Draft

Essays on Average Incremental Cost Pricing for Independent System Operators

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Abstract.

In a series of essays, we introduce average incremental cost (AIC) pricing and present examples to help understand its advantages. In non-convex markets, AIC pricing is the rough equivalent to marginal cost pricing in convex markets. We present a qualitative comparison of current ISO pricing methods and the AIC approach. We argue that AIC better meets the Federal Energy Regulatory Commission's stated goals. We compare incentives to deviate from efficiency-maximizing behavior for resources under current pricing methods and AIC pricing. We present models and examples of AIC Pricing and other pricing methods in single-period markets and compare approaches. In multi-period markets, non-convexities like startup costs, minimum operating levels, minimum run times and minimum down times present pricing issues that are resolved using cost causation principles. We develop the multi-period mathematical model for AIC pricing. In networks, non-convexities address issues involving the role of flowgate marginal prices in signaling for efficient transmission expansion. We develop the network mathematical models for AIC pricing. We discuss the role of Price-Responsive Demand and Ramsey-Boiteux pricing needed for market efficiency. We show that along with the LMP, the AIC is a better signal for efficient entry, but in some cases, it is too low.

Preface

In recent years, there has been growing interest in price formation in the ISO markets. The non-convexities of ISO markets invalidate the elegant properties of convex markets. Pricing is vital for the economic efficiency of liberalized electricity markets. Prices sends signals to potential new grid resources and for retirements. This paper explores a pricing scheme, called average incremental cost (AIC) pricing. Eleven essays provide background, justifications, examples, mathematical formulations, and extensions of average incremental cost (AIC) pricing.

The target audience for this paper is both analysts with significant mathematical skills and those without these skills. For the most part, we keep the mathematical analysis to the end of the chapter or in an appendix. In the beginning of the chapters, the simple numerical examples illustrate the concepts and calculations. Small examples are similar to the economics of load pockets that are import constrained.

The complexity of non-convex markets is initially explained through prose that presents background, describes and compares AIC pricing to other approaches, then through examples that can be easily understood and replicated, and lastly, generalized through mathematical formulations, propositions and conjectures. Each contributes to the understanding of the problem and the solution. The paper is roughly divided as follows:

	percent
prose	30
relatively simple examples	30
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Further research and analysis are needed on large scale actual ISO models. Also, the AIC approach has options where the more efficient choice is not obvious and needs more analysis.

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1 History of Independent System Operator Market Design

1.1 Introduction to ISO Market Design

Regional Transmission Organization (RTO)/Independent System Operator (ISO) market designs have evolved over time in response to changing ideas and theoretical debates over appropriate market designs. In addition, the increases in computational efficiency have allowed greater detail and accuracy in the market. This chapter reviews the history of ISO market design and pricing.

Early Market Design. The first century of determining electricity prices used the cost-of-service model and its cost allocation rules designed for monopolies. Historically, vertically integrated utilities with monopoly franchises forecasted demand, scheduled their own generation, and sent invoices to consumers once a month with a single energy price. Wholesale energy transactions took place in power pools or where negotiated and roughly reflected costs. Investment occurred under the cost-of-service regime held in check by state commissions.

The Federal Power Act of 1935 requires that rates (that is, prices) be just and reasonable and not be unduly discriminatory. This responsibility falls to the Federal Energy Regulatory Commission (aka FERC or Commission). Since 1935, this has been interpreted as prices devoid of market power.

ISOs grew out of the Commission Orders Nos. 888 and 889, issued in 1996, where the Commission outlined the concept of an ISO as one way for existing power pools to satisfy the requirement of providing non-discriminatory access to transmission.

Order Nos. 888 and 889 required each ISO to:

- Provide open access to the transmission system and all services under its control to all eligible users in a non-discriminatory manner
- Ensure short-term reliability
- Administratively control the operation of transmission facilities within its region
- Identify constraints on the system and take actions to relieve the constraints
- Promote efficient trading
- Have appropriate incentives for efficient management
- Promote the efficient use of and investment in generation, transmission, and consumption
- Conduct studies to identify operational problems or expansions
- Make transmission system information publicly available on a timely basis via an electronic information network
- Coordinate with neighboring control areas.

In 1999, FERC Order No. 2000 set forth minimum characteristics and functions for becoming an RTO. The minimum characteristics and functions essentially clarified ISO practices and specifically codified stronger management requirements to become an RTO. Compliance was voluntary. Four ISOs – PJM Interconnection, LLC (PJM), Midcontinent Independent System Operator (MISO), Southwest Power Pool (SPP), and ISO-New England (ISO-NE) became an RTO. California Independent System Operator (CAISO) and New York Independent System Operator (NYISO) declined to file for RTO status. An RTO is also an ISO. The discussions in this paper do not deal with the difference between ISOs and RTOs so we will simply refer to them as ISOs. In 2005, the Energy Policy Act required the Commission to 'promote reliable and economically efficient transmission and generation of electricity'.

Market Efficiency and Price Mechanisms. Market efficiency is a goal of auction market design. Market design has two parts: short-term design and long-term design. Short-term market efficiency as the name indicates is the efficiency markets without the possibility of major investments. ISOs strive to achieve this efficiency by operating a day-ahead and a real-time market. Long-term market efficiency is achieved by creating prices in the short-term markets that incent efficient sustainable investment.

In a market economy, the interaction of buyers and sellers enables goods, services, and resources to be allocated using prices. Price changes send contrasting signals to consumers and producers to enter or leave a market. Rising prices signal consumers to reduce demand and producers to increase supply. Higher prices provide an incentive to existing producers to supply more if they provide the possibility of increased profits. Conversely, falling prices signal consumers to increase consumption and producers to reduce output. In a convex market, LMPs are such prices. In a non-convex market, LMPs play a limited role as price signals. The paper is devoted to how AIC prices can better perform the above functions particularly in electricity markets.

1.2 The Evolution of ISO Market Designs

In 1982 and 1984, Caramanis, Bohn and Schweppe proposed the 'optimal spot pricing of electricity '-- a market. This become known as the locational marginal price (LMP) -- and spot wheeling charges -- later called congestion rent. In 1988, Schweppe et al popularized these concepts. The LMP is a marginal cost to serve load at a specific time and location in the network. As ISOs evolved, alternative pricing methods have been implemented. The Commission has also changed what costs should be considered as "marginal" costs and how the markets are settled.

Initial ISO Market Designs. In the 1990s after almost two decades of PURPA and some industry restructuring, ISOs formed, first from power pools. The Independent System Operators (ISOs) formed wholesale auction markets and moved to competition for generators with more granular prices. The ISOs adopted the LMP pricing concepts, but quickly realized that optimally dispatched generators were losing under LMP pricing and introduced make-whole payment to avoid these losses. The LMPs were made public, but the make-whole payments are considered private information and only aggregates were made public. For the most part, ISOs and load serving entities (LSEs) kept the practice of forecasting demand – a principal-agent problem. Creating a one-sided auction for what should be a two-sided auction. Ideally, the loads should be expressing their value of consumption and their willingness to shift their consumption. When loads bid into the market, the clearing prices and quantities are 'crowd sourced' and not the result of forecasts by less financially motivated agents.

In the mid-1990s, when the initial ISO market designs were developed, communications, auction market software was considerably less developed then today. Approximations and simplifications were necessary due to the limited capabilities. These capabilities have greatly increased over time. In 2018, the optimization software and hardware are over a million times faster, allow more detail to be modeled, and find better solutions. This technological improvement has prompted many changes in how ISO markets are designed and offer opportunities for further improvement. We provide two examples of how market design was limited by technology.

Initially PJM could only send one electronic signal to each transmission region, with additional dispatch instructions for individual units relayed by phone. The PJM operators relied on dispatch tools that identified resources that should be dispatched up or down but did not solve a complete optimization. A pricing module calculated the LMPs that were most consistent with the dispatch instructions. (Ott, 1998, p. 20.)

In 1999, NYISO's initial implementation relaxed any constraints that could not be solved in the 5-minute dispatch due to limitations in software. NYISO also designated capacity to provide reserves or regulation in advance, and then blocked this capacity from being dispatched. Further, NYISO's pricing incorporated fixed block pricing. "Fixed block" generation resources can only be dispatched in one of two states. They must either be turned completely off or run at their maximum capacity. If their entire output was not needed, a less expensive unit was backed down to make room in the dispatch. Initially, much of NYISO pricing was based on the New York Power Pool's then-current real-time dispatch tool. The early tariff history of NYISO fixed block pricing is somewhat unclear because NYISO did not explicitly discuss it in any its pre-startup tariff filings (see Harvey, 2018).

In a January 31, 1997 filing, there was no mention of fixed block pricing and the tariff itself had no details about LMP price calculations. On November 19, 1999, NYISO implemented fixed block pricing. In July 2000, FERC ordered NYISO to stop using fixed block pricing (NYISO, 92 FERC 61,037 (2000) pp. 18-20.) On August 9, 2002, FERC accepted a NYISO proposal to implement fixed block pricing, concluding that it "find[s] persuasive NYISO's arguments that precluding fixed block generation from setting day-ahead prices will have adverse effects on its markets." (NYISO, 100 FERC 61,182, P 8).

Zonal vs. Nodal Markets. Early ISO market design focused on the locational granularity of the market, that is, whether to price based on zones or nodes. Zones are aggregations nodes. Zonal pricing is based on a market separated into different zones connected by flowgates or interface constraints. The zonal representation assumes that power can flow without constraints and losses within the zone. In contrast, nodal pricing is based on more specific locations (nodes or busses) on the transmission system.

Initially, PJM, ISO-NE and CAISO chose zonal pricing partially because power pools, the predecessors to the ISOs, historically operated with little or no congestion. However, zonal markets quickly failed because they did not adequately represent the ISO's network constraints and produced dispatches that were physically infeasible. Eventually, each of the ISOs adopted nodal models and nodal energy pricing.

Weak Price Signals. By themselves, LMPs do not provide enough of a price signal to incent efficient entry, exit and new investment in facilities and equipment. This result, however, is more subtle than the failure of the zonal markets. The effect of weak price signals on investment and conservation is both hard to detect and may not be seen for years.

When costs are allocated too broadly, they dilute the price signal needed to stimulate investment in efficient alternatives. Below are two examples of cost allocation that was too broad and did not trigger the investment in efficient alternatives sooner. The economic investments would likely have been made sooner if efficient prices signaled the need for less expensive alternatives.

Canal Units on Cape Cod. The following account is from an ISO New England report (2009). Located in Sandwich, Massachusetts on Cape Cod, the Canal Generating Plant has two generators with a capacity of about 550 MW each. In 1968, the plant started using coal to generate power, but later relied mainly on oil. The Canal units are relatively expensive to run with long start times and high minimum-operating levels.

Prior to 2009, the lower Southeastern Massachusetts (SEMA) area was served by either the Canal generating units or power imported over two 345 kV transmission lines and lower capacity 115 kV lines. Canal generation had to be run whenever regional load levels exceeded 10,500 MW, which in combination with Canal's long start times required operation of one unit every day of the year.

In 2009, transmission upgrades in lower SEMA area resulted in the Canal units dispatched only when regional load exceeds 20,000 MW in the summer and 24,000 MW in the winter. In 2014, additional transmission upgrades were completed. The Canal generating units operated only during peak times (very hot or very cold weather) – 42 to 58 days a year. Canal receives capacity payments for being available to generate electricity. The ISO-New England consumers save by not running the Canal units at their minimum operating limits most of the year.

ISO-NE allocated Canal's make-whole payments to the entire SEMA area, but the Canal units supported only Cape Cod, not all lower SEMA. If the Cape Cod load was removed from the market, the Canal units would not have been needed. Therefore, make-whole payments costs caused by Canal generation should have been allocated primarily to Cape Cod. Had ISO-NE allocated make-whole payments more granularly, it would have increased costs allocated to Cape Cod, and stimulated Cape Cod to find a way to reduce their costs. Thus, the transmission upgrades would likely have been built earlier, saving costs to lower SEMA consumers.

Upper Peninsula of Michigan. In 1955, the Presque Isle Power Plant began operating in Marquette, Michigan, on Lake Superior. The plant has five active generating units with a net generating capacity of 431 MW. They generated about 90 percent of the Upper Peninsula's electricity and 12 percent of the electricity in the Wisconsin Energy system. Half of the plant's generating capacity was sold to the Empire and Tilden mines on the Marquette Iron Range. Like the case in Cape Cod, the Presque Isle units would not be needed without the load from the Upper Peninsula. Therefore, the costs of operating are primarily caused by the Upper Peninsula.

The Presque Isle units were needed for reliability in the Upper Peninsula. In MISO, resources designated as needed to ensure system reliability are called System Support Resources (SSRs). Originally, MISO required that the costs associated with the Presque Isle SSR agreement be allocated to all load-serving entities (LSEs) within the American Transmission Company (ATC) footprint (Wisconsin and the Upper Peninsula of Michigan). In turn, ATC allocated costs to its load on a pro rata basis. In July 2014, FERC found that MISO's policy of allocating the costs of SSR agreements to all load-serving entities within the ATC pricing zone on a pro rata basis was inconsistent with cost-causation principles and therefore, unjust and unreasonable. MISO changed its cost allocation based on cost causality. The change triggered a sequence of events that eventually lead to a lower cost solution.

1.3 Price Formation Circa 2019 and its Challenges

This section summarizes the primary market design used in the US ISO markets.

Efficient Scheduling and Dispatch. While each ISO market is different, we distill the ISOs' detailed rules to define a generic day-ahead market and real-time market with following steps:

(1) Market participants submit bids and offers. All ISOs allow generators to offer both marginal and avoidable fixed costs, and operating constraints.
(2) Mitigate offers, if needed, to an approximation of incremental (avoidable fixed and marginal) costs.
(3) Run the auction market to determine an economically efficient dispatch.
(4) Examines the results for reliability violations.
(5) If needed, the ISO corrects reliability violations by adding constraints. If additional constraints are added, go to step two
(6) ISO sends dispatch instruction to market participants.
(7) ISO calculates and posts energy prices.

The day-ahead market provides a daily commitment and schedule of resources hour-by-hour over the next day and a hedge against the real-time market prices. The real-time market is focused on maintaining physical balance and enough reserves to deal with contingencies. The real-time market is dispatched in five-minute periods. In addition, the ISO may employ advisory software that looks ahead several periods. ISO pricing schemes differ.

Pricing. Later, the ISO determines the actual metered energy and prices to settle the market for energy and reserves. The day-ahead and real time market settlements comprise three parts: energy, reserves and make-whole payments. In the real-time market, the energy component is settled based on nodal energy price on a 5-minute time interval and is published in real-time.

Make-whole payments are made to resources whose dispatch by the ISO results in a shortfall between the resource's offers and the revenues they earn based on energy and reserve prices. Resource-specific make-whole payments are not published due to concerns about compromising proprietary information. While some information about make-whole payments is published in market reports, it is aggregated to a level where payments to individual participants cannot be determined. Thus make-whole payments have limited transparency.

Make-whole payments are allocated to load on daily or monthly basis over a large region within the ISO. There is not enough granularity in the published make-whole payments to create a price signal that provides efficient incentives for entry and exit. A better approach would be to provide an allocation mechanism for make-whole payments that incents market participants to respond with efficient decisions to enter or exit the market.

Departures from LMP Pricing. The prices should clear the market, send entry and exit signals for both generation and load, and reward infra-marginal generators and load. Until 2005, Lagrangian relaxation (LR) 'solved' the unit commitment problem, but the solution was usually infeasible with a duality gap and rules were needed to find feasibility. The dual variables from the LR do not have a good economic interpretation. In 2005, after significant improvements in mixed integer programming (MIP) software and hardware, PJM introduced MIP as their unit commitment solver. MIP eliminated the infeasibility problem and improved the dispatch. By 2018, all ISO were using MIP with estimated cost savings more than five billion dollars per year (O'Neill, 2017).

In the MIP formulation, mathematically, the LMPs do not exist. For MIP problems, many commercial software packages create a linear program by fixing each binary to its MIP solution value. This is equivalent to assuming avoidable costs are sunk. The dual variables supply valuable information for economic analysis post-commitment, for example, LMPs are short-term marginal entry signals. The dual variables on the energy balance and reserve constraints alone have no claim to being market clearing (see Van Vyve, 2011).

In most ISOs, the LMP is defined in the tariff as the cost of supplying the next increment of load at a node or bus. In practice, the LMP is the value of the dual variable from the pricing run. As defined in the DC approximation, LMPs gradually disappeared from ISO markets as the settlement mechanism with the introduction of the pricing run. It modified the dispatch algorithm and produced energy prices different from the LMPs, but the prices are still called LMPs. In the pricing run, some ISOs relaxed the minimum operating level, some relaxed the binaries, and some modified the marginal energy costs by including some fixed costs. This generally led to higher prices and lower make-whole payments. Modified LMPs are neither fish nor fowl. They are too high to be a marginal entry signal and too low to eliminate make-whole payments and signal efficiency entry for generators with avoidable fixed costs.

LMP+ Pricing. Under LMP pricing, the highest marginal cost unit of the units scheduled or dispatched and not at its minimum operating level sets the energy price. However, the LMP revenues may not cover avoidable costs of dispatch. To solve this problem, LMP+ pricing was introduced to cover both a resource's marginal costs and its avoidable Fixed Costs. Avoidable Fixed Costs include Start-Up Costs and Fixed Operating Costs (aka No-Load Costs), but do not include investment or capital costs. LMP+ pricing guarantees that no generator that offers its avoidable costs into the market and is dispatched will lose money by following dispatch instructions. The make-whole payment is charged to a broad class of inelastic demand that dulls the price signal for market participants to react in an efficient manner.

Relaxed Minimum Operating Level (RMOL) Pricing. In the pricing run, some or all generators have their minimum operating level relaxed. If the minimum operating level is reduced in the pricing run, the higher marginal cost generator sets the energy price. Since avoidable fixed costs are not part of setting the energy price, Make-Whole Payments may still be necessary.

After relaxing the minimum operating levels, the energy price may diverge from the LMP. This creates incentives for some resources with marginal cost lower than the RMOL to "chase prices." In the real-time market, Price Chasing or self-dispatch reduces market efficiency and raises operating costs. To address this issue, the options vary from penalties for self-dispatch to paying generators their Lost Opportunity Costs (LOC) not to self-dispatch. However, the Lost Opportunity Costs approach may not be revenue adequate. We address this issue in more detail in chapter on Incentives.

Convex Hull Pricing (CHP). In 2007, Gribik, Hogan, and Pope proposed CHP. CHP follows from a hypothetical sequence of events different from the ISO auction processes with several unusual elements. The construction of the CHP problem requires a hypothetical auction that has little resemblance to the actual ISO auction. In CHP, each market participant solves an individual Walrasian-type optimization with a given set of prices to schedule its dispatch. The source of these prices is come from the solution to the convex hull problem. Calculation of the convex hull – an extremely difficult problem. Walrasian-type two-sided auction markets, especially with fixed costs, do not have good convergence properties and are mostly used in academic contexts. In 2016, Schiro et al, with simple examples, presented counterintuitive properties of Convex Hull Pricing. CHP does not eliminate make-whole payments, pays generators to stay on the dispatch signals and is not always revenue adequate. We are not aware of any other auction that pays market participants to comply with the auction results. MISO implemented a simple CHP approximation, but penalizes departures from dispatch signals (see Wang, et. al. 2016).

The goal of CHP is to identify uniform prices that minimize uplift payments defined as the sum of Make-Whole Payments and LOC (see Schiro et. al., 2016). However, Convex Hull Prices are difficult to calculate and the economic properties of Convex Hull Prices are not well understood. No ISO uses CHP. MISO and ISO-NE started their pricing discussions with CHP. ELMP and ELMPL are the simplified progeny of Convex Hull concepts. The simplifications made to facilitate computation may invalidate some of the pricing properties.

Extended LMP (ELMP) Pricing. In 2011, MISO implemented ELMP pricing noting that the computational intensity of Convex Hull Pricing was a limiting factor. ELMP relaxes some binary variables, which leaves a Make-Whole Payment if the generator is not operating at its maximum output. Make-Whole Payments are usually smaller than in RMOL.

MISO began with resources that had a start-up time of ten minutes or less, about 50 resources with less than 2% of capacity. Seven percent of real-time intervals had an approximate increase in prices of $1/MWh and a reduction of 1 percent in make-whole payments. In 2017, MISO increased to about 180 resources with less than 10% of capacity. About 23 percent of intervals had a price increases on average of $3/MWh and a 9 percent reduction in make-whole payments. MISO's ELMP method acknowledges Price Chasing and penalizes for deviations from the dispatch.

Extended LMP with LOC (ELMPL). In 2017, ISO-NE implemented a mechanism similar to MISO (ISO-New England, 2017). The most notable difference is rather than a penalty for not following the dispatch signal like MISO and other ISOs, ISO-NE provides LOC payments.

Fast-Start Pricing. In 2018, the Commission found that SPP, PJM, and NYISO's practices were unjust and unreasonable and ordered them to change their pricing because prices did not accurately reflect the cost of serving load. This debate motivates the average incremental cost (AIC) pricing proposal.

Average Incremental Cost (AIC) Pricing. In 2017, O'Neill introduced AIC pricing. It eliminates the need for make-whole payments, creates better incentives for infra-incremental generators, and sends better entry and exit price signals (see O'Neill 2017 and O'Neill et al, 2019 revised 2020). The AIC approach comes closest to satisfying desired economic properties of pricing. In 2019, O'Neill et al introduced the one-pass AIC approach. For dispatched generators, the energy and reserve prices are profitable without make-whole payments. (We define profitable to include breaking even.) Excursions from the dispatch signal pay at a minimum the cost of redispatch (aka liquidated damages) or receive a lower the energy price, thereby eliminating the incentive to self-dispatch.

The intuition for the AIC approach starts with the single-period, single-bus, unit-commitment model. The AIC approach finds the highest average incremental cost generator, it sets the clearing price for all generators, results in no make-whole payments to generators and is an exact entry and exit price. In markets with multiple buses, if a transmission capacity constraint binds, the problem is decomposed into two separate problems by fixing the transmission at the line capacity. In problems with multiple periods, binding ramp rate constraints and binary generator constraints tie periods together.

There is no simple pricing model that satisfies all the properties of the convex model's LMP, for example, transparent prices, no make-whole payments, good long and short-term entry and exit signals, and positive profits for infra-incremental generators. In a non-convex market, more information is required.

Pricing Methods Circa 2018. No ISO uses LMP pricing, where there is only a single energy payment without a make-whole payment. CAISO and SPP use LMP +.

NYISO and PJM relax minimum operating levels with make-whole payments (RMOL). Several ISOs allow resources at their minimum operating levels to set prices. NYISO became the first ISO to implement a version of this pricing. In 2014, PJM relaxed by 10 percent the Minimum operating level of units (PJM, 2016). In 2016, PJM begun relaxing block-loaded units' minimum operating level by 20 percent.

MISO's ELMP and ISO-NE's ELMPL are the progeny of CHP. Table 1.1 summarizes current ISO pricing methods and includes AIC pricing

Table 1.1 Current (2018) and Proposed AIC Pricing Methods

Existing ISO Practices	Energy price	Pricing run	Relaxed Minimum Operating Level	LOC payments	Make-Whole Payments	Modified Marginal Cost
CAISO	LMP	No	No	No	Yes	No
SPP	LMP	No	No	No	Yes	Yes
NYISO	RMOL	Yes	Yes	No	Yes	No
PJM	RMOL	Yes	Yes	No	Yes	No
MISO	ELMP	Yes	Yes	No	Yes	Yes
ISO-NE	ELMPL	Yes	Yes	Yes	Yes	Yes
AIC	LIP	Yes	Yes	No	No	Yes

Capacity Markets. After ISO energy markets were introduced, it was soon recognized that LMPs without price-responsive demand could not support energy prices that incent efficient investments (aka the 'missing money' problem, see Joskow, 2007). This may cause long-run inefficiencies. In four ISOs, the answer was a capacity market. Other ISOs have capacity assurance mechanisms. They incent capacity under the N-1 paradigm and less so flexibility. Higher energy prices result in lower procurement costs. Over time, capacity markets have become more complex. Capacity markets should be limited to risk management and not collect the missing money. In 2019, most ISO markets had excess capacity for several reasons: Federal and state incentives and subsidies favor certain technologies. Low natural gas prices have made a gas-fired generator less costly. Future demand estimates in capacity markets have been overestimated.

Price-Responsive Demand and Scarcity Pricing. Price-Responsive Demand in the ISO energy markets voluntarily bids consumption and may set the energy price. This results in a more efficient dispatch and better investment price signals because the energy price more accurately reflects demand's willingness to pay. Without Price-Responsive Demand and Scarcity Pricing, prices in the energy markets under the above methods are too low to incent efficient new investment. Price-Responsive Demand and Scarcity Pricing are different in two important ways. Price-Responsive Demand is a bid from a consumer that is an expression of values and quantities that it wants to consume and the flexibility of its consumption. Scarcity pricing is an administrative construct. It assumes how much load is willing to pay for energy. Scarcity pricing raises the energy and reserve prices.

1.4 Summary of the Forthcoming Chapters

Chapter 2 introduces AIC pricing and presents examples to help understanding of AIC pricing and its advantages. It discusses incentives for efficient outcomes under the pricing methods in non-convex ISO markets. Chapter 3 presents Price-Responsive Demand Ramsey-Boiteux pricing concepts and their influence on incentives, prices, and market efficiency. Chapter 4 presents a comparison of current pricing and the AIC methods. It argues that AIC better meets the Commission's stated goals and market efficiency. It presents single-period market models and examples of the pricing methods. Chapter 5 reviews the non-convex market pricing literature. Chapter 6 presents models and examples for AIC Pricing in multi-period markets. In multi-period markets, non-convexities like startup costs, minimum operating levels, minimum run times and minimum down times present pricing issues that are resolves using cost causation principles. The appendix develops the multi-period mathematical model for AIC pricing. Chapter 7 examines the AIC pricing model in networks. The non-convexities present issues involving congestion price signals and ramping. Numeric examples illustrate this issue and its resolution. The appendix develops the network mathematical model for AIC pricing and FTR markets. Chapter 8 presents entry and exit issues in Non-convex Markets. We show that the AIC is a better signal for efficient entry. A glossary and references are provided at the end.

2 Introduction to Average Incremental Cost Pricing

2.1 Introduction

The purpose of this chapter is to introduce Average Incremental Cost (AIC) pricing. It is designed to improve on the current pricing methodologies. The AIC pricing mechanism produces prices that incorporate both the Marginal costs and the avoidable fixed operating costs of a dispatched resource. In non-convex markets, AIC pricing is the rough equivalent to marginal cost pricing in convex markets. This introduction leaves out some of the details of AIC pricing and examines how it differs from textbook neoclassical convex markers. There is no best known approach to pricing in non-convex auction markets, but the attributes of AIC pricing are superior to other approaches. In future chapters, we will provide additional detail, examples and properties of AIC pricing. While the focus is on generation, analogous arguments can be made for all resources, such as load and storage.

AIC pricing applies to all resources and all periods in the network. The AIC pricing algorithm is a post UC optimal dispatch that relaxes some of the constraints of UC problem and allocates avoidable fixed costs over the optimal dispatch (energy and reserves) and adds this allocation to the marginal costs. Additional rules and variations are in subsequent chapters. The generator with the highest AIC and a non-zero AIC dispatch in the pricing run sets the energy price. AIC pricing ensures offer cost recovery, eliminates Make-whole Payments, and sends a better signal for entry and investment. Finally, the AIC method uses Ramsey-Boiteux pricing for price-responsive demand. The LIP is the dual variable on the energy balance constraint and is calculated after a complete startup/shutdown cycle or a reasonable approximation for cycles greater than the market horizon(for example, 24 hours in the day-ahead market) for the marginal resource so that costs are allocated to the time periods when high cost resources are needed.

Prices under the AIC pricing method are calculated using the ISO's linear program software. The linear program is run on a relaxed UC and the LIPs are the dual variables on the energy balance and reserve constraints. The resulting prices reflect the highest AIC of the dispatched generators. Total settlement revenue for the resource is the product of the efficient dispatch from the SCUC and the prices from the AIC pricing run. Under AIC pricing, the LMPs, LIPs at a feasible point of entry would be public information. Penalties for not following dispatch instructions are used instead of paying LOC payments.

In all examples tested, AIC pricing eliminates make-whole payments. For example, an incumbent generator with start-up costs of $40 and marginal costs of $30/MWh is dispatched at 15 MWh for one period. Its total incremental operating cost over the period is $490 (= 40+30x15) with an AIC of $32.67/MWh (= $490/15 MWh). If the incumbent generator described above set the price under AIC pricing method (at $32.67/MWh), that price would send a signal that any potential resource with 15 MWh of energy (the energy dispatched by the incumbent) whose average incremental cost is below $32.67 (the incumbent's AIC) could have efficiently entered the market.

Arbitrage Condition. Satisfying the Arbitrage Condition means that resources do not have an incentive to shift the relative energy and reserves dispatch. In a co-optimized market, reserve prices satisfy the Arbitrage Condition where prices differ by the marginal cost of energy less marginal cost of reserves. Other approaches do not or have not been shown to satisfy the Arbitrage Condition because a portion of the avoidable fixed costs are not allocated to reserves.

Capacity Procurement. AIC lowers the capacity price and may eliminate the need for ISO capacity auctions mechanisms. The AIC pricing mechanism would produce prices in the ISO energy markets that send better price signals for entry and exit, allow more infra-incremental generators to earn a profit, preserve the arbitrage between energy and reserve prices, and lower capacity costs.

Market Design Goals. Markets should be designed so that market participants have the incentive to provide actual cost, value, and technical constraint information to the market as part of their offers and bids. In all cases, the sufficient conditions for an efficient market outcome include bids at incremental value, offers at incremental costs, internalized externalities, and optimization software that can find the optimal solution to the auction problem. In this paper, we will generally assume these conditions are true.

To produce just and reasonable prices, the Commission's pricing goals are:

maximize market surplus for consumers and suppliers.
provide incentives to follow dispatch instructions.
maintain reliability.
provide market transparency.
ensure that all suppliers have an opportunity to recover their costs.
make efficient dynamic and locational decisions to enter the market; and
make efficient decisions to exit the market.

In addition, just and reasonable prices must have the ability to attract needed investment (see Hope v FPC). The current price formation rules in the ISO markets fail to meet these goals. For example, the make-whole payments are not transparent to other market participants and are allocated too broadly to provide correct price incentives for market participants to make efficient entry and exit decisions as well as efficient investments in facilities and equipment.

In contrast to current pricing rules, the AIC approach allocates costs that would have been included in make-whole payments with greater granularity in time and location. Thus, AIC prices would provide greater transparency as well as better price signals and greater confidence in the market.

2.2 Convex Markets and Non-Convex (Lumpy) Markets

In this section, we review the properties of convex and non-convex markets, and how non-convexity complicates market design, in particular, the day-ahead and real-time electricity markets. Convexity is a mathematical concept that has important implications for the study of markets. Much of microeconomics is viewed through the lens of convex markets. In convex markets, marginal cost (or value) prices clear the market, signal entry, provide fully compensatory settlements and have good incentives for efficiency enhancements. In convex markets, a local optimal solution is a global optimal solution. Convex markets are a special case of non-convex markets. Non-convex markets do not possess many of these properties, but are a more realistic representations of actual markets.

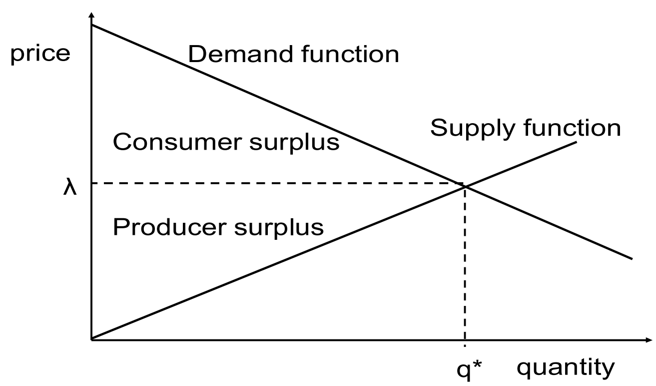
In convex markets, the efficient dispatch can be represented as a convex optimization problem. LMPs alone clear and settle the markets, and signal efficient entry and exit. The LMP settlement, except for degeneracy, is the singular price signal for market clearing, entry, and exit. In strictly convex markets, LMPs are unique and have desirable economic properties as prices. In convex markets, if the primal problem is degenerate, the dual problem may not have a unique solution. If the degeneracy involves the energy balance or reserve constraints, there may be multiple optimal dual variables that are the source of the prices. Linear program (LP) solvers produce only one set of prices. Over time, some analysts made the unwitting assumption that the LMP was more than just a spot price. In the presence of avoidable fixed costs, the LMP does not clear the market and is a weak signal for entry and exit. In vertically integrated utilities, these properties were mostly academic.

Because the power market auctions are non-convex, finding the efficient solution and pricing is more complicated. Early power market auctions discussions often assumed that power markets are convex since this allows for a simplified discussion of the market design and produces elegant mathematical results. Indeed, if actual power markets were convex, market design would be easier. When the fixed costs of operation and investment, that is, non-convexities, are introduced, the elegant theoretical pricing properties of convex markets quickly disappear.

Economically Efficient Convex Markets. If a cost function is non-decreasing, it is convex (technically quasiconvex). If a demand function is non-increasing, it is concave (technically quasiconcave). Maximizing the sum of concave demand functions less sum of convex supply functions subject to convex constraint sets is a convex optimization problem and solving the economic optimization problem results in maximum economic (economically efficient) benefits. If the problem is convex, the dual optimal variable (aka Lagrangian multipliers, LMP or λ) on the energy balance constraint is a price that clears the market, signals entry at the margin (with no information on the entry quantity), and provides fully compensatory settlements. Linear program (LP) solvers produce only one set of prices. When the linear program used for market clearing is degenerate, there may be multiple LMPs that result in arbitrary settlements and weaker entry and exit signals.

Strictly Convex Markets. In a strictly convex market, that is, a market with strictly downward-sloping (no flat areas) demand functions and strictly upward-sloping supply functions. A strictly convex market has only one market clearing and entry price at each point in time and space, see Figure 2.1.

Figure 2.1. Strictly Convex Market Clearing at q* with Entry Price is λ.



Convex Markets. If a cost function is non-decreasing, it is convex (technically quasiconvex). If a demand function is non-increasing, it is concave (technically quasiconcave), see Figure 2.2. Maximizing the sum of concave demand functions less sum of convex supply functions subject to convex constraint sets is a convex optimization problem. In this case, the LMP price signal for supplier 3's dispatch is ambiguous and requires a dispatch signal.

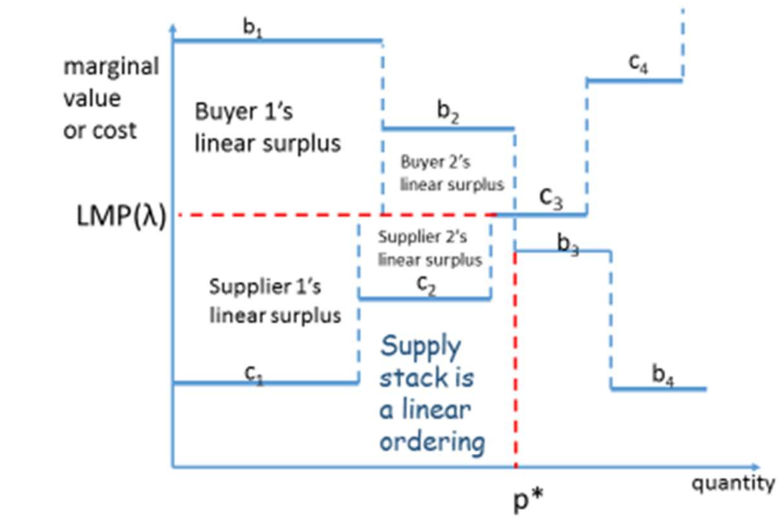


Figure 2.2. Market clearing for a convex auction market and its surplus.

Convex markets with the demand and supply functions that intersect at more than one point, see Figure 2.3, at all prices between high LMP (λ) and low LMP (λ) at quantity p* is the optimal dispatch, but there is no unique clearing price and the entry and exit signals are different. This convex market has a supply entry price (low LMP) set by the demand function for the marginal supply and a demand entry price (high LMP) for the marginal demand set by the supply function. When these two prices are different if the market is degenerate (a low probably event). In convex markets, when a new generator enters the highest cost generator exits the market and/or lower-valued consumption increases.

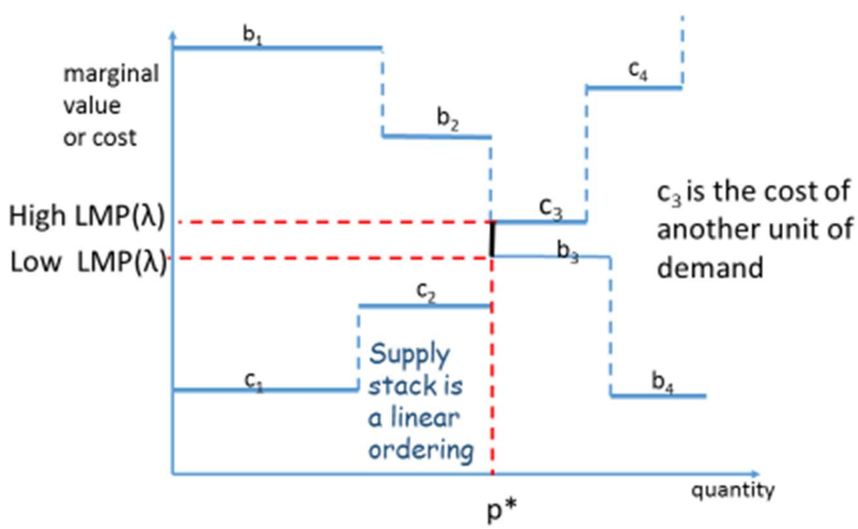


Figure 2.3. Multiple Market-Clearing Prices

Economically Efficient Non-Convex Markets. If any of the convexity assumptions is violated, the market is non-convex. Many properties of convex markets do not hold in non-convex markets. Non-convex markets may have local (but not global) optimal solutions that are not economically efficient. In non-convex markets, the marginal cost concepts are extended to include avoidable incremental costs incurred by binary decisions.

In non-convex markets, LMPs generally have no claim to be market clearing. Determining whether entry in a non-convex market is profitable and efficient requires more information than the single price that is the dual variable (LMP) on the energy balance constraint after fixing the binary variables to their optimal values and solving the resulting linear program. In a non-convex market, the LMP alone may not ensure offer cost recovery for all efficiently dispatched participants may not be compensatory and is a weak price signal. There may be many Pricing Points of Entry for a non-convex market. In non-convex markets, when a new generator enters the highest cost generator may not exit the market and/or consumption may not increase, but the market is more efficient.

In general, the entry signal is not just a price, but also a specific quantity (or vector of quantities) paired with an LIPs below which dispatched generator (or set of generators) can be displaced. For example, multiple entry price-quantity signals are LMP and LIP_i at p_i* where i is an incremental generator.

Non-Convex Markets. Convex markets are a special case of non-convex markets without avoidable fixed costs, or if the binary decision variables are not binding constraints. In the absence of convexity, LMPs may not support an optimal dispatch schedule which includes both commitment and dispatch. In 2005, O'Neill et al showed that LMPs plus make-whole payments are equilibrium prices.

Without price-responsive demand or high penalties for shorting reserves, there is not enough revenue in the ISO energy markets to support new investment. Some ISOs require LSEs to prove they have sufficient reserves committed to the market (owned or under contract) to satisfy demand most of the time, for example, all but one event in ten years. Other ISOs added capacity markets to make up for 'missing money' due to the lack of price-responsive demand. With price-responsive demand, ISOs no longer need to forecast demand or use capacity markets for the missing money.

Apart from infrequent blackouts, the US power system runs continuously. Due to the complexity of the system, operational constraints are approximated. From these approximations, rules, 'good utility' practice, intuitions (operator experience), shibboleths and myths result not fully understanding the approximations. For example, the distribution factor DC approximation has a reference bus with an LMP -- incorrectly called the system marginal price. Congestion that occurs only between two nodes or buses is often referred to as nodal due to the algorithmic approximation. By choosing a different reference bus these values change (see Litvinov, 2010).

Non-Convex Electricity Energy Markets. If actual power markets were convex, the SCUC would be unnecessary; the SCED would produce the optimal dispatch; and the LMPs would clear the market and send appropriate price signals for short-term and long-term production, entry and exit. In Convex Markets, the LIP is LMP. However, power systems and power markets are not convex, creating important differences between the simplified convex model of the power system and the physics of the market.

In electricity energy markets, a higher cost generator with a lower minimum operating level can replace a lower cost generator with a higher minimum operating level. More generally, a sufficient entry condition is any combination of generators that can produce the same amount at less cost than the current generators. The information needed to calculate entry into the market is usually not publicly available and complete information for entry is difficult to calculate. In non-convex markets, efficient supply entry may not displace the marginal supply, but an entire generator.

There are several characteristics arising in electricity markets that can render a function non-convex. An existing non-convex generator, for example, a fossil fuel generator, has startup costs, avoidable fixed operating costs per period, a non-zero minimum operating level, a minimum run time, and a minimum down time violating these constraints can cause damage to the equipment. Figure 2.4. illustrates a supply function. This supply function is non-convex because the function is undefined on (0, 5) and has a minimum operating level of 5.

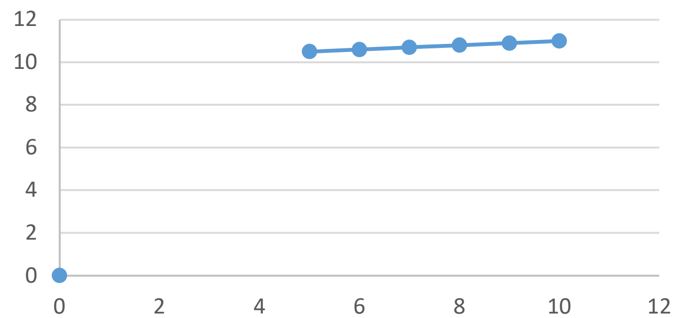


Figure 2.4. Non-Convex Supply Function. Marginal cost= 10+.1p with an operating range= 0, [5, 10]

Because suppliers incur start-up costs and have a minimum operating level, there will often be no single price that will clear the market. A price equal to the marginal supplier's marginal cost will be too low to fully cover the marginal supplier's total costs. The incremental supply may be a generator and not a unit output. However, a price high enough to cover every supplier's total costs may elicit more supply than that demanded by customers. The optimal dispatch depends on the other resources in the market. Intermittent resources are dependent on current weather and may need back up by resources not as dependent on the current weather.

Current market auctions allow for non-convex bids. Resources can offer marginal and avoidable fixed costs, such as startup and fixed operating costs. The SCUC considers these costs. The SCED considers the avoidable fixed costs for units as sunk and they are not considered in determining the LMPs. In these markets, marginal cost pricing does not ensure offer cost recovery and those unrecovered fixed costs are recovered via make-whole payments, which are paid by the entire load whether or not an individual market participant caused them.

Non-convexities inherent in the power markets may mean that there is no set of LMPs that supports a sustainable efficient equilibrium. In this case, resources would be better off taking actions different from those in the efficient solution. For instance, suppose the revenue a resource would earn if it followed the dispatch signals and was paid the prices (LMPs) resulting from the efficient solution is insufficient to cover that resource's commitment and variable costs of following dispatch signals. The resource would be better off not participating in the market. In AIC pricing, we would adjust the energy and reserve settlement prices such that every resource receives sufficient revenue to cover its costs at its efficient dispatch.

A comparison of Convex and Non-Convex Markets is in Table 2. Some similarities and differences of LMP and AIC pricing Table 2.2.

Table 2.1 of Convex Markets and Non-Convex Markets

Convex Markets	Non-Convex Markets
LMP clears the market	LMP may not clear the market
The highest marginal cost of supply dispatched, or the lowest value of demand dispatched sets the LMP. There is no uplift.	The highest incremental cost of supply dispatched or the lowest value of demand dispatched may not set an LMP. There may be confiscation at the LMP.
The supply 'stack' is a linear ordering.	The supply 'stack' is not a linear ordering.
LMP is consistent with the off-ISO markets	The LMP is not consistent with the off-ISO markets
An LMP is a signal for efficient entry and exit	The LMP is a limited signal for efficient entry and exit
infra-marginal generators make positive profits	infra-incremental (infra-marginal) generators do not always make positive profits
The LMP is non-confiscatory	The LMP may be confiscatory
there are no scale economies	there are scale economies

The efficient dispatch algorithm is simple	The efficient dispatch algorithm is complex
single entry price signal	multiple entry price signals
exist in textbooks and some financial markets	physical markets are non-convex. Binary decisions are ubiquitous.
In strictly convex markets, the 'law of one price' clears and decentralizes market.	non-convex markets cannot always be decentralized using only one price.
In non-strictly convex markets' multiple prices can clear and decentralize market.	many entry price-quantity pairs exist for entry. For each non-convex generator or asset, there may be a separate entry point.
have no local (not global) optima	have local optima.
transmission assets with no marginal value have no incremental value	transmission assets with no marginal value may have incremental value
has a non-empty core	may have an empty core

Table 2.2. A Comparison of Non-Convex Markets with LMP and with AIC.

Non-Convex Markets with LMP	Non-Convex Markets with AIC Pricing
LMP may require make-whole payment	LIP does not require a make-whole payment
The highest marginal cost of supply dispatched, or the lowest value of demand dispatched sets the LMP	The highest AIC of supply dispatched or the lowest value of demand dispatched sets LIP.
LMP is not consistent with the off-ISO markets	The AIC approach is consistent with the off-ISO markets
The LMP is not a limited signal for efficient entry and exit	The LMP is a limited signal for efficient entry and exit
infra-marginal generators may not make positive profits	infra-marginal (infra-incremental) generators make positive profits
The LMP is confiscatory	AIC is not confiscatory
the dispatched generator with the highest marginal cost (or the demand dispatched with the lowest value) sets the energy price. There is uplift.	the dispatched generator with the highest AIC (or the demand dispatched with the lowest value) sets the energy price. There is no uplift.

2.3 Risk Management with Bilateral and Multilateral Off-ISO Contracts

In addition to participating in the ISO markets, market participants may also enter into bilateral or multilateral contracts that are not a part of the auction market itself. We use bilateral contract to mean any bilateral or multilateral contract entered into outside of the ISO auction markets. Even if ISO markets are functioning well, bilateral contracts can usefully support ISO markets by providing a way for market participants to manage their individual risk. The ISO and bilateral markets should be in equilibrium.

The decision to invest involves estimating one or more future possible revenue streams and the probability of each. A risk-neutral seller will invest in generation or other assets if it expects to cover its costs and make a profit over the life of the investment. A risk-adverse seller will invest if it expects to cover its costs and make a larger profit over the life of the investment to compensate for the impact of potential losses.

Once an asset is constructed, a large part of the investment is usually a sunk cost. The sunk costs cause financial risk. To reduce its risk, a seller may sell some or all its future output through bilateral contracts, find investors willing to share the investment risk, buy insurance, or combinations of these options.

Efficient pricing in bilateral negotiations may result in contracts with complex terms to cover possible future outcomes, penalties for nonperformance, and can result in high transaction costs. The ISO's published information informs negotiations.

If the seller had scale economies, for example, declining average costs, it may offer a lower price for higher quantities. Startup and fixed operating costs create scale economies, most be factored into the prices.

Coalitions of buyers and sellers may form to reduce risks and to take advantage of scale economies. For example, generation and transmission electric cooperatives form to supply their members. The AIC pricing approach allocates costs in a way more consistent with efficient bilateral contracts.

Non-convex markets may present a conflict between bilateral contracts and ISO markets. This problem is called market without a core (see Owen, 1982). This problem is solved by market rules that eliminate the empty core.

The ISO's day-ahead market schedules energy and reserves over the next day in 24-hour periods and hedges the market participants against unexpected events in the real-time market. A generator scheduled by the day-ahead market is guaranteed not to lose money. The real time market is a physical delivery mechanism that dispatches the market and prices deviations from the day-ahead market. A generator scheduled by the real-time market is guaranteed not to lose money.

To be efficient, the longer-term Off-ISO markets must consider the full operating cost of generating plants, including longer-term avoidable fixed and variable costs. Clearing these markets using the LMP can lead to distortions since the LMP does not reflect avoidable fixed costs. AIC best promotes efficiency in Off-ISO markets, both in terms of contracts for energy, and entry and exit because the AIC pricing approach integrates the avoidable fixed costs of operation into the energy and reserves prices. A high volume of Make-Whole and other 'uplift' Payments can lead to underinvestment in Off-ISO markets. The Off-ISO markets should receive price signals from the ISO that reflect the highest average cost resource in the Efficient Dispatch. Make-Whole Payments can hide the cost of such resources, dull price signals, and prevent efficient investment.

2.4 Self-Commitment, Self-schedules and Self-dispatch

In ISO markets, in addition to market participants bidding and offering incremental costs into the day-ahead market, a market participant may also Self-Schedule or self-commit its resource, see Table 2.3. Self-Schedules and self-commits are not assured of cost recovery, lower the flexibility of the market, and may lower the market efficiency (surplus). A resource can self-commit, by self-scheduling a portion of its output, for example, its minimum operating level and submit a marginal cost function up to a maximum level. To self-schedule, a market participant fixes the quantity in its bid or offer. Because the ISO calculates and posts prices after it has determined the efficient dispatch, the energy price, for example, the LMP or the ELMP, is not a signal to market participants to change their dispatch level. In the real-time market, a market participant may self-dispatch causing energy imbalances and other constraint violations, but should be responsible for the costs of rebalancing.

Table 2.3. Self-scheduled and Self-Committed Offers by Type in PJM

Resource Type	Self-Scheduled	Self-Committed	Total
Natural gas-CC	0.6%	13.9%	14.5%
Natural gas-Steam	0.3%	4.5%	4.8%
Coal-CT	0.9%	44.8%	45.7%
Coal-Steam	3.1%	46.2%	49.3%
Nuclear	64.5%	25.5%	90.0%

Source: PJM 2018

2.5 Incentives

Good market design creates incentives for market participants to trade in ways that will produce economically efficient outcomes, that is, maximum market surplus. First, with the available information, potential market participants have to decide whether to participate in a market and to invest in new or existing generators when efficient and profitable to do so. Second, a market participant must decide whether the offer/bid submitted to the market will accurately reflect all economic and technical characteristics (costs, values, and technical constraints). Third, market participants need to decide whether to produce or consume at the market operator's dispatch signal. In an idealized, perfectly-competitive markets with no avoidable fixed cost or non-convex operating constraints, LMP provides proper incentives. However, the avoidable fixed cost and operating constraints of electric power markets and invalidate many of the properties of LMP pricing.

Decision to Enter the Market and Transparency. To participate in the market, a potential entrant considers the market rules and the published information in each market to evaluate the expected profit. A risk adverse potential entrant will need a higher expected profit or a hedge. The current approaches (LMP+, RMOL, ELMP, ELMPL) send weak signals to incentivize efficient decisions about which markets to participate and invest in.

By eliminating make-whole payment and allocating cost to the proper time and location, AIC pricing provides superior incentives for participation and to invest in new technology. The high prices in the capacity markets indicate the energy market prices are too low. AIC pricing would increase transparency and may also increase energy prices. Increasing energy prices does not necessarily increase consumer prices and may decrease consumer prices. Higher energy prices lead to lower capacity prices. Due to flaws in the capacity markets and the greater efficiency of energy markets, AIC pricing may lower consumer prices and produce more efficient outcomes.

Decision to Submit Truthful Bids and Offers. Under LMP pricing, market participants may suffer losses because their fixed operating costs are not always covered. The possibility of short-term economic losses produced by LMP pricing violates a market design principle of "non-confiscation." The problem with confiscatory mechanisms is that they reduce or eliminate the incentive for market participants to participate and follow dispatch instructions because they can improve their outcome by simply avoiding the market. To address this problem, ISO auction markets provide make-whole payments to ensure that no generators operate at a loss for offering incremental costs and following dispatch instructions. These payments fix the non-confiscation problem, but introduce a new problem.

The revenues a generator receives through make-whole payments are directly affected by the offer it submits to the market, such that submitting higher offer costs higher than an avoidable cost offer will often directly result in higher profits. This problem is addressed, albeit imperfectly, through mitigation that provides direct oversight and regulation of a market participant's offer so that it reflects the market monitor's estimate of incremental costs. AIC pricing eliminates the make-whole payments that are necessary in current mechanisms.

A market participant generally may have incentives to submit offers/bids that do not accurately reflect economic and technical characteristics in three circumstances: when market participants have market power, or when the bid or offer format does not allow the accurate expression of operating and cost parameters. Rules that provide incentives for market participants to submit offers/bids that accurately reflect their economic and technical characteristics are said to be incentive compatible.

All pricing methods need market monitoring and mitigation to ensure accurate energy costs are offered to the auction market. The market monitor screens offers that deviate from their estimated costs and mitigate offers that are considerably in excess of estimated costs. The best approach is to screen offers once before the SCUC and adjust offers that fail mitigation. If the market is perfectly competitive or contestable, mitigation is not necessary because market participants would not have the incentives to deviate from truthful bidding. Unfortunately, the requirements for perfectly competitive or contestable markets are far from being met in power markets.

To reduce gaming and provide incentives to offer incremental costs, the AIC approach could pay, with some exceptions, Self-Scheduled resources no more than the LMP. This approach creates an incentive for a Self-Schedule to instead offer its costs and operating constraints into the market creating additional flexibility leading to more efficient dispatch (including enhanced reliability and resilience).

An exception to this pricing rule is a unit with minimum run time greater than the horizon of the market that are certified by the ISO as part of the efficient dispatch by other analysis, for example, a week-ahead advisory dispatch. In the day-ahead market, nuclear and some coal plants fall into this category and would be eligible to receive the LIP.

Another exception is if a bid of incremental value or an offer of incremental cost cannot fit its technical operating requirements into the bid or offer format. For example, if the feasible configurations of a combined cycle or pumped hydro unit cannot be fully represented in the offer format, it may need to self-commit or self-schedule.

If a bid of incremental value or an offer of incremental cost that is the result of an off-ISO contract. This contract should be certified as necessary by the ISO or market monitor as necessary, for example, a take-or-pay fuel contract for natural gas that requires a corresponding physical dispatch in the power market.

Decision to Follow Dispatch Instructions. Market rules should discourage unnecessary self-dispatch. For example, a generator may receive a dispatch instruction to produce 150 MWh but decide to produce 175 MWh instead. The generator has essentially forced the market operator to take an additional 25 MWh. The market operator must then balance the market through deployment of frequency regulation resources. The rules of the market, such as those that determine when a participant will receive a LOC payment or pay penalties, will determine whether self-dispatch is profitable or costly to the market participants. Markets have little integrity if the dispatch instructions are not enforced. More generally, most auction markets penalize non-performance.

Self-dispatch after the auction outcome can force the market operator to redispatch the system and could cause reliability issues. If deviations are settled at the market price, generators may have an incentive to deviate from the dispatch signal if the market price is more than their marginal cost of operation. In nearly every ISO, the pricing mechanism has already produced prices higher than the LMP. Similarly, a dispatchable load generally have an incentive to deviate from dispatch if the market price is less than its marginal value of consumption.

Self-Dispatch (Uninstructed Deviation) Charges. The first option to correct self-dispatch imbalances is to implement uninstructed deviation charges on a market participant that does not follow dispatch instructions. Such penalties need to be high enough to remove any profit motive for over-generating due to higher energy prices. Since generators are not perfect machines, any deviation penalty approach would likely need to incorporate some deadband within which strong penalties are not assessed. Any such dead bands would create an incentive for generators to deviate as much as they could within the deadband, if they can control their output. NYISO penalizes market participants for over-generation (outside of a 3% deadband) by not paying for the excess energy, see NYISO Market Services Tariff Section 2.3.

The minimum charge for uninstructed deviation should be the liquidated damages, that is, the costs of frequency regulation and other actions to rebalance the system caused by the market participant's failure to follow dispatch instructions. Such an approach would usually align incentives to follow dispatch instructions with the cost incurred. Liquidated damages may be difficult to calculate.

Another option to correct for the imbalance issues to compensate generators that can profitably deviate from dispatch instructions for not acting on that incentive. Under this approach, such generators would receive a payment in the amount of the opportunity cost that was incurred as a result of the generator not profitably deviating from dispatch. While this approach effectively removes the incentive for over-generation, it does so at a significant expense and in general is not revenue adequate.

The AIC approach could pay self-dispatches no more than the LMP and the generator should pay the liquidated damages of the rebalancing. The AIC approach creates an incentive for market participants to stay on the efficient dispatch and eliminates the incentives to 'price chase'.

2.6 Conceptual Underpinnings of AIC Approach

There is no known best approach to pricing in non-convex markets. Analyzing and designing power markets under the assumption of convexity is one approach, but has failed and been abandoned. Analyzing and designing power markets without the assumption of convexity is closer to reality, but more difficult. AIC pricing is a fusion of theory and principles of auction market design. It implements beneficiaries pay cost allocation; provides signals needed for efficient entry and exit; allows for complementary goods; and is in equilibrium with bilateral contract markets.

In the presence of non-convexities, the AIC approach is an extension of the marginal cost concepts to the startup-shutdown cycle as it allocates the total costs of a startup-shutdown cycle over that cycle. This is also consistent with the cost allocation principle that costs be allocated to the time, location and need that caused them to be incurred.

Price-Responsive Demand and Passive Consumers. An important part of the AIC approach (and all pricing approaches) is price-responsive demand that creates the necessary entry and exit signal for all resources including consumption, storage and supply. The highest AIC of supply dispatched, or the lowest value of demand dispatched sets the LIP. Most consumers are passive, that is, they consume without being price responsive to, or even aware of, the wholesale or retail price.

In 2018, price-responsive demand was a small part of the ISOs' markets. Some level of price-responsive demand is necessary prices to promote long-term market efficiency through prices. To qualify as price-responsive demand, a load must bid some or all of its demand into the day-ahead and real-time markets at its incremental value. In return, the market participant has no capacity obligation or charge and no demand response "baseline" for participation in the reserve markets. Price-responsive demand would replace other demand response programs that are less efficient and avoid forced curtailment. When price-responsive demand can be dispatched based on expressed values, reliability and efficiency improve.

To achieve market efficiency, we introduce Ramsey-Boiteux pricing that avoids over charging price-responsive demand and creates incentives for load to participate as price-responsive demand. While some pricing methods can raise the price paid by demand above its expressed value, this has not been a serious problem in markets due to the inconsequential amount of price-responsive demand currently in the markets. In order to prevent equilibrium issues where some loads have incentive to consume at levels less than their dispatched amounts, Ramsey-Boiteux pricing reallocates demand surplus from low-valued consumers to high-valued consumers through efficeint price discrimination, creating prices that are lower for both types of consumers.

Illustrations and Numerical Examples. The prices that would result from AIC pricing are likely to be higher than the LMP. We illustrate the AIC methodology with a one period model and generators with different minimum operating levels, avoidable fixed costs and constant marginal costs. Figure 2.1 illustrates a multiple-resource market after running the SCUC and the LIP using the AIC pricing methodology. The flat (orange) steps are the marginal costs of the resources. The non-increasing (purple) functions show the AIC for each resource as its output increases. When operating at minimum operating level, the AIC costs are higher and when operating at maximum, AIC costs are lower

As the dispatch of the resource increases, the price on the resource's AIC function decreases because the fixed costs are now spread over more output. Eventually a generator reaches its maximum and a new generator may become the marginal generator serving load with declining AIC creating a 'saw tooth' pricing pattern. The efficient dispatch of generation (p*) is determined from the SCUC. Instead of clearing from the marginal cost function, LIP clears from the AIC cost function of each resource.

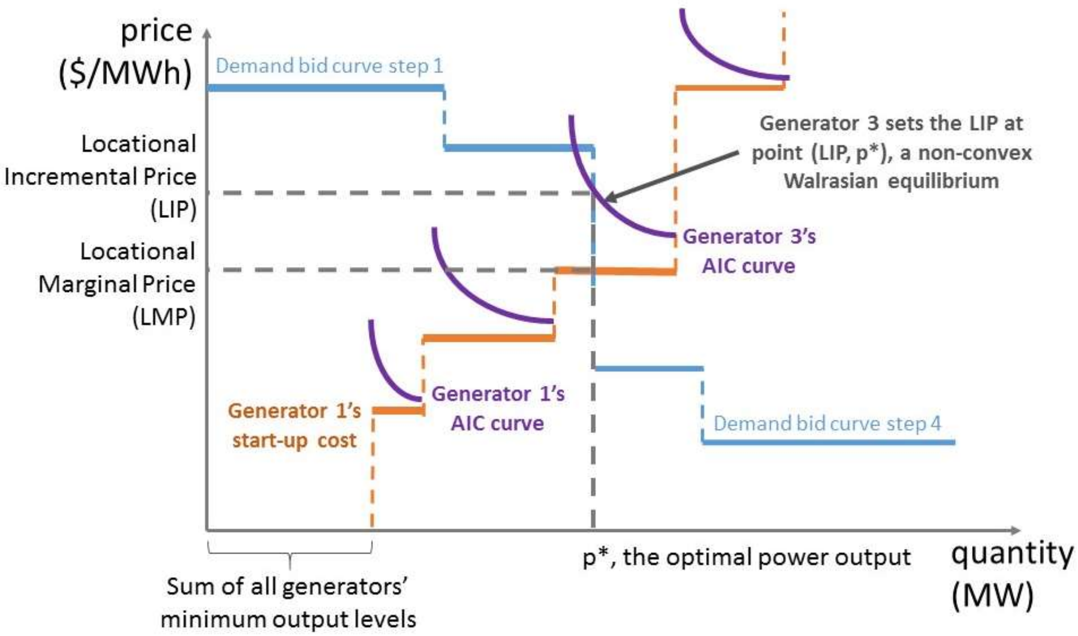


Figure 2.1 A multi-resource market cleared using the AIC pricing method.

Example 1. To illustrate AIC Pricing, we present an example with the three generators shown in Table 2.2 and demand shown in Table 2.3.

Table 2.2 Generator Parameters

Generator	Startup costs $	Minimum MW	Maximum MW	Marginal cost $/MWh	AIC at min $/MWh	AIC at max $/MWh
GA	200	80	95	10	12.50	12.11
GB	90	40	50	20	22.25	21.80
GC	40	15	20	30	32.67	32.00

Table 2.3 Load parameters

load	Minimum MW	Maximum MW	Marginal value $/MWh
LA	0	110	200
LB	0	49	150
LC	0	40	5

The SCUC yields the results shown in Table 2.4. All three generators are dispatched to serve loads LA and LB. The LMP from the SCED is $20/MWh set by GB. GA makes a positive profit. Under LMP pricing, GC and GB make negative profits totaling $280. Under LMP+ pricing, GB and GC receive a make-whole payment and breakeven even though GB has lower costs than GC. The make-whole payments are not public information lowering transparency.

Table 2.4 Efficient Dispatch and LMP Pricing. Optimal surplus is $26,640

Gen	startup	dispatch	value at max	profit
GA	1	95	10	750
GB	1	49	0	-90
GC	1	15	0	-190
Totals		159		470
Demand		dispatch	value at max	surplus
LA		110	180	19800
LB		49	130	6370
LC		0	0	0
Totals		159		26170

LMP is $20/MWh; make-whole payments are $280; average make-whole charge to load is $1.761/MWh.

The results of the AIC pricing run are in Table 2.5. The LIP is $32.67/MWh set by GC, the highest AIC generator dispatched. GC breaks even. There are no make-whole payments resulting in full price transparency. The infra-incremental generators, GA and GB make positive profits. Without the pricing rule for Self-Schedules and self-dispatch, GB would self-dispatch to 50 MW.

Table 2.5 AIC Pricing Run.

gen	startup	AIC dispatch	AIC	profit	Incremental Costs	settlement
GA	1	95	12.10	1953.33	1150	3103.33
GB	1	50	21.84	530.67	1070	1600.67
GC	1	14	32.67	0	490	490.00
totals		159		2484.00		5194.00
demand	startup	dispatch	value at max	Consumer surplus		settlement
LA	1	110	167.33	18406.67		3593.33
LB	1	49	117.33	5749.33		1600.67
LC	1	0	0	0		0
totals		159		24156.00		5194.00

LIP is $32.67/MWh; make-whole payments are 0; make-whole charge is 0.

Example 2. Example 2 has less demand than Example1 as shown in Table 2.6. The generators have the same characteristics as Example 1.

Table 2.6 Load Parameters

load	Minimum MW	Maximum MW	Marginal value $/MWh
LA	0	85	200
LB	0	49	150
LC	0	40	5

The SCUC/SCED results are in Table 2.7. Generators GA and GB are dispatched to serve loads LA and LB. The LMP is $10/MWh set by GA. Under LMP pricing, GA and GB earn negative profits. Under LMP+ pricing, GA and GB receive a make-whole payment and break even. The make-whole payments are not public information lowering price transparency.

Table 2.7 Efficient Dispatch and LMP Pricing. Optimal surplus is $22,320.

Gen	startup	dispatch	value at max	profit
GA	1	94	0	-200
GB	1	40	0	-490
GC	0	0	0	0
totals		134		-690
demand		dispatch	value at max	Consumer surplus
LA		85	190	16150
LB		49	140	6860
LC		0	0	0
totals		134		23010

LMP= $10/MWh; make-whole payments are $690; average make-whole charge is $5.15/MWh

The results of the AIC pricing run are in Table 2.8. The LIP is $22.25/MWh set by GB, the highest AIC generator dispatched. GB breaks even. The infra-incremental generator GA makes positive profits. There are no make-whole payments, resulting in greater price transparency. Without the pricing rule for self-dispatch, GA would self-dispatch to 95 MW.

Table 2.8 AIC Pricing Run.

gen	Start up	dispatch		profit	Incremental Costs	AIC	settlement
GA	1	95		951.50	1140	12.13	2091.50
GB	1	39		0	890	22.25	890.00
GC	0	0		0	490	30.00	0
totals		134		951.50			2981.50
demand	Start up	dispatch	value at max	Consumer surplus			settlement
LA	1	85	178	15108.75			1891.25
LB	1	49	128	6259.75			1090.25
LC	1	0	0	0			0
totals		134		21368.50			2981.50

LIP is $22.25/MWh; no make-whole payment

Example 3. Example 3 has less demand than Example 2 as shown in Table 2.9. The generators are the same as Example 1.

Table 2.9 Load Parameters

load	Minimum MW	Maximum MW	Marginal value $/MWh
LA	0	46	300
LB	0	46	300
LC	0	40	5

The SCUC/SCED yields results in Table 2.10. Generator GA is dispatched to serve loads LA and LB. The LMP is $10/MWh set by GA. Under LMP, pricing GA makes negative profits. Under LMP+ pricing, GA receives a make-whole payment and breaks even. The make-whole payments are not public information, lowering transparency.

Table 2.10 Efficient Dispatch and LMP Pricing. Optimal surplus is $26,480.

gen	startup	dispatch	value at max	profit
GA	1	92	0	-200
GB	0	0	0	0

GC	0	0	0	0
totals		92		-200
demand		dispatch	value at max	surplus
LA		46	290	13340
LB		46	290	13340
LC		0	0	0
totals		92		26680

LMP is $10/MWh; make-whole payments are $200; average make-whole charge to load is $2.17/MWh.

The results of the AIC pricing run are in Table 2.11. The LIP is $12.17/MWh set by GA, the highest AIC generator dispatched. In this case, the LIP is exactly equal to the LMP plus average make-whole payment. GA breaks even. There are no make-whole payments and full price transparency. Without the pricing rule for self-dispatch, GA would self-dispatch to 95 MW.

Table 2.11 AIC Pricing. Surplus is $26,480

gen	Start up	dispatch	value at max	profit	Incremental Costs	AIC	settlement
GA	1	92	0	0	1120	12.17	1120
GB	0	0	0	0	890	20.00	0
GC	0	0	0	0	490	30.00	0
totals		92	0	0			1120
demand		dispatch	value at max	Consumer surplus			settlement
LA		46	288	13240			560
LB		46	288	13240			560
LC		0	0	0			0
totals		92		26480			1120

LIP is $12.17/MWh; no make-whole payments.

Example 4. Example 4 has less demand than Example 3 as shown in Table 2.12. The generators are the same as Example 1.

Table 2.12 Load Parameters

load	Minimum MW	Maximum MW	Marginal value $/MWh
LA	0	40	200
LB	0	35	150
LC	0	40	6

The SCUC/SCED yields results in Table 2.13. Generator GA is dispatched to serve loads LA, LB, and LC. The LMP is $6/MWh set by LC. Even though the marginal value for LC is $6/MWh and the marginal costs of GA is $10/MWh without LC, there is not enough demand to overcome the minimum operating level of GA. LA and LB are willing to pay the for LC's participation and LC pays only the LMP. Under LMP pricing, GA earns negative profits. Under LMP+ pricing, GA receives a make-whole payment and breaks even. The make-whole payments are not public information resulting in partial transparency. While the average make-whole charge is $6.50 (=520/80), since the average make-whole charge is only allocated to LA and LB, LA and LB pay $6.933/MWh (=$520/75 MWh).

Table 2.13 Efficient Dispatch and LMP Pricing. Optimal surplus is $12,280.

gen	startup	dispatch	value at max	profit	Marginal Cost
GA	1	80	0	-520	10
GB	0	0	0	0	20

GC	0	0	0	0	30
totals		80		-520	
demand	**startup**	**dispatch**	**value at max**	**surplus**	**marginal value**
LA	1	40	194	7760	200
LB	1	35	144	5040	150
LC	1	5	0	0	6
totals		80		12800	

LMP=$6/MWh; total make-whole payment is $520; average make-whole charge is $6.5/MWh.

The results of the AIC pricing run are in Table 2.11. The LIP is $12.50/MWh set by GA, the highest AIC generator dispatched. GA breaks even. There are no make-whole payments and full price transparency. Without the pricing rule for self-dispatch, GA would self-dispatch to 95 MW.

Table 2.14 AIC pricing. Surplus is $12,280.

gen	startup	dispatch	AIC	profit	Incremental Costs	settlement
GA	1	80	12.50	0	1000	1000.00
GB	0	0	20.00	0		0
GC	0	0	30.00	0		0
totals		80		0		1000.00
demand	**startup**	**dispatch**	**value at max**	**consumer surplus**		**settlement**
LA	1	40	187.50	7500.00		500.00
LB	1	35	137.50	4812.50		437.50
LC	1	5	-7.50	-32.50		62.50
totals		80		12280.00		1000.00

LIP= $12.50/MWh; total make-whole payment and average make-whole charge is 0.

In the pricing scheme in Table 2.15, LC would need to pay $12.50/MWh, but it is only willing to pay $6/MWh. We use Ramsey-Boiteux pricing to reduce LC's settlement to $6/MWh.

Table 2.15 The Ramsey-Boiteux Settlement.

demand	surplus	settlement
LA	7483.75	516.25
LB	4796.25	453.75
LC	0	30.00
totals	12800	1000.00

2.7 Conclusions

From the work to date, AIC pricing would improve ISO price formation by:

- ensuring bid and offer cost recovery in energy and reserve prices.
- eliminating make-whole payments resulting in greater transparency.
- creating better signals for efficient entry, exit, and investment.
- encouraging incremental cost offers in energy markets.
- eliminating LOC incentives and payments.
- creating profit incentives for all infra-marginal resources to increase efficiency.
- providing better price signals for price-responsive demand.
- lowers capacity prices and investments costs.

2.8 APPENDIX. CONVEX AND NON-CONVEX SETS AND FUNCTIONS

Convex Sets. A set is convex if a line connecting any two points in the set is also in the set. In addition, a tangent plane (or hyperplane) through any point on the set's border intersects no interior points. See Figure 2.1. If the point of tangency is the optimal, the slope of the tangent plane is the price with the convex market properties

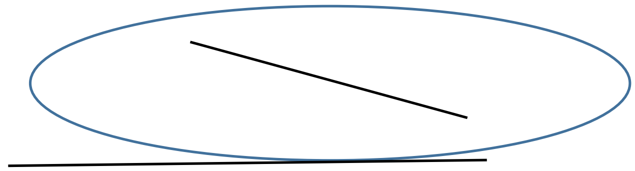


Figure 2.3. A convex set.

A set is non-convex if there is a line connecting two points in the set that is not completely in the set. In addition, a tangent plane through a point on the border may contain interior points. See Figure 2.2.

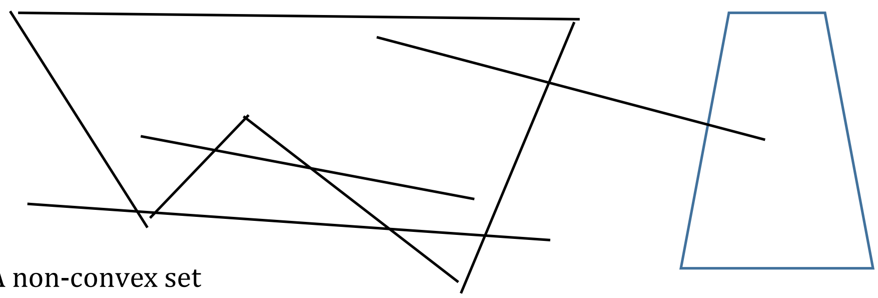


Figure 2.2. A non-convex set

Convex and Concave Functions. A mathematical function, such as a supply or demand function, if every possible straight line connecting any two points on the function lies above the function. Figure 2.3 illustrates convex function. If differentiable, the tangent plane lies below the function. If a straight line connecting any two points lies below the function at any point on the line, the function is concave.

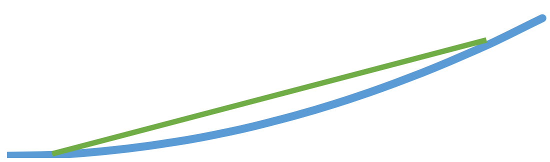


Figure 2.3. A Convex Function

However, if a straight line connecting any two points on the function does not lie completely above or completely below the function, then the function is non-convex. Figure 2.4 illustrates a non-convex function. For more detail see Luenberger and Ye, (2008) or Mangasarian (1969).

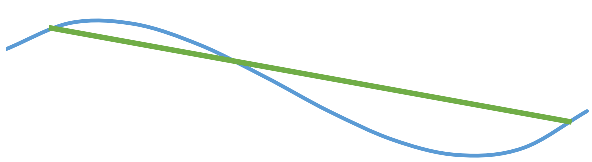


Figure 2.4. A Non-Convex Function

3 Price-Responsive Demand and Ramsey-Boiteux Pricing

3.1 Introduction

A goal of any market design should be to the maximize market efficiency (surplus). Today, in ISO markets, generators offer detailed costs and operating constraints and most load is represented by LSE forecasts. In the short term, demand is essentially passive. Historically, consumers have accepted a monthly bill and complained to their state regulator if they think the bill is too high. The development of price-responsive demand has been inhibited by the lack of computing, communications infrastructure, and appropriate retail rate structures. As part of efficient market design, efficient demand response allows direct participation of consumers (or load) similar to generation. Programs to subsidize participation and administratively intervene in the market should be a last resort with an expiration date.

Consumers can benefit from participating in the ISO markets. Price-responsive demand bids can save consumers money, prevent the exercise of market power, allow better risk management and hedging opportunities, lower capacity costs, and lower administrative intervention in markets, and, at times, eliminate the need to activate administrative scarcity pricing. Also, price-responsive demand allows the day-ahead market and real-time market to function more efficiently, stimulates investments in efficient load control, and is consistent with the bilateral markets.

Efficient Day-Ahead and Real-Time Energy Markets. Efficient prices are important to efficient markets. Efficient markets maximize benefits to society and energy prices should reflect the condition where incremental value of consumption is less than or equal to the Incremental Cost of supply. There is always an underlying demand function, that is, there is a price where load would not consume energy from the grid.

Under conditions of high wind generation, high solar generation and/or low load, the LMP may be near zero or negative. Negative prices encourage consumption by paying load to consume or charge batteries, for example, for electric vehicles, but prices are time dependent and hard to predict. If load is not price-responsive demand, the incentives to consume when prices are low are weaker.

In 2009, Centolella and Ott advocated a price-responsive demand scheme. The program completes the market, but was never fully implemented. By 2018, time-of-use metering installations have increased significantly. Real-time pricing is the default tariff for some larger consumers and can be an option available to other consumers. In ERCOT, a Controllable Load Resource (CLR) can submit a bid-to buy in the day-ahead market to hedge the CLR's consumption in real-time market. The CLR can also submit bids to real-time market and must be capable of controllably reducing or increasing consumption under Dispatch control by ERCOT. The CLR provides the telemetry and a Current Operating Plan to ERCOT, see http://www.ercot.com/mktrules/nprotocols/current.

Price-responsive demand provides the opportunity for load efficiently to voluntarily reduce their consumption when prices are high and increase consumption when prices are low. In addition, price-responsive demand can profit from supplying actual reserves without a baseline calculation. Price-responsive demand is its own 'capacity requirement' and does not need to buy capacity in the capacity markets or pay for capacity. Price-responsive demand should be part of the efficient resource mix that maximizes market efficiency including the prevention of cascading blackouts and forced curtailments.

With price-responsive demand, loads bid into the energy and reserves market and receive a schedule or dispatch signal. However, there is currently very little of this type of participation in today's ISOs. This is due to the LSEs acting as agents for load and retail prices that do not reflect marginal or incremental values of consumers. The load that bids into the market can offer to supply reserves. The day-ahead market gives load an opportunity to efficiently schedule consumption into the market and hedge against the volatility of the real-time market. In the real-time market, load can adjust its consumption based on market conditions by selling its purchases in the day-ahead market back to the real-time market when prices are high and making additional purchases when prices are low.

In this Chapter, we address how price-responsive demand can be efficiently integrated into wholesale power markets and is a necessary component of an efficient market design. The addition of price-responsive demand should increase the overall benefits to all market participants through lower peak/higher-price consumption, greater off-peak/lower-price consumption, lower capacity payments, and send better price signals for future decision-making. We also introduce Ramsey-Boiteux (RB) pricing that is a necessary component of price-responsive demand and efficient market design. Ramsey pricing is well-known to most economists, but not to most power systems people. Although Marcel Boiteux was president of Électricité de France and employed this pricing in France, his pricing concepts are not well-known in the US. We explore how the RB paradigm is applied in AIC pricing to improve efficiency of the market in the presence of price-sensitive demand.

3.2 Background on Wholesale Markets and Retail Markets.

The ISO wholesale energy markets and the retail markets are physically inseparable. Load and generation have a reflexive symmetry. At any point in time, supply must equal demand at each bus in the system. Load withdraws energy from the system. Generation injects power into the system. Load pays for energy withdrawn from the system. Generation is paid for injections into the system. System changes are immediately felt on both the transmission and distribution systems. Each time a load changes its consumption; it instantaneously changes the wholesale dispatch and affects the just and reasonable price. The ISO tries to keep the system in balance by purchasing reserves that must respond to the unanticipated changes in demand.

Losses can be over 40% higher from the transmission bus to a bus at the end of a feeder, see Perez-Arriaga (2016). The economics of location is important to future decisions such as the location of storage and generation in the distribution grid. Under locational marginal pricing, because losses are quadratic, the benefits of price-responsive demand are enjoyed by all consumers on the feeder. Price-responsive demand provides incentives to locate facilities efficiently on the system.

Consumption. Currently, most load does not actively participate in the energy markets as generation does. Load is represented in the day-ahead market by an agent, usually a load serving entity (LSE), and in the real-time market by the ISO in the form of a forecast. In a large ISO, a one percent load forecast error is the equivalent of a losing a large nuclear plant. Most LSEs have weak incentives to save money for consumers since the costs of energy are usually passed on to load.

The thermal envelop of each building is an energy storage facility. Today, most load is given few incentives to respond to wholesale energy prices due to inefficient retail rate designs. Schemes to incent efficient consumption without allowing market participation are a distant second best.

For decades, in capacity expansion models, the projected future load growths were based on projections of GDP or similar macro forecasts. Table 3.1 shows that electricity consumption has not been growing at the rate of GDP growth.

Table 3.1 Annual Consumption (in TWh) and GDP (in current $ trillion)

Year	Residential	Commercial	Industrial	Transport	Total	Direct Use	Total	GDP	MWh/GDP
2005	1359	1275	1019	8	3661	150	3811	13.09	0.29
2006	1352	1300	1011	7	3670	147	3817	13.86	0.28
2007	1392	1336	1028	8	3765	126	3890	14.48	0.27
2008	1381	1336	1010	8	3734	132	3866	14.72	0.26

2009	1365	1307	917	8	3597	127	3724	14.42	0.26
2010	1446	1330	971	8	3755	132	3887	14.96	0.26
2011	1423	1328	991	8	3750	133	3883	15.52	0.25
2012	1375	1327	986	7	3695	138	3832	16.16	0.24
2013	1395	1337	985	8	3725	143	3868	16.69	0.23
2014	1407	1352	998	8	3765	139	3903	17.39	0.22
2015	1404	1361	987	8	3759	141	3900	18.04	0.22

Source: EIA

Over the last decade, the industrial sector accounted for about 27% of total consumption. The average industrial consumer consumes over five times more electricity than other consumers, see Table 3.2. The industrial sector uses electricity primarily to operate machinery followed by process heating, see Table 3.3. Many process applications must startup, run for a minimum amount of time and then shut down similar to generators.

Table 3.2 Average consumption per customer class in kWh, 2015

	Residential	Commercial	Industrial
Customers in millions	1784	1558	216
Average consumption per customer	787	874	4573

Source: EIA

Table 3.3 Manufacturing Electric Energy Consumption by End Use Type, 2006

End-Use Type	Percent
Machine Drive	0.51
Process Heating	0.12
Heating, Ventilation, and Air Conditioning	0.09
Process Cooling and Refrigeration	0.07
Electrochemical Processes	0.07
Lighting	0.07
Data Centers	0.02
Other	0.05
Total	1.00

Source: EIA

Electric Vehicles Charging electric vehicles can require significant power compared to the average residential load. An average house consumes about 30 kWh/day. The charge for an electric vehicle is 60 to 85 kWh/day. Without price-responsive demand, incentives to reduce charging when wholesale prices are high, and increase.

Data Centers In 2014, data centers in the U.S. consumed an estimated 70 billion kWh about 1.8% of total U.S. electricity consumption. The "hyperscale shift" is an aggressive shift of data center activity from smaller data centers to larger data centers, see Shehabi, 2016. Large data centers can consume more than 100 MW with a significant portion of time-flexible load.

Types of Demand Response. Most load participates in the real-time market simply by consuming. Currently, there are three ways the loads can 'participate' in the ISO markets: ex-ante price response, ex-post price response, or price-responsive demand. Currently, the most common form of demand response is ex-post response. Ex-post prices are historical prices and do not necessarily reflect future prices. If load is responding to an ex-post price signal, the optimal dispatch needs rebalancing and the response can be counterproductive. Much of current DR is available only in emergencies, but is deemed comparable to a generator available almost year-round.

3.3 DEMAND PARTICIPATION IN MARKETS

Demand Response. In 2011, FERC in Order 745 introduced a demand response ("DR") program that was inefficient and invited manipulation. Order 745 inefficiently lowers prices to generators who are required to participate in the market and whose market power is mitigated while demand is paid to withhold and fails to send an efficient signal to load for conservation. Order 745 is the equivalent to requiring the ISO to operate a cartel on behalf of the consumers. On May 23, 2014, the DC Circuit Court in EPSA v. FERC vacated the Commission's Order 745 response program. On January 25, 2016, the Supreme Court reversed the DC Circuit Court decision making demand response FERC juristional.

Demand response baseline measures have created moral hazard gaming problems and have been the subject of gaming investigations. This process can create phantom reserves that do not actually exist. Two examples of gaming are the cases of Comverge and Rumford Paper. Comverge, a demand response services provider for Oriole Park at Camden Yards in Baltimore, during a September 2010 power emergency turned its lights on when no event was taking place on the field to increase its baseline consumption. In addition, Rumford Paper Co. in western Maine "artificially inflated" its baseline power needs for six months beginning in July 2007 by not running its 'behind the meter' generators.

Operating Reserves Demand Curve (ORDC). The ORDC is an administrative construct used to value and price reserves. Prices during reserves shortages should be substantial and reflected in the ORDC. A superior approach that could limit the impact of the administrative invention is price-responsive demand.

Price-Responsive Demand. A consumer with Price-Responsive Demand splits demand into two components: fix demand and bid-in demand. The fix portion incurs capacity costs. The biddable portion does not. The biddable portion must bid with finite value into the day-ahead and real-time markets. Price-responsive demand allows to compete on an equal basis with generation. For example, an synchronous motor can provide voltage support.

The best way a resource (generation, storage, transmission or load) can respond to prices is by expressing its intentions and actions (that is, bidding) in the market. Once the auction concludes and a price is announced, the price can effectively only serve as input to future price expectations. By bidding into the energy markets, the load will know in advance how much it desires to consume at a given price. For example, price-responsive demand could bid the opportunity cost of selling its natural gas to a generator instead of using it as am input to its own operations.

Bid function formats need to evolve. Using generator bid functions can be a short-term substitute for actual load bid functions such as minimum run time, minimum and maximum consumption, startup and shut down, and load shifting. Consumers should have the option to bid into the markets on a comparable basis (that is, the mirror image of supply is consumption). Advanced bid functions can include:

Load	Comparable Generator Parameters
time needed to reduce or increase consumption	startup time
bidding consumption in minimum time blocks	minimum run time
shutdown	startup
minimum consumption levels	maximum generation levels
maximum consumption levels	minimum consumption levels

Price-responsive demand needs no baseline measures. Consequently, there are no gaming issues in measuring it. It also has the 'limiting principle.' Load is simply bidding into the day-ahead market and real-time market to give the operator advanced notice of how it would consume at a given price.

3.4 Benefits of Price-Responsive Demand

Price-responsive demand responding to efficient prices is both beneficial to market participants who respond and society in general.

Price-responsive demand inhibits market power. In the Nash-Cournot equilibrium, the market power metric is $\sum_i (p-s_i mc_i)/p = H/e$, where p is the equilibrium price, s_i is the market share of i, mc_i is the marginal cost of i, H is the Herfindahl-Hirschman index and e is the demand elasticity. The greater the elasticity the lower the market power.

Risk Management. Risk management, like insurance, can be an individual decision. Good regulation does not hedge all price risk or assume uniform risk tolerance for market participants. Price-responsive demand allows better individual risk assessment and hedging. Most market participants are risk averse. Some are risk takers. Bring the two groups together creates a hedging market. Bilateral contracts can reduce risk. In power markets, risks are physical delivery and price. In ISOs, the risk of physical delivery is low leaving risk management mostly to hedge price.

Capacity Markets. Capacity markets exist to supply demand that is not price-responsive in the energy markets. Capacity markets value one attribute, that is, maximum operating level. Unlike energy markets, capacity markets do not value or consider other attributes, for example, startup time, minimum operating level, ramp rate or minimum run time. In addition, they have a simplified topology.

If energy market prices are capped, energy market prices may be too low to stimulate price-responsive demand and contribute to the "missing money" problem, that is, prices from the energy markets are not sufficient to compensate efficient investment in needed generation for reliable provision of electricity. When demand is high, price suppression may inhibit market clearing and result in voltage reduction and forced curtailment. In other markets, a price would increase due to scarcity in the short term and greater efficiency in the longer term.

Capacity markets may be needed to supply capacity for load that is not bidding into the market. Capacity market procurement depends on many assumptions with little empirical support. The capacity markets must implicitly estimate the value of lost load (VOLL), loss of load expectation (LOLE), and net cost of new entry (CONE). Table 3.4 shows the wide range of interpretations of these parameters.

Table 3.4 LOLEs for Various VOLL a Five-Hour Outage and Capital Cost Assumptions

VOLL $/MWh	capital cost (Net CONE) $/MW-yr	LOLE events/yr
4000	120000	6
4000	80000	4
4000	40000	2
2000	120000	12
2000	80000	8
2000	40000	4
20000	120000	1
20000	80000	1
20000	40000	0

Source: Wilson 2010

The current capacity markets present a moral hazard. The politics of capacity markets introduce inefficient biases into the design. Some market participants are subsidized by the assumptions needed to parameterize the market. Others are disadvantaged.

Price-responsive demand does not pay for capacity since it is willing to voluntarily reduce consumption when needed for reliability. By not participating in the capacity market, price-responsive demand drives down the capacity price. If needed demand can hedge energy prices in the bilateral markets. 'Avoided capacity costs can be the single largest cost savings in the business case for price-responsive demand.' (Centolella and Ott 2009). With an efficient market design, the expected market-clearing price for capacity is close to zero; otherwise there are energy market design flaws, for example, price caps.

Reserve Service. Price-responsive demand can participate fully and more efficiently in reserve markets where the load's response time satisfies the ISO's reserve requirements. For example, ramping down load is the equivalent to ramping up a generator. In many cases, load ramping down may be faster and more efficient (and more reliable) than generators ramping up. Many electrical appliances can perform these services that are individually almost unnoticeable, for example, heating and cooling appliances. Collectively, millions of appliances can constitute a large response that can be signaled by frequency changes.

Reliability. Reliability should focus on preventing cascading blackouts and little else. Price-responsive demand is not forcibly curtailed. In a reliability crisis, non-price-responsive demand could be forcibly (involuntarily) curtailed to avoid a cascading blackout. Price-responsive demand voluntarily reduces its consumption as part of the dispatch to avoid a reliability violation. Voluntary reductions are less costly and more efficient. When a market participant 'responds' ex-post to a market-clearing price, it may come too late.

3.5 Barriers to Price-Responsive Demand

Several reasons for not implementing price-responsive demand include: the lack of smart metering, retail rate design, and latency in communications, aggregation and computation capability. As described below, the issues are being resolved.

Smart Meters. Price-responsive demand requires time-of-use metering on a sub-hourly basis and the ability to communicate with the ISO. This allows for better participation in ISO energy and reserve markets. In 2011, more than 33 million or 23% of all U.S. electrical consumers had smart meters. By 2018, smart meter penetration was about 60% and projected to be 70% in 2020, see Table 3.5

Table 3.5 Number of Smart Meter Installations by Sector

Year	Residential	Commercial	Industrial	Transportation	Total
2015	57,107,785	7,324,345	310,889	813	64,743,832
2017	69,474,626	9,060,128	365,447	1,389	78,901,590
2018	76,498,388	9,932,993	411,287	1,489	86,844,157

Source EIA

Retail Rate Design. The most important part of the price-responsive demand is an efficient retail market design. Retail tariffs should have a part that reflects the Incremental Costs of energy, that is, directly tied to prices in the wholesale energy markets plus distribution losses.. Other parts of the tariffs could reflect a capacity or option call on the network infrastructure based on peak usage in $/MW. A third could be a fixed charge for residual costs in $. Many states and municipalities modify the wholesale market price at the retail level that do not vary with MWh. Retail rate designs are being challenged by retail consumers selling power back to the grid, for example, from rooftop solar power and batteries.

Transition to More Price-Responsive Demand. A transition from current DR programs to price-responsive demand is not difficult. Price-responsive demand is already available in some ISOs, but needs more development. There will be less forced curtailment and lower capacity prices. Capacity markets will be less contentious, and the day-ahead market and real-time market will better compensate flexibility and incent better performance.

Non-Price-Responsive Demand. With price-responsive demand, a virtuous cycle or feedback loop is created for non-price-responsive demand participant. When they discover that they would have preferred to have consumed less or would have preferred to consume more at the market price, they have a greater incentive to bid into the market. In the longer-term, more price-responsive demand should result greater efficiency. An increased amount of price-responsive demand is most likely the largest missing element of ISO market design.

3.6 Ramsey-Boiteux Pricing

In the electricity market (as in other markets where at least some suppliers have high fixed costs and low marginal costs), charging all customers a uniform price that covers suppliers' costs may discourage efficient consumption. This problem arises because a uniform price that is high enough to cover suppliers' costs may need to be above suppliers' marginal cost. Customers that value electricity less than this uniform price will not want to consume. However, if the value obtained by these consumers exceeds marginal cost, consumption by these customers would increase the economic efficiency (market surplus). As a result, charging a uniform price high enough to cover the avoidable supplier's costs to all customers would discourage consumption by some customers and efficiency is improved by charging lower prices to customers that value electricity less. In ISO markets, the software dispatches all demand where incremental value is greater than incremental costs. Current pricing rules may not reflect this.

The Natural Gas Act (NGA) and Federal Power Act (FPA) require just and reasonable prices that are 'not unduly discriminatory' and high enough for suppliers to recover prudently incurred fixed costs. Without price controls, a monopolist will maximize its profits and typically set prices above – potentially substantially above – the competitive level. Pricing based on the value of demand could be considered 'not unduly discriminatory' if it is efficiency enhancing.

3.7 Example of Ramsey-Boiteux Pricing applied to AIC pricing

Ramsey-Boiteux (RB) pricing is implemented when the demand function has one or more steps with values between the average incremental cost (AIC) function and the marginal cost function. In this case, a portion of demand that clears in the efficient dispatch may need a price below the to maintain the requirement that prices be non-confiscatory. .

Figure 3.1 presents a case without Ramsey-Boiteux pricing where marginal costs and marginal values are by step functions. Seller 1 receives LIP·p* and no make-whole payment. Buyer 1 pays LIP·d_1 for d_1 units. Buyer 2 pays LIP·d_2 for d_2 units. Since Buyer 3 values power less than c_1, as demonstrated by the line segment b_3 is below c_1, Buyer 3 is out-of-the-market. No RB pricing is needed.

Figure 3.1 One generator with no RB pricing

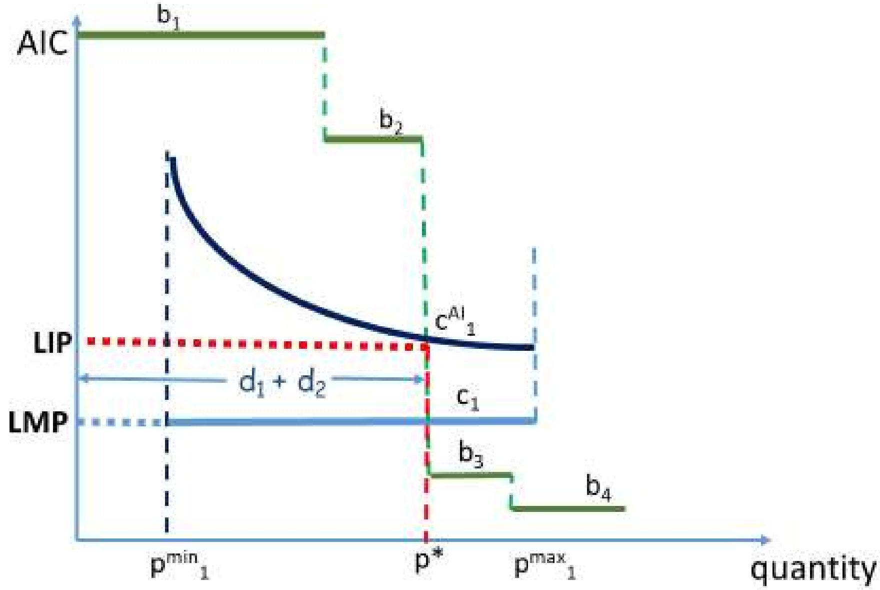
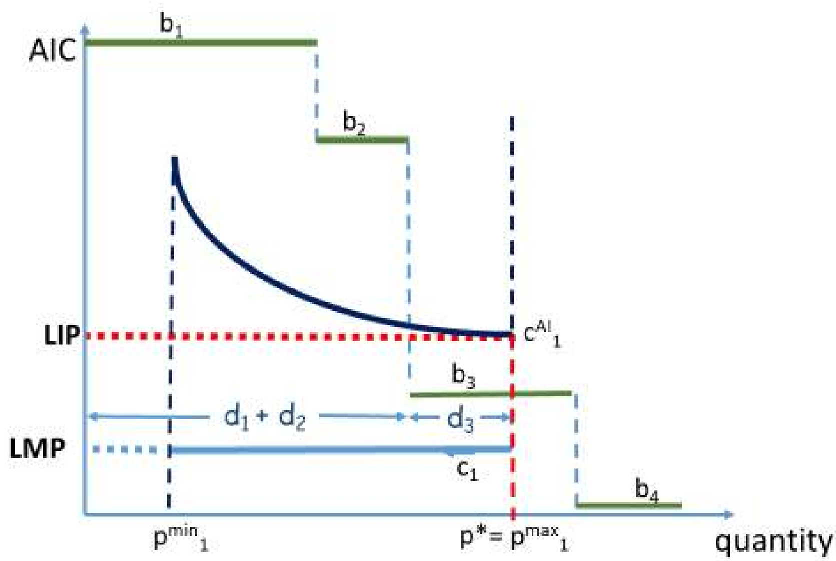


Figure 3.2 presents a case in which Buyer 3 is part of the efficient dispatch but values power at an amount less than the LIP. If Buyer 3 pays only its consumption value, Seller 1 will have a shortfall of (LIP – b_3)d_3. This dilemma can be resolved by charging a portion of the shortfall to Buyers 1 and 2 by adjusting their prices by θ_1 and θ_2, respectively, such that $\theta_1 d_1 + \theta_2 d_2 = (\text{LIP} - b_3)d_3$. Seller 1 then receives LIP·p*.

Figure 3.2 One generator with RB pricing



Multiple Generators and Price is Set by a Generator at Minimum Output. This example includes a larger set of generators. Importantly, this example will show that the efficient dispatch is not possible (or is confiscatory) unless discriminatory pricing is applied. The generator and load parameters are in Table Table 3.8 and Table 3.9.

Table 3.8. Generator Parameters

Gen	Startup cost	Marginal cost	Minimum operating level	AIC at minimum operating level	Maximum operating level
GA	1000	20	800	21.25	900
GB	200	30	200	31.00	300
GC	100	50	90	51.11	100

GD	0	21	800	21.00	900

Table 3.9. Load Parameters

Load	Startup value	Marginal value	Minimum consumption level	Maximum consumption level
LA	0	200	1	1250
LB	0	14	1	400

The efficient dispatch is in Table 3.10. Generator GD enters the market displacing GB and GC. GA and GD are at their minimum operating level and cannot set the LMP. LB enters the market to meet the minimum operating levels of GA and GD, even though its valuation is below either generator's marginal cost. The LMP is $14/MWh set by LB and the market surplus is $221,100.

Table 3.10 Efficient Dispatch. Optimal Market Surplus is $221,100

gen	startup	dispatch	marginal value	profit
GA	1	800	0	-5800
GB	0	0	0	0
GC	0	0	0	0
GD	1	800	0	-5600
totals	-	1600	-	-11400
demand				surplus
LA	1	1250	186	232500
LB	1	350	0	0
totals	-	1600		232500
LMP =	14			

AIC Pricing. The efficient dispatch is only agreeable to LA and LB if discriminatory pricing is applied. Under traditional LMP with make-whole payments, LB would pay $14/MWh plus a pro rata share of the fixed costs, making the effective payment higher than its valuation. By applying discriminatory pricing, LB enables the feasible commitment of GD. This lowers the amount paid by LA, so LA's surplus increases even though it is paying the highest discriminatory price in the market settlement. When LB is not included in the market, the optimal dispatch would be to dispatch both GB and GC (since it is not feasible to commit GD) with an optimal market surplus is $218,400. Including LB allows the feasible simultaneous commitment of GA and GD.

3.8 APPENDIX: DERIVATIONS OF RAMSEY-BOITEUX PRICING

Ramsey Pricing with Fixed Costs and Demand Function. We derive the Ramsey pricing formula with fixed costs and aggregate demand function. For convenience of the derivation, we assume that the value and cost functions are continuously differentiable.

Notation

D	is the demand
P	is supply
B(d)	is the value function for d
C(p)	is the cost function
F	is the fixed costs to be recovered
b(d)	$= \partial B(d)/\partial d$ is the inverse demand function
c(p)	$=\partial c(p)/\partial p$ is the marginal cost function
e	$= b(d)/(b'(d)d)$, elasticity of consumer at p
α	$= -\gamma/(1-\gamma)$, coefficient ensuring exactly fixed costs are recovered
λ^r	$= b(d)$
λ	is the dual variable for the supply balance constraint

γ is the dual variable for the fixed cost recovery constraint

The model for efficient pricing is:

Max	$MS = \int b(d)\partial d - C(p) - F$	Maximize market surplus = Consumer value - Supply costs		(1a)
		Dual variable		
	$p - d = 0$	λ	Quantity balance	(1b)
	$b(d)d = F + c(p)$	γ	Recovery of fixed costs	(1c)

To find the maximum market surplus, we form the Lagrangian:

$$L(d, p, \lambda, \gamma) = \int b(d)\partial d - C(p) - F - \lambda(d - p) - \gamma(b(d)d - F - C(p)) \quad (2a)$$

Substituting p =d from constraint (1b),

$$L(d, \gamma) = \int b(d)\partial d - C(d) - F - \gamma(b(d)d - F - C(d)) \quad (2b)$$

The first order condition for d is

$$\partial L(d, \gamma)/\partial d = b(d) - c(d) - \gamma(b(d) + b'(d)d - c(d)) = 0 \quad (2c)$$

Rearranging,

$$[b(d) - c(d)](1-\gamma) = \gamma b(d)d \quad (2d)$$

Dividing by b(d)(1-γ) and substituting $\lambda^r = b(d)$,

$$[\lambda^r - c(d)]/\lambda^r = \gamma b'(d)d/[(1-\gamma)b(d)] \quad (2e)$$

The elasticity of demand is $(\partial d/\partial d)/d) / (\partial b(d)/\partial(d)/b(d))$. Substituting $e = b(d)/(b'(d)d)$, $\alpha = \gamma/(1-\gamma)$ and $\lambda^r = b(d)$, we have the Ramsey result, often called the inverse elasticity pricing rule,

$$[\lambda^r - c(d)]/\lambda^r = \alpha/e \quad (2f)$$

Ramsey-Boiteux Pricing and Consumer Class Demand Functions. We extend the results to consumer classes.

Additional Notation

d_j is the consumption of the consumer class or consumer j

P is supply; $p = \sum_j d_j$

$b_j(d_j)$ is the inverse demand function for consumer j

F is the fixed costs to be recovered

Max	$MS = \sum_j \int b_j(d_j)\partial d_j - (C(p) + F)$	Maximize market surplus = Consumer value - Supply costs		(3a)
		Dual variable		
	$\sum_j d_j - p = 0$	λ	Quantity balance	(3b)
	$\sum_j b_j(d_j)d_j \geq F + C(p)$	γ	Recovery of fixed costs	(3c)

To find the maximum market surplus, we form the Lagrangian for each j:

$$L(d_j, p) = \sum_j \int b_j(d_j)\partial d_j - C(p) - \lambda(\sum_j d_j - p) - \gamma(\sum_j b_j(d_j)d_j - F - C(p))$$

Let $\partial b_j(d_j)/\partial d_j = b'_j(d_j)$,

$$\partial C(p)/\partial d_j = \partial C(p)/\partial p * \partial p/\partial d_j = c(p), \text{ where } \partial C(p)/\partial p = c(p) \text{ and } \partial p/\partial d_j = 1.$$

The first order condition is

$$\partial L(d_j, p)/\partial d_j = b_j(d_j) - c(p) - \gamma(d_j b'_j(d_j) + b_j(d_j) - c(p)) = 0$$

Rearranging, we obtain

$$\partial L(d_j, p)/\partial d_j = b_j(d_j) - c(p)(1- \gamma)\gamma(d_j b'_j(d_j) + b_j(d_j)) = 0$$

$$[b_j(d_j) - c(p)](1-\gamma) - \gamma d_j b'_j(d_j) = 0$$

Dividing by $b_j(d_j)$ and $(1-\gamma)$ and rearranging, , we obtain

$$[b_j(d_j) - c(p)]/b_j(d_j) = [\gamma/(1-\gamma)]d_j b'_j(d_j)/b_j(d_j)$$

Let $b_j(p_j) = \lambda_j$, $b'_j(p_j) = b'_j$ and $\alpha = \gamma/(1-\gamma)$ and the elasticity of consumer j at p_j be

$$e_j = b_j/p_j b'_j.$$

Ramsey-Boiteux result is

$[\lambda_j - c(p)]/\lambda_j = \alpha/e_j$

Or expressed as two-part pricing:

$\lambda_j = c(p) + \lambda_j(\alpha/e_j)$

The Monopolist. The monopolist maximizes profits by price discrimination. We drop the fixed cost recovery constraint and the problem becomes

Max	$\sum_j b_j(d_j)d_j$ - F - C(p)		Maximize profits	(4a)
		Dual variable		
	$\sum_j d_j - p = 0$	λ	Quantity balance	(4b)

To find the maximum profit, we form the Lagrangian:

$L(d_j, p) = \sum_j b_j(d_j)d_j - C(p)$

Let $\partial b_j(d_j)/\partial d_j = b'_j(d_j)$,

$\partial C(p)/\partial d_j = \partial C(p)/\partial p * \partial p/\partial d_j = c(p)$, where $\partial C(p)/\partial p = c(p)$ and $\partial p/\partial d_j = 1$.

The first order condition is

$\partial L(d_j, p)/d_j = d_j b'_j(d_j) + b_j(d_j) - c(p)) = 0$

Rearranging, we obtain

$\partial L(d_j, p)/\partial d_j = b_j(d_j) - c(p) = d_j b'_j(d_j)$

Dividing by $b_j(d_j)$ and rearranging, , we obtain

$[b_j(d_j) - c(p)]/b_j(d_j) = -d_j b'_j(d_j)/b_j(d_j)$

Let $b_j(d_j) = \lambda_j$, $b'_j(d_j) = b'_j$ and the elasticity of consumer j at p_j be

$e_j = -b_j/d_j b'_j$.

Monopolist's result is: $[\lambda_j - c(p)]/\lambda_j = 1/e_j$

4 Comparison of AIC and Other Pricing Methods

4.1 Introduction

All ISO energy markets have common elements that include a day-ahead market with unit commitment, financial market participants, and hourly balancing; and a real-time market with 5-minute dispatch and pricing. Each ISO has its market design idiosyncrasies. They differ in how or whether they co-optimize energy and reserves, use of residual unit commitment (RUC), make intraday offer adjustments, reserves products offered, and pricing. Current ISO market auctions allow for Non-convex offers with both marginal and avoidable fixed operating costs, such as startup and fixed operating costs. In these markets, marginal cost prices do not ensure offer cost recovery. While the focus of this chapter is on generation, analogous arguments can be made for all resources, such as load.

The chapter starts with an overview and comparison of the circa-2018 pricing methodologies with each other and the AIC approach based primarily on the goals expressed by the Commission and basic economic principles. The chapter then presents a set of numerical examples to illustrate the concepts of each approach to pricing. Following the examples, the chapter presents a single-node, single-period, unit commitment market model and its mathematical properties.

4.2 Pricing Methods Circa 2018

No ISO uses LMP pricing alone. Current ISO pricing techniques fall into four general categories described in Chapter 1. Each ISO methodology has an energy price and a make-whole payment component. The energy price is public information and highly granular in time and place. The individual make-whole payment is not public information and creates a lack of transparency since participants do not know exactly where or when the costs were incurred. The current uplift costs are allocated on a pro-rata and highly aggregated basis to load who may not have caused these costs thereby, dulling the price signal.

4.3 Comparative Analysis.

Table 4.1 provides a high-level comparative analysis of pricing mechanisms.

Table 4.1 Comparison of Current Pricing Methods and the AIC Method.

Attribute	Pricing Options				
	LMP+	RMOL	ELMP	ELMPL	AIC
Revenue adequate	Yes	Yes	Yes	No	Yes
Make-Whole payments	High	< LMP+	< RMOL	same as ELMP	None
Preserves Arbitrage Condition	?	?	?	?	Yes
signal for efficient entry, exit and investment	LMP[1]	> LMP+	> RMOL	same as ELMP	Best
Short-term energy prices	LMP	> LMP	> RMOL	same as ELMP	Best
capacity market prices	Highest	< LMP+	< RMOL	same as ELMP	Lowest
Discriminatory prices	Make-Whole Payments	Make-Whole Payments	Make-Whole Payments	Make-Whole and LOC payments	Ramsey-Boiteux prices
Full transparency of energy prices	No	No	No	No	Best

The Real-Time Market. Even though every ISO has some look-ahead component in its real-time market, all use essentially a single-period model for settlement purposes. The market auction software 'sees' some incremental costs in its dispatch decision. If the sum of a generator's start-up, operating, and marginal costs dispatched is greater than the value received by residual demand facing the generator, it is not dispatched. If a generator is dispatched at its minimum operating level (with the rare exception of degeneracy), the LMP will be lower than both the generator's marginal cost and the AIC. With the exception of the AIC prices, Demand will pay less than the incremental cost of supplying energy. The LMP will fail to send an efficient signal for investment decisions in consumption efficiency, storage, and generation.

Comparison to Current Practice and AIC. Table 4.1 shows the formulation and prices from different single-period energy pricing schemes. The appendix contains the derivations of these properties.

Pricing run	Traditional (LMP+)	Relaxed minimum operating level (RMOL)	Relaxed binary (ELMP)	Average Incremental Cost (AIC)
Simple math formulation	$\min \sum_{i \in G} c_i p_i + c_i^{SU} z_i$ $\sum_{i \in G} p_i = d$ $p_i - z_i p_i^{max} \leq 0$ $-p_i + z_i p_i^{min} \leq 0$ $z_i = z_i^*$	$\min \sum_{i \in G} c_i p_i + c_i^{SU} z_i$ $\sum_{i \in G} p_i = d$ $p_i - z_i p_i^{max} \leq 0$ $-p_i \leq 0$ $z_i = z_i^*$	$\min \sum_{i \in G} c_i p_i + c_i^{SU} z_i$ $\sum_{i \in G} p_i = d$ $p_i - z_i p_i^{max} \leq 0$ $-p_i + z_i p_i^{min} \leq 0$ $0 \leq z_i \leq 1$	$\min \sum_{i \in G} (c_i + c_i^{SU}/p_i^*) p_i$ $\sum_{i \in G} p_i = d$ $p_i - z_i p_i^{max} \leq 0$ $-p_i \leq 0$
energy price (λ)	c_j	$c_j + \beta_j^{min}$	$c_j + c_j^{SU}/p_j^{max}$	$c_j + c_j^{SU}/p_j^*$
make-whole payment	yes	yes	yes	no

where $j \in G$ and is generator that sets the price. λ is the dual variable on the energy balance constraint.

4.4 Pricing Examples

Example 1. AIC Pricing, RMOL and Relaxed Binaries Pricing. In Example 1, we compare AIC pricing to other approaches. We solve a unit commitment model with the generator parameters from Table 4.2 and load of 120 MWs.

Table 4.2 Generator Parameters

Generator	c^{su}	p^{min}	p^{max}	c
GA	100	50	100	20
GB	1000	50	100	10

Table 4.3 shows the Efficient Dispatch and LMP settlement results. Both generators are dispatched. Since GA has the highest marginal cost, it is dispatched to its minimum operating level of 50 MWh. GB with a lower marginal cost is dispatched to 70 MWh and sets the LMP at \$10/MWh. LMP pricing results in a Make-whole Payment for each generator that is charged to load.

Table 4.3 Efficient Dispatch and Settlement at the LMP

Generator	efficient dispatch	LMP payment	cost	Make-whole Payment	Total profit
GA	50	500	1100	600	0
GB	70	700	1700	1000	0
Demand	dispatch	LMP charge	value	Make-whole Payment cost	total cost to load
LA	120	1200	N/A	1600	2800
LMP	10				

In the AIC pricing run, the minimum operating level is relaxed to zero and the AICs replace the marginal cost coefficients. The AIC is based on each generator's dispatch level in the Efficient Dispatch. Table 4.4 shows the AIC calculations. The generator with highest AIC (GB) sets the Locational Incremental Price (LIP) at $24.29/MWh.

Table 4.4 AIC Pricing Calculations

generator	AIC	AIC dispatch
GA	20+100/50 = 22	100
GB	10+1000/70 = 24.29	20
Demand		120
LIP	24.29	

Table 4.5 shows the settlement at the Efficient Dispatch with AIC pricing. There are no Make-whole Payments. We cannot calculate the net value to demand since its value was not given and a fixed demand implies an infinite value.

Table 4.5 The Settlement under AIC Pricing.

generator	efficient dispatch	payment = LIP*dispatch	incremental cost = mc*dispatch + SUC	profit
GA	50	1214.3	1100	114.3
GB	70	1700	1700	0
Total	120	2914.3		
demand	efficient dispatch	charge = LIP*dispatch		value
total	120	2914.3		N/A
LIP	24.29			

Relaxed Minimum Operating Level. We now relax minimum operating level to zero and fix the commitments variables to their optimal solution. Since GB is the lowest cost generator, the pricing run tries to serve all demand from GB. Since total demand exceeds GB's capacity, GB is dispatched to its maximum in the pricing run. The remaining demand, 20 MW, is served from GA. Since GA is now marginal, it sets the price at $20/MWh -- its marginal cost and the highest marginal cost of a generator dispatched. Table 4.6 shows the settlement for GA is $20/MWh*50 MWh = 1000. This settlement does not cover the fixed cost and requires a $100 Make-whole Payment. The settlement for GB is $20/MWh*70 MWh = 1400. This settlement does not cover the fixed cost and requires a $600 Make-whole Payment.

Table 4.6 The Settlement under Relaxed Minimum Pricing.

generator	efficient dispatch	payment = RMOL*dispatch	incremental cost = mc*dispatch+SUC	Make-whole Payment	profit
GA	50	1000	1100	100	0
GB	70	1400	1700	300	0
total	120	2400			
demand	efficient dispatch	charge = RMOL*dispatch		make-whole payment cost	total cost to load
total	120	2400		400	2800
Price	20				

Relaxed Binaries. We now examine relaxed binary pricing or ELMP (they are equivalent for a single period market), where the binary variables are relaxed and bounded between 0 and 1 for generators receiving a Make-whole Payment. Generators' incremental costs under ELMP are computed by effectively averaging their fixed costs over their p^{max}, thus GA's incremental costs for ELMP purposes are \$21/MWh (=20+100/100) and GB's costs are \$20/MWh (=10+1000/100). Since GB is the lowest cost generator, we first try to serve all demand from GB. Since total demand exceeds GB's capacity, GB is dispatched to its maximum in the pricing run and the relaxed binary on GB is 1.0. The remaining demand, 20 MW, is served from GA and the relaxed binary on GA is relaxed to 0.2. The relaxed-binaries LMP or ELMP is \$21/MWh. The pricing run results are in Table 4.7. (The dispatch is displayed here for informational purposes only.) GA is relaxed below its p^{min} and sets the ELMP (the LMP from the relaxed binaries) at \$21/MWh.

Table 4.7 Relaxed Binaries Pricing Results

gen	dispatch	relaxed binaries	p^{min} dual value	p^{max} dual value
GA	20	0.2	0	1
GB	100	1	0	11
total	120			
demand	120			
ELMP = \$21/MWh				

The settlement for GA is \$21/MWh*50 MWh = 1050, see Table 4.8. This settlement does not cover the fixed cost and requires a \$50 Make-whole Payment. The settlement for GB is \$21/MWh*70 MWh = 1470. This settlement does not cover the fixed cost and requires a \$230 Make-whole Payment.

Table 4.8 The Settlement under Relaxed Binaries Pricing.

generator	efficient dispatch	payment = ELMP*dispatch	incremental cost = mc*dispatch+SUC	Make-whole Payment	profit
GA	50	1050	1100	50	0
GB	70	1470	1700	230	0
total	120	2520			
demand	efficient dispatch	charge = ELMP*dispatch		make-whole payment cost	total cost to load
total	120	2520		280	2800
ELMP	21				

Under relaxed binaries pricing or ELMP, changing the maximum operating level changes the ELMP. One would typically not expect p^{max} to change if the unit is offering based on its true operating parameters. In this example, slightly lowering or raising the p^{max} has no effect on the dispatch, but changes the price in the pricing run. Increasing the maximum operating level to 115 does not affect the optimal dispatch, but decreases the ELMP to \$20.87/MWh. Decreasing the maximum operating level to 60 does not affect the optimal dispatch but increases the ELMP to \$21.67/MWh.

A comparison of the three methods is in Table 4.9.

Table 4.9 Comparison of AIC and Relaxed Binary Pricing

	AIC Pricing		Relaxed Binary Pricing		Relaxed Minimum Operating Level	
Generator	Profit	Dispatch	Profit	Dispatch	Profit	Dispatch
GA	114.3	100	-50	20	-100	20

GB	0	20	-230	100	-300	100
Demand	Make-whole charge	Consumers Total cost	Make-whole charge	Consumers Total cost	Make-whole charge	Consumers Total cost
total	0	2914.3	280	2800	400	2800
Price	24.29		21		20	

Example 2. AIC Pricing with Price-Responsive Demand. We add price-responsive demand. The market parameters are in Table 4.10.

Table 4.10 Market Parameters

generator	c^{su}	p^{min}	p^{max}	c
GA	100	50	100	20
GB	1000	50	100	10
Demand	c^{su}	d^{min}	d^{max}	value
LA	0	0	120	200
LB	0	0	20	15

Table 4.11 shows the Efficient Dispatch and LMP settlement. Both generators are dispatched. Since it has the highest marginal cost, GA is dispatched to its minimum operating level of 50 MWh. GB is dispatched to 90 MWh and sets the LMP at $10/MWh.

Table 4.11 Efficient Dispatch and LMP Pricing

generator	efficient dispatch	payment = LMP*dispatch	cost = SU+mc*dispatch	benefit at the LMP
GA	50	500	1100	-600
GB	90	900	1900	-1000
Total	140	1400	3000	-1600
Demand		LMP charge	gross value	net value
LA	120	1200	24000	22800
LB	20	200	300	100
Total	140	1400	24400	22900
LMP	10		Make-Whole Payment	1600

In the pricing run, for generators with a Make-whole Payment, the marginal cost coefficients are set to the AIC and we relax their minimum operating level to zero. GA, which now has the highest AIC of $22/MWh sets the LIP. At an AIC of $22/MWh, LB would not want to consume. We solve this problem in the next section.

Example 3. AIC Pricing with Ramsey-Boiteux Pricing AIC pricing includes a Ramsey-Boiteux price construct as an efficient pricing approach to ensure Non-confiscation of load value. When determining the LIP, it is possible for the bid value, b_i, of a given load to lie between the LMP and the LIP (LMP $< b_i <$ LIP). In this case, the Ramsey-Boiteux price ensures that demand is not charged more than its willingness to pay, which may be lower than the LIP, but higher than the LMP.

In the presence of fixed costs, it is efficient to set prices in proportion to a consumer's demand elasticity or value. Such price setting would not be unduly discriminatory since it would enhance market efficiency and lower the price that demand must pay. Table 4.12 shows the pricing run results. There are no Make-whole Payments and Demand 2 pays its offer value of $15/MWh. The LIP is set by GA at $22/MWh.

Table 4.12. AIC Pricing with Ramsey-Boiteux Pricing Results

generator	AIC	AIC dispatch	Efficient dispatch

GA	22 (= 20+100/50)	40	50
GB	21.11 (= 10+1000/90)	100	90
Demand			
LA		120	120
LB		20	20
LIP	22		

The Ramsey-Boiteux pricing ensures that the efficient load does not face confiscatory prices; that is, it pays more than the value it receives. Table 4.13 shows the settlement. In this example, LA is higher valued than LB, meaning LA is able to pay a subsidy and receive more value.

The LIP is $22/MWh and results in no Make-whole Payment and GA has a zero profit. Demand 2 pays only its value of $15/MWh. LA pays the cost of LB's subsidy payment ($7/MWh*20 MWh = $140), which makes LA's payment $2640 + $140 = $2780 and an average price of $23.16/MWh =$2780/120 MWh. Since LB lowers the AIC, LA is still better off after paying a Ramsey-Boiteux price than if LB left the market. Without LB, the solution would be the same as in Table 4.4 and Table 4.5; assuming that LA in those examples also valued consumption at $200/MWh, LA's net value in that case would be 21085=(($200/MWh-$24.29/MWh)*120 MWh), which is less than its net value in this example.

Table 4.13 The AIC Pricing Settlement

Generator	Efficient Dispatch	payment	cost	profit	
GA	50	1100	1100	0	
GB	90	1980	1900	80	
Total	140	3080	3000	80	
Demand		charge	gross value	net value	Ramsey-Boiteux price $/MWh
LA	120	2780	24000	21220	23.16
LB	20	300	300	0	15
Total	140	3080	24300	21220	
LIP	22				

Example 4. Incremental Analysis of AIC with Price-Responsive Demand. In this example, we step through a dispatch sequence to show how that for a generator to be dispatched, the incremental value must exceed incremental costs. The market parameters are in Table 4.14.

Table 4.14 Market Parameters

demand	marginal value	startup cost	d^{min}	d^{max}	
LA	200	0	0	135	
LB	60	0	0	40	
Generator	marginal cost	startup cost	p^{min}	p^{max}	minimum cost
GA	25	0	130	150	0
GB	55	100	20	30	1220

We solve the dispatch problem. First, we satisfy LA because its value is higher than LB. The least cost way is to dispatch GA to 135 MWh with an LMP of $25. There are 15 MWh of GA at $25 available after satisfying LA. We satisfy 15 MWh of LB with the remaining capacity of GA.

The decision to dispatch GB requires an incremental cost analysis. 25 units of LB that are not satisfied with a marginal value of $60/MWh and a total value of $1500. At a marginal cost of $55/MWh, GB is dispatched because the incremental costs (25 MWh*$55/MWh + $100 = $1475) to satisfy the residual demand are less than the incremental value of $1500 from LB. The AIC of GB is $59/MWh (= $1475/25 MWh) and it sets the LIP.

If the AIC of GB was $57/MWh, the total costs would be $1525 and the average incremental cost would be $61/MWh. Even though the marginal cost of the unit is $55, which is lower than the value for LB of $60, the incremental cost of $1525 is higher than the incremental value of $1500. Therefore, the software will not dispatch GB to satisfy LB. LB sets the LMP at $60/MWh.

At a AIC of $56/MWh, the market is indifferent on dispatching GB because the incremental value equals the average incremental costs of $60/MWh. The results are in Table 4.15.

Since the dispatch decision is based on incremental costs and not marginal costs, AIC pricing is more appropriate than relaxed binaries pricing and consistent with neoclassical economic theory, i.e. convex market pricing.

Table 4.15 Incremental analysis

Demand	incremental value	marginal value
LA	25*60=1500	60
LB		

GB Marginal Cost	incremental costs	AIC	LMP	set by	market surplus
$55/MWh	100 + 25*55 = 1475	59	55	GB	24175
$56/MWh	100 + 25*56 = 1500	60	56	GB	24150
$57/MWh	100 + 25*57 = 1525	61	60	LB	24150

4.5 Conclusion

The AIC pricing approach offers features superior to other pricing approaches. Only the AIC approach satisfies all of the following: the Arbitrage Condition, no Make-Whole Payments, and no lost-opportunity-cost payments. It improves incentives to bid and offer into the energy market; uses price-responsive demand to achieve market efficiency; ensures that all infra-incremental generators make profits; and promotes better decisions in Off-ISO markets.

4.6 APPENDIX: Single-Period, Single-Node Real-Time Market Model

In this section, we formulate the AIC and binary relaxation pricing models. We show that the AIC approach has no Make-whole Payment for Single-Period and Single Node Market Model.

Notation

Sets		index
D	is the index set loads	i
G	is the set of generators	i
G^{-i}	is the set of generators without unit i	i
G^{+}	is set of generators dispatched	i
G^{mp}	is set of generators with Make-whole Payment and in the Efficient Dispatch	
G^{nmp}	is set of generators without Make-whole Payment in the Efficient Dispatch	

Parameters

b_i	is bid price per MWh for load i
c_i	is the marginal offer cost per MWh for generating unit i

c_i^{SU}	is offer start-up cost for generating unit i
c_i^{ai}	$= c_i + c_i^{SU}/p_i^*$;the average incremental cost of generator i
p_i^{max}	is maximum output(demand) of unit i ,
p_i^{min}	is minimum operating level of unit i

Primal Variables (dispatch)

MS	is the market surplus
MS_{-i}	is the market surplus without unit i
MS^{AIC}	is market surplus of the AIC pricing run
MS^{BR}	is market surplus of the binary relaxation pricing run
p_i	is supply from unit i
d_i	is demand by unit i
z_i	is 1 if unit i is operating; 0, otherwise
IC_i	is the incremental costs of unit i

Dual Variables (pricing)

RC	is resource opportunity cost of the dual problem
RC^{AIC}	is resource opportunity cost of the AIC dual problem
α_i^{max}	is marginal value of demand step i
α_i^{ram}	is marginal Ramsey-Boiteux discount of demand step i
β_i^{max}	is marginal value of capacity of generator i
β_i^{min}	is marginal cost of the minimum operating level of generator i
δ_i	is incremental cost of startup of generator i
δ_i^{mp}	is incremental cost of startup of generator i

Optimal Solutions

*	identifies the solution to optimal dispatch model
**	identifies optimal solution to the pricing run model
degeneracy	So as not to complicate the analysis we assume that there is no degeneracy (degeneracy occurs on a set of measure zero).

The Non-Convex Single-Period Auction Market Problem. The non-convex single-period auction market problem is:

$$MS = \max \sum_{i \in D} b_i d_i - \sum_{i \in G} (c_i p_i + c_i^{SU} z_i) \qquad \text{Maximize market surplus} \qquad (1a)$$

dual variable

$$\sum_{i \in D} d_i - \sum_{i \in G} p_i = 0 \qquad \lambda \qquad \text{energy balance} \qquad (1b)$$

demand constraints

$$d_i \le d_i^{max} \quad i \in D \qquad \alpha_i^{max} \qquad \text{maximum demand} \qquad (1c)$$

$$-d_i \le -d_i^{min} \quad i \in D \qquad \alpha_i^{min} \qquad \text{minimum demand} \qquad (1d)$$

generator constraints

$$p_i - z_i p_i^{max} \le 0 \quad i \in G \qquad \beta_i^{max} \qquad \text{maximum generation} \qquad (1e)$$

$$-p_i + z_i p_i^{min} \le 0 \quad i \in G \qquad \beta_i^{min} \qquad \text{minimum generation} \qquad (1f)$$

$$z_i \in \{0,1\} \quad i \in G \qquad \delta_i \qquad \text{z is binary} \qquad (1g)$$

Average Incremental Cost Pricing with Ramsey-Boiteux-Like Pricing. For the average incremental cost (AIC) pricing run, for any unit with a Make-whole Payment, for example, at its minimum operating level, we drop $\boldsymbol{c_i^{SU} z_i}$ and replace $\boldsymbol{c_i}$ with $\boldsymbol{c_i^{AIC} = c_i + c_i^{SU}/p_i^*}$ (the average incremental cost of generator i) and relax the minimum operating level. For generators without a Make-whole Payment we fix the binary variable to its optimal value.

We re-index the generators with a Make-whole Payment where $c_1^{AIC} = \max\{c_i^{AIC} | i \in G^{mp}\}$. We also replace the constraints, $d_i \geq d_i^{min}$, with $d_i \geq d_i^*$, to create a Ramsey-Boiteux-like price. This constraint is a proxy to ensure that lower valued demand from the efficient dispatch still consumes in the pricing run. If a subsidy is needed, it will be reflected in the shadow price of the constraint (α_i^{ram}).

The pricing run model is:

$$MS^{AIC} = \max \sum_{i \in D} b_i d_i - \sum_{i \in G^{nmp}} (c_i p_i + c_i^{SU} z_i^*) - \sum_{i \in G^{mp}} c_i^{ai} p_i \quad (2a)$$

		dual variable	
$\sum_{i \in D} d_i - \sum_{i \in G} p_i = 0$		λ	(2b)
demand constraints			
$d_i \leq d_i^{max}$	$i \in D$	α_i^{max}	(2c)
$-d_i \leq -d_i^*$	$i \in D$	α_i^{ram}	(2d)
constraints for generators with a Make-whole Payment			
$p_i \leq p_i^{max}$	$i \in G^{mp}$	β_i^{max}	(2e)
$p_i \geq 0$	$i \in G^{mp}$		(2f)
constraints for generators without a Make-whole Payment			
$p_i - z_i p_i^{max} \leq 0$	$i \in G^{nmp}$	β_i^{max}	(2g)
$-p_i + z_i p_i^{min} \leq 0$	$i \in G^{nmp}$	β_i^{min}	(2h)
$z_i = z_i^*$	$i \in G^{nmp}$	δ_i	(2i)
p_i, z_i free	$i \in G^{nmp}$		(2j)

Proposition 4.1. In the pricing run, $p^{**}_1 \leq p^*_1$ and G1 will become the marginal generator and set the LIP (λ in the pricing run). The dual of (2) is:

$$RC^{AIC} = \min \sum_{i \in D} (d_i^{max} \alpha_i^{max} - d_i^* \alpha_i^{ram}) + \sum_{i \in G^{nmp}} z_i^* \delta_i + \sum_{i \in G^{mp}} p_i^{max} \beta_i^{max} \quad (3a)$$

		dual variable	
demand constraints			
$\lambda + \alpha_i^{max} - \alpha_i^{ram} \geq b_i$		d_i	(3b)
$\alpha_i^{max}, \alpha_i^{ram} \geq 0$	$i \in D$		(3c)
constraints for generators with a Make-whole Payment			
$-\lambda + \beta_i^{max} \geq -c_i^{AIC}$	$i \in G^{mp}$	p_i	(3d)
$\beta_i^{max} \geq 0$	$i \in G^{mp}$		(3e)
constraints for generators without a Make-whole Payment			
$-\lambda + \beta_i^{max} - \beta_i^{min} = -c_i$	$i \in G^{nmp}$	p_i	(3f)
$\delta_i - p_i^{max} \beta_i^{max} + p_i^{min} \beta_i^{min} = -c_i^{SU}$	$i \in G^{nmp}$	z_i	(3g)
$\beta_i^{max}, \beta_i^{min} \geq 0$	$i \in G^{nmp}$		(3h)
δ_i, λ free			(3i)

Proposition 4.2. In the pricing run (2), if $p_1^{**} < p_1^{max}$, by complementary slackness, $\beta_1^{max**} = 0$. For $i = 1$, (3d) becomes,

$$-\lambda^{**} \geq -c_1^{AIC} \qquad 1 \in G^{mp} \qquad (4a)$$

Since $p_1^{**} > 0$, by complementary slackness, (4a) becomes

$$\lambda^{**} = c_1^{AIC} \qquad 1 \in G^{mp} \qquad (4b)$$

That is, the dual variable on the energy balance constraint is (4b).

Proposition 4.3. The settlement at LIP covers incremental costs and there is no Make-whole Payment:

$$\lambda^{**}p_1^* = c_1^{AIC}p_1^* = (c_1 + c_1^{SU}/p_1^*)p_1^* = c_1p_1^* + c_1^{SU} \qquad (4c)$$

For other generators since $c^{AI}{}_1 \geq c^{AI}{}_i$,

$$\lambda^{**}p_i^* = c_1^{AIC}p_i^* \geq (c_i + c_i^{SU}/p_i^*)p_i^* = c_ip_i^* + c_i^{SU} \qquad i \in G^{mp} \qquad (4d)$$

That is, all generators recover cost and there are no Make-whole Payment.

The Relaxation of the Binary Variables. In this section, we analyze the pricing model by the relaxation of the binary variables with a Make-whole Payment. From (1) the binary relaxation is:

$$MS^{BR} = max \sum_{i \in D} b_i d_i - \sum_{i \in G} (c_i p_i + c_i^{SU} z_i \;) \qquad (5a)$$

dual variable

$$\textstyle\sum_{i \in D} d_i - \sum_{i \in G} p_i = 0 \qquad \lambda \qquad (5b)$$

demand constraints

$$d_i \leq d_i^{max} \qquad i \in D \qquad \alpha_i^{max} \qquad (5c)$$

$$-d_i \leq -d_i^* \qquad i \in D \qquad \alpha_i^{ram} \qquad (5d)$$

constraints for generators without a Make-whole Payment

$$p_i - z_i p_i^{max} \leq 0 \qquad i \in G^{nmp} \qquad \beta_i^{max} \qquad (5e)$$

$$-p_i + z_i p_i^{min} \leq 0 \qquad i \in G^{nmp} \qquad \beta_i^{min} \qquad (5f)$$

$$z_i = z_i^* \qquad i \in G^{nmp} \qquad \delta_i \qquad (5g)$$

constraints for generators with a Make-whole Payment

$$p_i - z_i p_i^{max} \leq 0 \qquad i \in G^{mp} \qquad \beta_i^{max} \qquad (5h)$$

$$-p_i + z_i p_i^{min} \leq 0 \qquad i \in G^{mp} \qquad \beta_i^{min} \qquad (5i)$$

$$z_i \leq z_i^* \qquad i \in G^{mp} \qquad \delta_i \qquad (5j)$$

$$z_i \geq 0 \qquad i \in G \qquad (5k)$$

The dual of (5) is:

$$RC^{BR} = min \sum_{i \in D} d_i^{max} \alpha_i^{max} + \sum_{i \in G} z_i^* \delta_i \qquad (6a)$$

demand constraints — dual variable

$$\lambda + \alpha_i^{max} - \alpha_i^{ram} \geq b_i \qquad d_i \qquad (6b)$$

$$\alpha_i^{max} \geq 0 \qquad i \in D \qquad (6c)$$

constraints for generators without a Make-whole Payment

$$-\lambda + \beta_i^{max} - \beta_i^{min} = -c_i \qquad i \in G^{nmp} \qquad p_i \qquad (6d)$$

$$\delta_i - p_i^{max}\beta_i^{max} + p_i^{min}\beta_i^{min} = -c_i^{SU} \qquad i \in G^{nmp} \qquad z_i \qquad (6e)$$

$$\beta_i^{max}, \beta_i^{min} \geq 0, \delta_i \; free \qquad i \in G^{nmp} \qquad (6f)$$

constraints for generators with a Make-whole Payment

$$-\lambda + \beta_i^{max} - \beta_i^{min} = -c_i \qquad i \in G^{mp} \qquad p_i \qquad (6g)$$

$$\delta_i - p_i^{max}\beta_i^{max} + p_i^{min}\beta_i^{min} \geq -c_i^{SU} \qquad i \in G^{mp} \qquad z_i \qquad (6h)$$

$$\beta_i^{max}, \beta_i^{min}, \delta_i \geq 0 \qquad i \in G^{mp} \qquad (6i)$$

$$\lambda \; free \qquad (6j)$$

When the startup binaries are relaxed in (5j), since the objective function in (5) has $-c_i^{SU}z_i$, z_i is reduced to maximize the relaxed market surplus. It binds in the optimal solution and $\beta_i^{max} \geq 0$. Unless $p_i^{max} = p_i^{min}$, (5i) will not bind and $\beta_i^{min} = 0$. From (6g)

$$-\lambda^{**} + \beta_i^{max**} = -c_i \qquad i \in G^{mp} \qquad (7a)$$

And (6h) by complementary slackness if $z_i^{**} > 0$,

$$\delta_i^{**} - p_i^{max}\beta_i^{max**} + p_i^{min}\beta_i^{min**} = -c_i^{SU} \qquad i \in G^{mp}$$

Since $\beta_i^{min**} = 0$,

$$\delta_i^{**} - p_i^{max}\beta_i^{max**} = -c_i^{SU} \qquad i \in G^{mp} \qquad (7b)$$

Rearranging,

$$\beta_i^{max**} = c_i^{SU}/p_i^{max} + \delta_i^{**}/p_i^{max} \qquad i \in G^{mp} \qquad (7c)$$

Substituting (7c) into (7a), we obtain

$$\lambda^{**} = c_i + c_i^{SU}/p_i^{max} + \delta_i^{**}/p_i^{max} \qquad i \in G^{mp} \qquad (7d)$$

Since z_i is reduced, $z_i < z_i^*$, $\delta_i^{**} = 0$ and

$$\lambda^{**} = c_i + c_i^{SU}/p_i^{max} \qquad i \in G^{mp} \qquad (7e)$$

Proposition 4.3. If $p_i^* < p_i^{max}$, a make-whole payment is necessary. If $p_i^* = p_i^{max}$ (a set of measure zero), a make-whole payment is not necessary. The settlement is

$$\lambda^{**}p_i^* = c_i p_i^* + c_i^{SU}p_i^*/p_i^{max} \qquad i \in G^{mp} \qquad (7f)$$

If $p_i^* < p_i^{max}$, $p_i^*/p_i^{max} < 1$ and

$$\lambda^{**}p_i^* = c_i p_i^* + c_i^{SU}p_i^*/p_i^{max} < c_i p_i^* + c_i^{SU} \qquad i \in G^{mp} \qquad (7g)$$

The settlement does not cover offer costs.

In the relaxed binary settlement, the ELMP does not cover all incremental costs and does not eliminate Make-whole Payments. In the AIC settlement, the LIP covers all incremental costs, and eliminates Make-whole Payments and sends an efficient signal. Even though p_i^{max} does not bind in the optimal dispatch, it affects the ELMP.

Proposition 4.4. The relaxed binary pricing yields the same result in the pricing run:

$$c_i^{RB} = c_i + c_i^{SU}/p_i^{max} \qquad i \in G^{nmp} \qquad (7h)$$

5 Review of Economic Literature

5.1 Introduction

Recent interest in electricity pricing methods has led to several new proposals to change the pricing in US ISOs. In this literature survey, we introduce electricity market pricing. Work analyzing bidder behavior involves substantial theoretical complexity to sufficiently characterize interactions between sophisticated bidders in an already complicated ISO market. Extending this analysis quickly tests the limits of theoretical analysis if researchers begin to consider non-convexities caused by unit commitment (UC), fixed but avoidable costs, and generator minimum output constraints. We find that advances in this direction would be both practically important for the real-world application of new pricing methods. This does not provide an extensive list of all work published about electricity pricing, but provides context to the recent proposals.

5.2 Marginal and Second-Best Pricing

In the middle of last century, the “marginal cost Controversy” divided economists. on how prices should be determined in when average costs are decreasing. When average costs are decreasing, marginal costs are less than average costs and marginal cost pricing falls short of total costs. Some economists including Hotelling (1938), Lerner, Meade and Fleming argued that the price paid for each unit of the product should be the marginal cost and the difference between total costs and total receipts should be paid by general taxation. Other economists including Coase (1946) and Clemens (1941) disagreed. They argued that The Hotelling-Lerner approach had serious implementation and incentive problems and was inferior to a multi-part system of prices. If costs cannot be attributed to individual consumers, a better approach is to allocate these costs to the product’s consumers and not the public or uniformly over all buyers.

Frischmann and Hogendorn (2015) explain why Coase’s position eventually won out, though only in the sense that fixed cost recovery is better achieved through two-part pricing than through general taxation. They point out some of the pricing alternatives that address shortcomings of the two-part pricing framework, noting that whether this is the optimal approach to pricing has remained an open question.

Often this construct is cast as a ‘first best’ vs. ‘second best’. ‘First best’ pricing approaches are used when the assumptions underlying can be met, which requires the fixed costs disappear using the proverbial ‘trap door’ or are paid by someone else, often the government. Having the government absorb some or all fixed costs makes has poor incentive properties and violates the revenue adequacy principle of competitive markets. When standard assumptions cannot be met. the ‘first best’ often fails to be a realistic option and the ‘second best’ becomes the best achievable alternative.

Differential or Discriminatory Pricing. Differential (or discriminatory) pricing charges different prices to different buyers for the same product. It is usually presented in three categories. Personalized pricing (or first-degree price differentiation) is selling to each customer at a different price. Perfect price discrimination results in the supplier(s) appropriating the entire market surplus. Second-degree price differentiation offers a slightly different product or possible quantity discounts for the purpose of price differentiation. Group (or sector) pricing (third-degree price differentiation) divides the market in sectors charging the same price for everyone in each sector. As sectors get smaller, sector pricing approaches personalized pricing. As sectors get larger, sector pricing approaches Ramsey pricing. Typical examples of price discrimination include student and seniors' discounts and seasonal prices. For discriminatory pricing to be successful, the costs of trading in a secondary market must be high enough to inhibit trade.

In 1927, Ramsey presented a uniform pricing scheme (often called the inverse-elasticity pricing rule) for a market that requires taxes or fixed costs to be recovered from the product’s consumers. The pricing scheme requires full knowledge of cost and value functions, where the price paid based on the price elasticity of demand. Without fixed costs, the efficient market requires a uniform market-clearing price where marginal cost equal marginal value. In the presence of fixed costs, the efficient price is above marginal costs. In 1956, Boiteux extended the Ramsey pricing concept to sector prices of French electricity monopoly Électricité de France (EDF). In 1967, Boiteux became CEO of EDF.

Multi-Part Pricing. Two-part or multi-part pricing is a pricing technique where the price of a product or service is composed of two parts, for example, an access charge as well as a per-unit of consumption charge. This approach is often used to price club goods, for example, club dues and greens fees in country clubs. The Mickey Mouse tariff (see Oi) named after an amusement park that charges an entrance fee and may charge additional fee for some rides. Two-part pricing for English electricity goes back at least back to an 1892 proposal (Ng and Weisser 1974). O'Neill et al. (2005) show that the LMP and Make-Whole Payment pricing framework forms a nonlinear (two-part, discriminatory) efficient Price.

One of the main drawbacks to the two-part pricing approach is its inability to signal efficient entry of firms with non-convex costs. Hogan (1992) recognized shortcoming of funding transmission investment solely through LMP pricing.

For load, the optimal policy is that the deviation between Prices and marginal cost should be inversely proportional to the elasticity of demand. Ramsey (1927) proved this result for optimal taxation. Later Boiteux (1956, 1971 for English translation) generalized this result in the field of public utility pricing.

5.3 Electricity Market Foundation and Pricing Proposals

Spot Pricing of Electricity. Under strong assumptions, the total welfare of the economy, as well as that of all individual participants, can be maximized through the decentralized coordination of consumers and producers exchanging goods that are priced at marginal cost. Arrow and Debreu (1954) formalized this idea, but required assumptions that preclude indivisibilities. Scarf (1990, 1994) shows how startup costs or minimum output levels can result in the nonexistence of uniform Market Clearing Prices. Hogan et al. (1992) argued that the physical realities of AC power flow created the need to price transmission access. Harvey et al. (1996) considered bid-based rather than cost-based Dispatch. Under several strong assumptions, they showed that a centrally dispatched, bid-based system would possess the necessary characteristics to support a competitive Market Equilibrium.

While the ISO-based wholesale electricity markets did not develop until the late 1990s and early 2000s, Caramanis et al. (1982) and Schweppe et al. (1988) showed how an electricity pricing system based on the physics of power flow in the transmission network could be used to price electricity.

UK Electricity Reforms. The Electricity Act of 1989 called for the restructuring of the electricity industry, competitive generation markets and a new system of regulation. Green (1998) and Simmonds (2002) a characterization and early criticism of the England and Wales (EW) market. The EW prices had three components: The System Marginal Price (SMP), the Capacity Payment, and Uplift. The sum of the SMP, capacity payment and uplift produce the Pool Purchase Price (PPP) for all consumers. The SMP was about 85% of the PPP and the remainder split about evenly between capacity payment and uplift. A generator's offer price was the (incremental price*output + FOC*duration + startup price)/output. A generator's offer was not required to be its cost although their technical parameters were required to reflect actual operating characteristics. The generator dispatched with the highest offer price sets the SMP. SMP is the price paid for each unit of energy. Since the SMP Pool prices may not be enough to remunerate fully all the plant on the system, a capacity payment is equal to LOLP*(VOLL - SMP) was paid to every MW of capacity that was declared available in certain half-hours. The capacity payment was heavily criticized in part because it is a weak substitute for lack of load bidding. In 2016, Newbery argued that capacity auctions are principally the result of the 'missing money' problem and lead to over-procurement. For several reasons including market power of the generators, an excessively high capacity payment and the lack of locational pricing, the PPP was too high. In contrast, the energy prices in the US ISO energy markets were and are too low.

Pricing and Unit Commitment. Unlike assumptions for convex production capabilities in the early electricity pricing proposals, the SCUC process introduces important indivisibilities into the daily scheduling and pricing of electricity. Johnson et al. (1997) show how minor changes to the SCUC solution with negligible cost impact can significantly affect economic outcomes for individual participants. O'Neill et al. (2005) formalized the standard framework for calculating LMPs and Make-Whole Payments, showing that the system of LMPs with Make-Whole Payments constitutes a Walrasian competitive equilibrium. Sioshansi et al. (2008) replicates these results using efficient branch-and-bound solution techniques instead of Lagrangian Relaxation used by Johnson et al.

Bjørndal and Jörnsten (2008) modify the O'Neill et al. (2005) approach to improve price stability by fixing certain continuous variables, in addition to binary variables, at their optimal value. An extensive comparison of proposals is in Liberopoulos and Andrianesis (2016). Van Vyve (2011) allocates uplift costs to demand and minimizes the maximum uplift charged. O'Neill et al. (2017) calculate system Prices that minimize total uplift subject to incentive and profitability constraints. Motto and Galiana (2004), Araoz and Jörnsten (2011), Ruiz et al. (2012), and Huppmann and Siddiqui (2017) provide incentive compatibility, but do so with more complex modeling techniques.

Convex Hull Pricing (CHP). Gribik et al. (2007) proposed CHP to minimize uplift payments (defined as make-whole plus LOC payments) to provide participants with enough incentive to follow the Efficient Dispatch. Schiro et al. (2016) argue that the exact implementation of CHP comes with a heavy computational burden and that offline units or constraints that are nonbinding in the Efficient Dispatch can affect the price. Also, they argue that there is no widely accepted economic justification for Convex Hull Pricing. Cadwalader (2010) demonstrates that positive Financial Transmission Right (FTR) values in the absence of binding transmission constraints could result in FTR market revenue deficiency that constitutes one of the uplift payments. Ring (1995) and Hogan and Ring (2003) have all pointed out that the precise pricing effects are not well understood.

CHP spawned a series of papers on approximating the convex hull (for example, see Wang et al 2009, 2011, 2013 and 2016). In 2017, Hua and Baldick introduced specific conditions for solving CHP exactly through a linear program (LP), but the approximation does not include dynamic models. In 2018, Borokhov described a modification of CHP in which relaxes the requirement for a convex price-quantity curve while eliminating uplift. Fattahi et al. (2017) propose approximations that are exact under certain conditions.

Other alternative pricing methods include dual pricing (see O'Neill et al, 2016). In 2019, Chao introduced an LP approach to solve CHP with multi-step incremental energy functions. In 2020, Yu et al developed an extended convex hull approximation that can solve multi-interval CHP on MISO day ahead cases with LP approach. CHP discussions are muddled by the approximations, the time taken to compute the approximations (it is generally longer than solving the MIP dispatch problem), the assumptions made, relation to the actual auctions, and the economic properties of the prices.

CHP has two counterintuitive properties. It pays generators that are not part of the efficient dispatch an uplift payment for not self-dispatching and it pays market participants that are dispatched to behave as dispatched. Most auctions penalize market participants for not behaving as the auction results determine. We know of no other auction that does this. Practical pricing algorithms that faithfully produce the Convex Hull Price remain a difficult task and little is known of the properties of the approximations.

5.4 Mechanism Design and Incentive Compatibility

The choice of pricing methodology rests in the field of mechanism design where decision makers analyze how each market design effects the incentives of rational, profit maximizing producers and value maximizing consumers. Often not all the decision maker's goals can be met simultaneously.

Truthful Bidding. Truthful cost revelation is an important goal of market design because it helps ensure that the market outcome is efficient (that is, maximizes the market surplus). Vickrey (1961) introduced an auction design where winning bids do not influence the resulting prices. In an auction for a single item, the highest bidder pays the amount of the second-place bid, and therefore given the assumptions, there is no incentive to bid anything other than a true valuation. Clark (1971) and Groves (1973) generalized this idea widely referred to as the VCG auction mechanism.

While the VCG approach has become the theoretical standard for incentive compatibility in auction design, practical concerns have prevented it from being adopted. Hobbs et al. (2000b) analyze a VCG mechanism for electricity markets and discuss several problems associated including revenue sufficiency, fairness of the discriminatory pricing, and susceptibility to collusion. Ausubel and Milgrom (2006) and Rothkopf (2007) provide more concerns with the VCG auction mechanism.

Bidder Behavior. Borenstein et al. (2008) present evidence that fear of penalties from unclear restrictions was a problem leading up to the California energy crisis in 2000. Wolak and Patrick (2001) describe the use of seemingly benign market rules to gain strategically important information by suppliers in the EW electricity market. Klemperer and Meyer (1989) introduced supply function equilibrium (SFE) where firms compete by submitting individual offer functions. Green and Newbery (1992) applied SFE to electricity market reforms in England and Wales. Baldick et al. (2004) revisits industry reforms in England and Wales using more general functional forms for generator cost functions and constraints. Hortaçsu and Puller (2008) analyze bidding incentives in the Texas electricity market.

Equilibrium Analysis with Transmission Constraints. Oren (1997) uses a Cournot model to show implicit collusion in a two-node network and inefficient dispatch in a three-node network. Both problems can be mitigated through changes in how transmission rights are traded. Day et al. (2002) uses a generalized Cournot model and conjectured supply functions in which suppliers may have incorrect beliefs about the responses of other market participants. The equilibrium analysis of transmission constraints on bidding strategies has been modeled through the solution of non-convex bi-level optimization problems. Cardell et al. (1997) and Hobbs et al. (2000a) propose computational methods to solve for the equilibrium conditions.

Multi-Unit Auctions. Von der Fehr and Harbord (1993) apply a multi-part bid approach to electricity markets. Elmaghraby and Oren (1999) use this approach in discussing incentives and complications caused by UC in electricity auctions. Reguant (2014) analyzes block bidding in the Spanish electricity market as a multi-unit auction that allows firms to explicitly express time-coupled complementarities caused by commitment costs. Meeus et al. (2009) use a multi-unit auction framework to analyze similar block restrictions in European electricity markets and suggest that these mechanisms could see efficiency gains if there were fewer restrictions on supply offers.

Combinatorial Auctions. Xia et al. (2004) and Abrache et al. (2007) review pricing mechanisms proposed for combinatorial auctions that use multi-stage auctions in which participants can update and submit new bids or offers at each stage of the auction. In contrast, practical implementation in electricity markets usually entails a single round with sealed bids in an auction that repeats with sub-hourly or daily frequency. Examples of sealed bid combinatorial auction mechanisms include Rassenti et al. (1982) and O'Neill et al. (2005).

Combinatorial auction pricing mechanisms can roughly be divided into proposals that produce nondiscriminatory (linear) and discriminatory (nonlinear) Prices. When the auction uses linear Prices, that is the total Price for multiple items is the same as the Price for each item individually. When the auction uses discriminatory Prices, each market participant may receive a different Price for the same item. In electricity pricing algorithms, the inclusion of Make-Whole Payments causes discriminatory Prices. Analysis of these pricing mechanisms relies heavily on mathematical optimization theory, such as linear programming duality and nonlinear optimization techniques (see Luenberger and Ye (1984) or Bertsimas and Tsitsiklis (1997)).

Bikhchandani and Ostroy (2002) solve an auction assignment MIP at each iteration of a multi-stage auction and generate Prices using the dual variables of a linear relaxation of the assignment model. Parkes (1999) proposed the iBundle mechanism that generates discriminatory Prices through price update rules based on nonlinear optimization techniques. The dual variables on a linear relaxation of the auction assignment MIP are the Prices. Wurman (1999) proposed the Ascending k-Bundle Auction (AkBA). Porter et al. (2003) propose the Combinatorial Clock (CC) auction that consists of price update rules like the iBundle mechanism and results in linear Prices. Kwasnica et al. (2005) propose Resource Allocation Design (RAD) mechanism Bichler et al. (2009) propose the Approximate Linear Prices (ALPS) mechanism to calculate pseudodual Prices. Non-convexities in the allocation problem can prevent these methods from calculating Prices that separate winning and losing bids at each stage of the auction.

Experimental Economics. In the experimental approach, each participant plays the role of either a buyer or seller and is assigned a cost or value function. Researchers vary the conditions of the market and measure how human participants react to each experimental setting. Van Boening and Wilcox (1996) test a double auction. Sellers have an avoidable fixed cost component and no marginal component. Participants submit a one-part offer Price, which may be either accepted or rejected by participants. The experimental results show erratic bidder behavior with low efficiency and the allocation of market surplus that is unstable. Durham et al. (1996) assign identical production cost functions where buyers are simulated and sellers submit multi-part offers. Winners are selected using MIP. The results show that the ability to offer multi-part bids helps mitigate much of the inefficiency from the Van Boening Wilcox design. Rassenti et al. (2003a) apply this approach in electricity markets and include demand-side bidding that limits supply-side Market Power. Rassenti et al. (2003b) analyze uniform Price and pay-as-offered systems.

While typical combinatorial auction settings differ substantially from electricity markets, they use integer relaxation determine auction-clearing prices in experimental settings. DeMartini et al. (1999) and Porter et al. (2003) use experimental testing to support linear pricing mechanisms in combinatorial auctions which relax the auction assignment MIP to formulate a dual linear program that calculates prices. Scheffel et al. (2011) tests the iBundle, RAD and CC auctions and find that there are only small differences in efficiency between the different mechanisms. These studies find that better efficiencies are achieved when using linear rather than nonlinear discriminatory pricing.

Long-Term Investment Behavior. Herrero et al. (2015) compare electricity-pricing proposals with respect to investment incentives and find equilibrium investment behavior by enumerating all possible investment plans of profit-maximizing firms that earn revenue either through linear Prices or nonlinear Prices. They find that the linear Prices result in investment decisions that are closer to the social optimum, noting that the presence of binary decisions may make it impossible to prove that any set of Prices will correspond to the set of socially optimal investments. Better investment incentives in the linear pricing case is partially explained by stronger incentives for flexible, low marginal cost technologies to enter the market. However, the two equilibrium investment plans had only small differences in efficiency and changing from one pricing rule to another has equity concerns.

5.5 Conclusion

Given the nature of the electricity market, no pricing scheme can meet all competing policy objectives discussed. Full efficiency in electricity markets requires generators submit multi-part cost-based offers and the ISO finds the efficient SCUC satisfying each generator's non-convex operational constraints.

6 Pricing in Multi-Period Markets

6.1 Introduction

In this Chapter, we examine the AIC and several pricing approaches in multi-period multi-product auction markets. When the market auction takes place over multiple periods, the prices and quantities are coupled by ramp rates, startup costs, minimum operating levels, minimum run times, and minimum down times. The latter constraints introduce non-convexities. Minimum operating level limits may prevent the highest marginal cost generator from setting the LMP and cause the need for make-whole payments. When relaxing the minimum operating level in the pricing run, ramp-rate constraints that did not bind in the efficient dispatch run can bind in the pricing run, causing prices that are inconsistent with the efficient dispatch.

The incremental operating cost (IC) of a generator is the total avoidable costs of a startup/shut-down cycle (or simply, a cycle). The AIC for each generator is the marginal cost plus the avoidable fixed costs amortized over the total generation and reserves provided by the resource during the cycle allocated to the time and location in the network where the resource is operating.

The multi-period multi-product market examples may include contingency reserves and ramping 'up' reserves. Contingency reserves cover failures of generators. In addition, these markets have forecasted consumption and weather-dependent generation. Ramping or frequency reserves cover errors in forecast and uninstructed deviations. In addition, we allow price-responsive demand to supply both reserve categories. To simplify the exposition, we do not include the other reserves categories. The AIC approach maintains the arbitrage condition between energy and reserve prices.

To make the presentation easier to follow, we assume that all demand bids have a single marginal value and minimum and maximum purchases. A generator has a minimum operating level, a maximum operating level, minimum run time, and ramp rate constraints.

In multi-period model, the minimum run time (MRT) and minimum operating level constraint, may result in an efficient dispatch that has one or more periods where a generator is committed in the economic dispatch but not needed. If the MRT is a binding constraint, it is likely that there is a period that the unit would otherwise not be needed. With startup costs, a unit may be dispatched in periods where it is not needed to serve periods where it is needed. In these cases, the relaxation of the minimum operating level to 0 causes the pricing run dispatch to result in a zero dispatch for the highest incremental cost generator. In this period, the unit recovers none its allocated fixed costs and causes a make-whole payment. To eliminate a make-whole payment, AIC approach iterates to allocate the fixed cost to the periods where demand caused the resource to be dispatched. This results in improved price signals relative to other pricing methods.

To be an actionable signal, the price must be transparent (available to the public). Make-whole payments are not currently transparent. The announced prices are not a signal to market participants to change their output or consumption from their dispatch signal, but rather should serve as a guide for future decisions. For example, settlement prices may provide a signal of the potential profitability of future imports from outside the ISO, delays in maintenance to capture near term prices, or in the longer term, investment in new resources.

6.2 Multi-Period AIC Examples

Table 6.1 describes each example in this section.

Table 6.1 Example Summary

Description	Conclusion
Example 1 has two-period, one expensive inflexible and one cheaper flexible generator	LIP prices are higher resulting in no make-whole payment; ELMP prices are between LMP and LIP with a make-whole payment
Example 2 is Example 1 with longer minimum run times	the AIC process eliminates make-whole payments
Example 3 is Example 2 with storage	Storage eliminates the need for a generator
Example 4 has three-periods, three-generators and two demand steps	LIP prices in two periods result from incrementally marginal generators, differing from LMP pricing
Example 5 has an eight period, three-generator with two reserve classes.	AIC yields a multi-period prices without make-whole payment payments.
Example 6 is Example 5 with larger load	a binding ORDC yields non-zero reserve prices.
Example 7 has no binding ramp constraints in the dispatch	Removing non-binding ramp constraints in pricing run yields consistent prices.
Example 8 has a ramp constraint in both pricing and dispatch runs	Binding ramp constraints in the pricing run yields consistent prices.

Example 1. Two-Generator Example. This example demonstrates three pricing methods for a simple market with two-generators and two-periods. Each generator has a minimum run time of one period. The generator and load parameters are in Table 6.2 and Table 6.3

Table 6.2 Generator Parameters

Gen	Marginal Cost ($/MWh)	Min Capacity (MW)	Max Capacity (MW)	Min Run Time (h)	Startup Cost ($/start)
GA	10	0	170	1	0
GB	40	50	100	1	200

Table 6.3 Load Parameters

Period	Marginal Value ($/MWh)	Min Demand (MW)	Max Demand (MW)
1	700	0	75
2	900	0	200

The efficient (optimal) solution to the auction is in Table 6.4. GA is cheaper and more flexible than GB, since it has no minimum operating level. GA is be able to fulfill all needed demand in period 1, but does not have enough capacity to meet demand in period 2. GB must be turned on to accommodate demand in period 2. Because it is dispatched at its minimum operating level, it does not set the price. The LMP in both periods is set by GA at its marginal cost of $10/MWh.

Table 6.4 Optimal Dispatch and LMP. Market Surplus is $228,050.

Period	Demand (MW)	GA Dispatch (MW)	GB Dispatch (MW)	LMP ($/MWh)
1	75	75	0	10
2	200	150	50	10

The LMP+ settlement for generation and load is in Table 6.5. Load is charged $10/MWh, a small fraction of the marginal value. Revenue and costs for GB are:

Revenue = (50 MW)*($10/MWh)*(1 h) = $500

Costs = (50 MW)*($40/MWh)*(1 h) + $200/start = $2200

Due to the high costs and low revenue of GB, LMP+ requires a make-whole payment. Load will pay GB an additional payment of $1700, which is the difference of $500 in revenue and $2200 in costs.

Table 6.5 Settlement at an LMP of $10/MWh, the totals include startup costs

	Charge to	Revenue ($)		Costs ($)		Profit ($)	
Period	Load ($)	GA	GB	GA	GB	GA	GB
1	750	750	0	750	0	0	0
2	2000	1500	500	1500	2200	0	-1700
Total	2750	2250	500	2250	2200	0	-1700

AIC Process. GB's make-whole payment is $1700. For GB the average incremental cost of producing power (c_2^{ai}) is calculated as the marginal cost added to the startup cost amortized over the total dispatch:

$c_2^{ai} = \$40/\text{MWh} + (\$200/\text{start})/(50\ \text{MW}) = \$44/\text{MWh}.$

We replace the marginal cost coefficient for GB with c_i^{ai} and set the minimum operating level to zero. The locational incremental price (LIP) resulting from the AIC pricing run is in Table 6.6; the AIC dispatch is shown to provide insight for the prices. The settlement for the AIC pricing is in Table 6.7. GA makes a slightly higher profit and GB breaks even. There are no make-whole payments.

Table 6.6 AIC Pricing Run Results

Period	Demand (MW)	GA Dispatch (MW)	GB Dispatch (MW)	LIP ($/MWh)
1	75	75	0	10
2	200	170	30	44

Table 6.7 Settlement for AIC pricing, the totals include startup costs

	Charge to	Revenue ($)		Costs ($)		Profit ($)	
Period	Load ($)	GA	GB	GA	GB	GA	GB
1	750	750	0	750	0	0	0
2	8800	6600	2200	1500	2200	5100	0
Total	9550	7350	2200	2250	2200	5100	0

The Relaxed Binaries Pricing Run. The settlement and prices using the relaxed binary method are in Table 6.8. The relaxed binary approach produces prices that result in a make-whole payment of $100 to GB. The prices in period 2 are lower than the AIC method at $42/MWh, but still much higher than the LMP. The make-whole payment is reduced, but not eliminated.

Table 6.8 Prices and settlement for relaxed binary approach (ELMP)

	Prices	Charge to	Revenue ($)		Costs ($)		Profit ($)	
Period	($/MWh)	Load ($)	GA	GB	GA	GB	GA	GB
1	10	750	750	0	750	0	0	0
2	42	8400	6300	2100	1500	2200	4100	-100
Total		9150	7050	2100	2250	2200	4100	-100

The three methods are compared in Table 6.9. LIP prices are highest AIC of the units dispatched and needed and eliminate the need to allocate a make-whole payment.

Table 6.9 Settlement Prices for the LMP, LIP and Relaxed-Binaries Approach

Settlement Method	Period 1 ($/MWh)	Period 2 ($/MWh)	Make-Whole Payment ($)
LMP	10	10	1700
LIP	10	44	0
ELMP	10	42	100

Example 2. Example 1 with Longer Minimum Run Time. The generator and load characteristics are the same as those in Table 6.3, except GB's minimum run time is two periods. The optimal dispatch and LMPs are in Table 6.10. Unlike Example 1, GB must now operate in periods 1 and 2, running in each at its minimum operating level. Due to the two-hour minimum run time, GA is backed down in period 1 to accommodate the minimum operating level of GB.

Table 6.10 optimal dispatch and LMP. Market Surplus is $226,550.

Period	Demand (MW)	GA Dispatch (MW)	GB Dispatch (MW)	LMP ($/MWh)
1	75	25	50	10
2	200	150	50	10

The LMP settlement for generation and load is in Table 6.11. Prices are the same as those in Example 1; load is charged $10/MWh. Due to the high costs and low LMP revenue of GB, a make-whole payment is required. The make-whole payment is now higher than in Example 1 since GB is operating for two periods. Demand will pay GB a total of $3200, the difference of $1000 in LMP revenue and $4200 in costs.

Table 6.11 Settlement at an LMP of $10/MWh

	Charge to	Revenue ($)		Costs ($)		Profit ($)	
Period	Load ($)	GA	GB	GA	GB	GA	GB
1	750	250	500	250	2000	0	-1700
2	2000	1500	500	1500	2000	0	-1500
Total	2750	1750	1000	1750	4200	0	-3200

AIC Price Calculation. As in Example 1, GB's average incremental cost of producing power (c_i^{ai}) is the marginal cost added to the startup cost amortized over the total dispatch:

$c_2^{ai} = \$40/\text{MWh} + (\$200/\text{start})/(50\text{ MW} + 50\text{ MW}) = \$42/\text{MWh}$.

The AIC cost replaces the marginal cost coefficient in the cost function and the minimum operating level is set to zero. The AIC dispatch is in Table 6.12.

Table 6.12 optimal dispatch of AIC iteration 1 and LIP.

Period	Demand (MW)	GA Dispatch (MW)	GB Dispatch (MW)	LIP ($/MWh)
1	75	75	0	10
2	200	170	30	42

The settlement for GB at this point is based on the efficient dispatch in Table 6.4 with the prices from Table 6.12, where costs and revenues are calculated as follows.

Revenue = $10/MWh*50 MW + $42/MWh*50 MW = $2600

Cost = $200/start + $40/MWh*(50 MW + 50 MW) = $4200

The make-whole payment for GB is $1600 less than the make-whole payment with LMP+ but not 0. Therefore, another AIC iteration is performed.

Iteration 2. After iteration 1, GB has a residual make-whole payment and is dispatched to zero in the first period. Following the AIC procedure, we calculate the residual costs or total costs less revenue for each period. The residual costs from period 1 are the startup costs less the profit using the LIP from iteration 1 (since the GB was dispatched to 0), less the costs from period 2 (since GB was dispatched above 0).

Residual = $200/start – ($10/MWh–$40/MWh)*50 MW + $40/MWh*50 MW = $3700

The residual demand is the efficient dispatch from period two, since the first iteration of AIC dispatched above zero. We now recalculate the AIC for GB. In period 1, the AIC is $40/MWh. In period two, the AIC is the residual cost divided by the residual demand, $3700/50 MWh = $74/MWh. We run the pricing algorithm again with the new AIC in the objective. The optimal solution is in Table 6.13, and the settlement is in Table 6.14.

Table 6.13 optimal dispatch of AIC iteration 2 and LIP.

Period	Demand (MW)	GA Dispatch (MW)	GB Dispatch (MW)	LIP ($/MWh)
1	75	75	0	10
2	200	170	30	74

Table 6.14. Settlement at AIC iteration 2

	Charge to	Revenue ($)		Costs ($)		Profit ($)	
Period	Load ($)	GA	GB	GA	GB	GA	GB
1	750	250	500	250	2000	0	-1500
2	14800	11100	3700	1500	2200	9600	1500
Total	15550	11350	4200	1750	4200	9600	0

GB is not needed for period 1, but is needed to satisfy demand in period 2 and all residual make-whole payment costs are allocated to period 2. With AIC pricing, the inframarginal generator GA profits and GB the incremental generator in period 2 breaks even without the need for a make-whole payment.

The Relaxed Binaries Pricing (ELMP) Run. The startup cost of GB is ratably allocated over the minimum run time, which is now two intervals. Thus, for each interval startup cost for GB is now $100. The relaxed binary approach produces prices that result in a small make-whole payment of $1700 to GB. Like the two other approaches, the price in period 1 is $10/MWh. In period 2 the price increases to $57/MWh, lower than the LIP but much higher than the LMP of $10/MWh. The settlement and prices using this method are in Table 6.15.

Table 6.15 Prices and settlement for relaxed binary approach (ELMP)

	Prices	Charge to	Revenue ($)		Costs ($)		Profit ($)	
Period	($/MWh)	Load ($)	GA	GB	GA	GB	GA	GB
1	10	750	250	500	250	2100	0	-1600
2	41	8200	6150	2050	500	2100	6150	-50
Total	-	8950	6400	2550	750	4200	7050	-1650

Prices and make-whole payment across the three methods are in Table 6.16.

Table 6.16 Settlement prices for the LMP, LIP and relaxed binaries approach

Settlement Price	Period 1 ($/MWh)	Period 2 ($/MWh)	Make-Whole Payment ($)
LMP	10	10	3200
LIP	10	74	0
ELMP	10	41	1650

Example 3. Addition of Storage. From Example 2, we add a storage unit to the market with a charge and discharge capacity of 40 MWh and no losses. The storage device can discharge or charge to its full capacity in one period with a marginal cost of zero. Anytime it charges, it will pay the market price, and anytime it discharges, it will receive the energy price. Although an actual storage device would involve further complicating factors (efficiency loses and cycling costs), this example demonstrates the ability of a highly flexible device to absorb some of the inflexibilities of thermal generators. The generator and load parameters are in Table 6.17 and Table 6.18.

Table 6.17 Generator Parameters

Gen	Marginal Cost ($/MWh)	Min Capacity (MW)	Max Capacity (MW)	Min Run Time (periods)	Startup Cost ($/start)
A	10	0	170	1	0
B	40	50	100	2	200
Storage	0	-40	40	0	0

Table 6.18 Load Parameters

Period	Marginal Value ($/MWh)	Demand (MW)
1	700	75
2	900	200

The optimal solution is in Table 6.19 and settlement in Table 6.20. GB is not needed because storage is able to charge in the first period and discharge in the second, allowing more power from the cheaper GA. GA sets the price in both periods, and since it does not have startup costs, it breaks even and makes no profit. Storage must pay $300 in the first period and subsequently receives $300 in the second period. It also breaks even over the two periods.

Table 6.19 optimal dispatch and LMP.

Period	Demand (MW)	GA Dispatch (MW)	GB Dispatch (MW)	Storage Charge (MW)	Storage Discharge (MW)	State of Charge (beginning of period)	LMP ($/MWh)
1	75	105	0	30	0	0	10
2	200	170	0	0	30	30	10

Table 6.20 Settlement for LMP pricing, the totals include startup costs

	Revenue ($)			Costs ($)			Profit ($)		
Period	GA	GB	Storage	GA	GB	Storage	GA	GB	Storage
1	1050	0	-300	1050	0	0	0	0	-300
2	1700	0	300	1700	0	0	0	0	300
Total	2750	0	0	2750	0	0	0	0	0

Example 4. Example with Three Periods. In this example, we extend the market to include three periods, three generators, and two loads (or demand steps). The new generator and load characteristics are in Table 6.21 and Table 6.22.

Table 6.21 Generator Parameters

Gen	Marginal Cost ($/MWh)	Min Capacity (MW)	Max Capacity (MW)	Min Run Time (periods)	Startup Cost ($/start)
A	10	10	75	1	200
B	20	25	40	1	10
C	50	10	20	1	5

Table 6.22 Load Parameters

	Demand 1		Demand 2	
Period	maximum in MW	value in $/MWh	maximum in MW	value in $/MWh
1	50	100	20	90
2	60	100	35	90
3	70	100	50	90

The optimal dispatch and LMPs are in Table 6.23. The generator settlement is in Table 6.24, and demand value is in Table 6.25. GA sets the LMP in periods 1 and 2, while GB sets the LMP in period 3. Due to lower prices in the first two periods, GB and GC require a make-whole payment of $260 and $305 respectively. Demand value is positive for all periods.

Table 6.23 optimal dispatch and LMP, market surplus = $23,705

Period	GA Dispatch (MW)	GB Dispatch (MW)	GC Dispatch (MW)	Demand 1 (MW)	Demand 2 (MW)	LMP ($/MWh)
1	70	0	0	50	20	10
2	70	25	0	60	35	10
3	75	35	10	70	50	20

Table 6.24 Settlement for LMP pricing, the totals include startup costs

	Revenue ($)			Costs ($)			Profit ($)		
Period	GA	GB	GC	GA	GB	GC	GA	GB	GC
1	700	0	0	900	0	0	-200	0	0
2	700	250	0	700	510	0	0	-260	0
3	1500	700	200	750	700	505	750	0	-305
Total	2950	950	200	2400	1210	505	550	-260	-305

Table 6.25 Demand net value and payment (before make-whole payment)

	Payment ($)		Net Value ($)	
Period	Demand 1	Demand 2	Demand 1	Demand 2
1	500	200	4500	1600
2	600	350	5400	2800
3	1400	1000	5600	3500
Total	2500	1550	15500	7900

AIC Process. Iteration 1. GB and GC each require a make-whole payment, and the AIC cost coefficients are:

$c_2^{ai} = \$20/\text{MWh} + (\$10/\text{start})/(20\text{ MW} + 35\text{ MW}) = \$20.18/\text{MWh}$

$c_3^{ai} = \$50/\text{MWh} + (\$5/\text{start})/(10\text{ MW}) = \$50.50/\text{MWh}$

The AIC pricing model is run. The LIPs and settlement are in Table 6.26. The AIC settlement has no make-whole payment; the infra-incremental (infra-marginal in convex markets) generators make a profit, and the incremental (marginal in convex markets) generators makes no profit. The analogy to convex markets is the

LMP settlement has no make-whole payment, the infra-marginal generators make a profit, and the 'marginal' generator makes no profit.

Table 6.26 LIP Prices and LIP Settlement

Period	LIP ($/MWh)	Generator	Revenue ($)	Costs ($)	Profit ($)
1	10.00	GA	5900.10	2350	3550.10
2	20.18	GB	2271.70	1210	1061.70
3	50.50	GC	505.00	505	0

If there is a generator with an AIC lower than $50.50/MWh and a feasible dispatch at 10MW, it would have been dispatched instead of GC. If there is a generator with a marginal cost less than $10/MWh in periods t = 1 or 2 and feasible dispatch in (0, p_{At}^*], it would have been dispatched. If there is a generator with a marginal cost less than 20 in period 3 feasible dispatch in (0, p_{B3}^*], it would have been dispatched. Due to these non-convexities, there is more than one margin.

The Relaxed Binaries Pricing Run. The prices and settlement resulting from the relaxed binary pricing model are in Table 6.27. The prices are lower compared to AIC prices, resulting in a make-whole payment for GC.

Table 6.27 Settlement for ELMP pricing

Period	Price ($/MWh)	Generator	Revenue ($)	Costs ($)	Profit ($)
1	10.00	GA	5886.00	2350	3536.00
2	20.25	GB	2265.00	1210	1055.00
3	50.25	GC	502.50	505	-2.50

The AIC approach eliminates make-whole payments, but the relaxed binary approach does not. Table 6.28 shows a comparison of each pricing method.

Table 6.28 Settlement prices for the LMP, LIP and relaxed binaries approach

Settlement Price ($/MWh)	Period 1	Period 2	Period 3	Make-Whole Payment ($)
LMP	10.00	20.00	20.00	565.00
LIP	10.00	20.18	50.50	0
ELMP	10.00	20.25	50.25	2.50

Example 5. Dynamic Pricing in Light Load. We explore incremental cost pricing using an example modified from a MISO example with 8 periods (see MISO 2014). The example has 3 generators, GA, GB and GC with parameters shown in Table 6.29. GB and GC are fast start units and can provide non-spinning reserves without starting up. Demand is valued at $1000/MWh and its maximum levels are in

Table 6.29 Generator Parameters

Generator	Start-Up Cost ($)	Fixed Cost ($/Hr)	Min Level (MW)	Max Level (MW) Step 1	Max Level (MW) Step 2	Min Run Time (Hr)	Ramp Rate (MW/min)	Marginal Cost ($/MWh) Step 1	Marginal Cost ($/MWh) Step 2
GA	900	1	200	1000	200	1	1	30	40
GB	600	1	50	60	20	1	50	50	75
GC	360	1	25	40	10	1	50	60	85
total			275	1100	230				

Table 6.30. Demand by Period

period	1	2	3	4	5	6	7	8
maximum demand in MW	850	880	910	955	970	980	990	940

There two types of reserves: spinning and non-spinning. The sum of the two are operating reserves with an operating reserves demand curve/function (ORDC). The first quantity step on the ORDC is 10 percent of demand. As reserves increase beyond the first step, they still have value, for example, in a double contingency or greater. The second through sixth steps of the function are in Table 6.31. The maximum amount of valued reserves cleared in reserves market results are called 'ORDC-reserves'. The reserves marginal price (RMP) is the intersection of the ORDC and the cost of operating reserves. Spinning reserves are set at 90 percent of the first step on the ORDC. Beyond the last step of the ORDC, reserves have no value.

Table 6.31 Values of Steps of the Operating Reserves Demand Function

Step	1	2	3	4	5	6
Step Size	10% of Load	20	20	20	20	20
Value ($/MWh)	820	80	40	20	10	5

For reference, we define the following acronyms used in different pricing methods.

Name	Description
LMP	Dual variable on the energy balance in the efficient dispatch
LMPR	Dual variable on the reserve requirements in the efficient dispatch
LMPS	Dual variable on the spinning reserve requirements in the efficient dispatch
LIP	Dual variable on the energy balance in the AIC pricing run
LIPR	Dual variable on the reserve requirements in the AIC pricing run
LIPS	Dual variable on the spinning reserve requirements in the AIC pricing run

Efficient Dispatch. We also assume that generators offer incremental costs and load bids incremental values, so that the optimal dispatch is efficient. The dispatch including reserves, the LMPs and reserve prices are in Table 6.32. All energy comes from GA. GB and GC do not start up, but supply non-spinning operating reserves of 130 MW. The LMP is $30/MWh in each period. The excess supply of operating and spinning reserves results in zero prices for these products.

Table 6.32 Efficient Dispatch (in MW) and Marginal Cost Prices (in $/MWh)

period	1	2	3	4	5	6	7	8
Consumption	850	880	910	955	970	980	990	940
total generation	850	880	910	955	970	980	990	940
generation-GA	850	880	910	955	970	980	990	940
generation-GB	0	0	0	0	0	0	0	0
generation-GC	0	0	0	0	0	0	0	0
fast reserves	130	130	130	130	130	130	130	130
spin-reserves	77	79	82	86	87	88	89	85
total reserves	480	450	420	375	360	350	340	390
ORDC-reserves	185	188	191	196	197	198	199	194
LMP	30	30	30	30	30	30	30	30
LMPR	0	0	0	0	0	0	0	0
LMPs	0	0	0	0	0	0	0	0

Table 6.33 presents the market surplus, make-whole payments, LMP surplus of market participants and the non-linear surplus with make whole-costs included. Since GA sets the LMP at its step 1 marginal cost, the

LMP settlement does not cover its startup and fixed operating costs, causing a make-whole payment of $908. Demand receives 91.9% of the market surplus. Using prices from Table 6.31 and quantities from Table 6.32, the value of reserves is can be calculated as 8% of the market surplus with a zero cost.

Table 6.33 Marginal Cost Pricing Results

market surplus = $7,887,592; make-whole payment = $908

reserves value total= $637750 reserve %= 8.085 reserves cost total= $0

	buyer	total gen	GA	GB	GC
LMP surplus ($)	7250750	-908	-908	0	0
surplus %	91.926	-0.012	-0.012	0	0
LMP+ surplus ($)	7249842	0	0	0	0
surplus %	91.903	0	0	0	0

Average Incremental Cost Pricing. We allocate avoidable fixed costs to the energy output and reserves, and add them to marginal costs. The average incremental costs are in Table 6.34.

Table 6.34 Average Incremental Costs for Example 5 (in $/MWh)

	energy AIC step 1	energy AIC step 2	spinning reserves AIC	non-spinning reserves AIC
GA	30.09	40.09	0.09	0.09
GB	50	75	0	0
GC	60	85	0	0

We relax the minimum operating levels for generators with make-whole payment and run the AIC pricing algorithm to obtain the locational incremental prices (LIPs). Table 6.35 presents the pricing run results. The LIPs are $30.09/MWh, a $.09 increase in all periods and as a result, there are no make-whole payments. The spinning prices are $.09/MWh and operating reserves prices are zero.

Table 6.35 Pricing Run Results Using Average Incremental Costs

period	1	2	3	4	5	6	7	8
Demand	850	880	910	955	970	980	990	940
total generation	850	880	910	955	970	980	990	940
generation-GA	850	880	910	955	970	980	990	940
generation-GB	0	0	0	0	0	0	0	0
generation-GC	0	0	0	0	0	0	0	0
fast reserves	130	130	130	130	130	130	130	130
spin-reserves	77	79	82	86	87	88	89	85
total reserves	207	209	212	216	217	218	219	215
ORDC-reserves	185	188	191	196	197	198	199	194
LIP	30.09	30.09	30.09	30.09	30.09	30.09	30.09	30.09
LIPR	0	0	0	0	0	0	0	0
LIPS	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09

Table 6.36 presents the market surplus, make-whole payments, AIC surplus of market participants The settlement quantities are unchanged from the SCED. GA breaks even.

Table 6.36 Surplus using Average Incremental Cost Pricing Run Results

	Buyer surplus	total gen profit	gen A profit	gen B profit	gen C profit
LIP surplus ($)	7250043	0	0	0	0

surplus-%	91.92	0	0	0	0

Example 6. Dynamic Pricing in Heavier Load. We increase maximum consumption for each period as shown in Table 6.37. The value of demand remains $1000/MWh.

Table 6.37 Maximum Demand by Period (MW)

Period	1	2	3	4	5	6	7	8
Demand maximum	940	980	1000	1030	1070	1100	1130	1090

The optimal dispatch and LMPs are in Table 6.38. For periods 1 through 6, the LMP reflects that the first or second step of GA's cost function is marginal. In period seven, the highest load period, GC is started up and dispatched at its minimum operating level. From Table 6.31 total operating reserves needed are 10% of load plus 100 MW. However, in period 7,GA's total capacity (1330 MW) is insufficient to cover both demand (1130 MW) and the non-zero portion of the ORDC demand function (213 MW). The ORDC clears operating reserves at step 6 at $5/MWh. The total cost of reserves is then $5/MWh*200 MW = $1000. The LMP is $45/MWh. It is comprised of the marginal cost of $40/MWh and an opportunity cost of $5/MWh for being on reserve. At $45/MWh, GA is indifferent to being dispatched for energy or reserves. In period 7, GC is chosen over GB even though GC has higher marginal cost because GC has lower total incremental costs, including those of being committed and dispatched to meet demand.

Table 6.38 Marginal Cost Pricing (in $/MWh) and Efficient Dispatch (in MW)

Period	1	2	3	4	5	6	7	8
Demand	940	980	1000	1030	1070	1100	1130	1090
total generation	940	980	1000	1030	1070	1100	1130	1090
generation-GA	940	980	1000	1030	1070	1100	1105	1090
generation-GB	0	0	0	0	0	0	0	0
generation-GC	0	0	0	0	0	0	25	0
fast reserves	130	130	130	130	130	130	80	130
spin-reserves	85	88	90	93	96	99	102	98
total reserves	390	350	330	300	260	230	200	240
ORDC-reserves	194	198	200	203	207	210	200	209
LMP	30	30	40	40	40	40	45	40
LMPR	0	0	0	0	0	0	5	0
LMPS	0	0	0	0	0	0	0	0

Table 6.39 presents the market surplus, make-whole payments, linear or LMP surplus of market participants and the non-linear surplus with make whole-costs included. Demand receives 91.2% of the market surplus. The net value of reserves (value less cost of reserves) is 8.1% of the market surplus, while net generator profit is 0.7%.

Table 6.39 marginal cost Pricing Results from the SCED

market-surplus= $8,792,446; make-whole payment = -$611

reserves-value-total= $708,615; reserve-%= 8.059; reserves-cost-total= -$1000

	Buyer surplus	total gen profit	GA profit	GB profit	GC profit
linear (LMP) ($)	8,019,950	64,881	65,092	400	-611
surplus-%	91.214	0.738	0.74	0.005	-0.007
nonlinear (LMP+MWP) ($)	8,019,339	65,492	65,092	400	0
surplus-%	91.2	0.745	0.74	0.005	0

In an LMP settlement, GA receives profit of $65,092. GB receives profit of $400 for supplying non-spinning operating reserve. For GC, the LMP and LMPR settlement does not cover incremental costs, requiring a make-whole payment of $611. For GC, components of the make-whole payment calculation are in Table 6.40:

Table 6.40 The Make-Whole Payment Calculation for GC

45*25	+5*25		-60*25	-1	
1125	+125	-360	-1500	-1	=-611
revenue from energy LMP*minimum operating level	revenue from operating reserves LMPR*operating reserves	startup costs	Cost at the minimum operating level	Fixed operating cost	Make-whole payment

AIC Pricing Iteration 1. The average incremental costs are in Table 6.41.

Table 6.41 Average Incremental Costs for Iteration 1 ($/MWh)

	energy AIC step 1	energy AIC step 2	spinning reserves AIC	fast start reserves AIC
GA	30.09	40.09	0.09	0.09
GB	50.00	75.00	0	0
GC	60.90	85.90	0.90	0.90

Table 6.42 presents the LIP pricing results from the AIC pricing procedure. GA's average incremental cost add $.09/MWh to the AIC energy price (LIP) in periods 1 through 5. GC's average incremental cost adds $.90/MWh to the marginal prices in period 6 and 8. The ORDC sets the LIPR in period 7.

Table 6.42 Pricing Run Results Using AIC for Iteration 2 (in $/MWh).

period	1	2	3	4	5	6	7	8
LIP	30.09	30.09	40.09	40.09	40.09	40.90	45.00	40.90
LIPR	0.09	0.09	0.09	0.09	0.09	0.90	5.00	0.90
LIPS	0	0	0	0	0	0	0	0

Table 6.43 presents the LIP surplus and make-whole payments. The market surplus and value of reserves remain the same. The LIP settlement does not cover startup and fixed operating costs for GC, but has a reduced its make-whole payment from 611 to 497. Another iteration of the AIC procedure is required.

Table 6.43 Average incremental Cost Pricing Results Iteration 1.

market-surplus= $8,792,446 make-whole payment = $497

reserves-value-total= $708,615 reserve-%= 8.059 reserves-cost-total= $1578

	buyer surplus	total gen profit	GA profit	GB profit	GC profit
LIP surplus ($)	8,017,499	67,911	67,825	582	-497
surplus-%	91.186	0.772	0.771	0.007	-0.006

AIC procedure Iteration 2. In the AIC iteration run in periods where GC is not needed for energy, it is dispatched at 0 MW. We reallocate the fixed costs from periods where the GC is not needed (that is, with 0 MW dispatched in the pricing run) to periods where it is needed. The results are in Table 6.44.

Table 6.44 Average Incremental Costs (in $/MWh) for Iteration 2

	energy AIC step 1	energy AIC step 2	spinning reserves AIC	fast-start reserves AIC
GA	30.09	40.09	0.09	0.09
GB	50.00	75.00	0	0
GC	65.70	90.70	5.70	5.70

We run the AIC pricing procedure to obtain the LIPs, LIPRs and LIPSs. Table 6.45 presents the results. GA's average incremental cost adds $.09/MWh to the marginal price. In periods 1 through 5, the LIPs increase by $.09. GC's average incremental cost set the prices in periods 6 and 7. In period 8, step 6 of the ORDC sets the price for the LIPR, and the sum of the marginal cost of GC ($40/MWh) plus the opportunity cost of remaining on reserve ($5/MWh) yields the LIPS in period 8.

Table 6.45 AIC Pricing Run Results for Iteration 2 (in $/MWh)

period	1	2	3	4	5	6	7	8
LIP	30.09	30.09	40.09	40.09	40.09	45.70	45.70	45.00
LIPR	0.09	0.09	0.09	0.09	0.09	5.70	5.70	5.00
LIPS	0	0	0	0	0	0	0	0

Table 6.46 presents the LIP surplus, profits and make-whole payments. The LIP settlement does not cover startup and fixed operating costs for GC, but has a reduced its make-whole payment from $497 to $17.

Table 6.46 Average incremental Cost Pricing Results Iteration 2.

market-surplus= $8,792,446 make-whole payment = $17

reserves-value-total= $708,615 reserve-%= 8.059 reserves-cost-total= $3805

	buyer surplus	total genprofit	gen A profit	gen B profit	gen C profit
LIP surplus ($)	8,006,967	80,669	79,337	1,350	-17
surplus-%	91.066	0.917	0.902	0.015	0

AIC procedure Iteration 3. We reallocate the fixed costs from periods where the generator is not needed to periods where it is needed. The results are in Table 6.47.

Table 6.47 Average Incremental Costs for Iteration 3 (in $/MWh)

	energy AIC step 1	energy AIC step 2	spinning reserves AIC	fast-start reserves AIC
GA	30.09	40.09	0.09	0.09
GB	50.00	75.00	0	0
GC	65.93	90.93	5.93	5.93

We run the AIC pricing procedure to obtain the LIPs. Table 6.48 presents the pricing results. GC's average incremental cost adds $.90/MWh to the marginal price. In periods 6 and 7, the LIPs increase by $5.93.

Table 6.48 AIC Pricing Run for Iteration 3 (in $/MWh)

period	1	2	3	4	5	6	7	8
LIP	30.09	30.09	40.09	40.09	40.09	45.93	45.93	45.00
LIPR	0.09	0.09	0.09	0.09	0.09	5.93	5.93	5.00
LIPS	0	0	0	0	0	0	0	0

Table 6.49 presents the LIP surplus and make-whole payments. The LIP settlement covers the startup and fixed operating costs for GC and has reduced GC's make-whole payment from 17 to 0 with a profit of $6. After

three iterations of the AIC procedure using fixed cost allocation principles, the make-whole payment is zero and prices are fully transparent.

Table 6.49 Average Incremental Cost Pricing Results Iteration 3

market-surplus= $8,792,446 make-whole payment = 0

reserves-value-total= $708,615 reserve-%= 8.059 reserves-cost-total= -$3,905

	Buyer surplus	Total gen profit	GA profit	GB profit	GC profit
LIP surplus ($)	8,006,447	81,289	79,896	1,387	6
surplus-%	91.061	0.925	0.909	0.016	0

Example 7. Dynamic Pricing with Binding Ramp Rates. Example 7 demonstrates the effect on prices when ramp rate constraints are not active in the SCED, but active in the pricing run. The maximum demand at a value of $1000/MWh is in Table 6.50.

Table 6.50 Maximum Demand by Period.

period	1	2	3	4	5	6	7	8
Maximum Demand (MW)	940	980	990	1055	1085	1100	1130	1040

Efficient Dispatch (SCUC). The dispatch, LMP and settlement results are in Table 6.52. The increase in load from period 3 to 4 is 65 MW. The ramp rate limit for GA is 60 MW/hour. Therefore, GA cannot satisfy the increase in load. In period 4, GC is committed and dispatched at its minimum operating level and the ramp rate constraint on GA does not bind. In period 7, GC is dispatched at its minimum operating level to satisfy the spinning reserves requirement.

In each period, GA is the marginal generator and sets the LMP at its marginal cost. In period 7, the ORDC sets the reserves price and as described before, in order for GC to remain indifferent between generating and providing reserves requires that the sum of the marginal cost of GC ($40/MWh) plus the opportunity cost of remaining on reserve ($5/MWH) yields the LMP of $45 in period 7.

Table 6.51. Efficient Dispatch (in MW) and Marginal Cost Prices (in $/MWh)

period	1	2	3	4	5	6	7	8
consumption	940	980	990	1055	1085	1100	1130	1040
total generation	940	980	990	1055	1085	1100	1130	1040
generation-GA	940	980	990	1030	1085	1100	1105	1040
generation-GB	0	0	0	0	0	0	0	0
generation-GC	0	0	0	25	0	0	25	0
fast reserves	130	130	130	80	130	130	80	130
spin-reserves	85	88	89	95	98	99	102	94
total reserves	390	350	340	275	245	230	200	290
ORDC-reserves	194	198	199	206	209	210	200	204
LMP	30	30	30	40	40	40	45	40
LMPR	0	0	0	0	0	0	5	0
LMPS	0	0	0	0	0	0	0	0

Table 6.52 presents the market surplus and make-whole payments, linear or LMP surplus of market participants and the non-linear surplus with make whole-costs included. The LMP settlement does not cover startup and no-load costs for GC causing make-whole payments of $1,472.

Table 6.52 Marginal Cost Pricing Results

market-surplus= $8,770,645 make-whole payment = $1,472 reserves-value-total= $706,975 reserve-%= 8.061 reserves-cost-total= $1000

	Buyer surplus	total gen profit	GA profit	GB profit	GCprofit
LMP	8,010,650	54,020	55,092	400	-1,472
surplus-%	91.335	0.616	0.628	0.005	-0.017
LMP+	8,009,178	55,492	55,092	400	0
surplus-%	91.301	0.649	0.628	0.005	0

GC is started up and shutdown in periods 4 and 7. In period 4, the make-whole payment is

40*25 =	+0	-360	-60*25 =	-1	=-861
1000	+0	-360	-1500	-1	=-861
LMP*(min operating level)	RMP* (operating reserves)	startup costs	minimum operating level cost	Fixed operating cost	Make-whole payment

In period 7, the make-whole payment is

45*25	+5*25		-60*25	-1	
1125	+125	-360	-1500	-1	=-611
LMP*(min operating level)	RMP*(operating reserves)	startup costs	minimum operating level cost	Fixed operating cost	Make-whole payment

GC's make-whole payments over the entire horizon are: $861+$611 = $1,472.

AIC Pricing with Binding Ramp Rates. Table 6.53 shows the AICs.

Table 6.53 Average Incremental Costs for Iteration 1 (in $/MWh)

	energy AIC step 1	energy AIC step 2	spinning reserves AIC	fast-start reserves AIC
GA	30.09	40.09	0.09	0.09
GB	50	75	0	0
GC	61.81	86.81	1.81	1.81

In the pricing run, we do not relax the ramp rate constraints that did not bind in the efficient dispatch. The results are in Table 6.55. The energy price is $8.38/MWh in period 3 and $61.81/MWh in period 4 caused by GA's binding ramp rate constraint that did not bind in the dispatch. The prices are inconsistent with the dispatch.

Table 6.54 AIC Pricing Run with Binding Ramp Rates, Iteration 1

period	1	2	3	4	5	6	7	8
demand	940	980	990	1055	1085	1100	1130	1040
total-generation	940	980	990	1055	1085	1100	1130	1040
generation-GA	940	980	990	1050	1085	1100	1130	1040
generation-GB	0	0	0	0	0	0	0	0
generation-GC	0	0	0	5	0	0	0	0
fast reserves	80	80	80	80	94	110	80	80
spin-reserves	85	88	89	95	98	99	102	94
total reserves	194	198	199	206	209	210	200	204
ORDC-reserves	194	198	199	206	209	210	200	204
LIP	30.09	30.09	8.38	61.81	41.81	41.81	45.00	40.09
LIPR	0.09	0.09	0.09	0.09	1.81	1.81	5.00	0.09
LIPS	0	0	0	0	0	0	0	0

AIC Pricing with Relaxed Ramp Rates Iteration 1. The derived AICs are in Table 6.55. When we drop the ramp rate constraint in the pricing run, AIC pricing run becomes time-decoupled, that is, equivalent to eight separate optimization problems. The price anomaly between periods 3 and 4 vanishes, shown in Table 6.56.

Table 6.55 Average Incremental Costs Iteration 1(in $/MWh)

	energy AIC step 1	energy AIC step 2	spinning reserves AIC	fast start reserves AIC
GA	30.10	40.10	0.10	0.10
GB	50.00	75.00	0.00	0.00
GC	67.17	92.17	7.17	7.17

Table 6.56. Example 7 AIC Pricing Run Results with Relaxed Ramp Rate, Iteration 1.

period	1	2	3	4	5	6	7	8
demand	940	980	990	1055	1085	1100	1130	1040
demand2	0	0	0	0	0	0	0	0
total generation	940	980	990	1055	1085	1100	1130	1040
generation-GA	940	980	990	1055	1085	1100	1130	1040
generation-GB	0	0	0	0	0	0	0	0
generation-GC	0	0	0	0	0	0	0	0
fast reserves	80	80	80	80	80	90	80	80
spin-reserves	85	88	89	95	98	99	102	94
d2-reserves	0	0	0	0	0	0	0	0
total reserves	194	198	199	206	195	190	193	204
ORDC-reserves	194	198	199	206	195	190	193	204
LMP	30.09	30.09	30.09	40.09	45.00	47.17	47.17	40.09
LMPR	0.09	0.09	0.09	0.09	5.00	7.17	7.17	0.09
LMPS	0	0	0	0	0	0	0	0

Table 6.57 presents the LIP surplus and make-whole payments. The LIP settlement does not cover startup and fixed operating costs for GC, but has a reduced its make-whole payment from $1472 to $731.

Table 6.57 Average incremental Cost Pricing Results Iteration 1.

market-surplus= $8,770,645 make-whole payment = $731

reserves-value-total= $706,975 reserve-%= 8.061 reserves-cost-total= $4,464

	Buyer surplus	gen surplus	gen A surplus	gen B surplus	gen C surplus
LIP surplus ($)	7,994,409	73,725	72,871	1,585	$-731
surplus-%	91.15	0.84	0.831	0.018	-0.008

Iteration 2. We reallocate the make-whole payments from periods where the generator is not needed to periods where it is needed. The results are in Table 6.58.

Table 6.58 Average Incremental Costs for Iteration 2 (in $/MWh)

	energy AIC step 1	energy AIC step 2	spinning reserves AIC	fast-start reserves AIC
GA	30.09	40.09	0.09	0.09
GB	50.00	75.00	0.00	0.00
GC	76.92	101.92	16.92	16.92

The AIC procedure results are in Table 6.59. The LIPs in periods 1 through 4 and 8 are set by GA's AIC. The LIPs increase by reserve prices in periods 5 through 7.

Table 6.59 Pricing Run Results Using AIC for Iteration 2 (in $/MWh)

period	1	2	3	4	5	6	7	8
LIP	30.09	30.09	30.09	40.09	45.00	50.00	56.92	40.09
LIPR	0.09	0.09	0.09	0.09	5.00	10.00	10.00	0.09
LIPS	0.00	0.00	0.00	0.00	0.00	0.00	6.92	0.00

Table 6.60 presents the LIP surplus and make-whole payments. The LIP settlement does not cover startup and fixed operating costs for GC, but has a reduced its make-whole payment is $102.

Table 6.60 Average Incremental Cost Pricing Results Iteration 2.

market-surplus= $8,770,645 make-whole payment = $102

reserves-value-total= $706,975 reserve-%= 8.061 reserves-cost-total= $6,511

	Buyer surplus	total gen profit	gen A profit	gen B profit	gen C profit
LIP surplus ($)	$7980281	$89900	$87965	$2038	$-102
surplus-%	90.99	1.03	1.00	0.02	0.00

AIC Iteration 3. The AIC approach reallocates the make-whole payments from periods where the generator is not needed to periods where it is needed. The AICs are in Table 6.61. We rerun the market to obtain the LIPs. Table 6.62 presents the results. GC's reallocation of costs changes the price in period 7. Table 6.63 presents the LIP surplus. The LIP settlement covers startup and fixed operating costs for GC and makes a profit of $102. After three iterations of the AIC procedure, the make-whole payment is zero and prices are fully transparent.

Table 6.61 Average Incremental Costs for Iteration 3 (in $/MWh)

	energy AIC step 1	energy AIC step 2	spinning reserves AIC	fast-start reserves AIC
GA	30.09	40.09	0.09	0.09
GB	50.00	75.00	0	0
GC	81.01	106.01	21.01	21.01

Table 6.62 Pricing Run Results Iteration 3 (in $/MWh)

period	1	2	3	4	5	6	7	8
LIP	30.09	30.09	30.09	40.09	45.00	50.00	61.01	40.09
LIPR	0.09	0.09	0.09	0.09	5.00	10.00	10.00	0.09
LIPS	0	0	0	0	0	0	11.01	0

Table 6.63 Average incremental Cost Pricing Results Iteration 3.

market-surplus= $8,770,645 make-whole payment = 0

reserves-value-total= $706,975 reserve-%= 8.061 reserves-cost-total= $7,002

	Buyer surplus	total-gen profit	gen-A profit	gen-B profit	gen-C profit
LIP surplus ($)	7,975,657	95,016	92,875	2,038	102
surplus-%	90.9	1.083	1.059	0.023	0.001

Example 8. SCUC with Binding Ramp Rates. Example 8 demonstrates the effect on prices when ramp rate constraints are active in the dispatch (SCUC). The maximum demand is in Table 6.64.

Table 6.64 Maximum Demand by Period

Period	1	2	3	4	5	6	7	8
Maximum demand	940	980	990	1076	1085	1100	1130	1040

Efficient Dispatch (SCUC). Consumption, commitment, dispatch, LMP and settlement results are in Table 6.65. In this example, the increase in load from period 3 to 4 is 65 MW. Due to the ramp rate limit for GA of

60 MW/hr. GA cannot satisfy the increase in load. In period 4, GC is committed and dispatched at its minimum operating level and the ramp rate constraint on GA binds. The price separates from period 3 and 4 caused by the binding ramp rate on GA. In period 7, GC is committed and dispatched at its minimum operating level to satisfy the spinning reserves requirement.

Table 6.65 Efficient Dispatch (in MW) and Prices (in $/MWh)

period	1	2	3	4	5	6	7	8
Consumption	940	980	990	1076	1085	1100	1130	1040
total generation	940	980	990	1076	1085	1100	1130	1040
generation-GA	940	980	990	1050	1085	1100	1105	1040
generation-GB	0	0	0	0	0	0	0	0
generation-GC	0	0	0	26	0	0	25	0
fast reserves	130	130	130	80	130	130	80	130
spin-reserves	85	88	89	95	98	99	99	94
total reserves	390	350	340	254	245	230	200	290
ORDC-reserves	194	198	199	206	209	210	200	204
LMP	30	30	10	60	40	40	45	40
LMPR	0	0	0	0	0	0	5	0
LMPS	0	0	0	0	0	0	0	0

Table 6.66 presents the market surplus, make-whole payments, linear or LIP surplus of market participants and the non-linear surplus with make whole-costs included. The LIP settlement does not cover startup and fixed operating costs for GC, and it receives a make-whole payment of 972.

Table 6.66 Average incremental Cost Pricing Results Iteration 1.

market-surplus= $8,788,665; make-whole payment = $972

reserves-value-total= $704,855; reserve-%= 8.02; reserves-cost-total= $1,000

	buyer	total gen	gen A	gen B	gen C
linear-LMP ($)	8,029,090	55,720	56,292	400	-972
surplus-%	91.357	0.634	0.641	0.005	-0.011
nonlinear-LMP+mwp ($)	8,028,118	56,692	56,292	400	0
surplus-%	91.335	0.645	0.641	0.005	0

AIC Pricing Iteration 1. We do not relax the ramp rate because it occurred in the SCUC. We reallocate the fixed costs from periods where the generator is not needed to periods where it is needed.

Table 6.67Average Incremental Costs for Iteration 1 (in $/MWh)

	energy AIC step 1	energy AIC step 2	spinning reserves AIC	fast-start reserves AIC
GA	30.09	40.09	0.09	0.09
GB	50.00	75.00	0	0
GC	61.81	86.81	1.81	1.81

We run the AIC procedure to obtain the LIPs. Table 6.68 presents the results. GA's average incremental cost add $.09/MWh to its marginal price. Since it is marginal in periods 1-4 and 8, the LIPs increase by $.09 to in those periods. GC's average incremental cost add $.90/MWh to the marginal price. Through the arbitrage condition discussed in Example 3, the LIPs increase by the reserve prices in periods 5 through 7. GC's AIC sets the spinning reserve price in period 7.

Table 6.68 Pricing Run Results Using Average Incremental Costs for Iteration 1 (in $/MWh)

Period	1	2	3	4	5	6	7	8
LMP	30.09	30.09	8.38	61.81	41.81	41.81	45.00	40.09
reserves-price	0.09	0.09	0.09	0.09	1.81	1.81	5.00	0.09
spin-res-price	0	0	0	0	0	0	0	0

Table.6.69 presents the LIP surplus and make-whole payments. The LIP settlement does not cover startup and fixed operating costs for GC, which receives a make-whole payment of 723.

Table.6.69 AIC Pricing Results Iteration 1.

market-surplus= $8788665; make-whole payment = -$723

reserves-value-total= $704855; reserve-%= 8.02; reserves-cost-total= -$2011

	Buyer surplus	Total gen profit	gen-A profit	gen-B profit	gen-C profit
LIP surplus ($)	8,024,524	61,297	61,294	727	-723
surplus-%	91.30	0.697	0.697	0.008	-0.008

Iteration 2. We reallocate the fixed costs from periods where the generator is not needed to periods where it is needed. The average incremental costs are in Table 6.70.

Table 6.70 Average Incremental Costs for Iteration 2 ($)

	energy AIC step 1	energy AIC step 2	spinning reserves AIC	fast-start reserves AIC
GA	30.09	40.09	0.09	0.09
GB	50.00	75.00	0.00	0.00
GC	67.12	92.12	7.12	7.12

We run the AIC procedure to obtain the LIPs. Table 6.71 presents the results. GA's average incremental cost add $.09/MWh to the marginal price. The LIPs increase by $.09 in periods 1 through 4 and 8. GC's average incremental cost adds $7.12/MWh to the marginal price in periods 6 and 7. The ORDC sets the price in period 5. Due to the arbitrage condition discussed above, the LIPs increase by the reserve prices in periods 5 through 7.

Table 6.71 Pricing Run Results Using AICs for Iteration 2 (in $/MWh)

period	1	2	3	4	5	6	7	8
LIP	30.09	30.09	3.06	67.12	45.00	47.12	47.12	40.09
LIPR	0.09	0.09	0.09	0.09	5.00	7.12	7.12	0.09
LIPS	0	0	0	0	0	0	0	0

Table 6.72 presents the LIP surplus and make-whole payments. The LIP settlement does not cover startup and fixed operating costs for GC, which has a make-whole payment of $53.

Table 6.72 Average Incremental Cost Pricing Results Iteration 2.

market-surplus= $8788665 make-whole payment = $972

reserves-value-total= $704855 reserve-%= 8.02 reserves-cost-total= $4442

	consumer	total-gen	gen-A	gen-B	gen-C
LIP surplus ($)	8012347	75905	74380	1578	-53
surplus-%	91.16	0.864	0.846	0.018	-0.001

AIC Iteration 3. We reallocate the fixed costs from periods where the generator where it is not needed to periods where it is needed.

Table 6.73 Average Incremental Costs Iteration 3 (in $/MWh)

	energy AIC step 1	energy AIC step 2	spinning reserves AIC	fast-start reserves AIC
GA	30.09	40.09	0.09	0.09
GB	50.00	75.00	0.00	0.00
GC	67.65	92.65	7.65	7.65

We run the AIC procedure to obtain the LIPs. Table 6.74 presents the results. GA's average incremental cost adds $.09/MWh to the marginal price in periods 1 through 4 and 8. GC's average incremental cost add $.90/MWh to the marginal price in periods 6 and 7. The ORDC sets the price in period 5.

Table 6.74 Average Incremental Cost Pricing Run Results Iteration 3 (in $/MWh)

Period	1	2	3	4	5	6	7	8
LMP	30.09	30.09	2.54	67.65	45.00	47.65	47.65	40.09
reserves-price	0.09	0.09	0.09	0.09	5.00	7.65	7.65	0.09
spin-res-price	0	0	0	0	0	0	0	0

Table 6.75 presents the LIP surplus. The LIP settlement covers the startup and fixed operating costs for GC, and GC makes a profit of $13. No more re-allocation is needed.

Table 6.75 Average incremental Cost Pricing Results Iteration 3.

market-surplus= $8788665; make-whole payment = $972

reserves-value-total=; $704855; reserve-%= 8.02; reserves-cost-total= $4668

	Consumer surplus	total-gen profit	gen-A profit	gen-B profit	gen-C profit
LIP surplus ($)	8011129	77349	75674	1662	13
surplus-%	91.15	0.88	0.861	0.019	0

6.3 APPENDIX: Formulations of Multiple-Period, Single-Node Model

In this section, we formulate commitment, dispatch and pricing models for multiple-period, single-node model with two reserve products. We assume that ramp rates are uniform over the dispatch range.

Notation

System-Wide

Sets		index
T	is the set of time periods and the last time period, tote	t

Parameters

r^{lf}_t is reserves 'up' requirement for low frequency excursions

r^{rr}_t is the ramping reserves requirement in t

F is minimum operating level relaxation factor; $0 \leq f < 1$

M is 0 if the constraint was binding in SCED and is a very large number if it was not binding in the SCED

Primal Variables (dispatch)

MS is the market surplus

MS^R is market surplus of the RMOL pricing run

MS^E is market surplus of the ELMP pricing run

MS^{EL} is market surplus of the ELMPL pricing run

MS^{AIC} is market surplus of the AIC pricing run

Dual Variables (prices and values)

RC is resource cost of the dual problem

RC^{AIC} is resource cost of the AIC dual problem

λ^{lf}_t is system low frequency reserves marginal value

λ^{rr}_t is ramping reserve marginal value

Optimal Solutions

*	identifies the solution to efficient (optimal) dispatch model
**	identifies optimal solution to the pricing run model
degeneracy	to not to complicate the analysis we assume no degeneracy (or degeneracy occurs on a set of measure zero).

Load

Sets		index
D	is the index set loads	i

Parameters

b_{it} is bid price or value per MWh for load i ∊ D in t

c^{lf}_{it} is the cost of reserves from load i ∊ D I

c^{rr}_{it} is the cost of ramping reserves from load by demand i ∊ D in t

Primal Variables (dispatch)

d_{it} is demand satisfied (or load) by unit i ∊ D in t

d^{lf}_{it} is the low frequency reserves from load i ∊ D in t

d^{rr}_{it} is the ramping reserves from load i ∊ D in t

Dual variables

α^{max}_{it} is marginal value of demand step i ∊ D

α^{min}_{it} is marginal value of demand step i ∊ D

α^{ram}_{it} is marginal value of Ramsey-Boiteux discount of demand step i ∊ D in t

Generators

Sets		index
G	is set of generators	i
G^{mp}	is set of generators that qualify for a make-whole payment at the efficient dispatch LMP	i
G^{nmp}	is set of generators that do not qualify for a make-whole payment at the LMP of the efficient dispatch	i
G^{br}	is set of binding ramp rate constraints in the efficient dispatch	

Parameters

c_{it} is offer cost per MWh for generating unit i in t in the SCUC

c^{E}_{it} is offer cost per MWh for generating unit i in t in the ELMP

c^{A}_{it} is offer cost per MWh for generating unit i in t in the AIC

c^{su}_{it} is offer start-up cost for generating unit i in t

c^{op}_{it} is the fixed operating cost unit i in t not including the fuel costs

c^{rr}_{it} is offer cost per MWh for ramp reserves from generating unit i ∊ G in t

c^{sr}_{it} is offer cost per MWh for self-ramp from generating unit i ∊ G in t

c^{lf}_{it} is offer cost per MWh for low frequency reserves from unit i ∊ G in t

c^{Efc}_{it}

c^{Asr}_{it} is offer cost per MWh for self-ramp from unit i ∊ G in t in the AIC

c^{Alf}_{it} is offer cost per MWh for low frequency reserves from i ∊ G in t in the AIC

p^{max}_{it} is the maximum output(demand) of unit i ∊ G in t

p^{min}_{it} is the minimum operating level of unit i ∊ G in t

MRT_i is the minimum run time in a startup/shut-down cycle i ∊ G

MDT_i is the minimum down time after shut-down cycle

r^r_i is the maximum ramp rate for unit $i \in G$

Primal Variables (Unit Commitment and Dispatch)

p_{it} is the supply from unit i and t

p^{lf}_{it} is the low frequency reserves from unit i and t

p^{rr}_{it} is the ramp reserves from unit i and t

p^{sr}_{it} is the self-ramp reserves from unit i and t

z_{it} is 1 if unit i is started up in t or 0 otherwise (relaxed in the pricing run)

z^d_{it} is 1 if unit i is shut down in t or 0 otherwise (relaxed in the pricing run)

u_{it} is 1 if unit i is running in t or 0 otherwise (relaxed in the pricing run)

Dual variables (pricing)

β^{max}_{it} is marginal value of capacity of generator i

β^{min}_{it} is marginal cost of the minimum operating level of generator i

δ_{it} is incremental cost of startup of generator i

β^{ramp}_{it} is ramp rate marginal value

β^{rr}_{it} is load following marginal value

δ_{it} is binary logic marginal value

μ_{it} is min run logic marginal value

ω_{it} is fixed startup marginal value

ω^d_{it} is fixed shut down marginal value

Post-Dispatch Notation

Sets

Ti is the startup/shut-down cycle for generator I. Ti = {t', …, t''}. We assume that a generator starts up only once in t' and shuts down in t''+1. $Ti \subset T$

Summary Statistics

$p_i = \sum_{t \in T} (p_{it} + p^{rr}_{it} + p^{lf}_{it})$

$d_i = \sum_{t \in T} (d_{it} + d^{rr}_{it} + d^{lf}_{it})$

IC_i is the incremental costs of unit i for d_i^*

π_i profit of unit i

π^{rmp}_i residual make-whole payment

Multi-Period Multi-Product Security Constrained Unit Commitment Model. The security constrained unit commitment (SCUC) market auction model provides both efficient unit commitment and dispatch for settlement purposes, and to determine which resources participate in the pricing solution. Ramping resources is only in the "up" direction, i.e., capability to increase production in the next period. An analogous ramping product in the "down" direction is omitted for brevity. Price-responsive demand can participate in the reserves markets. The formulation for the SCUC is:

SCUC MIP Formulation (1)

$MS = \max \sum_{t \in T} [\sum_{i \in D} (b_{it}d_{it} - c^{lf}_{it}d^{lf}_{it} - c^{rr}_{it}d^{rr}_{it})$ Maximize market surplus (1a)

$-\sum_{i \in G} (c_{it}p_{it} + c^{op}_{it}u_{it} + c^{su}_{it}z_{it} + c^{rr}_{it}p^{rr}_{it} + c^{sr}_{it}p^{sr}_{it} + c^{lf}_{it}p^{lf}_{it})]$

		<u>dual var</u>	<u>constraints</u>	
system balancing constraints				
$\sum_{i \in D} d_{it} - \sum_{i \in G} p_{it} = 0$	$t \in T$	λ_t	energy balance	(1b)
$-\sum_{i \in G} p^{lf}_{it} - \sum_{i \in D} d^{lf}_{it} \leq -r^{lf}_t$	$t \in T$	λ^{lf}_t	freq reserves	(1c)
$-\sum_{i \in G} p^{rr}_{it} - \sum_{i \in D} d^{rr}_{it} \leq -r^{rr}_t$	$t \in T$	λ^{rr}_t	ramp reserves	(1d)

demand constraints

$d_{it} \le d^{max}_{it}$	$i \in D$ $t \in T$	α^{max}_{it}	Max load	(1e)
$-d_{it} - d^{lf}_{it} - d^{rr}_{it} \le -d^{min}_{it}$	$i \in D$ $t \in T$	α^{min}_{it}	Min load	(1f)
generator constraints				
$p_{it} + p^{rr}_{it} + p^{sr}_{it} + p^{lf}_{it} - p^{max}_{it} u_{it} \le 0$	$i \in G$ $t \in T$	β^{max}_{it}	max capacity	(1g)
$p^{rr}_{it} + p^{sr}_{it} + p^{lf}_{it} \le r^{r}_{i}$	$i \in G$ $t \in T$	β^{ramp}_{it}	Ramp reserves	(1h)
$p_{it+1} - p_{it} - p^{sr}_{it} \le 0$	$i \in G$ $t \in T$	β^{sr}_{it}	Ramp limits	(1i)
$-p_{it} + p^{min}_{it} u_{it} \le 0$	$i \in G$ $t \in T$	β^{min}_{it}	Minimum operating level	(1j)
$u_{it} - u_{it-1} - z_{it} + z^{d}_{it} = 0$	$i \in G$ $t \in T$	δ_{it}	binary logic	(1k)
$-u_{it} + \sum_{t-MRT+1,\ldots,t} z_{it} \le 0$	$i \in G$ $t \in T$	μ^{MRT}_{it}	Min run logic	(1l)
$u_{it} + \sum_{t-MDT+1,\ldots,t} z^{d}_{it} \le 1$	$i \in G$ $t \in T$	μ^{MDT}_{it}	Min down logic	(1m)
$p^{sr}_{it}, p^{rr}_{it}, p^{lf}_{it}, z_{it}, z^{d}_{it} \ge 0$, $u_{it} \in \{0, 1\}$, $i \in G$, $t \in T$				(1n)

We refer to the market model, expressed through equations (1a) through (1n), as (1).

Binary Logic. From (1g), $z_{it} + u_{it-1} - u_{it} - z^{d}_{it} = 0$, If u_{it} is fixed for $t \in T$, z_{it} and z^{d}_{it} for $t \in T$ are uniquely determined at 0 or 1. Therefore, there is no need to explicitly require the z_{it} and z^{d}_{it} to be binary. The combinations in the startup/shut-down cycle are:

Table 6.76 Start-up and Shut-down Binary Combinations

	z_{it}	u_{it-1}	u_{it}	z^{d}_{it}
startup	1	0	1	0
continue operations	0	1	1	0
shut down	0	1	0	1

Multi-Period Multi-Product Security Constrained Economic Dispatch (SCED) Model. We fix u_{it} to the optimal values. Minimum run and minimum down time constraints become redundant and are dropped. The SCED model is:

SCED LP Formulation with dual variables (2)

(1a)				(2a)
(1b) –(1k)				(2b) –(2k)
		dual var	constraints	
$u_{it} = u_{it}^{*}$	$i \in G$ $t \in T$	ω_{it}	fixed binary	(2l)
$p^{it}, p^{rr}_{it}, p^{sr}_{it}, p^{lf}_{it}, z_{it}, z^{d}_{it} \ge 0$	$i \in G$, $t \in T$			(2m)

We refer to the SCED market model (2a) through (2m) as (2). By design an optimal solution to SCUC (1) that is also feasible in (2). Since (2) has more restrictions than (1), the SCUC solution is optimal for (2). The linear program yields the optimal set of dual variables and is referred to as the pricing run. The optimal dual variables give us prices and marginal values. The prices are λ_t (energy), λ^{lf}_t (low freq reserves) and λ^{rr}_t (ramp reserves). In combination with the SCUC dispatch yield revenues for resources. The make-whole payments under LMP+ pricing are enough for all resources to recover their fixed and variable costs.

Relaxed Minimum Operating Level Pricing Model. This model is a form of pricing in PJM and NYISO. The u_{it} are fixed to the SCUD solution. The relaxed minimum operating level pricing methods relax the minimum operating level by a factor f ($0 \le f < 1$) for generators with a make-whole payment under LMP+. The model provides prices for products and reserves through the dual variables. The avoidable fixed costs are fixed and do not impact the dual prices. The multi-period relaxed minimum operating level pricing (RMOL) model is:

RMOL Model (3)

(1a)			market surplus	(3a)
(1b)-(1f)				(3b)-(3k)
		dual var	constraints	
$u_{it} = u_{it}^*$	$i \in G^{nmp}\ t \in T$	ω_{it}	fixed binary	(3l)
$p^{rr}_{it}, p^{sr}_{it}, p^{lf}_{it}, z_{it}, z^{d}_{it} \ge 0$	$i \in G, t \in T$			(3m)
generators with a make-whole payment				
$p_{it} + p^{rr}_{it} + p^{sr}_{it} + p^{lf}_{it} - p^{max}_{i} u_{it} \le 0$	$i \in G^{mp}\ t \in T$	β^{max}_{it}	max capacity	(3n)
$p^{rr}_{it} + p^{sr}_{it} + p^{lf}_{it} \le r^{r}_{i} + M$	$i \in G^{mp}\ t \in T$	β^{ramp}_{it}	Ramp rate	(3o)
$p_{it+1} - p_{it} - p^{sr}_{it} \le 0$	$i \in G^{mp}\ t \in T$	β^{sr}_{it}	Self-ramping	(3p)
$-p_{it} \le -p^{min}_{it} f$	$i \in G^{mp}\ t \in T$	β^{min}_{it}	min supply	(3q)
$u_{it} - u_{it-1} - z_{it} + z^{d}_{it} = 0$	$i \in G^{mp}\ t \in T$	δ_{it}	binary logic	(3r)
$u_{it} = u_{it}^*$	$i \in G^{mp}\ t \in T$	ω_{it}	fixed binary	(3s)
$p^{rr}_{it}, p^{sr}_{it}, p^{lf}_{it}, z_{it}, z^{d}_{it} \ge 0$	$i \in G^{mp}, t \in T$			(3t)

We refer to (3a) through (3t) as (3). Since (3) is a relaxation of (2), $MS \le MS^R$, but may not be a feasible solution to the SCUC. For RMOL, the relaxation is in the minimum operating level, that is, (3q). This allows the generator with the highest marginal cost to set the energy price.

Multi-Period ELMP Pricing Run. In addition to relaxing the minimum operating level, ELMP and AIC methods modify the cost coefficients in the pricing run objective function. Ideally, the pricing run should encompass the entire commitment horizon of every resource committed, with each period connected through equations of the type 1(k) through 1(m). However, extending this formulation to relaxing the binary variables has numerous technical challenges.

In this formulation, startup costs are amortized over the product of the minimum run time and maximum operating level . Using this approximation, we can then solve the pricing run one period at a time, rather than over the commitment horizon of any resource. In the formulation below, we demonstrate such a multi-period pricing algorithm while incorporating startup costs.

Rather than a fractional unit commitment variable, ELMPL eliminates the minimum generation constraint for resources with a make-whole payment and modifies their incremental energy function by adding a portion of commitment cost in each period. By amortizing the commitment costs across a fixed number of periods, each period can be solved separately.

The price covers incremental costs only if it was dispatched at its p^{max}. If not, the price does not cover incremental costs and the generator receives a make-whole payment. The highest cost incremental generator dispatched is often at its p^{min} in the dispatch run and requires a make-whole payment. Since the ELMP procedure is not fully defined for a multi-period market. We propose the following approach.

Procedure

Step	Description
1	For a generator with a make-whole payment, relax u_{it} to $u_{it} \le u_{it}^*$
2	For a generator with a make-whole payment, amortize the fixed costs: $c^{Efc}_{it} = c^{su}_{it}/MRT + c^{op}_{it}$ for $t = t^{su}, \ldots\ t^{su+MRT-1}$ (during minimum run time) and $c^{Efc}_{it} = c^{op}_{it}$ for $t > t^{su+MRT-1}$ (after minimum run time)
3	Replace c^{op}_{it} (the coefficient on u_{it} in the objective function) with c^{fc}_{it}
4	For a generator without a make-whole payment, replace u_{it} with u_{it}^*
5	Remove constraints (1k), (1l) and (1m)
6	Run the resulting linear program for pricing only

The ELMP pricing formulation (4)

$MS^E = \max \sum_{t\in T} [\sum_{i\in D,\, t\in T} (b_{it}d_{it} - c^{dlf}_{it}d^{lf}_{it} - c^{rr}_{it}d^{rr}_{it})$			market	(4a)
$-\sum_{i\in Gnmp} (c_{it}p_{it}+c^{op}_{it}u_{it}+c^{su}_{it}z_{it}+c^{rr}_{it}p^{rr}_{it}+c^{sr}_{it}p^{sr}_{it}+c^{lf}_{it}p^{lf}_{it})$			surplus	
$-\sum_{i\in Gmp} (c^{E}_{it}p_{it}+c^{fc}_{it}u_{it}+ c^{sr}_{it}p^{sr}_{it} + c^{lf}_{it}p^{lf}_{it})$				
(1b)-(1h)				(4b)-(4h)
		dual var	constraints	
$u_{it} = u_{it}^*$	$i\in G^{nmp}$ $t\in T$	ω_{it}	fixed binary	(4i)
$p^{rr}_{it}, p^{sr}_{it}, p^{lf}_{it}, z_{it}, z^{d}_{it} \geq 0$	$i\in G^{nmp}, t\in T$			(4j)
generators with make-whole payment constraints				
$p_{it}+p^{rr}_{it}+ p^{sr}_{it} + p^{lf}_{it} - p^{max}_{it}u_{it} \leq 0$	$i\in G^{mp}$ $t\in T$	β^{max}_{it}	max capacity	(4m)
$p^{rr}_{it} + p^{sr}_{it} + p^{lf}_{it} \leq r^{r}_{i}$	$i\in G^{mp}$ $t\in T$	β^{ramp}_{it}	Ramp rate	(4n)
$p_{it+1} - p_{it} - p^{sr}_{it} \leq 0$	$i\in G^{mp}$ $t\in T$	β^{sr}_{it}	Ramp limit	(4o)
$-p_{it} \leq 0$	$i\in G^{mp}$ $t\in T$	β^{min}_{it}	min supply	(4p)
$u_{it} - u_{it-1} - z_{it}+z^{d}_{it} = 0$	$i\in G^{mp}$ $t\in T$	δ_{it}	binary logic	(4h)
$u_{it} \leq u_{it}^*$	$i\in G^{mp}$ $t\in T$	ω_{it}	relaxed binary	(4q)
$-u_{it} + \sum_{t-MRT+1,\ldots,t} z_{it} \leq 0$	$i\in G$ $t\in T$	μ^{MRT}_{it}	Min run logic	(4r)
$u_{it} + \sum_{t-MDT+1,\ldots,t} z^{d}_{it} \leq 1$	$i\in G$ $t\in T$	μ^{MDT}_{it}	Min down logic	(4s)
$p^{rr}_{it}, p^{sr}_{it}, p^{lf}_{it}, z_{it}, z^{d}_{it} \geq 0$	$i\in G, t\in T$			(4t)

ELMPL with Lost Opportunity Costs (LOC) Payments. LOCs can be calculated ex post and paid as make-whole payment without influencing the prices derived in the pricing run. ELMPL also pays a LOC to generator to stay on the efficient dispatch. Let $c^{lo}_{it} = \max\{\lambda_t - c_{it}, 0\}$. $LOC_i = c^{lo}_{it}(p^{max}_{it} - p_{it}^*)$

The above calculation is simplified for the case of a constant marginal cost function. The LOC is more generally the difference between the linear profit of a generator's actual dispatch and the maximum linear profit a generator can produce. This payment can cause revenue inadequacy. That is, payments to generators could exceed what is collected from load.

Average Incremental Cost Pricing Procedure. Unlike other approaches, AIC amortizes fixed costs using the optimal dispatch for energy and reserves. We also relax the ramp rate constraint if it did not bind in the SCED.

AIC Pricing Algorithm (5)

Steps

1. for generator with a make-whole payment, relax u_{it} to $u_{it} \leq u_{it}^*$, set $p^{min}_{i} = 0$, and calculate $p^{T}_{it}{}^* = p_{it}^* + p^{rr}_{it}{}^* + p^{lf}_{it}{}^*$; $p^{T}_{i}{}^* = \sum_{t\in Ti} p^{T}_{it}{}^*$ and
   $\pi_i = \sum_{t\in T} [(\lambda_t^* - c_{it})p_{it}^* + (\lambda^{rr}_t{}^* - c^{r}_{it})p^{lf}_{it}{}^* + (\lambda^{lf}_t{}^* - c^{lf}_{it})p^{lf}_{it}{}^* - c^{op}_{it}u_{it}^* - c^{su}_{it}z_{it}^*]$
   for all $i\in G$ (Negative linear profits result in make-whole payments)
2. for generators with a make-whole payment, $\pi_i < 0$, calculate the average incremental costs: $c^{afc}_{it} = c^{op}_{it}u_{it}^*/p^{T}_{it}{}^* + \sum_{t\in T} c^{su}_{it}z_{it}^{**}/p^{T}_{i}{}^*$.
3. replace c_{it} with $c^{a}_{it} = c_{it} + c^{afc}_{it}$; c^{rr}_{it} with $c^{arr}_{it} = c^{rr}_{it} + c^{afc}_{it}$; and c^{lf}_{it} with $c^{alf}_{it} = c^{lf}_{it} + c^{afc}_{it}$;
4. for a generator without a make-whole payment, fix u_{it} with $u_{it} = u_{it}^*$
5. remove constraints (1k), (1l) and (1m)
6. run the resulting linear program for pricing

The formulation for the AIC pricing run is then:

$MS = \max \sum_{t\in T} [\sum_{i\in D} (b_{it}d_{it} - c^{dlf}_{it}d^{lf}_{it} - c^{rr}_{it}d^{rr}_{it})$ $- \sum_{i\in Gnmp} (c_{it}p_{it}+c^{op}_{it}u_{it}+c^{su}_{it}z_{it}+c^{rr}_{it}p^{rr}_{it}+c^{sr}_{it}p^{sr}_{it}+c^{rlf}_{it}p^{rlf}_{it})$ $- \sum_{i\in Gmp} (c^{a}_{it}p_{it} + c^{arr}_{it}p^{rr}_{it} + c^{sr}_{it}p^{sr}_{it} + c^{alf}_{it}p^{lf}_{it})]$ — Maximize market surplus (5a)

(1b)-(1f) — (5b)-(5f)

		dual var	constraints	
constraints for generators without a make-whole payment				
$p_{it} + p^{rr}_{it} + p^{sr}_{it} + p^{lf}_{it} - p^{max}_{i}u_{it} \le 0$	$i\in G^{nmp}$ $t\in T$	β^{max}_{it}	max capacity	(5g)
$p^{rr}_{it} + p^{sr}_{it} + p^{lf}_{it} \le r^{r}_{i}$	$i\in G^{br}$ $t\in T$	β^{ramp}_{it}	Ramp rate	(5h)
$p_{it+1} - p_{it} - p^{sr}_{it} \le 0$	$i\in G^{nmp}$ $t\in T$	β^{sr}_{it}	Self-ramping	(5i)
$-p_{it} + p^{min}_{i}u_{it} \le 0$	$i\in G^{nmp}$ $t\in T$	β^{min}_{it}	min supply	(5j)
$u_{it} - u_{it-1} - z_{it} - z^{d}_{it} = 0$	$i\in G^{nmp}$ $t\in T$	δ_{it}	binary logic	(5k)
$u_{it} = u_{it}^{*}$	$i\in G^{nmp}$ $t\in T$	ω_{it}	fixed binary	(5l)
$p^{rr}_{it}, p^{sr}_{it}, p^{lf}_{it}, z_{it}, z^{d}_{it} \ge 0$	$i\in G$, $t\in T$			(5m)
constraints for generators with a make-whole payment				
$p_{it} + p^{rr}_{it} + p^{sr}_{it} + p^{lf}_{it} - p^{max}_{i}u_{it} \le 0$	$i\in G^{mp}$ $t\in T$	β^{max}_{it}	max capacity	(5n)
$p^{rr}_{it} + p^{sr}_{it} + p^{lf}_{it} \le r^{r}_{i}$	$i\in G^{br}$ $t\in T$	β^{ramp}_{it}	Ramp rate	(5o)
$p_{it+1} - p_{it} - p^{sr}_{it} \le 0$	$i\in G^{mp}$ $t\in T$	β^{sr}_{it}	Self-ramping	(5p)
$-p_{it} \le 0$	$i\in G^{mp}$ $t\in T$	β^{min}_{it}	min supply	(5q)
$u_{it} \le u_{it}^{*}$	$i\in G^{mp}$ $t\in T$	ω_{it}	fixed binary	(5r)
$p^{rr}_{it}, p^{sr}_{it}, p^{lf}_{it}, z_{it}, z^{d}_{it} \ge 0$	$i\in G^{mp}$, $t\in T$			(5s)

Iterative Procedure for Calculating LIPs. If the first iteration of the AIC does not result in non-negative profit for dispatched generators at the LIP, we reallocate the make-whole payment to the periods where the resource was needed. The resource is not needed if $p_{it}^{**} = 0$ in the pricing run. In this case, another unit will set the price and generator i's remaining costs will be allocated to another period.

Step 1. Solve the AIC pricing program, obtaining $\lambda_t^{**}, \lambda^{rr}_t{}^{**}$, and $\lambda^{lf}_t{}^{***}$. For units with no make-whole payment, $u_{it} = u_{it}^{*}$. Calculate the LMP settlement (linear profits) for each startup/shutdown cycle from the efficient dispatch: for $i\in G$, the profits at the LMP, λ_t^{*}, are equal to π_i defined above.

Step 2. If $\pi_i \ge 0$ for all $i\in G$, stop, the settlement has no make-whole payments.

Step 3. Let $G^{mp} = \{i|\ \pi_i < 0 \text{ for } i\in G\}$. Calculate average fixed costs, c^{afc}_{it}, for each $i\in G^{mp}$ and replace c_{it} with $c^{a}_{it} = c_{it} + c^{afc}_{it}$; c^{rr}_{it} with $c^{arr}_{it} = c^{rr}_{it} + c^{afc}_{it}$; and c^{lf}_{it} with $c^{alf}_{it} = c^{lf}_{it} + c^{afc}_{it}$;

Run the pricing algorithm and calculate profits π^{AI}_i using $\lambda_t^{**}, \lambda^{rr}_t{}^{**}, \lambda^{lf}_t{}^{**}$

$\pi^{AI}_i = \sum_{t\in Ti} [(\lambda_t^{**} - c_{it})p_{it}^{*} + (\lambda^{rr}_t{}^{**} - c^{r}_{it})p^{lf}_{it}{}^{*} + (\lambda^{lf}_t{}^{**} - c^{lf}_{it})p^{lf}_{it}{}^{*} - c^{op}_{it}u_{it}^{*} - c^{su}_{it}z_{it}^{*}]$ for all $i\in G$

Step 4. If there are no make-whole payments, that is, $\pi^{AI}_i \ge 0$ for all $i\in G$, stop and settle the market using λ_t^{**}, $\lambda^{rr}_t{}^{**}$, and $\lambda^{lf}_t{}^{**}$.

If there is a make-whole payment, that is, if $\pi^{AI}_i < 0$ for any $i\in G$, go to step 5.

Step 5. If $\pi^{AI}_i < 0$ for any $i\in G$. The residual make-whole payment represents costs not recovered in the dispatch run using the pricing run prices. The residual dispatch represents the total dispatch in the pricing run. The quotient represents the additional increment added to the offer cost needed to ensure that make-whole payments are zero. The residual make-whole payment and dispatch are:

$\pi^{rmp}_i = \sum_{t\in T} (c^{op}_{it}u_{it}^{*} + c^{su}_{it}z_{it}^{*})$; $p^{rd}_i = 0$;

For $t\in Ti$, if $p_{it}^{**} = 0$, $\lambda_t^{**} > c^{a}_{it}$ and $u_{it}^{*} = 1$, then $\pi^{rmp}_i = \pi^{rmp}_i - (\lambda_t^{**} - c^{a}_{it})p_{it}^{*}$;

$p^{min}_{it} = p_{it}^{*}$)

if $p_{it}^{**} > 0$ and $u_{it}^{*} = 1$, then $\pi^{rmp}_i = \pi^{rmp}_i + c^{a}_{it}p_{it}^{*}$; $p^{rd}_i = p^{rd}_i + p_{it}^{*}$;

end t loop

For $i\in G$, recalculate the AIC coefficients, c^{a}_{it}.

For $t \in Ti$,
If ($p_{it}^{**} > 0$ and $u_{it}^{*} = 1$, $c^{a}_{it} = \pi^{rmp}_{i}/p^{rd}_{i}$)
end t loop
If $p_{it}^{*} = p^{min}_{it}$, set $p^{min}_{it} = 0$.
Rerun the dispatch algorithm obtaining λ_t^{**}, $\lambda^{rr}{}_t^{**}$, and $\lambda^{lf}{}_t^{**}$
Calculate the profit for each startup/shutdown cycle using λ_t^{**} and p_{it}^{*}.
For $i \in G^{mp}$ calculate profits;
$\pi^{AI}_{i} = \sum_{t\in Ti} [(\lambda_t^{**} - c_{it})p_{it}^{*} + (\lambda^{rr}{}_t^{**} - c^{r}_{it})p^{lf}{}_{it}^{*} + (\lambda^{lf}{}_t^{**} - c^{lf}_{it})p^{lf}{}_{it}^{*} - c^{op}_{it}u_{it}^{*} - c^{su}_{it}z_{it}^{*}]$ for all $i \in G$
Step 6. For $i \in G$ If $\pi^{AI}_{i} < 0$, go to step 5. If for all i, $\pi^{AI}_{i} \geq 0$, stop.

The SCUC determines a solution that has non-negative total welfare. That is, resources would not have been committed if there was not consumer surplus to support their costs. The AIC adjusts prices such that they completely cover the fixed costs of the marginal resource in the pricing run, eliminating make-whole payment. If the dispatch decisions remain constant, any change in price can only transfer surpluses from consumer surplus to supplier surplus: there can be no change overall. Therefore, AIC pricing is revenue adequate.

Proposition: The LIP settlement is revenue adequate.

ELMP and ELMPL pricing model equivalence with no reserve constraints and no LOC payments. The following proofs show equivalence between the ELMP pricing procedure used by MISO and the ELMPL procedure used by ISO-NE, referred to as RMOL. We use a multi-period model with minimum run time constraints, but we assume that reserves are not included in the model.

ELMP Pricing Run. The MISO ELMP pricing procedure without reserves is formulated as before, except that now constraints which connect the periods are removed.

ELMP Pricing Formulation (8)

$MS^{E} = \max \sum_{i\in D,\, t\in T} b_{it}d_{it} - \sum_{i\in Gnmp,\, t\in T} (c_{it}p_{it} + c^{op}_{it}u_{it}^{*} + c^{su}_{it}z_{it}^{*})$			Maximize	(8a)
$- \sum_{i\in Gmp,\, t\in T} (c^{afc}_{it}u_{it} + c_{it}p_{it})$			market surplus	
		<u>dual variables</u>	<u>Technology constraints</u>	
system balancing constraints				
$\sum_{i\in D} d_{it} - \sum_{i\in G} p_{it} = 0$	$t \in T$	λ_t	market clearing	(8b)
demand constraints				
$d_{it} \leq d^{max}_{it}$	$i \in D$ $t \in T$	α^{max}_{it}	upper bound on step	(8c)
$-d_{it} \leq -d_{it}^{*}$	$i \in D$ $t \in T$	α^{ram}_{it}	upper bound on step	(8d)
constraints for generators with no make-whole payment				
$p_{it} - p^{max}_{i}u_{it}^{*} \leq 0$	$i \in G^{nmp}$ $t \in T$	β^{max}_{it}	upper bound on supply	(8e)
$-p_{it} + p^{min}_{i}u_{it}^{*} \leq 0$	$i \in G^{nmp}$ $t \in T$	β^{min}_{it}	lower bound on supply	(8f)
$u_{it} = u_{it}^{*}$	$i \in G^{mp}$ $t \in T$	ω_{it}	fixed binary	(8g)
constraints for generators with a make-whole payment				
$p_{it} - p^{max}_{i}u_{it} \leq 0$	$i \in G^{mp}$ $t \in T$	β^{max}_{it}	upper bound on supply	(8h)
$-p_{it} + p^{min}_{i}u_{it} \leq 0$	$i \in G^{mp}$ $t \in T$	β^{min}_{it}	lower bound on supply	(8i)
$u_{it} \leq u_{it}^{*}$	$i \in G^{mp}$ $t \in T$	ω_{it}	integer relaxation	(8j)
$-u_{it} \leq 0$	$i \in G^{mp}$ $t \in T$			(8k)

The dual equations for u_{it} for generators with make-whole payment are,
dual equation primal variable

$\omega_{it} + p^{min}_i\beta^{min}_{it} - p^{max}_i\beta^{max}_{it} \geq -c^{afc}_{it}$ $\qquad u_{it}$ (9a)

If $u_{it} > 0$, then, by complementary slackness, the inequality becomes an equality:

$$\omega_{it} + p^{min}_i\beta^{min}_{it} - p^{max}_i\beta^{max}_{it} = -c^{afc}_{it} \quad (9b)$$

If $u_{it} < u_{it}^*$, then, by complementary slackness, $\omega_{it} = 0$ and can be dropped from (9b),

$$p^{min}_i\beta^{min}_{it} - p^{max}_i\beta^{max}_{it} = -c^{afc}_{it} \quad (9c)$$

If we assume for simplicity that $p^{min}_i < p^{max}_i$ from (8g) and (8h), then $\beta^{max}_{it} > 0$ implies that $\beta^{min}_{it} = 0$, and (9c) becomes,

$$-p^{max}_i\beta^{max}_{it} = -c^{afc}_{it} \quad (9d)$$

Although the dual problem formulation of the pricing model (8) is omitted here for brevity, the result above remains valid if $p^{min}_i = p^{max}_i$ because $\beta^{max}_{it}{}^{**} > 0$ and $\beta^{min}_{it}{}^{**} > 0$ will contradict optimality of the dual solution. Rearranging (9d), we obtain

$$\beta^{max}_{it} = c^{afc}_{it}/p^{max}_i \quad (9e)$$

From duality and complementary slackness, the dual equations for p_{it}:.

dual equation	primal variable	
$-\beta^{min}_{it} + \beta^{max}_{it} - \lambda_t \geq -c_{it}$	p_{it}	(9f)

From (9b) and complementary slackness, if $p_{it} > 0$,

$$-\beta^{min}_{it} + \beta^{max}_{it} - \lambda_t = -c_{it} \quad (9g)$$

The assumption that $\beta^{max}_{it} > 0$ implies that $\beta^{min}_{it} = 0$,

$$\beta^{max}_{it} - \lambda_t = -c_{it} \quad (9h)$$

Substituting (9e) into (9h), we obtain

$$\lambda_t = c_{it} + c^{afc}_{it}/p^{max}_i \quad (9i)$$

The results above assume that $\beta^{max}_{it} > 0$, so cases where $\beta^{max}_{it} = 0$ and $\beta^{min}_{it} > 0$ or $\beta^{max}_{it} = \beta^{min}_{it} = 0$ remain to be discussed. If constraint (8h) is binding at the optimal solution (so that $\beta^{min}_{it} > 0$), then the objective function can be improved by decreasing the value of u_{it}. Therefore, the case when $\beta^{max}_{it} = 0$ and $\beta^{min}_{it} > 0$ contradicts optimality of the ELMP pricing model (8) and can be ignored. The same logic holds if neither (8g) nor (8h) bind at the optimal solution, so $\beta^{max}_{it} = \beta^{min}_{it} = 0$ can also be ignored. The results above also assume that $u_{it} > 0$. If $u_{it}^{**} = 0$, then generator i is not needed in period t. It does not set the price but will require a make-whole payment.

ELMPL Pricing Run. The ELMPL pricing procedure without reserves is formulated as before, except that now the multi-period connecting constraints (7g), (7h), and (7i) are not included. The ISO-NE pricing model is:

ELMPL Pricing Formulation (9)

$MS^R = \max \sum_{i\in D,\, t\in T} b_{it}d_{it} - \sum_{i\in Gnmp,\, t\in T} (c_{it}\, p_{it} + c^{op}_{it}u_{it}^* + c^{su}_{it}z_{it}^*)$ $-\sum_{i\in Gmp,\, t\in T} (c^{afc}_{it}/p^{max}_i + c_{it})p_{it}$ — Maximize market surplus (10a)

		dual variables	Technology constraints	
system balancing constraints				
$\sum_{i\in D} d_{it} - \sum_{i\in G} p_{it} = 0$	$t\in T$	λ_t	market clearing	(10b)
demand constraints				
$d_{it} \leq d^{max}_{it}$	$i\in D$ $t\in T$	α^{max}_{it}	upper bound on step	(10c)
$-d_{it} \leq -d_{it}^*$	$i\in D$ $t\in T$	α^{ram}_{it}	upper bound on step	(10d)
constraints for generators with no make-whole payment				
$p_{it} - p^{max}_i u_{it}^* \leq 0$	$i\in G^{nmp}$ $t\in T$	β^{max}_{it}	upper bound on supply	(10e)
$-p_{it} + p^{min}_i u_{it}^* \leq 0$	$i\in G^{nmp}$ $t\in T$	β^{min}_{it}	lower bound on supply	(10f)
constraints for generators with a make-whole payment				
$p_{it} \leq p^{max}_i$	$i\in G^{mp}$ $t\in T$	β^{max}_{it}	upper bound on supply	(10g)

$-p_{it} \leq 0 \quad i \in G^{mp}\ t \in T \qquad \beta^{min}_{it} \qquad$ lower bound on supply (10h)

The resulting prices can be calculated from the dual constraint for p_{it}:

dual equation	primal variable	
$-\beta^{min}_{it} + \beta^{max}_{it} - \lambda_t = -(c^{afc}_{it}/p^{max}_i + c_{it})$	p_{it}	(11)

We want to show that the system lambda equals the cost (as represented in the objective function of (11) of the marginal resource; that is, the resource which would serve the next increment of load, and that it is identical to the relation in (9i). We assume that there is one such resource which is dispatched below its maximum value in the pricing run, and that it is the highest cost per MW in the optimal solution. We show below that the cost of this resource sets the system price and that in this model all other less expensive resources should be dispatched at their maximum operation level in this pricing run. We can rank the resource costs represented in the objective function from smallest to largest, with $c^{afc}_{i't}/p^{max}_{i'} + c_{i't}$ the largest. By complementarity, for each $p_{it} > 0$ in (11) the inequality must then be an equality:

$$-\beta^{min}_{it} + \beta^{max}_{it} - \lambda_t = -(c^{afc}_{it}/p^{max}_i + c_{it}).$$

By inspection of the optimization model (10), the solution will be such that minimizing the output of generator i' will take precedence over all other generators. Thus, complementarity of constraints (10e) and (10g) implies that $\beta^{max}_{it} > 0$ for all $i \neq i'$. By assumption that generator i' is needed in period t, $p_{i't} > 0$ and thus $\beta^{min}_{i't} = 0$ by complementarity of constraint (10h). Equation (11) becomes,

$$\lambda_t = c^{afc}_{i't}/p^{max}_{i'} + c_{i't}$$

If $p_{it}^{**} = 0$, then generator i' is not needed in period t and does not set the price but will require a make-whole payment.

ELMP and ELMPL yield the same pricing result. The price covers incremental costs only if it was dispatched at its p^{max}. If not, the price does not cover incremental costs and the generator receives an additional make-whole payment. As a practical matter, the highest cost incremental generator dispatched is often dispatch at its p^{min}_i

In addition, there may be an interval where the generator is not needed, which becomes apparent when the output level of the generator is zero in the pricing run. This creates an additional make-whole payment since the generator does not set the price. In such a case the fixed cost is amortized over the periods where it is needed.

Other results will be obtained when reserves are added into the dispatch and pricing models. Notice that most formulations with reserves will include a constraint with the following form,

$$p_{it} + p^{rr}_{it} + p^{sr}_{it} + p^{lf}_{it} - p^{max}_{it} u_{it} \leq 0 \qquad i \in G\ t \in T$$

In the ELMP pricing model, $u_{it} \leq u_{it}^*$. This relaxation allows the pricing model to reduce costs by lowering the value of u_{it} to be less than 1, but this also has the effect of lowering the amount of reserves that can be procured in the model, possibly raising the reserve price. The ELMPL pricing model removes the u_{it} variable, and in effect, the pricing model will "see" the same constraint on reserves based on p^{max}_i. The two pricing models (8) and (10) are not necessarily equivalent in terms of the resulting prices when reserves are included in the model.

7 The One-Pass AIC Pricing Approach for Multi-Period Markets

7.1 Introduction

We extend the one-pass AIC pricing approach to the dynamic model with multi-step marginal cost functions, ramp constraints, and a co-optimized reserves market. The reserve requirement includes contingencies and energy balancing (aka 'net load' following) in the up direction. Excursions from the dispatch signal are charged at a minimum the cost of redispatch (aka liquidated damages). For dispatched generators, the prices are profitable without make-whole payments.

Multi-Period Unit Commitment Economic Dispatch (UCED).

Acronyms and Abbreviations

MIP	mixed integer program
UCED	unit commitment, security constrained economic dispatch
SCED	security constrained economic dispatch after fixing the binaries
AICOPR	AIC One-pass Pricing Run
LMP	locational marginal energy price from the energy-balance dual variable of the fixed-binary linear program
rLMP	locational marginal reserve price from the reserves-requirement dual variable of the fixed-binary linear program
AIC	average incremental cost
LIP	locational incremental energy price from the energy-balance dual variable of the AICOPR linear program
rLIP	locational incremental reserves price from the reserves-requirement dual variable of the AICOPR linear program

Definitions

MIP gap	is the distance between the best MIP feasible, LP optimal objective and the best-remaining-node objective. It is a bound on the how close the current best MIP feasible, LP optimal solution is to an optimal solution. If the MIP gap = 0, the solution is optimal. For computational reasons, the MIP gap maybe be greater than 0.
degeneracy	An optimal linear program is degenerate if an optimal basic feasible solution has one or more variables with a zero value. Primal optimal degeneracy implies multiple optimal dual solutions. So as not to overly complicate the analysis unless otherwise stated, we assume that there is no degeneracy or degeneracy occurs on a set of measure zero.
valid optimality cut	is any constraint that does not eliminate all optimal solutions.
in-market	dispatched or scheduled by the system operator.
market-clearing	is the process by which the auction quantities are computed.
market-clearing price	is the price of a good or service at which quantity supplied equals quantity demanded, also called the equilibrium price. in non-convex markets, a single market clearing pri e may not exist.
out-of-market	not dispatched or scheduled by the system operator.
dispatched	the unit is generating or off-line but on reserve
profitability	profitability includes breaking even in each independent up-down cycle

incremental generator	breaks even
infra-incremental generator	has positive profit

Notation

System-Wide

Sets index

$\mathcal{T}$ is the set of time periods; $\mathcal{T} = \{t|\ t = 1, \ldots, T\}$ t

$\mathcal{T}'$ is the set of time periods without t = 1; $\mathcal{T}' = \{t|\ t = 2, \ldots, T\}$ t

$\mathcal{T}i$ is the startup/shut-down cycle for generator I. $\mathcal{T}i = \{t', t'+1, \ldots, t''\} = \{t|u_{it}^* = 1\}$. $\mathcal{T}i \subset \mathcal{T}$ t

System Parameters

r^{us}_t is the minimum system ramping up reserves

Primal Variables (dispatch)

MS is the market surplus

MS^{AIC} is market surplus of the AIC pricing run

Dual Variables (prices and values)

RC^{AIC} is resource cost of the AIC dual problem; at optimality; $MS^{AIC} = RC^{AIC}$

Optimal Solutions

* identifies the solution to efficient (optimal) dispatch model (1)

** identifies optimal solution to the pricing run model (2)

Load

Sets index

D is the index set loads i

Parameters

b_{it} is the bid price per MWh for load i ∊ D in t

Primal Variables (dispatch)

d_{it} is demand by unit i ∊ D in t

Dual variables

α^{max}_{it} is the marginal value of maximum demand on step i ∊ D

α^{min}_{it} is marginal value of minimum demand reduction on step i ∊ D

Generators

Sets index

G is set of generators i

G^* is set of generators dispatched i

G^{mp} is set of generators that qualify for a make-whole payment at the efficient dispatch LMP i

G^{nmp} is set of generators that do not qualify for a make-whole payment at the LMP of the efficient dispatch i

G^u is set of generators operating at the beginning of the market horizon

G^f is set of generators that can start in time to satisfy a short term need

Ji Are the steps on the marginal cost function where $c_{jit} < c_{j+1it}$. $Ji = \{j|\ j = 1, \ldots, j^{max}_i\}$ j

Parameters

c_{jit} is offer cost per MWh of step j for generating unit i in t in the SCUC. $c_{1it} = 0$ and $c_{jit} < c_{j+1it}$

c^{ru}_{it} is offer cost per MWh ramping for generating unit i in t in the SCUC

c^{su}_{it} is offer start-up cost for generating unit i in t

c^{op}_{it} is the fixed operating cost unit i in t not including the fuel costs

r^{up}_{it} is the maximum ramping capability for generating unit i in t

p^{max}_{jit} is the maximum output of step j of unit i ∈ G in t. $p^{max}_{1it} = p^{min}_{it}$

p^{max}_{it} is the maximum capacity of unit i ∈ G in t. $\sum_{j\in Ji} p^{max}_{jit} = p^{max}_{it}$

p^{sua}_{it} is the adjustment to p^{max}_{it} for the startup period.

r^{sua}_{it} is the adjustment to r^{up}_{it} on startup (assume for one period only)

p^{min}_{it} is the minimum operating level of unit i ∈ G in t

mr_i is the minimum run time in a startup/shut-down cycle for unit i ∈ G

Primal Variables (Unit Commitment and Dispatch)

p_{jit} is the supply from step j of unit i and t

p^{ru}_{it} is the supply of ramp rate reserves from unit i and t

z_{it} is 1 if unit i starts up in t or 0 otherwise (relaxed in the pricing run)

z^d_{it} is 1 if unit i shuts down in t or 0 otherwise (relaxed in the pricing run)

u_{it} is 1 if unit i is running in t or 0 otherwise (relaxed in the pricing run)

Dual variables (pricing)

β^{max}_{jit} is marginal value of capacity of step j for generator i in t

β^{max}_{it} is marginal value of total capacity for generator i in t.

β^{min}_{it} is marginal cost of the minimum operating level of generator i in t

ρ^{up}_{it} Is the marginal value of ramp from generator i in t

δ_{it} is binary logic marginal value for generator i in t.

μ_{it} is dual variable on the summation of energy steps for generator i in t.

ω_{it} is fixed startup variable's marginal value for generator i in t.

ω^d_{it} is fixed shut down variable's marginal value for generator i in t.

The multi-period unit-commitment security-constrained optimal power flow (UCSCOPF) with reserves, multi-step marginal costs, and ramp rate constraints is

MS = max $\sum_{\mathcal{T}}$	$[\sum_{i\in D} b_{it}d_{it}$	$-\sum_{i\in G} [\sum_{j\in Ji} (c_{jit}p_{jit}) + c^{ru}_{it}p^{ru}_{it} + c^{op}_{it}u_{it} + c^{su}_{it}z_{it}]]$	(1a)
Maximize	= Consumer value	- Producer costs	

system balancing constraints		<u>description</u>	
$\sum_{i\in D} d_{it} - \sum_{i\in G} p_{it} = 0$	$t\in\mathcal{T}$	energy balance	(1b)
$-\sum_{i\in G} p^{ru}_{it} \le -r^{us}_t$	$t\in\mathcal{T}$	System ramp requir.	(1c)
demand constraints			
$d_{it} \le d^{max}_{it}$	$i\in D$ $t\in\mathcal{T}$	Max load	(1d)
$-d_{it} \le -d^{min}_{it}$	$i\in D$ $t\in\mathcal{T}$	Min load	(1e)
generator constraints			
$p_{jit} - p^{max}_{jit}u_{it} \le 0$	$i\in G$ $t\in\mathcal{T}$ $j\in Ji$	max step capacity	(1f)
$p_{it} - \sum_{j\in Ji} p_{jit} = 0$	$i\in G$ $t\in\mathcal{T}$	summation	(1g)
$p_{it} + p^{ru}_{it} - p^{max}_{it}u_{it} - p^{sua}_{it}z_{it} \le 0$	$i\in G$ $t\in\mathcal{T}$	max capacity	(1h)
$p^{ru}_{it} - p^{rumax}_{it}u_{it} \le 0$	$i\in G$ $t\in\mathcal{T}$	max ramp capacity	(1i)
$-p_{it} + p^{min}_{it}u_{it} \le 0$	$i\in G$ $t\in\mathcal{T}$	min supply	(1j)
$p_{it} - p_{it-1} - r^{up}_{it}u_{it} - r^{sua}_{it}z_{it} \le 0$	$i\in G$ $t\in\mathcal{T}'$	Ramp up limits	(1k)
$u_{it} - u_{it-1} - z_{it} + z^d_{it} = 0$	$i\in G$ $t\in\mathcal{T}$	binary logic	(1l)
$\sum_{t'\in[t-mr+1,t]} z_{it'} - u_{it} \le 0$	$i\in G$ $t\in\mathcal{T}$	Minimum run time	(1m)
$p^{ru}_{it}, z_{it}, z^d_{it} \ge 0, u_{it} \in\{0, 1\}$	$i\in G$, $t\in\mathcal{T}$		(1n)

We denote the optimal solution to the UCSCOPF with *. Later in the paper, we address the UCSCOPF returning a suboptimal solution. Each feasible MIP with an optimal linear program is a local optimal solution.

Since p_{i0} and u_{i0} define the state of the generators at the beginning of period 1, they are parameters in the model. If $u_{i0} = 1$, c^{su}_{i1} is set to zero and p_{i0} and r^{up}_{i1} are used to adjust p^{max}_{i1} to reflect the ramp rate constraint for p_{i1}.
In the startup sequence of a generator, it may not be able to achieve its steady-state maximum operating level in a single period. The one-period adjustment is in (1h). To represent a multi-period startup sequence, let Js be a multi-period sequence, Js = {0, 1,…, jsu}; p^{sua}_{jit} be the adjustment in period j after startup; and $p^{sua}_{it}z_{it}$ is replaced with $\sum_{j\in Js} p^{sua}_{jit}z_{it-j}$. For simplicity, we use the (1h) formulation.

After fixing the binary variables, the linear program usually has redundant constraints that were not redundant in the MIP, for example, minimum run time and minimum down time. Dropping redundant constraints, retains the optimal MIP solution in the linear program. After fixing the binaries to their optimal values and solving the linear program, the dual variables on the energy balance equations (1b), are the called the LMPs. The dual variables on the reserve constraints (1c) are marginal reserves prices and are called rLMPs. The LMPs and rLMPs provide marginal information about low cost entry, but do not signal the possible higher-cost unit-replacement entry when units have avoidable fixed costs. The LIP prices from the AIC pricing run (below) and optimal quantities of the incremental generators from the MIP provide entry information for units with avoidable fixed costs.

In the market horizon, for convenience, we assume that a generator starts up only once in t', operates through t" and is shut down in t"+1. Multiple startup-shutdown cycles are priced independently, as if it were owned by a different market participant.

AIC One-Pass Pricing Run (AICOPR).
The AICOPR eliminates the binary constraints and adds the constraints: $0 \le z_{it} \le z_{it}^*$; $0 \le u_{it} \le u_{it}^*$;and $0 \le z^d_{it} \le z^{d*}_{it}$. This relaxation eliminates the non-dispatched generators from influencing the prices since the optimal binary variables are 0. We retain the binary logic constraints (2l). When the binaries are relaxed, the dual variables on the binary logic constraints play an important role in distributing the startup and fixed operating costs across the up/down cycle. When the binaries are fixed, the minimum run/down time constraints are redundant. In the AIC formulation (2), although it is not necessary, we drop the minimum run time constraints.

The optimal solution to (1) can be used to tighten the constraints of the pricing problem. For dispatched generators with a make-whole payment, we add valid optimality cuts that retain the optimal solution by setting $p^{max}_{it} = p_{it}^* + p^{ru}_{it}{}^* - p^{sua}_{it}z_{it}^* + \varepsilon$ and dropping $p^{sua}_{it}z_{it}^*$ since the new p^{max}_{it} in the startup period contains the $p^{sua}_{it}z_{it}$ adjustment; and $p^{max}_{jit} = p_{jit}^* + \varepsilon$, where $\varepsilon > 0$ but small. In addition, we add $p^{rumax}_{it} = p^{ru}_{it}{}^* + \varepsilon$, $r^{up}_{it}u_{it} = (r^{up}_{it} + r^{sua}_{it}z_{it}^*)u_{it}$ and $p^{min}_{it} = p_{it}^* - \varepsilon$. These cuts keep the pricing problem in the neighborhood of the optimal solution.

Two sets of constraints couple the time periods: the startup/shutdown cycle binary logic constraints (2l) and ramp rate constraints (2k). Both are important to dynamic pricing. In the iterative AIC, the ramp rates that did not bind in the dispatch model (1) are relaxed in the pricing run. Here we continue that practice for the one-pass AIC. The AICOPR becomes:

$$MS^{AIC} = \max \sum_{t\in\mathcal{T}} [\sum_{i\in D} b_{it}d_{it} - \sum_{i\in G} [\sum_{j\in Ji} (c_{jit}p_{jit}) + c^{ru}_{it}p^{ru}_{it} + c^{op}_{it}u_{it} + c^{su}_{it}z_{it}]] \quad (2a)$$

<u>dual var</u> <u>constraints</u>

system balancing constraints

$\sum_{i\in D} d_{it} - \sum_{i\in G} p_{it} = 0$	$t\in\mathcal{T}$	λ_t	energy balance	(2b)
$-\sum_{i\in G} p^{ru}_{it} \le -r^{us}_t$	$t\in\mathcal{T}$	λ^{us}_t	System ramp	(2c)
demand constraints				
$d_{it} \le d^{max}_{it}$	$i\in D$ $t\in\mathcal{T}$	α^{max}_{it}	Max load	(2d)
$-d_{it} \le -d^{min}_{it}$	$i\in D$ $t\in\mathcal{T}$	α^{min}_{it}	Min load	(2e)
generator constraints				
$p_{jit} - p^{max}_{jit}u_{it} \le 0$	$i\in G$ $t\in\mathcal{T}$ $j\in Ji$	β^{max}_{jit}	max step capacity	(2f)
$p_{it} - \sum_{j\in Ji} p_{jit} = 0$	$i\in G$ $t\in\mathcal{T}$	μ_{it}	Summation	(2g)
$p_{it}+p^{ru}_{it}-p^{max}_{it}u_{it} \le 0$	$i\in G$ $t\in\mathcal{T}$	β^{max}_{it}	max capacity	(2h)
$p^{ru}_{it} - p^{rumax}_{it}u_{it} \le 0$	$i\in G$ $t\in\mathcal{T}$	β^{rumax}_{it}	max ramp reserve	(2i)
$-p_{it} + p^{min}_{it}u_{it} \le 0$	$i\in G$ $t\in\mathcal{T}$	β^{min}_{it}	min supply	(2j)
$p_{it} - p_{it-1} - r^{up}_{it}u_{it} \le 0$	$i\in G$ $t\in\mathcal{T}'$	ρ^{up}_{it}	Ramp up limits	(2k)
$u_{it} - u_{it-1} - z_{it} + z^{d}_{it} = 0$	$i\in G$ $t\in\mathcal{T}$	δ_{it}	binary logic	(2l)
$z_{it} \le z_{it}^*$	$i\in G$ $t\in\mathcal{T}$	ω_{it}	Relaxed binaries	(2m)
$u_{it} \le u_{it}^*$	$i\in G$ $t\in\mathcal{T}$	γ_{it}	Relaxed binaries	(2n)
$z^{d}_{it} \le z^{d}_{it}{}^*$	$i\in G$ $t\in\mathcal{T}$	ω^{d}_{it}	Relaxed binaries	(2o)
$p^{ru}_{it}, z_{it}, z^{d}_{it}, u_{it} \ge 0$	$i\in G$ $t\in\mathcal{T}$		Lower bounds	(2p)

Dual of AICOPR is

$$RC^{AIC} = \min \sum_{t\in\mathcal{T}} [-r^{us}_t\lambda^{us}_t + \sum_{i\in D} (d^{max}_{it}\alpha^{max}_{it} - d^{min}_{it}\alpha^{min}_{it}) + \sum_{i\in G} (z_{it}{}^*\omega_{it} + u_{it}{}^*\gamma_{it} + z^{d}_{it}{}^*\omega^{d}_{it})] \qquad (3a)$$

		<u>dual var</u>	
demand constraints			
$\lambda_t + \alpha^{max}_{it} - \alpha^{min}_{it} \ge b_{it}$	$i\in D$ $t\in\mathcal{T}$	d_{it}	(3b)
generator constraints			
$\rho^{up}_{it} - \rho^{up}_{it+1} - \lambda_t + \mu_{it} - \beta^{min}_{it} + \beta^{max}_{it} \ge 0$	$i\in G$ $t\in\mathcal{T}$	p_{it}	(3c)
$\omega_{it} - \delta_{it} \ge -c^{su}_{it}$	$i\in G$ $t\in\mathcal{T}$	z_{it}	(3d)
$-\mu_{it} + \beta^{max}_{jit} \ge -c_{jit}$	$i\in G$ $t\in\mathcal{T}$ $j\in Ji$	p_{jit}	(3e)
$\beta^{rumax}_{it} + \beta^{max}_{it} - \lambda^{us}_t \ge -c^{ru}_{it}$	$i\in G$ $t\in\mathcal{T}$	p^{ru}_{it}	(3f)
$\gamma_{it}+\delta_{it}-\delta_{it+1}-r^{up}_{it}\rho^{up}_{it}+p^{min}_{it}\beta^{min}_{it}-\sum_{j\in Ji} (p^{max}_{ji}\beta^{max}_{jit})-p^{max}_{i}\beta^{max}_{it} - p^{rumax}_{i}\beta^{rumax}_{it} \ge -c^{op}_{it}$	$i\in G$ $t\in\mathcal{T}$	u_{it}	(3g)
$\omega^{d}_{it} + \delta_{it} \ge 0$	$i\in G$ $t\in\mathcal{T}$	z^{d}_{it}	(3h)
$\alpha^{max}_{it}, \alpha^{min}_{it} \ge 0$	$i\in D$ $t\in\mathcal{T}$		(3i)
$\beta^{max}_{jit} \ge 0$	$i\in G$ $t\in\mathcal{T}$ $j\in Ji$		(3j)
$\rho^{up}_{it}, \beta^{max}_{it}, \beta^{rumax}_{it}, \beta^{min}_{it}, \omega_{it}, \gamma_{it}, \omega^{d}_{it} \ge 0$	$i\in G$ $t\in\mathcal{T}$		(3k)
$\lambda^{up}_t \ge 0$	$t\in\mathcal{T}$		(3l)

In (3c), since (2k) does not exist for t=1, neither does ρ^{up}_{i1} and it is set it to 0.

In (4) through (8) below, we assume the variables are at the optimal value; * indicates an optimal solution to (1); ** indicates an optimal solution to (2). The dual variables do not exist in (1). Therefore, they do not need to be distinguished as are the primal variables.

Lemma 1. $MS \le MS^{AIC}$.
Proof: Since the optimal solution to (1) is a feasible solution to (2), $MS \le MS^{AIC}$.
:-)

Lemma 2. In the one-pass AIC pricing run, as $\varepsilon \to 0$, the upper bounds, (2f), (2h), and (2i), and lower bounds, (2j), in the pricing run are constrained by the relaxation of u_{it} and the optimal solution to (1).

Proof: since

Since $p^{max}_{it} + p^{sua}_{it}z_{it}^* = p_{it}^* + p^{ru}_{it}{}^* + \varepsilon$, from (2h) making the adjustment for $p^{sua}_{it}z_{it}^*$

$$p_{it}^{**} + p^{ru}_{it}{}^{**} \leq p^{max}_{it} = p_{it}^* + p^{ru}_{it}{}^* + \varepsilon \quad (4a)$$

from (2f),

$$p_{jit}^{**} \leq p^{max}_{jit}u_{it}^{**} = (p_{jit}^* + \varepsilon)u_{it}^{**} \quad (4b)$$

and from (2i),

$$p^{ru}_{it}{}^{**} \leq p^{rumax}_{it}u_{it}^{**} = (p^{ru}_{it}{}^* + \varepsilon)u_{it}^{**} \quad (4c)$$

and from (2j),

$$-p_{it} \leq p^{min}_{it}u_{it}^{**} = -(p_{it}^* - \varepsilon)u_{it}^{**} \quad (4c)$$

As $\varepsilon \rightarrow 0$,

$$p_{it}^{**} + p^{ru}_{it}{}^{**} \leq (p_{it}^* + p^{ru}_{it}{}^*)u_{it}^{**} \quad (4d)$$

$$p_{jit}^{**} \leq p_{jit}^* u_{it}^{**} \quad (4e)$$

$$p^{ru}_{it}{}^{**} \leq p^{ru}_{it}{}^* u_{it}^{**} \quad (4f)$$

$$-p_{it} \leq -p_{it}^* u_{it}^{**} \quad (4g)$$

As $\varepsilon \rightarrow 0$, by complementary slackness

If $\beta^{max}_{it} > 0$, $\quad p_{it}^{**} + p^{ru}_{it}{}^{**} = (p_{it}^* + p^{ru}_{it}{}^*)u_{it}^{**}$ (4d)

If $\beta^{max}_{jit} > 0$, $\quad p_{jit}^{**} = p_{jit}^* u_{it}^{**}$ (4e)

If $\beta^{rumax}_{it} > 0$, $\quad p^{ru}_{it}{}^{**} = p^{ru}_{it}{}^* u_{it}^{**}$ (4f)

If $\beta^{min}_{it} > 0$, $\quad -p_{it} = p_{it}^* u_{it}^{**}$ (4g)

As $\varepsilon \rightarrow 0$, absent degeneracy,

If $\beta^{max}_{it} = 0$, $\quad p_{it}^{**} + p^{ru}_{it}{}^{**} < (p_{it}^* + p^{ru}_{it}{}^*)u_{it}^{**}$ (4h)

If $\beta^{max}_{jit} = 0$, $\quad p_{jit}^{**} < p_{jit}^* u_{it}^{**}$ (4i)

If $\beta^{rumax}_{it} = 0$, $\quad p^{ru}_{it}{}^{**} < p^{ru}_{it}{}^* u_{it}^{**}$ (4j)

If $\beta^{min}_{it} = 0$, $\quad -p_{it} < p_{it}^* u_{it}^{**}$ (4k)

:-)

Lemma 3. Reserves are profitable.

Proof: By complementary slackness of (3f),

$$(\beta^{rumax}_{it} + \beta^{max}_{it} + c^{ru}_{it} - \lambda^{us}_{t})p^{ru}_{it}{}^{**} = 0 \quad (4l)$$

Rearranging,

$$\lambda^{us}_{t}p^{ru}_{it}{}^{**} - c^{ru}_{it}p^{ru}_{it}{}^{**} = \beta^{rumax}_{it}p^{ru}_{it}{}^{**} + \beta^{max}_{it}p^{ru}_{it}{}^{**} \quad (4m)$$

As $\varepsilon \rightarrow 0$, $p^{ru}_{it}{}^{**} \rightarrow p^{ru}_{it}{}^*$ and

$$\lambda^{us}_{t}p^{ru}_{it}{}^{*} - c^{ru}_{it}p^{ru}_{it}{}^{*} = \beta^{rumax}_{it}p^{ru}_{it}{}^{*} + \beta^{max}_{it}p^{ru}_{it}{}^{*} \quad (4n)$$

Since $\beta^{rumax}_{it} \geq 0$ and $\beta^{max}_{it} \geq 0$, the revenue from reserves may exceed the costs of reserves

$$\lambda^{us}_{t}p^{ru}_{it}{}^{*} \geq c^{ru}_{it}p^{ru}_{it}{}^{*} \quad (4o)$$

If $\beta^{rumax}_{it} = 0$ and $\beta^{max}_{it} = 0$,

$$\lambda^{us}_{t}p^{ru}_{it}{}^{*} = c^{ru}_{it}p^{ru}_{it}{}^{*} \quad (4p)$$

If $p^{ru}_{it}{}^* > 0$, dividing by $p^{ru}_{it}{}^*$,

$$\lambda^{us}_{t} = c^{ru}_{it} \quad (4q)$$

the price of reserves is set by the marginal reserve cost of generator i.

:-)

Proposition 1. From AICOPR using the LIP and rLIP prices, all dispatched units are profitable, that is, no generator needs a make-whole payment.

Proof:

The static constraints. From complementary slackness of (3e),

$$(-\mu_{it} + \beta^{max}_{jit})p_{jit}^{**} = -c_{jit}\, p_{jit}^{**} \tag{5a}$$
Summing over j,
$$\textstyle\sum_{j\in Ji} (-\mu_{it} + \beta^{max}_{jit})p_{jit}^{**} = -\sum_{j\in Ji} c_{jit}p_{jit}^{**} \tag{5b}$$
Rearranging and substituting $p_{it}^{**} = \sum_{j\in Ji} p_{jit}^{**}$
$$\mu_{it}p_{it}^{**} = \textstyle\sum_{j\in Ji} (c_{jit} + \beta^{max}_{jit})p_{jit}^{**} \tag{5c}$$
By complementary slackness of (3c),
$$(\rho^{up}_{it} - \rho^{up}_{it+1} - \lambda_t + \mu_{it} - \beta^{min}_{it} + \beta^{max}_{it})p_{it}^{**} = 0 \tag{5d}$$
Rearranging,
$$\mu_{it}p_{it}^{**} = -(\rho^{up}_{it} - \rho^{up}_{it+1} - \lambda_t - \beta^{min}_{it} + \beta^{max}_{it})p_{it}^{**} \tag{5e}$$
Combining (5c) and (5d), and rearranging,
$$\lambda_t p_{it}^{**} = \textstyle\sum_{j\in Ji} c_{jit}p_{jit}^{**} + \sum_{j\in Ji} \beta^{max}_{jit}p_{jit}^{**} + \beta^{max}_{it}p_{it}^{**} - \beta^{min}_{it}p_{it}^{**} + (\rho^{up}_{it} - \rho^{up}_{it+1})p_{it}^{**} \tag{5f}$$
Rearranging,
$$\textstyle\sum_{j\in Ji} \beta^{max}_{jit}p_{jit}^{**} + \beta^{max}_{it}p_{it}^{**} - \beta^{min}_{it}p_{it}^{**} = \lambda_t p_{it}^{**} - \sum_{j\in Ji} c_{jit}p_{jit}^{**} - (\rho^{up}_{it} - \rho^{up}_{it+1})p_{it}^{**} \tag{5g}$$
From (2f) by complementary slackness,
$$(p_{jit}^{**} - p^{max}_{jit}u_{it}^{**})\beta^{max}_{jit} = 0 \tag{5h}$$
From (2h) by complementary slackness,
$$(p_{it}^{**} + p^{ru}_{it}{}^{**} - p^{max}_{it}u_{it}^{**})\beta^{max}_{it} = 0 \tag{5i}$$
From (2j) by complementary slackness,
$$(-p_{it}^{**} + p^{min}_{it}u_{it}^{**})\beta^{min}_{it} = 0 \tag{5j}$$
Substituting (5h), (5i), and (5j) into (5o) and rearranging
$$[\textstyle\sum_{j\in Ji} (p^{max}_{jit}\beta^{max}_{jit}) + p^{max}_{it}\beta^{max}_{it} - p^{min}_{it}\beta^{min}_{it}]u_{it}^{**} = \lambda_t p_{it}^{**} - \sum_{j\in Ji} c_{jit}p_{jit}^{**} + p^{ru}_{it}{}^{**}\beta^{max}_{it} - (\rho^{up}_{it} - \rho^{up}_{it+1})p_{it}^{**} \tag{5k}$$
By complementary slackness of (2k),
$$(p_{it}^{**} - p_{it-1}^{**} - r^{up}_{it}u_{it}^{**})\rho^{up}_{it}{}^{**} = 0 \tag{5l}$$
Rearranging,
$$(p_{it}^{**} - p_{it-1}^{**})\rho^{up}_{it} = r^{up}_{it}u_{it}^{**}\rho^{up}_{it} \tag{5m}$$
By complementary slackness of (3g),
$$[\gamma_{it} + \delta_{it} - \delta_{it+1} - r^{up}_{it}\rho^{up}_{it} + p^{min}_{it}\beta^{min}_{it} - \textstyle\sum_{j\in Ji}(p^{max}_{jij}\beta^{max}_{jit}) - p^{max}_{it}\beta^{max}_{it} - p^{rumax}_{it}\beta^{rumax}_{it} + c^{op}_{it}]u_{it}^{**} = 0 \tag{5n}$$
Rearranging,
$$[\gamma_{it} + \delta_{it} - \delta_{it+1} - r^{up}_{it}\rho^{up}_{it} - p^{rumax}_{it}\beta^{rumax}_{it} + c^{op}_{it}]u_{it}^{**} = [p^{min}_{it}\beta^{min}_{it} - \textstyle\sum_{j\in Ji}(p^{max}_{jij}\beta^{max}_{jit}) - p^{max}_{it}\beta^{max}_{it}]u_{it}^{**} \tag{5o}$$
Substituting (5k), (5m), into (5o) and rearranging
$$\lambda_t p_{it}^{**} = \textstyle\sum_{j\in Ji} c_{jit}p_{jit}^{**} - p^{ru}_{it}{}^{**}\beta^{max}_{it} + [\gamma_{it} + \delta_{it} - \delta_{it+1} - p^{rumax}_{it}\beta^{rumax}_{it} + c^{op}_{it}]u_{it}^{**} - (p_{it}^{**} - p_{it-1}^{**})\rho^{up}_{it} + (\rho^{up}_{it} - \rho^{up}_{it+1})p_{it}^{**} \tag{5p}$$
By complementary slackness of (2i),
$$(p^{ru}_{it} - p^{rumax}_{it}u_{it})\beta^{rumax}_{it} = 0 \tag{5q}$$
By complementary slackness of (3f),
$$(\beta^{rumax}_{it} + \beta^{max}_{it} - \lambda^{us}_t + c^{ru}_{it})p^{ru}_{it} = 0 \tag{5r}$$
Subtracting (5r) and (5q) and rearranging,
$$-p^{rumax}_{it}u_{it}\beta^{rumax}_{it} = (\beta^{max}_{it} - \lambda^{us}_t + c^{ru}_{it})p^{ru}_{it} \tag{5s}$$
Substituting (5s) into (5v) and rearranging,
$$\lambda_t p_{it}^{**} + \lambda^{us}_t p^{ru}_{it}{}^{**} = \textstyle\sum_{j\in Ji} c_{jit}p_{jit}^{**} + c^{ru}_{it}p^{ru}_{it}{}^{**} + [\gamma_{it} + \delta_{it} - \delta_{it+1} + c^{op}_{it}]u_{it}^{**} - (p_{it}^{**} - p_{it-1}^{**})\rho^{up}_{it} + (\rho^{up}_{it} - \rho^{up}_{it+1})p_{it}^{**} \tag{5t}$$

Ramp rate dynamics. Summing $(\rho^{up}_{it-1} - \rho^{up}_{it})p_{it}$ over $\mathcal{T}$.
$$\textstyle\sum_{\mathcal{T}} (\rho^{up}_{it} - \rho^{up}_{it+1})p_{it}^{**} = \rho^{up}_{i1}p_{i1}^{**} - \rho^{up}_{i1}p_{i2}^{**} + \rho^{up}_{i2}p_{i2}^{**} - \rho^{up}_{i23}p_{i3} + \ldots + \rho^{up}_{iT}p_{iT}^{**} - \rho^{up}_{iT+1}p_{iT}^{**} \tag{5u}$$
Summing $(p_{it+1}^{**} - p_{it}^{**})\rho^{up}_{it}$ over $\mathcal{T}$
$$\textstyle\sum_{\mathcal{T}} (p_{it}^{**} - p_{it-1}^{**})\rho^{up}_{it} = p_{i1}^{**}\rho^{up}_{i1} - p_{i0}^{**}\rho^{up}_{i1} + p_{i2}^{**}\rho^{up}_{i2} - p_{i1}^{**}\rho^{up}_{i2} + \ldots + p_{iT}^{**}\rho^{up}_{iT} - p_{iT-1}^{**}\rho^{up}_{iT} \tag{5v}$$

Subtracting (5u) from (5v) and since p_{iT+1}^{**} not in the model we set it to 0.

$$\sum_{\mathcal{T}} (p_{it}^{**} - p_{it-1}^{**})\rho^{up}_{it} - \sum_{\mathcal{T}'} (\rho^{up}_{it} - \rho^{up}_{it+1})p_{it}^{**} = -p_{i0}^{**}\rho^{up}_{i1} - \rho^{up}_{iT+1}p_{iT}^{**} \quad (5w)$$

In the first term, ρ^{up}_{i0} is undefined in the model and set to 0; p_{i0}^{**} is a parameter from the previous operating period. we make an adjustment to p^{max}_{i1} to account for the ramp constraint in period one. In the last term, ρ^{up}_{iT+1} is outside the model horizon and set to 0. This is a general problem with finite horizon models. Initial conditions are specified and in practice, the horizon extends several periods beyond the auction horizon to minimize the effect of the end of the horizon.

Summing (5t) over $\mathcal{T}$, canceling terms and rearranging,

$$\sum_{\mathcal{T}} [\lambda_t p_{it}^{**} + \lambda^{us}_t p^{ru}_{it}^{**}] = \sum_{\mathcal{T}} [\sum_{j\in Ji} c_{jit}p_{jit}^{**} + c^{ru}_{it}p^{ru}_{it}^{**} + (\gamma_{it}+\delta_{it}-\delta_{it+1} + c^{op}_{it})u_{it}^{**}] \quad (5x)$$

The binary relaxation dynamics. Since the binary variables must satisfy the equality, (1l).
For binary variables, if $c^{op}_{it} > 0$ and $c^{su}_{it} > 0$, the following must hold:
If $z_{it}^* = 1$, then $u_{it-1}^* = 0$, $u_{it}^* = 1$, and $z^d_{it}^* = 0$
If $z^d_{it}^* = 1$, then $u_{it-1}^* = 1$, $u_{it}^* = 0$, and $z_{it}^* = 1$.

Since the relaxed binary variables must satisfy the equality, (2l), the following must hold:
From (2m), if $z_{it}^* = 0$, $z_{it}^{**} = 0$.
From (2n), if $u_{it}^* = 0$, $u_{it}^{**} = 0$.
From (2o), if $z^d_{it}^* = 0$, $z^d_{it}^{**} = 0$.
This eliminates out-of-market generators from the pricing algorithm.

Since the relaxed binary variable must satisfy the equality, (2j),
From (2m), if $z_{it}^*=1$, $0 \le z_{it}^{**} \le 1$.
Since $c^{su}_{it} > 0$ and generator i is part of the optimal solution, $z_{it}^{**} > 0$ because if $z_{it}^{**} > 0$, a less costly solution would be $z_{it}^* = 0$ which is a contradiction.
If $z_{it}^*=1$, $u_{it}^* = 1$, from(2l) $u_{it}^{**} = z_{it}^{**}$.
If $u_{it}^* = 1$ and $u_{it+1}^* = 1$, that is, was not shut down or started up in t+1, $u_{it+1}^{**} = u_{it}^{**}$
If $z^d_{it}^* = 1$, $0 \le z^d_{it}^{**} \le 1$.
If $z^d_{it}^* = 1$, $u_{it-1}^* = 1$ and $u_{it}^* = 0$. If $u_{it}^* = 0$, $u_{it-1}^{**} = z^d_{it}^{**}$.
For the up-down cycle in (1), $\mathcal{T}i = \{t', ..., t''\} = \{t|u_{it}^* = 1\} = \{t|u_{it}^{**} > 0\}$.
For $t \in \mathcal{T}i$, $z_{it}^{**} = u_{it}^{**}$. For t"+1, $u_{it''}^{**} = z^d_{it''+1}^{**}$ and if t" > t', $z_{it}^{**} = u_{it}^{**} = u_{it+1}^{**} = ... = u_{it''}^{**} = z^d_{it''+1}^{**}$
For $t \in \mathcal{T}i$, let $u_i = u_{it}^{**} = u_i^{**}$. For $t \notin \mathcal{T}i$, $u_{it}^{**} = 0$.

Since $u_{it}^{**} = u_i^{**}$ for $t \in \mathcal{T}i = \{t', ..., t''\}$ and $u_{it}^{**} = 0$ for $t \notin \mathcal{T}i$.

$$\sum_{\mathcal{T}} [(\delta_{it} - \delta_{it+1})u_{it}^{**}] = \sum_{\mathcal{T}i} [(\delta_{it} - \delta_{it+1})u_i^*] = (\delta_{it'} - \delta_{it''+1})u_i^{**} \quad (6a)$$

From complementary slackness of (2o), if $z^d_{it''+1}^{**} < z^d_{it''+1}^*$, $\omega^d_{it''+1} = 0$
From complementary slackness of (3h), if $z^d_{it''+1}^{**} < z^d_{it''+1}^*$, then $(\omega^d_{it''+1} + \delta_{it''+1}) = 0$ and

$$\delta_{it''+1} = 0^* \quad (6b)$$

From complementary slackness of (3d),

$$(\omega_{it} - \delta_{it} + c^{su}_{it})z_{it}^{**} = 0 \quad (6c)$$

For $t \notin \mathcal{T}i$, $u_{it}^{**} = 0$ and $z_{it}^{**} = 0$.
For $t \in \mathcal{T}i$, if $z_{it'}^{**} < z_{it'}^*$, from complementary slackness of (2m), $\omega_{it'} = 0$ and

$$(-\delta_{it'} + c^{su}_{it'})z_{it'}^{**} = (-\delta_{it'} + c^{su}_{it'})u_{it'}^{**} = 0 \quad (6d)$$

$\delta_{it'}u_{it'}^{**} = c^{su}_{it'}u_{it'}^{**,}$ (6e)

Substituting (6b) and (6e) into (6a),

$(\delta_{it'} - \delta_{it''+1})u_i^{**} = c^{su}_{it'}u_i^{**}$ (6f)

And (5x) becomes

$\sum_{\mathcal{T}} [\lambda_t p_{it}^{**} + \lambda^{us}_t p^{ru}_{it}^{**}] = \quad \sum_{\mathcal{T}} [\sum_{j\in Ji} c_{jit}p_{jit}^{**} + c^{ru}_{it}p^{ru}_{it}^{**} + c^{op}_{it}u_i^{**} + \gamma_{it}u_i^{**}] + c^{su}_{it'}u_i^{**}$ (6g)

Case 1. $u_i^{**} = 1$

Since $\gamma_{it} \geq 0$, if $u_i^{**} = 1$, as $\varepsilon \to 0$, (6g) becomes

$\sum_{\mathcal{T}i} (\lambda_t p_{it}^{*} + \lambda^{us}_t p^{ru}_{it}^{*})$	$-\sum_{\mathcal{T}} [\sum_{j\in Ji} c_{jit}p_{jit}^{*} + c^{op}_{it}] - c^{su}_{it'} - \sum_{\mathcal{T}} c^{ru}_{it}p^{ru}_{it}^{*}$	≥ 0	(6h)
Energy and reserves revenues	Incremental energy and reserves costs	profits	

That is, all dispatched units are profitable with only the LIP energy and reserve prices, that is, no generator needs a make-whole payment.

If $\gamma_{it} = 0$ and $u_i^{**} = 1$, (6g) becomes

$\sum_{\mathcal{T}i} (\lambda_t p_{it}^{*} + \lambda^{us}_t p^{ru}_{it}^{*})$	$-\sum_{\mathcal{T}} [\sum_{j\in Ji} c_{jit}p_{jit}^{*} + c^{op}_{it}] - c^{su}_{it'} - \sum_{\mathcal{T}} c^{ru}_{it}p^{ru}_{it}^{*}$	$= 0$	(6i)
Energy and reserves revenues	Incremental energy and reserves costs	profits	

Case 2. for $t \in \mathcal{T}i$, $u_i^{**} < u_{it}^{*}$. For $t \notin \mathcal{T}i$, if $u_{it}^{**} = 0$, $p_{it}^{**} = 0$.

From(5x),

$\sum_{\mathcal{T}i} [\lambda_t p_{it}^{**} + \lambda^{us}_t p^{ru}_{it}^{**}] = \quad \sum_{\mathcal{T}} [\sum_{j\in Ji} c_{jit}p_{jit}^{**} + c^{ru}_{it}p^{ru}_{it}^{**} + (\gamma_{it} + \delta_{it} - \delta_{it+1} + c^{op}_{it})u_{it}^{**}]$ (6j)

If as $\varepsilon \to 0$, since $p_{it}^{**} \leq p_{it}^{*}u_i^{**}$,

$\sum_{\mathcal{T}i} [\lambda_t p_{it}^{*}u_i^{**} + \lambda^{us}_t p_{it}^{*}u_i^{**}] \geq \quad [\sum_{j\in Ji} c_{jit}p_{jit}^{*}u_i^{*} + c^{ru}_{it}p^{ru}_{it}^{*}u_i^{**} + (\delta_{it} - \delta_{it+1} + c^{op}_{it})u_i^{**}]$ (6k)

Dividing by u_i^{**} for $t \in \mathcal{T}i$

$\sum_{\mathcal{T}i} [\lambda_t p_{it}^{*} + \lambda^{us}_t p_{it}^{*}] \geq \quad [\sum_{j\in Ji} c_{jit}p_{jit}^{*} + c^{ru}_{it}p^{ru}_{it}^{*} + (\delta_{it} - \delta_{it+1} + c^{op}_{it})]$ (6l)

Since $\sum_{\mathcal{T}i} (\delta_{it} - \delta_{it+1}) = c^{su}_{it'}$

$\sum_{\mathcal{T}i} [\lambda_t p_{it}^{*} + \lambda^{us}_t p_{it}^{*}] \geq \quad \sum_{\mathcal{T}i} [\sum_{j\in Ji} (c_{jit}p_{jit}^{*}) + c^{ru}_{it}p^{ru}_{it}^{*} + c^{op}_{it})] + c^{su}_{it'}$ (6m)

That is, all dispatched units are profitable using only the LIP energy and reserve prices, that is, no generator needs a make-whole payment.

:-)

We define two concepts. Generator i is an incremental (aka marginal) generator if it breaks even, that is,

$\sum_{\mathcal{T}} (\lambda_t p_{it}^{*} + \lambda^{us}_t p^{ru}_{it}^{*})$	$-\sum_{\mathcal{T}} [\sum_{j\in Ji} c_{jit}p_{jit}^{*} + c^{op}_{it}u_{it}^{*} + c^{ru}_{it}p^{ru}_{it}^{*})] + c^{su}_{it}z_{it}^{*}$	$= 0$	(7a)
Energy revenues	Incremental energy and reserves costs	profits	

Generator i is an infra-incremental (aka infra-marginal) generator if it has positive profit, that is,

$\sum_{\mathcal{T}} (\lambda_t p_{it}^{*} + \lambda^{us}_t p^{ru}_{it}^{*})$	$-\sum_{\mathcal{T}} [\sum_{j\in Ji} c_{jit}p_{ji}^{*} + c^{op}_{it}u_{it}^{*} + c^{ru}_{it}p^{ru}_{it}^{*})] + c^{su}_{it}z_{it}^{*}$	> 0	(7b)
Energy revenues	Incremental energy and reserves costs	profits	

We will demonstrate this in example 3.

Proposition 2. Absent degeneracy, in the optimal solution either the market clears on the demand function or there is an incremental generator

Proof by contradiction. The solution is not degenerate, therefore, the dual solution is unique.

Suppose there is no incremental generator and the market does not clear on the demand function and (7b) holds for all generators $i \in G^*$. We fix the binaries and solve the linear program. Since $p_i^* = \sum_{j\in Ji} p_{ji}^*$ and let c_{it} be the marginal cost of gen i at p_i^*. For any $t \in \mathcal{T}$, if the $LMP_t > c_{it}$, for any $i \in G^*$ then the solution $LMP_t = b_{it}$ for some $t \in \mathcal{T}$. If $LMP_t < b_{it}$, $c_{it} < LMP_t < b_{it}$, $p_{it}^* + \varepsilon$ where $\varepsilon > 0$ is a feasible solution with a higher market surplus which a contradiction.

:-)

Proposition 2. For an incremental generator, then λ_t and λ^{us}_t for $t \in \mathcal{T}$ is a set of prices that are minimal in the sense that higher prices are not necessary to eliminate make-whole payments for the generator and maximal in the sense that lower prices that do not eliminate make-whole payments for the incremental generator.

Proof: Let λ^- be a set of prices such that λ^-_t and λ^{us-}_t for $t \in \mathcal{T}$ be a set of prices such that $\lambda^-_t \leq \lambda_t$, $\lambda^{us-}_t \leq \lambda^{us}_t$, and for one or more t, $\lambda^-_t < \lambda_t$ or $\lambda^{us-}_t < \lambda^{us}_t$ since at revenue of $\sum_{\mathcal{T}} (\lambda_t p_{it}^* + \lambda^{us}_t p^{ru}_{it}{}^*)$ the generator breaks even, the revenue at λ^-_t and λ^{us-}_t is

$$\sum_{\mathcal{T}} (\lambda^-_t p_{it}^* + \lambda^{us-}_t p^{ru}_{it}{}^*) \leq \sum_{\mathcal{T}} (\lambda_t p_{it}^* + \lambda^{us}_t p^{ru}_{it}{}^*) \quad (7c)$$

If $\lambda^-_t < \lambda_t$ or $\lambda^{us-}_t < \lambda^{us}_t$ is paired with $p_{it}^* > 0$ or $p^{ru}_{it} > 0$, then

$$\sum_{\mathcal{T}} (\lambda^-_t p_{it}^* + \lambda^{us-}_t p^{ru}_{it}{}^*) < \sum_{\mathcal{T}} (\lambda_t p_{it}^* + \lambda^{us}_t p^{ru}_{it}{}^*) \quad (7d)$$

and requires a make-whole payment.

Let λ^+ be a set of prices such that $\lambda^+_t \geq \lambda_t$, $\lambda^{us+}_t \geq \lambda^{us}_t$ for $t \in \mathcal{T}$ and for one or more t, $\lambda^+_t > \lambda_t$, or $\lambda^{us+}_t > \lambda^{us}_t$ then total revenue for resource i. since at revenue $\sum_{\mathcal{T}} \lambda_t p_{it}^*$ the generator breaks even, the revenue at λ^+_t and λ^{us+}_t is

$$\sum_{\mathcal{T}} (\lambda^+_t p_{it} p_{it}^* + \lambda^{us+}_t p^{ru}_{it}{}^*) \geq \sum_{\mathcal{T}} (\lambda_t p_{it}^* + \lambda^{us}_t p^{ru}_{it}{}^*). \quad (7e)$$

If $\lambda^+_t < \lambda_t$ or $\lambda^{us+}_t < \lambda^{us}_t$ is paired with $p_{it}^* > 0$ or $p^{ru}_{it} > 0$, then

$$\sum_{\mathcal{T}} (\lambda^+_t p_{it}^* + \lambda^{us+}_t p^{ru}_{it}{}^*) > \sum_{\mathcal{T}} (\lambda_t p_{it}^* + \lambda^{us}_t p^{ru}_{it}{}^*) \quad (7f)$$

and produces a positive profit.

:-)

Cost Allocation and Settlement

From (5t), for each time period

$\lambda_t p_{it}^{**} + \lambda^{us}_t p^{ru}_{it}{}^{**} =$	$\sum_{j \in J_i} c_{jit} p_{jit}^{**} + c^{ru}_{it} p^{ru}_{it}{}^{**}$	$+\gamma_{it} u_{it}^{**}$	$+c^{op}_{it} u_{it}^{**} + [\delta_{it} - \delta_{it+1}] u_{it}^{**}$	$+(p_{it+1}^{**} - p_{it}^{**}) \rho^{up}_{it} - (\rho^{up}_{it} - \rho^{up}_{it-1}) p_{it}^{**}$
revenues	Marginal costs incurred	'profits'	Reallocation of costs due to the relaxed binaries	Changes in temporal prices due to binding ramp rates

As $\varepsilon \to 0$, $p_{it}^{**} = p_{it}^* u_i^{**}$, for $t \in \mathcal{T}i$,

$\lambda_t p_{it}^* + \lambda^{us}_t p^{ru}_{it}{}^* =$	$\sum_{j \in J_i} c_{jit} p_{jit}^* + c^{ru}_{it} p^{ru}_{it}{}^*$	$+\gamma_{it}^*$	$+c^{op}_{it} + \delta_{it}^* - \delta_{it+1}^*$	$+(p_{it+1}^* - p_{it}^*) \rho^{up}_{it} - (\rho^{up}_{it} - \rho^{up}_{it-1}) p_{it}^*$
revenues	Marginal costs incurred	'profits'	Reallocation of costs due to the relaxed binaries	Changes in temporal prices due to binding ramp rates rates

Revenues in period t from LIPs can be divided into four parts: From (6f),The term, $[\delta_{it} - \delta_{it+1}] u_{it}^*$, summed over $\mathcal{T}$ is c^{su}_{it}. The individual terms distribute the startup cost to other periods where the generator is needed. The term, $(p_{it+1}^* - p_{it}^*) \rho^{up}_{it} - (\rho^{up}_{it} - \rho^{up}_{it-1}) p_{it}^*$, reallocate costs due to ramp rate constraints. The term, $\gamma_{it} u_{it}^*$, is nonnegative since both components are nonnegative. If $\gamma_{it} = 0$ for $t \in \mathcal{T}i$, the generator i is an incremental generator. If any $\gamma_{it} > 0$, the generator i is an infra-incremental generator. If $\varepsilon = 0$, (2) becomes degenerate or more degenerate and may produce different dual variables including the prices.

Proposition 3. In AIC pricing, the arbitrage conditions holds for energy and reserves. If the ramp rate constraint does not bind, the static arbitrage condition holds, that is,

$$\lambda_t - \lambda^{us}_t = c_{j'it} - c^{ru}_{it} \quad (8a)$$

If the ramp rate constraint does bind, the dynamic arbitrage condition holds, that is,

$$\lambda_t - \lambda^{us}_t = c_{j'it} - c^{ru}_{it} + + \rho^{up}_{it} - \rho^{up}_{it+1} \quad (8b)$$

where j' denotes the highest active marginal cost step , that is, if $j > j'$, $p_{jit}^* = 0$.

Proof: $p_{it}{}^* > 0$, from complementary slackness of (3c),

ρ^{up}_{it} -ρ^{up}_{it+1} -λ_t + μ_{it} -β^{min}_{it} + β^{max}_{it} = 0 (8c)

From complementary slackness of (3f), if $p^{ru}_{it}{}^{**} > 0$,

$(\beta^{rumax}_{it} + \beta^{max}_{it} + c^{ru}_{it} - \lambda^{us}_t) = 0.$ (8d)

From complementary slackness of (3e), if $p_{jit} > 0$,

$\mu_{it} = c_{jit} + \beta^{max}_{jit}$ (8e)

If there is a j'∈Ji where $p^{min}_{it} < p_{j'it}{}^* < p^{max}_{j'it}$, then $\beta^{max}_{j'it} = 0$ $\mu_{it} = c_{j'it}$. For j >j', $p_{jit}{}^* = 0$, and.

$\mu_{it} = c_{j'it}$ (8f)

For j >j', $p_{jit}{}^* = 0$ and $p_{it}{}^* = \sum_{j\in Ji} p_{jit}{}^* < p^{max}_{it}$

if $p^{min}_{it} < p_{it}{}^* < p^{max}_{it}$, $\beta^{max}_{it} = 0$, $\beta^{min}_{it} = 0$.

Combining (8c) and (8f),

$(\rho^{up}_{it} - \rho^{up}_{it+1} - \lambda_t + c_{j'it}) = 0$ (8g)

Rearranging,

$\lambda_t = c_{j'it} + \rho^{up}_{it} - \rho^{up}_{it+1}$ (8h)

By complementary slackness of (3f), if $0 < p^{ru}_{it}{}^* < p^{rumax}_{it}$, $\beta^{rumax}_{it} = 0$, and

$\lambda^{us}_t = c^{ru}_{it}$ (8i)

Adding (8g) and (8i),

$\lambda_t - \lambda^{us}_t = c_{j'it} - c^{ru}_{it} + \rho^{up}_{it} - \rho^{up}_{it+1}$ (8j)

If there is no binding ramp constraint, $\rho^{up}_{it} - \rho^{up}_{it+1} = 0$ and

$\lambda_t - \lambda^{us}_t = c_{j'it} - c^{ru}_{it}$ (8k)

:-)

Proposition 4. When each generator is in the optimally dispatch at its maximum operating level, the market clears at a point on the price-responsive demand function, that is, the price-responsive demand function sets the energy price and absent degeneracy, all generators dispatched make positive profits.

Proof: if each generator is dispatched at its maximum operating level, there is no possibility of Ramsey–Boiteux (RB) prices. The price-responsive demand function sets the energy price above all AICs and the LMP = LIP

Proposition 5. Without a price-responsive demand and if there is positive fixed cost investment for new generation, there may not be enough profit to stimulate efficient investment.

Proof: obvious.

Examples

Small examples serve three purposes. First, they allow the reader to track and replicate the example results. Second, they help with intuition. Third, they may show pathologies that are hidden or less pronounced in larger problems, for example, degeneracy and horizon effects. Degeneracy can produce unusual pricing results, fortunately degeneracy is not known to be a serious problem in practice. In practice, the initial conditions are specified, and the horizon is extended several periods beyond the settlement periods to dampen any end-of-horizon influence. Nevertheless, large examples on actual problems are the acid test for implementation. In this section, we focus on small examples with two variations of AIC pricing. Variation 1 (V1) sets $p^{max} = p^*+\varepsilon$ only for generators with negative profits under LMP prices. Variation 2 (V2) sets $p^{max} = p^*+\varepsilon$ for all dispatched generators.

At some point as $\varepsilon\to 0$, ε becomes numerically zero and the problem becomes numerically degenerate. We show a series of results as $\varepsilon\to 0$ and $\varepsilon=0$. At $\varepsilon=0$, more degeneracy occurs, that is, degeneracy in addition the degeneracy that may already be present, enlarging the set of optimal dual variables, in particular the prices. Even though the solvers choice of dual variables is not known to be predictable, they are repeatable.

From the examples below, it appears the V1 with ε=10^{-4} is the more stable pricing scheme Nevertheless, in these AIC pricing schemes, the incremental generators always breakeven (close to zero profit) and the infra-incremental generators make a positive profit.

Monetary units are dollars and a period is one hour, but could be any time interval. The results for examples 1 and 2 were produced by the GAMS program: MinRunRampPricDyn20200530.gms. The results for example 3 were produced by the GAMS program: MinRunRampPricDyn20200521.gms.

Example 1. Three period market with one-step marginal costs functions.
The load parameters are in Table 1.1. The generators' parameters are in Table 1.2. Table 1.3 has the market results.

Gen2 with a startup p^{max} = 26. For Gen2, the startup p^{max} = 26 and the ramp rate constraints combine to force it to startup in period 2 to be at 30 MW in period 3. Gen1 sets the price in periods 1 and 2. The binding ramp rate constraint for Gen2 from period 2 to 3 adds $40/MWh to the LMP in period 3. With an additional unit of ramp, the dispatch of Gen2 would be 24 in period 2, and 30 in period 3 saving $40. At ε=$10^{-4}$, each AIC variation allocates all residual costs of Gen2 to the LIP in period 3 – the period that caused the dispatch of Gen2. At ε=0, each AIC variation the program chooses $900/MWh as the clearing price in period 3, because Gen1 is at its maximum and Gen2 is constrained by its ramp rate.

Table 1.4 contains the avoidable costs, the settlement at the LMPs, and the LIPs settlements at ε=10^{-4} and ε=0. The LIP settlements need no make-whole payments. The sequence of AIC objective function values converge to optimal dispatch (1) objective function value because the pmax constraints bind. For problem (1) and AIC problem (2) with ε=10^{-4} and ε=0, the objective function is the same to 7 digits (see Table 1.7). At ε=0, the LIP price in period 3 is $900/MWh, the settlement changes, but all generation remains profitable. The market surplus does not change much.

Table 1.1. Load

Period	1	2	3
Value	900	900	900
Max Load	95	100	130

Table 1.2. Generation

Gen	Marg Cost $/MWh	Min Gen MW	Max Gen MW	Max at Startup MW	Start Cost $	Min Run Time hrs	Fix Oper Cost $/per	Ramp Up Rate MW/per	Ramp Dn Rate MW/per
1	10	0	100	0	0	1	0	200	900
2	50	20	35	26	1000	1	30	5	900
3	302	0	31	31	0	1	0	200	200

Table 1.3. Optimal Dispatch and Prices without Reserves for startup p^{max}_2 at 26 MW

period	1		2		3	
	Energy	Marginal Value	Energy	Marginal Value	Energy	Marginal Value
Load	95	890	100	890	130	810
Gen1	95	0	75	0	100	80

Gen2	0	0	25	0	30	0
LMP		10.00		10.00		90.00
LIP1 (ε=10^{-4})		10.00		10.00		118.67
LIP1 (ε=0)		10.00		10.00		900.00
LIP2 (ε=10^{-4})		10.00		10.00		118.67
LIP2 (ε=0)		10.00		10.00		900.00

Table 1.4. Settlement at LMP and LIP (ε=10^{-4} and ε =0) without Reserves

	Avoidable	Profit/Value at				
	cost	LMP	LIP1(ε=10^{-4})	LIP1(ε=0)	LIP2(ε=10^{-4})	LIP2(ε=0)
Gen1	2700	8000	10866.67	89000	10866.67	89000
Gen2	3810	-860	0	23440	0	23440
total	6510	7140	10866.67	112440	10866.67	112440
load		278850	275123	173550	275123	173550
MS		285990	285989.67	285990	285989.67	285990

Gen3 Entry. If we lower the marginal of Gen3 to 118.66, Table 3b contains the results. Gen3 enters the market and displaces 4 MW of Gen2. Gen2 starts up in period 3 at its startup maximum (26 MW). Gen3 sets the price in period 3, makes up the difference to satisfy the demand of 130 MW, and breaks even. The saving for dispatching Gen2 in only period 3 is $1230 (= $40/MWh×25 MWh plus $30 fix cost in period 2 and $50/MWh×4 MWh in period 3). For Gen3, at $307.5/MWh for the 4 MWh cost is $1230 and the solver is indifferent about dispatching Gen3. At $307.4/MWh, gen3 enters the market and sets the price in period 3 at 307.4. At $307.6/MWh, gen3 does not enter the market.

Table 1.3b. Optimal Dispatch, Prices and Settlement for Gen3 to 118.66 with startup p^{max} at 26 MW

period	1		2		3			
	Energy	Marginal Value	Energy	Marginal Value	Energy	Marginal Value	Avoidable cost	Value/Profit at LMP
Load	95	890	100	890	130	781.00		275132.00
Gen1	95	0	100	0	100	108.60	2950.00	8000.00
Gen2	0	0	0	0	26	68.600	2330.00	753.60
Gen3	0	0	0	0	4	0	474.40	0
LMP		10.00		10.00		118.60		
total							5754.40	283885.60

Gen2 with a startup p^{max} = 23. If the startup p^{max} is 23 MW, Gen2 would need to startup in period 1 to ramp to 30 MW in period 3. Table 1.3a has the market results and prices. The market surplus declines from the example above by $830 ($800 for extra marginal cost and $30 for fixed operating costs in period 1). The LMP in period 3 is $130/MWh set by Gen2 due to ramp rate constraints from periods 1 to 2 and 2 to 3.

An additional unit of ramp rate is worth $80/MWh ($40/MWh from period 1 to 2 and $40/MWh from period 1 to 2. With an additional unit of ramp, the dispatch of Gen2 would be 18, 24, and 30 in periods 1, 2 and 3 saving $120. The fixed-binary linear problem is degenerate. In Period 1, the p^{min} and the ramp rate constraints of Gen2 simultaneously bind blocking the dispatch of 18 in period 1. The solver sets the LMP to $130/MWh (=50+80). If the p^{min}_2 is increased to 21 in period 1, the LMP in period 3 decreases to $90/MWh.

At ε=0, V1 produces \$900/MWh as the clearing price in period 3 and \$10/MWh in periods 1 and 2. At ε=0, V2 produces a \$900/MWh price in period 1 and \$50/MWh in periods 2 and 3.

Table 1.4a contains the avoidable costs, the settlement at the LMPs, and the settlement the LIPs at $\varepsilon=10^{-4}$ and ε=0. At $\varepsilon=10^{-4}$, the settlement for V1 and V2 are the same and Gen2 is incremental. At ε=0, the settlements are considerably different since with V1 load sets the price in period 3 and in V2 sets the price in period 1 as a result of additional degeneracy. For the optimal dispatch (1) and AIC problem (2) with $\varepsilon=10^{-4}$ and ε=0, the objective function is the same to 7 digits and converges to the efficient solution (see Table 1.7).

Table 1.3a. Optimal Dispatch and Prices without Reserves for startup p^{max}_2 at 23 MW

period	1		2		3	
	Energy	Marginal Value	Energy	Marginal Value	Energy	Marginal Value
Load	95	890	100	890	130	810
Gen1	75	0	75	0	100	80
Gen2	20	0	25	0	30	0
LMP		10.00		10.00		130.00
LIP1 ($\varepsilon=10^{-4}$)		10.00		10.00		146.33
LIP1 (ε=0)		10.00		10.00		900.00
LIP2 ($\varepsilon=10^{-4}$)		10.00		10.00		146.33
LIP2 (ε=0)		900.00		50.00		50.00

Table 1.4a. Settlement at LMP and LIP ($\varepsilon=10^{-4}$ and ε =0) without Reserves

	Avoidable	Profit/Value at				
	cost	LMP	LIP1($\varepsilon=10^{-4}$)	LIP1(ε=0)	LIP2($\varepsilon=10^{-4}$)	LIP2(ε=0)
Gen1	2500	12000	13633.33	89000	13633.33	73750
Gen2	4840	-490	0	22610	0	15910
total	7340	11510	13633.33	111610	13633.33	89660
load		273650	271527.00	173550	271527.00	195500
MS		285160	285160.00	285160	285160.00	285160

Example 1 with Reserves. We add a reserves requirement of 1 MW per period to the previous problems. Reserve offer is \$1/MWh for Gen1 and \$1.5/MWh for Gen2.

Startup P^{max}_2 = 26. The startup p^{max}_2 = 26. Table 1.5 has the market results. Reserves are provided by Gen1 in periods 1 and 2 and Gen2 in period 3 when all Gen1 capacity is used for energy. The degeneracy produced by the binding ramp rates for Gen2 and a binding pmax for Gen1 results in different prices. Each AIC approach allocates all avoidable fixed costs to the period 3 – the period that caused the dispatch of Gen2. For both $\varepsilon=10^{-4}$ and ε=0, AIC V1 maintains the arbitrage condition of \$88.50 (= 90-1.5 = 117.74-29.24). In AIC V2, the relaxation of Gen1 allows it to supply ε of reserves and causes different prices at $\varepsilon=10^{-4}$, but it arbitrage condition is preserved at ε=0.

Table 1.6 contains the avoidable costs, the settlement at the LMPs, and the settlement the LIPs at $\varepsilon=10^{-4}$ and ε=0. For $\varepsilon=10^{-4}$ and ε=0, the LIP settlements need no make-whole payments and Gen2 is the incremental generator. For problem (1) and problem (2) with $\varepsilon=10^{-4}$ and ε=0, the objective function is the same to 7 digits (see Table 1.7).

Table 1.5. Optimal Dispatch and Prices with Reserves of 1 MW per period.

period	1			2			3		
	energy	marg	resrv	energy	marg	resrv	energy	marg	resrv
Load	95			100			130		
Gen1	95	0	1	75	0	1	100	1	0
Gen2	0	0	0	25	0	0	30	0	1
LMP/rLMP		10.00	1.00		10.00	1.00		116.46	1.50
LIP1/rLIP1 ($\varepsilon=10^{-4}$)		10.00	1.00		10.00	1.00		117.74	29.24
LIP1/rLIP1 ($\varepsilon=0$)		10.00	1.00		10.00	1.00		117.74	29.24
LIP2/rLIP2 ($\varepsilon=10^{-4}$)		10.00	1.00		10.00	1.00		118.67	1.50
LIP2/rLIP2 ($\varepsilon=0$)		10.00	1.00		10.00	1.00		117.74	29.24

marg is the marginal value of energy; resrv is the reserves.

Table 1.6. Settlement at LMP and LIP ($\varepsilon=10^{-4}$ and $\varepsilon=0$) with Reserves

	Avoidable	Profit/Value at				
	cost	LMP	LIP1($\varepsilon=10^{-4}$)	LIP1($\varepsilon=0$)	LIP2($\varepsilon=10^{-4}$)	LIP2($\varepsilon=0$)
Gen1	2702.00	8000	10774.18	10774.19	10866.67	10774.19
Gen2	3811.50	-860	0	0	0	0
total	6313.50	7140	10774.18	10774.19	10866.67	10774.19

Table 1.7. Market Surplus (Objective Function Value) with p^{max}_2 at startup is 26 MW

	Variation 1 without reserves	Variation 1 with reserves	Variation 2 without reserves	Variation 2 with reserves
optimal	285990.00	285986.50	285990.00	285986.50
$\varepsilon=1$	286017.74	286013.38	286017.74	286014.74
$\varepsilon=.01$	285990.29	285986.79	285990.29	285986.79
$\varepsilon=10^{-4}$	285990.00	285986.50	285990.00	285986.79
$\varepsilon=0$	285990.00	285986.50	285990.00	285986.50

Startup P^{max}_2 Is 23 MW. If the startup p^{max}_2 is 23 MW, Gen2 would need to startup in period 1 to ramp to 30 MW in period 3. Table 1.5a. has the market results and prices. The LMP in period 3 to $116.46/MWh due to ramp rate constraints and p^{max} at startup. In period 1, Gen2 is both at its minimum operating level and constrained by its ramp rate creating a degeneracy. At $\varepsilon=0$, V1 and V2 produce a $900/MWh clearing price in period 3 and $10/MWh in periods 1 and 2. In V2, the relaxation of Gen1 allows it to supply an ε of reserves and causes different prices.

Table 1.6a contains the avoidable costs, the settlement at the LMPs, and the settlement the LIPs at $\varepsilon=10^{-4}$ and $\varepsilon=0$. The LIPs produce no make-whole payments. For $\varepsilon=10^{-4}$, the incremental generator breaks even, and the infra-incremental generator makes a positive profit that is slightly different. For $\varepsilon=0$, the settlements are considerably different and both generators make positive profits. The market surplus declines from the example above by $830 ($800 for extra marginal cost and $30 for fixed operating costs in period 1. For the optimal dispatch (1) and AIC problem (2) with $\varepsilon=10^{-4}$ and $\varepsilon=0$, the objective function is the same to 7 digits and the sequence converges to the efficient solution (see Table 1.7).

Table 1.5a. Optimal Dispatch and Prices with Reserves of 1 MW per period for startup p^{max}_2 = 23 MW

period	1			2			3		
	energ	marg	reserv	energ	marg	reserv	energ	marg	reserv
Load	95			100			130		
Gen1	75	0	1	75	0	1	100	1	0
Gen2	20	0	0	25	0	0	30	0	1
LMP/rLMP	10.00		1.00	10.00		1.00	116.46		1.50
LIP1/rLMP1 ($\varepsilon=10^{-4}$)	10.00		1.00	10.00		1.00	145.80		17.30
LIP1/rLMP1($\varepsilon=0$)	10.00		1.00	10.00		1.00	900		1.50
LIP2/rLMP ($\varepsilon=10^{-4}$)	10.00		1.00	10.00		1.00	146.33		1.50
LIP2/rLMP2($\varepsilon=0$)	10.00		1.00	10.00		1.00	900		1.50

energ is energy; marg is the marginal value of energy; reserv is the reserves.

Table 1.6a. Settlement at LMP and LIP ($\varepsilon=10^{-4}$ and $\varepsilon=0$) with Reserves for startup p^{max}_2 at 23 MW

	Avoidable	Profit/Value at				
	cost	LMP	LIP1($\varepsilon=10^{-4}$)	LIP1($\varepsilon=0$)	LIP2($\varepsilon=10^{-4}$)	LIP2($\varepsilon=0$)
Gen1	2702.00	10646.15	13580.64	89000	13633.33	89000
Gen2	4841.50	-896.15	0	22610	0	22610
total	7543.50	9750	13580.64	111610	13633.33	111610

Table 1.7a. Market Surplus (Objective Function Value) with p^{max}_2 at start = 23

	Variation 1 without reserves	Variation 1 with reserves	Variation 2 without reserves	Variation 2 with reserves
optimal	285160.00	285156.50	285160.00	285156.50
$\varepsilon=1$	285175.81	285175.81	285175.81	285172.81
$\varepsilon=.01$	285160.16	285156.66	285160.16	285156.67
$\varepsilon=10^{-4}$	285160.00	285156.50	285160.00	285156.50
$\varepsilon=0$	285160.00	285156.50	285160.00	285156.50

Example 2. Five-period market with two-step marginal costs functions.
The load parameters are in Table 2.1. The generator parameters are in Table 2.2. Table 2.3 has the market results. In period 1, Gen2 step 2 sets the LMP at \$9/MWh because Gen1's ramp-up rate constraint from period 1 to 2 binds and it cannot supply any more energy in period 1. Gen1's ramp-up rate constraint shows up in period 2's LMP = 10 [= 9.5 (the marginal cost of Gen1 step2) +.5 (marginal value of the ramp-up constraint)]. In periods 3 through 5, the LMP is set by Gen1 step l. Both AIC variations $\varepsilon=10^{-4}$ and V1 at $\varepsilon=0$ allocate all avoidable fixed costs of Gen1 to period 2, the period of peak need, and produce the same prices. The exception is V2 with $\varepsilon=0$ that raises the LIP2 by \$.50/MWh in period 1 and lowers the LIP1 by \$.45/MWh in period 2.

Table 2.4 contains the settlement at the LMPs and the LIPs. The LIP settlements need no make-whole payments. Gen1 breaks even and is an incremental generator. Gen2 makes a positive profit and is an infra-incremental generator. The energy prices change for LIP2 ($\varepsilon=0$) and lowers Gen2's profits by about .5% compared to Gen2's profits under V1. The sequence of AIC objective function values converge to optimal dispatch (1) objective function. For the optimal solution, $\varepsilon=10^{-4}$ and $\varepsilon=0$ solutions, the objective function is the same to 8 digits (see Table 2.7).

Table 2.1. Load

Period	1	2	3	2	3
Value	900	900	900	900	900
Max load	140	165	90	80	70

Table 2.2. Generation

gen	Marg Cost1 $/MWh	Max Gen MW	Marg Cost2 $/MWh	Max Gen MW	Start Cost $	Start adjust MW	Min Gen MW	Min run Time $/Per	Fix cost Period $/Hr	Ramp Up Rate MW/Hr
1	4.5	80	9.5	20	800	0	0	1	50	10
2	2	30	9	40	0	0	20	1	0	200

Table 2.3. Optimal Dispatch and Prices without Reserves

period	1	2	3	4	5
	energy	energy	energy	energy	energy
load	140	165	90	80	70
Gen1	85	95	60	50	40
Gen2	55	70	30	30	30
LMP	9	10	4.5	4.5	4.5
LIP1 ($\varepsilon=10^{-4}$)	9	12.58	4.50	4.50	4.50
LIP1 ($\varepsilon=0$)	9	12.58	4.50	4.50	4.50
LIP2 ($\varepsilon=10^{-4}$)	9	12.58	4.50	4.50	4.50
LIP2 ($\varepsilon=0$)	9.5	12.13	4.50	4.50	4.50

Table 2.4. Settlement at LMP and LIP ($\varepsilon=10^{-4}$ and $\varepsilon=0$) without Reserves

	Avoidable	Profit/Value at				
	cost	LMP	LIP1($\varepsilon=10^{-4}$)	LIP1($\varepsilon=0$)	LIP2($\varepsilon=10^{-4}$)	LIP2($\varepsilon=0$)
Gen1	2635	-245	0	0	0	0
Gen2	885	715	895.53	895.53	895.53	891.71
Total	3520	470	895.53	895.53	895.53	891.71

Example 2 with Reserves. We add a reserves requirement of 1 MW per period to the above problem. Gen1's reserve cost is $1/MWh and Gen2's reserve cost is $1.5/MWh. Table 2.5 has the market optimal dispatch and prices. Reserves are provided by Gen1 in all periods. The LIPs and rLIPs move around. V1 at $\varepsilon=10^{-4}$ allocate all avoidable fixed costs of Gen1 to period 2, the period of peak need, and maintains the reserves arbitrage condition. V2 at $\varepsilon=0$ the prices are considerably.

Table 2.6 contains the avoidable costs, the settlement at the LMPs, and the settlement the LIPs at $\varepsilon=10^{-4}$ and $\varepsilon=0$. Even though the LIPs and rLIPs move around, the LIPs settlements have no make-whole payments, Gen1 always breaks even, and the settlement for Gen2 has a maximum variation of 3%. The sequence of AIC objective function values converge to optimal dispatch (1) objective function. For problem (1) and problem (2) with $\varepsilon=10^{-4}$ and $\varepsilon=0$, the objective function is the same to 8 digits (see Table 2.7).

Table 2.5. Optimal Dispatch and Prices with Reserves

period	1		2		3		4		5	
	energ	resrv	energ	resrv	energ	resrv	energ	resrv	energ	resrv
load	140		165		90		80		70	
Gen1	85	1	95	1	60	1	50	1	40	1

Gen2	55	0	70	0	30	0	30	0	30	0
LMP/rLMP	9	1	10	1	4.5	1	4.5	1	4.5	1
LIP1/rLIP1($\varepsilon=10^{-4}$)	9	1	12.55	3.55	4.50	1.00	4.50	1.00	4.50	1.00
LIP1/rLIP1 ($\varepsilon=0$)	9	1.50	12.55	3.05	4.50	1.00	4.50	1.00	4.50	1.00
LIP2/rLIP2($\varepsilon=10^{-4}$)	9	1.50	11.97	2.47	5	1.50	5	1.50	4.50	1.00
LIP2/rLIP2 ($\varepsilon=0$)	12.41	3.91	9.50	1.00	4.50	1.00	4.50	1.00	4.50	1.00

Table 2.6. Settlement at LMP and LIP ($\varepsilon=10^{-4}$) with Reserves

	Total cost	Profit at LMP	Profit at LIP1 ($\varepsilon=10^{-4}$)	Profit at LIP1 ($\varepsilon=0$)	Profit at LIP2 ($\varepsilon=10^{-4}$)	Profit at LIP2 ($\varepsilon=0$)
Gen1	2640	-245	0	0	0	0
Gen2	885	715	893.65	893.65	882.81	867.38
Total	3525	470	893.65	893.65	882.81	867.38

Table 2.7. Market Surplus (Objective Function Values).

	without reserves	with reserves
optimal	486980.00	486975.00
$\varepsilon=1$	486982.55	486977.53
$\varepsilon=.01$	486980.03	486975.03
$\varepsilon=10^{-4}$	486980.00	486975.00
$\varepsilon=0$	486980.00	486975.00

Example 3. Five period market with no reserves, $c^{op}_{it} = 0$, no ramp rate constraints and one-step marginal costs functions.

In this example, we simplify the problem and present details on the LIP calculations to give additional insight to AIC pricing. If there no reserves; $c^{op}_{it} = 0$; a single-step marginal cost; no ramp rate constraints bind; $p^{max}_{it} = (p_{it}^*+\varepsilon)$; and $p^{min}_{it} = (p_{it}^*-\varepsilon)$; (2) becomes (9)

$$MS^{AIC} = \max \sum_{t\in\mathcal{T}} [\sum_{i\in D} b_{it}d_{it} - \sum_{i\in G} (c_{it}p_{it}+c^{su}_{it}z_{it})] \quad (9a)$$

		dual var	constraints	
system balancing constraints				
$\sum_{i\in D} d_{it} - \sum_{i\in G} p_{it} = 0$	$t\in\mathcal{T}$	λ_t	energy balance	(9b)
demand constraints				
$d_{it} \le d^{max}_{it}$	$i\in D$ $t\in\mathcal{T}$	α^{max}_{it}	Max load	(9c)
$-d_{it} \le -d^{min}_{it}$	$i\in D$ $t\in\mathcal{T}$	α^{min}_{it}	Min load	(9d)
generator constraints				
$p_{it}-(p_{it}^*+\varepsilon)u_{it} \le 0$	$i\in G$ $t\in\mathcal{T}$	β^{max}_{it}	max capacity	(9e)
$-p_{it} + (p_{it}^*-\varepsilon)u_{it} \le 0$	$i\in G$ $t\in\mathcal{T}$	β^{min}_{it}	min supply	(9f)
$u_{it} - u_{it-1} - z_{it} + z^d_{it} = 0$	$i\in G$ $t\in\mathcal{T}$	δ_{it}	binary logic	(9g)
$z_{it} \le z_{it}^*$	$i\in G$ $t\in\mathcal{T}$	ω_{it}	Relaxed binaries	(9h)
$u_{it} \le u_{it}^*$	$i\in G$ $t\in\mathcal{T}$	γ_{it}	Relaxed binaries	(9i)
$z^d_{it} \le z^{d*}_{it}$	$i\in G$ $t\in\mathcal{T}$	ω^d_{it}	Relaxed binaries	(9j)
$z_{it}, z^d_{it}, u_{it} \ge 0$	$i\in G$ $t\in\mathcal{T}$		Lower bounds	(9k)

(3g) reduces to

$$\gamma_{it}+\delta_{it}-\delta_{it+1}+p^{min}_{it}\beta^{min}_{it}-p^{max}_{i}\beta^{max}_{it} \ge 0 \quad (10a)$$

By complementary slackness

$(\gamma_{it}+\delta_{it}-\delta_{it+1})u_{it}^{**} = (p^{max}{}_{i}\beta^{max}{}_{it} - p^{min}{}_{it}\beta^{min}{}_{it})u_{it}^{**}$ (10b)

for $t \in \mathcal{T}i$, if there are no binding constraints, $\rho^{up}{}_{it} = 0$ $u_i^{**} = u_{it}^{**}$ and (5t) becomes

$\lambda_t p_{it}^{**}=$	$c_{it}p_{it}^{**}$	$+[\gamma_{it}+\delta_{it}-\delta_{it+1}]u_i^{**}$	(10c)
price in period t	Marginal cost	Adjustment to the price	

Dividing by p_{it}^{**}

$\lambda_t =$ c_{it} $+[\gamma_{it}+\delta_{it}-\delta_{it+1}]u_i^{**}/p_{it}^{**}$ (10d) As $\varepsilon \rightarrow 0$, $p_{it}^{**} = p_{it}^{*}u_i^{**}$ and (10c) becomes

$\lambda_t =$ c_{it} $+(p^{max}{}_{it}\beta^{max}{}_{it} - p^{min}{}_{it}\beta^{min}{}_{it})/p_{it}^{*}$ (10e)

For $t \in \mathcal{T}i$, if $u_{it}^{**} < u_{it}^{*}$, $\gamma_{it} = 0$. Since (9e) and (9f) cannot simultaneously bind, $\beta^{min}{}_{it}\beta^{max}{}_{it} = 0$. For $t \notin \mathcal{T}i$, $u_{it}^{**} = 0$ and $p_{it}^{*} = p_{it}^{**} = 0$. If $p_{it}^{**} < (p_{it}^{*}+\varepsilon)u_{it}^{**}$, $\beta^{max}{}_{it} = 0$, and if $p_{it}^{**} = (p_{it}^{*}-\varepsilon)u_{it}^{**}$, $\beta^{min}{}_{it} \geq 0$ and (10e) becomes

$\lambda_t = c_{it} - (p_{it}^{*}-\varepsilon)\beta^{min}{}_{it}/p_{it}^{*}$ (10f)

If $p_{it}^{**} = (p_{it}^{*}+\varepsilon)u_{it}^{**}$, $\beta^{max}{}_{it} \geq 0$, and if $p_{it}^{**} > (p_{it}^{*}-\varepsilon)u_{it}^{**}$, $\beta^{min}{}_{it} = 0$ and (10e) becomes

$\lambda_t = c_{it} - (p_{it}^{*}+\varepsilon)\beta^{max}{}_{it}/p_{it}^{*}$ (10g)

To demonstrate the above properties, we present the following results. The load parameters are in Table 3.1. The generators' parameters are in Table 3.2. Table 3.3 has the market results. The AIC approach allocates startup costs and all avoidable fixed costs (when $p^{min} = p^{max}$, all costs are avoidable fixed costs) in periods 1, 2, and 3 to the peak period 4. Table 3.4 contains the avoidable costs, the settlement at the LMPs and the settlement LIPs at $\varepsilon=10^{-4}$ and $\varepsilon=0$. The LIP settlement produces no make-whole payments, is revenue neutral, and allocates all fixed costs to the period with the highest demand (period 4).

Table 3.1. Load

Period	1	2	3	4	5
Value	900	900	900	900	900
Max Load	260	270	400	430	200

Table 3.2. Generation

Gen	Marg Cost $/MWh	Min Gen MW	Max Gen MW	Start adjust MW	Start Cost $	Min Run Time hrs	Fix Cost $/per	Ramp Up Rate MW/per	Ramp Dn Rate MW/per
1	10	0	300	0	0	1	0	500	0
2	53.1	250	250	0	2020	4	0	200	0
3	206.18	0	131			1		200	

Table 3.3. Optimal Dispatch and Prices

period	1		2		3		4		5	
	ener	marg	ener	marg	ener	marg	ener	marg	ener	marg
load	260	890	270	890	400	890	430	890	200	890
Gen1	10	0	20	0	150	0	180	0	200	0
Gen2	250	-43.10	250	-43.10	250	-43.10	250	-43.10	0	-43.10
LMP		10		10		10		10		10
LIP1($\varepsilon=10^{-4}$)		10		10		10		190.48		10
LIP1($\varepsilon=0$)		10		10		10		190.48		10

Table 3.4. Settlement at LMP and LIP ($\varepsilon=10^{-4}$ and $\varepsilon=0$)

Avoidable	Value/Profit at		
cost	LMP	LIP1 ($\varepsilon=10^{-4}$)	LIP1 ($\varepsilon=0$)

Gen1	5600	0	32486.40	32486.40
Gen2	55120	-45120	0	0
Total Gen	60720	-45120	32486.40	32486.40
load		1388400	1310793.60	1310793.60
Total value		1343280	1343280.00	1343280.00
Obj. Value		1343280	1364937.61	1364937.60

We demonstrate in two different ways the relocation of Gen2 costs to the period 4. For gen2, $\gamma_{2t} = 0$. In Table 3.5, using $\lambda_t = c_{it} + [\gamma_{it}+\delta_{it}-\delta_{it+1}]u_i^{**}/p_{it}^{**}$, we see how the LIPs are calculated. The solver lowers u_2 until the p^{min} constraints binds. In this example, $\delta_{it}-\delta_{it+1}$ can be interpreted as the savings of not starting in period 1, that is, $10775 = 250\times(53.1\text{-}10)$, to satisfy demand in period 4. In period 4 where gen2 is needed the p^{min} constraint does not bind and p^{max} constraint does.

Table 3.5. The relocation of cost for Gen2 using $\lambda_t = c_{2t} + [\gamma_{2t}+\delta_{2t}-\delta_{2t+1}]u_2^{**}/p_{2t}^{**}$.

period	1	2	3	4	5	sum
$\delta_{2t}-\delta_{2t+1}$	-10775	-10775	-10775	34345	-2020	-0.09
p_{2t}^{**}	129.9999	129.9999	129.9999	130.0000	0	
u_2^{**}	.52	.52	.52	.52	0	
$\lambda_t = c_{2t} + [\gamma_{2t}+\delta_{2t}-\delta_{2t+1}]u_2^{**}/p_{2t}^{**}$	9.999876	9.999876	9.999876	190.48	n/a	

In Table 3.6 using $\lambda_t = c_{it} + (p^{max}_{it}\beta^{max}_{it} - p^{min}_{it}\beta^{min}_{it})/p_{it}^*$, we see a more economic interpretation. In the AIC relaxation, in periods 1 through 3 where Gen2 is not needed, the AIC solver minimizes the generation from the most expensive marginal cost generator, Gen2 and the p^{min}_{it} constraint binds and $\beta^{min}_{it} = 43.1$, the marginal savings for reducing p^{min}_{it} . in period 4, p^{max}_{24} binds and $\beta^{max}_{24}. = 190.48$. In period 4, the dual variable on the p^{max}_{24} constraint contains the cost of running at the minimum operating level in the three previous periods due to the minimum run time and the startup costs:

$(3\times250\times43.1$	$+2020)$	$/250$	$= 137.38$
The opportunity cost of running at the minimum operating level in the three previous periods.	startup costs	Optimal dispatch	Dual variable β^{max}_{24}

Table 3.6. The relocation of cost for Gen2 using $\lambda_t = c_{2t} + (p^{max}_{2t}\beta^{max}_{2t} - p^{m2n}_{2t}\beta^{min}_{2t})/p_{2t}^*$

period	1	2	3	4	5
β^{max}_{2t}	0	0	0	137.38	
β^{min}_{2t}	43.10	43.10	43.10	0	0
p_{2t}^*	250	250	250	250	0
$(p^{max}_{2t}\beta^{max}_{2t} - p^{min}_{2t}\beta^{min}_{2t})/p_{2t}^*$	43.10	43.10	43.10	137.38	
c_{2t}	53.10	53.10	53.10	53.10	0
λ_t	10	10	10	190.48	n/a

For $\varepsilon = 10^{-4}$, the results for Gen1, $\gamma_{1t} = 1$ are in tables 3.7 and 3.8.

Table 3.7. the relocation of cost for Gen1 using $\lambda_t = c_{1t} + [\gamma_{1t}+\delta_{1t}-\delta_{1t+1}]u_1^{**}/p_{1t}^{**}$.

period	1	2	3	4	5
$\delta_{1t}-\delta_{1t+1}$	0	-54144.00	0	54144.00	0
γ_{1t}	0	54144.00	0	0	0
p_{1t}^{**}	130.0001	140.0001	270.0001	300	200
u_1^{**}	1	1	1	1	1

$\lambda_t = c_{1t} + [\gamma_{1t} + \delta_{1t} - \delta_{1t+1}]u_1^{**}/p_{1t}^{**}$	10	10	10	190.48	10

In period 4, the system dispatches Gen2 to its maximum and it is infra-incremental.

Table 3.8 the relocation of cost for Gen1 using $\lambda_t = c_{1t} + (p^{max}_{1t}\beta^{max}_{1t} - p^{m1n}_{1t}\beta^{min}_{1t})/p_{1t}^*$

period	1	2	3	4	5
β^{max}_{1t}	0	0	0	180.48	
β^{min}_{1t}	0	0	0	0	0
p_{1t}^*	10	10	10	180	10
$(p^{max}_{1t}\beta^{max}_{1t} - p^{min}_{1t}\beta^{min}_{1t})/p_{1t}^*$	10	20	150	180.48	
c_{1t}	10	10	10	10.00	10
λ_t	10	10	10	190.48	n/a

If we add Gen3, a convex generator, to the market with $p^{max}_3 \geq p^{max}_2$and $p^{min}_3 \geq 0$. With a one-step marginal cost of 206.18, Gen3 will not enter the market, but at a marginal cost of 206.17 or less, Gen3 replaces Gen2, see Table3.9. Therefore, at a marginal cost of 190.48, Gen3 will replace Gen2, see Table3.10.

Table 3.9. Optimal Dispatch and Prices at Gen3's marginal costs = 206.17

period	1	2	3	4	5		
	energy	energy	energy	energy	energy	Avoidable cost	Value/Profit at LMP
load	260	270	400	430	200		1225578.90
Gen1	260	270	300	300	200	13300.00	117702.00
Gen2	0	0	0	0	0	0	0
Gen3	0	0	100	130		47419.10	0
Total						60719.10	1343280.90
LMP	10	10	206.17	206.17	10		

Table 3.10. Optimal Dispatch and Prices at Gen3's marginal costs = 190.48

period	1	2	3	4	5		
	energy	energy	energy	energy	energy	Avoidable cost	Value/Profit at LMP
load	260	270	400	430	200		1238601.60
Gen1	10	20	150	180	200	13300.00	108288.00
Gen2	0	0	0	0	0	0	0
Gen3	0	0	100			43810.40	0
Total						57110.40	1346889.60
LMP	10	10	190.48	190.48	10		

Carryover from the Previous Market. If Gen2 is a carryover from the previous market with a minimum run time of four periods remaining, we set $mr_2 = 4$ and startup costs = 0 (incurred in the previous day). The results are in table 3.3a and the settlement is Table 3.4a and reflect the lack of startup costs that was allocated in the last market horizon.

Table 3.3a. Optimal Dispatch and Prices

period	1	2	3	4	5
	energy	energy	energy	energy	energy
load	260	270	400	430	200
Gen1	10	20	150	180	200

Gen2	250	250	250	250	0
LMP	10	10	10	10	10
LIP ($\varepsilon=10^{-4}$)	10	10	10	182.40	10
LIP ($\varepsilon=0$)	10	10	10	182.40	10

Table 3.4a. Settlement at LMP and LIP ($\varepsilon=10^{-4}$ and $\varepsilon=0$)

	Avoidable	Value/Profit at		
	cost	LMP	LIP1 ($\varepsilon=10^{-4}$)	LIP1 ($\varepsilon=0$)
Gen1	5600	0	31032	31032
Gen2	53100	-43100	0	0
Total Gen	58700	-43100	31032	31032
Load value less mwp		1345300	1314268	1314268
Total value		1345300	1345300	1345300
Objective Function Value		1345300	1365988	1365988

In both cases, the total market value is preserved, and the surplus is reallocated from the LMP settlement both with and without make-whole payments. The incremental generator breaks even, and the infra-incremental generator makes a positive profit.

Example 4. Sub-Optimal Solutions. In practice, the MIP solver may return an integer feasible sub-optimal solution in the MIP gap or the solver simply times out. The solution is not known to be sub-optimal. Only that it has not been proven optimal. We propose an approach address this issue. Suppose the solver terminates in a sub-optimal solution. If $p_{it}^{**} > 0$ for some t and the AIC V1 solution relaxes u_{it}^{**} to 0 and $p_{it}^{**} = 0$, for all t generator i is not in the relaxation and may not be needed in the MIP. A branch and bound node child is created and resolved with $p_{it} = 0$ to see if gen i is needed.

To illustrate consider the following problem in Table 4. Suppose the solver terminates in a sub-optimal solution, $p_1 = 70$, $p_2 = 20$ and the LMP = 0. AIC V1 set $u_2 = 0$ and $p_2 = 0$. Create a new node in the in the B&B tree. The MIP solver returns a solution with $p_1 = 90$, $p_2 = 0$ and the LMP = 0.

Table 4. Feasible but Sub-Optimal Dispatch.

Unit	Pmin (MW)	Pmax (MW)	marginal value/cost ($/MWh)	Startup costs ($)	Suboptimal solution MWh	Optimal solution MWh
G1	0	100	0	10^{-8}	70	90
G2	20	20	50	100	20	0
Load	90	90	500			90
LMP					0	0
LIP					55	0

Conclusions. For a dynamic market with a multi-step marginal costs and ramp rate constraints, we modified the AIC one-pass pricing algorithm by replacing p^{max} with $p^{*}+\varepsilon$ where ε is a small positive number and obtain a set of prices that result in profits for all dispatched generators and no make-whole payments. These results continues to bridge the differences between AIC pricing and approximate CHPs.

8 AIC Pricing in Markets with Networks

8.1 Introduction

In this Chapter, we extend AIC pricing approach to auction markets with a network. The network brings additional issues to pricing. The general framework presents a cost allocation scheme that can be derived from cost causation principles. For ease of presentation we assume lossless networks. First, the examples have two buses and then three buses. In these examples, the AIC settlement is revenue neutral, non-confiscatory with no make-whole payment. The prices are transparent and send better entry and exit signals.

The auction first solves for the efficient dispatch in SCUC. The SCED yields nodal LMPs for energy (and reserves). The AIC pricing method relaxes some generators' minimum operating levels, modifies the offer parameters, and the modified auction dispatch problem is resolved to obtain AIC prices. The settlement uses the efficient dispatch quantities from the dispatch run and the prices from the AIC pricing run. The approach easily integrates into current ISO market software.

We use the standard distribution-factor approximations for power flow and define a flowgate as a transmission element (such as a transmission line or device connecting to buses) or collection of transmission elements. Flowgate Marginal Prices (FMPs) are calculated for each flowgate in the network and are defined as the change in system surplus if the flowgate capacity is increased by a small amount. FMPs are nonnegative and only positive if the flowgate is at (or beyond) its steady-state capacity. Prices at each bus are based on the price at the reference bus, the FMPs, and network distribution factors that define how power injections or withdrawals at a bus affect flowgates in the network.

Congestion rents occur when flowgate constraints are binding and produce price separation between busses (or nodes). In LMP and LMP+ pricing, the congestion rents are the sum of the product of each transmission flow and FMP. In the pricing run, if the generator's minimum operating level is relaxed, the flowgate incremental price (FIP) may not be the same as the FMP.

We show examples in which the efficient dispatch has no congestion rents and therefore, no nodal price separation, but in the pricing run, nodal price separation occurs. We show that there can be efficient incremental expansion of transmission with the FMP of zero. That is, in non-convex markets, flowgates can have no marginal value, but have incremental value. For example, when an incremental (lumpy) transmission capacity is needed to overcome a generator's minimum operating level, market surplus can increase with enough incremental transmission even if the FMP is zero in LMP pricing. This creates pricing results that give insight into the incremental value of transmission investment. Transmission entry,(as is entry in most markets) is not 'marginal', but incremental. Electric power is no exception.

In networks, any pricing scheme that relaxes the generators minimum operating level may result in 'flow reversals'. We present options for addressing this issue.

Two three-bus network examples are taken from a PJM presentation (PJM 2017). We add price-responsive demand that does not change the dispatch, but does allow the calculation of consumer and market surplus. We first reproduce PJM's efficient dispatch and then compare the PJM pricing run to the AIC pricing run. In each example, the LIP entry signal is a better than the PJM's pricing method.

The appendix addresses the transmission rights market.

8.2 Average Incremental Cost Pricing Examples in Two-Node Markets

Example 1. Two Bus, Two Generators. We start the discussion of pricing with a simple static two-bus two-generator example shown in Figure 8.1. Table 8.1 contains the market parameters.

Figure 8.1 Example 1

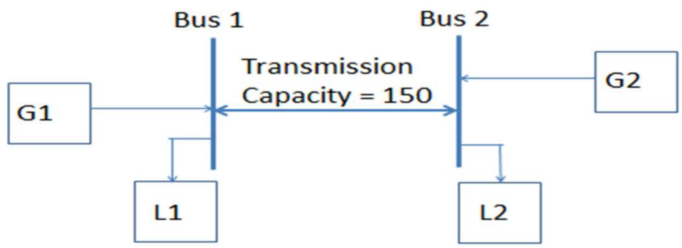


Table 8.1 Market Parameters

	Bus 1	Flowgate	Bus 2
Demand/load (MW)	30		200
generator	G1A		G2B
Startup cost	0	0	0
Minimum operating level (MW)	0	0	100
Maximum operating level (MW)	250	150	150
Marginal cost (in $/MWh)	40	0	80

Efficient Dispatch and LMPs. In Table 8.2, the efficient dispatch is to dispatch G2B at its minimum operating level and flow 100 MW from bus 1 to bus 2 to satisfy bus-2 demand of 200 MW. There is no congestion and the LMP is $40/MWh at both buses. FMP is zero.

Table 8.2 Efficient Dispatch, LMPs and FMPs.

	bus 1	Flowgate	bus 2
Efficient generation dispatch (MW)	130	100	100
load	30		200
LMP /FMP (in $/MWh)	40	0	40

The LMP+ settlement is in Table 8.3 with make-whole payments are allocated to load based on consumption. Load at bus 1 gets a make-whole-payment charge despite having no role in causing it.

Table 8.3 LMP+ Settlement (in $)

	bus 1	Flowgate	bus 2
Congestion rent		0	
Load payment	1200		4000
Load uplift	521.70		3478.30
Generator LMP payment	5200		4000
Generator Cost	5200		8000
Generator Make-whole payment	0		4000

LMP Pricing. A generator with a marginal cost less than $40/MWh and no fixed costs will enter the market at bus 1 or bus 2. The LMP at bus 2 does not reflect the avoidable costs of G2B. G2B needs a non-transparent make-whole payment of $4000. The LMP prices alone do not give a complete price signal for economic entry at bus 2. A new generator at bus 2 with an AIC of less than $80/MWh at a feasible output of 100 MW is a less expensive than G2B and it would displace G2B and increase market efficiency.

The flowgate marginal price (FMP) is 0 and indicates that a marginal unit of flowgate capacity has no value. The FMP gives a misleading entry signal for additional flowgate capacity. An increase in flowgate capacity of at least 50 MW would allow the complete displacement of G2B by G1A and increase market efficiency, but the FMP does not signal this upgrade. When making an incremental entry decision, marginal information may be insufficient.

Average Incremental Cost Pricing. For AIC pricing, we relax the minimum operating level of G2B, replace marginal costs with AICs, fix the commitment from the SCUC and rerun the SCED. The results are in Table 8.4. The Flowgate Incremental Price (FIP) is $40/MWh.

Table 8.4 AIC Pricing

	Bus 1	Flowgate	Bus 2
LIP/FIP ($/MWh)	40	40	80
AIC implied Dispatch (MW)	180	150	50

The settlement is in Table 8.5. Locational incremental price (LIP) reflects the average incremental costs of G2B and reduces make-whole payment to zero, but creates nodal price separation without a physically binding flowgate constraint and an implied dispatch of only 50 MW for G2B that is physically infeasible.

Table 8.5 AIC Settlement (in $)

	Bus 1	flowgate	Bus 2	Total
'congestion rents'		4,000		4,000
Load LIP payments	1,200		16,000	17,200
Load uplift			0	
Generator LIP revenues	5,200		8,000	13,200
Generator Cost	5,200		8,000	13,200
Make-whole payment	0		0	0

Flowgate Upgrade. Price separation without binding flowgate constraints shows that there can be an efficient incremental expansion of transmission without congestion rents. In a convex market, this would not be possible, but occurs due the non-convexities. There is no simple or correct answer to this problem, but price separation without congestion is a signal to search for incremental upgrades.

A flowgate capacity upgrade of 50 MW allows the full displacement of G2B and an increase in the market surplus of $4000 (=8000-4000) benefiting the load at bus 2. Therefore, any upgrade cost less than $4000 is an efficient investment and load at bus 2 is willing to pay for it. If the flowgate rights are fully subscribed, the FTR settlement using the AIC prices is $4000 (=100*$40), the settlement is revenue neutral.

Example 2. Two Bus, Three Generators. We present a simple static two-bus three-generator example as shown in Figure 8.2 with market parameters in Table 8.6.

Figure 8.2 Example 2 Network

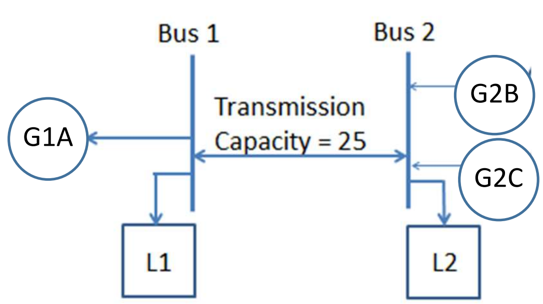


Table 8.6 Generator, Load and Flowgate Parameters

	Marginal cost ($/MWh)	Startup costs ($)	Minimum operating level (MW)	Maximum operating level (MW)
G1A	30	0	0	100
G2B	80	0	50	70

G2C	20	0	0	100
L1A		0	50	50
L2B		0	130	130
Flowgate	0	0	0	25

SCUC. Efficient dispatch is in Table 8.7. G2B operates its minimum operating level of 50 MW. The fixed cost of operating the generator at minimum operating level is $4000. G2C is dispatched to its maximum of 100 MW. G1A is dispatched to 30 MW. The flow from bus 2 to bus 1 is 20 MW. With 5 MW of spare capacity and the LMP is $30/MWh at both buses. The settlement is in Table 8.8.

Table 8.7 Efficient Dispatch, LMPs and FMPs

	bus 1		Flowgate		bus 2	
	dispatch (MW)	Cost ($)	Flow (MW)		dispatch (MW)	Cost ($)
Demand	50				130	
			20←			
G1A	30	900		G2B	50	2400
				G2C	100	2000
LMP/FMP ($/MWh)	30		0		30	

Table 8.8 Settlement (in $) using LMPs

	Bus LMP payment	Flowgate	bus 2 LMP payment	Make-whole payment
demand	1500		3900	
flowgate		0		
G1A	900			0
G2B			1500	900
G2C			3000	0

AIC Run. The AIC Pricing run results are in Table 8.9 and Table 8.10. We relax G2B's minimum operating level of 50 MW to zero. The AIC process results in an implied 'flow reversal' and price separation between buses without physical congestion. For investment purposes, this sends a signal that expansion may be economic. The flow from bus 1 to bus 2 is at its maximum of 25 MW. The LIP at bus 1 is $30 and at bus 2 is $80/MWh. The FIP indicates that there may be an efficient transmission upgrade. A transmission upgrade of 5 MW allows G2C at bus 2 to be displaced.

Table 8.9 AIC 'Dispatch' and LIP

	bus 1	Flowgate		bus 2
	dispatch (MW)	flow		dispatch (MW)
demand	50			130
		25→		
G1A	75		G2B	5
			G2C	100
LIP/FIP ($/MWh)	30	50		80

Table 8.10 AIC Settlement using LIP.

	Bus 1 LIP payment	Flowgate FIP payment		Bus 2 LIP payment
demand	1500		demand	10400
		1250		
G1A	900		G2B	8000
			G2C	4000
Make-whole payment ($)	0	0		0

LIP/FIP ($/MWh)	30	0	80

AIC Pricing Run with Fixed Flows at the Efficient Dispatch. As an alternative to flow reversals, the flow from bus 2 to 1 is fixed at its optimal dispatch to 20. Table 8.11 contains the efficient dispatch and LMPs. The LIP at bus 1 is $30 and at bus 2 is $80/MWh. Even though the LIP at bus 2 is $80/MWh, 20 MW of power flows from bus 2 to bus 1 where the LIP is $30/MWh. Both approaches yield the same AIC prices.

Table 8.11 AIC prices with the Flowgate Fixed at the Efficient Dispatch

Efficient dispatch	Bus 1		fixed	Bus 2		
LIP/FIP ($/MWh)	30		50	80		
	dispatch (MWh)	Payment ($)			dispatch (MWh)	Payment ($)
Load	50	1500		Load	130	10400
Transmission			20←			
G1A	30	900		G2B	50	4000
				G2C	100	8000

Make-whole payments ($) = 0

Example 3. Two Bus, Three Generators. Example 3 examines the question of how to handle transmission constraints and flows in the pricing run. The market parameters are in Table 8.12.

Table 8.12 Market Parameters

	Bus 1		Flowgate	Bus 2
Fixed Load (MWh)		80		100
Flowgate capacity (MW)			50	
	G1A	G1B		G2C
Offer cost ($/MWh)	20	40		70
Capacity (MW)	110	80		120
Minimum operating level (MW)	20	30		80

Efficient Dispatch and LMPs. The efficient dispatch is in Table 8.13. Since the flowgate capacity alone is not enough to meet all load at bus 2, G2A must run at its minimum of 80 MW. The LMPs at both buses are $20/MWh. The offer cost of power at bus 2 is $70. The next MWh of energy comes from bus 1 and costs only $20. The make-whole payment for G2C is $4000 (=80*(70-20)). If the make-whole payment costs are allocated across all load uniformly, the charge is $22.22/MWh of consumption. Using granular beneficiaries pay cost allocation principles, the make-whole payment cost should have been allocated to load at bus 2.

Table 8.13 Efficient Dispatch and LMPs

	Bus 1	flowgate 1-2	Bus 2
G1A dispatch (MWh)	100		
G1B dispatch (MWh)	0		
G2C dispatch (MWh)			80
Flowgate flow		20 →	
Load (MWh)	80		100
Price ($/MWh)	20	0	20
Make-whole payment ($)			4000
Make-whole payment charge ($/MWh)	22.22		22.22

AIC Pricing. We relax the minimum operating level constraint, fix each binary to its optimal level and rerun the SCED. The results are in Table 8.14. The LIPs change to $40/MWh and $70/MWh respectively and the flowgate constraint binds at a flowgate price of $30/MWh. From the AIC results, the market would be more efficient with a generator with a feasible dispatch at 50 MW with marginal cost less than $70/MWh would displace G2C and result in a more efficient dispatch.

Table 8.14 AIC pricing run without fixed flowgate flows

	Bus 1	flowgate 1-2		Bus 2
	dispatch (MWh)			dispatch (MWh)
G1A	110		G2C	50
G1B	20			
Flowgate dispatch		50→		
Load	80			100
LIP/FIP price ($/MWh)	$40	$30		$70

If we relax the minimum load constraint and fix the flowgate flow, the results are in Table 8.145 LIPs change to $20/MWh and $70/MWh respectively and the marginal flowgate price from the efficient dispatch is zero, but the price difference between bus 1 and 2 is $50/MWh. An increase in flowgate capacity of 50 MW or more would eliminate the need for G2C.

Table 8.15 AIC pricing run with fixed flowgate flow

	Bus 1	flowgate 1-2		Bus 2
	dispatch (MWh)			dispatch (MWh)
G1A	100		G2C	80
G1B	0			
Flowgate dispatch		20		
Load	80		Load	100
LIP/FIP price($/MWh)	$20	$50		$70

Example 4. Two-Bus Four-Generators. We examine different options for the pricing methodology. We explore a two-bus four-generator example shown in Figure 8.3 with the generators in Table 8.16. The demand at bus 1 is 50MW. All demand is valued at $900/MWh. Flowgate capacity is 25 MW in both directions. The demand at bus 2 increases from 90 to 190 MW in increments of 20 MW.

Figure 8.3 Two-Bus, Four-Generator Example

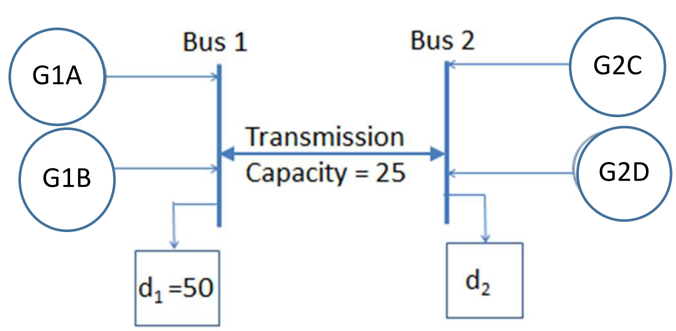


Table 8.16 Two-Bus Four-Generator Example Parameters

bus	generator	Marginal cost ($/MWh)	Startup cost ($)	Minimum operating level (MW)	Maximum operating level (MW)
1	G1A	30	0	0	45

1	G1B	45	0	0	40
2	G2C	80	0	50	70
2	G2D	20	0	0	100

Example 4.1. Demand at bus 2 is 90 MW. The efficient dispatch and LMPs results are in Table 8.17. The flow is 10 MW from bus 2 to bus 1, the FMP is 0, and the LMP at both buses is $30/MWh. Since there is no startup cost and no generators at their minimum operating level, LMPs and LIPs are the same. The value of capacity is the difference between the LMP and the marginal cost of the resource.

Table 8.17 Efficient Dispatch and LMPs. Efficient surplus is $122,800

bus1						flowgate	bus2					
gen	start up	profit	marg cost	dispatch	capacity value		gen	start up	profit	marg cost	dispatch	capacity value
G1A	1	0	30	40	0		G2C	0	0	80	0	0
G1B	1	0	45	0	0		G2D	1	1000	20	100	10
total dispatch =				40		10					100	
demand		surplus	value	dispatch					surplus	value	dispatch	
		43500	900	50					78300	900	90	
Energy LMP/FMP = $				30		0				30		

Example 4.2. Demand at bus 2 is 110 MW. The efficient dispatch and LMPs that results are in Table 8.18. The flow is 10 MW from bus 1 to bus 2, the FMP is 0, and the LMP at both buses is $45/MWh. Since there is no startup cost and no generators at their minimum operation level, LMPs and LIPs are the same.

Table 8.18 Efficient Dispatch and LMPs. Efficient surplus is $139,975

bus 1						flowgate	bus 2					
gen	Start Up	profit	marg cost	dispatch	capacity value		gen	start up	profit	marg cost	dispatch	capacity value
G1A	1	675	30	45	15		G2C	0	0	80	0	0
G1B	1	0	45	15	0		G2D	1	2500	20	100	25
total dispatch =				60		10					100	
demand		surplus	value						surplus	value		
		42750	900	50					94050	900	110	
Energy LMP/FMP = $				45		0					45	

Example 4.3. In Example 4.3, demand at bus 2 is 130 MW. The efficient dispatch and LMPs results are in Table 8.19. G2D at bus 2 is the least expensive generator and is dispatched to its maximum. G2C at bus 2 starts up and dispatched to its minimum operating level of 50 MW. Since demand at bus 2 is 130 MW, there is 20 MW of inexpensive power to send to bus 1 lowering the LMP at bus 1. G1A at bus 1 is dispatched to 30 MW to fill demand at bus 1. The flow is from bus 2 to bus 1 is 20 MW and the LMP at both buses is $30/MWh. The Make-whole payment to G2C at bus 2 is $2500.

Table 8.19 Efficient Dispatch and LMPs. Efficient surplus is $155,100

bus 1						flowgate	bus 2					
gen	Start Up	profit	marg cost	dispatch	capacity value		gen	start up	profit	marg cost	dispatch	capacity value
G1A	1	0	30	30	0		G2C	1	-2500	80	50	0
G1B	1	0	45	0	0		G2D	1	1000	20	100	10

total dispatch =			30	20			150
demand	surplus	value	dispatch		surplus	value	dispatch
	43500	900	50		113100	900	130
Energy LMP = $			30	0			30

The transmission from bus 2 to bus 1 is fixed at the efficient dispatch of 20 MW. To calculate the AIC prices, we set the minimum operating level for G2C to zero, fix each binary to its optimal value, and rerun the SCED. The results are in Table 8.20. The LIP at bus 2 is $80/MWh and the LIP at bus 1 is $30/MWh. G2D at bus 1 has a profit of $6000. Make-whole payments are zero. 'Congestion rents' are ($80-$30)*20 = $1000, but the flowgate constraint does not bind in the efficient dispatch indicating a possible efficient incremental transmission expansion.

Table 8.20 AIC Dispatch, LIPs and Settlements.

bus 1						flowgate	bus 2					
gen	Start Up	profit	marg cost	dispatch	capacity value		gen	start up	profit	marg cost	dispatch	capacity value
G1A	1	0	30	30	0		G2C	1	0	80	50	0
G1B	1	0	45	0	0		G2D	1	6000	20	100	60
		total dispatch =		30		20					150	
	demand	surplus	value						surplus	value	load	
		43500	900	50					106600	900	130	
		Energy LIP = $		30		50					80	

An alternative approach to calculate the AIC prices is to not set the non-dispatched generators to 0. We set the minimum operating level for G1B at bus 1 to zero and rerun the SCED. The results are in Table 8.21. The transmission from bus 1 to bus 2 is at capacity (25 MW) in the opposite direction of the efficient dispatch. The LIP at bus 2 is $80/MWh and the LIP at bus 1 is $45/MWh (set by G1B at bus 1). G1B at bus 1 was not in the efficient dispatch. G1A at bus 1 has a profit of $675. Make-whole payments are zero. 'Congestion rents' are ($80-$45)*20 = $700.

Table 8.21 AIC pricing and LIPs.

bus 1						flowgate	bus 2					
gen	Start Up	profit	marg cost	efficient dispatch	capacity value		gen	start up	profit	marg cost	dispatch	capacity Value
G1A	1	675	30	45	15		G2C	1	0	80	5	0
G1B	1	0	45	30	0		G2D	1	6000	20	100	60
		total generation =		75		25					105	
demand		surplus	value	dispatch					surplus	value	dispatch	
		42750	900	50					106600	900	130	
		Energy LIP/FIP = $		45		35					80	

Example 4.4. Demand at bus 2 is 150 MW. The results are in Table 8.22. GB at bus 1 is started up and dispatched at 5 MW. There is no flow on the flowgate and the LMP at both buses is $45/MWh. Make-whole payment to GA at bus 2 is $1750. The SCED solution is degenerate.

Table 8.22 Efficient Dispatch and LMPs. Efficient surplus is $172,425

bus 1						flowgate	bus 2					
gen	start up	profit	marg cost	dispatch	capacity value		gen	start up	profit	marg cost	dispatch	capacity value

gen	start up	profit	marg cost	dispatch	cap value	flowgate	gen	start up	profit	marg cost	dispatch	cap value
G1A	1	675	30	45	15		G2C	1	-1750	80	50	0
G1B	1	0	45	5	0		G2D	1	2500	20	100	25
		total dispatch =		50		0					150	
	demand	surplus	value						surplus	value		
		42750	900	50					128250	900	150	
		Energy LMP/FMP = $		45		0					45	

AIC Pricing. Set the minimum operating level for G2C to zero, and rerun the SCED. The results are in Table 8.23. The LIP at bus 2 is $80/MWh and the LIP at bus 1 is $45/MWh. Make-whole payments are zero. 'Congestion rents' are $875 (=25*35) indicating there may be an efficient incremental expansion.

Table 8.23 AIC Pricing Run. Surplus is $173,300

bus 1						flowgate	bus 2					
gen	start up	profit	marg cost	disp	cap value		gen	start up	profit	marg cost	disp	cap value
G1A	1	675	30	45	15		G2C	1	0	80	25	0
G1B	1	0	45	30	0		G2D	1	6000	20	100	60
		total generation =		75		25					125	
	demand	surplus	value						surplus	value		
		42750	900	50					123000	900	150	
		Energy LIP/FIB = $		45		35					80	

Alternative AIC prices. The transmission from bus 1 to bus 2 is fixed at the efficient dispatch of 0 MW. We set the minimum operating level for GA at bus 2 to zero. The results are in Table 8.24. The G1B is dispatched to its minimum operating level. The LMP at bus2 is $80/MWh and the LMP at bus 1 is $45/MWh. Make-whole payments are zero.

Table 8.24 AIC Pricing Run. Surplus is $172,425

bus 1						flowgate	bus 2					
gen	start up	profit	marg cost	dispatch	capacity value		gen	start up	profit	marg cost	dispatch	capacity value
G1A	1	675	30	45	15		G2C	1	0	80	50	0
G1B	1	0	45	5	0		G2D	1	6000	20	100	60
		total dispatch = 50				0					150	
	demand	surplus	value	dispatch					surplus	value	dispatch	
		42750	900	50					123000	900	150	
		Energy LIP/FIP = $		45		35					80	

Example 4.5. Demand at bus 2 is 170 MW. The results are in Table 8.25. The G2C is dispatched to its minimum operating level. The flow on the transmission line from bus 1 to bus 2 is 20. The LMP at each bus is $45/MWh. Make-whole payments to G1B are $1750.

Table 8.25 Efficient Dispatch and LMPs. Efficient surplus is $189,525.

bus 1							bus 2					
Gen	start up	profit	marg cost	dispatch	capacity value	flowgate	gen	start up	profit	marg cost	dispatch	capacity value
G1A	1	675	30	45	15		G2C	1	-1750	80	50	0
G1B	1	0	45	25	0		G2D	1	2500	20	100	25

			total dispatch =	70		20					150	
	demand	surplus	value	dispatch					surplus	value	dispatch	
		42750	900	50					145350	900	170	
			Energy LMP/FMP = $	45		0					45	

AIC Pricing. We set the minimum operating level for G2C to zero. The results are in Table 8.26. The transmission from bus 1 to bus 2 is fixed at the efficient dispatch of 20 MW. The G2A is dispatched to its minimum operating level. There is no flow on the transmission line and the LMP at both buses is $45. The LIP at bus 2 is $80/MWh and the LIP at bus 1 is $45/MWh. Make-whole payments are zero. There is no congestion, but 'congestion rents' are $700 (= (80-45)*20).

Table 8.26 AIC Pricing Run. Surplus is $189,525

		bus 1				flowgate			bus 2			
	Start	profit	marg		capacity			start	profit	marg		capacity
gen	Up		cost	dispatch	value		gen	up		cost	dispatch	value
G1A	1	675	30	45	15		G2C	1	0	80	50	0
G1B	1	0	45	25	0		G2D	1	6000	20	100	60
			total generation =	70		20					150	
	demand	surplus	value	dispatch					surplus	value	dispatch	
		42750	900	50					139400	900	170	
			Energy LIP/FIP = $	45		35					80	

An alternative AIC pricing run. We set the minimum operating level for generator to zero. The results are in Table 8.27. The GA at bus 2 is started up and dispatched to 45 (below its minimum operating level of 50). The flow on the transmission line from bus 1 to bus 2 is 25. The LMP at bus2 is $80/MWh and the LMP at bus 1 is $45/MWh. Make-whole payments are zero.

Table 8.27 AIC pricing run. Surplus is $189,700

		bus 1				flowgate			bus 2			
	Start	profit	marg		capacity			start	profit	marg		capacity
gen	Up		cost	dispatch	value		gen	up		cost	dispatch	value
G1A	1	675	30	45	15		G2C	1	0	80	45	0
G1B	1	0	45	30	0		G2D	1	6000	20	100	60
			total generation =	75		25					145	
	demand	surplus	value	dispatch					surplus	value	dispatch	
		42750	900	50					139400	900	170	
			Energy LMP/FMP = $	45		35					80	

Example 4.6. Demand at bus 2 is 190 MW. The results are in Table 8.28. The G2C is dispatched 65 MW. The flow on the transmission line from bus 1 to bus 2 is 25. The LMP at bus 2 is $80/MWh and the LMP at bus 1 is $45/MWh. There are no make-whole payments and the LIPs are the LMPs..

Table 8.28 Efficient Dispatch and LMPs. Efficient surplus is $206,100.

		bus 1				flowgate			bus 2			
	Start	profit	marg		capacity			start	profit	marg		capacity
gen	Up		cost	dispatch	value		gen	up		cost	dispatch	value
G1A	1	675	30	45	15		G2C	1	0	80	65	0
G1B	1	0	45	30	0		G2D	1	6000	20	100	60

	total dispatch =		75	25			165
demand	surplus	value	dispatch		surplus	value	dispatch
	42750	900	50		155800	900	190
	Energy LMP = $		45	35			80

Example 5. We demonstrate the incremental transmission expansion and the role of price-responsive demand. The market parameters are in Table 8.29.

Table 8.29 Two Bus Network Model Parameters

	Bus 1	flowgate	Bus 2
	L1A		L2B
Min in MW	0	0	0
Max in MW	40	120	150
Value in $/MWh	999		999
	G1A		G2B
Marginal cost in $/MWh	10		30
Min in MW	0		50
Max in MW	200		80
Startup cost in $	0		100

Example 5.1. Table 8.30 presents the efficient dispatch and LMPs from the SCED results. The LMPs at each bus are the same and the FMP is 0, that is, there is no congestion.

Table 8.30 Efficient Dispatch and LMPs. Market surplus = $186,910

	Bus 1	flowgate	Bus 2
Load (MWh)	40		150
Dispatch (MWh)	140	100	50
LMP ($/MWh)	10	0	10

We replace marginal costs with AICs and relax the minimum operating level and rerun the SCED. We obtain LIPs and AIC flowgate prices, seen in Table 8.31. There is nodal price separation without congestion, FIP (= 22) indicates there may be a beneficial incremental but not a marginal expansion.

Table 8.31 AIC pricing run and LIPs

	Bus 1	flowgate	Bus 2
LMP/FMP	10	0	10
LIP/FIP	10	22	32
AIC 'dispatch'	160	120	30

We increase the flowgate capacity by 28 MW to 148 MW and run the SCUC. There is no change in dispatch, market surplus or pricing. The LIPs and FIP are the same.

Example 5.2. Increase the flowgate capacity 149 MW. Table 8.32 has the efficient dispatch and LMPs. No make-whole payments are needed. At bus 2, demand sets the price-responsive price at bus 2 where LMP = LIP = $999/MWh.

Table 8.32 Efficient Dispatch and LMPs. Market surplus = 186,921

	Bus 1	flowgate	Bus 2
Efficient dispatch	189	149	149
LMP=LIP/FMP=FIP	10	989	999

The incremental value of an additional unit of demand at bus 2 is $999/MWh. The incremental cost of an additional unit of demand at bus 2 is

100	+30*50	-10*49	= $1110
GB Startup cost	GB Energy cost	bus 1 Dispatch savings	bus 2 Incremental cost

Example 5.3. Increase the flowgate capacity to 150 MW. The results are in Table 8.33. At bus 2, LMP = LIP = $10/MWh. Since there are no Make-Whole payments, an additional pricing run is not needed. The market surplus increases by $989.

Table 8.33 Efficient Dispatch and LMPs. Market surplus is $187,910

	Bus 1	flowgate	Bus 2
Efficient dispatch	190	150	150
LMP	10	0	10

The summarized results for transmission capacity expansion are in Table 8.34.

Table 8.34 Summary of Transmission Capacity Expansion

Transmission capacity in MW	120		148		149		150	
Market surplus in $	186910		186910		186921		187910	
Change in market surplus			0		11		989	
	Bus 1	Bus 2	Bus 1	Bus 2	Bus 1	Bus 2	Bus 1	Bus 2
LMP in $/MWh	10	10	10	10	10	999	10	10
FMP in $/MWh	0		0		989		0	
LIP in $/MWh	10	32	10	32	10	999	10	32
FIP in $/MWh	22		22		989		22	

8.3 THREE BUS EXAMPLES

PJM Example 1. This example is adapted from PJM (PJM 2017, p 36). The loads are variable and assigned values in Table 8.35. The generator parameters are in Table 8.35. The network parameters are in Table 8.37. The Optimal SCUC Dispatch is in Figure 8.4 and Figure 7.38.

Table 8.35 Load Parameters

bus	step	Value in $/MWh	Maximum consumption in MWh
1	1	500	600
2	1	500	0
3	1	500	100

Table 8.36 Generator Parameters

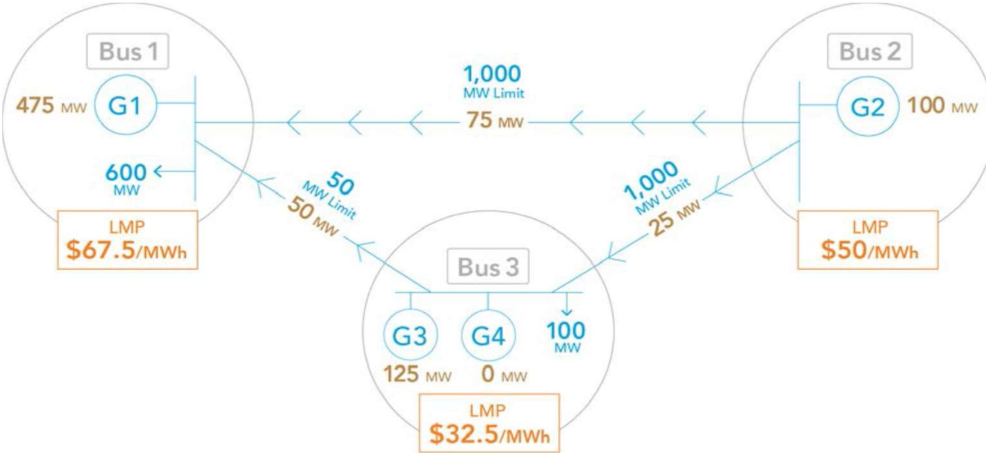
Gen	Bus	a	b	Startup cost	minimum	maximum
G1	1	20	0.1	0	0	500
G2	2	75	0	100	100	100
G3	3	20	0.1	0	0	1000
G3	3	40	0	100	100	100

The marginal cost is a + b*p where p is the dispatch.

Table 8.37 Network Distribution Factors and Capacity in MW

Flowgate	21	23	31
Bus 1	0	0	0
Bus 2	0.6667	0.3333	0.3333
Bus 3	0.3333	-0.3333	0.6667
Capacity	1000	1000	50

Figure 8.4 PJM Three Bus Example 1 and Optimal Solution



We replicate PJM's dispatch with make-whole payments and allocate them to load based on consumption. The results are in Table 8.38, Table 8.38 and Table 8.40.

Table 8.38 optimal dispatch. The optimal market surplus is $318,338

Load	Bus	Marg value	dispatch	value	surplus	LMP Charge	MWP charge	Total charge
	1	432.5	600	300000	259500	40500	2229	42729
	2	450	0	0	0	0	0	0
	3	467.5	100	50000	46750	3250	371	3621
Total			700	350000	306250	43750	2600	46350
Gen	bus	Marg cost		Value at max	profit	cost	MWP	Total Payment
G1	1	67.5	475	0	11281	20781	0	32062
G2	2	75	100	0	0	7600	2600	7600
G3	3	32.5	125	0	781	3281	0	4063
G4	3	40	0	0	0	0	0	0
total			700		12062	31662	2600	43725

Table 8.39 LMP in $/MWh

Bus 1	Bus 2	Bus 3
67.5	50	32.5

Table 8.40 LMP Flowgate Results with Flowgate Marginal Price (FMP)

Flowgate	21	23	31
Flow (MWh)	75	25	50
Maximum flow (MW)	1000	1000	50
FMP ($/MWh)	0	0	52.5
Congestion ($)	0	0	2625

AIC Pricing Run. The AIC Pricing Run results are in Table 8.41. The LIP prices and PJM ELMPs are in Table 8.42. AIC Pricing Run Transmission Flows and Flowgate Incremental Prices are in Table 8.43

Table 8.41 AIC Pricing Run Results

load bus	AIC dispatch	marginal value	total surplus	Consumer surplus	total charge
1	600	380.5	300000	228301	71699
2	0	424	0	0	0
3	100	467.5	50000	46750	3250
total	700		350000	275051	74949

Gen	bus	AIC		Value at max	total cost	profit	Total Payment
G1	1	67.5	500	52	20781	35981	56762
G2	2	75	50	0	7600	000	7600
G3	3	32.5	150	0	3281	781	4063
G4	3	40	0	32.5	0	0	0
Total			700		31662	36762	68424

Table 8.42 Nodal AIC Energy Prices (in $/MWh)

	bus_1	bus_2	bus_3
LIP	119.50	76	32.50
PJM ELMP	117.00	76	35.00
LMP	67.50	50	32.50

Table 8.43 AIC Pricing Run Transmission Flows and Flowgate Incremental Prices

Flowgate	21	23	31
Flow (MWh)	50	0	50
Maximum flow (MWh)	1000	1000	50
FIP (in $/MWh)	0	0	130.50
Congestion ($)	0	0	6524.86

In Table 8.44, we compare the resulting prices, revenues, costs, and profit of LMP+, PJM's ELMP, and AIC pricing. PJM ELMP's are the energy prices from the PJM pricing run. PJM's pricing run has make-whole payments. The AIC prices are slightly higher with no make-whole payments and G2 receives a small profit. Load payment increases as does the profit of G1.

Table 8.44 Summary Table in $.

	LMP +	ELMP	AIC
Load Payment	46,350	73,700	74,949
Uplift	2600	650*	0
G1 Profit	11,282	35,413	35,981
G2 Profit	0	0	0
G3 Profit	781	1,125	781
G4 Profit	0	0	0

*This value is published in the PJM example; however, it could not be replicated.

Entry at Bus 1. To demonstrate the value of the LIP signal, we add a new convex generator, Gen Ea at bus 1 with parameters in Table 8.45. Gen Ea's marginal cost is above PJM ELMP of $117/MWh, but below the LIP of $119.5/MWh. AIC signals entry for Gen Ea. The PJM's ELMP is too low to signal entry for Gen Ea.

Table 8.45 Gen Ea Parameters

gen	bus	a	b	Startup cost	minimum	maximum
GEa	1	119	0	0	0	200

Results are in Table 8.46. GEa's flexibility with a minimum operating level of zero, allows it to enter the market at 25 MWh at a cost of $2,975. G2 is replaced (from 75 MWh) at bus 2. G1 is dispatched at 500 MWh (an additional 25 MWh). G3 is dispatched at 175 MWh (an additional 25 MWh). Optimal-surplus increases $1056 from $318,438 without Gen Ea at bus 1 to $319,494 with Gen Ea. LIP signals entry for Gen Ea. The PJM's ELMP is too low to signal entry for Gen Ea.

Table 8.46 optimal dispatch, LMP Pricing. Optimal-Surplus is $319,494

Load bus			dispatch	value	total surplus	surplus	charge
1		-	600	381	300000	228600	71400
2		-	0	421.75	0	0	0
3		-	100	462.5	50000	46250	3750
Total	-		700	-	350000	274850	75150
Gen	bus	marginal cost	dispatch	value at max	cost	profit	Payment
G1	1	70	500	49	22500	37000	59500
G2	2	75	0	3.25	0	0	0
G3	3	37.5	175	0	5031	1531	6562
G4	3	40	0	0	0	0	0
GEa	1	119	25	0	2975	0	2975
Total	-	-	700		30506	38531	69038

Entry at Bus 2. If gen Eb with parameters below in Table 8.47 is placed at bus 2.

Table 8.47 Gen Eb Parameters

gen	bus	a	b	Startup cost	minimum	Maximum
Eb	2	77	0	0	0	200

The results are in Table 8.48. The flexibility (minimum operating level of zero) of Gen Eb allows it to replace G2 at 2 and generate 50 MWh. This allows G1 to be dispatched at 500 MWh. Optimal-surplus increases $1087 from $318,438 without Gen Eb to $319,525 with Gen Eb. PJM's ELMP is too low to signal entry for Gen Eb. AIC signals entry for Gen Eb. The PJM's ELMP is too low to signal entry for Gen Eb.

Table 8.48 optimal dispatch and LMP Pricing. Optimal-Surplus is $319,525

Load Bus	step		dispatch	marginal value	consumer surplus	surplus	total charge
1	1		600	381	228601	300000	71399
2	1		0	423	0	0	0
3	1		100	465	46500	50000	3500
total	-	-	700		275101	350000	74899
gen	bus	Marg. Cost	dispatch	value at maximum	profit	cost	Payment
G1	1	70	500	49	36999	22500	59499
G2	2	75	0	2	0	0	0
G3	3	35	150	0	1125	4125	5250
G4	3	40	0	0	0	0	0
GEb	2	77	50	0	0	3850	3850
total	-	-	700	-	38124	30475	68599

PJM Example 2. This example is adapted from (PJM 2017, p 39). The loads, generator parameters and network parameters are in Tables 8.49, 8.50 and 8.51. The optimal dispatch is in Table 8.52 and Figure 8.5.

Table 8.49 Load Parameters

bus	step	Value in $/MWh	Maximum consumption in MWh
1	1	500	600
2	1	500	0

3	1	500	100

Table 8.50 Generator Parameters

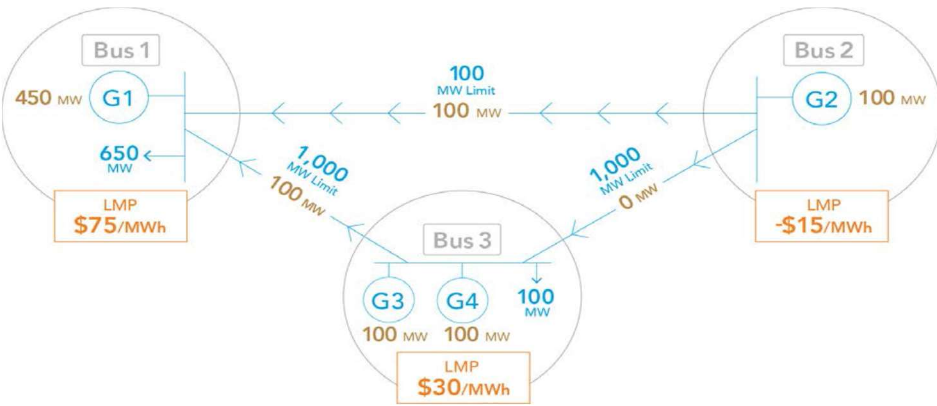
Gen	Bus	a	b	Startup cost	minimum	maximum
G1	1	30	0.1	0	0	451
G2	2	10	0	100	100	100
G3	3	20	0.1	0	0	250
G4	3	10	0	100	100	100

The marginal cost is a + b*p where p is the dispatch.

Table 8.51 Network Distribution Factors and Capacity in MW

Flowgate	21	23	31
Bus 1	0	0	0
Bus 2	0.6667	0.3333	0.3333
Bus 3	0.3333	-0.3333	0.6667
Capacity	100	1000	1000

Figure 8.5 PJM Three Bus Example 2



PJM Example 2's optimal dispatch has a degenerate solution that allows for a range of pricing (dual) solutions. To avoid degeneracy, we increase the maximum operation level for G1 at bus 1 to 451 from 450. The SCUC and SCED results are the same as PJM's and are presented in Table 8.52, 8.53 and Table 8.

Table 8.52. Efficient Dispatch. Optimal Surplus is $346,675

load bus	value	dispatch	Marg value	Consumer surplus	LMP charge	Pro-rata mwp	total charge	
1	500	650	425	276250	48750	2253	51003	
2	500	0	515	0	0	0	0	
3	500	100	470	47000	3000	347	3347	
total		750		323250	51750	2600	54350	
bus	Gen	dispatch	cost	profit	cost	mwp	Payment	
1	G1	450	75	10125	23625	0	33750	
2	G2	100	10	0	1100	2600	-1500	
3	G3	100	30	500	2500	0	3000	
3	G4	100	10	1900	1100	0	3000	
total	-	750		-	12525	28325	2600	40850

Table 8.53. LMP in $/MWh

bus_1	bus_2	bus_3
75	-15	30

Table 8.54. Flowgate Information with Flowgate Marginal Price (FMP)

Flowgate	21	23	31
Flow (MWh)	100	100	
Maximum flow (MWh)	100	1000	1000
FMP (in $/MWh)	135	0	0
Congestion ($)	13500	0	

AIC pricing run. The AIC results are in Table 8.49 through Table 8.58. The AIC prices are slightly higher with no make-whole payments. PJM's pricing run has a residual make-whole payment.

Table 8.49 AIC Pricing Run. Optimal Surplus is $338,000

load bus	step	dispatch	marg value	value	Consumer surplus	LIP charge	total charge
1	1	650	425	325000	276250	48750	48750
2	1	0	489	0	0	0	0
3	1	100	457	45700	45700	4300	4300
total		750		375000	321950	53050	53050
gen	bus		marg cost	Value at max	profit	cost	total
G1	1	500	67.5	52	10125	23625	33750
G2	2	50	75	0	000	1100	1100
G3	3	150	32.5	0	1800	2500	4300
G4	3	0	40	32.5	3200	1100	4300
total		700		-	15125	28325	43450

Table 8.50. Energy Prices in $/MWh

	bus_1	bus_2	bus_3
LIP	75	11	43
PJM ELMP	69.80	11	40.40
LMP	75	-15	30

Table 8.51. Flowgate Information with Flowgate Incremental Price (FIP)

Flowgate	21	23	31
Flow (MWh)	-100	-75	175
Maximum flow (MWh)	100	1000	1000
FIP (in $/MWh)	-96		
Congestion ($)	9600	0	0

Table 8.52 Summary Table in $

	LMP +	ELMP	AIC
Load Payment	51750	49410	53050
G1Profit	10125	7785	10125
G2Profit	0	0	0
G3Profit	500	1540	1800
G4Profit	1900	2940	3200

Entry at Bus 3. To demonstrate the value of the AIC price signal, we add Gen Ec (a convex generator) at bus 3 with parameters below in Table 8.53.

Table 8.53. Gen Ec Parameters

Gen	bus	a	b	Startup cost	minimum	maximum
GEc	3	42	0	0	0	200

Table 8. and Table 8.61 show GEc's entry at a marginal cost of $42/MWh. It is below the LIP of $43/MWh, but above the PJM ELMP of $40.40/MWh. Optimal market surplus increases. The flexibility of GEc with a minimum operating level of 0 allows it to enter the market at 80 MWh. Load remains the same. G2 at bus 2 is completely replaced to 0 from 100 MWh, G3 increases to 220 MWh from 100, and G1 decreases to 350 MWh from 450. Optimal-surplus increases $420 from $346,675 without GEc to $347,095 with GEc. LIP signals entry for Gen Eb. PJM's ELMP is too low to signal entry for Gen Eb.

Table 8.60. Efficient Dispatch and LMP Pricing. Optimal-Surplus is $347,095

bus	Load step	dispatch	marginal value	surplus	consumer surplus	Total Charge
1	1	650	435	325000	282750	42250
2	1	0	481	0	0	0
3	1	100	458	50000	45800	4200
total	-	750	-	375000	328550	46450
bus	Gen			cost	profit	Payment
1	G1	350	0	16625	6125	22750
2	G2	0	9	0	0	0
3	G3	220	0	6820	2420	9240
3	G4	100	32	1100	3100	4200
3	GEc	80	0	3360	0	3360
Total	-	750		27905	11645	39550

Table 8.61 LMPs in $/MWh

Bus	1	2	3
LMP	65.00	19.00	42.00

8.4 Prorating for Revenue Neutrality of FTRs

For any pricing mechanism that relaxes the minimum operation level (for example, RMOL, ELMP, ELMPL, and AIC), the FTR settlement can become revenue inadequate. We propose a solution by prorating FTRs.

Two Bus Example. The generator and fixed load parameters are in Table 8.62. The transmission capacity between bus 1 and bus 2 is 200 MW. Since the generators have no fixed costs, any method that relaxes the minimum operation level yields the same prices. The efficient dispatch, LMPs and FMP from the SCUC and prices from the SCED are in Table 8.63. Table 8.64 has the revenue adequate LMP/FTR settlement.

Table 8.62. Generator and Load Parameters

	bus 1				bus 2		
	Marginal cost	min	max		Marginal cost	min	max
Gen A	10	0	250	Gen B	20	50	100
	Marginal value				Marginal value		
Load 1	0	0	0	Load 2	50	230	230

Table 8.63. Efficient Dispatch. Optimal Market surplus is $8700

	bus 1 energy	Transmission Flow →	bus 2 energy
Gen	180	180	50
Load	0		230
Total	180		180
LMP	10		10
FMP		0	

Table 8.64 Settlement at the LMPs and FMP.

	bus 1 settlement	Transmission FTR settlement	bus 2 settlement
Gen	1800	0	500
Load			-2300

Total	1800		0	-1800

A negative payment is a charge

Relaxed Minimum Operation Level. The pricing run results are in Table 8.54. The unadjusted relaxed minimum operation level/FTR settlement is in Table 8.55. Payments are $4800 (= 1800+1000+2000) and receipts are $4600. It is not revenue adequate because the uncongested flowgate from the SCUC becomes 'congested' in the pricing run.

Table 8.54. Relaxed Minimum Operation Level Pricing Solution

	bus 1 energy	Transmission	bus 2 energy
Gen	200	200	30
Load	0		230
Total	200		180
LIP/ FIP	10	10	20

Table 8.55. Relaxed Minimum Operation Level Energy and FTR settlement

	bus 1	Transmission	bus 2
Gen	1800		1000
Load	0		-4600
FTR		2000	

In the efficient dispatch, only 180 MW flows from bus 1 to bus 2 in the SCED, therefore only 180 MWs need to be hedged. One approach to achieve revenue adequacy is to prorate the flowgate limits to the actual flows. Table 8.56 has the adjusted settlement for the relaxed minimum operating level pricing run. The prorating method can be applied to any pricing method that relaxes the original dispatch problem. It is revenue neutral.

Table 8.56. AIC Pricing Settlement with the Adjusted Minimum Operating Level

	bus 1	Transmission	bus 2
Gen	1800		1000
Load	0		-4600
FMP revenues		1800	

Transmission Expansion. In the LMP and LMP+ pricing methods, the FMP is zero in the SCED. This means that the line has no marginal value, but if the transmission capacity is increased by 30 MWs or more, Gen B can be completely displaced, and the market efficiency increases.

If we remove the transmission capacity constraint and rerun the SCUC, the results are in Table 8.57 and Table 8.58. The optimal solution dispatches Gen A to 230 MW and eliminates Gen B from the optimal solution. The flow from bus 1 to bus 2 is 230 MW. The new solution has a market surplus of $9200. The market with a flowgate capacity of 200 MW has market surplus of $8700. With an additional transmission capacity of 30 MWs, the market surplus increases by $500, indicating that a 30 MW upgrade to the capacity costing less than $500 is an efficient transmission investment. This example points to the failure of marginal cost pricing to give complete signals for efficient transmission expansion. This approach can be extended to networks with more than two nodes. In transmission planning, an unconstrained network scenario may help to see the possibilities for transmission expansion.

Table 8.57 Efficient Dispatch with a 30 MW expansion. Market surplus of $9200

	bus 1	Transmission	bus 2
	energy	Flow →	energy
Gen	230	230	0
Load	0		230
Total	230		230

LMP	10		10
FMP		0	

Table 8.58. Settlement with an incremental 50 MW expansion.

	bus 1 settlement	Transmission settlement	bus 2 settlement
Gen	2300	0	0
Load			2300

8.5 APPENDIX: FINANCIAL TRANSMISSION RIGHTS (FTRS)

Single-Period, Unit Commitment, Network Auction Market Notation

Sets		Associated Index
K	is the set of flowgates in the network, $k \in K$	k
N	is a set of buses or nodes in the network, $n \in N$	n
$\mathcal{N}$	$= \{(df_{kn}, p^{max}_k) \mid k \in K, n \in N\}$, a collection of distribution factors and flowgate capacities define the network.	k, n
D	is the set of nodes with demand, $n \in D \subset N$	n
G	is the set of generators $n \in G \subset N$	n
Indices		**Associated Sets**
k	is a flowgate or transmission element	K
n	is a bus or node in the network	N

Network

Parameters

df_{kn} is the energy market distribution factor, that is, is the approximate flow across k for a unit injection (or $-df^{ed}_{kn}$ for a withdrawal) at bus n and withdrawal (or injection) at the reference bus. $-1 \leq df_{kn} \leq 1$.

p^{max}_k is the minimum of thermal, voltage and stability constraints associated with flowgate k

Primal Variables

p_k is the real power flowing on $k \in K$; $p_k = \sum_{n \in N} df^{ed}_{kn}(p_n - d_n) \geq 0$.

Dual variables

τ_k is the marginal value of transmission capacity on k

ν_j is the marginal value of nomogram constraint j

Superscripts

$[\ ]^{tr}$ Denotes parameters and sets in the FTR market

$[\ ]^{ed}$ Denotes parameters and sets used in the day-ahead market

Generator

Parameters		Associated Dual Variable
p^{max}_n	is the maximum injection for $n \in G$; $p^{max}_n \geq 0$.	β^{max}_n
p^{min}_n	is the minimum injection for $n \in G$; $p^{min}_n \geq 0$.	β^{min}_n
c_n	is the marginal operating cost for $n \in G$	λ_n
c^{su}_n	is the fixed cost to startup	δ_n
c^{ai}_n	is the AIC cost for $n \in G$	λ^{ai}_n
Primal Variables		
p_n	Cleared energy for $n \in G$	λ_n
z_n	Startup binary variable for $n \in G$	δ_n

Dual Variables		Associated Primal Variable
β^{max}_n	is the marginal value of generator capacity at n	p_n
β^{min}_n	is the marginal value of generator minimum operating level of n or the marginal value of reducing generation by one unit at n.	p_n
δ_n	is dual for the startup binary. If $\delta_n \geq 0$, unit n makes a short-term profit of δ_n. If $\delta_n < 0$, unit n receives a make-whole payment of $-\delta_n$	z_n

Demand

Parameters | Associated Dual Variable

d^{max}_n	is the maximum withdrawal for $n \epsilon D$; $d^{max}_n \geq 0$.	α^{max}_n
d^{min}_n	is the minimum withdrawal for $n \epsilon D$; $d^{min}_n \geq 0$.	α^{min}_n
b_n	is the bid value per unit of withdrawal for $n \epsilon D$	λ_n
Primal Variables		Associated Dual Variable
d_n	is the cleared demand for segment for $n \epsilon D$	λ_n
Dual variables		Associated Primal Variable
α^{max}_n	is the marginal value of maximum demand $n \epsilon D$	d_n
α^{min}_n	is the marginal value of minimum demand of $n \epsilon D$	d_n

System

Primal Variables

MS is the Market surplus

Dual Variables

RC are resource costs

λ_n $= \lambda - \sum_{k \epsilon K} df_{kn}\tau_k$, marginal energy price at each node

FTR Market

B	is the set of buyers in the FTR market, $i \in B$
S	is the set of sellers in the FTR market, $i \in S$
congestion revenues (CR)	$= \sum_{k \epsilon K} p^{max}_k \tau_k$ from the FTR market
df^{tr}_{knm}	$= df^{tr}_{kn} - df^{tr}_{km}$, the distribution factor for an injection at m and a withdrawal at n in the FTR market topology $\mathcal{N}^{tr}$
$(df^{tr}_{1nm},\ldots, f^{tr}_{Knm})$	is a portfolio or set of flowgate rights for the ftr_{nm} in the FTR market topology $\mathcal{N}^{tr}$
ftr_{nmi}	is amount of purchase or sale of $(\lambda_n - \lambda_m)$ by market participant i. An ftr_{nmi} is the right and obligation of buyer i to receive or pay, $(\lambda_m - \lambda_n)ftr_{nmi}$ in the day-ahead market
b_{nmi}	is the bid from $i \in B$ to buy an FTR from n to m
c_{nmi}	is the bid from $i \in S$ to buy a counterflow FTR from m to n
η^{max}_i	is the marginal value of bid or offer i
τ_k	is the marginal value of transmission capacity

The Single-Period, Network Auction Market. We assume that each resource is located at its own bus. Some buses are electrically equivalent and will have the same distribution factor (df). Flow in the negative direction is modeled as a separate flowgate with a distribution factor that is the negative of the positive flow. The flowgate SCUC (1) is a mixed integer program (MIP):

$\max MS = \sum_{n \epsilon D} b_n d_n - \sum_{n \epsilon G} (c_n p_n + c^{su}_n z_n)$		market surplus	(1a)
system constraints		<u>description</u>	
$\sum_{n \epsilon D} d_n - \sum_{n \epsilon G} p_n = 0$		Power balance	(1b)
load constraints			
$d_n \leq d^{max}_n$	$n \epsilon D$	Upper bound on demand	(1c)
$-d_n \leq -d^{min}_n$	$n \epsilon D$	Lower bound on demand	(1d)
generator constraints			
$p_n - p^{max}_n z_n \leq 0$	$n \epsilon G$	Upper bound on supply n	(1e)
$-p_n + p^{min}_n z_n \leq 0$	$n \epsilon G$	Lower bound on supply n	(1f)
$z_n = \{0, 1\}$	$n \epsilon G$	Supply committed	(1g)
transmission constraints			
$\sum_{n \epsilon N} df^{ed}_{kn}(p_n - d_n) \leq p^{maxed}_k$	$k \epsilon K$	Max flow on k	(1h)
where the network is $\mathcal{N}^{ed}$			

SCED. From the SCUC, all $z_n \in \{0, 1\}$ are replaced by $z_n = z_n^*$, and the SCED problem becomes a linear program which yields the dual variables:

$$\max MS = \sum_{n \epsilon D} b_n d_n - \sum_{n \epsilon G} (c_n p_n + c^{su}_n z_n) \qquad \text{market surplus} \qquad (2a)$$

		dual variables	description	
$\sum_{n\in D} d_n - \sum_{n\in G} p_n = 0$		λ	Power balance	(2b)
load constraints				
$d_n \le d^{max}_n$	$n\in D$	α^{max}_n	Upper bound on demand	(2c)
$-d_n \le -d^{min}_n$	$n\in D$	α^{min}_n	Lower bound on demand	(2d)
generator constraints				
$p_n - p^{max}_n z_n \le 0$	$n\in G$	β^{max}_n	Upper bound on supply n	(2e)
$-p_n + p^{min}_n z_n \le 0$	$n\in G$	β^{min}_n	Lower bound on supply n	(2f)
$z_n = z_n^*$	$n\in G$	δ_n	Supply committed	(2g)
transmission constraints				
$\sum_{n\in N} df^{ed}_{kn}(p_n-d_n) \le p^{maxed}_k$	$k\in K$	τ_k	Max flow on k	(2h)

where the network is $\mathcal{N}^{ed}$

The optimal solution to the SCED is an optimal solution to the SCUC. The SCED dual minimizes the resource cost:

SCED (Economic) Dual. The dual problem minimizes the resource value to achieve an efficient equilibrium.

$RC = \min \sum_{n\in D} (d^{max}_n\alpha^{max}_n - d^{min}_n\alpha^{min}_n) + \sum_{n\in G} z_n^*\delta_n + \sum_{k\in K} p^{max}_k\tau_k$ resource costs (3a)

		dual variables	Equilibrium conditions	
$-\lambda + \sum_{k\in K} df_{kn}\tau_k + \alpha^{max}_n - \alpha^{min}_n = b_n$	$n\in D$	d_n	withdrawal	(3b)
$\lambda - \sum_{k\in K} df_{kn}\tau_k + \beta^{max}_n - \beta^{min}_n = -c_n$	$n\in G$	p_n	injection	(3c)
$\delta_n - p^{max}_n\beta^{max}_n + p^{min}_n\beta^{min}_n = -c^{su}_n$	$n\in G$	z_n	startup	(3d)

$\tau_k, \beta^{max}_i, \beta^{min}_n, \alpha^{max}_n, \alpha^{min}_n \ge 0; \lambda, \delta_n$ free (3e)

where the network is $\mathcal{N}^{ed}$

We define $\lambda_n = \lambda - \sum_{k\in K} df_{kn}\tau_k$

$\lambda_n = b_n - \alpha^{max}_n + \alpha^{min}_n$	$n\in D$	d_n	withdrawal	(4a)
$\lambda_n = c_n + \beta^{max}_n - \beta^{min}_n$	$n\in G$	p_n	injection	(4b)

From (4a), if $\alpha^{max}_n = \alpha^{min}_n = 0$, $\lambda_n = b_n$, that is, price-responsive demand sets the energy price at n. From (4b), if $\beta^{max}_n = \beta^{min}_n = 0$, $\alpha^{max}_n = \alpha^{min}_n = 0$, $\lambda_n = c_n$, that is, generator n sets the energy price at n.

From (3e) and duality of the primal (2) and dual (3), the surplus distribution from linear prices.

$\sum_{n\in D} b_n d_n^* - \sum_{n\in G} (c_n p_n^* + c^{su}_n z_n^*) =$	$\sum_{n\in D} (d^{max}_n\alpha^{max}_n{}^* - d^{min}_n\alpha^{min}_n{}^*)$	$+\sum_{n\in G} z_n^*\delta_n^*$	$+ \sum_{k\in K} p^{max}_k\tau_k^*$	(4d)
Market surplus	Buyer surplus	Producer surplus	Congestion revenue	

Note that producer surplus, δ_n^*, can be negative. If so, in LMP+ pricing, generator n receives a make-whole payment and make-whole payments to producers are charged to buyers.

FTR Market Model. Prior to the SCUC, the ISO assigns FTRs (or ARRs) to LSEs that pay the transmission revenue requirements. Next, the ISO conducts an FTR auction to reconfigure these rights. The network for the FTR is the ISO's best guess of day-ahead market network. In the FTR market, an FTR from n to m purchased is a financial obligation contract to receive the energy price at m and pay the energy price at n in the day-ahead SCED. The FTR Market Model (a linear program) is:

$MS = \max \sum_{i\in B} b_{nmi}ftr_{nmi} - \sum_{i\in S} c_{nmi}ftr_{nmi}$ Max surplus (5a)

		dual variables	Equilibrium conditions	
$\sum_{n,m} df^{tr}_{knm}(\sum_{i\in B} ftr_{nmi} - \sum_{i\in S} ftr_{nmi}) \le p^{maxtr}_k$	$k\in K$	τ^{tr}_k	flowgate k max flow	(5b)
$ftr_{nmi} \le ftr^{max}_{nmi}$	$i\in B\cup S$	η^{max}_i	Buyers and sellers	(5c)
$ftr_{nmi} \ge 0$	$i\in B\cup S$		Buyers and sellers	(5d)

where the network is $\mathcal{N}^{tr}$

Note that there is no appearance of an energy price in the FTR market and no energy balance constraint. Quantities purchased by buyers and sold by naked short sellers plus the sellers with the physical flowgate capacity need to balance each other.

From complementary slackness of (5b), if $\tau^{tr}_k > 0$, by definition, it is a congested flowgate and

$$\sum_{n,m\in(NxN)} df^{tr}_{knm}(\sum_{i\in B} ftr_{nmi} - \sum_{i\in S} ftr_{nmi})\tau^{tr}_k = p^{maxtr}_k\tau^{tr}_k$$

Since $p^{max}_k > 0$ and if $\tau^{tr}_k > 0$, a congested flowgate has positive value.

The dual of (5) is:

$MS = \min \sum_{i\in B} ftr^{max}_{nmi}\eta^{max}_i + \sum_{k\in K} p^{maxtr}_k\tau^{tr}_k$			Max surplus	(6a)
	<u>dual variables</u>		<u>Equilibrium</u>	
$\sum_{n,m\in(NxN)} df^{tr}_{knm}\tau^{tr}_k + \eta^{max}_i \geq b_{nmi}$	$i \in B$	ftr_{nmi}	Buyers	(6b)
$\sum_{n,m\in(NxN)} -df^{tr}_{knm}\tau^{tr}_k - \eta^{max}_i \geq -c_{nmi}$	$i \in S$	ftr_{nmi}	Sellers	(6c)
$ftr_{nmi} \geq 0 \quad i \in B\cup S$			max bid	(6d)

At optimality, duality requires that (5a) = (6a). (6a) splits the market surplus into two components: gains from trade and flowgate capacity marginal value.

$\sum_{i\in B} b_{nmi}ftr_{nmi} - \sum_{i\in S} c_{nmi}ftr_{nmi}$	$=\sum_{i\in B} ftr^{max}_{nmi}\eta^{max*}_i$	$+ \sum_{k\in K} p^{maxtr}_k\tau^{tr}_k$	(6a)
Max surplus	= Gains from trade +	flowgate capacity marginal value	

From complementary slackness of (6b), if $ftr_{nmi} > 0$ for $i \in B$,

$$\sum_{n,m\in(NxN)} df^{tr}_{knm}\tau^{tr}_k + \eta^{max}_i = b_{nmi} \qquad (7a)$$

From complementary slackness of (6c), if $ftr_{nmi} > 0$ for $i \in S$,

$$\sum_{n,m\in(NxN)} df^{tr}_{knm}\tau^{tr}_k + \eta^{max}_i = -c_{nmi} \qquad (7b)$$

If $\eta^{max}_i = 0$, bidder $i \in B\cup S$ sets the FTR price.

In the FTR market from (7a) and (7b), we can see that the FTR acquired or liquidated is a portfolio of distribution factors, df_{knm}, $k\in K$, with an underlining flowgate marginal value, τ^{tr}_k, that is set by a buyer or a seller.

From the definition of λ_n in the day-ahead market with topology $\mathcal{N}^{ed}$, an FTR from n to m is

$$\lambda_m - \lambda_n = \lambda - \sum_{k\in K} df^{ed}_{km}\tau^{ed}_k - (\lambda - \sum_{k\in K} df^{ed}_{kn}\tau^{ed}_k) \qquad (7aa)$$

Rearranging,

$$\lambda_m - \lambda_n = \sum_{k\in K} (df^{ed}_{kn} - df^{ed}_{km})\tau^{ed}_k = \sum_{k\in K} df^{ed}_{knm}\tau^{ed}_k \qquad (7bb)$$

In the day-ahead market, from (7aa) and (7bb), we can see that the FTR acquired or liquidated is a portfolio of distribution factors, df^{ed}_{knm}, $k\in K$, with an underlining flowgate marginal value, τ^{ed}_k.

For sellers for each unit of ftr_{nmi}, seller i sold a flowgate right on k if $df^{tr}_{knm} > 0$ or purchased a flowgate right on k if $df^{tr}_{knm} < 0$. Seller i receives $ftr_{nmi}\sum_{k\in K} df_{knm}\tau^{tr}_k$. Buyer i pays $ftr_{nmi}\sum_{k\in K} df^{tr}_{knm}\tau^{tr}_k$.

The Cashout from the SCED. The cashout prices of the FTRs use the nodal prices from the SCED. The congestion revenue from the SCED is

$$\sum_{k\in K} p^{maxed}_k\tau^{ed*}_k \qquad (7d)$$

The Cashout for a buyer i is:

$$(\lambda_m - \lambda_n)ftr_{nmi} \qquad (7e)$$

Proposition 7.1. If the topologies of the FTR market and the day-ahead market SCED Cashout market have the same distribution factors, $df^{tr}_{km} = df^{ed}_{km}$, and the property $p^{maxtr}_k \leq p^{maxed}_k$, for $k\in K$, the FTR market is revenue adequate.

Proof: In the SCED, the energy price at each node is

$$\lambda_n = \lambda - \sum_{k\in K} df^{ed}_{kn}\tau_k \qquad (7f)$$

The energy price difference between two nodes, m and n, is

$$\lambda_m - \lambda_n = \lambda - \sum_{k\in K} df^{ed}_{km}\tau_k - (\lambda - \sum_{k\in K} df^{ed}_{kn}\tau_k) = -\sum_{k\in K} df^{ed}_{km}\tau_k + \sum_{k\in K} df^{ed}_{kn}\tau_k \quad (7g)$$

Rearranging and substitution,

$$\lambda_m - \lambda_n = \sum_{k\in K} (df^{ed}_{kn} - df^{ed}_{km})\tau_k = \sum_{k\in K} df^{ed}_{knm}\tau_k \quad (7h)$$

The cashout for $i\in B$ is

$$(\lambda_m - \lambda_n)ftr_{nmi} = ftr_{nmi}\sum_{k\in K} df^{ed}_{knm}\tau_k \quad (7i)$$

Summing over i,

$$\sum_{i\in B} (\lambda_m - \lambda_n)ftr_{nmi} = \sum_{i\in B} ftr_{nmi} \sum_{k\in K} df^{ed}_{knm}\tau_k \quad (7k)$$

From (5b) and complementary slackness, we have the following for any branches that are binding in the assumed direction of flow.

$$\sum_{i\in B} ftr_{nmi} \sum_{n,m\in(NxN)} df^{tr}_{knm} - \sum_{i\in S} ftr_{nmi} \sum_{n,m\in(NxN)} df^{tr}_{knm} = p^{maxtr}_k \quad (7l)$$

Multiplying by the flowgate price in the economic dispatch, τ_k,

$$\sum_{i\in B} ftr_{nmi} \sum_{n,m\in(NxN)} df^{tr}_{knm}\tau_k - \sum_{i\in S} ftr_{nmi} \sum_{n,m\in(NxN)} df^{tr}_{knm}\tau_k = p^{maxtr}_k\tau_k \quad (7m)$$

If $df^{ed}_{knm} = df^{tr}_{knm}$,

$$\sum_{i\in B} ftr_{nmi} \sum_{n,m\in(NxN)} df^{ed}_{knm}\tau_k - \sum_{i\in S} ftr_{nmi} \sum_{n,m\in(NxN)} df^{ed}_{knm}\tau_k = p^{maxtr}_k\tau_k \quad (7n)$$

If $p^{maxtr}_k \le p^{maxed}_k$

$$\sum_{i\in B} ftr_{nmi} \sum_{n,m\in(NxN)} df^{ed}_{knm}\tau_k - \sum_{i\in S} ftr_{nmi} \sum_{n,m\in(NxN)} df^{ed}_{knm}\tau_k = p^{maxtr}_k\tau_k \le p^{maxed}_k\tau_k \quad (7o)$$

From (3e),

$$\sum_{k\in K}\sum_{n,m} [df^{ed}_{knm} (\sum_{i\in B} ftr_{nmi} - \sum_{i\in S} ftr_{nmi})\tau_k] \le \sum_{k\in K} p^{maxed}_k\tau_k \quad (7r)$$

Cashout payments ≤ Congestion revenues

☺

Proposition 7.2. If $\mathcal{N}^{tr} = \mathcal{N}^{ed}$, that is, the topologies of the FTR market and the day-ahead market are the same, the FTR market is revenue neutral.

Proof: From (7i) and (7j), if $p^{maxtr}_k = p^{maxed}_k$

$$\sum_{k\in K}\sum_{n,m\in(NxN)} [df^{ed}_{knm} (\sum_{i\in B} ftr_{nmi} - \sum_{i\in S} ftr_{nmi})\tau_k] = p^{maxtr}_k\tau_k = p^{maxed}_k\tau_k \quad (7m)$$

or

$$\sum_{k\in K}\sum_{n,m\in(NxN)} [df^{ed}_{knm} (\sum_{i\in B} ftr_{nmi} - \sum_{i\in S} ftr_{nmi})\tau_k] = \sum_{k\in K} p^{maxed}_k\tau_k \quad (7n)$$

Cashout payments Congestion revenues

☺

AIC Pricing Run. From the SCED, we form the AIC Pricing Run:

$$\max MS = \sum_{n\in D} b_n d_n - \sum_{n\in Gmp} c^{ai}_n p_n - \sum_{n\in Gnmp} (c_n p_n + c^{su}_n z_n) \text{ market surplus} \quad (8a)$$

Constraint	Set	<u>dual variables</u>	<u>Constraint description</u>	
$\sum_{n\in D} d_n - \sum_{n\in G} p_n = 0$		λ	Power balance	(8b)
load constraints				
$d_n \le d^{max}_n$	$n\in D$	α^{max}_n	Upper bound on demand	(8c)
$-d_n \le -d^{min}_n$	$n\in D$	α^{min}_n	lower bound on demand	(8d)
generator constraints without make-whole payment				
$p_n - p^{max}_n z_n \le 0$	$n\in G^{nmp}$	β^{max}_n	upper bound on supply n	(8d)
$-p_n + p^{min}_n z_n \le 0$	$n\in G^{nmp}$	β^{min}_n	lower bound on supply n	(8e)
$z_n = z_n^*$	$n\in G^{nmp}$	δ_n	supply committed	(8f)
generator constraints with make-whole payment				(8g)
$p_n - p^{max}_n z_n \le 0$	$n\in G^{mp}$	β^{max}_n	upper bound on supply n	(8h)
$-p_n \le 0$	$n\in G^{mp}$	β^{min}_n	lower bound on supply n	(8i)
$z_n = z_n^*$	$n\in G^{mp}$	δ_n	supply committed	(8j)
transmission constraints				(8k)
$\sum_{n\in N} df_{kn}(p_n - d_n) \le p^{max}_k$	$k\in K$	τ_k	Max flow on k	(8l)

where the network is $\mathcal{N}^{ed}$

AIC Dual. The AIC dual problem is:

$RC = \min \sum_{n\in D} (d^{max}{}_n\alpha^{max}{}_n - d^{min}{}_n\alpha^{min}{}_n) + \sum_{n\in G} z_n{}^*\delta_n + \sum_{k\in K} p^{max}{}_k\mu_k$			resource cost	(9a)
		<u>dual variables</u>	<u>Equilibrium conditions</u>	
$-\lambda + \sum_{k\in K} df_{kn}\tau_k + \alpha^{max}{}_n - \alpha^{min}{}_n = b_n$	$n\in D$	d_n	withdrawal	(9b)
$\lambda - \sum_{k\in K} df_{kn}\tau_k + \beta^{max}{}_n - \beta^{min}{}_n = -c_n$	$n\in G^{nmp}$	p_n	injection	(9c)
$\delta_n - p^{max}{}_n\beta^{max}{}_n + p^{min}{}_n\beta^{min}{}_n = -c^{su}{}_n$	$n\in G^{nmp}$	z_n	startup	(9d)
$\lambda - \sum_{k\in K} df_{kn}\tau_k + \beta^{max}{}_n - \beta^{min}{}_n = -c^{ai}{}_n$	$n\in G^{mp}$	p_n	injection	(9e)
$\delta_n - p^{max}{}_n\beta^{max}{}_n + p^{min}{}_n\beta^{min}{}_n = 0$	$n\in G^{mp}$	z_n	startup	(9f)
$\tau_k, \beta^{max}{}_i, \beta^{min}{}_n, \alpha^{max}{}_n, \alpha^{min}{}_n \geq 0, \lambda$ free				(9h)

where the network is $\mathcal{N}^{ed}$

From the definition of $\lambda_n = \lambda - \sum_{k\in K} df_{kn}\tau_k$

$\lambda_n = b_n - \alpha^{max}{}_n + \alpha^{min}{}_n$	$n\in D$	d_n	withdrawal	(10a)
$\lambda_n = c_n + \beta^{max}{}_n - \beta^{min}{}_n$	$n\in G^{nmp}$	p_n	injection	(10b)
$\lambda_n = c^{ai}{}_n + \beta^{max}{}_n - \beta^{min}{}_n$	$n\in G^{mp}$	p_n	injection	(10c)

From (10c) if $z_n = 1$ and $\beta^{max}{}_n = \beta^{min}{}_n = 0$, gen n sets the LIP, that is,

$$\lambda_n = c^{ai}{}_n$$

By complementary slackness of (8i), $p^{min}{}_n\beta^{min}{}_n = 0$, (9f) becomes: $\delta_n - p^{max}{}_n\beta^{max}{}_n = 0$, $n\in G^{mp}$.

Since $p^{max}{}_n \geq 0$ and $\beta^{max}{}_n \geq 0$, $\delta_n = p^{max}{}_n\beta^{max}{}_n \geq 0$, $n\in G^{mp}$.

That is, δ_n represents producer surplus or porfits and is non-negative, that is, there are no make-whole payments. The energy price at bus n is set at highest average incremental cost in an unconstrained subregion of the ISO market.

$\sum_{n\in D} b_n d_n{}^* - \sum_{n\in G} (c_n p_n + c^{su}{}_n z_n{}^*) =$	$\sum_{n\in D} (d^{max}{}_n\alpha^{max}{}_n - d^{min}{}_n\alpha^{min}{}_n)$	$+\sum_{n\in G} z_n{}^*\delta_n$	$+\sum_{k\in K} p^{max}{}_k\tau_k$	(10e)
Market surplus	Buyer surplus	Producer surplus	Congestion revenue	

Alternative AIC Pricing Run. From the SCED, we form an alternative AIC Pricing Run including generators without a make-whole payment in the AIC calculations

$\max MS = \sum_{n\in D} b_n d_n - \sum_{n\in G} c^{ai}{}_n p_n$			market surplus	(11a)
		<u>dual variables</u>	<u>description</u>	
$\sum_{n\in D} d_n - \sum_{n\in G} p_n = 0$		λ	Power balance	(11b)
load constraints				
$d_n \leq d^{max}{}_n$	$n\in D$ *	$\alpha^{max}{}_n$	Upper bound on demand	(11c)
$-d_n \leq -d^{min}{}_n$	$n\in D$ *	$\alpha^{min}{}_n$	lower bound on demand	(11d)
generator constraints				
$p_n \leq p^{max}{}_n$	$n\in G$ *	$\beta^{max}{}_n$	upper bound on supply n	(11e)
$-p_n \leq 0$	$n\in G$ *	$\beta^{min}{}_n$	lower bound on supply n	(11f)
transmission constraints				
$\sum_{n\in N} df_{kn}(p_n - d_n) \leq p^{max}{}_k$	$k\in K$	τ_k	Max Flow on k	(11g)

where the network is $\mathcal{N}^{ed}$

AIC Dual. The dual problem minimizes the resource value to achieve an efficient equilibrium.

$RC = \min \sum_{n\in D} (d^{max}{}_n\alpha^{max}{}_n - d^{min}{}_n\alpha^{min}{}_n) + \sum_{n\in G} p^{max}{}_n\beta^{max}{}_n + \sum_{k\in K} p^{max}{}_k\tau_k$			resource cost	(12a)
		<u>dual variable</u>	<u>Equilibrium conditions</u>	
$\lambda - \sum_{k\in K} df_{kn}\tau_k + \alpha^{max}{}_n - \alpha^{min}{}_n = b_n$	$n\in D$	d_n	withdrawal	(12b)
$-\lambda + \sum_{k\in K} df_{kn}\tau_k + \beta^{max}{}_n \geq -c^{ai}{}_n$	$n\in G$	p_n	injection	(12c)
$\tau_k, \beta^{max}{}_i, \alpha^{max}{}_n, \alpha^{min}{}_n \geq 0, \lambda$ free				(12g)

where the network is $\mathcal{N}^{ed}$

Conjecture. The two methods produce the same result.

Modified AIC Pricing Run for Transmission Rights. Any pricing procedure that relaxes minimum operating traditional level constraints can result in price separation between two nodes without physical congestion and result in FTR settlement being revenue inadequate. To solve this problem, in the AIC pricing run, we set the actual flowgate capacity to the actual physical dispatch, that is, $p^{maxic}_k = p_k^* \leq p^{max}_k$. (An alternative is to set $p^{maxic}_k = p_k^*+\varepsilon \leq p^{max}_k$ where ε is a small positive number to avoid degeneracy). In addition, we prorate flowgate rights by p_k^*/p^{max}_k.

The Modified AIC Pricing Run model is

$\max MS = \sum_{n\in D} b_n d_n - \sum_{n\in G} c^{ai}_n p_n$		market surplus		(14a)
		<u>dual variables</u>	<u>description</u>	
$\sum_{n\in D} d_n - \sum_{n\in G} p_n = 0$		λ	Power balance	(14b)
load constraints				
$d_n \leq d^{max}_n$	$n\in D$	α^{max}_n	Upper bound on demand	(14c)
$-d_n \leq -d^{min}_n$	$n\in D$	α^{min}_n	lower bound on demand	(14d)
generator constraints without make-whole payment				
$p_n - p^{max}_n z_n \leq 0$	$n\in G^{nmp}$	β^{max}_n	upper bound on supply n	(14d)
$-p_n + p^{min}_n z_n \leq 0$	$n\in G^{nmp}$	β^{min}_n	lower bound on supply n	(14e)
$z_n = z_n^*$	$n\in G^{nmp}$	δ_n	supply committed	(14f)
generator constraints with make-whole payment				(14g)
$p_n - p^{max}_n z_n \leq 0$	$n\in G^{mp}$	β^{max}_n	upper bound on supply n	(14h)
$-p_n \leq 0$	$n\in G^{mp}$	β^{min}_n	lower bound on supply n	(14i)
$z_n = z_n^*$	$n\in G^{mp}$	δ_n	supply committed	(14j)
transmission constraints				
$\sum_{n\in N} df_{kn}(p_n-d_n) \leq p^{maxic}_k$	$k\in K$	τ^{ic}_k	Flow on k	(14k)

If flowgate k was not a binding constraint in the SCED, $p_k^* < p^{maxed}_k$ and $\tau^{ed}_k = 0$. In the AIC pricing run, $p^{maxic}_k = p_k^* \leq p^{maxed}_k$ and $\tau^{ic}_k \geq 0$.

Proposition 3. Since (p_n^*, d_n^*) is an optimal solution to the SCED, (p_n^*, d_n^*) is a feasible solution to the AIC pricing model and the modified AIC model.

Proof: for (14k) and (14l)

$$\sum_{n\in N} df_{kn}(p_n^*-d_n^*) = p^{maxic}_k \quad k\in K \qquad \tau^{ic}_k \qquad \text{Flow on k} \qquad (15a)$$

Since (14b) through (14j) is a relaxation of the SCED constraints and (p_n^*, d_n^*) is feasible to (14k) and (14l), (p_n^*, d_n^*) is feasible solution in the AIC run. ☺

Proposition 4. Using the modified FTR rights, the FTR cashout is revenue neutral.

Proof: From proposition 2, the prorated flowgate model is revenue neutral. ☺

The Cashout from the AIC run. The Cashout prices of the FTRs use the nodal price from the AIC. The congestion revenue from the SCED is $\sum_{k\in K} p^{max}_k \tau_k$.

Proposition 5. With $p^{maxic}_k = p_k^*$, the SCED dispatch $(p_i^*\ d_i^*)$ is still feasible and in the AIC run.

Proof: Since (p_i^*, d_i^*) is a feasible solution to the SCED, with $p^{maxic}_k = p_k^*$, the SCED dispatch $(p_i^*\ d_i^*)$ is still feasible in the AIC run.

9 Entry and Exit in Non-Convex Electricity Markets

9.1 Introduction

An important role for prices is to signal entry and exit. In strictly-convex markets, the necessary and sufficient condition for generator entry is its average incremental cost (AIC) be less the LMP. In convex markets, the necessary and sufficient condition for generator entry is its AIC be less the lowest possible LMP. In convex markets, locational incremental price (LIP) = LMP. In non-convex markets, a sufficient condition for generator entry is its AIC be less than the lowest possible LMP. In addition, in non-convex markets, a sufficient condition for generator entry is its AIC at specific level of output be less than the LIPs at the same level of an incremental generator. As explained in this chapter, the generalization is the AIC of a group of generators at a specific level of output is less than the LIPs of a group of generators at the same level.

In ISO markets, the necessary and sufficient condition for generator entry is to run the SCUC with the generator and see if it results in a more efficient dispatch.. Unfortunately, this test is not readily available to market participants, requires significant information about other market participants and can be computationally intensive.

In ISO auction markets, prices play no explicit role in determining the efficient dispatch. Prices are a post-dispatch calculation that determines how the efficient market surplus is distributed among the market participants –- settling the market. the LIPs leave no make-whole payments or uplift creating additional transparency.

Prices are not signaling self-dispatch since at the LMP there is not enough information to . To slightly over-simplify, LMPs are determined by fixing the binary variables at their optimal values and rerunning the dispatch model. This is the equivalent to assuming avoidable fixed costs are sunk. LIPs are also determined by relaxing the binary variables and reallocating the fixed costs to the periods and place where the demand required them. LMPs and LIPs are necessary for a more transparent entry and exit price signals. The LMPs give no assurance of the quantity of entry since they are marginal. LIPs require is minimum level of entry to guarantee successful entry.

Signals or incentives for entry come from the public information provided by the market operator enabling a current or future participant to evaluate its potential to enter or exit the market profitably. It would be efficient for a potential new entrant to enter the market if the potential entrant had a lower average incremental cost (AIC) equivalently average avoidable costs at an incumbent's optimal dispatch.

If a sufficiently large entrant can fully displace an incumbent generator, the decision of the entrant includes not only the incumbent's marginal energy costs, but also its avoidable fixed costs. In this case, the AIC of the entrant at the dispatch of the incremental generator is a sufficient signal for entry.

The Locational Incremental Price (LIP) at a feasible operating level of the incremental generator would provide sufficient information to evaluate whether the entry of a resource is efficient at a higher cost level than the LMP, because the LIP incorporates avoidable fixed costs in addition to marginal costs. Of course, there may be multiple combinations of costs and characteristics embodied in different resources whose entry might increase the market surplus. Therefore, using both the LMP and LIP together can better signal efficient entry and should be publicly available.

Entry can take place for infra-incremental generators by displacing a generator with a generator with similar parameters, for example, a nuclear plant displacing a more efficient nuclear plant of similar size and operating cost and not displacing the marginal generator or an incremental generator because two inflexible generators could not efficiently serve load..

This Chapter examines the prices as signals for entry and exit in electricity auction markets. We argue that a pricing method with non-transparent uplift payments is an signal too low for efficient entry and AIC pricing provides a signal for 'guaranteed' efficient entry and profitability due to its inclusions of avoidable fixed operating costs such as start-up costs in prices that all sellers receive. We illustrate this point with the aid of examples. We shall assume a convex demand function although it is not necessary. We examine the conditions where entry is profitable and efficient in non-convex markets, in particular, the ISO day-ahead and real-time markets, but the principles extend to any investment with fixed costs. We use a retrospective approach, that is, we use information from the efficient dispatch to see if entry decisions would have changed because of public information posting and settling based on the highest average incremental cost of all suppliers dispatched in the market. Since future entry depends on many uncertainties. We confine the analysis to simply rerunning the market with the entrant. First, we compare entry in convex and non-convex markets. Next, through a series of examples, we illustrate entry in non-convex markets. Finally, we generalize entry conditions in non-convex markets with a discussion of 'surrogate' generators.

We conclude that both the LMPs and the LIPs should be publicly available prices. In addition to the LIP, the output of the incremental generator being displaced a good entry decision and would also provide additional transparency. Entry is not only dependent on costs but also size and flexibility of the entrant.

9.2 Entry and Economic Efficiency

Market Design Objectives. A principal objective of auction market design is to maximize efficiency (market surplus). The announcement of prices and market settlement occurs after the efficient dispatch is determined. The settlement must not confiscate the offers (covers all avoidable costs) and must be revenue neutral (pay out equals pay in). If the settlement is confiscatory, the market participant will not want to enter again. In addition, price-responsive demand with Ramsey-Boiteux pricing is needed to ensure that load's bid is not confiscated by charging more than it bid. The market rules should allow off-ISO (bilateral and multilateral) trading for financial management, for example, hedging.

Entry can take place in two ways. Either it can occur by satisfying demand growth or through replacing less efficient generators. Whether entry is efficient can depend on uncertainty of output, operating range, flexibility of the entrant, and its ramp rate whether the entrant can displace an incumbent generator. Incumbent generators can be competitive or complementary with other units. An entrant may be able to displace, at most, a portion of an incumbent's energy production. In this case, the value of the entrant is measured by the incumbent's marginal energy costs that are avoided by the entry. This could occur if, for example, the entrant is an intermittent resource with low marginal costs.

Under pricing methods with make-whole payments, a chosen market participant with lower costs may receive less revenue than an incumbent market participant with higher costs that provides the same service at the same time and location.

Since AIC pricing will usually produce higher prices compared to pricing methods with make-whole uplift payments, a resource is more likely to make a profit commensurate with benefit it creates. AIC pricing will reduce the chances of early retirement or defer retirement when compared to pricing methods with make-whole uplift payments.

Entry, exit and the threat of entry are important to economic efficiency. Therefore, markets should send information to facilitate efficient entry and exit. In strictly-convex markets, the strong entry signal is the market-clearing price, that is, the LMP. Entry by any potential entrant with costs lower than the LMP will increase the total market surplus. Beating the LMP is a necessary and sufficient signal for entry. A necessary but not sufficient signal for convex markets. In convex markets, a necessary and sufficient condition for entry is having a marginal cost less than the lowest possible LMP. In non-convex markets, a sufficient, but not necessary condition for entry is having an AIC less than the lowest possible LMP.

However, generators in electricity markets have avoidable fixed costs and operating constraints. This complicates the clearing mechanism so that re-running the unit commitment model is the only guaranteed way to establish if an entrant would have cleared the market. Not only do the characteristics of the entrant impact its ability to clear the market, the characteristics of the other generators in the market impact an entrant's ability to clear.

Having an AIC at a certain level of output that less than the LMP is a sufficient, but not necessary signal for entry and gives no information about the level of entry. A complete signal for market entry gives prospective market entrants all the necessary information to determine if entry would have been profitable. Complete signals are multi-faceted, including average incremental costs and feasible operating level. This information is usually not available to ISO market participants. This property of electricity markets and more generally non-convex markets makes it impossible to send a single-entry signal or price that conveys complete entry information.

Power markets must also balance energy supply and demand physically. In addition, they must keep voltage and frequency within limits that allow stable operations. Therefore, power markets need reserves to be able to stabilize the system after unexpected events and before the system becomes unstable. One function of the ISO is to facilitate efficient trades that would have happened in a competitive market with better coordination among market participants and lower transaction costs. another function of the ISO is to facilitate efficient hedging in the off-ISO markets. This requires good entry signals.

9.3 Examples of Entry

Example 1. Consider a market with generation resources described in Table 9.1, and a demand that is not price responsive at 2350 MW. In this market, there are two large and flexible generators, A and B, and four small generators, C, D, E, and F, all of which are inflexible and must be dispatched at their maximum capacity (Pmax) if they are to be dispatched. The avoidable fixed costs of the four inflexible generators ranges from $10100 to $10400.

Table 9.1 Generation Resources and Efficient Dispatch. LMP is $60/MWh; LIP = $104/MWh

Gen	Pmax	Pmin	Marginal cost	Fixed cost	AIC	optimal dispatch	Make-Whole Payment
GA	1000	0	30	0	30	1000	0
GB	1000	0	60	0	60	950	0
GC	100	100	0	10100	101	100	4100
GD	100	100	0	10200	102	100	4200
GE	100	100	0	10300	103	100	4300
GF	100	100	0	10400	104	100	4400
Total	2400					2350	17000

To satisfy demand for the inflexible generator, GB, which is less costly and more flexible than Generators C, D, E, and F, is dispatched at 950 MW, less than its maximum operating level and sets the LMP at $60/MWh that is the marginal cost of the generator B. Under LMP+, the four inflexible generators need the make-whole payments shown in Table 9.1, in addition to LMP revenues.

A potential new entrant, N, is considering whether to offer into this market and has the characteristics shown in Table 9.2.

Table 9.2 Characteristics of a Potential New Entrant

Gen	Pmax	Pmin	Marginal Cost	Startup Costs
N	100	0	70	1000

If Generator N were to enter the market, it would efficiently displace Generator F, that is, Generator N's AIC of $80/MWh at 100 MW could displace Generator F's AIC of $104/MWh. The information N needs is not publicly available.

If Generator N were to enter, the dispatch is shown in Table 9.3. LMP increases to $70/MWh, set by the marginal cost of N. The LIP decreases to $103/MWh due to the replacement of generator F. However, while the $70 LMP would cover N's marginal costs, it would not cover N's startup costs, which N must incur in order to enter the market. Since entering would result in a financial loss, LMP pricing would fail to provide an efficient incentive for N to enter the market, even though N's entry would be efficient. Under current pricing methods, a market participant with a make-whole payment breaks even.

Table 9.3. Efficeint Dispatch with Generator N. LMP is $70/MWh. LIP = $103/MWh

Generator	Pmax	Pmin	Marginal cost	Avoidable Fixed cost	AIC	optimal dispatch (P*)	Make-Whole Payment at LMP
A	1000	0	30	0	30	1000	0
B	1000	0	60	0	60	1000	0
N	100	0	70	1000	90	50	1000
C	100	100	0	10100	101	100	3100
D	100	100	0	10200	102	100	3200
E	100	100	0	10300	103	100	3300
F	100	100		0	0	0	0
Total	2500	400				2350	10600

By contrast, the LIP would provide enough revenue for N to cover its costs and enter the market. In addition, since the post-entry LIP of $103/MWh would be above the average incremental costs of every infra-marginal generator, the LIP would also allow all infra-marginal generators to receive a profit, thereby providing a positive incentive for them to remain in the market and reduce their costs.

a potential new entrant, N2, is considering whether to offer into this market and has the characteristics shown in Table 9.2.

Table 9.4 Characteristics of a Potential New Entrant

Gen	Pmax	Pmin	Marginal Cost	Startup Costs	Avg Cost at Pmax
N	200	50	84	2000	94

The potential new entrant has an AIC of $94/MWh at Pmax that are less than the average incremental Costs of each of the inflexible incumbents. If Generator N were to enter the market, it would efficiently displace Generators E and F, that is, Generator N2's AIC of $90/MWh could displace Generator F.'s AIC of $104/MWh. The information N2 needs is not publicly available.

If Generator N2 were to enter, the dispatch is shown in Table 9.3. LMP is $84/MWh. LIP = $102/MWh. The LMP is $70/MWh, the marginal cost of the new entrant, N2, which is now the most expensive flexible generator needed to efficiently meet load. However, while the LMP would cover N2's marginal costs, it would not cover N2's startup costs, which N2 must incur to enter the market. Since entering would result in a financial loss, LMP pricing would fail to provide an efficient incentive for N2 to enter the market, even though N2's entry would be efficient.

Table 9.5. Efficeint Dispatch with Generator N2.

Generator	Pmax	Pmin	Marginal cost	Avoidable Fixed cost	Average incremental Cost	optimal dispatch (P*)	Make-Whole Payment
A	1000	0	30	0	30	1000	0
B	1000	0	60	0	60	1000	0
N2	100	0	84	2000	97.33	150	2000
C	100	100	0	10100	101	100	1700
D	100	100	0	10200	102	100	1800
E	100	100	0	10300	103	0	0
F	100	100		0	104	0	0
Total	2500	400				2350	5500

By contrast, the LIP is based on the average incremental cost of the dispatched generator with the highest average incremental cost needed to meet demand and would provide enough revenue for N2 to cover its costs and enter the market. In addition, since the post-entry LIP of $110/MWh would be above the average incremental costs of every infra-marginal generator, the LIP would also allow all infra-marginal generators to receive a profit, thereby providing a positive incentive for them to remain in the market and reduce their costs.

Example 2: Entry over a Range of Operating Characteristics. In this example, we analyze potential entry over a range of operating characteristics. It shows the complexities that can exist with entry in a non-convex market. The incumbent generator parameters are in Table 9.6. The load parameters are in Table 9.7. The potential entrant only has marginal costs and no minimum operating level.

Table 9.6 Generator Parameters

generators	Startup cost	Marginal cost	Minimum operation	Maximum operation
GA	1000	20	800	900
GB	200	30	200	300
GC	100	50	90	100

Table 9.7 Load Parameters

Load	Startup value	Marginal value	Minimum consumption	Maximum consumption
LA	0	200	0	1050
LB	0	14	0	17

Efficient Dispatch. The efficient dispatch is in Table 9.8. The market surplus is $185,800. The LMP is $20/MWh. Make-whole payment (negative profit at the LMP) to generators GA and GB totals $3200.

Table 9.8 Efficient Dispatch with an optimal market surplus is $185,800.

	startup	dispatch	Profit at LMP+	Profit at LMP	AIC at dispatch
GA	1	850	0	-1000	21.18
GB	1	200	0	-2200	31.00
GC	0	0	0	0	N/A
totals	-	1050	-	-3200	
demand	startup	dispatch	marginal value	total value	consumer surplus
LA	1	1050	180	189000	185800
LB	0	0	0	0	
totals	-	1050	-	189000	185800

AIC Pricing. The AIC Pricing Run results are in Table 9.7. The LIP is $31/MWh, set by GB, the highest AIC generator dispatched. No make-whole payments are necessary.

Table 9.9 AIC Settlement with No Make-Whole Payments. LIP is $31/MWh

gen	AIC dispatch	efficient dispatch	profit at LIP	Settlement at LIP
GA	900	850	7800	26350
GB	150	200	0	6200
total	1050	1050	7800	32550
demand			surplus	
LA	1050	1050	169000	32550

The conditions under which entry would be efficient depend, in part, on the size of the new entrant. Under current rules, while a potential entrant would not know this information, the operating range does impact its potential to clear the market. There are at least six size categories to consider. After the description in each category, a plot shows comparing marginal costs for a potential entrant, GD, against the quantity that would clear the market. The dispatch quantity was determined by running a unit commitment model with each new marginal cost and a capacity of 0 to the maximum listed in the category. The dashed lines show the LMP and LIP corresponding to each marginal cost/dispatch quantity pair.

Entry with Capacity Less Than or Equal to 50 MW. A potential generator with a capacity no greater than 50 MW could not displace GB, but it could displace up to 50 MWh of energy from GA. The value of this displaced energy is measured by the LMP of $20/MWh, since GA's energy cost sets the LMP. If the potential entrant's average incremental costs are less than the $20 LMP, its entry would be both efficient and profitable under either LMP pricing or AIC pricing.

On the other hand, if the average incremental costs of the entrant were higher than the LMP, its entry would not be efficient. Figure 9.1 shows GD with a maximum capacity of 50 MW and minimum of 0 MW. It would be included in the efficient dispatch, shown on the left axis, given the range of marginal costs on the x-axis. The right axis shows the resulting LMP and LIP prices.

For any AIC cost under $20/MWh, the unit would dispatch to its capacity, and any costs above $20/MWh would not clear the market. However, its entry could appear to be profitable at the LIP if the entrant's average incremental costs were below the LIP of $31/MWh. The entrant would not be dispatched since its average incremental costs were higher than the marginal costs of the marginal generator, and thus, entry would not be profitable.

The most transparent and full entry signal that can be provided is announcing both the LMP, LIP and a dispatch point at the LIP. If the potential entrant only sees the LMP and LIP, it would not be aware that entry is only possible for average incremental costs under $20/MWh. Without knowing the dispatch point, AIC pricing could encourage inefficient entry while LMP pricing would establish incentives to enter in this range only when it is efficient. However, revealing the dispatch point also discloses information about the marginal generator. Full transparency is a tradeoff with access to information not currently available to the public.

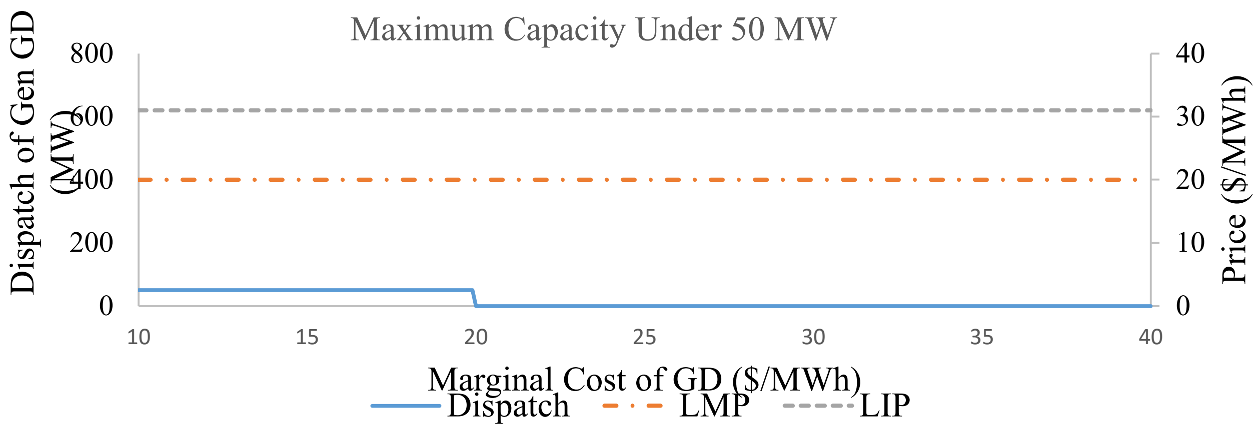


Figure 9.1 Dispatch and prices resulting from GD at varying marginal costs.

Entry with Capacity Greater 50 MW and Less Than 150 MW. If the capacity of the potential entrant was greater than 50 MW but less than 150 MW, the potential entrant could not displace any additional energy above 50 MW from either GA or GB, because of the incumbents' minimum operating level. Thus, increasing the entry to between 50 MW and 150 MW would bring no additional efficiency benefit without price responsive demand.

If the potential generator were to enter at this size, the post-entry LMP would be set no higher than the entrant's marginal cost. As a result, entry of this additional capacity under LMP pricing would not be profitable. By contrast, AIC pricing could make it appear that the additional entry would be profitable (if the entrant's AIC is less than the AIC of GA, the generator that could set the LIP). In fact, the additional entry would not be profitable since the entrant would not be dispatched between 50 and 150 MW. However, the entrant may not have the dispatch information. If not, LIP alone could inefficiently encourage entry, while LMP pricing may not.

Due to the price responsiveness of LA, entry can occur when GD's marginal costs are less than $20/MWh (less than the marginal cost of GA). This occurs for resources with maximum capacity near 150 MW. Figure 9.2 shows entry with a maximum capacity of 145 MW. In this case, LA sets the price at its bid, $200/MWh and GD clears at its maximum. Both the LMP and LIP are set by demand, at $200/MWh. In this case, both might encourage inefficient entry, since resources with high costs (e.g., $150/MWh) would not clear the market. Although demand is still responsive when GD's costs are higher than $20/MWh, it is cheaper at those prices to get the next MW of demand from GD (i.e., GD sets the price rather than demand). At low marginal costs, GD would enter the market and load would set the price (meaning less load would be cleared).

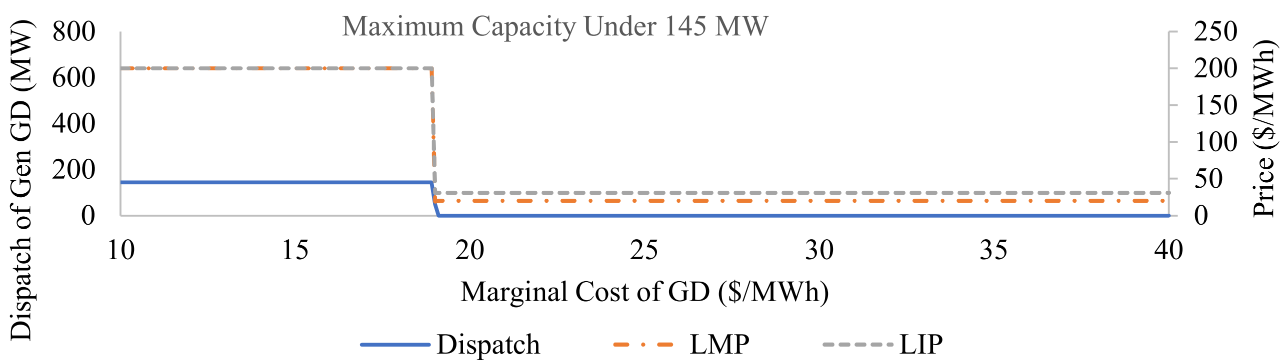


Figure 9.2 Dispatch and prices resulting from GD at varying marginal costs.

Entry at 150 MW. If GD had a maximum capacity of 150 MW, the possible entry is in Figure 9.3. For any price below $34.67/MWh, the unit is dispatched to 150 MW. Above $34.67/MWh, the generator would not clear. If LA required 1051 instead of 1050, the last MWh would not clear the market.

Entry of 150 MW by a new generator, combined with increasing the output of GA by 50 MW (to 900 MW) would provide enough energy to fully displace GB's 200 MW of capacity. The net savings from displacing GB is $5200, i.e., the elimination of GB's costs ($200 + [200x$30]) less the $1000 increase in GA's costs from increasing its output by 50 MW (i.e., 50x$20). Spread over the entrant's 150 MW capacity, this cost savings of $5200 amounts to an average benefit of $34.67/MWh. Thus, entry of this amount would be efficient if the new entrant's average costs were less than $34.67/MW, which is higher than both the pre-entry LMP and the pre-entry LIP.

However, by displacing GB (the generator that previously would have set the LIP) the post-entry LIP would be calculated as the greater of $21.11 or the entrant's AIC. As a result, entry at this level would be profitable only if the entrant's average costs were less than $21.11/MW, although an entrant with average costs between $21.11/MW and $34.67/MW would break even in the market. (Note, this example is degenerate, meaning the price can alternatively be set by demand at $200/MWh.)

Here both LMP and LIP understate the potential costs that would clear the market.

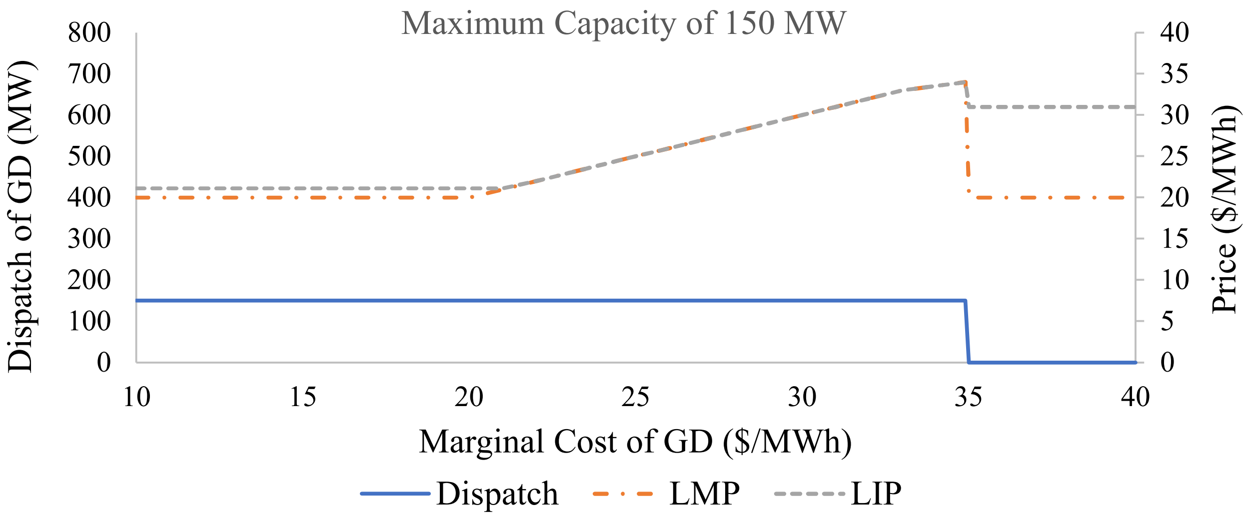


Figure 9.3 Dispatch and prices resulting from GD at varying marginal costs.

Entry with a Maximum Capacity Less Than 200 MW. Figure 9.4 shows entry at or below a maximum capacity of 200 MW. Under $20/MWh, GD would be dispatched to 200 MW, where the LMP is $20/MWh and the LIP is $21.18/MWh (GA's AIC). Above $20/MWh and below $34.67/MWh, GD would be dispatched to 150 MW and GD's marginal cost sets the price. Above $34.67/MWh, the unit would not be dispatched.

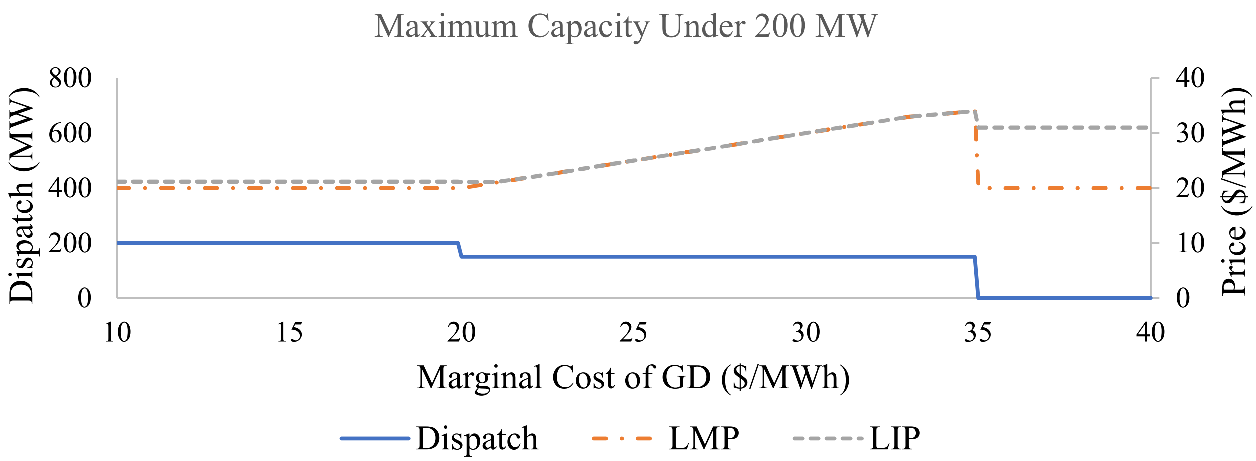


Figure 9.4 Dispatch and prices resulting from GD at varying marginal costs.

Entry greater than 200 MW and less than 250 MW. If the potential entrant were to enter with capacity of 200 MW or more, it could fully displace GB and possibly displace a portion of GA's energy. Fully displacing GB would avoid GB's fixed operating costs associated with its minimum operating level. Thus, the value of fully displacing GB is its AIC of $31/MWh. Entry of 200 MW would be efficient if the entrant's AIC is less than $31/MWh. However, by completely displacing GB, the LIP would decrease to the higher of $21.18 or the entrant's AIC. Thus, under AIC pricing, entry at this size would allow the entrant either to break even (if its AIC is between $21.18 and $31) or to earn a profit (if its AIC is below $21.18). By contrast, under LMP pricing, the LMP would remain at $20 before and after entry, which would suggest that entry would be profitable only if the entrant's AIC was below $20. Thus, AIC pricing would provide a better signal than LMP pricing to encourage efficient entry at this size level.

While entry of 200 MW with AIC of less than $31 would be efficient, additional entry above 200 MW up to 250 MW would displace GA's energy, whose marginal cost is only $20. Thus, additional entry between 200

MW and 250 MW would be efficient only if the entrant's marginal energy cost in this size range were less than $20/MWh, shown in Figure 9.5.

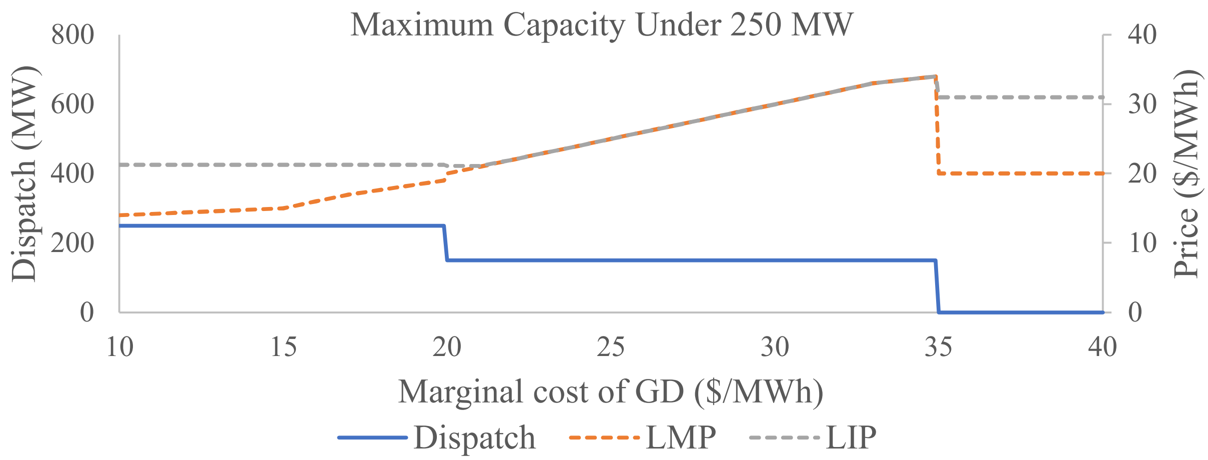


Figure 9.5 Dispatch and prices resulting from GD at varying marginal costs.

Entry above 250 MW and less than 800 MW. Entry at this size level would not displace any additional energy production from the incumbent generators, because of their minimum operating level constraints and costs. Thus, entry in this size range would provide no additional efficiency benefits. It would also provide no additional profits to the entrant under either pricing mechanism, since the entrant would not be dispatched to produce any energy from this additional capacity. However, this information may not be made public. For potential entrants to make efficient entry decisions, potential entrants would need to be made aware that their potential resources would not be dispatched at this size level.

At a maximum capacity of 800 MW, dispatch is possible when costs are below $16.91/MWh since it would displace GA, as shown in Figure 9.6. See the next section for a discussion of entry at $16.91. Above $16.91/MWh, the trends are the same as Figure 9.5, and entry would occur at or below 250 MW.

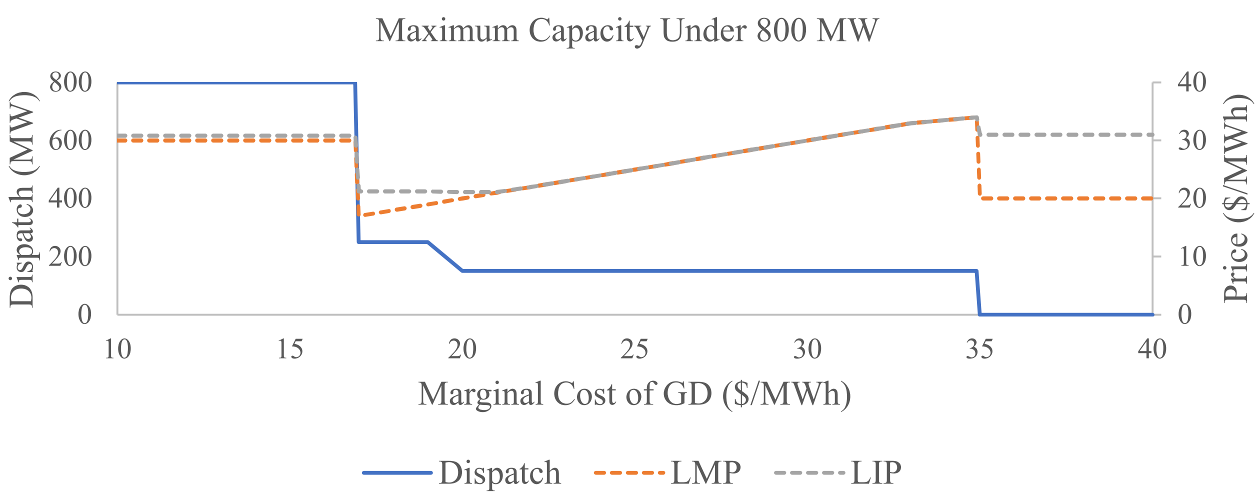


Figure 9.6 Dispatch and prices resulting from GD at varying marginal costs.

Entry at or Above 800 MW and Less Than 1050 MW. Figure 9.7 shows the maximum output of potential entrant GD with a $20/MWh marginal costs on the x-axis and each series shows a different minimum operating level for the potential entrant. The y-axis shows the dispatch of GD; if the dispatch is nonzero, then it was selected in the optimal dispatch and increases the market surplus. The black dotted line is a convex generator, since its minimum is 0 and it has no fixed costs.

Each category described above shows the Pricing Point of Entry for the simple market in Example 2. The Pricing Point of Entry depends on both price and generator characteristics. Many potential generators could enter this market and fully or partially displace existing generation. When demand sets the price, both the LMP and LIP might signal inefficient entry. A generator with costs of $150/MWh (below the $200/MWh price set by demand) would be unlikely to clear the market. There are also cases when the LMP is too low to signal entry (there is a Pricing Point of Entry above the LMP). For instance, a generator with a maximum capacity above 150 could enter the market with marginal cost below $34/MWh, but it would not enter if its only entry signal is LMP which is $20/MWh. However, there are also instances when the LIP might signal inefficient entry. With an announced LIP of $31/MWh, a generator with capacity smaller than 50 MW could not enter the market unless its marginal costs were less than $20/MWh. These cases show the complexity of entry signals in non-convex markets, and occasions when both LMP and LIP fail to signal efficient entry.

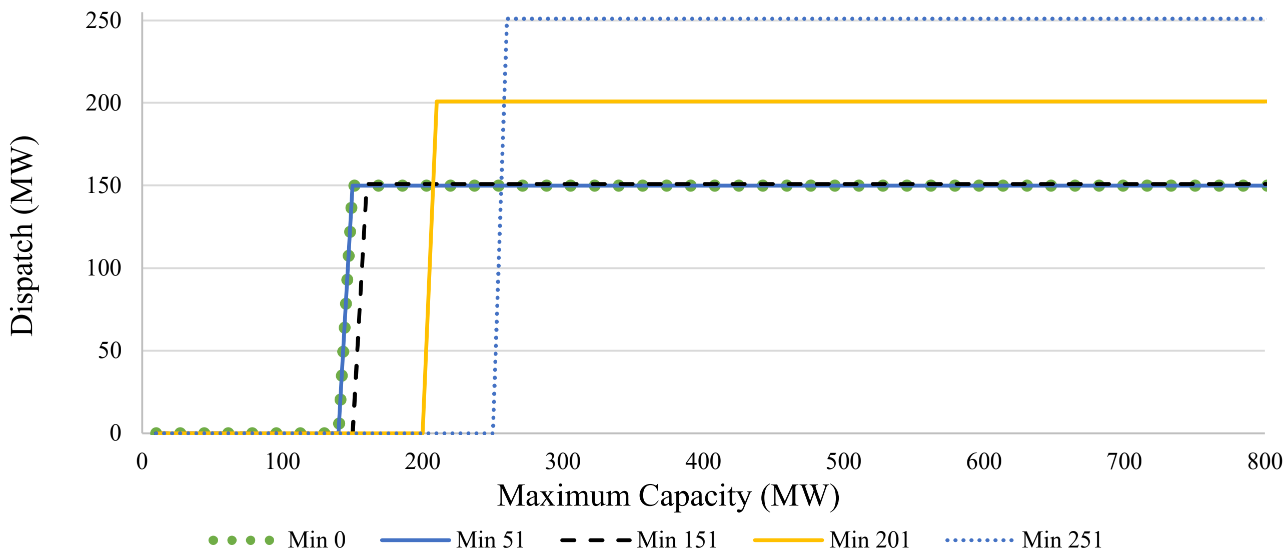


Figure 9.7 Entry points for GD assuming a Marginal Cost of $20/MWh.

We conclude that both the LMP and the LIP should be publicly available prices. In addition to the LIP, the output of the generator being displaced is needed to make a good entry decision and would also provide additional transparency. Entry is not only dependent on costs but also size and flexibility.

Example 3. Entry that would be efficient for an existing generator may not be efficient for a potential generator that must incur construction costs. In the following example, with generator characteristics in Table 9.10. Demand is 180 MWh and is not responsive to price.

Table 9.10 Generator Parameters, Efficient Dispatch, Prices and Settlements

Gen	Pmax	Pmin	Startup Cost	Marginal Cost	Dispatch	Profit at LMP	AIC	LIP Profit
GA	100	0	3000	30	100	-2000	60	0
GB	100	0	1200	40	80	-1200	55	400
Demand	180				180			

LMP = $40/MWh, set by GB; LIP = $60/MWh, set by GA

The LMP would be insufficient to cover either GA's or GB's avoidable costs, so make-whole payments of $2000 (= $30x100+$3000-$40x100) would be paid to GA and $1200 (= $40x80+$1200-$40x80) to GB. Under AIC pricing, no generator would receive a make-whole payment. The incremental generator, GA, would break even, and the infra-incremental generator, GB, would receive a profit of $400 (= $60x80-$40x80-$1200). GB has an incentive to invest in greater operating efficiency since it results in higher profits that does not happen under LMP or LMP+. The market operator announces that LMP = $40 set by GB and the LIP = $60 at 100 MWh set by GA. Entry at less than 100 MW needs an AIC of $40/MWh. Entry at 100 MW needs an AIC of 60 $/MWh.

Suppose that a developer is considering whether to build a new generator, E, to enter this market. Generator GE's characteristics are in Table 9.11. there are no sc ale economies.

Table 9.11 Parameters of a Potential Entrant

Generator	Pmin	Marginal Cost	Start-Up Cost	Avg Construction Cost	Avg Total Cost
GE	0	20	0	30	50

The developer is considering two different capacity levels – 50 MW and 100 MW. In either size, the new entrant would be included in the efficient dispatch since its marginal costs of $20/MWh would be less than those of either incumbent generator. The construction costs for the new entrant would be $30/MWh, so that its total average costs would be $50/MWh that is less than the AIC of either incumbent generator. If GE is built at 100 MW, it would fully displace GA in the dispatch, thereby allowing the grid operator to avoid GA's $60 average operating costs. Since E's average total costs of $50/MWh are lower than A's AIC, building the 100 MW unit would be efficient.

Under LMP pricing, the post 100-MW-entry LMP would be $40/MWh as shown in Table 9.11. Since this LMP is less than GE's average total cost (including construction cost), entry would not be profitable, and thus, the LMP price signal would discourage this efficient entry. By contrast, under AIC pricing, the post-entry LIP would be $55. Since this LIP is higher than GE's average total cost, entry would be profitable, and thus, AIC pricing would provide an incentive for this efficient entry.

If the developer were to build GE at a 50 MW size, it could not fully displace either incumbent generator. It would displace 50 MWh of energy from GB in the dispatch, thereby allowing the grid operator to avoid 25 MWh of GB at a $40 marginal energy cost, but not any start-up costs. Since the average total cost of building a 50 MW unit would be $50/MWh, which is more than B's $40 avoided cost, building a 50 MW unit would not be efficient.

GB's AIC is higher when GB produces 30 MWh than when it produces 80 MWh, because spreading GB's start-up costs over a smaller output increases GB's average incremental costs. Since this AIC is greater than GE's average total costs, entry would be profitable at 30 MWh or greater, and thus AIC pricing would encourage entry of another generator with the characteristics of GE.

Table 9.12 Post-Entry Dispatch under Alternative-Sized Entrants. Demand = 180.

Gen	Pmax	Pmin	Marginal Cost	Start-up Cost	Dispatch	Average Incremental Cost	Profit at LIP
Entry of 50 MW							
GA	100	0	30	3000	100	60	2000
GB	100	0	40	1200	30	80	0
GE	50	0	20	0	50	20	3000
LMP = $40/MWh; LIP = $80/MWh at 30 MWh							
Entry of 100 MW							
GA	100	0	30	0	0	0	0
GB	100	0	40	1200	80	55	0
GE	100	0	20	0	100	20	3500

Demand = 180; LMP = $40/MWh set by GB ; LIP = $55/MWh at 80 MWh set by GB.

Example 4. From example 2, we add generator GD. The market parameters are in Table 9.13 and Table 9.14.

Table 9.13 Generator Parameters

generators	Startup cost	Marginal cost	Minimum operation	Maximum operation
GA	1000	20	800	900
GB	200	30	200	300
GC	100	50	90	100
GD	0	21	800	900

Table 9.14 Load Parameters

Load	Startup value	Marginal value	Minimum consumption	Maximum consumption
LA	0	200	1	1050
LB	0	14	1	17

Efficient Dispatch. Generator GD enters the market displacing GA and sets the LMP at $21/MWh, seen in Table 9.8. The optimal market surplus increases from $185,800 to $185,950.

Table 9.15 Efficient Dispatch. The optimal surplus is $185,950 and LMP= 21 $/MWh

	startup	dispatch	Profit
GA	0	0	0
GB	1	200	-2000
GC	0	0	0
GD	1	850	0
totals	-	1050	-2000
demand			surplus
LA	1	1050	187950
LB	0	0	0
LC	0	0	0
totals	-	1050	187950

LMP. The LMP increases from $20/MWh to $21/MWh with a $2000 in make-whole payment to GB. GB, the generator with higher marginal costs remains in the optimal dispatch because GA and GD cannot both be dispatched because the sum of their minimum operating levels is greater than demand. If GD's marginal cost were $22/MWh, it would not enter the market, because at 850 MWh GA's AIC is $21.18/MWh (= $20/MWh + $1000/850 MWh).

AIC Pricing. The AIC Pricing Run results are in Table 9.16. The LIP is $31/MWh with no make-whole payments. The most expensive generator (GB) sets the LIP. A sufficient condition to replace GB is a generator with an AIC of less than $31/MWh and a feasible dispatch of 200 MWh.

Table 9.16 AIC Pricing Run settlement with LIP = $31/MWh

gen	startup	AIC dispatch	efficient dispatch	profit	AIC	Payment
GB	1	150	200	0	31	6200
GD	1	900	850	8500	21	26350
totals	-	1050	1050	8500	-	32550
demand				surplus	AIC	Charge
LA	1	1050	1050	177450	200	32550

9.4 Examining Entry Points with Surrogate Generators

We end the discussion on entry by examining two cases of potential entrants, or surrogate generators. Each surrogate generator is a variation of a generator currently in the market, with one or more non-convexities removed. These generators allow us to analyze bounds of potential entry. Determining the exact bounds of entry would be difficult, but the surrogate generators provide intuition for the range of entry possible. These entry points can be calculated after the fact and show that marginal cost pricing does not provide a clear signal for the range of entry points available to generators, especially those with high marginal costs and low or no fixed operating costs.

We assume that the surrogate generators did not offer into the market. Rather, we examine the market with existing generation, and then rerun the market with a surrogate generator to see if the generator would be able to clear. Two surrogate generators are compared to a current market participant in Table 9.17. The current participant, GenX, has both startup costs and marginal costs, and operates between a minimum and maximum operating level. The first surrogate generator, GenY has no startup costs, and its marginal costs are slightly lower than the average incremental cost of GenX. GenZ, a convex generator, has no startup costs, but additionally has a minimum operating level of 0, making it the most flexible of the three. GenX and GenY are non-convex generators. A surrogate generator can be a generator in a neighboring market or a generator not currently participating in the market, for example, due to maintenance.

Table 9.17 Generator Parameters

generator	Marginal Costs	startup costs	minimum operation	maximum operation	constraint sets
GenX	c_x	$c^{su}{}_x$	$p^{min}{}_x$	$p^{max}{}_x$	P_x
GenY	$c_y < c^{ai}{}_x$	0	$p^{min}{}_x$	$p^{max}{}_x \leq p^{max}{}_y$	$P_x \subset P_y$
GenZ	$c_z \geq c_x$	0	0	$p^{max}{}_x \leq p^{max}{}_z$	$P_x \subset P_z$

If GenX and GenY are both in the market, for efficiency, GenY is preferred to GenX because it is less expensive to operate without startup costs. If GenY and GenZ are both in the market, for efficiency, GenZ is preferred to GenY because it wider operating range.

Non-Convex Surrogate Generator Example (GenY). The market consists of two generators and a load with the parameters in Table 9.18; the resulting efficient solution is also in Table 9.18.

Table 9.18 Market Parameters. Efficient dispatch with an LMP of \$10/MWh

	Marginal value/cost	startup cost	minimum	maximum	Efficient dispatch	AIC
GA	10	0	0	100	75	10
GB	20	500	50	80	50	30
load	90		125	125	125	

From the dispatch, we can calculate the average incremental costs of both generators: GA = \$10/MWh and GB = \$30 (=\$20/MWh + \$500/50 MW). We can GBY to the market, see Table 9.19. The new surrogate generator, GBY, fully displaces GB and sets the price at \$29.99/MWh. Even though GBY has a higher marginal cost, its AIC is lower and therefore, it enters the market and increases the market surplus.

Table 9.19 Market Parameters and Dispatch with a Surrogate GBY

	Marginal value/cost	startup cost	minimum	maximum	Efficient dispatch
GA	10	0	0	100	75
GB	20	500	50	80	0
GBY	29.99	0	50	100	50
load	90		125	125	125

Convex Surrogate Generator Example (GenZ). A surrogate convex generator could be battery storage, wind generator, solar generator or a generator started up in a neighboring market. Instead of adding GenY, this example adds GenZ with the characteristics in Table 9.20. The generator has costs higher than GB, but no minimum operating level or startup cost.

Table 9.20 Market Parameters and Dispatch with a Surrogate GB^z

Gen	Marginal value/cost	startup cost	minimum	Maximum	Efficient dispatch without GBZ	Efficient dispatch with GBZ
GA	10	0	0	100	75	100
GB	20	500	50	80	50	0
GBZ	c^z	0	0	100		25
load	90		125	125	125	
Market surplus						
LIP					30	

With GBZ, the dispatch will be 100 MW from GA and 25 MW from GBZ and a total cost with GBZ is \$10/MWh*100MW + \$$c^z$/MWh*25 MW = \$1000 + 25 MW*c^z. Without GBZ, we know the dispatch will be 75 MW from GA, 50 MW from GBZ and Total cost without GBZ is \$2250 (=\$10/MWh*75 MW + \$500 + \$20/MWh*50 MW).

The highest marginal cost for GBZ. at which it will enter the market is \$50/MWh (= (2250 - \$1000)/25 MW) due to the additional dispatch of GA. If the market is rerun with the cost of GBZ below \$50/MWh, GBZ will displace GB. With a cost above \$50/MWh, GB will remain in the dispatch. These two examples are in **Error! Reference source not found.**, with costs of \$49.99/MWh and \$50.01/MWh. This example shows that the average incremental costs of GB signal entry, but entry can occur at a higher price.

9.5 SUFFICIENT CONDITIONS FOR ENTRY

In this section we formalize the sufficient conditions for entry a generator and groups of generators. A generator's parameters include startup costs, minimum operating level, and fixed operating costs per period, minimum run time and minimum downtime constraints. These parameters create non-convexities and present entry conditions that may involve multiple generators.

In this section, we prove sufficient entry conditions for a surrogate convex generator e in a generic market with non-convex generators.

Notation

Indices and symbols

i, j	are generators	
e	potential generator entrant	
t	time period	
*	denotes the optimal solution without generator e	
^{e}	denotes a solution with generator e included	
Sets		index
G	is a set of incumbent generators	i, j
G*	is a set of incumbent generators in the optimal dispatch; $G^* \subset G$	i, j
D	is the set of loads	i
E	is the set of generators seeking entry	e
J	is the set of generators to be replaced by new entrants	j

P_i is the feasible set of generator i

T is the set of time periods; T = {1,2,…tmax} t

Generators

c_i is the marginal cost of gen i

c^{su}_i is the startup cost of gen i

c^{op}_t is the avoidable fixed cost of operating gen i

p_{it} is the dispatch of gen i in period t

z_i is the startup decision for gen i

IC_i is incremental costs of gen i at p_i^*; $IC_i = c_i p_i + c^{su}_i z_i$

c^{ai}_i is average incremental costs of gen i; $c^{ai}_i = IC_i^*/p_i^*$

Load

b_{it} is the marginal value of load i in period t

d_{it} is the consumption of load i in period t

d^{max}_i is the maximum dispatch of load i

System

MS is the market surplus

MS^e is the market surplus with generator e

$\Delta MS = MS^e - MS$

Gen e is a surrogate convex generator that wishes to enter the market. It has no minimum operation level or startup costs.

Market Formulation without the Surrogate Convex Generator e

		Dual variable	constraint	
Max $MS = \sum_t [\sum_{i\in D} b_{it}d_{it} - \sum_{i\in G} (c_i p_{it} + c^{op}_i u_{it} + c^{su}_i z_{it})]$			Max surplus	(1a)
$\sum_i d_{it} - \sum_i p_{it} = 0$	t∈T	λ_t	energy balance	(1b)
$d_{it} \le d^{max}_{it}$	i∈D t∈T	α^{max}_{it}	maximum load	(1c)
$(p_{it}, z_{it}, u_{it}) \in P_i$	i∈G t∈T		generator feasible set	(1d)
$d_{it} \ge 0$	i∈D t∈T		Demand nonnegativity	(1e)

Below is the market with surrogate convex generators e∈E and existing generator j, j∈G*.

		Dual variable	constraint	
Max $MS^e = \sum_t [\sum_{i\in D} b_{it}d_{it} - \sum_{i\in G} (c_i p_{it} + c^{op}_i u_{it} + c^{su}_i z_{it}) - \sum_{e\in E} c_e p_{et}]$			Market surplus	(2a)
$\sum_i d_{it} - \sum_i p_{it} = 0$	t∈T	λ_t	energy balance	(2b)
$d_{it} \le d^{max}_{it}$	i∈D t∈T	α^{max}_{it}	maximum load	(2c)
$(p_{it}, z_{it}, u_{it}) \in P_i$	i∈G t∈T		generator feasible set	(2d)
$(p_{et}, z_{et}, u_{et}) \in P_e$	e∈E t∈T		entrant feasible set	(2e)
$d_{it} \ge 0$	i∈D t∈T		nonnegativity	(2f)

ΔMS is the value added by entry of generators e∈E.

Proposition 8.1. Suppose E is a singleton set, E=e and J is a set of generators in the market, J ⊂ G*. If $c_e < \sum_t\sum_{j\in J} (c_j p_{jt}^* + c^{su}_j z_{jt}^* + c^{op}_j u_{it}^*)/\sum_t\sum_{j\in J} p_{jt}^*$ and $p_{et}=\sum_{j\in J} p_{jt}^*$ for each t∈T is a feasible solution for generator e, then generator e will enter the market.

Proof: If $p_{et}=\sum_{j\in J} p_{jt}^*$ p_{jt}^* is a feasible solution for generator e and $c_e < \sum_t\sum_{j\in J} (c_j p_{jt}^* + c^{su}_j z_{jt}^* + c^{op}_j u_{it}^*)/\sum_t\sum_{j\in J} p_{jt}^*$, the market with e replacing J is feasible with a higher surplus.☺

Propositions 8.2 generalizes Propositions 8.1 to a set of nonconvex generator entrants e ∈ E

Propositions 8.2. if there exists some $(p_{et}, z_{et}, u_{et}) \in P_e$ such that $\{p_{it}^*, z_{it}^*, u_{it}^* \mid j\in J\} \ni \cup_{e\in E} P_e$ with a cost of $\sum_t\sum_{e\in E} (c_e p_{et} + c^{su}_e z_{et} + c^{op}_e u_{et}) < \sum_t\sum_{j\in J} (c_j p_{jt}^* + c^{su}_j z_{jt}^* + c^{op}_j u_{it}^*)$ and $\sum_{e\in E} p_{et} = \sum_{j\in J} p_{jt}^*$ for each t∈T then there is a solution with $MS^e > MS^*$.

Proposition 8.3. Suppose E is a singleton set, E=e. and J is any set of generators in the market, $J \subset G^*$. Generator e may enter the market for some c_e such that $c_e \geq \sum_t\sum_{j\in J} (c_j p_{jt}^* + c^{su}_j z_{jt}^* + c^{op}_j u_{it}^*)/\sum_t\sum_{j\in J} p_{jt}^*$ and $p_{et}=\sum_{j\in J} p_{jt}^*$ is a feasible solution for generator e.
Proof: Example 4 confirms that c_e can be greater than the AIC of J and $c_e > \sum_t\sum_{j\in J} (c_j p_{jt}^* + c^{su}_j z_{jt}^* + c^{op}_j u_{it}^*)/\sum_t\sum_{j\in J} p_{jt}^*$ and $p_{et}=\sum_{j\in J} p_{jt}^*$ is a feasible solution for generator e could enter the market. ☺

Proposition 8.4 generalize to a set of nonconvex generator entrants e ∈ E if there may some $(p_{et}, z_{et}, u_{et}) \in P_e$ such that $\cup_{e\in E} P_e \ni \{p_{it}^*, z_{it}^*, u_{it}^* \mid j\in J\}$ with a cost of $\sum_t\sum_{e\in E} (c_e p_{et} + c^{su}_e z_{et} + c^{op}_e u_{et}) > \sum_t\sum_{j\in J} (c_j p_{jt}^* + c^{su}_j z_{jt}^* + c^{op}_j u_{it}^*)$ when $\sum_{e\in E} p_{et} = \sum_{j\in J} p_{jt}^*$ for each t∈T.
Proof: see example 4.

10 Glossary

Term	Definition
RMOL	Relaxed Minimum Operating Level
Arbitrage Condition	The Arbitrage Condition is a property of ISO markets in which resources do not have an incentive to withhold offers from the energy market and instead make offers in the reserves market. Some market mechanisms attain the Arbitrage Condition while others do not.
Auction Revenue Right (ARR)	A right to receive revenues from the financial transmission rights (FTR) auctions
Auctions	Auctions are ubiquitous and have rules that must be accepted to participate. Some auction markets are simplified to avoid complexities and lower transaction costs. Most organized spot markets are continuous bid-ask markets (for example, future and option markets). If markets have daily openings and closings prior to the opening, a sealed bid auction is usually conducted to determine the opening price. The spot market auctions make the off-ISO markets more competitive and efficient via price transparency. In many cases, the market design must be approved by the government. For example, the CFTC approves market designs for futures and options markets. Interior sells the rights to exploit Federal Government property. The FCC designs markets for spectrum. FERC approves the market design for power markets, transmission, and pipelines.
Average Incremental Cost (AIC)	The average cost incurred in operating an asset over a defined time period that would not have been incurred if the asset had not operated during the time period, per MWh of Energy and Operating Reserves provided during the time period
Average Total Cost (ATC)	Total cost, including construction and other fixed cost and total variable cost, divided by the quantity of output produced.
Average Variable Cost (AVC)	Total variable cost divided by the quantity of output produced. Total variable cost includes all costs that vary with output and excludes all costs that do not vary with output.
Avoidable Costs	are expenses that not incurred if a decision is not taken and are incurred if the decision is taken.
Black Market	A Market where goods or services are traded illegally.
Blocked Loaded Unit	is a generator where the minimum and maximum operating limits are the same
Block-Loaded Resource	A resource whose maximum operating level is equal its minimum operating level
Branch (or line)	Physical equipment connecting two Buses
Bus	A strip or bar of copper, brass or aluminum that conducts electricity within a substation. A Bus is a terminal for one or more transmission or distribution elements. The terms Bus, Bus Bar, and Node are used interchangeably.
Cascading Blackout	An unintended and uncontrolled loss of electric power, generally across a wide area.
Clearing House	A financial institution that provides clearing and settlement services for transactions. Its purpose is to reduce the Risk of Market Participants defaulting on its settlement obligations. A Clearing House reduces the settlement Risks by netting offsetting transactions between multiple counterparties, requiring margin deposits and monitoring credit worthiness, providing independent valuation of trades and collateral, and often, providing a guarantee fund that can be used to cover defaulting losses.
Closed-Loop Interface Constraint	An Interface defined by a set of transmission lines that form a "pocket" with Load and Generation. Closed-Loop Interface Constraints are used to translate voltage conditions into thermal constraints that can be used by the Dispatch and pricing algorithms. (see Bresler, PJM 2014)
Club Goods	Goods that are Excludable and Non-rivalrous (unless congested), for example, private golf courses, private parks, satellite television and local police.
Common Pool Goods	Goods that are Non-excludable and Rivalrous, for example, fish stocks and grazing land.
Competition	The act or process of rivalry as the effort of two or more parties acting independently to secure the business of a third party by offering the most favorable terms. In economic theory, Competition sometimes refers to Market conditions in terms of how much control sellers can exercise over the Market Price. It ranges from a great deal of control (in monopoly) to no control whatever (under perfect Competition). See also, Market Power.

Conduct and Impact Test	A two-part test used in ISOs to determine whether a seller has attempted to exercise enough Market Power to warrant applying Market Power mitigation to the seller's offer. The first test, the conduct test, compares the seller's desired offer with a reference Price that approximates the seller's marginal cost. If the desired bid exceeds the reference Price by a specified threshold, an impact test is applied, which estimates the impact of raising the desired bid above marginal cost on the Market Price. If the impact of the bid raises the Market-clearing Price by a threshold amount, the bid is mitigated to an estimate of Incremental Costs.
Contract for Difference (CFD)	A contract between a buyer and seller stipulating that the seller will pay to the buyer the difference between the current value of an asset and its value at contract time (If the difference is negative, then the buyer pays seller).
Convex Hull Pricing	Convex Hull Prices identify uniform prices that minimize defined uplift payments based on the convex hull of the supply functions, see Schiro et. al. (2016)
Convex Market	A market where all components (such as supply and demand) are convex. A mathematical function describing a market component is convex if every possible straight line connecting any two points on the function lies above the function. For example, convex Markets require supply functions be monotonically non-decreasing, and demand functions be monotonically non-increasing. When the functions are differentiable, the marginal cost is the first derivative. For convex markets, the market-clearing price creates a market equilibrium and allocates costs over time and topology based on causality. In economics, convex markets are popular because they are easy to analyze, are easy to solve, and have elegant properties. Convex markets are more of a pedagogical tool than an actual phenomenon, as often the assumptions do not hold.
Core	A set of Market outcomes where no participant can improve its outcome with additional bilateral or multilateral agreements.
Cost-Of-Service Regulation	A form of regulation where Prices are set based on an estimate of total costs of the quantity of service expected to be provided.
Cut Set	See definition for Interface
Cut Set Constraint	is a surrogate for a constraint that are not explicitly modeled, for example, voltage constraints. See also Closed-Loop Interface Constraint.
degeneracy	An optimal linear program is degenerate if an optimal basic feasible solution has one or more variables with a zero value. Primal optimal degeneracy implies multiple optimal dual solutions. So as not to overly complicate the analysis unless otherwise stated, we assume that there is no degeneracy or degeneracy occurs on a set of measure zero.
Demand Response	The Commission defines "demand response" as a "reduction in the consumption of electric energy by customers from their expected consumption in response to an increase in the price of electric energy or to incentive payments designed to induce lower consumption of electric energy" (18 CFR § 35.28)
Dispatch	A process for determining the real-time level of output for each resource in an ISO market. Alternative usage: The quantity of Energy instructed by the ISO to be produced by each resource in the real-time Market.
dispatched	the unit is generating or off-line but on reserve
Economic Dispatch	The process by which an ISO instructs a resource to operate in a manner that is economically efficient.
Economic Efficiency	A Market is efficient if it provides more goods and services for society without using more resources or provide the same output with fewer resources.
Economic Maximum (Ecomax)	The highest level of electric Energy a generating resource can produce.
Economic Minimum (Ecomin)	The minimum amount of electric Energy available from a generating resource for economic Dispatch.
Efficient Dispatch	see efficient dispatch.
Efficient Resource	Is a resource that is contributing to market efficiency
Electrical Energy	Generation or use of electric power over a period expressed in kilowatt-hours (kWh), megawatt-hours (MWh) or gigawatt-hours (GWh).
Emergency Max	The highest short-term MW level a generating unit can produce. The Energy level at which the operating company operates the generating unit if the ISO requests Maximum Emergency Generation:

	Emergency Minimum Limit ≤ Economic Minimum Limit ≤ Economic Maximum Limit ≤ Emergency Maximum Limit
Emergency Min	The lowest level of Energy in MW the unit can produce and maintain a stable level of operation. Emergency Minimum Limit ≤ Economic Minimum Limit ≤ Economic Maximum Limit ≤ Emergency Maximum Limit
End-Of-Horizon Effects	In dynamic models, there are a limited number of periods; the model needs to know the conditions in the last period. If the conditions are not specified properly, the model may shut the market down at the end of the horizon. Often the model will be extended several periods in order to avoid the shutdown effects. It is also important to specified ending inventories.
Equilibrium Settlement	A settlement that, if offered, a rational Market Participant will accept
Equilibrium Settlements	A set of individual settlements that result in Market Clearing
Ex-Ante	before an event
Ex-Ante Price	A Price calculated and announced in advance of the Dispatch period, that reflects the anticipated physical operation of the system during the period.
Ex-Ante Regulation	Regulation of the Market before the transaction. Price or Price cap regulation is ex-ante regulation since it limits in advance the Price that can be paid. Antitrust merger regulation is ex-ante. In common parlance, the term 'regulated Market' is associated with ex-ante Price regulation
Excludable	A good is excludable when its owners can prevent others from consuming.
Ex-Post	after the event.
Ex-Post Price	A Price calculated after the Dispatch period, that reflects actual physical operation of the system during the period.
Ex-Post Regulation	Regulation that imposes fines and penalties for undesired behavior after the behavior is observed. The fines and penalties are deterring the future undesired behavior, but usually does not fully correct the actual Market Failure. Market fraud and antitrust violations such as collusive behavior and 'attempts' to monopolize are ex-post regulation.
Extended Locational Marginal Price (ELMP)	A modification of the LMP that allows selected Generators at minimum operating level to set the settlement Price by modifying the Dispatch algorithm and recalculating the LMP.
ELMPL	ELMP with LOC payments
Externality	A cost or a benefit which results from an activity or transaction and which affects a party outside the transaction who did not choose to incur that cost or benefit.
Financial Derivative	A financial instrument that derives its value from the value of an underlying asset. Derivative transactions include swaps, futures, and options.
Financial Market Participant	A Market Participant that does not own or control physical Market assets and cannot take or supply physical quantities. A financial Market Participant cannot participate in the real-time Market.
Financial Transmission Right (FTR)	A financial right that entitles the holder to the Price difference between two busses, usually in the Day-Ahead Market.
First Price Auction	An auction where the winner pays or receives its bid or offer. (aka a 'pay as bid' auction) In a first Price auction, the incentives for sellers are to guess what the highest winning bid will be. A Risk-adverse seller will offer lower because if a seller guesses too high it may not be Dispatched at all
Fixed Operating Costs	Fixed Operating Costs or Avoidable Fixed Costs are defined as Start-Up Costs and Fixed Operating Costs per period (aka No-Load Costs. Fixed Operating Costs do not include investment or capital costs.
Flowgate (FG)	A transmission element or a collection of transmission elements
Flowgate Marginal Price (FMP)	is the change in market surplus if the flowgate capacity is increased by one MW.
Flowgate, Modeled	A Flowgate that is modeled for overloads
Flowgate, Monitored	A Flowgate that is being monitored for overloads incurred by normal operating conditions or for loss of another flowgate.

Futures Contract	A contract for future delivery.
Futures or Options Market	An exchange where Market Participants trade Futures or Options contracts. All contracts are with the exchange. Until the 1990s, trading occurred in 'pits' on the floor of the exchange using the 'open outcry' method. Now, almost all trades are electronic.
Generation	Electrical Energy injected into the network
Generator	A resource that converts mechanical or other power to electric power.
Generator Synchronization	The process of matching the speed and phase position of the Generator.
Gold-Plating	Engaging in unnecessary capital expenditures
Good Utility Practice	Any of the practices, methods, and acts engaged in by a significant portion of the electric utility industry or the exercise of reasonable judgment considering the facts known at the time the decision was made. (see FERC Order 888)
Hub	A representative selection of Nodes to facilitate trading.
Hub Price	An average of the prices at all hub nodes. Hub Prices are calculated after the model is run with no effect on LMPs.
Incentive Compatible	A process is incentive-compatible if all the participants fare best when they truthfully reveal any private information asked for by the mechanism.
Incremental Cost	is the change in the total costs for resulting from a given change in output. (Baumol, 1982, p 67). Some economists consider marginal cost and Incremental Cost to be identical. This paper defines them to be potentially different. marginal cost is measured with respect to an infinitesimally small change in output. Incremental Cost can be measured with respect to a larger change in output. Incremental Costs depend on the time horizon. For short periods, the Incremental Costs may be equal to marginal costs. For longer periods, the Incremental Costs may be greater than marginal costs. For long periods, the Incremental Costs include new investment costs. The average incremental cost is the change in total costs divided by the change in output (Δtotal costs/(Δoutput)).
incremental generator	A generator that breaks even under AIC pricing
Infra-Incremental Generator	A dispatched generator with lower average costs than the dispatched generator with the highest average costs and has and has positive profit
Infra-Marginal Resource	A dispatched resource with lower average costs than the dispatched resource with the highest average costs
Injection	is the flow of power into the network or Bus.
in-market	dispatched or scheduled by the system operator.
Interface	A set of Branches that, when opened, split network into two separate islands. An Interface is also known as a Cut Set
Intermittent	Occurring at irregular intervals; not continuous or steady.
Linear Settlement	For each Market Participant, the settlement Price multiplied by the settlement quantity
Linear Surplus	The difference between the bid value or cost and the Linear Settlement revenues. It can be either positive or negative
Liquidated Damages	compensation usually monetary stipulated in a contract for a loss created by a breach of the contract
Liquidity	The ability to sell or buy rapidly at an efficient Price. A Market is liquid if there are many ready-and-willing buyers and sellers and large quantities of trades can occur without causing large Price movements with low Transactions Costs. Financial Market Participants can provide liquidity – the willingness to take the other side of any transaction.
LMP Plus or LMP+	A pricing mechanism whereby each seller receives an energy price equal to the LMP at the relevant location and time, plus any necessary Make-Whole Payments
Load	An entity or entities that withdraw Energy from the network. Also known as Demand.
Load Pocket	An area where congestion limits the imports
Locational Incremental Price (LIP)	The Price used in settlement under AIC pricing. The LIP is the value of the dual variable on the Energy balance constraint in the AIC pricing run.
Locational Marginal Price (LMP or λ)	The Energy Price used in settlement under LMP pricing. The LMP for a given location and time period is either the marginal cost of serving one more unit of load at the location and period or the value of one more unit of demand to customers. That is, the LMP is the value

	of the dual variable on the Energy balance constraint in the Dispatch run. LMPs are calculated through the Security Constrained Economic Dispatch (SCED) algorithm that also determines the Energy and reserve Dispatches for both real time and day-ahead Markets
Locational Settlement Price (LSP)	is the energy price used in the Market settlement. The energy price is multiplied by the Market quantity of each Market Participant in the settlement calculations.
Long	A Market Participant is 'long' if it has a contract position beyond its physical capability to take delivery.
Long Term Emergency (LTE) Line Rating	The maximum loading for four hours without damaging the line beyond normal physical depreciation, about 120% of the normal rating
Loop Flow	An unscheduled flow over a neighboring system
Losses	See Transmission Line Losses.
Lost Opportunity Costs	When a choice is between mutually exclusive alternatives, the lost opportunity cost of one choice is the economic value or profit of the best-foregone alternative. In the case of Dispatch, the lost opportunity cost associated with following Dispatch instructions is the forgone profit from the most profitable alternative level of Energy production.
Make-Whole Payment	A payment to a resource to cover any positive different between its offer costs and the revenues received from selling at the Energy Price. That is, it is a payment when the Market Participant's Linear Surplus is negative.
Marginal Cost	The additional cost incurred to produce an additional unit of output. The additional cost can include both out-of-pocket and opportunity cost. marginal cost is a concept from differential calculus. In theoretical models, marginal cost is the first derivative of the cost function with respect to the Dispatch quantity. When cost functions are not differentiable, the derivative may not exist, or the right and left derivative are not the same. Marginal costs are 'instantaneous' costs (in the first derivative sense). Incremental costs, on the other hand, are variable costs incurred over a longer interval. Historical investment or sunk costs are not included in marginal costs.
Marginal Opportunity Cost	see marginal cost
Market	The act, instance or a collection of buying and selling transactions (Webster's). It includes a single transaction between two Market Participants and larger auction Markets with many Market Participants. The definition of a Market says nothing about regulation, Market power or Competition. Bilateral Markets are Markets where each transaction is between an individual buyer and an individual seller. Bilateral contract Markets can be very idiosyncratic. Auction Markets are organized Markets with defined products, defined procedures and the usually multiple buyers and/or multiple sellers.
Market Clearing	The state that occurs when the quantity supplied equals the quantity demanded.
Market Clearing Price or Prices	A price or set of Prices that cause quantities supplied and demanded to be equal.
Market Equilibrium	A state that exists when a set of Prices (including penalties for non-performance) and quantities offered to rational Market Participants, who can accept or reject the offers, the rational Market Participants will accept the offers, and no Market Participant has the ability and incentive to obtain a different set of Prices and quantities.
Market Failure	A condition that occurs when the Market does not achieve Economic Efficiency.
Market Microstructure	The details of how exchange occurs.
Market Power	The ability of a firm to alter the Market Price of a good or service (Wikipedia). For practical reasons, some include in the definition 'profitability' to exclude irrational behavior from the definition of Market power. We do not include profitability in the definition of Market power. Regardless of intent, results are the same. Sustainability is included for practical reasons, since the cost of regulatory intervention often exceeds the damage done in short-term exercises of Market power. We define market power as the ability to lower the market efficiency (surplus). There is no known revenue adequate pricing mechanism that solves the market power problem except for strong mitigation; that is, market participants bid incremental values and offer incremental costs. No price-

	formation proposal can fully eliminate market power. Efficient markets require mitigation.
market-clearing	is the process by which the auction quantities are computed.
market-clearing price	is the price of a good or service at which quantity supplied equals quantity demanded, also called the equilibrium price. in non-convex markets, a single market clearing price may not exist.
Merit Order	The ranking of resources in order of marginal costs, without consideration of other constraints.
MIP gap	is the distance between the best MIP feasible, LP optimal objective and the best-remaining-node objective. It is a bound on the how close the current best MIP feasible, LP optimal solution is to an optimal solution. If the MIP gap = 0, the solution is optimal. For computational reasons, the MIP gap maybe be greater than 0.
Mitigation	The actions taken by a regulator to constrain the behavior of market participants so that they behave efficiently an efficient market is achieved if the bids are at incremental values and offers are at incremental costs.
Monitored Line	A transmission line that is explicitly represented in the Dispatch model. These are lines with a high probability of being operated at its maximum operating level.
Moral Hazard	The incentive for a rational Market Participant to take excessive Risks because the costs incurred will not be felt fully by a Market Participant.
Motor	A device that converts electrical power to mechanical power and can produce or consume reactive power and produces torque
N-1 reliable	a state of reliability can survive any single contingency.
N-1-1 reliable	a state of reliability that can survive any contingency and returned to N-1 reliability in 30 minutes
N-2 reliable	a state of reliability that can survive any two simultaneous contingencies
Natural Monopoly	A market for which it is efficient with only one firm.
Natural Oligopoly	a market for which it is efficient with a few firms
Neoclassical Economics	An economic theory developed at the end of the 19th and the beginning of the 20th century by (among others) Léon Walras, Alfred Marshall and Vilfredo Pareto -- all of whom were engineers) that places greater emphasis on physics and mathematics. In order to prove theorems, assumptions created greater abstraction from reality, for example convexity and differentiability. Discrete decisions were treated in an ad hoc way. Here, neoclassical economics and convex economics are synonymous.
Node	The connection point of two or more devices
Nomogram Constraint	is a set of weighted elements whose function is to impose a flow limit on the sum of its members (Abdurrahman, PJM, 2012). A nomogram constraint is a surrogate for constraints that are not explicitly modeled, for example, voltage constraints
Non-Confiscation	is a Market condition that requires payments to be made to a seller to cover incremental offer costs and payment from a buyer not to exceed its incremental bid value (offer to pay).
Non-Convex Markets	A market where at least one of the components (such as supply and demand) is not Convex. A mathematical function describing a market component is Non-convex if at least one possible straight line connecting any two points on the function does not lie above the function. ISO markets are non-convex. There is no perfect analogy between convex markets and non-convex markets, but many of the efficiency concepts are analogous, but difference is important. Convex markets do not allow binary decisions. Unlike convex markets, there is no known perfect pricing scheme for non-convex markets. It is difficult to explain non-convex markets with a convex model. The incremental cost increases with demand, but the average incremental costs may decline. With binary decisions (variables), the efficient market is a combinatorial optimization with market participants' bidding incremental values and offering incremental costs. In electricity markets, some of the causes of non-convexity in the short-term include startup costs, minimum operating

	level, and minimum run time. Over the longer term, the causes of non-convexity include maintenance and investment decisions.
Non-Excludable	A characteristic of a good or service where the owners cannot prevent others from consuming it.
Non-Incumbent Transmission Developer	A transmission developer that seeks to develop transmission facilities in an area where it does not have a retail distribution service territory. Such a developer is either: a transmission developer that does not have a retail distribution service territory or a public utility transmission provider that proposes a transmission project outside of its existing retail distribution service territory where it is not the incumbent for purposes of that project. See Transmission Planning and Cost Allocation by Transmission Owning and Operating Public Utilities, Order No. 1000, 136 FERC ¶ 61,051, at P 225
Non-Rivalrous	A characteristic of a good or service where consumption of the good or service by one does not prevent consumption by another
Normal Transmission Line Rating (NR)	The maximum loading that the conductor can carry continuously without damaging the line beyond normal physical depreciation.
Off-ISO Market	A Market that trades in and around the ISO Markets, but not operated by an ISO. Off-ISO Markets often cash out using ISO Market Prices and/or use the ISO Market as a physical delivery mechanism. They are essentially financial (settled without a physical exchange) since the real-time Market provides the physical product reliability at a just and reasonable Price. These Markets are essentially for Risk management bet on ISO Market outcomes
One in Ten	A resource adequacy standard that specifies that enough Generation capacity is procured so that firm load is shed involuntarily no more than one time in 10 years. The standard does not specify the duration or size of the load shedding event.
Operating Reserve	is a reserve of real power available in the current Market.
Operating Reserve Shortage	A condition where the amount of available supply falls short of demand plus the Operating Reserve requirement.
Operator Action	An action taken by an operator
optimal dispatch	The Dispatch that maximizes the Market surplus, that is, the producer (offer) plus consumer (bid) surplus. If a consumer does not bid, the optimal dispatch is the least cost Dispatch.
Option Contract	A contract that gives the buyer the right, but not the obligation to buy (a call) or sell (a put) an underlying asset at a specific Price on or before a certain date.
Off-Market	refers to transactions that are made outside of the ISO's auction
out-of-market	not dispatched or scheduled by the system operator.
Out-Of-Merit	Usually refers to a Generator that is part of the efficient commitment and Dispatch, but appears to be uneconomic in the real power Dispatch if transmission constraints are ignored, because its marginal costs are higher than one or more other Generators that are not dispatched. This condition usually arises because the undispatched Generator is located on the export side of a congested transmission interface. Most of the time these Generators are actually 'in merit', that is, part of the efficient commitment and Dispatch
Out-Of-Merit Order	A Dispatch in which one or more Generators are dispatched Out of Merit.
Pareto Improvement	A change in the Market that makes one or more Market Participants better off without making someone worse off
Pareto Optimality or Efficiency	A Market condition where no one can be made better off without making someone worse off
Pass-Through	The act of offsetting increased costs by raising Prices
Physical Market Participant	A Market Participant who bids or offers a physical asset into the Market and physically exchanges for a product.
Pivotal Supplier	A supplier who can create a shortage in the Market by withdrawing. A supplier becomes pivotal when its total capacity exceeds the total surplus supply (i.e., the difference between total supply and total demand) in the Market. The m Pivotal Supplier test is when the m largest suppliers can create a shortage in the Market. Generally, as m gets small, the potential for Market power increases, but we have no comparison to the other more traditional tests. This test is unique to the power Markets since it assumes consumers have no Price elasticity. PJM and CAISO use the Pivotal Supplier test.

Potential Energy	Stored Energy
Power (P)	The work in a unit of time. P = E/t. watt = joule/second
Price	The compensation given by one party to another party in return for goods or services.
Price Chasing	Responding to the ex-post LMP without receiving a Dispatch instruction from the grid operator to do so. The LMP ex post is not a Price for sale or purchase in the immediately forthcoming period. Rather it is the Price that was applied for sales and purchases in the period that has just closed.
Price Taker	is an entity that must accept the prevailing Price in the Market for its products, since it lacks Market power to be able to influence the Market Price.
Price-Responsive Demand	Demand that bids into the day-ahead Market and real-time Market and pays no capacity price.
Pricing Point of Entry	The minimum price at which a potential entrant can cover its costs and be willing to enter the market.
Principal-Agent Problem	The problem concerning the difficulties in motivating one party (the agent), to act in the best interests of another (the principal) rather than in its own interests. LSEs act on behalf of its retail customers in ISO Markets
Private Good	A good or service that is Excludable and Rivalrous.
profitability	profitability includes breaking even in each independent up-down cycle
Public Good	A good that is Non-Excludable and Non-Rivalrous, that is, consumption by one does not prevent consumption by another. Usual examples include national defense, clean air and clean water. Public Goods are often not free goods and have the free-rider problem, that is, a free rider may consume a good without paying for it.
Ramsey-Boiteux prices	a pricing scheme that requires fixed costs to be recovered from consumers based on the willingness to pay.
Revenue Neutral Market	A Market that allocates all Market revenues to the Market Participants and needs no additional revenues.
Risk	The exposure to the possibility of loss, injury, or other adverse or unwelcome circumstance (The Oxford English Dictionary)
Rivalrous	A good is rivalrous when consumption by one prevents consumption by another
Rogue Trading	Trading irrationally often due to poor internal incentives and controls.
Rolling Blackout	A controlled series of forcibly curtailing load
Scarcity Pricing	A pricing approach whereby when reserves are scarce or becoming scarce, Prices increase to reflect the scarcity and thereby incent Market Participants to respond to the scarcity in an efficient (and reliable) way
SCED	Security Constrained Economic Dispatch is an economic Dispatch
SCUC	Security Constrained unit commitment is UC with reliability constraints
Self-Commitment	The decision to commit a resource made by the owner of the resource rather than by the grid operator.
Self-Schedule	Self-scheduling by a market participant fixes the quantity in its bid or offer. The decision by the owner of the resource to schedule the Energy produced, rather than by the grid operator. . Self-schedules and self-commits may lower the market surplus and lower the flexibility of the market.
Short	A Market Participant is 'short' if it has a net contract position beyond its physical capability to supply.
Short-Time Emergency (STE) Line Rating	The maximum loading that a conductor can carry for fifteen (15) minutes without damaging the line beyond normal physical depreciation.
State Estimator (SE)	A component of the Energy management system (EMS) that estimates the current state of the system based on data from supervisory control and data acquisition (SCADA) system. SE finds the solution 'closest' to the metered data and runs every few minutes and may produce an estimated topology and an estimated power flow for the system.
Static Var Compensator (SVC)	A device for providing reactive power in transmission networks. SVCs can regulate voltage and harmonics stabilizing the system with no significant moving parts.
Stranded Costs	Costs incurred by a cost-of-service regulated company, which the company is not able to recoup

Superpositioning	The assumption that the individual effects are algebraically added. For example, in linear models, each power Injection or Withdrawal is algebraically added. In addition, if I is a vector of complex current, Y is the complex admittance matrix and V is a vector of complex voltage, then I = YV. If $I^1 = YV^1$ and $I^2 = YV^2$, then $I^1 + I^2 = Y(V^1 + V^2)$
Swap	A Financial Derivative in which counterparties exchange cash flows of one party's financial instrument for those of the other party's financial instrument.
Synchronous Condenser	A rotating electrical machine identical to a synchronous motor that consumes real power, can produce or consume reactive power and produces no torque. Its field is controlled by a voltage regulator to generate or absorb reactive power continuously as needed to adjust the voltage. Increasing the device's field excitation results in more reactive power (Vars) to the system. The kinetic Energy stored in the rotor of the machine can help stabilize a power system during short circuits or rapidly fluctuating loads such as electric arc furnaces. A decoupled turbine can stay in place, eliminating demolition costs and allowing a seasonal change between Generator and condenser modes if required.
Tangent	A straight line or plane that touches a function at a point, but does not cross the function
Transactions Costs	The costs of executing a transaction or contract.
Transmission Line Losses (Losses)	The losses of Energy occurring when transmitting Energy over a transmission system due to heating of assets
Transmission Line Rating	A level of power flow on a transmission line at which the transmission operator determines that actions may be necessary to reduce line flow.
Transparency	The public information available for the ISO
True Up	The process of revising charges to reflect actual costs incurred after the occurrence.
truthful bidding	Bidding incremental values or offering incremental costs
Uninstructed Deviation	Production in real time by a resource that differs from the amount instructed by the grid operator to be produced.
Unit Commitment (UC)	a mixed integer program that determines of the resource dispatch and synchronized to the grid to produce Energy over a specified period. The software considers each unit's startup costs, a minimum operating level, minimum run time, minimum down times and shutdown schedules. .
Uplift	The sum of the Make-Whole Payment and the Lost Opportunity Costs. .
Uplift Charge	A charge to a Market Participant in addition to the energy price charge.
Uplift Payment	A payment to a Market Participant in addition to the energy and reserve price payments.
valid optimality cut	is any constraint that does not eliminate all optimal solutions.
Voltage	A measure of the difference in electric potential between two points in space, a material, or an electric circuit, expressed in volts. One volt is the difference in electric potential between two points of a conducting wire when an electric current of one ampere dissipates one watt of power between those points. It is also the potential difference between two parallel, infinite planes spaced 1 meter apart that create an electric field of 1 newton per coulomb. Additionally, it is the potential difference between two points that will impart one joule of Energy per coulomb of charge that passes through it. $v = kg \times m^2/(A \times s^2)$
Walrasian Auction	The classic Walrasian auction (or tatonnement) starts with an announced price. The announced price is a signal to consider changing your bid or offer in response to the announced price. The auction ends when supply equals demand. Walrasian auctions are mostly pedagogical. They do not work well in practice because they may not converge to market-clearing prices. Over time, the concept of a Walrasian auction has evolved to market participants submitting bids and offers and clear markets by computer algorithm.
Wheel	Transmission of electricity. Before transmission lines networks of wooden rods or fast-moving ropes conveyed power from water wheels to machinery. Later, steel cables replaced wooden rods for the transmission of power.
Withdrawal	Flow of power from Bus aka load or demand
Work	In physics, a force applied through a distance. W=Fd
Zonal Price	The load-weighted average of all Nodal Prices in the Zone.
Zone	A collection of Buses (often contiguous in a single area).

11 References

The references are arranged in alphabetical order by the first author's last name. The blue references are intended for background information, while the black references are directly mentioned in the text.

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