathcal{N}_{SL}}\sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{N}_{TD}}\mu_{ik1}F_{ijk1}+\sum_{i\in\mathcal{N}_{\mathcal{SI}}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\theta_{ikt}F_{ijkt} \\
& +\mathrm{TC}+\mathrm{HC}+\mathrm{SC} && \text{(8a)}\\
\text{s.t.} \quad & (\boldsymbol{\mu}+\boldsymbol{A}\boldsymbol{\zeta})^\top \boldsymbol{F}^P\leq q \qquad \forall\boldsymbol{\zeta}\in\mathcal{U} && \text{(8b)}\\
& \text{(5b)}-\text{(5j)},
\end{aligned}$$

where $\boldsymbol{\mu}$, $\boldsymbol{F}^P\in\mathbb{R}^{(N-N_{SI})K(T-1)}$, with $F^P_{ikt}=\sum_{j\in\mathcal{N}_{\mathcal{TD}}}F_{ijkt}$ restricted to $i\in\mathcal{N}_{\mathcal{SR}}\cup\mathcal{N}_{\mathcal{SL}}$ for $t\geq 2$, as defined before, and $\boldsymbol{A}\in\mathbb{R}^{(N-N_{SI})K(T-1)\times MK(T-1)}$ is a linear transformation matrix such that $(\boldsymbol{A}\boldsymbol{\zeta})_{ikt}=\zeta_{m_ikt}$ for every $i\in\mathcal{N}_{\mathcal{SR}}\cup\mathcal{N}_{\mathcal{SL}}$, $k\in\mathcal{K}$, and $t\in\mathcal{T}$. In this way, $\boldsymbol{A}\boldsymbol{\zeta}$ has the same dimension as $\boldsymbol{F}^P$. We can rewrite constraint (8b) to:

$$\boldsymbol{\mu}^\top\boldsymbol{F}^P+\Omega\sqrt{(\boldsymbol{A}^\top\boldsymbol{F}^P)^\top\boldsymbol{\Sigma}\boldsymbol{A}^\top\boldsymbol{F}^P}\leq q. \tag{9}$$

Hence, we can consider the LHS of (8b) as a random variable with expected value $\boldsymbol{\mu}^\top\boldsymbol{F}^P$ and standard deviation $\sqrt{(\boldsymbol{A}^\top\boldsymbol{F}^P)^\top\boldsymbol{\Sigma}\boldsymbol{A}^\top\boldsymbol{F}^P}$. Thus, Constraint (8b) can be equivalently written as:

$$\boldsymbol{\mu}^\top\boldsymbol{F}^P+\Omega\left\|\boldsymbol{L}^\top\boldsymbol{A}^\top\boldsymbol{F}^P\right\|_2\leq q, \tag{10}$$

where $\boldsymbol{L}$ denotes the Cholesky factor for $\boldsymbol{\Sigma}$. In this way, we obtain an equivalent second order cone optimization problem without uncertainty. Under the assumption that $\boldsymbol{\mu}^\top\boldsymbol{F}^P$ is normally distributed, using this ellipsoidal uncertainty set with Ω representing the $(1-\epsilon)$-percentile of a standard normal distribution, hence leads to the probability guarantee

$$\mathbb{P}\left((\boldsymbol{\mu}+\boldsymbol{\xi})^\top \boldsymbol{F}^P\right) \leq q\right) \geq 1-\epsilon, \tag{11}$$

see also Ben-Tal et al. (2009).

3.2. Finding a Pareto robust optimal solution

The robust formulation has in practice usually multiple optimal solutions. This optimality is measured with respect to the worst-case scenarios of the uncertain parameters. This means that there might exist alternative RO optimal solutions which yield different objective values based on the average scenario (see Iancu and Trichakis (2014)). Consequently, in case of multiple optimal RO solutions, among those solutions we want to find a Pareto robustly optimal solution, that is, the one which yields the best objective value regarding the average scenario.

As proposed by Iancu and Trichakis (2014), we first solve the RO formulation to obtain the optimal objective value q^* with respect to the worst case scenario. Subsequently, we solve RO once more, in which we change the objective to the nominal objective (5a), since the expected value of the robust objective (8a) equals the nominal objective. In this way, we minimize the costs for the expected scenario, instead of the worst case scenario. Furthermore, we add the constraint:

$$q = q^*,$$

to make sure that the robust objective value regarding the worst case scenario is the same as after solving RO the first time. This is called the Pareto Robust Optimization model (PRO).

4. Adaptive robust optimization formulation

In the robust optimization formulation, decisions regarding the flow variables $\boldsymbol{F}$ and the food basket design variables $\boldsymbol{R}$ are done at the start of the planning period, they are here-and-now decisions. However, suppose, that one commodity suddenly becomes cheaper than was considered at the start of the planning period. As a result of the change in price, one might want to change the ration or transport of commodities in the future. This means that the flow and food basket design variables are actually wait-and-see variables, since each variable can depend on the procurement costs of (different) commodities at (different) suppliers. In this section we will adapt the nominal optimization model of Section 2 to an adaptive robust optimization model.

4.1. Linear decision rules

In general, problems that contain adaptive robust inequality constraints are NP-hard (see Ben-Tal et al. (2004)). Therefore, decision rules are often restricted to a certain class of functions to make the problem tractable. We are restricting ourselves to decision rules that are affinely dependent on $\boldsymbol{\zeta}$ as previously used in Ben-Tal et al. (2004); Bertsimas and Goyal (2009); Rocha and Kuhn (2012); Iancu et al. (2013); Gounaris et al. (2013). To this end, we define a linear decision rule for F_{ijkt}, with $t \geq 2$, as follows:

$$F_{ijkt} = \begin{cases} \bar{F}_{ijkt} + \sum\limits_{\substack{m \in \mathcal{M}; \\ k^* \in \mathcal{K}}} v_{ijkt,mk^*t} \zeta_{mk^*t}, & \text{if } i \in \mathcal{N}_{\mathcal{S}}, \\ \bar{F}_{ijkt} + \sum\limits_{\substack{m \in \mathcal{M}; \\ k^* \in \mathcal{K}; \\ t^* \leq t}} v_{ijkt,mk^*t^*} \zeta_{mk^*t^*} & \text{if } i \in \mathcal{N}_{\mathcal{T}}, \end{cases}$$
$$= \begin{cases} \bar{F}_{ijkt} + (\boldsymbol{v}_{ijkt}^t)^\top \boldsymbol{\zeta}^t, & \text{if } i \in \mathcal{N}_{\mathcal{S}}, \\ \bar{F}_{ijkt} + \sum\limits_{t^* \leq t} (\boldsymbol{v}_{ijkt}^{t^*})^\top \boldsymbol{\zeta}^{t^*}, & \text{if } i \in \mathcal{N}_{\mathcal{T}}, \end{cases} \tag{12}$$

where $\bar{F}_{ijkt}$ and v_{ijkt,mk^*t} are the coefficients to be determined, and $\boldsymbol{v}_{ijkt}^t, \boldsymbol{\zeta}^t \in \mathbb{R}^{|\mathcal{M}|K}$ for every $t \in \{2, \ldots, T\}$. Observe that the flow from a supply node to a transshipment or delivery node does not depend on the uncertainty in procurement costs at earlier points in time since the actual price at the moment of procurement is known. However, the flow from a transshipment node to another transshipment or delivery node does depend on the uncertainty in procurement costs at earlier points in time since time t is not the same as the time of procurement ($t^* \leq t$). Since transport between transshipment nodes is possible, it is hard to keep track of the time of procurement, hence we take into account all uncertainties at earlier points in time.

As with the Robust Optimization formulation, we assume that the uncertain parameters lie within the ellipsoidal uncertainty set given in (7).

Both $\boldsymbol{F}$ and $\boldsymbol{R}$ are wait-and-see variables. However, defining a decision rule for the adjustable variable $\boldsymbol{R}$ is superfluous since we can eliminate this variable from problem (5) by substituting the expression for $\boldsymbol{R}$ obtained from equality constraint (5c) in constraint (5g) for all $j \in \mathcal{N}_{\mathcal{D}}, k \in \mathcal{K}$ and $t \in \mathcal{T}$ except for those j and t for which $d_{jt} = 0$. Observe that as long as there exists a $j \in \mathcal{N}_{\mathcal{D}}$ for all $t \in \mathcal{T}$ such that $d_{jt} \neq 0$, we do not need any constraint for j and t for which d_{jt} equals zero. Moreover, if this is the case, the nonnegativity of R_{kt}, i.e., constraint (5j), is already satisfied for all $k \in \mathcal{K}$ and $t \in \mathcal{T}$. Hence, we can replace constraints (5c), (5g) and (5j) by constraints (13a) and (13b):

$$\sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}_{\mathcal{ST}}} \beta_{k\ell} F_{ijkt} \geq \frac{d_{jt}\delta_t}{\alpha} \eta_\ell (1 - S_{\ell t}) \qquad j \in \mathcal{N}_{\mathcal{D}}, \ell \in \mathcal{L}, t \in \mathcal{T}, \tag{13a}$$

$$d_{j't} \sum_{i \in \mathcal{N}_{\mathcal{ST}}} F_{ijkt} = d_{jt} \sum_{i \in \mathcal{N}_{\mathcal{ST}}} F_{ij'kt} \qquad j, j' \in \mathcal{N}_{\mathcal{D}}, d_{j't}, d_{jt} \neq 0, k \in \mathcal{K}, t \in \mathcal{T}. \tag{13b}$$

There are two difficulties with imposing a decision rule for the adaptive variables $\boldsymbol{F}$. First of all, equality constraints cannot be satisfied for all uncertain variables. Secondly, since our problem is non-fixed recourse (procurement prices depend linearly on the uncertain variables), using just a linear decision rule for $\boldsymbol{F}$ results in quadratic uncertainty. One can deal with equality constraints (5b) by first substituting the linear decision rule for $\boldsymbol{F}$ and subsequently grouping terms multiplied with $\boldsymbol{\zeta}$ and setting them equal to zero (Gorissen et al., 2015), to obtain the following equivalent set of constraints:

$$\sum_{j\in\mathcal{N}_{\mathcal{TD}}} \boldsymbol{v}_{ijkt}^{t^*} = \sum_{j\in\mathcal{N}_{\mathcal{ST}}} \boldsymbol{v}_{jik,t-\tau(j,i)}^{t^*}, \qquad i\in\mathcal{N}_{\mathcal{T}}, k\in\mathcal{K}, t^*<t-\tau(j,i), t\in\{2,\dots,T\}, \tag{14a}$$

$$\sum_{j\in\mathcal{N}_{\mathcal{TD}}} \boldsymbol{v}_{ijkt}^{t-\tau(j,i)} = \sum_{j\in\mathcal{N}_{\mathcal{ST}}} \boldsymbol{v}_{jik,t-\tau(j,i)}^{t-\tau(j,i)}, \qquad i\in\mathcal{N}_{\mathcal{T}}, k\in\mathcal{K}, t\in\{2,\dots,T\}, \tag{14b}$$

$$\sum_{j\in\mathcal{N}_{\mathcal{TD}}} \boldsymbol{v}_{ijkt}^{t^*} = 0, \qquad i\in\mathcal{N}_{\mathcal{T}}, k\in\mathcal{K}, t-\tau(j,i)<t^*\leq t, t\in\{2,\dots,T\}, \tag{14c}$$

$$\sum_{j\in\mathcal{N}_{\mathcal{ST}}} \bar{F}_{jik,t-\tau(j,i)} = \sum_{j\in\mathcal{N}_{\mathcal{TD}}} \bar{F}_{ijkt}, \qquad i\in\mathcal{N}_{\mathcal{T}}, k\in\mathcal{K}, t\in\mathcal{T}. \tag{14d}$$

In the same way one can deal with equality constraints (13b) to obtain the following equivalent set of constraints:

$$d_{jt}\sum_{i\in\mathcal{N}_{\mathcal{T}}} v_{ij'kt}^{t^*} = d_{j't}\sum_{i\in\mathcal{N}_{\mathcal{T}}} v_{ijkt}^{t^*}, \qquad j,j'\in\mathcal{N}_{\mathcal{D}}, d_{jt}, d_{j't}\neq 0, k\in\mathcal{K}, 2\leq t^*<t, t\in\{3,\dots,T\}, \tag{15a}$$

$$d_{jt}\sum_{i\in\mathcal{N}_{\mathcal{ST}}} v_{ij'kt}^{t} = d_{j't}\sum_{i\in\mathcal{N}_{\mathcal{ST}}} v_{ijkt}^{t}, \qquad j,j'\in\mathcal{N}_{\mathcal{D}}, d_{jt}, d_{j't}\neq 0, k\in\mathcal{K}, t\geq 2, \tag{15b}$$

$$d_{j't}\sum_{i\in\mathcal{N}_{\mathcal{ST}}} \bar{F}_{ijkt} = d_{jt}\sum_{i\in\mathcal{N}_{\mathcal{ST}}} \bar{F}_{ij'kt}, \qquad j,j'\in\mathcal{N}_{\mathcal{D}}, d_{jt}, d_{j't}\neq 0, k\in\mathcal{K}, t\in\mathcal{T}. \tag{15c}$$

To deal with the fact that our problem is non-fixed recourse, we use the exact S-lemma to obtain an exact tractable robust counterpart of the robust linear constraint with quadratic uncertainty, which will be explained in more detail in Section 4.2.

Adaptive robust formulation of the HSC model

The nominal formulation (5) can be extended to a fully adaptive robust formulation, by first of all substituting expression (12) for the flow variables in objective (5a). This leads to the following objective function for the adaptive robust formulation:

$$\begin{aligned} \min_{\bar{\boldsymbol{F}},\boldsymbol{V},q} \quad & q \\ \text{s.t.} \quad & r(\bar{\boldsymbol{F}}) + \boldsymbol{s}(\bar{\boldsymbol{F}},\boldsymbol{V})^T\boldsymbol{\zeta} + \boldsymbol{\zeta}^T\boldsymbol{Q}(\boldsymbol{V})\boldsymbol{\zeta} \leq q \quad \forall \boldsymbol{\zeta}\in\mathcal{U}, \end{aligned} \tag{16}$$

where $\bar{\boldsymbol{F}}$ is the vector comprising all $\bar{F}_{ijkt}$, $\boldsymbol{V}$ is the matrix consisting of all $\boldsymbol{v}_{ijkt}^{t^*}$ with $i\in\mathcal{N}_{\mathcal{ST}}$, $j\in\mathcal{N}_{\mathcal{TD}}$, $k\in\mathcal{K}$, $t\in T$, $t^*\leq t\in\{2,\dots,T\}$, and $\boldsymbol{\zeta} = \left((\boldsymbol{\zeta}^2)^\top,\dots,(\boldsymbol{\zeta}^T)^\top\right)^\top$. We refer to Appendix A for a complete derivation of the reformulation of the objective and the explicit expressions for $r(\bar{\boldsymbol{F}}), \boldsymbol{s}(\bar{\boldsymbol{F}}, \boldsymbol{V})$ and $\boldsymbol{Q}(\boldsymbol{V})$.

Secondly, constraints (5b) are replaced by constraints (14a) - (14d), and constraints (5c), (5h), and (5j) are replaced by constraints (13a), (15a) - (15c). Furthermore, we substitute the linear decision rule (12) for the flow variables, where $t \in \{2, \ldots, T\}$, into constraints (5d) - (5f), (5i), (13a) and (13b). We then obtain robust linear constraints, which we can rewrite as constraints without uncertainty, using the KKT conditions and Cholesky factor $\hat{\boldsymbol{L}}$ as is done in Section 3. This leads in total to the following adaptive robust optimization problem with quadratic uncertainty:

$$\min_{\bar{\boldsymbol{F}}, \boldsymbol{V}, \boldsymbol{S}, q} \quad q \tag{17a}$$

$$\text{s.t.} \quad r(\bar{\boldsymbol{F}}) + \boldsymbol{s}\left(\bar{\boldsymbol{F}}, \boldsymbol{V}\right)^\top \boldsymbol{\zeta} + \boldsymbol{\zeta}^\top \boldsymbol{Q}(\boldsymbol{V}) \boldsymbol{\zeta} \leq q \qquad \forall \boldsymbol{\zeta} \in \mathcal{U} \tag{17b}$$

$$(14\text{a}) - (14\text{d})$$

$$(15\text{a}) - (15\text{c})$$

$$\sum_{j \in \mathcal{N}_{\mathcal{TD}}} \bar{F}_{ijk1} \leq c^P_{ik1} \qquad i \in \mathcal{N}_{\mathcal{S}}, k \in \mathcal{K} \tag{17c}$$

$$\sum_{k \in \mathcal{K}} \bar{F}_{ijk1} \leq c^T_{ij1} \qquad i \in \mathcal{N}_{\mathcal{S}}, j \in \mathcal{N}_{\mathcal{TD}} \tag{17d}$$

$$\sum_{i \in \mathcal{N}_{\mathcal{ST}}} \sum_{k \in \mathcal{K}} \bar{F}_{ijk1} \leq c^H_{j1} \qquad j \in \mathcal{N}_{\mathcal{TD}} \tag{17e}$$

$$\sum_{j \in \mathcal{N}_{\mathcal{TD}}} \bar{F}_{ijkt} + \Omega \left\| \hat{\boldsymbol{L}}^\top \left(\sum_{j \in \mathcal{N}_{\mathcal{TD}}} \boldsymbol{v}^{0,t,0}_{ijkt} \right) \right\|_2 \leq c^P_{ikt} \qquad i \in \mathcal{N}_{\mathcal{S}}, k \in \mathcal{K}, t \in \{2, \ldots, T\} \tag{17f}$$

$$\sum_{k \in \mathcal{K}} \bar{F}_{ijkt} + \Omega \left\| \hat{\boldsymbol{L}}^\top \left(\sum_{k \in \mathcal{K}} \boldsymbol{v}^{0,t,0}_{ijkt} \right) \right\|_2 \leq c^T_{ijt} \qquad i \in \mathcal{N}_{\mathcal{S}}, j \in \mathcal{N}_{\mathcal{TD}}, t \in \{2, \ldots, T\} \tag{17g}$$

$$\sum_{k \in \mathcal{K}} \bar{F}_{ijkt} + \Omega \left\| \hat{\boldsymbol{L}}^\top \left(\sum_{k \in \mathcal{K}} \boldsymbol{v}^{1-t,0}_{ijkt} \right) \right\|_2 \leq c^T_{ijt} \qquad i \in \mathcal{N}_{\mathcal{T}}, j \in \mathcal{N}_{\mathcal{TD}}, t \in \{2, \ldots, T\} \tag{17h}$$

$$\sum_{i \in \mathcal{N}_{\mathcal{ST}}} \sum_{k \in \mathcal{K}} \bar{F}_{ijkt} + \Omega \left\| \hat{\boldsymbol{L}}^\top \left(\sum_{i \in \mathcal{N}_T} \sum_{k \in \mathcal{K}} \boldsymbol{v}^{1-t_S,0}_{ijkt} \right) \right\|_2 \leq c^H_{jt} \qquad j \in \mathcal{N}_{\mathcal{TD}}, t \in \{2, \ldots, T\} \tag{17i}$$

$$\sum_{i \in \mathcal{N}_{\mathcal{ST}}} \sum_{k \in \mathcal{K}} \beta_{kl} \bar{F}_{ijk1} \geq \frac{\eta_\ell (1 - S_{\ell 1}) d_{j1} \delta_1}{\alpha} \qquad j \in \mathcal{N}_{\mathcal{D}}, \ell \in \mathcal{L} \tag{17j}$$

$$\sum_{i\in\mathcal{N}_{\mathcal{ST}}}\sum_{k\in\mathcal{K}}\beta_{kl}\bar{F}_{ijkt}-\Omega\left\|\hat{\boldsymbol{L}}^\top\left(\sum_{i\in\mathcal{N}_{\mathcal{T}}}\sum_{k\in\mathcal{K}}\beta_{kl}\boldsymbol{v}_{ijkt}^{1-t_S,0}\right)\right\|_2 \geq \frac{\eta_\ell(1-S_{\ell t})d_{jt}\delta_t}{\alpha} \qquad j\in\mathcal{N}_{\mathcal{D}},\ell\in\mathcal{L},t\in\{2,\ldots,T\} \tag{17k}$$

$$S_{\ell t}\leq sf_\ell \qquad \ell\in\mathcal{L},t\in\mathcal{T} \tag{17l}$$

$$\bar{F}_{ijk1}\geq 0 \qquad i\in\mathcal{N}_{\mathcal{ST}},j\in\mathcal{N}_{\mathcal{TD}},k\in\mathcal{K} \tag{17m}$$

$$\bar{F}_{ijkt}-\Omega\left\|\hat{\boldsymbol{L}}^\top\left(\boldsymbol{v}_{ijkt}^{0,t,0}\right)\right\|_2\geq 0 \qquad i\in\mathcal{N}_{\mathcal{S}},j\in\mathcal{N}_{\mathcal{TD}},k\in\mathcal{K},t\in\{2,\ldots,T\} \tag{17n}$$

$$\bar{F}_{ijkt}-\Omega\left\|\hat{\boldsymbol{L}}^\top\left(\boldsymbol{v}_{ijkt}^{1-t,0}\right)\right\|_2\geq 0, \qquad i\in\mathcal{N}_{\mathcal{T}},j\in\mathcal{N}_{\mathcal{TD}},k\in\mathcal{K},t\in\{2,\ldots,T\} \tag{17o}$$

where

$$\begin{aligned}
\boldsymbol{v}_{ijkt}^{0,t,0} &= \left(0,\ldots,0,\left(\boldsymbol{v}_{ijkt}^{t}\right)^\top,0,\ldots,0\right)^\top\in\mathbb{R}^{|\mathcal{M}|KT}\\
\boldsymbol{v}_{ijkt}^{1-t,0} &= \left(\left(\boldsymbol{v}_{ijkt}^{1}\right)^\top,\ldots,\left(\boldsymbol{v}_{ijkt}^{t-1}\right)^\top,\left(\boldsymbol{v}_{ijkt}^{t}\right)^\top,0,\ldots,0\right)^\top\in\mathbb{R}^{|\mathcal{M}|KT}\\
\boldsymbol{v}_{ijkt}^{1-t_S,0} &= \left(\left(\boldsymbol{v}_{ijkt}^{1}\right)^\top,\ldots,\left(\boldsymbol{v}_{ijkt}^{t-1}\right)^\top,\sum_{i\in\mathcal{N}_{\mathcal{S}}}\left(\boldsymbol{v}_{ijkt}^{t}\right)^\top,0,\ldots,0\right)^\top\in\mathbb{R}^{|\mathcal{M}|KT}.
\end{aligned}$$

We refer the reader to Appendix B for a more detailed description of the reformulation of the capacity constraints.

4.2. Exact reformulation using S-lemma

Since $\mathcal{U}$ is an ellipsoidal uncertainty set, we can use the exact S-lemma to obtain an exact tractable reformulation of constraint (17b).

Lemma 1 **(S-lemma)**. *Let $q_a, q_b:\mathbb{R}^n\rightarrow\mathbb{R}$ be two quadratic functions such that*

$$\begin{aligned}
q_a(\boldsymbol{z}) &= \boldsymbol{z}^T\boldsymbol{Q}_a\boldsymbol{z}+\boldsymbol{u}_a^T\boldsymbol{z}+c_a\\
q_b(\boldsymbol{z}) &= \boldsymbol{z}^T\boldsymbol{Q}_b\boldsymbol{z}+\boldsymbol{u}_b^T\boldsymbol{z}+c_b,
\end{aligned}$$

and suppose there is a $\bar{\boldsymbol{z}}\in\mathbb{R}^n$ such that $q_a(\bar{\boldsymbol{z}})>0$. If

$$q_a(\boldsymbol{z})\geq 0 \implies q_b(\boldsymbol{z})\geq 0 \quad \forall\boldsymbol{z},$$

then

$$\exists\lambda\geq 0,\ s.t.\ q_b(\boldsymbol{z})\geq\lambda q_a(\boldsymbol{z}) \quad \forall\boldsymbol{z}.$$

A proof can be found in Polik and Terlaky (2007).

Theorem 1. *$\boldsymbol{F}$ and $\boldsymbol{V}$ satisfy constraints* (17b) *if and only if there exists a $\lambda \in \mathbb{R}$ such that $\boldsymbol{F}$ and $\boldsymbol{V}$ satisfy*

$$\begin{pmatrix} \lambda\hat{\boldsymbol{\Sigma}}^{-1} - \boldsymbol{Q}(\boldsymbol{V}) & -\frac{1}{2}\boldsymbol{s}(\bar{\boldsymbol{F}},\boldsymbol{V}) \\ -\frac{1}{2}\boldsymbol{s}(\bar{\boldsymbol{F}},\boldsymbol{V})^T & -r(\bar{\boldsymbol{F}}) - \lambda\Omega^2 + q \end{pmatrix} \succeq \mathrm{O}.$$

A proof can be found in Appendix C.

Using Theorem 1, we obtain the following exact semidefinite programming (SDP) reformulation of problem (17):

$$\begin{aligned} \min_{\bar{\boldsymbol{F}},\boldsymbol{V},\boldsymbol{S},q,\lambda} \quad & q \\ \text{s.t.} \quad & \begin{pmatrix} \lambda\hat{\boldsymbol{\Sigma}}^{-1} - \boldsymbol{Q}(\boldsymbol{V}) & -\frac{1}{2}\boldsymbol{s}(\bar{\boldsymbol{F}},\boldsymbol{V}) \\ -\frac{1}{2}\boldsymbol{s}(\bar{\boldsymbol{F}},\boldsymbol{V})^T & -r(\bar{\boldsymbol{F}}) - \lambda\Omega^2 + q \end{pmatrix} \succeq \mathrm{O}. \\ & (14a) - (14d) \\ & (17d) - (17j) \\ & \lambda \geq 0. \end{aligned} \tag{18}$$

4.3. Finding the best ARO solution

Similar as for the robust formulation, the adaptive robust formulation also might have multiple optimal solutions. Those optimal solutions may yield possibly better objective values for scenarios other than the worst-case scenarios. Hence we want to find the solution which yields the best objective value regarding the average scenario. To this end, we first solve the ARO formulation (18) to obtain the optimal objective value q^* with respect to the worst case scenario. Subsequently, we solve ARO once more, in which we change the objective to

$$\begin{aligned} \mathbb{E}\left[r(F) + s(F,V)^\top \zeta + \zeta^\top Q(V)\zeta\right] &= r(F) + s(F,V)^\top \mu + tr(Q(V)\Sigma) + \mu^\top Q(V)\mu \\ &= r(F) + tr(Q(V)\Sigma). \end{aligned} \tag{19}$$

By changing the objective to (19), we minimize the costs for the expected scenario, instead of the worst case scenario. Furthermore, we also add the constraint:

$$q \leq q^*,$$

to make sure that the objective value regarding the worst case scenario is just as good as after solving ARO the first time. We term this model the Pareto Adaptive Robust Optimization model (PRO-A).

5. Flexible commitment contract

An important aspect of the ARO approach is that new decisions can be taken, once part of the uncertainty has been revealed, whereas both the NO and RO approaches deal with complete uncertainty. In order to have a relatively fair comparison between the NO, RO and ARO approaches we introduce a Folding Horizon (FH) approach for NO and RO. In the first time period procurement commitments are made based on price estimations for future time periods, whereafter part of the uncertainty is revealed in the later time periods and the commitments can be altered to a certain degree.

The procurement prices of the first period are known (p^P_{ik1}), and there is an estimate on the procurement prices for future time periods: $p^{*P}_{ikt}, t \geq 2$. With this information we solve the Nominal and Robust optimization problem to obtain solutions, F_{ijk1}, for $t = 1$ and commitments F^*_{ijkt} for $t \geq 2$. At the beginning of each following period, the actual procurement prices of that period are revealed and we solve the nominal and robust problem again with the additional constraint that one can only deviate from the commitments made in the first period by a certain percentage *per*:

$$\left| \sum_{j \in \mathcal{N}_{TD}} F_{ijkt} - \sum_{j \in \mathcal{N}_{TD}} F^*_{ijkt} \right| \leq \text{per} \sum_{j \in \mathcal{N}_{TD}} F^*_{ijkt} \qquad i \in \mathcal{N}_S, k \in \mathcal{K}, t \in \mathcal{T},$$

The ARO approach does not have restrictions towards commitments made in the first period. Thus, in order to have a fair comparison, such restrictions need to be included. To that end, we solve the ARO approach with the following additional constraint:

$$\Omega \left| \boldsymbol{L}^\top \sum_{j \in \mathcal{N}_{TD}} \boldsymbol{v}^{0,t,0}_{ijkt} \right| \leq \text{per} \sum_{j \in \mathcal{N}_{TD}} \bar{F}_{ijkt} \qquad i \in \mathcal{N}_S, k \in \mathcal{K}, t \in \mathcal{T},$$

Here, we restrict the ARO approach to deviate from commitments made in the first period by at most a *per* percentage, similar as in the NO and RO FH approach.

6. Case study

In this section we discuss results of applying the nominal model (equations (5)-(5j)), the robust optimization model (equations (8a),(10), (5d) - (5j)), and the adaptive robust optimization model (18) to a real-life case, based on the food assistance operation during the aftermath of the civil war and humanitarian disaster in Syria in 2017. Section 6.1 describes the case study in further detail, whereafter Section 6.2 presents general results of the three different methods. Finally, Section 6.3 shows the results of the three methods when we use a folding horizon approach.

6.1. Case description

There are 7 relevant demand places where beneficiaries are located (Ar Raqqa, Hassakeh, Idleb, Jubb al Jarrah, Daraa, Dayr Az Zor, Hassakeh, and Qamishli) of which 3 locations have local markets (Hassakeh, Daraa and Dayr Az Zor). Furthermore, there are 5 regional suppliers (Aleppo, As Suweida, Damascus, Hama, and Homs), and we have three international suppliers (Amman, Beirut, and Gaziantep). All regional suppliers and demand locations are transshipment points as well. Figure 1 shows the 15 different nodes and the available connections between the nodes. As can be seen, not all nodes are connected to each other, since plenty of border crossings were closed during the civil war (see WFP (2017); USAID (2020)).

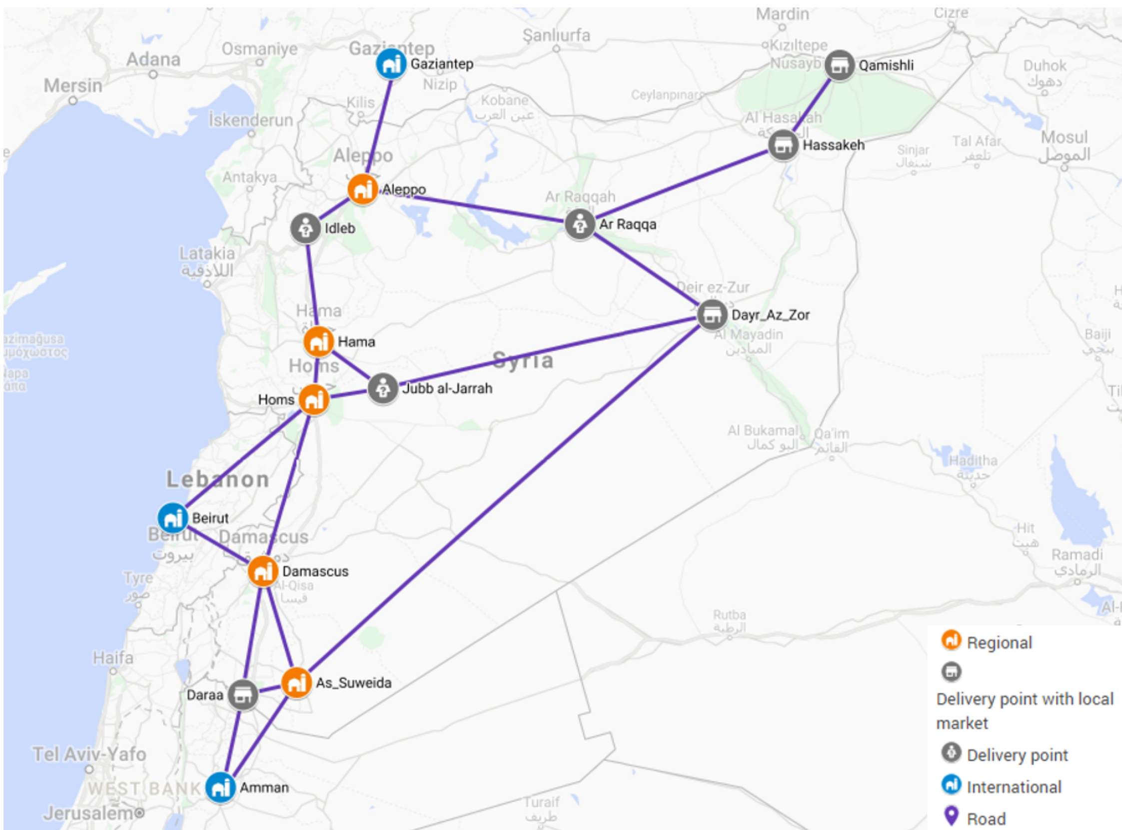


Figure 1 Map of Syria with all supply and demand nodes.

Table 4 gives an overview of the number of beneficiaries present at each of the seven demand locations and Table 5 shows the nutrient requirements that each beneficiary needs on average per day. Together, these tables translate to the total demand at a location.

In total we consider 24 different types of food, all with their own nutritional values. As explained in Section 2, we assume there is negligible uncertainty in procurement prices at international suppliers and the procurement prices at international suppliers are constant over time. We refer

Location	Number of beneficiaries
Ar Raqqa	10,000
Hassakeh	10,000
Dara	20,000
Dayr_Az_Zor	25,000
Qamishli	5,000
Jubb al-Jarrah	2,000
Idleb	5,000

Table 4

Demand in example cases.

Nutrient	Requirements
Energy (kcal)	2,100
Protein(g)	52.50
Fat (g)	89.25
Calcium (mg)	1,100
Iron (mg)	22
Vitamin A (ug)	500
Thiamine B1 (mg)	0.9
Riboflavin B2 (mg)	1.4
Nicacin B3 (mg)	12
Folate (ug)	160
Vitamin C (mg)	28

Table 5 Main nutrient requirements

to the online dataset for an overview of the time to traverse and the cost of using an edge, the nutritional values of each commodity, and the procurement prices used in our case study[a].

6.2. General results

All numerical experiments for this case study are conducted on an Intel Core i7-8665U 1.90GHz Windows computer with 32.0GB of RAM. All computations are implemented using YALMIP (Löfberg, 2004) in MATLAB (R2022b). Computations involving the nominal model are conducted with Gurobi 9.1.1, while computations involving the robust and the adaptive robust model are conducted with Mosek 9.3.18 (MOSEK ApS, 2022).

We compare the nominal model (NO), the robust model (RO), the Pareto robust model (PRO), the adaptive robust model (ARO), and the Pareto adaptive robust model (PRO-A) with each other based on performance in the nominal, worst-case and expected objective value. Moreover, we explore the differences in procurement and food baskets between the nominal model and the robust model. We do this for different levels of uncertainty, i.e., we vary the safety parameter Ω from 0 to 5, with $\Omega = 0$ indicating there is no uncertainty present and the higher the value of Ω the more uncertainty is considered when applying one of the robust models.

Table 6 shows that when solving the nominal scenario, i.e., $\boldsymbol{\zeta} = \mathbf{0}$, NO has the lowest objective value. This makes sense since NO optimizes the problem without taking uncertainty into account. We see that RO performs worst under the nominal scenario which can be explained due to the fact that RO only considers here-and-now decisions, opposed to ARO and PRO-A also considering wait-and-see decisions. We see that there is not much difference between the performance of RO and PRO under the nominal scenario. Moreover, the more uncertainty there is included in the model ($\Omega \geq 1$), the bigger the difference between the robust approaches and the nominal approach

[a] `http://doi.org/10.5281/zenodo.8091487`

become. This is as expected since the robust approaches consider the worst-case scenario across the uncertainty and the NO approach only considers the nominal situation.

In the worst case situation, both ARO and PRO-A perform best, followed by the robust models and NO respectively. NO does not take any uncertainty into account making its solution non-robust against the worst-case scenario, resulting in the highest objective value. This result is amplified in case more uncertainty is included. Since, opposed to RO and PRO, ARO, and PRO-A can wait with certain decisions until some of the uncertainty is revealed, ARO and PRO-A have the lowest worst-case objective. Note that the objective values of ARO and PRO-A under the nominal and worst-case scenario are the same. This is because the nominal scenario is in fact the worst-case scenario for ARO in our situation.

The solutions found of each of the models are also evaluated against the expected objective value over the uncertainty. For NO, RO, and PRO, this means that the found solutions are evaluated against the nominal scenario, since in taking the expectation of both the objectives of the nominal and the robust models we obtain the nominal objective. For ARO we have substituted the worst-case solution of ARO in the expectation of the objective as given by (19). Finally, for PRO-A, we have applied the method that as in Section 4.2. As can be seen, the ARO and PRO-A approaches are approximately as good as the NO approach in terms of expected behaviour, whereas they perform much better on the worst-case scenario.

Ω	Nominal value					Worst-case value					Expected value				
	NO	RO	PRO	ARO	PRO-A	NO	RO	PRO	ARO	PRO-A	NO	RO	PRO	ARO	PRO-A
0	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29
1	3.29	3.30	3.29	3.30	3.30	3.71	3.71	3.71	3.30	3.30	3.29	3.30	3.29	3.07	3.01
2	3.29	3.53	3.51	3.31	3.31	4.14	4.08	4.08	3.31	3.31	3.29	3.53	3.51	3.19	3.18
3	3.29	4.05	4.05	3.31	3.31	4.56	4.23	4.23	3.31	3.31	3.29	4.05	4.05	3.23	3.23
4	3.29	4.12	4.12	3.32	3.32	4.98	4.28	4.28	3.32	3.32	3.29	4.12	4.12	3.26	3.26
5	3.29	4.29	4.29	3.34	3.34	5.40	4.29	4.29	3.34	3.34	3.29	4.29	4.29	3.33	3.33

Table 6 Nominal (NO), robust optimization (RO), Pareto robust optimization (PRO), adaptive robust optimization (ARO), and Pareto adaptive robust optimization (PRO-A) approach are evaluated at the nominal, worst-case, and expected scenarios based on their costs in millions.

Table 7 shows the computation times of the four approaches. As can be seen, NO, RO, and PRO, are fast in finding solutions, whereas ARO and PRO-A are significantly slower, but still have computation times that are acceptable for usage in practice.

In order to gain more insight in how the solutions differ under different levels of uncertainty, we have created Figures 2-4.

Figure 2 shows the distribution between international, regional and local procurement for the NO approach ($\Omega = 0$) and the RO approach for different levels of uncertainty. As can be seen in

Ω	NO	RO	PRO	ARO	PRO-A
0	0.06	0.06	0.12	0.30	0.30
1	0.06	0.11	0.20	240	547
2	0.06	0.13	0.24	196	532
3	0.06	0.13	0.18	301	629
4	0.06	0.13	0.22	331	695
5	0.06	0.13	0.20	364	644

Table 7 Computation times in seconds of the nominal (NO), robust optimization (RO), Pareto robust optimization (PRO), adaptive robust optimization (ARO) and Pareto adaptive robust optimization (PRO-A) approach for $\Omega = \{0, \dots, 5\}$.

Figure 2a, the aid mainly comes from local and regional markets in case of little uncertainty in the procurement prices, and by increasing the uncertainty factor, there is a higher dependency on the international markets. More specifically, the uncertainty leads to Gaziantep and Amman becoming more and more important, see Figure 2b.

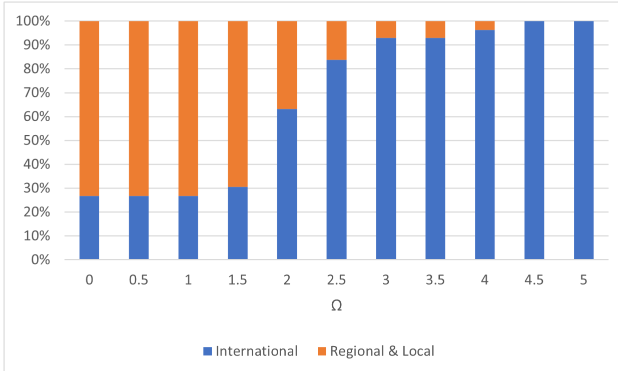
Figure 3 shows the food basket composition for different levels of uncertainty. With increasing safety level Ω, bulgur and beans are replaced by wheat. This means that bulgur and beans are procured regionally and locally and are therefore volatile to uncertainty. With increasing level of uncertainty, as we can see from Figure 2, the commodities are procured more from international suppliers, in which case wheat is the better option to procure.

Finally, Figure 4 shows how much of the objective cost is attributed to procurement and transportation for different levels of uncertainty with respect to $\Omega = 0$. The proportion of the objective cost attributed to transportation increases with increasing level of uncertainty. Since most of the food is bought at international markets for larger levels of uncertainty, this makes sense since this leads to longer transportation routes and hence more expensive transportation.

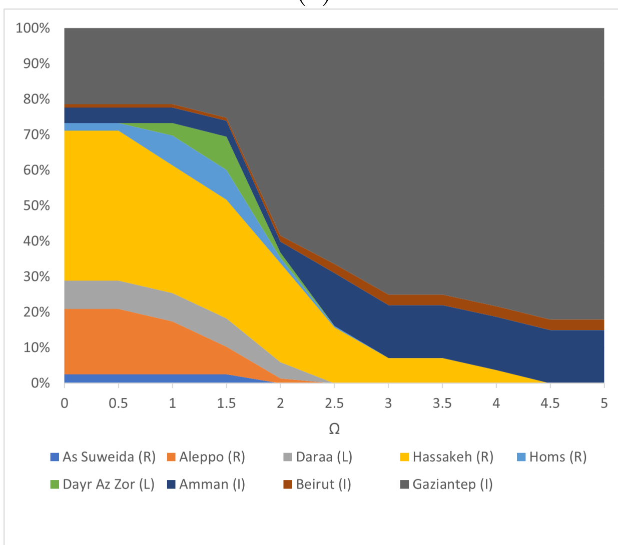
6.3. Folding Horizon results

In this section we will present results on the Folding Horizon approach as presented in Section 5. All methods make a commitment, based on price expectations, in the first time period about procurement's in future time periods. The actual procurement prices are only known just before the actual time period, and then small adjustments (*per* percentage) can be made towards the commitments. In this situation, the RO and PRO approach, as well as the ARO and PRO-A approach gave almost identical results, and thus we choose to only show the results for the RO and ARO approach respectively.

The three methods are evaluated on a nominal scenario, a worst-case scenario, and the average scenario. In the nominal scenario, the procurement prices in periods $t > 1$ are identical to the expected prices which were used in period $t = 1$ to make the commitments. A worst-case scenario is



(a)



(b)

Figure 2 Proportion of total weight procured attributed to international, regional and local suppliers at $t = \{2, 3\}$ for $\Omega \in \{0, \ldots, 7\}$ using the Nominal Approach ($\Omega = 0$).

obtained by uniformly drawing 500 scenarios from the uncertainty set and then solving the three approaches for each of the 500 scenarios and identifying per method the worst-case scenario. In previous results we have used a theoretical worst-case objective, however, this objective cannot be formulated in case the decisions can be altered. Consequently, we identify the worst-case based on drawing scenarios from the uncertainty set. For an overview of how we generate the scenarios uniformly from the ellipsoidal uncertainty set, we refer to Dezert and Musso (2001). The average scenario is obtained by computing the average objective value over those 500 scenarios.

Table 8 shows the results for different values of per, the percentage by which one can deviate from the commitments made in the first time period. First of all, if $per = 0$, the results are comparable to Table 6. The results for the nominal scenario are identical, but not for the worst-case and average

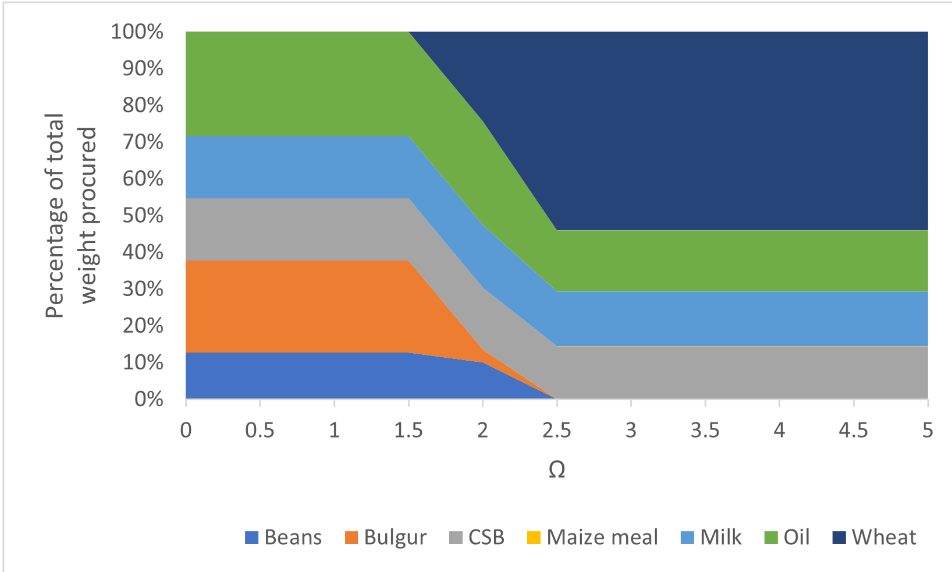


Figure 3 Food basket composition in percentage of total weight procured for different levels of uncertainty at $t = \{2, 3\}$.

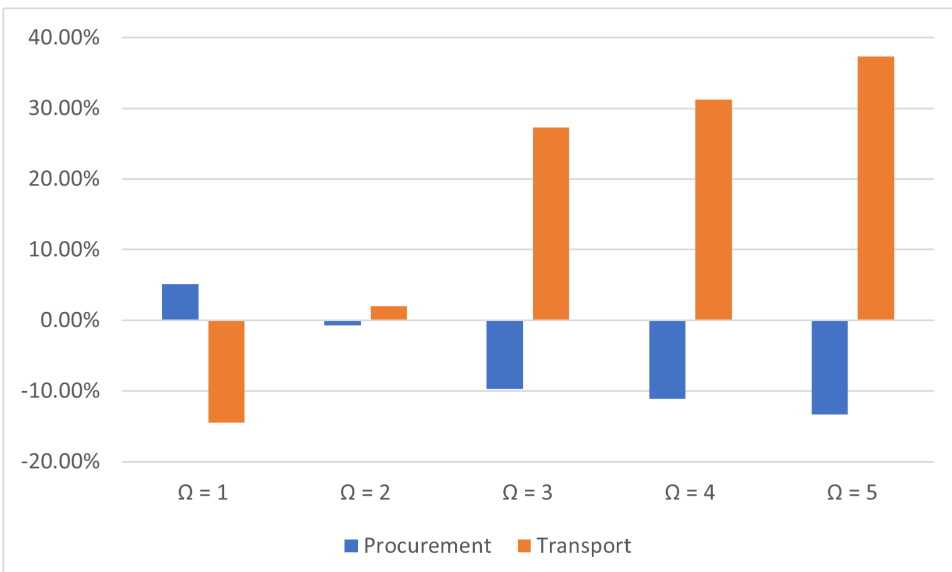


Figure 4 Relative change in percentage of objective costs attributed to procurement versus transport for $\Omega = \{1, \ldots, 5\}$ with respect to $\Omega = 0$.

scenario. This is due to the fact that we now consider 500 scenarios, instead of the theoretical worst-case and average scenario.

The nominal approach only considers the nominal scenario when making decisions, thus those decisions do not change in case the nominal scenario turns out to be the true scenario. The commitments made by the RO and ARO approaches change, even if the nominal scenario is true.

This is because both approaches include the possibility that other scenarios are true, and thus make more conservative decisions. Furthermore, as can be seen, the more uncertainty there is, the larger the difference in objective value between the RO approach and the other two approaches.

A second observation that can be made is regarding the RO approach. For this approach, the nominal scenario turns out to be the worst-case scenario as well. This indicates that the solution will be better or identical in all other tested scenarios, as also shown by the average scenario, as the average objective is always smaller or equal to the nominal/ worst-case scenario.

Finally, the NO and ARO approach outperform the RO approach both on the nominal, the worst-case, and the average scenario for all levels of uncertainty and percentages on which the methods can differ from commitments.

7. Conclusion and future research

Humanitarian organizations often operate on budgets originating from donors and efficiency is thus key. Therefore, we have developed a novel robust optimization model taking into account different levels of uncertainty in procurement prices of food at local and regional suppliers. This robust optimization model is extended to an adaptive and a pareto adaptive robust optimization model, in which the flow and ration variables are considered to be wait-and-see variables instead of here-and-now variables. Since humanitarian operations are often multi-period and dynamic, this reflects reality in a better way.

We test our four different robust approaches and a nominal approach (no uncertainty included) on a case study based on the aftermath of the civil war in Syria in 2017. We show that the (pareto) adaptive robust optimization approach performs approximately as good as the nominal model on the nominal scenario. However, our adaptive approach outperforms the nominal approach on the worst-case scenario, showing the added value of using robust optimization during food aid operations.

In actual operations of WFP, commitments are made in the first period, which can later on be changed, based on the procurement prices at that time. To that end, we analyze a Folding Horizon approach for the nominal, robust, and adaptive robust optimization models. It turns out that in our situation, where transportation times are less than a day, it is better to optimize decisions every month instead of only once at the start. Consequently, WFP could use a relatively simple approach in practice and apply a folding horizon approach each month to optimize decisions.

In this paper we observed that the nominal approach applied in a folding horizon way is at least as good as the ARO approach. We conjecture that this might be the case more general, especially when there is only uncertainty in the objective function. It is striking that in most papers on ARO there is no comparison with the nominal folding horizon approach.

per	Ω	Nominal scenario			Worst-case scenario			Average scenario		
		NO	RO	ARO	NO	RO	ARO	NO	RO	ARO
0.00	0	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29
	1	3.29	3.30	3.30	3.45	3.46	3.46	3.29	3.30	3.30
	2	3.29	3.53	3.31	3.52	3.68	3.54	3.28	3.52	3.30
	3	3.29	4.05	3.31	3.52	4.08	3.60	3.28	4.05	3.30
	4	3.29	4.12	3.32	3.65	4.16	3.68	3.29	4.12	3.33
	5	3.29	4.29	3.34	3.66	4.29	3.71	3.28	4.29	3.33
0.05	0	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29
	1	3.29	3.30	3.29	3.45	3.46	3.46	3.29	3.30	3.29
	2	3.29	3.52	3.29	3.52	3.68	3.53	3.28	3.52	3.29
	3	3.29	4.04	3.30	3.58	4.09	3.59	3.28	4.04	3.29
	4	3.29	4.12	3.30	3.65	4.15	3.65	3.29	4.12	3.29
	5	3.29	4.29	3.30	3.66	4.29	3.66	3.28	4.29	3.29
0.1	0	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29
	1	3.29	3.30	3.29	3.45	3.46	3.45	3.29	3.30	3.29
	2	3.29	3.52	3.29	3.52	3.68	3.53	3.28	3.51	3.29
	3	3.29	4.04	3.30	3.58	4.08	3.60	3.28	4.03	3.31
	4	3.29	4.11	3.30	3.65	4.15	3.65	3.29	4.11	3.29
	5	3.29	4.29	3.30	3.66	4.29	3.66	3.28	4.29	3.28
0.25	0	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29
	1	3.29	3.30	3.29	3.45	3.46	3.45	3.29	3.30	3.29
	2	3.29	3.51	3.29	3.52	3.67	3.53	3.28	3.50	3.29
	3	3.29	4.02	3.29	3.58	4.07	3.60	3.28	4.01	3.30
	4	3.29	4.10	3.29	3.65	4.14	3.65	3.29	4.10	3.29
	5	3.29	4.29	3.29	3.66	4.29	3.66	3.28	4.29	3.28
0.5	0	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29
	1	3.29	3.30	3.29	3.45	3.45	3.45	3.29	3.30	3.29
	2	3.29	3.49	3.29	3.52	3.66	3.53	3.28	3.49	3.29
	3	3.29	3.98	3.29	3.58	4.05	3.59	3.28	3.98	3.29
	4	3.29	4.07	3.29	3.65	4.12	3.65	3.29	4.07	3.29
	5	3.29	4.29	3.29	3.66	4.29	3.66	3.28	4.29	3.29

Table 8 Nominal (NO), robust optimization (RO), and adapative robust optimization (ARO) approach are evaluated at the nominal, worst-case, and expected scenario based on their costs in millions by means of a Folding Horizon.

An interesting avenue for further research is to find robust solutions for uncertain delivery times of the food aid due to delays in the harbor. Moreover, it could be interesting to use a different uncertainty set than the ellipsoidal uncertainty set for the procurement prices. Finally, in our current approach the procurement prices estimated for future periods are identical, because we use the average over the past prices for all future periods. It would be more realistic to estimate future procurement prices using a more advanced forecasting method. Note that this would only change our results, but would not influence our methodology.

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Appendices

A. ARO objective formulation

$$
\begin{aligned}
\mathrm{PC} &= \sum_{j\in\mathcal{N}_{\mathcal{TD}}} \bar{F}_{ijk1}\left(\sum_{i\in\mathcal{N}_{\mathcal{SI}}}\sum_{k\in\mathcal{K}}\theta_{ik1} + \sum_{i\in\mathcal{N}_{\mathcal{S}}\setminus\mathcal{N}_{\mathcal{SI}}}\sum_{k\in\mathcal{K}}\mu_{ik1}\right) + \\
&\sum_{i\in\mathcal{N}_{\mathcal{SI}}}\sum_{k\in\mathcal{K}}\sum_{t\geq 2}\left(\theta_{ikt}\sum_{j\in\mathcal{N}_{TD}}\left(\bar{F}_{ijkt} + \left(\boldsymbol{v}_{ijkt}^{t}\right)^{\top}\boldsymbol{\zeta}^{t}\right)\right) + \\
&\sum_{i\in\mathcal{N}_{\mathcal{S}}\setminus\mathcal{N}_{\mathcal{SI}}}\sum_{k\in\mathcal{K}}\sum_{t\geq 2}\left((\mu_{ikt} + \zeta_{m_i kt})\sum_{j\in\mathcal{N}_{TD}}\left(\bar{F}_{ijkt} + \left(\boldsymbol{v}_{ijkt}^{t}\right)^{\top}\boldsymbol{\zeta}^{t}\right)\right) \\
&= \sum_{i\in\mathcal{N}_{\mathcal{SI}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\theta_{ikt}\bar{F}_{ijkt} + \sum_{i\in\mathcal{N}_{\mathcal{S}}\setminus\mathcal{N}_{\mathcal{SI}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\mu_{ikt}\bar{F}_{ijkt} + \\
&\underbrace{\left[\sum_{i\in\mathcal{N}_{\mathcal{SR}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\bar{F}_{ij12},\ldots,\sum_{i\in\mathcal{N}_{\mathcal{SL}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\bar{F}_{ijK2},\ldots,\sum_{i\in\mathcal{N}_{\mathcal{SR}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\bar{F}_{ij1T},\ldots,\sum_{i\in\mathcal{N}_{\mathcal{SL}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\bar{F}_{ijKT}\right]}_{\left(\bar{\boldsymbol{F}}_{RL}^{P,M}\right)^{\top}}\boldsymbol{\zeta} + \\
&\sum_{i\in\mathcal{N}_{\mathcal{SI}}}\sum_{k\in\mathcal{K}}\sum_{t\geq 2}\left(\theta_{ikt}\sum_{j\in\mathcal{N}_{TD}}\left(\boldsymbol{v}_{ijkt}^{t}\right)^{\top}\boldsymbol{\zeta}^{t}\right) + \sum_{i\in\mathcal{N}_{\mathcal{S}}\setminus\mathcal{N}_{\mathcal{SI}}}\sum_{k\in\mathcal{K}}\sum_{t\geq 2}\left(\mu_{ikt}\sum_{j\in\mathcal{N}_{TD}}\left(\boldsymbol{v}_{ijkt}^{t}\right)^{\top}\boldsymbol{\zeta}^{t}\right) + \\
&\sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}}\sum_{t\geq 2}\left(\zeta_{mkt}\sum_{i\in m}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\left(\boldsymbol{v}_{ijkt}^{t}\right)^{\top}\boldsymbol{\zeta}^{t}\right) + \\
&= \sum_{i\in\mathcal{N}_{\mathcal{SI}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\theta_{ikt}\bar{F}_{ijkt} + \sum_{i\in\mathcal{N}_{\mathcal{S}}\setminus\mathcal{N}_{\mathcal{SI}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\mu_{ikt}\bar{F}_{ijkt} + \left(\bar{\boldsymbol{F}}_{RL}^{P,M}\right)^{\top}\boldsymbol{\zeta} + \\
&\underbrace{\left[\sum_{i\in\mathcal{N}_{\mathcal{S}}\setminus\mathcal{N}_{\mathcal{SI}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\mu_{ik2}(\boldsymbol{v}_{ijk2}^{2})^{\top},\ldots,\sum_{i\in\mathcal{N}_{\mathcal{S}}\setminus\mathcal{N}_{\mathcal{SI}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\mu_{ikT}(\boldsymbol{v}_{ijkT}^{T})^{\top}\right]}_{(\boldsymbol{v}_{\mu})^{\top}}\boldsymbol{\zeta} + \\
&\underbrace{\left[\sum_{i\in\mathcal{N}_{\mathcal{SI}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\theta_{ik2}(\boldsymbol{v}_{ijk2}^{2})^{\top},\ldots,\sum_{i\in\mathcal{N}_{\mathcal{SI}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\theta_{ikT}(\boldsymbol{v}_{ijkT}^{T})^{\top}\right]}_{(\boldsymbol{v}_{\theta})^{\top}}\boldsymbol{\zeta} + \\
&\sum_{t\geq 2}(\boldsymbol{\zeta}^{t})^{\top}\underbrace{\left[\sum_{i\in\mathcal{N}_{\mathcal{SR}}}\sum_{j\in\mathcal{N}_{TD}}\boldsymbol{v}_{ij1t}^{t},\ldots,\sum_{i\in\mathcal{N}_{\mathcal{SR}}}\sum_{j\in\mathcal{N}_{TD}}\boldsymbol{v}_{ijKt}^{t},\sum_{i\in\mathcal{N}_{\mathcal{SL}}}\sum_{j\in\mathcal{N}_{TD}}\boldsymbol{v}_{ij1t}^{t},\ldots,\sum_{i\in\mathcal{N}_{\mathcal{SL}}}\sum_{j\in\mathcal{N}_{TD}}\boldsymbol{v}_{ijKt}^{t}\right]^{\top}}_{(\boldsymbol{V}^{t})^{PM}}\boldsymbol{\zeta}^{t} \\
&= \sum_{i\in\mathcal{N}_{\mathcal{SI}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\theta_{ikt}\bar{F}_{ijkt} + \sum_{i\in\mathcal{N}_{\mathcal{S}}\setminus\mathcal{N}_{\mathcal{SI}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\mu_{ikt}\bar{F}_{ijkt} + \boldsymbol{\zeta}^{\top}\bar{\boldsymbol{F}}_{RL}^{PM} + (\boldsymbol{v}_{\mu} + \boldsymbol{v}_{\theta})^{\top}\boldsymbol{\zeta} + \\
&\boldsymbol{\zeta}^{\top}\underbrace{\begin{bmatrix}(\boldsymbol{V}^{2})^{PM} & & \\ & \ddots & \\ & & (\boldsymbol{V}^{T})^{PM}\end{bmatrix}}_{\boldsymbol{Q}(\boldsymbol{V})}\boldsymbol{\zeta},
\end{aligned}
$$

where the first equation follows from substitution of (6) and (12) in the expression for PC as given by (1). The second, third and fourth equations follow from organizing the terms and rewriting every product term including ζ_{mkt} or $\boldsymbol{\zeta}^t$ in a product term including $\boldsymbol{\zeta}$.

$$TC = \sum_{i\in\mathcal{N}_\mathcal{S}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}} p^T_{ijk1}\bar{F}_{ijk1} +$$

$$\sum_{i\in\mathcal{N}_\mathcal{S}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\geq 2} p^T_{ijkt}\left(\bar{F}_{ijkt} + (\boldsymbol{v}^t_{ijkt})^\top\boldsymbol{\zeta}^t\right) + \sum_{i\in\mathcal{N}_\mathcal{T}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\geq 2} p^T_{ijkt}\left(\bar{F}_{ijkt} + \sum_{2\leq t^*\leq t}(\boldsymbol{v}^{t^*}_{ijkt})^\top\boldsymbol{\zeta}^{t^*}\right)$$

$$= \bar{\boldsymbol{F}}^\top\boldsymbol{p}^T + \underbrace{\left[\sum_{i\in\mathcal{N}_\mathcal{S}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}} p^T_{ijk2}\left(\boldsymbol{v}^2_{ijk2}\right)^\top, \ldots, \sum_{i\in\mathcal{N}_\mathcal{S}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}} p^T_{ijkT}\left(\boldsymbol{v}^T_{ijkT}\right)^\top\right]}_{\left(\boldsymbol{v}^S_{p^T}\right)^\top}\boldsymbol{\zeta} +$$

$$\underbrace{\left[\sum_{i\in\mathcal{N}_\mathcal{T}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\geq 2} p^T_{ijkt}\left(\boldsymbol{v}^2_{ijkt}\right)^\top, \sum_{i\in\mathcal{N}_\mathcal{T}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\geq 3} p^T_{ijkt}\left(\boldsymbol{v}^3_{ijkt}\right)^\top, \ldots, \sum_{i\in\mathcal{N}_\mathcal{T}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}} p^T_{ijkT}\left(\boldsymbol{v}^T_{ijkT}\right)^\top\right]}_{\left(\boldsymbol{v}^T_{p^T}\right)^\top}\boldsymbol{\zeta},$$

where the first equation follows from substitution of (12) in (2). The second equation follows from rewriting all terms in vector notation.

$$HC = \sum_{i\in\mathcal{N}_\mathcal{S}}\sum_{j\in\mathcal{N}_{TD}}\sum_{k\in\mathcal{K}} p^H_j\bar{F}_{ijk1} + \sum_{i\in\mathcal{N}_\mathcal{S}}\sum_{j\in\mathcal{N}_{TD}}\sum_{k\in\mathcal{K}}\sum_{t\geq 2} p^H_j\left(\bar{F}_{ijkt} + \left(\boldsymbol{v}^t_{ijkt}\right)^\top\boldsymbol{\zeta}^t\right) +$$

$$\sum_{i\in\mathcal{N}_T}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\geq 2} p^H_j\left(\bar{F}_{ijkt} + \sum_{2\leq t^*\leq t}\left(\boldsymbol{v}^{t^*}_{ijkt}\right)^\top\boldsymbol{\zeta}^{t^*}\right)$$

$$= \sum_{i\in\mathcal{N}_{\mathcal{ST}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} p^H_j\bar{F}_{ijkt} + \underbrace{\left[\sum_{i\in\mathcal{N}_\mathcal{S}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}} p^H_j\left(\boldsymbol{v}^2_{ijk2}\right)^\top, \ldots, \sum_{i\in\mathcal{N}_\mathcal{S}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}} p^H_j\left(\boldsymbol{v}^T_{ijkT}\right)^\top\right]}_{\left(\boldsymbol{v}^S_{p^H}\right)^\top}\boldsymbol{\zeta} +$$

$$\underbrace{\left[\sum_{i\in\mathcal{N}_\mathcal{T}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\geq 2} p^H_j\left(\boldsymbol{v}^2_{ijkt}\right)^\top, \sum_{i\in\mathcal{N}_\mathcal{T}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\geq 3} p^H_j\left(\boldsymbol{v}^3_{ijkt}\right)^\top, \ldots, \sum_{i\in\mathcal{N}_\mathcal{T}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}} p^H_j\left(\boldsymbol{v}^T_{ijkT}\right)^\top\right]}_{\left(\boldsymbol{v}^T_{p^H}\right)^\top}\boldsymbol{\zeta},$$

where the first equation follows from substitution of (12) in (3). The second equation follows from rewriting all terms including the uncertain parameter in vector notation.

$$SC = \sum_{i\in\mathcal{N}_\mathcal{T}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}} p^S_i\bar{F}_{ijk1} + \sum_{i\in\mathcal{N}_\mathcal{T}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\geq 2} p^S_i\left(\bar{F}_{ijkt} + \sum_{2\leq t^*\leq t}\left(\boldsymbol{v}^{t^*}_{ijkt}\right)^\top\boldsymbol{\zeta}^{t^*}\right)$$

$$= \sum_{i\in\mathcal{N}_\mathcal{T}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} p^S_i\bar{F}_{ijkt} +$$

$$\underbrace{\left[\sum_{i\in\mathcal{N}_\mathcal{T}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\geq 2} p_i^S\left(\boldsymbol{v}_{ijkt}^2\right)^\top, \sum_{i\in\mathcal{N}_\mathcal{T}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\geq 3} p_i^S\left(\boldsymbol{v}_{ijkt}^3\right)^\top, \ldots, \sum_{i\in\mathcal{N}_\mathcal{T}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}} p_i^S\left(\boldsymbol{v}_{ijkT}^T\right)^\top\right]}_{\left(\boldsymbol{v}_{p^S}\right)^\top}\boldsymbol{\zeta},$$

where the first equation follows from substitution of (12) in (4) and the second equation follows from rewriting all terms including the uncertain parameter in vector notation.

Hence, if we take

$$\begin{aligned} r(\bar{\boldsymbol{F}}) = &\sum_{i\in\mathcal{N}_{\mathcal{SI}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\theta_{ikt}\bar{F}_{ijkt} + \sum_{i\in\mathcal{N}_\mathcal{S}\setminus\mathcal{N}_{\mathcal{SI}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\mu_{ikt}\bar{F}_{ijkt} + \\ &\sum_{i\in\mathcal{N}_\mathcal{T}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}} p_i^S\bar{F}_{ijkt} + \sum_{i\in\mathcal{N}_{\mathcal{ST}}}\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\sum_{k\in\mathcal{K}}\sum_{t\in\mathcal{T}}\left(p_{ijkt}^T + p_j^H\right)\bar{F}_{ijkt} \end{aligned}$$

and

$$\boldsymbol{s}\left(\bar{\boldsymbol{F}}, \boldsymbol{V}\right) = \bar{\boldsymbol{F}}_{RL}^{PM} + \boldsymbol{v}_\mu + \boldsymbol{v}_\theta + \boldsymbol{v}_{p^T}^S + \boldsymbol{v}_{p^T}^T + \boldsymbol{v}_{p^H}^S + \boldsymbol{v}_{p^H}^T + \boldsymbol{v}_{p^S},$$

using the epigraph formulation we obtain problem (16).

B. ARO reformulation

Capacity constraints (17f):

$$\begin{aligned} &\sum_{j\in\mathcal{N}_{\mathcal{TD}}} F_{ijkt} \leq c_{ikt}^P && i\in\mathcal{N}_S, k\in\mathcal{K}, t\in\{2,\ldots,T\}, \forall\boldsymbol{\zeta}\in\mathcal{U} \\ \iff &\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\bar{F}_{ijkt} + \sum_{j\in\mathcal{N}_{\mathcal{TD}}}(\boldsymbol{v}_{ijkt}^t)^\top\boldsymbol{\zeta}^t \leq c_{ikt}^P && i\in\mathcal{N}_S, k\in\mathcal{K}, t\in\{2,\ldots,T\}, \forall\boldsymbol{\zeta}\in\mathcal{U} \\ \iff &\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\bar{F}_{ijkt} + \sum_{j\in\mathcal{N}_{\mathcal{TD}}}\left[0,\ldots,0,(\boldsymbol{v}_{ijkt}^t)^\top,0,\ldots,0\right]\boldsymbol{\zeta} \leq c_{ikt}^P && i\in\mathcal{N}_S, k\in\mathcal{K}, \in\{2,\cdots,T\}, \forall\boldsymbol{\zeta}\in\mathcal{U} \\ \iff &\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\bar{F}_{ijkt} + \Omega\left\|\boldsymbol{L}^\top\left(\sum_{j\in\mathcal{N}_{\mathcal{TD}}}\boldsymbol{v}_{ijkt}^{0,t,0}\right)\right\|_2 \leq c_{ikt}^P && i\in\mathcal{N}_\mathcal{S}, k\in\mathcal{K}, t\in\{2,\cdots,T\}. \end{aligned}$$

Capacity constraints (17g) **-** (17h):

$$\begin{aligned} &\sum_{k\in\mathcal{K}} F_{ijkt} \leq c_{ijt}^T && i\in\mathcal{N}_{\mathcal{ST}}, j\in\mathcal{N}_{\mathcal{TD}}, t\in\{2,\ldots,T\}, \forall\boldsymbol{\zeta}\in\mathcal{U} \\ \iff &\begin{cases} \sum_{k\in\mathcal{K}}\bar{F}_{ijkt} + \left(\sum_{k\in\mathcal{K}}\boldsymbol{v}_{ijkt}^t\right)^\top\boldsymbol{\zeta}^t \leq c_{ijt}^T & i\in\mathcal{N}_S, j\in\mathcal{N}_{\mathcal{TD}}, t\in\{2,\cdots,T\}, \forall\boldsymbol{\zeta}\in\mathcal{U} \\ \sum_{k\in\mathcal{K}}\bar{F}_{ijkt} + \sum_{t^*\leq t}\left(\sum_{k\in\mathcal{K}}\boldsymbol{v}_{ijkt}^{t^*}\right)^\top\boldsymbol{\zeta}^{t^*} \leq c_{ijt}^T & i\in\mathcal{N}_T, j\in\mathcal{N}_{\mathcal{TD}}, t\in\{2,\cdots,T\}, \forall\boldsymbol{\zeta}\in\mathcal{U} \end{cases} \\ \iff &\begin{cases} \sum_{k\in\mathcal{K}}\bar{F}_{ijkt} + \sum_{k\in\mathcal{K}}\left[0,\ldots,0,(\boldsymbol{v}_{ijkt}^t)^\top,0,\ldots,0\right]^\top\boldsymbol{\zeta} \leq c_{ijt}^T & i\in\mathcal{N}_S, j\in\mathcal{N}_{\mathcal{TD}}, t\in\{2,\cdots,T\}, \forall\boldsymbol{\zeta}\in\mathcal{U} \\ \sum_{k\in\mathcal{K}}\bar{F}_{ijkt} + \sum_{k\in\mathcal{K}}\left[(\boldsymbol{v}_{ijkt}^1)^\top,\ldots,(\boldsymbol{v}_{ijkt}^{t-1})^\top,0,\ldots,0\right]^\top\boldsymbol{\zeta} \leq c_{ijt}^T & i\in\mathcal{N}_T, j\in\mathcal{N}_{\mathcal{TD}}, t\in\{2,\cdots,T\}, \forall\boldsymbol{\zeta}\in\mathcal{U} \end{cases} \end{aligned}$$

$$\iff \begin{cases} \sum_{k\in\mathcal{K}} \bar{F}_{ijkt} + \Omega \left\| \boldsymbol{L}^\top \left(\sum_{k\in\mathcal{K}} \boldsymbol{v}_{ijkt}^{0,t,0} \right) \right\|_2 \le c_{ijt}^T & i\in\mathcal{N}_S, j\in\mathcal{N}_{\mathcal{TD}}, t\in\{2,\cdots,T\} \\ \sum_{k\in\mathcal{K}} \bar{F}_{ijkt} + \Omega \left\| \boldsymbol{L}^\top \left(\sum_{k\in\mathcal{K}} \boldsymbol{v}_{ijkt}^{1-t,0} \right) \right\|_2 \le c_{ijt}^T & i\in\mathcal{N}_T, j\in\mathcal{N}_{\mathcal{TD}}, t\in\{2,\cdots,T\}. \end{cases}$$

Capacity constraints (17i):

$$\sum_{i\in\mathcal{N}_{\mathcal{ST}}} \sum_{k\in\mathcal{K}} F_{ijkt} \le c_{jt}^H \qquad i\in\mathcal{N}_{\mathcal{TD}}, t\in\{2,\ldots,T\}, \forall\boldsymbol{\zeta}\in\mathcal{U}$$

$$\iff \sum_{i\in\mathcal{N}_{\mathcal{ST}}} \sum_{k\in\mathcal{K}} \bar{F}_{ijkt} + \left(\sum_{i\in\mathcal{N}_{\mathcal{T}}} \sum_{k\in\mathcal{K}} \boldsymbol{v}_{ijkt}^t \right)^\top \boldsymbol{\zeta} \le c_{jt}^H \qquad i\in\mathcal{N}_{\mathcal{TD}}, \in\{2,\cdots,T\}, \forall\boldsymbol{\zeta}\in\mathcal{U}$$

$$\iff \sum_{i\in\mathcal{N}_{\mathcal{ST}}} \sum_{k\in\mathcal{K}} \bar{F}_{ijkt} + \sum_{i\in\mathcal{N}_{\mathcal{T}}} \sum_{k\in\mathcal{K}} \left(\boldsymbol{v}_{ijkt}^{1-t_S,0} \right)^\top \boldsymbol{\zeta} \le c_{jt}^H \qquad i\in\mathcal{N}_{\mathcal{TD}}, t\in\{2,\cdots,T\}, \forall\boldsymbol{\zeta}\in\mathcal{U}$$

$$\iff \sum_{i\in\mathcal{N}_{\mathcal{ST}}} \sum_{k\in\mathcal{K}} \bar{F}_{ijkt} + \Omega \left\| \boldsymbol{L}^\top \left(\sum_{i\in\mathcal{N}_{\mathcal{T}}} \sum_{k\in\mathcal{K}} \boldsymbol{v}_{ijkt}^{1-t_S,0} \right) \right\|_2 \le c_{jt}^H \qquad i\in\mathcal{N}_{\mathcal{TD}}, t\in\{2,\cdots,T\}.$$

C. Proof of Theorem 1

Let $q_b(\boldsymbol{\zeta}) = -\boldsymbol{\zeta}^\top \boldsymbol{Q}(\boldsymbol{V})\boldsymbol{\zeta} - \boldsymbol{s}(\bar{\boldsymbol{F}}, \boldsymbol{V})^\top \boldsymbol{\zeta} - r(\bar{\boldsymbol{F}}) + q$ and let $q_a(\boldsymbol{\zeta}) = \Omega^2 - \boldsymbol{\zeta}^\top \boldsymbol{\Sigma}^{-1}\boldsymbol{\zeta}$, where $r(\bar{\boldsymbol{F}})$, $\boldsymbol{s}(\bar{\boldsymbol{F}}, \boldsymbol{V})$ and $\boldsymbol{Q}(\boldsymbol{V})$ are given in Appendix A. Then for all $\boldsymbol{\zeta}$: $q_a(\boldsymbol{\zeta}) \ge 0$ implies $q_b(\boldsymbol{\zeta}) \ge 0$. Therefore, we can apply the S-lemma and we get $\exists\lambda \ge 0$ such that $q_b(\boldsymbol{\zeta}) \ge \lambda q_a(\boldsymbol{\zeta})$ for all $\boldsymbol{\zeta}$. Replacing $q_a(\boldsymbol{\zeta})$ and $q_b(\boldsymbol{\zeta})$ by their expressions we get

$$\exists\lambda\ge 0 \text{ s.t. } -\boldsymbol{\zeta}^\top \boldsymbol{Q}(\boldsymbol{V})\boldsymbol{\zeta} - \boldsymbol{s}(\bar{\boldsymbol{F}}, \boldsymbol{V})^\top \boldsymbol{\zeta} - r(\bar{\boldsymbol{F}}) + q \ge \lambda(\Omega^2 - \boldsymbol{\zeta}^\top \boldsymbol{\Sigma}^{-1}\boldsymbol{\zeta}) \qquad \forall\boldsymbol{\zeta}$$

$$\iff \exists\lambda\ge 0 \text{ s.t. } \boldsymbol{\zeta}^\top (\lambda\boldsymbol{\Sigma}^{-1} - \boldsymbol{Q}(\boldsymbol{V}))\boldsymbol{\zeta} - \boldsymbol{s}(\bar{\boldsymbol{F}}, \boldsymbol{V})^\top \boldsymbol{\zeta} - r(\bar{\boldsymbol{F}}) - \lambda\Omega^2 + c \ge 0 \qquad \forall\boldsymbol{\zeta}$$

$$\iff \exists\lambda\ge 0 \text{ s.t. } \begin{pmatrix} \zeta \\ 1 \end{pmatrix}^\top \begin{pmatrix} \lambda\boldsymbol{\Sigma}^{-1} - \boldsymbol{Q}(\boldsymbol{V}) & -\frac{1}{2}\boldsymbol{s}(\bar{\boldsymbol{F}}, \boldsymbol{V}) \\ -\frac{1}{2}\boldsymbol{s}(\bar{\boldsymbol{F}}, \boldsymbol{V})^\top & -r(\bar{\boldsymbol{F}}) - \lambda\Omega^2 + q \end{pmatrix} \begin{pmatrix} \zeta \\ 1 \end{pmatrix} \ge 0 \qquad \forall\boldsymbol{\zeta}$$

$$\iff \exists\lambda\ge 0 \text{ s.t. } \begin{pmatrix} \lambda\boldsymbol{\Sigma}^{-1} - \boldsymbol{Q}(\boldsymbol{V}) & -\frac{1}{2}\boldsymbol{s}(\bar{\boldsymbol{F}}, \boldsymbol{V}) \\ -\frac{1}{2}\boldsymbol{s}(\bar{\boldsymbol{F}}, \boldsymbol{V})^\top & -r(\bar{\boldsymbol{F}}) - \lambda\Omega^2 + q \end{pmatrix} \succeq \mathrm{O}.$$

To see why the last equivalence holds, let $\boldsymbol{X} = \begin{pmatrix} \lambda\boldsymbol{\Sigma}^{-1} - \boldsymbol{Q}(\boldsymbol{V}) & -\frac{1}{2}\boldsymbol{s}(\bar{\boldsymbol{F}}, \boldsymbol{V}) \\ -\frac{1}{2}\boldsymbol{s}(\bar{\boldsymbol{F}}, \boldsymbol{V})^\top & -r(\bar{\boldsymbol{F}}) - \lambda\Omega^2 + q \end{pmatrix}$. Suppose there exists a $\boldsymbol{y}$ such that $\boldsymbol{y}^\top \boldsymbol{X}\boldsymbol{y} < 0$ and suppose the last entry of $\boldsymbol{y}$ is nonzero, say α. Take $\lambda = \frac{1}{\alpha}$. Then $(\lambda\boldsymbol{y})^\top \boldsymbol{X}(\lambda\boldsymbol{y}) < 0$. This is a contradiction since $\begin{pmatrix} \zeta \\ 1 \end{pmatrix}^\top \boldsymbol{X} \begin{pmatrix} \zeta \\ 1 \end{pmatrix} \ge 0$ for all $\boldsymbol{\zeta}$. Now suppose that the last entry of $\boldsymbol{y}$ equals zero. Since the mapping $\boldsymbol{y} \mapsto \boldsymbol{y}^T \boldsymbol{X}\boldsymbol{y}$ is continuous, there exists a $\bar{\boldsymbol{y}} \ne 0$ such that $\bar{\boldsymbol{y}}^\top \bar{X}\boldsymbol{y} < 0$ and the last entry of $\bar{\boldsymbol{y}}$ is nonzero. By the previous argument, we again obtain a contradiction (Hall., n.d.).