One-Pass Average Incremental Cost Pricing

Richard P O'Neill, Yonghong Chen & Peter Whitman
July 2023

Abstract

Since the inception of ISOs, Locational Marginal Prices (LMPs) alone were not incentive compatible because an auction winner who offered its avoidable costs could lose money at the LMP. ISOs used make-whole payments to ensure market participants did not lose money. Make-whole payments were not public creating transparency issues. Over time, the ISOs employed methods to raise the price. They reduced but did not eliminate make-whole payments. In addition, the new 'LMPs' were too high to send a good marginal entry signal and too low to send an incremental entry signal. Some ISOs introduced capacity markets to remedy this problem. Capacity markets brought their own set of issues. The objective of the paper is to introduce a pricing scheme that improves on the current schemes and is more aligned with the theory of markets with scale economies. It eliminates make-whole payments, increases transparency, allocates the costs to period that caused them, sends better price signals, and lowers capacity market prices. We introduce the one-pass average incremental cost (AIC) pricing methodology to the multi-period model with multi-step marginal cost functions, ramp and transmission constraints, and a co-optimized reserves market. AIC prices address these issues in a positive economic way. Market rules, locational incremental energy price (LIPs) along with LMPs produce incentive compatibility. No market participant dispatched who offered its avoidable costs losses money. LIPs eliminate make-whole payments making the market more transparent. Generally, LIPs and LMPs send a good entry signals and have better economic design properties. These properties are proved theoretically, demonstrated on small examples, and demonstrated on actual ISO market problems.

1 Introduction

For the last century of US electric power markets, prices were calculated using the cost-of-service model and its cost allocation rules designed for monopolies. Vertically integrated utilities with monopoly franchises, forecasted demand, scheduled their own generation, and sent invoices to consumers once a month with a single energy price. Many still do. Over time, technology advances and government subsidies reduced prices and markets grew. Utilities made money through a regulated return on owned capital investments. Over-investment that occurs under the cost-of-service regime was held in check by state commissions. Wholesale energy transactions took place in power pools or were negotiated, but needed Federal Energy Regulatory Commission (FERC) approval.

Corresponding author(s): Yonghong Chen or Richard O'Neill

In 1984, Bohn et al proposed optimal spot pricing of electricity – later called the locational marginal price (LMP), and spot wheeling charges – later called transmission congestion rent (see Bohn et al. (1984)). In 1988, Schweppe et al popularized these concepts (see Schweppe et al. (2013)).

In the U.S. wholesale power markets, prices and procedures are regulated by the FERC under the 'just and reasonable' and 'not unduly discriminatory' standard of the Federal Power Act (FPA) of 1935. The double negative implies possible due discrimination. In 1989, Varian finds price discrimination is an ubiquitous phenomenon and examined some schemes to enhance economic welfare. For public utilities, he found that pricing schedules had become so complex that households often make the "wrong" choice electricity use (see Varian (1989)). For centuries, the 'just' price has been debated by Aristotle, Aquinas and others in the Roman Church, John Locke, and more recently, regulatory commissions (see Mueller & Gerber (2020)). The courts have agreed with FERC that efficient competition can set just and reasonable prices.

In 1996, FERC issued Order 888 that required open access to transmission and listed the requirements to become an Independent System Operators (ISO). ISOs formed, first from power pools, with wholesale auction markets repeated daily with hourly and later sub-hourly prices. In 2022, nine ISOs/RTOs (here after called ISOs) serve over two-thirds of US electricity consumers and more than 50% of Canada's population (see ISO/RTO COUNCIL).

In the day-ahead market, market participants submit bids and offers to the ISO. To satisfy the just and reasonable requirements, the ISO's market monitor mitigates the offers if they do not reflect the unit's estimated avoidable (aka incremental) cost. The ISO solves the market auction for efficient commitment and dispatch with 36 to 48 hourly periods. The ISO schedules the first 24 periods. Next, the ISO runs a pricing algorithm to obtain the 24-hourly nodal prices and settles the market with nodal prices and make-whole payments creating a hedge against the real-time market prices. Energy and other prices are announced to the public after the schedule is determined.

ISO real-time markets are a repeated series of next-period-only (for example, every 5-minutes) markets. The operator either uses judgement and/or a look-ahead model to commit units for future periods, if necessary. The system operator sends dispatch signals and announces the energy prices for each period. If the generator's real-time dispatch signal deviates from day-ahead schedule, it can buy back or sell from its schedule based on real time price. If the generator does not perform according to its real-time dispatch signal, it may be subject to a monetary penalty. Since the inception of ISO markets and the first England-Wales market, the question has been what should the prices be when there are avoidable fixed costs and passive consumers represented by a point forecast of demand?

Auction Market Mechanisms. An auction market mechanism can be viewed as an institution with rules including tariffs governing the market outcome. (see Mas-Colell et al. (1995) and Royal Swedish Academy of Sciences). In a two-sided ISO auction, auction rules are designed to have the following properties: Truth telling/incentive compatibility enforced by, if needed, mitigation, monetary penalties for rule violations, enforcement, and merger authority. Individual rationality is satisfied since the choice to go off-grid or self-schedule can be very expensive. Economic efficiency is satisfied by the software designed for Pareto efficiency (aka maximum market surplus). Revenue neutrality/balanced transfers is satisfied by market rules. Unfortunately, it is difficult to achieve all these requirements (see Myerson & Satterthwaite (1983)). Although possibly implied by the above properties, we would add additional desirable properties such as prices that are understandable and transparent, and sustain the market into the future. FERC rules are designed to achieve the above desired properties and achieving these properties satisfy the 'just and reasonable' and the 'not unduly discriminatory' pricing requirements. Actual ISO market design is a blend of economic theory, properties of AC power flow, approximations, computability, and compromise.

In game theory and economics, incentive compatibility occurs when the incentives that motivate the actions of individual participants are consistent with following the rules established by the group. Truthful bids and offers are needed for economic efficiency. In the day-ahead and real-time market, bids and offers are mitigated. Table 1 shows the recent mitigation (called capping) in PJM. Table 1 does not reflect the market participants that offered truthfully because they would not have been mitigated. Other ISOs report similar statistics. In addition, from 2007 to 2021, FERC has ordered over $1.3 billion in penalties under the manipulation mandate and rules violations (see FERC (2021)). Virtual bidding in the day-ahead market also helps prevent market power.

Table 1. Offer Mitigation in PJM from 2016 to 2020.

	Real-Time		Day-Ahead	
Year	Unit Hours Capped	MWh Capped	Unit Hours Capped	MWh Capped
2016	0.4%	0.3%	0.1%	0.1%
2017	0.4%	0.4%	0.1%	0.2%
2018	1.0%	0.8%	0.2%	0.3%
2019	1.7%	1.3%	1.3%	0.9%
2020	1.0%	1.1%	1.6%	1.3%

Source: Monitoring Analytics, Annual State of the Market, 2020

There are at least three legitimate reasons why generators should be allowed to deviate from truthful bidding: if the minimum run time spans more than the market horizon; if a unit needs more than the market horizon to recover its costs; or if the participation model does not allow the unit to represent its full incremental costs, for example, the configuration decision of combined-cycle gas turbines (CCGTs) or the generation-pump cycle decision of a pumped-storage unit. These conditions are easily checked. The remaining reasons involve strategic behavior that may result in inefficient dispatch, for example, an attempt to exercise market power by withholding. With self-scheduling, the resulting dispatch may not be efficient. Self-schedules receive only the LMPs and no make-whole payments.

In ISOs, self-scheduling is a misnomer. In ISO-NE, self-schedules are placed into the market as flexible resources with a maximum operating level at the self-schedule and a marginal cost offer at the minimum allowable (currently -$150/MWh). In PJM, self-scheduling rules are pages long (see PJM (2021)). Acting alone, if the self-schedule is greater than the unit's optimal dispatch, the system LMPs will be lower than the efficient price; if lower, the LMPs are higher.

An ISO may have several problems if too many market participants self-schedule.

As the fraction of self-scheduled resources increases, it becomes increasingly challenging for the ISO to balance the system; the solution efficiency decreases, and the market becomes an arbitrary non-economic physical balancing mechanism. In addition, it may cause renewable curtailments. Markets with significant self-scheduling achieve efficient dispatch and efficient prices only by chance. The future markets are projected to need more flexibility from market participants (see Orvis & Aggarwal (2018)).

The overall goal of pricing is to produce efficient two-sided markets with as little government intervention as possible. Energy is a private good and reserves (to prevent blackouts) are a public good. Energy and reserves are joint products. Prices should perform several basic functions. They should settle the market; signal entry and exit; and create incentives for greater efficiency of existing assets. In addition, they should be revenue neutral and prevent inefficient arbitrage.

Ideally, short-term market efficiency is achieved in day-ahead and real-time markets. Longer-term market efficiency is achieved by prices in the short-term markets that incent efficient sustainable investment for both producers and consumers. To achieve this, prices must be transparent and understandable and non-confiscatory. In the day-ahead and real-time markets, investments costs are sunk (not avoidable). With truthful bids and offers and the ability of software to solve the auction market, the market yields an efficient dispatch. In the long run, investments costs are avoidable and should be covered by discounted short-term market profits.

Initially, the ISOs adopted the LMP pricing concepts, but quickly realized that some optimally dispatched generators were losing money under LMP pricing and introduced make-whole payments to avoid these losses. The LMPs were made public, but the make-whole payments are considered private information and only aggregates were made public, diluting price transparency. ISOs and load serving entities (LSEs) kept the practice of forecasting demand – a principal-agent problem – creating a one-sided less-flexible auction for what should be a two-sided auction. Ideally, each consumer should have been expressing its value of consumption, its willingness to shift consumption to other periods, and constraints on its consumption. When consumers bid into the market, the scarcity prices and consumption quantities are 'crowd sourced' and not the result of estimates by less financially motivated entities. Since the inception of ISOs, the pricing rules have changed constantly with the purpose of increasing the price because the revenues from LMPs were too low to sustain the markets.

Until 2005, Lagrangian relaxation (LR) 'solved' the unit commitment problem, but the dispatch was usually infeasible. Heuristics were needed to find a feasible, not necessarily optimal, dispatch. The dual variables from the LR process have weak, possibility misleading, economic properties. In 2005, after significant improvements in mixed integer programming (MIP), PJM introduced MIP as their unit commitment and dispatch solver. MIP eliminated the infeasibility problem of LR and improved the dispatch efficiency. By 2018, all ISOs were using MIP with estimated cost savings more than five billion dollars per year (see O'Neill (2017b)).

In the MIP formulation, mathematically, the LMPs do not exist due to binary variables. For MIP problems, many commercial software packages create a linear program by fixing each binary to its MIP solution value. This is equivalent to assuming avoidable fixed costs are sunk. The dual variables on the energy balance and reserve constraints alone have no claim to being market clearing (see Vyve (2011)). LMPs are short-term low-level marginal entry signals.

Basic microeconomics assumes convexity and differentiability and provides only a naïve siren song for understanding non-convex markets. In convex markets, the efficient dispatch problem can be represented as a convex optimization problem. In strictly convex markets, dual variables (LMPs) are unique and have desirable economic properties as prices, that is, LMPs alone clear and settle the markets, and signal efficient entry and exit. In convex markets, if primal degeneracy involves the energy balance or reserve constraints, there may be multiple optimal dual variables that are the source of the prices. Linear program (LP) solvers produce only one set of prices that can result in arbitrary settlements, and weaker entry and exit signals. For more information on convexity (see Mangasarian (1969), Rockafellar (1970)).

In the absence of convexity, LMPs may not support an optimal commitment and dispatch schedule. Markets may have empty cores. ISO short-term markets may not be sub-market (sub-game) efficient. In 2005, O'Neill et al showed that LMPs plus make-whole payments are market clearing for generation (see O'Neill et al. (2005)). That is, efficient discriminatory prices are necessary to achieve efficiency of a two-sided non-convex markets with avoidable fixed costs.

In the presence of avoidable fixed costs, no single price performs the function that LMPs perform in strictly convex markets. Most fossil generators and large industrial consumers have scale economies. For a more discussion of pricing with declining average costs (see Baumol et al. (1982)).

Over time, LMPs gradually disappeared from ISO markets as modified pricing algorithms produced energy prices (dual variables) different from the LMPs, but these prices are still called LMPs (now an umbrella term). In the pricing run, some ISOs relaxed generator minimum operating levels, some relaxed the binaries, and some modified the marginal energy costs by including some fixed costs. This led to higher prices and lower make-whole payments. The modified LMPs are neither fish nor fowl. They are usually too high to be a low-level marginal entry signal and too low to eliminate make-whole payments.

In 2007, Gribik et al proposed Convex Hull Pricing (CHP) (see Gribik et al. (2007)). In 2015, Schiro et al presented some counterintuitive properties of CHP. CHP does not eliminate make-whole payments, pays generators to stay on the dispatch signals and is not always revenue adequate (see Schiro et al. (2015)). In addition, CHP spawned a series of papers on approximating the convex hull (see Wang et al. (2013) and Wang et al. (2016)). In 2016, Hua and Baldick introduced conditions for solving CHP exactly (see Hua & Baldick (2016)). In 2016, MISO implemented a single interval CHP approximation that allocates some avoidable fixed costs to the marginal cost function, but penalizes departures from dispatch signals (see Wang et al. (2016)). In 2018, Borokhov described a modification of CHP that relaxes the requirement for a convex price-quantity curve (see Borokhov (2018). In 2019, Chen and Wang proposed single period approximate CHP for piecewise linear incremental energy function (see Chen & Wang (2019)). In 2019, Chao introduced an LP approach to solve CHP with multi-step incremental energy functions (see Chao (2019)). In

2020, Yu et al developed an extended convex hull formulation approximation that can solve multi-interval CHP on MISO day-ahead cases (see Yu et al. (2020)). In 2021, Andrianes et al used Dantzig-Wolfe Decomposition to solve CHP problem (see Andrianesis et al. (2021)). In 2018, Eldridge et. al. examine pricing properties of near-optimal unit commitment solutions resulting in potentially large wealth transfers from sub-optimal solutions. Results on a selected set of problems demonstrate that approximate Convex Hull Pricing (aCHP) may eliminates most erratic price behavior (Eldridge et al. (2018)).

In 2016, for single-period markets with fixed costs and inelastic demand, Liberopoulos and Andrianes reviewed several pricing schemes for the price, uplifts, and profits and compared these schemes along these three dimensions. They present results for supplier strategic bidding behavior in the context of the considered pricing schemes (see Liberopoulos & Andrianesis (2016)).

In 2016, O'Neill et al present an approach to efficient prices and cost allocation for a revenue neutral and non-confiscatory day-ahead market. They propose an ex-post multi-part pricing scheme, called the Dual Pricing Algorithm, that can be incorporated into current day-ahead markets without altering the the efficient market equilibrium (see O'Neill et al. (2016)).

In 2017, Eldridge et. al proposed a method for updating the loss approximation. If the update procedure converges, it gives a solution to a nonlinear problem and shows rapid convergence properties on all networks tested. (see Eldridge et al. (2017a) and Eldridge et al. (2017b)).

In 2020, Hytowitz et al examine impacts of price formation efforts considering high renewable penetration levels and system resource adequacy targets and highlight the importance of scarcity price assumptions. (see Hytowitz et al. (2020)). In 2022, FERC posted: 'Use of uplift payments can undermine the market's ability to send actionable price signals ... and should be priced in the market.' (see Topping (2022)). In a broad sense, uplift includes make-whole payments and capacity market payments.

2 Average Incremental Cost (AIC) Pricing

In 2017, O'Neill introduced AIC prices and an iterative process for their calculation (see O'Neill (2017a)). AIC prices eliminate the need for make-whole payments, create better incentives for infra-incremental generators, and send better entry and exit price signals (see O'Neill et al. (2020)). Most fossil and some non-fossil generators have declining AICs. This paper introduces the AIC One-Pass Pricing (AICOP) methodology to solve for AIC prices through a linear program relaxation of the security constrained unit commitment (SCUC) optimal (or near optimal) solution. For dispatched generators, the one-pass AIC methodology results in profitable energy and reserve prices without make-whole payments. (Here we define profitable to include breaking even.) Excursions from the dispatch signal pay at a minimum the cost of redispatch (aka liquidated damages) or receive a lower energy price, thereby reducing the incentive to self-schedule.

The intuition for the AIC methodology starts with the single-bus, unit-commitment model (see examples in Appendix C). In the AIC methodology, dispatched incremental generators and bid-in demand set the clearing prices, that is, the LIPs and rLIPs are the optimal dual variables for energy balance constraints (2b) and reserve constraints (2c) below, respectively.

In markets with multiple buses, if a transmission capacity constraint binds, the problem can be decomposed into separate problems by fixing the transmission line flow at the line at its usage in the optimal dispatch or simply retain the existing transmission system capacity (see O'Neill et al. (2020)). In the AIC methodology, 'congestion' on the transmission system occurs on two levels: the LMP (lower) and the LIP (upper) level. At the lower level, the dual variable on the transmission constraint is the marginal value of another small amount of transmission capacity – the flowgate marginal price (FMP). At the upper level, the dual variable on the transmission constraint signals the potential of an incremental expansion for the complete displacement of a generator with avoidable fixed costs –the flowgate incremental price (FIP). In problems with multiple periods, binding ramp rate constraints and binary generator constraints, i. e., minimum run time, tie periods and prices together. Non-binding ramp constraints are dropped in the pricing run.

In a non-convex market absent degeneracy, the LMPs reflect the marginal cost of a decision to dispatch a small additional quantity. The LIPs reflect the incremental binary decision to commit and dispatch a generator (or resource) with possibly more capacity than necessary due to the binary nature of commitment decisions, but it will only be committed and dispatched if it is part of the efficient dispatch (aka optimal market surplus). As a result, the optimal dispatch contains a set of incremental generators that make zero profits and a set of infra-incremental generators that make positive profits at the LIPs and rLIPs. No generators dispatched have negative profits. They are roughly equivalent to marginal and infra-marginal generators in convex markets.

Absent degeneracy, the LMP can be a valid entry price for small amounts of energy. The LMP is an entry signal for a generator with an average incremental cost below the LMP, but may not be an exit signal for a generation with an AIC above the LMP. Also, it is an entry signal for a consumer with a value to consume above LMP, but less than the LIP.

The LIP is generally higher than the LMP. The LIPs are an entry signal for a generator with an average incremental cost below the LIPs with a feasible dispatch at an incremental generator's optimal dispatch. In the real-time market, the LMPs are announced immediately after the dispatch and the LIPs at the end of the market horizon. With bid-in demand if all available generation is scheduled, the prices are set by bid-in demand without a reserve shortage – a market-sourced scarcity price. If a convex bid or offer clears the market at a given time and place, the LIP = LMP. The examples in Appendix C illustrate these one-pass AIC properties.

Contract (aka Bilateral) Market and AIC Pricing Compatibility. No rational seller would agree to a price below its AIC. If the market had two or more sellers and a seller had a superior technology with a lower AIC, it could make a positive profit because the price would be set by the inferior technology at the higher AIC. In a world with stochastic information and risk-neutral market participants, no rational risk-neutral seller would agree to a price below its expected AIC — E(AIC). If a seller had a superior technology, i. e., a lower E(AIC) and the market had two or more sellers, it could make a positive profit because the price would be set by the inferior technology at the higher E(AIC). The conclusion is that AIC pricing is compatible with a rational contract market.

Reserves Pricing. The amount and location of reserves needed in ISO markets are mostly offline calculations. Reserves come in various flavors including energy balancing, single-generator contingencies, and more recently ramping reserves (both up and down) to address moderate weather events. The ability to ramp may limit a generator's output and reserves. The AIC prices preserve the arbitrage condition between energy and reserves and between periods.

3 Multi-Period, Multi-step Marginal Costs, Single-Node AIC Pricing Scheme

In this section we present the one-pass multi-period AIC pricing scheme. We assume that a generator starts up at most once in the horizon. We do not include down reserves and minimum down times to avoid over complicating the presentation and leave them for future work.

Notation

System Sets

T is the set of time periods; T = {t| t = 1, ..., tmax}

T' is the set of time periods without t = 1; T' = {t| t = 2, ..., tmax}

T_i is generator i's startup/shut-down cycle; $T_i = \{t', t'+1, \ldots, t''\} = \{t | u^*_{it} = 1\}$.

System Parameters

r^{us}_t is the minimum system ramping up reserves in t.

System Primal Variables (dispatch)

MS is the market surplus of the dispatch run.

MS^{AIC} is the market surplus of the AIC pricing run

System Dual Variables (prices)

RC^{AIC} is resource cost of the AIC dual program; at optimality, $MS^{AIC} = RC^{AIC}$

λ_t is dual variable on the energy balance constraint in t.

λ^{us}_t is dual variable on the energy reserve up constraint in t.

Consumption Sets (indexed by i)

D is the set of consumers

Consumption Parameters

b_{it} is the value (bid price) per MWh for consumer $i \in D$ in t

Consumption Primal Variables (dispatch)

d_{it} is demand by unit $i \in D$ in t

Consumption Dual Variables

α^{max}_{it} is the marginal value of maximum demand for $i \in D$ in t

α^{min}_{it} is the marginal value of minimum demand reduction for $i \in D$ in t

Generator Sets (indexed by i)

G is set of generators; G* is set of generators dispatched

G^{mp} is set of generators that qualify for a make-whole payment at the *LMP*

Ji are unit i's marginal cost function steps where $c_{jit} < c_{(j+1)it}$. $J_i = \{j | j = 1, \ldots, j^{max}_i\}$

Generator Parameters

c_{jit} is the offer cost/MWh of step j for unit i in t and $c_{jit} < c_{j+1it}$.
c_{it}^{ru} is the offer cost per MWh ramping up for generating unit i in t
c_{it}^{su} is offer start-up cost for generating unit i in t
c_{it}^{op} is the fixed operating cost of unit i in t
r_{it}^{up} is the maximum ramping capability for generating unit i in t
p_{jit}^{max} is the maximum output of step j for unit i $\in$ G in t
p_{it}^{max} is the maximum capacity of unit i $\in$ G in t. $\sum_{j \in J_i} p_{jit}^{max} = p_{it}^{max}$.
p_{it}^{sua} is the one-period adjustment to p_{it}^{max} for the startup period.
r_{it}^{sua} is the one-period adjustment to r_{it}^{up} on startup
p_{it}^{min} is the minimum operating level in t of unit $i \in G$
mr_i is the minimum run time in a startup/shut-down cycle for unit $i \in G$

Generator Primal Variables (Unit Commitment and Dispatch)

p_{jit} is the supply from step j of unit i and t
p_{it}^{ru} is the supply of ramp rate reserves from unit i and t
z_{it} is 1 if unit i starts up in t or 0 otherwise (relaxed in the pricing run)
z_{it}^{d} is 1 if unit i shuts down in t or 0 otherwise (relaxed in the pricing run)
u_{it} is 1 if unit i is running in t or 0 otherwise (relaxed in the pricing run)

Generator Dual variables (Prices and Values) in AIC pricing

β_{jit}^{max} is the marginal value of capacity of step j for generator i in t
β_{it}^{max} is the marginal value of total capacity for generator i in t.
β_{it}^{min} is the marginal cost of the minimum operating level of generator i in t
ρ_{it}^{up} is the marginal value of ramp from generator i in t
δ_{it} is the binary logic marginal value for generator i in t.
μ_{it} is the dual variable on the summation of energy steps for generator i in t.
ω_{it} is the fixed startup variable's marginal value for generator i in t.
ω_{it}^{d} is the fixed shut down variable's marginal value for generator i in t.

Multi-Period Security-Constrained Unit-Commitment Optimal Power Flow

$$MS = max \sum_{t \in T} [(\sum_{i \in D} b_{it} d_{it}) - \sum_{i \in G} ((\sum_{j \in J_i} c_{jit} p_{jit}) + c_{it}^{ru} p_{it}^{ru} + c_{it}^{op} u_{it} + c_{it}^{su} z_{it})] \quad (1a)$$

Market surplus = Consumer Value - Producer Costs

System balancing constraints	Description
$\sum_{i \in D} d_{it} - \sum_{i \in G} p_{it} = 0,\ t \in T$	Energy balance (1b)
$-\sum_{i \in G} p_{it}^{ru} \le -r_t^{us},\ t \in T$	System ramp up requirement (1c)

Demand constraints

$$d_{it} \leq d_{it}^{max},\ i \in D, t \in T \qquad \text{Maximum load (1d)}$$

$$-d_{it} \leq -d_{it}^{min},\ i \in D, t \in T \qquad \text{Minimum load (1e)}$$

Generator constraints

$$p_{jit} - p_{jit}^{max} u_{it} \leq 0, i \in G, t \in T, j \in J_i \qquad \text{Max step capacity (1f)}$$

$$p_{it} - \sum_{j \in Ji} p_{jit} = 0, i \in G, t \in T, \qquad \text{Summation (1g)}$$

$$p_{it} + p_{it}^{ru} - p_{it}^{max} u_{it} - p_{it}^{sua} z_{it} \leq 0, i \in G, t \in T, \qquad \text{Maximum capacity (1h)}$$

$$p_{it}^{ru} - p_{it}^{rumax} u_{it} \leq 0, i \in G, t \in T, \qquad \text{Maximum ramp capacity (1i)}$$

$$-p_{it} + p_{it}^{min} u_{it} \leq 0, i \in G, t \in T, \qquad \text{Minimum capacity (1j)}$$

$$p_{it} - p_{it-1} - r_{it}^{up} u_{it} - r_{it}^{sua} z_{it} \leq 0, i \in G, t \in T', \qquad \text{Ramp up limits (1k)}$$

$$u_{it} - u_{it-1} - z_{it} + z_{it}^{d} = 0, i \in G, t \in T, \qquad \text{Binary commitment logic (1l)}$$

$$\sum_{t' \in [t-mr+1, t]} z_{it'} - u_{it} \leq 0, i \in G, t \in T, \qquad \text{Minimum run time (1m)}$$

$$p_{it}^{ru} \geq 0; z_{it}, z_{it}^{d}, u_{it} \in \{0, 1\}, i \in G, t \in T \qquad \text{(1n)}$$

We denote the optimal solution to the SCUC (1) with *. Each feasible MIP with an optimal linear program is a local optimal solution. Parameters p_{i0} and u_{i0} define the state of the generators at the beginning of period 1. If $u_{i0} = 1$, c_{i1}^{su} is set to zero and p_{i0} and r_{i1}^{up} are used to adjust p_{i1}^{max} to reflect the ramp rate constraint for p_{i1}. Define $p_{it}^* = \sum_{j \in J_i} p_{jit}^*$ and let c_{it} be the marginal cost of gen i where j' denotes the highest active marginal cost step , that is, if $j > j'$, $p_{jit}^* = 0$.

The startup sequence of a generator may not be able to achieve its steady-state maximum operating level in a single period. The one-period adjustment is in (1h). To represent a multi-period startup sequence, let J_s be a multi-period sequence, $J_s = \{0, 1, \ldots, j_{su}\}$; p_{jit}^{sua} be the adjustment in period j after startup; and $p_{it}^{sua} z_{it}$ is replaced with $\sum_{j \in J_s} p_{jit}^{sua} z_{jit}$.

After fixing the binaries to their optimal values, the linear program usually has redundant constraints that were not redundant in the MIP, for example, minimum run time and minimum down time. Dropping redundant constraints retains the optimal MIP solution in the resulting linear program. The dual variables on the energy balance equations (1b), are called the LMPs. The dual variables on the reserve constraints (1c) are marginal reserves prices and are called rLMPs. The LMPs and rLMPs provide marginal information about low-cost entry, but do not signal the possible higher-cost unit-replacement entry when units have avoidable fixed costs. The LIPs from the AIC pricing run (below) and optimal quantities of the incremental generators from the MIP provide entry information for units with avoidable fixed costs.

AIC One-Pass Pricing (AICOP). The AICOP (2) eliminates the binary constraints and adds the constraints: $0 \leq z_{it} \leq z^*_{it}$; $0 \leq u_{it} \leq u^*_{it}$; and $0 \leq z^d_{it} \leq z^{d*}_{it}$. This relaxation eliminates the generators not in the optimal solution from influencing the prices since the optimal binary variables are 0. When the binaries are relaxed, the dual variables on the binary logic constraints play an important role in distributing the avoidable fixed costs across the generator up/down cycle. Although it is not necessary, we drop the minimum run time constraints to remove unnecessary notation.

The optimal solution to (1) is used to tighten the constraints of the pricing problem in neighborhood of the optimal solution. For dispatched generators, we add valid optimality cuts that retain the optimal solution by setting $p^{max}_{it} = p^*_{it} + p^{ru*}_{it} - p^{sua}_{it} z^*_{it} + \epsilon$; dropping $p^{sua}_{it} z^*_{it}$ since the new p^{max}_{it} in the startup period contains the $p^{sua}_{it} z_{it}$ adjustment; $p^{max}_{jit} = p^*_{jit} + \epsilon$; $p^{rumax}_{it} = p^{ru*}_{it} + \epsilon, r^{up}_{it} u_{it} = (r^{up}_{it} + r^{sua}_{it} z^*_{it}) u_{it}$ and $p^{min}_{it} = p^*_{it} - \epsilon$ where $\epsilon > 0$, but small. In addition, we add $d^{max}_{it} = d^*_{it} + \epsilon$ and $d^{min}_{it} = d^*_{it} - \epsilon$. A similar approach for the LMP calculation was used in PJM (Ott (2003)).

Two sets of constraints couple the time periods: the startup/shutdown cycle binary logic constraints (2l) and ramp rate constraints (2k). Both are important to multi-period pricing. The ramp rates that did not bind in the dispatch model (1) are relaxed in the pricing run. The AICOP becomes a linear program:

$$MS^{AIC} = max \sum_{t \in T} [\sum_{i \in D} b_{it} d_{it} - \sum_{i \in G} [(\sum_{j \in Ji} c_{jit} p_{jit}) + c^{ru}_{it} p^{ru}_{it} + c^{op}_{it} u_{it} + c^{su}_{it} z_{it}]] \quad (2a)$$

Equation		Dual Var	Constraints
system balancing constraints			
$\sum_{i \in D} d_{it} - \sum_{i \in G} p_{it} = 0$	$t \in T,$	λ_t	Energy Balance (2b)
$-\sum_{i \in G} p^{ru}_{it} \leq -r^{us}_t$	$t \in T,$	λ^{us}_t	System Ramp (2c)

demand constraints

$$d_{it} \leq d_{it}^{max} \qquad i \in D, t \in T \qquad \alpha_{it}^{max} \qquad \text{Max load} \tag{2d}$$

$$-d_{it} \leq -d_{it}^{min} \qquad i \in D, t \in T \qquad \alpha_{it}^{min} \qquad \text{Min load} \tag{2e}$$

generator constraints

$$p_{jit} - p_{jit}^{max} u_{it} \leq 0 \qquad i \in G, t \in T, j \in J_i \qquad \beta_{jit}^{max} \qquad \text{Max step capacity} \tag{2f}$$

$$p_{it} - \sum_{j \in J_i} p_{jit} = 0 \qquad i \in G, t \in T \qquad \mu_{it} \qquad \text{Summation} \tag{2g}$$

$$p_{it} + p_{it}^{ru} - p_{it}^{max} u_{it} \leq 0 \qquad i \in G, t \in T \qquad \beta_{it}^{max} \qquad \text{Max capacity} \tag{2h}$$

$$p_{it}^{ru} - p_{it}^{rumax} u_{it} \leq 0 \qquad i \in G, t \in T \qquad \beta_{it}^{rumax} \qquad \text{Max ramp reserve} \tag{2i}$$

$$-p_{it} + p_{it}^{min} u_{it} \leq 0 \qquad i \in G, t \in T \qquad \beta_{it}^{min} \qquad \text{Minimum supply} \tag{2j}$$

$$p_{it} - p_{it-1} - r_{it}^{up} u_{it} \leq 0 \qquad i \in G, t \in T \qquad \rho_{it}^{up} \qquad \text{Ramp up limit} \tag{2k}$$

$$u_{it} - u_{it-1} - z_{it} + z_{it}^{d} = 0 \qquad i \in G, t \in T \qquad \delta_{it} \qquad \text{Binary commitment} \tag{2l}$$

$$z_{it} \leq z_{it}^{*} \qquad i \in G, t \in T \qquad \omega_{it} \qquad \text{Relaxed binaries} \tag{2m}$$

$$u_{it} \leq u_{it}^{*} \qquad i \in G, t \in T \qquad \gamma_{it} \qquad \text{Relaxed binaries} \tag{2n}$$

$$z_{it}^{d} \leq z_{it}^{d*} \qquad i \in G, t \in T \qquad \omega_{it}^{d} \qquad \text{Relaxed binaries} \tag{2o}$$

$$p_{it}^{ru}, z_{it}, z_{it}^{d}, u_{it} \geq 0 \qquad i \in G, t \in T \qquad \text{Lower bounds} \tag{2p}$$

Dual of the AICOP is:

$$RC^{AIC} = min \sum_{t \in T} [r_t^{us} \lambda_t^{us} + \sum_{i \in D} (d_{it}^{max} \alpha_{it}^{max} - d_{it}^{min} \alpha_{it}^{min}) + \sum_{i \in G} (z_{it}^{*} \omega_{it} + u_{it}^{*} \gamma_{it} + z_{it}^{d*} \omega_{it}^{d})] \tag{3a}$$

Equation Dual Var

Demand constraints

$$\lambda_t + \alpha_{it}^{max} - \alpha_{it}^{min} \geq b_{it}, \qquad i \in D, t \in T \qquad d_{it} \tag{3b}$$

Generator constraints

$$\rho_{it}^{up} - \rho_{it+1}^{up} - \lambda_t + \mu_{it} - \beta_{it}^{min} + \beta_{it}^{max} \geq 0, \qquad i \in G, t \in T \qquad p_{it} \tag{3c}$$

$$\omega_{it} - \delta_{it} \geq -c_{it}^{su}, \qquad i \in G, t \in T \qquad z_{it} \tag{3d}$$

$$-\mu_{it} + \beta_{jit}^{max} \geq \; - c_{jit}, \qquad i \in G, t \in T, j \in Ji \quad p_{jit} \tag{3e}$$

$$\beta_{it}^{rumax} + \beta_{it}^{max} - \lambda_t^{us} \geq -c_{it}^{ru}, \qquad i \in G, t \in T \qquad p_{it}^{ru} \tag{3f}$$

$$\gamma_{it} + \delta_{it} - \delta_{it+1} - r_{it}^{up}\rho_{it}^{up} + p_{it}^{min}\beta_{it}^{min} - \sum_{j \in J_i}(p_{ji}^{max}\beta_{jit}^{max}) - p_i^{max}\beta_{it}^{max} - p_i^{rumax}\beta_{it}^{rumax} \geq -c_{it}^{op}, \qquad i \in G, t \in T \qquad u_{it} \tag{3g}$$

$$\omega_{it}^{d} + \delta_{it} \geq 0, \qquad i \in G, t \in T \qquad z_{it}^{d} \tag{3h}$$

$$\alpha_{it}^{max}, \alpha_{it}^{min} \geq 0, \qquad i \in D, t \in T \tag{3i}$$

$$\beta_{jit}^{max} \geq 0, \qquad i \in G, t \in T, j \in Ji \tag{3j}$$

$$\rho_{it}^{up}, \beta_{it}^{max}, \beta_{it}^{rumax}, \beta_{it}^{min}, \omega_{it}, \gamma_{it}, \omega_{it}^{d} \geq 0, \qquad i \in G, t \in T \tag{3k}$$

$$\lambda_t^{up} \geq 0, \qquad t \in T \tag{3l}$$

In (3c), since (2k) does not exist for t=1, neither does ρ_{i1}^{up} and it is set it to 0. In (4) through (8) below, we assume the variables are at their optimal value; * indicates an optimal solution to (1); ** indicates an optimal solution to (2). The dual variables do not exist in (1). Therefore, they do not need to be distinguished as are the primal variables. Longer proofs are in Appendix B.

Lemma 1. $MS \leq MS^{AIC}$.

Proof: The optimal solution to (1) is a feasible solution to (2), therefore, $MS \leq MS^{AIC}$. □

Lemma 2. In the one-pass AIC pricing run, if $\epsilon = 0$, then $p_{it}^{**} = p_{it}^{*}u_{it}^{**}$, $p_{jit}^{**} = p_{jit}^{*}u_{it}^{**}$, $p_{it}^{ru**} = p_{it}^{ru*}u_{it}^{**}$, for $t \in T$. Moreover, if $p_{it}^{*} > 0$ or $p_{it}^{ru*} > 0$, then $u_{it}^{**} = u_{it}^{*} = 1$, for $t \in T$.

Proof: see Appendix B. Lemma 2 is central in establishing the crucial link between the primal and dual solution of AICOP and SCUC.

Lemma 3. In the one-pass AIC pricing run, reserves are profitable.

Proof: By complementary slackness of (3f), $(\beta_{it}^{rumax} + \beta_{it}^{max} + c_{it}^{ru} - \lambda_t^{us})p_{it}^{ru**} = 0$. Rearranging, $\lambda_t^{us}p_{it}^{ru**} - c_{it}^{ru}p_{it}^{ru**} = \beta_{it}^{rumax}p_{it}^{ru**} + \beta_{it}^{max}p_{it}^{ru**}$. From Lemma 2, , substituting $p_{it}^{ru**} = p_{it}^{ru*}u_{it}^{**}$ and dividing by $u_{it}^{**} > 0$ for $p_{it}^{ru*} > 0$,

$$\lambda_t^{us}p_{it}^{ru*} - c_{it}^{ru}p_{it}^{ru*} = \beta_{it}^{rumax}p_{it}^{ru*} + \beta_{it}^{max}p_{it}^{ru*}. \tag{4a}$$

Since $\beta_{it}^{rumax} \geq 0$ and $\beta_{it}^{max} \geq 0$, the revenue from reserves may exceed the costs of reserves,

$$\lambda_t^{us}p_{it}^{ru*} \geq c_{it}^{ru}p_{it}^{ru*} \qquad \square \tag{4b}$$

If β_{it}^{rumax} = 0 and β_{it}^{max}= 0, $\lambda_t^{us}p_{it}^{ru*} = c_{it}^{ru}p_{it}^{ru*}$. If $p_{it}^{ru*} > 0$, dividing by p_{it}^{ru*}, $\lambda_t^{us} = c_{it}^{ru}$. The price of reserves is set by the marginal reserve cost of generator i. If β_{it}^{max}> 0, price of reserve may incorporate opportunity cost for energy.

Proposition 1. From AICOP, using the λ_t (LIPs) and λ_t^{us} (rLIPs) for $t \in T$, all dispatched units are profitable, that is, no make-whole payments are needed.

Proof: See Appendix B.

From Proposition 1, we define two concepts. Generator i is an incremental generator if it breaks even, that is,

$$\sum\nolimits_{t\in T}(\lambda_t p_{it}^{*} + \lambda_t^{us}p_{it}^{ru*}) \qquad - \sum\nolimits_{t\in T}[(\sum\nolimits_{j\in Ji}c_{jit}p_{jit}^{*}) + c_{it}^{op}u_{it}^{*} + c_{it}^{ru}p_{it}^{ru*}] + c_{it}^{su}z_{it}^{*} = 0 \tag{4c}$$

Energy revenues Incremental energy and reserves cost profits

Generator i is an infra-incremental generator if it has positive profit, that is,

$$\sum\nolimits_{t\in T}(\lambda_t p_{it}^{*} + \lambda_t^{us}p_{it}^{ru*}) \quad - \sum\nolimits_{t\in T}[(\sum\nolimits_{j\in Ji}c_{jit}p_{ji}^{*}) + c_{it}^{op}u_{it}^{*} + c_{it}^{ru}p_{it}^{ru*}] + c_{it}^{su}z_{it}^{*} \quad > 0 \tag{4d}$$

Energy revenues Incremental energy and reserves costs profits

We demonstrate these properties in Appendix C examples and actual MISO problems in the next section.

Proposition 2. Absent degeneracy, in any period t, there is a marginal generator, an incremental generator operating or the market clears off the demand function bid (b_{it}) and all generators are infra-incremental.

Proof. By contradiction. Assume the solution is not degenerate and the dual solution is unique. Suppose there is no incremental generator, and the market does not clear on the demand function and also there is no marginal generator, then (4d) holds for all generators $i \in G^*$. We solve the linear program (2). For any $t \in T$, if the $\lambda_t >$ c_{it}, for all $i \in G^*$ then either $\lambda_t = b_{it}$ and the market clears on the demand function or if $c_{it} < \lambda_t < b_{it}$, $p^*_{it} + \epsilon$ and $d^*_{it} + \epsilon$ where $\epsilon > 0$ is a feasible solution with a higher market surplus which a contradiction. □

Proposition 3. For an incremental generator, λ_t and λ^{us}_t for $t \in T$ is a set of prices that are minimal in the sense that higher prices are not necessary to eliminate make-whole payments for the generator and maximal in the sense that lower prices do not eliminate make-whole payments for the incremental generator.

Proof: From (4c), if any λ_t or λ^{us}_t for $t \in T$ is increased and its associated with a $p^*_{it} > 0$ or $p^{ru*}_{it} > 0$, (4c) becomes (4d). From (4c), if any λ_t or λ^{us}_t for $t \in T$ is decreased and its associated $p^*_{it} > 0$ or $p^{ru*}_{it} > 0$, (4c) is negative and requires a make-whole payment. □

This proposition is also true for bid-in demand.

Cost Allocation and Settlement. From (A6t) in Appendix B, for each period,

$$\lambda_t p^{**}_{it} + \lambda^{us}_t p^{ru**}_{it} = \sum\nolimits_{j \in Ji} c_{jit} p^{**}_{jit} + c^{ru}_{it} p^{ru**}_{it} + \gamma_{it} u^{**}_{it} + [c^{op}_{it} + \delta_{it} - \delta_{it+1}] u^{**}_{it} - r^{up}_{it} u^{**}_{it} \rho^{up}_{it} + (\rho^{up}_{it} - \rho^{up}_{it-1}) p^{**}_{it}] \quad (4e)$$

From Lemma 2, substituting $p^{**}_{it} = p^*_{it} u^{**}_{it}$, $p^{**}_{jit} = p^*_{jit} u^{**}_{it}$, $p^{ru**}_{it} = p^{ru*}_{it} u^{**}_{it}$,

$$\lambda_t p^*_{it} u^{**}_{it} + \lambda^{us}_t p^{ru*}_{it} u^{**}_{it} = (\sum\nolimits_{j \in Ji} c_{jit} p^*_{jit} u^{**}_{it}) + c^{ru}_{it} p^{ru*}_{it} u^{**}_{it} + \gamma_{it} u^{**}_{it} + [c^{op}_{it} + \delta_{it} - \delta_{it+1}] u^{**}_{it} - r^{up}_{it} u^{**}_{it} \rho^{up}_{it} + (\rho^{up}_{it} - \rho^{up}_{it-1}) p^*_{it} u^{**}_{it}] \quad (4f)$$

If $u^{**}_{it} = 0$, (4f) vanishes, 0 = 0. Dividing by $u^{**}_{it} > 0$,

$$\lambda_t p^*_{it} + \lambda^{us}_t p^{ru*}_{it} = (\sum\nolimits_{j \in Ji} c_{jit} p^*_{jit}) + c^{ru}_{it} p^{ru*}_{it} + \gamma_{it} + [c^{op}_{it} + \delta_{it} - \delta_{it+1}] - r^{up}_{it} \rho^{up}_{it} + (\rho^{up}_{it} - \rho^{up}_{it-1}) p^*_{it} \quad (4g)$$

Revenue in t for i in G* from LIPs and rLIPs is: $\lambda_t p^*_{it} + \lambda^{us}_t p^{ru**}_{it}$. Marginal costs incurred in t are:
$(\sum_{j \in J_i} c_{jit} p^*_{jit}) + c^{ru}_{it} p^{ru*}_{it}$. Profits in t are: γ_{it}. Temporal cost reallocation due to relaxed binaries is: $c^{op}_{it} + \delta_{it} - \delta_{it+1}$. Temporal change in prices due to binding ramp rates is: $-r^{up}_{it} \rho^{up}_{it} + (\rho^{up}_{it} - \rho^{up}_{it-1}) p^*_{it}$.

From (A7f), the term, $c^{op}_{it} + \delta_{it} - \delta_{it+1}$, allocates the startup cost and fixed operating costs to periods where the generator is needed most, creating cost-causal prices (see Appendix C Example 7). The term, $r^{up}_{it} \rho^{up}_{it} - (\rho^{up}_{it} - \rho^{up}_{it-1}) p^*_{it}$, allocates costs due to ramp rate constraints. The unit profit term, γ_{it}, is non-negative since both components are non-negative. If $\gamma_{it} = 0$ for $t \in T_i$, generator i is an incremental generator. If any $\gamma_{it} > 0$ for $t \in T_i$, generator i is an infra-incremental generator. If $\epsilon = 0$, (2) becomes degenerate or more degenerate and may produce larger set of dual variables that include the prices. These properties are demonstrated in Appendix C examples.

Proposition 4. In AIC pricing, the arbitrage conditions holds for positive energy,

$p_{it}^{min} < p_{it}^{*} < p_{it}^{max}$, and reserves, $p_{it}^{ru**} > 0$. If the ramp rate constraints do not bind, the static arbitrage condition holds, that is,

$$\lambda_t - \lambda_t^{us} = c_{it} - c_{it}^{ru} \tag{4h}$$

If one or more ramp rate constraints bind, the multi-period arbitrage condition holds, that is,

$$\lambda_t - \lambda_t^{us} = c_{it} - c_{it}^{ru} + \rho_{it}^{up} - \rho_{it+1}^{up} \tag{4i}$$

Proof: See Appendix B.

Proposition 5. Without bid-in demand and if there is positive fixed cost investment for new generation, there may not be enough short-term profit to stimulate efficient investment.

Proof: Without bid-in demand, demands are all fixed points. If demand is a fixed point, it can never set the clearing price because its implied value is infinite. The marginal or incremental unit may not make enough profit in the short-term to cover its long term investment costs. If the demand has a finite value, the efficient market may clear off the demand function producing positive profits for all dispatched generators. □

Proposition 6. The LIPs have the property that if, absent primal degeneracy, generators are paid at prices lower than LIPs, for example, LMPs, one or more incremental generators will have negative profits and need a make-whole payment.

Proof: There is at least one incremental generator active in each period. Since incremental generators makes zero profit at the LIPs, lowering one LIP causes negative profits. □

Real-Time Market Pricing with Look Ahead. The real-time market is a one-period market with a look-ahead. The dispatch signals and LMPs are available almost simultaneously. If the LMP is announced, it is a correct low-cost entry signal for the previous market period. Announcing a real time price that is not the LMP gives little and possibly misleading short-term economic information. Generally, a non-LMP price, for example, from relaxed minimum operating level, is too low for a full incremental entry signal because it contains some, but not all fixed costs and is too high for marginal entry because it is higher than the LMP. LMP is the marginal entry signal, but there is no public information on the magnitude of entry at the LMP.

The AIC methodology announces the LMP at the end of each time interval. For single periods, the LMP is a short-term entry and exit price signal. Because the LIPs are not calculated until the end the market horizon, they cannot be announced at the end of each period due in part to binding ramp rates and minimum run times. The LIPs are calculated after the generator up-down cycle or ideally the market horizon is completed, possibly 24 hours of 5-minute prices for the real-time market. The LIP prices can be calculated at the end of the horizon using the actual dispatch quantities as if it were a day-ahead market. The LIPs and clearing quantities for incremental generators are announced. The LIPs and LMPs are used to settle the market.

4 MISO Case Studies

In this section, we study two variations of AIC pricing on MISO cases. Variation 1 (LIP1) sets $p^{max}=p^{*}+\epsilon$ only for generators with negative profits under *LMP* prices. Variation 2 (LIP2) sets $p^{max}=p^{*}+\epsilon$ for all dispatched generators. Since large production MIP problems usually solve with a non-zero gap, $p^{min}=p^{*}-\epsilon$ is not applied so that non-optimal commitments may be relaxed to 0 in the LIP1 run. For LIP2 , when $p^{max}=p^{*}+\epsilon$ is set for all dispatchable generators, we have $p^{**} \geq p^{*} - (n-1)\epsilon$ where n is the total number of dispatchable generators. It's equivalent to set $p^{min} = p^{*} - \epsilon'$ where ϵ' is proportional to ϵ.

Polishing Sub-Optimal Solutions using the AIC Methodology. In practice, the MIP solver may return an integer-feasible sub-optimal solution in the MIP gap or the solver simply times out. The solution is not known to be sub-optimal, only that it has not been proven optimal. We propose a polishing methodology that may improve the solution. Suppose the solver terminates in a positive MIP gap. If $p_{it}^{*} > 0$ for any t and the AIC LIP1 solution relaxes u_{it}^{**} to 0 and $p_{it}^{**} = 0$, for all t, generator i is not in the relaxation solution and may not be needed in the MIP. A branch-and-bound child node is created and solved with p_{it} = 0 to see if Gen i is needed.

To illustrate, consider the following problem in Table 4.1. Suppose the solver terminates in a sub-optimal solution, p_1 = 70, p_2 = 20 and the LMP = 0. AIC LIP1 sets u_2 = 0 and p_2 = 0 and creates a new node in the branch-and-bound tree, the MIP solver returns a better solution with p_1 = 90, p_2 = 0 and the LMP = 0.

Table 4.1. Feasible but Sub-Optimal Dispatch to Better (Optimal) solution.

Unit	Pmin (MW)	Pmax (MW)	Marginal value/cost ($/MWh)	Startup costs ($)	Suboptimal solution (MWh)	Optimal solution (MWh)
G1	0	100	0	10^{-8}	70	90
G2	20	20	50	100	20	0
Load	90	90	500			90
LMP					0	0
LIP1					0	
LIP2					55	

We test this method on seven MISO day-ahead market cases mostly from January 2014 during the polar vortex with very tight system conditions. After solving the SCUC with a 1200 second time limit or 0.1% MIP gap tolerance, the results are in Table 4.2. The incumbent solution is used to solve for AIC LIPs takes 40 seconds or less. We then check the AIC solution to identify any generators that are backed down to 0, set the commitment solution of those generators to 0, and solve. For the MIP solver to find the same incumbent solution takes from less than a second up to almost as much time to solve the original problem. The savings range from 0 to $17,121. Although small in comparison to the total market size, it should not be ignored. If the seven cases were a representative sample of days, the savings would be $1.9 million/year.

Table 4.2. MISO Cases Results.

Case	MIP GAP (%)	MIP Time (s)	AIC time (s)	Objective increase ($)	Solver Time (s)	New GAP (%)
1	0.092	154	20	1301	55	0.089
2	0.068	1110	40	1	0	0.068
3	0.128	1201	31	17121	1116	0.096
4	0.074	883	22	0	0	0.074
5	0.151	1200	29	10591	1090	0.133
6	0.065	356	29	1097	274	0.064
7	0.150	1200	27	7500	910	0.135
average	0.104	872	28	5373	492	0.094

MISO Case Studies with One-Pass AIC Pricing. For a day-ahead market case from February 2020 where there were regional emergency alerts, we change the case by changing all must-run and self-schedule units to economic units, excluding virtual transactions and the small amount of dispatchable demand, removing reserve requirements, maximum daily energy, and maximum daily start constraints. All MISO generators are required to submit economic offers that include the parameters required for solving SCUC. Some of the parameters are ignored or revised for must-run and/or self-scheduled generators. By changing must-run and self-schedule to economic, the full set of economic offers are used for solving SCUC. Most steam units are run for multiple days and have no expressed startup costs. Some hydro units have small costs. There are over 100 generators (mainly renewables) with negative or zero costs. In Figures 1a and 1b, under One-Pass AIC pricing, all generators have non-negative profits at the LIPs. That is not the case under LMPs. For most generators, the difference in profits/MWh between LMP and LIP pricing is not large. The greatest difference occurs when the LMPs yields negative profits. Table 4.3 shows the summary of all on-line generators (both incremental and infra-incremental). The average increase in profits from LMPs to LIPs is 1.34% for all classes. Combustion Turbines (CTs) as a class have the highest increase in profits under AIC pricing followed by combined cycle aggregates (CCAs). As a result, in the capacity markets, the prices will decrease since CTs and CCGTs are often the price-setting units in capacity markets.

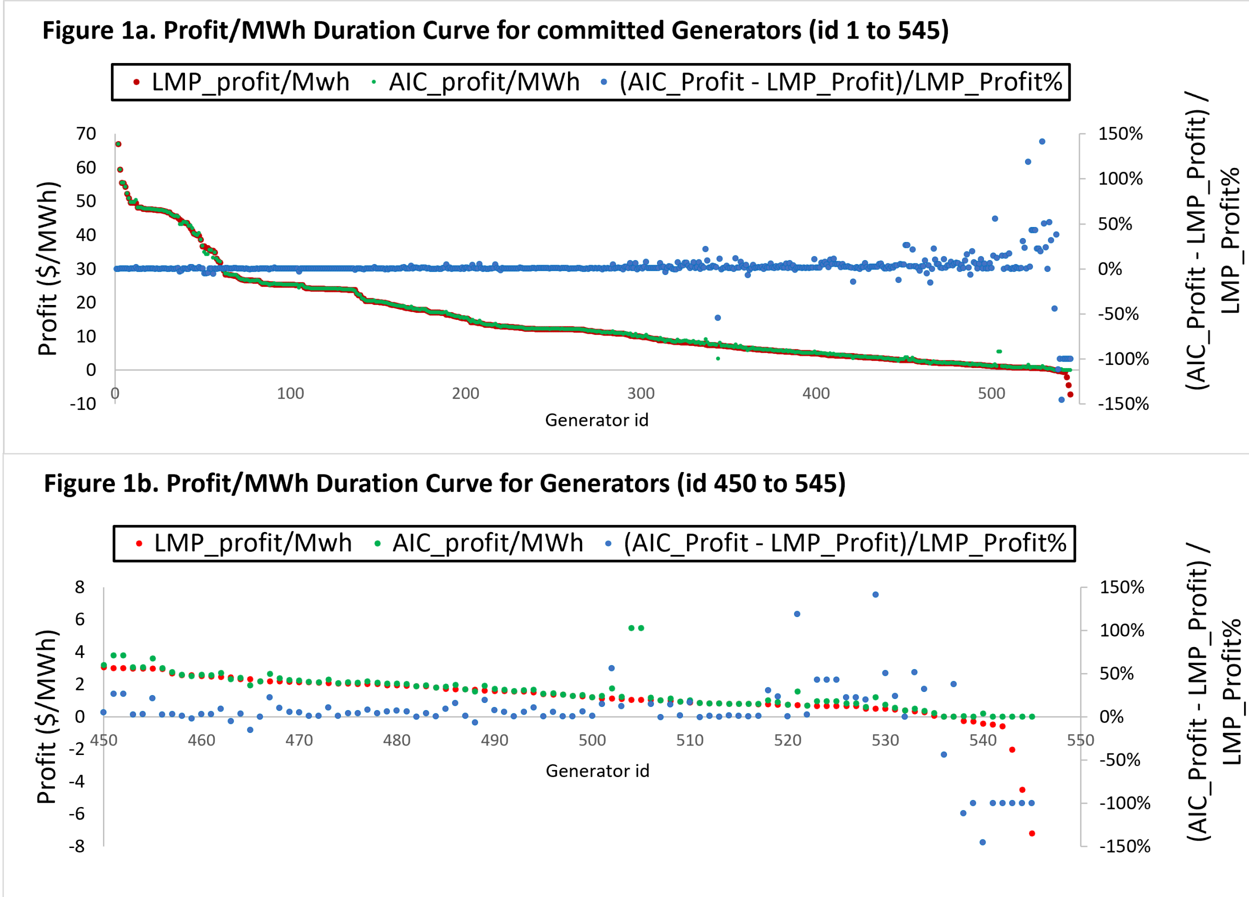
Table 4.3. All Dispatched Generators

class	Average Profit at *LMP*s $/MWh	Average Profit at *LIP*s $/MWh	Profit in-crease (%)	Average AFC $/MWh	Dispatch Energy TWh	Units Dispatched	Average En-ergy MWh	No. of units
CT	5.93	6.40	7.80	9.32	.289	65	4,448	378
CCA	6.72	6.88	2.48	5.00	.459	34	13,510	59
Steam	8.98	9.09	1.21	3.36	1.490	142	10,517	270
Renew	22.67	22.74	0.28	0.07	.474	224	2,115	233
Hydro	22.31	22.39	0.34	1.71	.065	65	996	67
Diesel	0	0	0	0	0	0	0	52
Other	6.05	6.12	1.14	4.32	.014	15	907	26
Totals						545	5,126	1,085

Renew is wind+solar. AFC is avoidable fixed costs. Average Energy is the average MWh energy from units dispatched.

Table 4.4 presents the thirteen generators with the lowest LMP profits per MWh. One steam generator operates for only one hour. Three generators operate beyond the entire 24-hour pricing horizon and are priced for the entire 36. Eight are numerically incremental. The largest energy producer of the group, Gen 8 with over 500MW capacity, goes from a negative profit under *LMP* pricing to a positive profit under AIC pricing becoming an infra-incremental generator. The remaining four generators see increases to positive profits and no make-whole payment. Make-whole payments lower incentives for resources to improve efficiency because reducing fixed cost may not result in higher profits. Under AIC pricing, infra-incremental generators that reduce their cost can make additional profits.

Figure 1



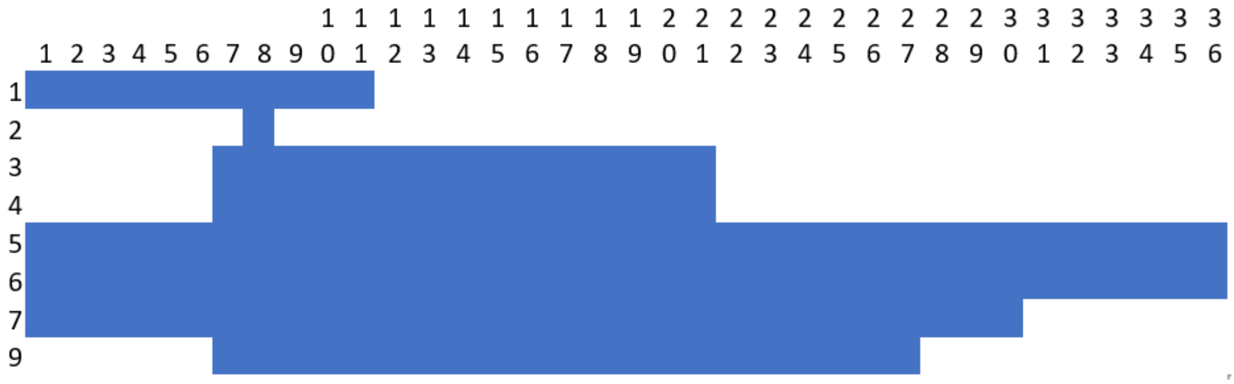
Not shown is a generator with profits of $513/MWh and three generators with (AIC Profit - LMP Profit)/LMP Profit between 286% and 422%.

Table 4.4. Incremental and Near-Incremental Generators.

Gen	type	time periods	Avoidable cost (AC)	MWh	LMP profit	AIC profit	LMP profit/MWh	AIC profit/MWh
1	CCA	[1,11]	54485.00	3798.51	-7736.9	-2E-04	-2.04	-5E-08
2	Steam	[8,8]	485.00	8.50	-4.99	6E-14	-0.59	7E-15
3	Wind	[7,22]	7074.00	785.97	0.00	4E-13	0.00	5E-16
4	CCA	[7,22]	81084.00	4399.93	-1291.5	-3E-5	-0.29	-7E-09
5	Steam	[1,36]	22947.00	2545.27	-1198.0	-8E-6	-0.47	-3E-09
6	CT	[1,36]	197825.00	8107.54	-36449	-2E-3	-4.50	-2E-07
7	CT	[1,30]	21457.00	950.64	-6847.4	-4E-4	-7.20	-4E-07
8	Steam	[1,36]	434525.00	14633.17	-3854.1	454.24	-0.26	0.031
9	Wind	[7,23]	0	1549.66	0.00	-3E-12	0.00	-2E-16
10	CCA	[1,36]	149516.80	9910.99	606.21	2339.58	0.06	0.24
11	CT	[6,36]	133430.10	5063.60	1408.48	1857.04	0.28	0.37
12	CT	[1,36]	87466.08	4183.77	1590.67	1587.75	0.38	0.38
13	CT	[7,15]	36984.99	1342.00	608.89	754.36	0.45	0.56

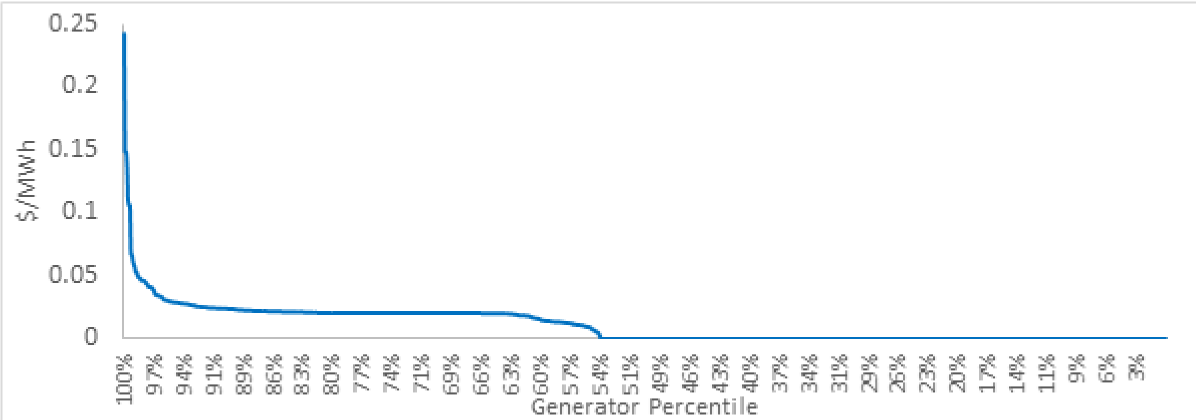
Table 4.5 shows the coverage of each incremental generator. Each time period has at least one incremental generator. Period eight has nine incremental generators.

Table 4.5. Thirty-six Hourly Period Coverage of Each Incremental Generator (G1-G7 and G9).



When transmission constraints bind, nodal prices include the network effects that can be counterintuitive at first. In the above MISO case, about 61 generators had higher profits under the LMP prices than profit under the LIPs. Example 8 in Appendix C shows how this can happen in a network with binding transmission constraints. Twenty generators had no costs with profit reduced less than 1%. Seven generators had negative total cost with profit reduced from 0.33% to 5.63%. Twenty generators had positive total cost and less than 1% profit reduction. Twelve generators with positive total cost and more than 1% profit reduction. Eight had profit reduction less than 8%. Three had profit reduction between 12% and 16%. One CT had profit reduction of 54% at \$190. When we removed the transmission constraints on several MISO cases, the AIC profits of all generators were higher than LMP profit. Figure 2 shows AIC profits/MWh less LMP profit/MWh with no transmission constraints for the same February 2020 regional emergency alerts case described above. For about 54% of the 1085 generators shown in Table 4.3, there is no difference. For about 45%, the difference is less than \$.05/MWh. For about 1%, the difference is between \$.05 and \$.25/MWh. The total number of 'watchlist' transmission constraints for 36 hourly intervals is 6636. These constraints are selected based on their history of binding in the distribution factor model. The number of binding transmission constraints is 339 in LMP run and 300 in LIP1 run. Binding transmission constraints can create larger differences between AIC and LMP profits.

Figure 2. AIC profits less LMP profits per MWh with no transmission constraints.



AIC Performance on 14 MISO Cases. For the same 7 cases in Table 4.2 and for another 7 cases from randomly selected normal days in 2018 (numbered 8 to 14), SCUC was set with 0.1% MIP gap. All 2018 cases reached the gap in less than 1200s. Table 4.6 contains average generator profit ranging from 1.2 to 14% higher under LIP1 and 4.6 to 30.9% higher under LIP2 than the *LMP* profit.

Table 4.6. Gen Profit (as a percent higher than the *LMP* Gen profit) for the 14 MISO Cases

Case	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Avg
LIP1	4.6	4.5	6.5	11.0	8.4	7.2	9.8	9.5	10.6	14.0	1.2	13.4	4.2	12.9	8.4
LIP2	4.6	5.0	9.4	9.4	12.0	8.9	9.5	9.1	30.9	28.8	9.9	13.2	6.8	12.7	12.2

Table 4.7 contains solve time under SCUC, LIP1 and LIP2. LIP1 and LIP2 solution takes mostly two minutes or less. The fixed binary (LMP) times are negligible. The increase in AIC prices will be offset lower capacity market prices.

Table 4.7. Solve Time (in 1000s) for the 14 MISO Cases.

Case	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Avg
SCUC	.31	2.3	3.8	1.5	2.6	.37	6.8	.11	.21	.46	.18	.18	.51	.18	1.4
LIP1	.03	.03	.03	.03	.04	.03	.03	.02	.03	.02	.03	.03	.03	.02	.03
LIP2	.30	.08	.09	.04	.11	.09	.09	.08	.08	.09	.08	.09	.11	.22	.11

The results on MISO cases are solved with ϵ=0.0001. Such a small ϵ may cause numerical issues and longer solving time for LIP2.

In LIP1, only the maximum limits of the generators with negative profits under LMP are adjusted to be within ϵ around the SCUC solution. Other generators can move up to allow the ones with negative profit under LMP to back down to zero MW in the LIP1 run. If the generator with negative profit under LMP is committed to relieve a transmission constraint, the transmission constraint may not bind in SCUC but it can bind in LIP1 run. The FIP of the binding constraint reflects potential value of an incremental expansion of the transmission constraint.

In LIP2, all generators can only move up within ϵ around the SCUC solution. Therefore, the generators with negative profit under LMP can collectively reduce their generation by no more than the product of ϵ and the total number of generators. To properly reflect the cost of commitment for transmission, we need to set the transmission constraint limit at SCUC flow plus a small number ϵ_1. Here we set $\epsilon_1 = \epsilon/10$ for those not binding in SCUC but binding in LIP1. LIP2 price and solving time can be sensitive to ϵ and ϵ_1. For example, for cases 9 and 10, by changing ϵ=0.01 and ϵ_1=0.025 , the LIP2 profit increases (as a percentage higher than the LMP Gen profit) change to 23.5 and 16.6 respectively (versus 30.9 and 28.8 shown in Table 4.6). Total MWPs for case 9 and 10 change from less than \$0.01 to \$5.3 and \$6.2 respectively.

For case 1, LIP2 solving time can be reduced to 0.08*1000s with larger ϵ=0.01 and $\epsilon_1 = \epsilon/10$ compared to 0.30*1000s in Table 4.7. The trade-off is that the total MWP for case 1 increases from \$0.005 to \$4.536. In general, LIP1 with ϵ=0.0001 works better and it can also polish sub-optimal solutions. LIP1 doesn't need to change transmission constraint limit and is more numerically stable.

5 Summary and Conclusions

AIC pricing addresses issues of pricing in non-convex markets with declining-average-costs generators with positive economic results. LIPs along with LMPs are incentive compatible since no dispatched unit that offered its avoidable costs loses money at the LIP. LIPs eliminate make-whole payments, making the market more transparent. LIPs and LMPs send valid entry signals and capacity market prices are reduced. These properties are proved theoretically, and demonstrated on small examples and actual MISO market problems.

For a multi-period market with a multi-step marginal costs and ramp rate constraints, the AIC one-pass pricing algorithm eliminates make-whole payments by relaxing each binary to be continuous between 0 and the optimal binary value and replaces p^{max} with $p^*+\epsilon$ and p^{min} with $p^*-\epsilon$ where ϵ is a small positive number to obtain prices that result in profits for all dispatched generators. Setting at $\epsilon > 0$ avoids possible degeneracy problems for prices. Some small examples illustrate issues that may be hard to find in large actual problems. As shown in the examples the set of entry prices (both LMPs and LIPs) in the presence of AFCs are not unique, but LIPs are in this set. As shown in the small examples and MISO test problems, LIP1 at $\epsilon = 10^{-4}$ seems to work best. Small examples and ISO scale problems validate the theory. The AIC algorithm solves quickly on actual MISO problems. Further testing is needed to better understand the results. The AIC methodology comes closer to satisfying many desired economic properties of pricing.

References

Andrianesis, Panagiotis, Dimitris J Bertsimas, Michael Caramanis & William Hogan. 2021. Computation of convex hull prices in electricity markets with non-convexities using dantzig-wolfe decomposition. *IEEE Transactions on Power*

Systems 1–1. doi:10.1109/TPWRS.2021.3122000.

Baumol, William J, John C Panzar & Robert Willig. 1982. *Contestable markets and the theory of industry structure*. Harcourt Brace Javanovich.

Bohn, Roger E, Michael C Caramanis & Fred C Schweppe. 1984. Optimal pricing in electrical networks over space and time. *The Rand Journal of Economics* 360–376.

Boiteux, M. 1956. Sur la gestion des monopoles publics astreints a líequilibre budgetaire. *Econometrica* 24(1).

Borokhov, Vadim. 2018. Modified convex hull pricing for power markets with price-sensitive load. *Inter J Electrical Power & Energy Systems* 100. 422–437.

Chao, Hung-Po. 2019. Incentives for efficient pricing mechanism in markets with non-convexities. *Journal of Regulatory Economics* 56(1). 33–58.

Chen, Y & C Wang. 2019. Enhancements of extended locational marginal pricing—advancing practical implementation. In *Proc. grid future papers*, .

Dierker, Egbert. 1991. The optimality of boiteux-ramsey pricing. *Econometrica* 31(1). 99–121. https://doi.org/10.2307/2938242.

Eldridge, B, RP O'Neill & AR Castillo. 2017a. An improved method for the dcopf with losses. *IEEE Transactions on Power Systems* 33(4). 3779–3788.

Eldridge, B, RP O'Neill & AR Castillo. 2017b. Marginal loss calculations for the dcopf. *FERC Technical Report on Loss Estimation* .

Eldridge, B, RP O'Neill & BF Hobbs. 2018. Pricing in day-ahead electricity markets with near-optimal unit commitment. *Cambridge Working Papers in Economics* .

FERC. 2021. Ferc enforcement activity increases in fiscal year 2021. https://www.ferc.gov/news-events/news/ferc-enforcement-activity-increases-fiscal-year-2021.

Gribik, Paul R, William W Hogan & Susan L Pope. 2007. Market-clearing electricity prices and energy uplift. *Cambridge, MA* 1–46.

Hua, Bowen & Ross Baldick. 2016. A convex primal formulation for convex hull pricing. *IEEE Transactions on Power Systems* 32(5). 3814–3823.

Hytowitz, Robin Broder, Bethany Frew, Gord Stephen, Erik Ela, Jessica Lau, Nikita Singhal & Aaron Bloom. 2020. Impacts of price formation efforts considering high renewable penetration levels and system resource adequacy targets https://www.nrel.gov/docs/fy20osti/74230.pdf.

ISO/RTO COUNCIL. 2022. https://isorto.org/.

Liberopoulos, G & P Andrianesis. 2016. Critical review of pricing schemes in markets with non-convex costs. *Operations Research* 64(1). 17–31.

Mangasarian, O. 1969. *Nonlinear programming*. McGraw-Hill.

Mas-Colell, Andreu, Michael Dennis Whinston, Jerry R Green et al. 1995. *Microeconomic theory*, vol. 1. Oxford University, Press New York.

Mueller, P & J Gerber. 2020. What is a just price. https://www.econlib.org/library/Columns/y2020/Muellerjustprice.html.

Myerson, Roger B & Mark A Satterthwaite. 1983. Efficient mechanisms for bilateral trading. *Journal of economic theory* 29(2). 265–281.

O'Neill, R P. 2017a. Pricing methods in iso day-ahead and real-time auction markets.

O'Neill, RP. 2017b. Computational issues in iso market models. https://sites.gatech.edu/workshop-pes-or/program/abstracts.

O'Neill, RP, A Castillo, B Eldridge & RB Hytowitz. 2016. Dual pricing algorithm in iso markets. *IEEE Transactions on Power Systems* 32(4). 3308–3310.

O'Neill, RP et al. 2020. Average incremental cost pricing for independent system operators.

Orvis, Robbie & Sonia Aggarwal. 2018. Refining competitive electricity market rules to unlock flexibility. *The Electricity Journal* 31(5). 31–37.

Ott, Andrew L. 2003. Experience with pjm market operation, system design, and implementation. *IEEE Transactions on Power Systems* 18(2). 528–534.

O'Neill, Richard, Paul Sotkiewicz, Benjamin Hobbs, Michael Rothkopf & William

Stewart. 2005. Efficient market-clearing prices in markets with nonconvexities. *European Journal of Operational Research* 164. 269–285. doi:10.1016/j.ejor.2003.12.011.

PJM. 2021. Pjm manual 11: Scheduling operations. https://www.pjm.com/~/media/documents/manuals/m11.ashx.

Ramsey, Frank P. 1927. A contribution to the theory of taxation. *The Economic Journal* 37(145). 47–61.

Rockafellar, R. T. 1970. *Convex analysis*. Princeton University Press.

Royal Swedish Academy of Sciences. 2007. Mechanism design theory. https://www.nobelprize.org/uploads/2018/06/advanced-economicsciences2007.pdf.

Schiro, Dane A, Tongxin Zheng, Feng Zhao & Eugene Litvinov. 2015. Convex hull pricing in electricity markets: Formulation, analysis, and implementation challenges. *IEEE Transactions on Power Systems* 31(5). 4068–4075.

Schweppe, Fred C, Michael C Caramanis, Richard D Tabors & Roger E Bohn. 2013. *Spot pricing of electricity*. Springer Science & Business Media.

Topping, Elizabeth. 2022. Energy price formation. https://www.ferc.gov/industries-data/electric/electric-power-markets/energy-price-formation.

Varian, HR. 1989. Price discrimination. *Handbook of Industrial Organization, Edited by R. Schmalensee and R.D. Willig* .

Vyve, Mathieu Van. 2011. Linear prices for non-convex electricity markets: models and algorithms. https://cdn.uclouvain.be/public/Exports%20reddot/core/documents/coredp2011_50web.pdf.

Wang, Congcong, Peter B Luh, Paul Gribik, Tengshun Peng & Li Zhang. 2016. Commitment cost allocation of fast-start units for approximate extended locational marginal prices. *IEEE Transactions on Power Systems* 31(6). 4176–4184.

Wang, Congcong, Tengshun Peng, Peter B Luh, Paul Gribik & Li Zhang. 2013.

The subgradient simplex cutting plane method for extended locational marginal prices. *IEEE Transactions on Power Systems* 28(3). 2758–2767.

Yu, Yanan, Yongpei Guan & Yonghong Chen. 2020. An extended integral unit commitment formulation and an iterative algorithm for convex hull pricing. *IEEE Transactions on Power Systems* 35(6). 4335–4346.

Appendix A. Definitions, Acronyms and Abbreviations

AIC is average incremental cost.

AICOPR is AIC One-pass Pricing Run linear program.

AFC is average fixed costs in $/MWh.

Arbitrage condition is when prices result in no desire to change dispatch, for example, absent ramp rate constraints, the price for reserves and energy differ by their marginal costs.

Average incremental cost (AIC) is avoidable costs divided by dispatched energy + reserves.

Avoidable costs are costs that can avoided by not operating.

Avoidable fixed costs are fixed costs that can avoided by not operating.

Degeneracy occurs when a linear program is degenerate. Its dual may have multiple optima.

Dispatched occurs a unit is generating or off-line but on reserve.

Economic efficiency (EE) is maximum social welfare, i.e., market surplus.

FBLP is fixed-binary linear program.

Flowgate incremental price (FIP) is the transmission-limit dual variable from the AICOPR.

Flowgate marginal price (FMP) is the transmission-limit dual variable from the FBLP.

Incentive compatibility occurs when the incentives that motivate the actions of individual market participants are consistent with following the rules established by the group.

Incremental generator breaks even at the LIP prices, that is, $\pi_i = 0$.

Individual rationality (IR) occurs when no person loses from joining the auction.

Infra-incremental generator has positive profit at the LIP prices, that is, $\pi_i > 0$.

Infra-marginal generator is a generator that makes positive profit under LMP prices.

In-market unit is a unit dispatched or scheduled by the system operator.

Locational incremental price (LIP) is the the energy-balance dual variable of the AICOPR.

Locational incremental reserves price (rLIP) from the reserves-requirement dual variable of the AICOPR.

Locational marginal price (LMP) from the energy-balance dual variable of the FBLP.

Locational marginal reserves price (rLMP) from the reserves-requirement dual variable of the FBLP.

Marginal consumer is a consumer that sets the LMP and makes zero profit at the LMP price.

Market-clearing is the process by which the auction quantities are computed.

Market-clearing price is the price at which the quantity supplied equals quantity demanded. In some convex and non-convex markets, a single market clearing price may not exist.

MIP is mixed integer linear program.

MIP gap is the distance between the best MIP feasible solution and the best bound.

One-event-in-ten-years is an arbitrary standard for reliability where an event is undefined.

ORDC is Operating Reserve Demand Curve.

Out-of-market unit is a unit not dispatched or scheduled by the system operator.

Profitability occurs when a market participant's gross value less costs is non-negative in each up-down cycle.

Revenue neutrality (RN)/balanced transfers (BT) occurs when money transfers net to zero.

rLMP is locational marginal reserve price from the reserves-requirement FBLP dual variable.

SCED is security constrained economic dispatch with fixed the binaries.

SCUC is security constrained, unit commitment and economic dispatch – the primal problem.

Self-schedule is any bid or offer that is other than avoidable costs or value.

Sunk or unavoidable costs are costs that cannot be avoided even if the unit is not operating.

Sustainable occurs when there is enough short-term market profit to invest in efficient new devices.

Truth telling (TT)/Incentive compatible (IC) is occurs when a market participant reveals its true values and costs in its bids and offers.

Valid optimality cut is any constraint that does not eliminate all optimal solutions.

π_i is the profit for generator i under AIC pricing.

Appendix B. Proofs

Proof of Lemma 2.: From (2f), (2h), (2i), and (2j),

$$p_{it}^{**} + p_{it}^{ru**} \leq p_{it}^{max} u_{it}^{**} = (p_{it}^{*} + p_{it}^{ru*} + \epsilon) u_{it}^{**} \tag{A5a}$$

$$p_{jit}^{**} \leq p_{jit}^{max} u_{it}^{**} = (p_{jit}^{*} + \epsilon) u_{it}^{**} \tag{A5b}$$

$$\text{summing over j, } p_{it}^{**} \leq p_{it}^{max} u_{it}^{**} = (p_{it}^{*} + \epsilon) u_{it}^{**} \tag{A5c}$$

$$p_{it}^{ru**} \leq p_{it}^{rumax} u_{it}^{**} = (p_{it}^{ru*} + \epsilon) u_{it}^{**} \tag{A5d}$$

$$-p_{it}^{**} \leq -p_{it}^{min} u_{it}^{**} = -(p_{it}^{*} - \epsilon) u_{it}^{**} \tag{A5e}$$

For $\epsilon = 0$,

$$p_{it}^{**} + p_{it}^{ru**} \leq (p_{it}^{*} + p_{it}^{ru*}) u_{it}^{**} \tag{A5f}$$

$$p_{jit}^{**} \leq p_{jit}^{*} u_{it}^{**} \tag{A5g}$$

$$p_{it}^{**} \leq p_{it}^{*} u_{it}^{**} \tag{A5h}$$

$$p_{it}^{ru**} \leq p_{it}^{ru*} u_{it}^{**} \tag{A5i}$$

$$-p_{it}^{**} \leq -p_{it}^{*} u_{it}^{**} \tag{A5j}$$

From (A5h) and (A5j), $p_{it}^{**} = p_{it}^{*} u_{it}^{**}$.

For (A5g), if there exists any $p_{jit}^{**} < p_{jit}^{*} u_{it}^{**}$, then summing over j, we have the contradiction $p_{it}^{**} < p_{it}^{*} u_{it}^{**}$. Hence, $p_{jit}^{**} = p_{jit}^{*} u_{it}^{**}$.

Without degeneracy, both (1c) and (2c) bind with $\sum_{i \in G} p_{it}^{ru*} = \sum_{i \in G} p_{it}^{ru**} = r_t^{us}$.

For (A5i), if there exists any $p_{it}^{ru**} < p_{it}^{ru*} u_{it}^{**} \leq p_{it}^{ru*}$, then $\sum_{i \in G} p_{it}^{ru**} < \sum_{i \in G} p_{it}^{ru*} = r_t^{us}$, also a contradiction. Hence, $p_{it}^{ru**} = p_{it}^{ru*} u_{it}^{**}$.

Moreover, with (1b), (2b) and $d_{it}^{**} = d_{it}^{*}$ with the construction of AICOP, we have $\sum_{i \in G} p_{it}^{**} = \sum_{i \in G} p_{it}^{*}$. We can prove using a similar argument that if $p_{it}^{*} > 0$ or $p_{it}^{ru*} > 0$, then $u_{it}^{**} = u_{it}^{*} = 1$. Therefore, $p_{it}^{**} = p_{it}^{*}$.

At the optimum the inequalities hold as equalities □

Proof of Proposition 1

With the construction of AICOP and Lemma 2, all units with $p^*_{it} > 0$ in SCUC solution have $p^{**}_{it} = p^*_{it} > 0$ in AICOP. We can prove that the price from AICOP will result in no make-whole payment for the solution from SCUC.

The static constraints. From complementary slackness of (3e),

$$(-\mu_{it} + \beta^{max}_{jit})p^{**}_{jit} = -c_{jit}p^{**}_{jit} \tag{A6a}$$

Summing over j,

$$\sum\nolimits_{j\in Ji}(-\mu_{it} + \beta^{max}_{jit})p^{**}_{jit} = -\sum\nolimits_{j\in Ji} c_{jit}p^{**}_{jit} \tag{A6b}$$

Rearranging and substituting $p^{**}_{it} = \sum_{j\in J_i} p^{**}_{jit}$

$$\mu_{it}p^{**}_{it} = \sum\nolimits_{j\in Ji} (c_{jit} + \beta^{max}_{jit})p^{**}_{jit} \tag{A6c}$$

By complementary slackness of (3c),

$$(\rho^{up}_{it} - \rho^{up}_{it+1} - \lambda_t + \mu_{it} - \beta^{min}_{it} + \beta^{max}_{it})p^{**}_{it} = 0 \tag{A6d}$$

Rearranging,

$$\mu_{it}p^{**}_{it} = -(\rho^{up}_{it} - \rho^{up}_{it+1} - \lambda_t - \beta^{min}_{it} + \beta^{max}_{it})p^{**}_{it} \tag{A6e}$$

Combining (A6c) and (A6e), and rearranging,

$$\lambda_t p^{**}_{it} = (\sum\nolimits_{j\in Ji} c_{jit}p^{**}_{jit}) + (\sum\nolimits_{j\in J_i} \beta^{max}_{jit}p^{**}_{jit}) + \beta^{max}_{it}p^{**}_{it} - \beta^{min}_{it}p^{**}_{it} + (\rho^{up}_{it} - \rho^{up}_{it+1})p^{**}_{it} \tag{A6f}$$

Rearranging,

$$(\sum_{j \in Ji} \beta_{jit}^{max} p_{jit}^{**}) + \beta_{it}^{max} p_{it}^{**} - \beta_{it}^{min} p_{it}^{**} = \lambda_t p_{it}^{**} - (\sum_{j \in Ji} c_{jit} p_{jit}^{**}) - (\rho_{it}^{up} - \rho_{it+1}^{up}) p_{it}^{**} \quad \text{(A6g)}$$

From (2f) by complementary slackness,

$$(p_{jit}^{**} - p_{jit}^{max} u_{it}^{**}) \beta_{jit}^{max} = 0 \quad \text{(A6h)}$$

From (2h) by complementary slackness,

$$(p_{it}^{**} + p_{it}^{ru**} - p_{it}^{max} u_{it}^{**}) \beta_{it}^{max} = 0 \quad \text{(A6i)}$$

From (2j) by complementary slackness,

$$(-p_{it}^{**} + p_{it}^{min} u_{it}^{**}) \beta_{it}^{min} = 0 \quad \text{(A6j)}$$

Substituting (A6h), (A6i), and (A6j) into (A6g) and rearranging

$$[(\sum_{j \in Ji} p_{jit}^{max} \beta_{jit}^{max}) + p_{it}^{max} \beta_{it}^{max} - p_{it}^{min} \beta_{it}^{min}] u_{it}^{**} = \lambda_t p_{it}^{**} - (\sum_{j \in Ji} c_{jit} p_{jit}^{**}) + p_{it}^{ru**} \beta_{it}^{max} - (\rho_{it}^{up} - \rho_{it+1}^{up}) p_{it}^{**} \quad \text{(A6k)}$$

By complementary slackness of (2k),

$$(p_{it}^{**} - p_{it-1}^{**} - r_{it}^{up} u_{it}^{**}) \rho_{it}^{up**} = 0 \quad \text{(A6l)}$$

Rearranging,

$$(p_{it}^{**} - p_{it-1}^{**}) \rho_{it}^{up} = r_{it}^{up} u_{it}^{**} \rho_{it}^{up} \quad \text{(A6m)}$$

By complementary slackness of (3g),

$$[\gamma_{it}+\delta_{it}-\delta_{it+1}-r_{it}^{up}\rho_{it}^{up}+p_{it}^{min}\beta_{it}^{min}-(\sum_{j\in Ji} p_{jij}^{max}\beta_{jit}^{max})-p_{it}^{max}\beta_{it}^{max}-p_{it}^{rumax}\beta_{it}^{rumax}+c_{it}^{op}]u_{it}^{**}=0 \tag{A6n}$$

Rearranging,

$$\begin{aligned}[\gamma_{it}+\delta_{it}-\delta_{it+1}-r_{it}^{up}\rho_{it}^{up}-p_{it}^{rumax}\beta_{it}^{rumax}+c_{it}^{op}]u_{it}^{**}= \\ -[p_{it}^{min}\beta_{it}^{min}-(\sum_{j\in Ji} p_{jit}^{max}\beta_{jit}^{max})-p_{it}^{max}\beta_{it}^{max}]u_{it}^{**}\end{aligned} \tag{A6o}$$

Substituting (A6k) into (A6o) and rearranging,

$$\begin{aligned}\lambda_t p_{it}^{**} &= (\sum_{j\in Ji} c_{jit}p_{jit}^{**})-p_{it}^{ru**}\beta_{it}^{max} \\ &+[\gamma_{it}+\delta_{it}-\delta_{it+1}-p_{it}^{rumax}\beta_{it}^{rumax}+c_{it}^{op}]u_{it}^{**}-r_{it}^{up}u_{it}^{**}\rho_{it}^{up}+(\rho_{it}^{up}-\rho_{it+1}^{up})p_{it}^{**}\end{aligned} \tag{A6p}$$

By complementary slackness of (2i),

$$(p_{it}^{ru}-p_{it}^{rumax}u_{it}^{**})\beta_{it}^{rumax}=0 \tag{A6q}$$

By complementary slackness of (3f),

$$(\beta_{it}^{rumax}+\beta_{it}^{max}-\lambda_t^{us}+c_{it}^{ru})p_{it}^{ru**}=0 \tag{A6r}$$

Subtracting (A6r) and (A6q) and rearranging,

$$-p_{it}^{rumax}u_{it}^{**}\beta_{it}^{rumax}=(\beta_{it}^{max}-\lambda_t^{us}+c_{it}^{ru})p_{it}^{ru**} \tag{A6s}$$

Substituting (A6s) and (A6m) into (A6p) and rearranging,

$$\lambda_t p_{it}^{**} + \lambda_t^{us} p_{it}^{ru**}$$
$$= (\sum_{j \in J_i} c_{jit} p_{jit}^{**}) + c_{it}^{ru} p_{it}^{ru**} + [\gamma_{it} + \delta_{it} - \delta_{it+1} + c_{it}^{op}] u_{it}^{**} + (\rho_{it}^{up} - \rho_{it+1}^{up}) p_{it}^{**} - (p_{it}^{**} - p_{it-1}^{**}) \rho_{it}^{up}$$
$$= (\sum_{j \in J_i} c_{jit} p_{jit}^{**}) + c_{it}^{ru} p_{it}^{ru**} + [\gamma_{it} + \delta_{it} - \delta_{it+1} + c_{it}^{op}] u_{it}^{**} - r_{it}^{up} u_{it}^{**} \rho_{it}^{up} + (\rho_{it}^{up} - \rho_{it+1}^{up}) p_{it}^{**} \tag{A6t}$$

Ramp rate dynamics. Summing $(\rho_{it-1}^{up} - \rho_{it}^{up}) p_{it}^{**}$ over T.

$$\sum_T (\rho_{it}^{up} - \rho_{it+1}^{up}) p_{it}^{**} = \rho_{i1}^{up} p_{i1}^{**} - \rho_{i2}^{up} p_{i1}^{**} + \rho_{i2}^{up} p_{i2}^{**} - \rho_{i3}^{up} p_{i2} + \ldots + \rho_{itmax}^{up} p_{itmax}^{**} - \rho_{itmax+1}^{up} p_{itmax}^{**} \tag{A6u}$$

Summing (p_{it}^{**}-p_{it-1}^{**})$\rho^{up}{}_{it}$ over T

$$\sum_T (p_{it}^{**} - p_{it-1}^{**}) \rho_{it}^{up} = p_{i1}^{**} \rho_{i1}^{up} - p_{i0}^{**} \rho_{i1}^{up} + p_{i2}^{**} \rho_{i2}^{up} - p_{i1}^{**} \rho_{i2}^{up} + \ldots + p_{itmax}^{**} \rho_{itmax}^{up} - p_{itmax-1}^{**} \rho_{itmax}^{up} \tag{A6v}$$

Subtracting (A6u) from (A6v) and since $p_{itmax+1}^{**}$ is not in the model we set it to 0.

$$\sum_T (p_{it}^{**} - p_{it-1}^{**}) \rho_{it}^{up} - \sum_{T'} (\rho_{it}^{up} - \rho_{it+1}^{up}) p_{it}^{**} = -p_{i0}^{**} \rho_{i1}^{up} + \rho_{itmax+1}^{up} p_{itmax}^{**} \tag{A6w}$$

In the first term, ρ_{i0}^{up} is undefined in the model and set to 0; p_{i0}^{**} is a parameter from the previous operating period. Given t=0 is the interval before a commitment block T, we have $u_{i0}^{**} = 0$ and $p_{i0}^{**} = 0$. We make an adjustment to p_{i1}^{max} to account for the ramp constraint in period one. In the last term, $\rho_{(itmax+1)}^{up}$, is outside the model horizon and set to 0. For finite horizon models, initial conditions are specified and in practice, the horizon extends several periods beyond the auction horizon to minimize the end of the horizon effect.

Summing (A6t) over τ, canceling terms and rearranging,

$$\sum_{t\in T}[\lambda_t p_{it}^{**}+\lambda_t^{us} p_{it}^{ru**}] = \sum_{t\in T}[\sum_{j\in J_i} c_{jit}p_{jit}^{**}+c_{it}^{ru}p_{it}^{ru**}+(\gamma_{it}+\delta_{it}-\delta_{it+1}+c_{it}^{op})u_{it}^{**}] \tag{A6x}$$

The binary relaxation dynamics. Binary variables must satisfy the equality, (1l). If $c_{it}^{op} > 0$ and $c_{it}^{su} > 0$, the following must hold: If z_{it}^{*} = 1, then $u_{it-1}^{*} = 0, u_{it}^{*} = 1$, and $z_{it}^{d*} = 0$. If z_{it}^{d*} = 1, then u_{it-1}^{*} = 1, u_{it}^{*} =0, and z_{it}^{*} =0. Since the relaxed binary variables must satisfy the equality, (2l), the following must hold: From (2m), if z_{it}^{*} = 0, z_{it}^{**} = 0. From (2n), if u_{it}^{*} = 0, u_{it}^{**} = 0. From (2o), if $z_{it}^{d\,*}$ = 0, z_{it}^{d**} = 0. This eliminates non-operating generators from the pricing algorithm.

Since the relaxed binary variable must satisfy the equality, (2l), from (2m), if z_{it}^{*}=1, $0 \leq z_{it}^{**} \leq 1$.

Since $c_{it}^{su} > 0$ and generator i is part of the optimal solution, $z_{it}^{**} > 0$ because if $z_{it}^{**} > 0$, a less costly solution would be z_{it}^{*} = 0 which is a contradiction.

If z_{it}^{*}=1, u_{it}^{*}= 1, from (2l) u_{it}^{**}= z_{it}^{**}. If $u_{it}^{*} = 1$ and u_{it+1}^{*} = 1, that is, unit i was not shut down or started up in t+1, u_{it+1}^{**} = u_{it}^{**}. If z_{it}^{d*}= 1, $0 \leq z_{it}^{d**} \leq 1$. If z_{it}^{d*}= 1, u_{it-1}^{*}= 1 and u_{it}^{*} = 0. If u_{it}^{*} = 0, u_{it-1}^{**}= z_{it}^{d**}.

For the up-down cycle in (1), $z_{it}^{**} = u_{it}^{**}$ for $t \in T_i$. For $t''+1, u_{it''}^{**} = z_{it''+1}^{d**}$, and if $t'' > t'$, $z_{it}^{**} = u_{it}^{**} = u_{it+1}^{**} = \ldots = u_{it''}^{**} = z_{it''+1}^{d**}$. For $t \in T_i$, let $u_i = u_{it}^{**} = u_i^{**}$. For $t \notin T_i$, $u_{it}^{**} = 0$. Since $u_{it}^{**} = u_i^{**}$ for $t \in T_i = \{t', \ldots, t''\}$ and $u_{it}^{**} = 0$ for $t \notin T_i$.

$$\sum_{t\in T}[(\delta_{it}-\delta_{it+1})u_{it}^{**}] = \sum_{t\in T_i}[(\delta_{it}-\delta_{it+1})u_i^{**}] = (\delta_{it'}-\delta_{it''+1})u_i^{**} \tag{A7a}$$

From complementary slackness of (2o), if $z_{it''+1}^{d**} < z_{it''+1}^{d*}$, $\omega_{it''+1}^{d}$ = 0.

From complementary slackness of (3h), if $z_{it''+1}^{d**} < z_{it''+1}^{d*}$, then ($\omega_{it''+1}^{d}$ + $\delta_{it''+1}$) =0 and

$$\delta_{it''+1} = 0 \tag{A7b}$$

From complementary slackness of (3d),

$$(\omega_{it} - \delta_{it} + c_{it}^{su})z_{it}^{**} = 0 \tag{A7c}$$

For $t \notin T_i$, $u_{it}^{**} = 0$ and $z_{it}^{**} = 0$. For $t \in T_i$, if $z_{it'}^{**} < z_{it'}^{*}$, from complementary slackness of (2m), $\omega_{it'} = 0$,

$$(-\delta_{it'} + c_{it'}^{su})z_{it'}^{**} = (-\delta_{it'} + c_{it'}^{su})u_{it'}^{**} = 0 \tag{A7d}$$

and

$$\delta_{it'}u_{it'}^{**} = c_{it'}^{su}u_{it'}^{**} \tag{A7e}$$

Substituting (A7b) into (A7e),

$$(\delta_{it'} - \delta_{it''+1})u_{i}^{**} = c_{it'}^{su}u_{i}^{**} \tag{A7f}$$

And (A6x) becomes,

$$\sum\nolimits_{t\in T} [\lambda_t p_{it}^{**} + \lambda_t^{us} p_{it}^{ru**}] = \sum\nolimits_{t\in T} [(\sum\nolimits_{j\in J_i} c_{jit}p_{jit}^{**}) + c_{it}^{ru}p_{it}^{ru**} + c_{it}^{op}u_i^{**} + \gamma_{it}u_i^{**}] + c_{it'}^{su}u_i^{**} \tag{A7g}$$

Case 1. $u_i^{**} = 1$, $t \in T_i$ and $u_i^{**} = 0$, $t \notin T_i$,

Since $\gamma_{it} \geq 0$, if $u_i^{**} = 1$, as $\epsilon = 0$, (A7g) becomes

$$\sum\nolimits_{t\in T_i} (\lambda_t p_{it}^{*} + \lambda_t^{us} p_{it}^{ru*}) - \sum\nolimits_{t\in T_i} [(\sum\nolimits_{j\in J_i} c_{jit}p_{jit}^{*}) + c_{it}^{op}] - c_{it'}^{su} - \sum\nolimits_{t\in T} c_{it}^{ru}p_{it}^{ru*} \geq 0. \tag{A7h}$$

Energy and reserves revenues are greater or equal to incremental energy and reserves costs, that is, all dispatched units are profitable with only the LIP energy and reserve prices, that is, no generator needs a make-whole payment.

If γ_{it} = 0 and u_i^{**} = 1, (A7g) becomes

$$\sum_{t\in Ti} (\lambda_t p_{it}^* + \lambda_t^{us} p_{it}^{ru*}) \qquad -\sum_{t\in Ti} [(\sum_{j\in J_i} c_{jit} p_{jit}^*) + c_{it}^{op}] - c_{it'}^{su} - \sum_{t\in T} c_{it}^{ru} p_{it}^{ru*} = 0 \tag{A7i}$$

Energy and reserves revenues - Incremental energy and reserves costs = 0.

Case 2. For $t \in T_i, u_i^{**} < u_{it}^*$. For t $\notin T_i$, if $u_{it}{}^{**} = 0$, $p_{it}^{**} = 0$. Let c_{it} be the marginal cost at p_{it}^*. If (2j) binds, absent degeneracy, $\beta_{it}^{min} > 0$, $p_{it}^* = p_{it}^{min}$, and $c_{it} > \lambda_t$. Relaxing u_i increases the objective function until another constraint binds. As ϵ= 0, we have $p_{it}^{**} = p_{it}^* u_i^{**}$ and $p_{it}^{ru**} = p_{it}^{ru*} u_i^{**}$, and

$$\sum_{t\in T_i} [\lambda_t p_{it}^* u_i^{**} + \lambda_t^{us} p_{it}^{ru*} u_i^{**}] = \sum_{t\in T_i} [\lambda_t p_{it}^{**} + \lambda_t^{us} p_{it}^{ru**}] \tag{A7j}$$

Substituting p_{it}^{ru**}=$p_{it}^{ru*}u_i^{**}$, p_{jit}^{**}=$p_{jit}^* u_i^{**}$, p_{jit}^{**}=$p_{jit}^* u_i^{**}$, and (A7j) into (A6x),

$$\sum_{t\in T_i} [\lambda_t p_{it}^* u_i^{**} + \lambda_t^{us} p_{it}^{ru*} u_i^{**}] = \sum_{t\in T_i} [\sum_{j\in Ji} c_{jit} p_{jit}^* u_i^{**} + c_{it}^{ru} p_{it}^{ru*} u_i^{**} + (\gamma_{it} + \delta_{it} - \delta_{it+1} + c_{it}^{op}) u_i^{**}] \tag{A7k}$$

Dividing (A7k) by u_i^{**} and since $\sum_{t\in T_i} (\delta_{it'} - \delta_{it''+1}) = c_{it'}^{su}$,

$$\sum_{t\in T_i} [\lambda_t p_{it}^* + \lambda_t^{us} p_{it}^{ru*}] = \sum_{t\in T_i} [(\sum_{j\in J_i} c_{jit} p_{jit}^*) + c_{it}^{ru} p_{it}^{ru*} + c_{it}^{op} + \gamma_{it}] + c_{it'}^{su} \tag{A7l}$$

Since $\gamma_{it} \geq$ 0, all dispatched units are profitable using only the *LIP* energy and reserve prices, that is, no generator needs a make-whole payment. □

Proof of Proposition 4: If $p_{it}^{min} < p_{it}^* < p_{it}^{max}, \beta_{it}^{min} = 0$. From complementary slackness of (3c),

$$\rho_{it}^{up} - \rho_{it+1}^{up} - \lambda_t + \mu_{it} + \beta_{it}^{max} = 0 \tag{A8a}$$

From complementary slackness of (3f), if $0 < p_{it}^{ru**} < p_{it}^{rumax**}, \beta_{it}^{rumax} = 0$,

$$\beta_{it}^{max} + c_{it}^{ru} - \lambda_t^{us} = 0. \tag{A8b}$$

Subtracting (A8a) from (A8b) , $\lambda_t - \lambda_t^{us} = \mu_{it} - c_{it}^{ru} + \rho_{it}^{up} - \rho_{it+1}^{up}$

From complementary slackness of (3e), if $p_{jit} > 0$, $\mu_{it} = c_{jit} + \beta_{jit}^{max}$.

If there is a $j' \in Ji$ where $p_{j'it}^{*} = p_{it}^{*} < p_{j'it}^{max}$, then $\beta_{j'it}^{max} = 0$ and $\mu_{it} = c_{j'it} = c_{it}$.

Substituting $\mu_{it} = c_{it}$, (A8a) becomes, $\lambda_t - \lambda_t^{us} = c_{it} - c_{it}^{ru} + \rho_{it}^{up} - \rho_{it+1}^{up}$.

If there is no binding ramp constraint, ρ_{it}^{up} - ρ_{it+1}^{up} = 0 and $\lambda_t - \lambda_t^{us} = c_{it} - c_{it}^{ru}$.

□

Appendix C. Illustrative Small Examples of AIC Pricing

Small examples serve several purposes. They help with intuition. ISO's use them as educational material. They allow the reader to track and replicate the example results, for example, the allocation of incremental costs to the period that caused them. They may show pathologies that are hidden or less pronounced in larger problems, for example, cost allocation, complementarities, degeneracy, and horizon effects. Fortunately, degeneracy is not known to be a serious problem in practice. In practice, the conditions in the initial period are specified, and the horizon is extended multiple periods beyond the settlement periods to dampen any end-of-horizon influence.They can demonstrate entry and exit conditions. Nevertheless, large examples on actual problems are the acid test for implementation.

To show the choice of ϵ, we show the dual variables on the energy balance and reserves constraints from a series of decreasing ϵ and at $\epsilon = 0$ along with two different linear program codes and two possible variations of the AIC pricing. In variation 1, LIP1 sets $p^{max} = p^{*} + \epsilon$ only for generators with negative profits under LMP prices. In variation 2, LIP2 sets $p^{max} = p^{*} + \epsilon$ for all dispatched generators.

At some point as $\epsilon \to 0$, ϵ becomes numerically zero and the problem becomes numerically degenerate. At ϵ=0, degeneracy in addition to the degeneracy that may already be present occurs, enlarging the set of optimal dual variables, in particular, the prices for energy and reserves. This can produce unusual and unusable pricing results. For most solvers, for example, GUROBI, the choice of dual variables under primal degeneracy is not known to be predictable, but they are repeatable.

From the examples below, it appears the LIP1 with $\epsilon = 10^{-4}$ is the more stable pricing scheme. In these AIC pricing schemes with two or more generators, there is always an incremental generator that breaks even (zero profit) and usually infra-incremental generators that make a positive profit. No generator needs a make-whole payment. These are similar to the properties of a convex market.

Monetary units are dollars. A period is one hour, but could be any time interval.

The results were produced by the GAMS program:

MinRunRampPricDyn20200530.gms and MinRunRampPricDyn20200521.gms

on a laptop. The 'alt' solutions are from the Excel solver on a different laptop.

Example 1. Three Period Market with One-Step Marginal Costs Functions.

The load parameters are in Table 1.1. The generators' parameters are in Table 1.2. Table 1.3 has the LMP and AIC market results.

Table 1.1. Load

Period	1	2	3
Value	900	900	900
Max Load	95	100	130

Table 1.2. Generation

Gen	Marg Cost $/MWh	Min Gen MW	Max Gen MW	Max at Start MW	Start Cost $	Min Run Time hrs	Fix Oper Cost $/per	Ramp Up Rate MW/per	Ramp Down Rate MW/per
1	10	0	100	0	0	1	0	200	900
2	50	20	35	26	1000	1	30	5	900
3	320	0	31	31	0	1	0	200	200

Table 1.3. Optimal Dispatch and Prices without Reserves.

period	1		2		3	
	Energy	MValue	Energy	MValue	Energy	MValue
Load	95	890	100	890	130	810
Gen1	95	0	75	0	100	80
Gen2	0	0	25	0	30	0
LMP		10.00		10.00		90.00
LIP1($\epsilon = 10^{-4}$)		10.00		10.00		118.67
LIP1(ϵ=0)		10.00		10.00		900.00
LIP1(ϵ=0)alt		10.00		10.00		118.67
LIP2($\epsilon = 10^{-4}$)		10.00		10.00		118.67
LIP2(ϵ=0)		10.00		10.00		900.00
LIP2(ϵ=0)alt		10.00		92.40		50.00

MValue is marginal value.

For Gen2, the startup p^{max} = 26 and the ramp rate constraints combine to force it to startup in period 2 to be at 30 MW in period 3. Gen1 sets the LMP in periods 1 and 2. Gen2 sets the LMP in period 3. The binding ramp rate constraint for Gen2 from period 2 to 3 adds $40/MWh to the LMP in period 3. With an additional unit of Gen2 ramp, the Gen2 dispatch would be 24 in period 2, and 30 in period 3 saving $40. At ϵ=10^{-4}, in period 3, the LIP is $118.67/MWh (= (3810-250)/30), each AIC variation with $\epsilon \geq 0$ allocates all residual costs (cost above those recovered by the LMP) of Gen2 to the period 3 LIP – the period that demand caused the dispatch of Gen2 in period 2.

At ϵ=0, two variations, LIP1(ϵ=0) and LIP2(ϵ=0), the program choose $900/MWh (the value of demand) as the clearing price in period 3, because Gen1 is at its maximum and Gen2 is constrained by its ramp rate. LIP2(ϵ=0)alt results in LIPs of $10, $92.4, $50, Gen2 also breaks even with the same market surplus. The redistribution of market surplus occurs between load and Gen1.

Table 1.4 contains the avoidable costs, the settlements at the LMPs and LIPs for $\epsilon = 10^{-4}$ and ϵ=0. the LMP settlement needs a make-whole payment. The LIP settlements need no make-whole payments. The sequence of AIC objective function values converge to optimal dispatch because the p^{max} constraints bind. For the dispatch problem and AIC problem with ϵ=10^{-4} and ϵ=0, the market surplus is the same rounded to 6 digits. At ϵ=0, the LIP price in period 3 is $900/MWh, the settlement changes, but all generation remains profitable and load breaks even.

Table 1.4. Settlement at LMP and LIP (ϵ=10^{-4} and ϵ =0) without Reserves

	Avoidable	Profit/Value at				
	cost	LMP	LIP1($\epsilon = 10^{-4}$)	LIP1($\epsilon = 0$)	LIP2($\epsilon = 10^{-4}$)	LIP2($\epsilon = 0$)
Gen1	2700	8000	10866.67	89000	10866.67	89000
Gen2	3810	-860	0	23440	0	23440
Total	6510	7140	10866.67	112440	10866.67	112440
load		278850	275123	173550	275123	173550
MS		285990	285990	285990	285990	285990

Gen3 Entry.

The LMP is a valid signal for generators with costs lower than the LMP to enter the market. But, in non-convex markets, entry at costs greater than the LMP can occur. To illustrate, we introduce Gen3, a convex generator with a maximum output of 31 MW and a constant marginal cost. If we set the marginal costs of Gen3 to \$118.66/MWh (just below the LIP1 price) , Gen3 enters the market and displaces 4 MW of Gen2 in period 3. Table 1.5 contains the optimal results. Gen2 now starts up in period 3 at its startup maximum (26 MW). Gen3 sets the price in period 3 and breaks even. The saving for dispatching Gen2 in only period 3 is \$1230 (= \$40/MWh×25 MWh plus \$30 fix cost in period 2 and \$50/MWh×4 MWh in period 3). At a marginal cost of \$118.66/MWh, Gen3 enters, and therefore, \$118.67/MWh is a valid entry signal, and the market surplus increases to \$286,745.36.

If Gen3 marginal costs are \$307.5/MWh, for 4 MWh cost is \$1230 and the solver is indifferent about dispatching Gen3. At \$307.4/MWh, Gen3 enters the market and sets the price in period 3 at \$307.4/MWh and breaks even. Gen1 and Gen2 are more profitable. At \$307.6/MWh, Gen3 does not enter the market. These results are due to the lumpiness of fixed costs, the high demand value, and the non-convex complementarities.

Table 1.5. Optimal Dispatch, Prices and Settlement for Gen3 marginal cost of \$118.66/MWh

period	1		2		3			
	Energy	MValue	Energy	MValue	Energy	MValue	Cost	Profit/Value
Load	95	890	100	890	130	781.34		275124.20
Gen1	95	0	100	0	100	108.66	2950.00	10866.00
Gen2	0	0	0	0	26	68.66	2330.00	755.16
Gen3	0	0	0	0	4	0	474.64	0
LMP		10.00		10.00		118.66		
Total							5754.64	286745.36

Example 2. Changing the Startup Value of p_2^{max}

If the startup p_2^{max} is 23 MW (instead of 26), Gen2 would need to startup in period

1 to ramp to 30 MW in period 3. Table 2.1 has the market results and prices. The market surplus declines from the example 1 by \$830 (\$800 for extra marginal cost and \$30 for fixed operating costs in period 1). An additional unit of ramp rate is worth \$80/MWh (\$40/MWh from period 1 to 2 and \$40/MWh from period 1 to 2. The LMP in period 3 is \$130/MWh (=50+40+40) set by Gen2 due to ramp rate constraints from periods 1 to 2 and 2 to 3 and and the marginal energy cost of \$50/MWh. In period 3, the LIP is \$146.33. With an additional unit of ramp, the dispatch of Gen2 would be 18, 24, and 30 in periods 1, 2 and 3 saving \$120. The fixed-binary linear problem is degenerate. In Period 1, the p^{min} and the ramp rate constraints of Gen2 simultaneously bind blocking the dispatch of 18 MWh in period 1. If the p_2^{min} is increased to 21 in period 1, the LMP in period 3 decreases to \$90/MWh.

Table 2.1. Optimal Dispatch and Prices without Reserves for startup Gen2 p_2^{max} at 23 MW

period	1		2		3	
	Energy	Value	Energy	MValue	Energy	MValue
Load	95	890	100	890	130	810
Gen1	75	0	75	0	100	80
Gen2	20	0	25	0	30	0
LMP		10.00		10.00		130.00
LIP1 (ϵ=10^{-4})		10.00		10.00		146.33
LIP1 (ϵ=0)		10.00		10.00		900.00
LIP1 (ϵ=0)alt		10.00		10.00		146.33
LIP2 (ϵ=10^{-4})		10.00		10.00		146.33
LIP2 (ϵ=0)		900.00		50.00		50.00
LIP2 (ϵ=0)alt		10.00		125.6		50.00

Table 2.2 contains the avoidable costs, the settlement at the LMPs, and LIPs with $\epsilon=10^{-4}$ and $\epsilon=0$. At $\epsilon=10^{-4}$, the settlement for LIP1 and LIP2 are the same and Gen2 is incremental. At $\epsilon=0$, the settlements are different since with LIP1, load sets the price in period 3 and in LIP2 sets the price in period 1 as a result of additional degeneracy. For the optimal dispatch and AIC problem with $\epsilon=10^{-4}$ and $\epsilon=0$, the

objective function (maximize market surplus) is the same and converges to the efficient market surplus.

Table 2.2. Settlement at LMP and LIP ($\epsilon = 10^{-4}$ and $\epsilon = 0$) without Reserves

	Avoidable	Profit/Value at				
	Cost	LMP	LIP1($\epsilon = 10^{-4}$)	LIP1($\epsilon = 0$)	LIP2($\epsilon = 10^{-4}$)	LIP2($\epsilon = 0$)
Gen1	2500	12000	13633.33	89000	13633.33	73750
Gen2	4840	-490	0	22610	0	15910
Total	7340	11510	13633.33	111610	13633.33	89660
load		273650	271527.00	173550	271527.00	195500
MS		285160	285160.00	285160	285160.00	285160

Example 3. Example 1 with Reserves.

We add a reserves requirement of 1 MW per period to Example 1. Reserve offer is \$1/MWh for Gen1 and \$1.5/MWh for Gen2. Table 3.1 has the market results. Reserves are provided by Gen1 in periods 1 and 2 and Gen2 in period 3 when all Gen1 capacity is used for energy. The degeneracy produced by the binding ramp rates for Gen2 and a binding p^{max} for Gen1 results in different prices. Each AIC methodology allocates all avoidable fixed costs of Gen 2 to period 3 – the period that caused the dispatch of Gen2. For both ϵ=10^{-4} and ϵ=0, AIC LIP1 maintains the arbitrage condition between energy and reserve prices of \$88.50 (= 90-1.5 = 117.74-29.24). In AIC LIP2, the relaxation of Gen1 allows it to supply ϵ of reserves and causes different prices at $\epsilon = 10^{-4}$. At $\epsilon = 0$, primal degeneracy produces many prices. Any convex combination of valid prices is a valid set of prices.

Table 3.1. Optimal Dispatch and Prices with Reserves of 1 MW per period.

period	1			2			3		
	en	marg	resrv	en	marg	resrv	en	marg	resrv
Load	95	0	0	100	0	0	130	0	0
Gen1	95	0	1	75	0	1	100	1	0
Gen2	0	0	0	25	0	0	30	0	1
LMP/rLMP		10.0	1.00		10.0	1.00		90.00	1.50
LIP1/rLIP1($\epsilon = 10^{-4}$)		10.0	1.00		10.0	1.00		117.7	29.24
LIP1/rLIP1(ϵ=0)		10.0	1.00		10.0	1.00		117.7	29.24
LIP1/rLIP1(ϵ=0)alt		10.0	1.00		10.0	1.00		900.0	1.50
LIP2/rLIP2(ϵ=10^{-4})		10.0	1.00		10.0	1.00		117.7	29.24
LIP2/rLIP2(ϵ=10^{-4})alt		10.0	1.00		10.0	1.00		118.7	1.50
LIP2/rLIP2(ϵ=0)		10.0	1.00		10.0	1.00		117.7	29.24
LIP2/rLIP2(ϵ=0)alt		10.0	1.00		10.5	1.50		900.0	1.50

En is energy; marg is the marginal value of energy; resrv is the reserves.

Table 3.2 contains the generator avoidable costs, the settlement at the LMPs, and the settlement the LIPs at $\epsilon = 10^{-4}$ and $\epsilon = 0$. The LMP payments result in a make-whole payment of $860 to Gen2. The LIP settlements need no make-whole payments, Gen1 is infra-incremental and Gen2 is incremental.

Table 3.2. Settlement at LMP and LIP (ϵ=10^{-4} and ϵ=0) with Reserves

	Avoidable	Profit/Value				
	cost	LMP	LIP1(ϵ=10^{-4})	LIP1(ϵ=0)	LIP2(ϵ=10^{-4})	LIP(ϵ=0)
Gen1	2702.00	8000	10774.18	10774.19	10866.67	10774.19
Gen2	3811.50	-860	0	0	0	0
Total	6313.50	7140	10774.18	10774.19	10866.67	10774.19

For dispatch problem and AIC pricing problem with ϵ=10^{-4} and ϵ=0, the objective function is the same to 7 digits (see Table 3.3).

Table 3.3. Market Surplus (Objective Function Value) with $p^{max}{}_2$ at startup is 26 MW.

	Variation 1 w/o reserves	Variation 1 w/reserves	Variation 2 w/o reserves	Variation 2 w/ reserves
optimal dispatch	285990.00	285986.50	285990.00	285986.50
ϵ=1	286017.74	286013.38	286017.74	286014.74
ϵ=.01	285990.29	285986.79	285990.29	285986.79
$\epsilon=10^{-4}$	285990.00	285986.50	285990.00	285986.79
ϵ=0	285990.00	285986.50	285990.00	285986.50

Example 4. Example 3 with Startup p_2^{max} is 23 MW.

If the startup p_2^{max} is 23 MW, Gen2 would need to startup in period 1 to ramp to 30 MW in period 3. Table 4.1 has one set of market results and prices. The LMP in period 3 to \$130/MWh due to ramp rate constraints and p^{max} at startup. In period 1, Gen2 is both at its minimum operating level and constrained by its ramp rate creating a degeneracy. At ϵ=0, LIP1 and LIP2 produce a \$900/MWh clearing price in period 3 and \$10/MWh in periods 1 and 2. In LIP2, the relaxation of Gen1 allows it to supply an ϵ of reserves and causes different prices. At $\epsilon=10^{-4}$, two sets of prices recover exactly the avoidable cost for Gen2, the incremental generator, but the profit for Gen1 changes.

Table 4.1. Optimal Dispatch and Prices with 1 MW/period Reserves for startup p$_2^{max}$ = 23 MW

period	1			2			3		
	energ	marg	reserv	energ	marg	reserv	energ	marg	reserv
Load	95			100			130		
Gen1	75	0	1	75	0	1	100	1	0
Gen2	20	0	0	25	0	0	30	0	1
LMP/rLMP	10.00		1.00	10.00		1.00	130		1.50
LIP1/rLMP1($\epsilon=10^{-4}$)	10.00		1.00	10.00		1.00	145.80		17.30
LIP1/rLMP1(ϵ=0)	10.00		1.00	10.00		1.00	900		1.50
LIP2/rLMP($\epsilon=10^{-4}$)	10.00		1.00	10.00		1.00	146.33		1.50
LIP2/rLMP2(ϵ=0)	10.00		1.00	10.00		1.00	900		1.50

energ is energy; marg is the marginal value of energy; reserv is the reserves.

Table 4.2 contains the avoidable costs, the settlement at the LMPs, and LIPs with $\epsilon=10^{-4}$ and ϵ=0. The LIPs produce no make-whole payments. For $\epsilon=10^{-4}$, the incremental generator, Gen2, breaks even, and the infra-incremental generator makes a positive profit that is slightly different. For ϵ=0, the settlements are different and both generators make positive profits. The market surplus declines from Example 3 by \$830 (\$800 for extra marginal cost and \$30 for fixed operating costs in period 1. For the optimal dispatch and AIC problem with $\epsilon=10^{-4}$ and ϵ=0, the objective function is the same to 7 digits and the sequence converges to the efficient solution (see Table 4.3).

Table 4.2. Settlement at LMP and LIP with Reserves for startup p_2^{max} at 23 MW

	Avoidable	Profit/Value					
	Cost	LMP	LIP1($\epsilon = 10^{-4}$)	LIP1($\epsilon = 0$)	LIP2($\epsilon = 10^{-4}$)	*LIP2*(ϵ=0)
Gen1	2502	12000	13580.64	89000	13633.33	89000
Gen2	4841.5	-490	0	22610	0	22610
Total	7543.5	11510	13580.64	111610	13633.33	111610

Table 4.3. Market Surplus (Objective Function Value)

	Variation 1 w/o reserves	Variation 1 w/ reserves	Variation 2 w/o reserve	Variation 2 w/ reserves
optimal	285160.00	285156.50	285160.00	285156.50
ϵ=1	285175.81	285175.81	285175.81	285172.81
ϵ=.01	285160.16	285156.66	285160.16	285156.67
$\epsilon=10^{-4}$	285160.00	285156.50	285160.00	285156.50
ϵ=0	285160.00	285156.50	285160.00	285156.50

Example 5. Five-Period Market with Two-Step Marginal Cost Functions.

The load parameters are in Table 5.1. The generator parameters are in Table 5.2. Table 5.3 has the market results.

Table 5.1. Load

Period	1	2	3	4	5
Value	900	900	900	900	900
Max load	140	165	90	80	70

Table 5.2. Generation

	Marginal				Start	Min	Min	Fixed	Ramp
Gen	Cost1 $/MWh	Max1 MW	Cost2 $/MWh	Max2 MW	Cost $	Gen MW	run hr	cost $/hr	Up MW/hr
1	4.5	80	9.5	20	800	0	1	50	10
2	2.0	30	9.0	40	0	20	1	0	200

Table 5.3. Optimal Dispatch and Prices without Reserves

period	1 energy	2 energy	3 energy	4 energy	5 energy
load	140	165	90	80	70
Gen1	85	95	60	50	40
Gen2	55	70	30	30	30
LMP	9.00	10.00	4.50	4.50	4.50
LIP1(ϵ=10^{-4})	9.00	12.58	4.50	4.50	4.50
LIP1(ϵ=0)	9.00	12.58	4.50	4.50	4.50
LIP1(ϵ=0)alt	9.50	9.50	5.33	5.50	25.75
LIP2(ϵ=10^{-4})	9.00	12.58	4.50	4.50	4.50
LIP2(ϵ=0)	9.50	12.13	4.50	4.50	4.50
LIP2(ϵ=0)alt	9.50	9.50	5.33	5.50	25.75

In period 1, Gen2 step 2 sets the LMP at \$9/MWh because Gen1's ramp-up rate constraint from period 1 to 2 binds and it cannot supply any more energy in period 1. Gen1's ramp-up rate constraint shows up in period 2's LMP = \$10/MWh [= 9.5 (the marginal cost of Gen1 step2) +.5 (marginal value of the ramp-up constraint)]. In periods 3 through 5, the LMP is set by Gen1 step l. Both AIC variations ϵ=10^{-4} and LIP1 at ϵ=0 allocate all avoidable fixed costs of Gen1 to period 2, the period of peak need, and produce the same prices. The exception is LIP2 with ϵ=0 that raises the LIP2 by \$.50/MWh in period 1 and lowers the LIP1 by \$.45/MWh in period 2.

Table 5.4 contains the settlement at the LMPs and the LIPs. The LIP settlements need no make-whole payments. Gen1 breaks even and is an incremental generator. Gen2 makes a positive profit and is an infra-incremental generator. The energy prices change for LIP2 ($\epsilon = 0$) and lowers Gen2's profits by about .5% compared to Gen2's profits under LIP1. The sequence of AIC objective function values converge to optimal dispatch (1) objective function. For the optimal solution, $\epsilon=10^{-4}$ and $\epsilon=0$ solutions, the objective function is the same to 8 digits.

Table 5.4. Settlement at LMP and LIP ($\epsilon=10^{-4}$ and $\epsilon=0$) without Reserves

	Avoidable cost	Profit/Value				
		LMP	LIP1($\epsilon = 10^{-4}$)	LIP1(ϵ=0)	LIP2($\epsilon = 10^{-4}$)	LIP2($\epsilon = 0$)
Gen1	2635	-245	0	0	0	0
Gen2	885	715	895.53	895.53	895.53	891.71
Total	3520	470	895.53	895.53	895.53	891.71

Example 6. Example 5 with Reserves.

We add a reserves requirement of 1 MW per period to the above problem. Gen1's reserve cost is \$1/MWh and Gen2's reserve cost is \$1.5/MWh. Table 6.1 has the optimal dispatch and prices. Reserves are provided by Gen1 in all periods. The LIPs and rLIPs move around due to primal degeneracy. At LIP1/rLIP1($\epsilon = 10^{-4}$), all avoidable fixed costs of Gen1 are allocated to period 2, the period of peak need, and the prices maintain the reserves arbitrage condition.

Table 6.1. Optimal Dispatch and Prices with Reserves

Period	1		2		3		4		5	
	ener	resr	ener	resr	ener	resr	ener	resr	ener	resr
load	140		165		90		80		70	
Gen1	85	1	95	1	60	1	50	1	40	1
Gen2	55	0	70	0	30	0	30	0	30	0
LMP/rLMP	9.00	1.00	10.00	1.00	4.50	1.00	4.50	1.00	4.50	1.00
LIP1/rLIP1($\epsilon = 10^{-4}$)	9.00	1.00	12.55	3.55	4.50	1.00	4.50	1.00	4.50	1.00
LIP1/rLIP1($\epsilon = 0$)	9.00	1.50	12.55	3.05	4.50	1.00	4.50	1.00	4.50	1.00
LIP1/rLIP1($\epsilon = 0$)alt	9.00	1.50	16.31	1.00	4.50	1.50	4.50	1.50	4.50	1.00
LIP2/rLIP2(ϵ=10^{-4})	9.00	1.50	11.97	2.47	5.00	1.50	5.00	1.50	4.50	1.00
LIP2/rLIP2($\epsilon = 0$)	12.41	3.91	9.50	1.00	4.50	1.00	4.50	1.00	4.50	1.00
LIP2/rLIP2($\epsilon = 0$)alt	9.50	1.00	9.50	1.00	4.50	4.00	4.50	4.00	26.90	1.00

Table 6.2 contains the avoidable costs, the settlement at the LMPs, and the settlement the LIPs at $\epsilon=10^{-4}$ and $\epsilon=0$. Even though the LIPs and rLIPs move around, the LIPs settlements have no make-whole payments, Gen1 always breaks

even, and the settlement for Gen2 has a maximum variation of 3%.

Table 6.2. Settlement at LMP and LIP (ϵ=10^{-4}) with Reserves

	Avoidable	Profit at				
	Cost	LMP	LIP1($\epsilon = 10^{-4}$)	LIP1($\epsilon = 0$)	LIP2($\epsilon = 10^{-4}$)	LIP2($\epsilon = 0$)
Gen1	2640	-245	0	0	0	0
Gen2	885	715	893.65	893.65	882.81	867.38
Total	3525	470	893.65	893.65	882.81	867.38

The sequence of AIC objective function values converge to optimal dispatch objective function. With ϵ=10^{-4} and ϵ=0, the objective function is the same to 8 digits (see Table 6.3).

Table 6.3. Market Surplus (Objective Function Values).

	without reserves	with reserves
optimal	486980.00	486975.00
ϵ=1.00	486982.55	486977.53
ϵ=.01	486980.03	486975.03
ϵ=.0001	486980.00	486975.00
ϵ=0	486980.00	486975.00

Example 7. The AIC Methodology Allocates Avoidable Fixed Costs to the Peak.

The load parameters are in Table 7.1. The generators' parameters are in Table 7.2. Table 7.3 has the market results. To meet load in periods 3 and 4 and a minimum run time of 4 periods, Gen2 starts up in period 1. In period 4 where Gen2 is needed because Gen1 alone cannot satisfy demand. All costs of the minimum run time are allocated to the peak period of greatest demand.

The AIC methodology allocates all avoidable fixed costs that is, startup costs and operating costs at its minimum operating level in periods 1, 2, and 3 to the peak period 4. The LIP in period 4 creates a strong incentive to shift demand to an off-peak period or conserve. The LMP does not present this incentive because the fixed cost go to make-whole payments.

Table 7.1. Load

Period	1	2	3	4	5
Value	900	900	900	900	900
Max Load	260	270	400	430	200

Table 7.2. Generation

Gen	Marg Cost $/MWh	Min Gen MW	Max Gen MW	Start Cost $	Min Run hr	Fixed Cost $/per	Ramp Up MW/per	Ramp Down MW/per
1	10.00	0	300	0	1	0	500	0
2	53.10	250	250	2020	4	0	200	0
3	206.18	0	131		1		200	

Table 7.3. Optimal Dispatch and Prices

period	1		2		3		4		5	
	Ener	Marg	Ener	Marg	Ener	Marg	Ener	Marg	Ener	Marg
Load	260	890	270	890	400	890	430	890	200	890
Gen1	10	0	20	0	150	0	180	0	200	0
Gen2	250	-43.10	250	-43.10	250	-43.10	250	-43.10	0	-43.10
LMP		10		10		10		10		10
LIP1(ϵ=10^{-4})		10		10		10		190.48		10
LIP1(ϵ=0)		10		10		10		190.48		10

Table 7.4 contains the avoidable costs, the settlement at the LMPs and the settlement LIPs at $\epsilon = 10^{-4}$ and $\epsilon = 0$. The LMP settlements needs $45,120 in make whole payments that are charged to consumers, but do not show up in the LMPs that are all set at $10/MWh. The LIP settlement produces no make-whole payments, is revenue neutral, and allocates all fixed costs to the period with the highest demand that caused the costs (period 4).

Table 7.4. Settlement at LMP and LIP (ϵ=10^{-4} and ϵ=0)

	Avoidable Cost	Value/Profit (in $) at LMP	LIP1($\epsilon = 10^{-4}$)	LIP1($\epsilon = 0$)
Gen1	5600	0	32486.40	32486.40
Gen2	55120	-45120	0	0
Total Gen	60720	-45120	32486.40	32486.40
load		1388400	1310793.60	1310793.60
Total value		1343280	1343280.00	1343280.00

For Example 7, $p_{it}^{ru*} = 0$; since there is only a one step supply curve, $c_{jit} = c_{it}$,

$c_{it}^{op} = 0$; $\rho_{it} = 0$; and $\rho_{it}^{up} = 0$; dividing (4g) by p_{it}^* reduces to

$$\lambda_t \quad = c_{it} \qquad + \gamma_{it}/p_{it}^* \qquad + (\delta_{it} - \delta_{it+1})/p_{it}^*$$

revenues = marginal costs + profits/MWh + fixed cost allocation.

Substituting u_{it}^{**}/p_{it}^{**} for $1/p_{it}^*$,

$$\lambda_t = c_{it} \quad + (\gamma_{it} \quad + \delta_{it} - \delta_{it+1})u_{it}^{**}/p_{it}^{**}.$$

We demonstrate in two different explanations for the allocation of Gen2 costs to the period 4. For Gen2, γ_{2t} = 0. The solver lowers u_2 until the p_2^{min} constraints bind. In this example, δ_{it}-δ_{it+1} can be interpreted as the savings of not starting in period 1, that is,

\$10775 =	250	×(53.1-10)
min run time operating cost	Min Gen in MW	Gen2- Gen1 marginal costs

to satisfy demand in period 4. In Table 7.5, we see how the LIPs are calculated.

Table 7.5. The relocation of cost for Gen2 using
$\lambda_t = c_{2t} + (\gamma_{2t} + \delta_{2t} - \delta_{2t+1})u_2^{**}/p_{2t}^{**}$.

δ_{2t}-δ_{2t+1}	-10,775	-10,775	-10,775	34,345	0
$p_{2t}{}^{**}$	129.9999	129.9999	129.9999	130.0000	0
u_2^{**}	.52	.52	.52	.52	0
$\lambda_t = c_{2t} + (\gamma_{2t} + \delta_{2t} - \delta_{2t+1})u_2^{**}/p_{2t}^{**}$	9.999876	9.999876	9.999876	190.48	10

From the complementary slackness of (3c) and the above simplifications,

$$\lambda_t = c_{it} + (p_{it}^{max}\beta_{it}^{max} - p_{it}^{min}\beta_{it}^{min})/p_{it}^*,$$

we see a different economic interpretation. In Table 7.6, in periods 1 and 2 where Gen2 is not needed, the AIC solver minimizes the generation from the most expensive marginal cost generator, Gen2, and the p_{2t}^{min} constraint binds and $\beta_{2t}^{min} = -43.1$, the marginal savings for reducing p_{it}^{min}. In period 4, p_{24}^{max} binds; $\beta_{24}^{max} = 137.38$; and the dual variable on the p_{24}^{max} constraint contains the opportunity cost of running at the minimum in the three previous periods is 3×250 MWh×\$43.1/MWh, the startup cost is \$2020; Optimal dispatch is 250 MWh; dual variable β_{24}^{max} is \$137.38 = (3×250×43.1+2020)/250, λ_t = 53.1+137.38.

Table 7.6. The relocation of cost for Gen2 using

$\lambda_t = c_{2t} + (p_{2t}^{max}\beta_{2t}^{max} - p_{2t}^{min}\beta_{2t}^{min})/p_{2t}^*$

period	1	2	3	4
β_{2t}^{max}	0	0	0	137.38
β_{2t}^{min}	43.10	43.10	43.10	0
p_{2t}^*	250.00	250.00	250.00	250.00
$(p_{2t}^{max}\beta_{2t}^{max} - p_{2t}^{min}\beta_{2t}^{min})/p_{2t}^*$	-43.10	-43.10	-43.10	137.38
c_{2t}	53.10	53.10	53.10	53.10
λ_t	10.0	10.00	10.00	190.48

Note Gen2 is shut down in period 5.

For $\epsilon = 10^{-4}$, the results for Gen1 are in Tables 7.7 and 7.8.
Table 7.7. The relocation of cost for Gen1 using $\lambda_t = c_{1t} + [\gamma_{1t} + \delta_{1t} - \delta_{1t+1}]u_1^{**}/p_{1t}^{**}$

period	1	2	3	4	5
δ_{1t}-δ_{1t+1}	0	-54144.00	0	54,144.00	0
γ_{1t}	0	54144.00	0	0	0
p_{1t}^{**}	130.0001	140.0001	270.0001	300	200
u_1^{**}	1	1	1	1	1
$\lambda_t = c_{1t} + [\gamma_{1t} + \delta_{1t} - \delta_{1t+1}]u_1^{**}/p_{1t}^{**}$	10	10	10	190.48	10

In period 4, the system dispatches Gen2 to its maximum and Gen2 is an incremental generator.

Table 7.8. The relocation of cost for Gen1 using λ_t=c_{1t} + ($p_{1t}^{max}\beta_{1t}^{max}$-$p_{1t}^{min}\beta_{1t}^{min}$)/$p_{1t}^*$

period	1	2	3	4	5
β_{1t}^{max}	0	0	0	180.48	
β_{1t}^{min}	0	0	0	0	0
p_{1t}^*	10	10	10	180.00	10
$(p_{1t}^{max}\beta_{1t}^{max} - p_{1t}^{min}\beta_{1t}^{min})/p_{1t}^*$	10	20	150	180.48	
c_{1t}	10	10	10	10.00	10
λ_t	10	10	10	190.48	10

Load Shifting and Flexibility. If load can reduce its total consumption to 1495 MWh or less and shift consumption from period 4 to 1, 2, 3 and 5, the new dispatch is shown in Table 7.9, the LIP = LMP = 10 will clear to the market in each period and load pays $14,950 with a net value of $1,330,550 compared to $1,310,794 in table 7.4.

Table 7.9. Optimal Dispatch and Prices with More Flexible Demand

period	1		2		3		4		5	
	Ener	Marg	Ener	Marg	Ener	Marg	Ener	Marg	Ener	Marg
Load	299	890	299	890	299	890	299	890	299	890
Gen1	299	0	299	0	299	0	299	0	299	0
Gen2	0	0	0	0	0	0	0	0	0	0
LMP		10		10		10		10		10
LIP1($\epsilon=10^{-4}$)		10		10		10		10		10

Entry of Gen3 at the LIP. We add Gen3, a convex generator with a one-step marginal cost of at \$190.48/MWh to the market. Gen3 replaces Gen2 (see Table 7.10). Gen3's dispatch allows greater use of the more flexible Gen1. Gen3 is the marginal generator in periods 3 and 4. Gen1 is the marginal generator in periods 1, 2 and 5. Gen1's profits increase from \$32,486.40 to \$108,288.00 due to the greater use of Gen1. The LIPs are the same as the LMPs because Gen2 is not in the dispatch and the market is convex.

Table 7.10. Optimal Dispatch and Prices at Gen3's marginal costs of \$190.48/MWh.

period	1	2	3	4	5	Avoidable Cost	Value/Profit at LMP
Load	260	270	400	430	200		1,238,601.60
Gen1	260	270	300	300	200	13,300.00	108,288.00
Gen2	0	0	0	0	0	0	0
Gen3	0	0	100	130	0	43,810.40	0
Total						57,110.40	1,346,889.60
LMP	10	10	190.48	190.48	10		
LIP	10	10	190.48	190.48	10		

Entry of Gen3 at a Higher Cost than the LIP. If we add Gen3, a convex generator, with a one-step marginal cost of \$206.18/MWh to the market, Gen3 will not enter the market, but at a marginal cost of \$206.16/MWh, Gen3 replaces Gen2. Due to the complementary nature of Gen1 and Gen3, Gen3 allows higher Gen1 dispatch in periods 1 through 4 with higher profits (see Table 7.11). The higher cost entry level is due to the savings from not dispatching Gen2 and the less expensive generator, Gen1, to be dispatch at a higher level.

Table 7.11. Optimal Dispatch and Prices at Gen3's marginal costs of

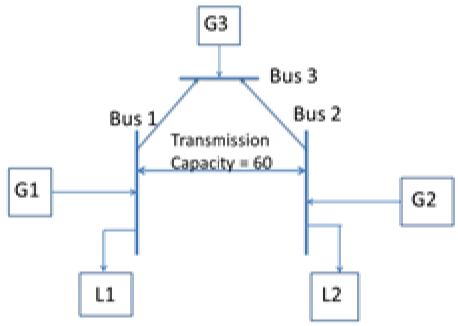
$206.17/MWh.

period	1	2	3	4	5	Avoidable Cost	Value/Profit at LMP
Load	260	270	400	430	200		1,225,587.20
Gen1	260	270	300	300	200	13,300.00	117,696.00
Gen2	0	0	0	0	0	0	0
Gen3	0	0	100	130	10	47,416.80	0
Total						60,716.80	1,343,283.20
LMP	10	10	206.16	206.16	10		
LIP	10	10	206.16	206.16	10		

Example 8. Three-bus Example of Generator Profit Reduction under AIC.

In this example we show that the AIC profits can be lower than LMP profits due to a transmission constraint.

Figure 4. Three Bus Network.



The problem parameters are in Tables 8.1 and 8.2. Table 8.3 presents the optimal one-period dispatch, the LMPs and the generation profits at the LMPs. Line 1-to-2 is at its 60 MW capacity with a flowgate marginal price (FMP) of $90/MWh. Gen1 has positive profits of $500. Gen2 and Gen3 have negative profits of -$1200 and -$500. Both would receive a make-whole payment to break-even using LMP pricing.

Table 8.1. Generator and Load Parameters

	Bus 1	Bus 2	Bus 3
Load (MWh)	30	300	0
Generator	G1	G2	G3
Fixed Costs ($)	0	1200	500
p^{min} (MW)	0	0	0
p^{max} (MW)	50	150	300
Marginal Costs ($/MWh)	10	80	50
Sensitivities of flow 1-to-2 to bus	0.666667	0	0.333333

Table 8.2 Transmission Line Limits.

Line	1-to-2	2-to-3	1-to-3
line limit (MW)	60	500	500

Table 8.3 Optimal Dispatch, LMPs, profits and make-whole payment under LMP

Bus	1	2	3	Total
LMP ($/MWh)	20	80	50	
Dispatch (MWh)	50	140	140	
Gen Revenue ($)	1000	11200	7000	19200
Gen Avoidable Cost ($)	500	12400	7500	20400
Gen Profit ($)	500	-1200	-500	-1200
Make-whole Payments ($)	0	1200	500	1700

Table 8.4 presents the AIC solution, the LIPs with $\epsilon = 10^{-4}$, and the generation profits at the LIPs. Line 1 to 2 is at its 60 MW capacity with a flowgate incremental price (FIP) of $105.00/MWh. Gen1 profit is $428.57 – less than the profit under LMP. Gen2 and 3 profits are -$0.09 and -$0.04 at ϵ=0.01. At $\epsilon = 10^{-4}$, Gen2 and 3 profits are -$0.000857 and -$0.000357 and round to zero. Gen2 and Gen3 essentially breakeven without a make-whole payment. Compared to the LMP, the LIP on bus 1 decreases to $18.57/MWh; on bus 2 the LIP increases to $88.57/MWh; and on bus 3, the LIP increases to $58.57/MWh. The profits change accordingly.

Table 8.4 AIC pricing solution and profit at LIP($\epsilon = 10^{-4}$)

LIP	18.57	88.57	53.57	
Bus	1	2	3	Total
Dispatch	50.00	140.00	140.00	
Gen Revenue	928.57	12400.00	7500.00	20828.57
Gen Marginal Cost	500.00	12400.00	7500.00	20400.00
Gen Profit	428.57	-.000857	-.000357	428.57

We change Gen1 marginal energy costs to $19/MWh. The results are in Table 8.5. The LIP at bus 1 is $19/MWh is slightly higher than the LIP of $18.57/MWh. Gen1 and Gen2 break even. Gen3 makes a profit of $30. Line 1 to 2 is at capacity and the FIP is $104.36/MWh.

Table 8.5. Optimal Dispatch, LIPs and AIC profit if Gen1 marginal energy cost is $19/MWh

LIP	19.00	88.57	53.79	
Bus	1	2	3	Total
Dispatch	50.00	140.00	140.00	
Gen Revenue	950.00	12400.00	7530.00	20880.00
Gen cost	950.00	12400.00	7500.00	20850.00
Gen Profit	0.00	0.00	30.00	30.00

Example 9. Single-period Market with Complementary Dispatchable Demand.

The load parameters are in Table 9.1. The generator parameters are in Table 9.2.

Table 9.1. Load

Load	Marg Value $/MWh	Min Load MW	Max Load MW
L1	200	0	130
L2	21	0	140

Table 9.2. Generation

Gen	Marg Cost $/MWh	Min Gen MW	Max Gen MW	Start Cost $
G1	10	80	95	200
G2	20	40	50	90

There are three local optimal solutions. They are neighbors in the sense that they are only one binary change apart, but differ significantly in optimal market surplus. If G1 is off and G2 is on, the optimal market surplus of $8910. If G1 is on and G2 is off, the optimal market surplus of $17850. If G1 is on and G2 is on, the optimal market surplus of $24075. The latter is the global optimal market solution. Alone L2 cannot cover the AIC costs of G2. At the local optima with G1 on and G2 off, L1 has 35 MWs of additional demand, but cannot cover the 40 MW min gen of G2. Together L1 and L2 can achieve the min gen of G2, L1 is satiated and L2 consumes the remainder of G2's capacity at the marginal cost of G2. L1 and L2 are complementary consumers, i.e., they are both needed to achieve the optimal dispatch. Table 9.3 has the global optimal market and pricing results.

Table 9.3. Global Optimal Dispatch and Prices

	G1	G2	G3	L1	L2
SCUC MW	95	50	0	130	15
Cost/Value	1150	1090	0	26000	315
LMP	21	21	21	21	21
LMP Revenue/Payment	1995	1050	0	2730	315
LMP Profit/Value	845	-40	0	23270	0
LIP2	21.8	21.8	21.8	21.8	21.8
LIP2 Revenue/payment	2071	1090	0	2834	327
LIP2 Profit/Payment	921	0	0	23166	-12

Under LMP pricing, L2 sets LMP at $21/MWh, G2 revenue is less than its cost and requires a $40 make whole payment. Under AIC pricing, LIP2 is $21.8/MWh is set by G2 whose revenue equals to its cost. However, LIP2 is higher than the marginal value of L2 and L2 requires a $12 make-whole payment.

To eliminate the make-whole payment to L2, we invoke the Ramsey–Boiteux pricing rule. Roughly it states that the most inelastic market participants are assigned the make-whole payments (see Ramsey (1927), Boiteux (1956), Dierker (1991)). In the broader sense since step functions are not differentiable, this rule assigns the make-whole payment to the market participants with highest surplus or in proportion to its surplus and roughly in the time and space the make-whole payments were incurred in such a way that market participants remain infra-marginal, infra-incremental, marginal and incremental, respectively. This is not a unique pricing rule (see O'Neill et al. (2016)).

Here, we allocate the make-whole payment to the market participant with the highest market surplus, L1. The LIP is $21.8/MWh. G1 and G2 get paid the LIP. G2 breaks even and is an incremental unit. L2 pays the LMP = $21/MWh, breaks even and is marginal. If we pay G1 the LIP, we charge L1 the difference in L2's payment $(21.8-21)/MWh*15MWh = $12 increasing its price by $12/130 MWh = $.09231/MWh or $21.8923/MWh. The L1 payment is $2845.999 (21.8923*130). The L2 payment is $315 (21*15). Payments sum to $3160.999. G1 is paid $2071 (21.8*95). G2 is paid $1090 (21.8*50). The generators are paid $3161. The transfers are balanced to six digits.