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Robust Workforce Management with Crowdsourced Delivery

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We investigate how crowdsourced delivery platforms with both contracted and ad-hoc couriers can effectively manage their workforce to meet delivery demands amidst uncertainties. Our objective is to minimize the hiring costs of contracted couriers and the crowdsourcing costs of ad-hoc couriers while considering the uncertain availability and behavior of the latter. Due to the complication of calibrating these uncertainties through data-driven approaches, we instead introduce a basic reduced information model to estimate the upper bound of the crowdsourcing cost and a generalized reduced information model to obtain a tighter bound. Subsequently, we formulate a robust satisficing model associated with the generalized reduced information model and show that a binary search algorithm can tackle the model exactly by solving a modest number of convex optimization problems. Our numerical tests using Solomon's data sets show that reduced information models provide decent approximations for practical delivery scenarios. Simulation tests further demonstrate that the robust satisficing model has better out-of-sample performance than the empirical optimization model that minimizes the total cost under historical scenarios.

Key words: Workforce management, crowdsourced delivery, uncertain ad-hoc couriers, data-driven robust satisficing

1. Introduction

The rapid expansion of e-commerce has stimulated e-retailers and local businesses to enhance their logistics operations and deliver goods promptly and reliably in a cost-effective way. As a result, many have turned to crowdsourced delivery, which involves independent individuals using their own vehicles to deliver goods. For instance, Amazon launched the Amazon Flex program in 2015, which utilizes crowdsourced couriers for last-mile deliveries. Similarly, Walmart piloted crowdsourced delivery in two US cities in 2018 under the name Spark Delivery. Many third-party logistics companies that rely on crowdsourced delivery have also emerged, such as Postmates, Uber

Eats, and Deliv. These platforms vary in terms of payment methods, target markets, and the types of items delivered. Leveraging crowdsourced delivery resources allows platforms to swiftly adjust their delivery capacity to cope with fluctuating demand while also providing cost advantages.

The use of independent couriers for crowdsourced delivery poses challenges due to their unknown availability and job bidding behavior. To mitigate the adverse effects caused by this type of uncertainty, many platforms have opted for a hybrid workforce model that combines ad-hoc crowdsourced couriers with pre-hired couriers, such as employees or crowdsourced couriers who agree to work for a certain period with a guaranteed minimum payment (Yildiz and Savelsbergh 2019, Ulmer and Savelsbergh 2020). In the Amazon Flex program, for instance, deliveries are organized into blocks lasting from 2 to 6 hours, and crowdsourced couriers search for available delivery blocks through the platform's app, make requests for interested offers, and receive confirmation from Amazon. When couriers' block times are approaching, they go to the pick-up sites, load packages, and deliver them to customers. A hybrid model enables platforms to manage their workforce more effectively to provide reliable customer service. Nevertheless, the employment decision of pre-hired couriers presents a challenge as future customer orders remain uncertain to the platform. Over-hiring couriers incurs unnecessary costs for the platform, while under-hiring may require the hiring of ad-hoc couriers at premium prices to ensure timely delivery.

In this study, we present a robust satisficing framework for addressing delivery platforms' workforce management problem, with the aim of maximizing the robustness of achieving cost targets under uncertainty. Our approach distinguishes between two types of couriers: *contracted* couriers, who are hired before the planning horizon, and *ad-hoc* couriers, who are hired during the operational stage. The proposed framework accounts for the uncertainty of ad-hoc couriers' availability and job bidding behavior, as well as the cost associated with hiring ad-hoc couriers when necessary. By formulating the problem as a robust satisficing model, our framework provides a tool for decision-making that enables platforms to effectively manage their workforce resources and balance their cost objectives with service quality requirements.

Literature review

The field of crowdsourced last-mile delivery has garnered increasing attention recently. Alnaggar et al. (2021) analyze the current industry status of crowdsourced delivery platforms based on matching mechanisms, target markets, and compensation schemes. They indicate that for a centralized system with an hourly compensation scheme, a significant challenge is forecasting delivery needs and the number of couriers required to fulfill them. Although platforms can provide on-demand delivery blocks in case contracted couriers are insufficient, there is a higher level of risk involved since the availability of on-demand couriers is not guaranteed. Savelsbergh and Ulmer

(2022) identify the challenges and opportunities in crowdsourced delivery planning and operations. The tactical challenge is ensuring that the required crowdsourced delivery capacity is available, while the operational question is how to adjust delivery capacity if the anticipated capacity is not materialized or if demand exceeds expectations. Besides uncertain availability, couriers' behavior is also uncertain, as they may accept or reject a delivery task and deviate from planned routes (Liu et al. 2021). To reduce uncertainties in delivery capacity, the authors suggest that planners determine a set of delivery shifts and offer them to couriers for commitment before the operational period begins. Additionally, a dynamic compensation mechanism can be designed to adjust the availability of ad-hoc couriers.

Behrendt et al. (2022) study the crowdsourced same-day delivery problem, deciding the fleet sizing of contracted couriers, pricing of ad-hoc couriers in the planning stage, and order allocation in the operational phase. They assume that order and ad-hoc courier arrivals follow Poisson processes and focus on evaluating the benefits of utilizing a hybrid workforce and various order allocation policies. Goyal et al. (2023) address a multistage problem involving the determination of contracted courier fleet sizes at each warehouse in the first stage, followed by assignment and routing decisions for both contracted and ad-hoc couriers once orders and ad-hoc couriers have arrived. They formulate the problem as a multistage stochastic integer program and develop an approximate dynamic programming method. These two studies assume that contracted couriers are available for the entire planning horizon once hired. In contrast, our work determines fleet sizes for each work shift, accounting for varying uncertainty in each period.

The works of Ulmer and Savelsbergh (2020) and Behrendt et al. (2023) are highly relevant to our paper. Specifically, Ulmer and Savelsbergh (2020) address the workforce scheduling problem for contracted couriers, determining the optimal number of shifts and their start time and duration. The authors consider stochastic arrivals of orders and ad-hoc couriers, as well as the duration an ad-hoc courier is willing to work. Their objective is to minimize working hours while ensuring a minimum percentage of orders are fulfilled. The authors employ continuous approximation and value function approximation methods, utilizing scenarios to represent realizations of random variables. Behrendt et al. (2023) propose a prescriptive machine learning method for a similar problem. They leverage the sample average approximation (SAA) method offline to generate solutions for various instances, then use a machine learning model to produce online solutions for new demand and ad-hoc courier arrival forecasts. Their objective is to minimize courier payments and penalty costs for late deliveries. We note that besides the uncertain availability, our work also considers ad-hoc couriers' uncertain job bidding behavior. Moreover, we do not assume any probability information of random variables and use a data-driven robust optimization method to mitigate the impact of distributional ambiguity on the risk-based objective function.

Many studies on crowdsourced delivery have focused solely on addressing operational-level allocation and routing problems without considering tactical decisions. For example, Archetti et al. (2016) address the vehicle routing problem (VRP) with occasional drivers, referring to drivers who are willing to make deliveries with a detour on their way to their destination. Dayarian and Savelsbergh (2020) investigate a same-day delivery problem that involves dynamically arriving online orders and in-store customers who, in addition to shopping, also deliver online orders as a supplement to company drivers. The authors propose rolling horizon dispatching approaches, with and without incorporating probabilistic information about future arrivals of orders and in-store customers. Similarly, Torres et al. (2022) tackle the VRP with stochastic crowd vehicles and customer presence, formulating the problem as a two-stage stochastic model and developing a column generation heuristic for solving large-size instances.

To the best of our knowledge, we are the first to use a data-driven decision framework to make tactical and operational planning decisions for the crowdsourced delivery problem with a hybrid workforce, considering ad-hoc couriers' uncertain availability and job bidding behavior. Our work is built upon the recent development of data-driven robust optimization, which aims to address issues in optimization under uncertainty. One prominent issue is the *Optimizer's curse* (Smith and Winkler 2006), a phenomenon that inferior results are always expected in the out-of-sample test if one uses the empirical distribution from training data to solve an optimization problem. A similar issue also appears in SAA (Birge and Louveaux 2011) through which the objective estimated in stochastic optimization is optimistically biased. Although the bias can be reduced with more samples, it would be prohibitive to do so in a data-driven setting where data is collected over time.

Data-driven robust optimization approach has been developed recently to address this issue (Bertsimas et al. 2018, Mohajerin Esfahani and Kuhn 2018, Gao and Kleywegt 2023). This method can effectively mitigate the risk of uncertainty by optimizing the worst-case objective within an ambiguity set that includes probability distributions of certain properties, such as having moment information matching the empirical distribution or being close to it based on specific distance metrics. Notably, Mohajerin Esfahani and Kuhn (2018) and Gao and Kleywegt (2023) propose the ambiguity set based on the Wasserstein distance that enjoys statistical guarantee to capture the true distribution. Nonetheless, the practical issue of determining the radius of the Wasserstein ambiguity set has been acknowledged. Instead of relying on the theoretical value of the statistical guarantee, cross-validation is often required to tune the radius parameter for better out-of-sample performance.

More recently, Long et al. (2022) propose the robust satisficing model, which is based on target-oriented optimization instead of conventional utility maximization. Contrary to robust optimization that specifies an ambiguity set of probability distributions, the robust satisficing model has no restriction on the distributions and minimizes the uncertainty risk of not achieving a specific target. Targets play an important role in the human decision process, especially in complex environments full of uncertainty and risks (Simon 1955, Mao 1970, Chen and Tang 2022). In articulating preferences, the robust satisficing model requires the decision-maker to set performance targets, which are more interpretable and practical to specify. The idea of robust satisficing has been emerging recently in both theoretical developments (Chen et al. 2022, Liu et al. 2023, Sim et al. 2021) and practical applications (Goh and Hall 2013, Zhang et al. 2021, Zhou et al. 2022).

Our contributions

We propose using the robust satisficing framework to address the workforce management problem, which involves hiring contracted couriers for each work shift before the planning horizon. Whenever the contracted couriers are insufficient during the operational stage, the platform hires ad-hoc couriers based on their bidding for delivery jobs. Because the bidding strategy of ad-hoc couriers is unknown, it would be prohibitive to precisely determine the expected crowdsourcing costs associated with the delivery workforce in the planning horizon. The robust satisficing model aims to meet the specified expected cost target and perform as well as possible under distributional ambiguity. The contributions of our work are summarized as follows.

1. To characterize the empirical distribution of the crowdsourcing costs using historical bidding records made by the ad-hoc couriers, we first propose a basic reduced information model that evaluates the upper bound of the crowdsourcing cost. We prove that the bound is tight under the single payment value bidding scheme. We further introduce a generalized reduced information model to obtain a tighter bound on the crowdsourcing cost that would improve with the number of breakpoints.
2. Based on the basic reduced information model, we propose a data-driven robust satisficing model that incorporates a time additive Wasserstein metric to characterize distributional ambiguity in the workforce management problem. We can reformulate the proposed robust satisficing model as a single deterministic tractable convex optimization problem. We also extend this to the generalized reduced information model and demonstrate that a binary search algorithm can be applied to tackle the robust satisficing model exactly via solving a modest number of convex optimization problems.
3. We provide statistical justifications for the robust satisficing model that are based on target attainment. For small deviations from the target, we provide a target attainment confidence guarantee based on the probability bound of the Wasserstein distance. For large shortfalls from the target, we provide a concise expression of probability guarantee, which does not depend on the number of breakpoints of the generalized reduced information model.

4. Numerical tests on Solomon's data sets show that the basic reduced information model can provide decent approximations of the crowdsourcing costs for practical delivery problems when multiple payment values are allowed. The generalized reduced information model further improves the bounds, controlling all the gaps within 5%. Moreover, simulated tests demonstrate that the proposed robust satisficing model provides better out-of-sample performance than the empirical model, especially in the cases of high levels of uncertainty and risk.

Notation. We use boldface lowercase letters for vectors (*e.g.*, $\boldsymbol{\theta}$), and calligraphic letters for sets (*e.g.*, $\mathcal{X}$). We use $|\cdot|$ to denote the cardinality of a finite set. We use $[N]$ to denote the running index $\{1, 2, \dots, N\}$ for $N \in \mathbb{N}$, and $[0] = \emptyset$. A random variable $\tilde{v}$ is denoted with a tilde sign such as $\tilde{v} \sim \mathbb{P}, \mathbb{P} \in \mathcal{P}_0$, where $\mathcal{P}_0$ represents the set of all possible distributions. For a multivariate random variable, we use $\mathcal{P}_0(\mathcal{Z})$ to represent the set of all distributions for the multivariate random variable that has support $\mathcal{Z} \subseteq \mathbb{R}^n$. Specifically, we use $\tilde{\boldsymbol{z}} \sim \mathbb{P}$, $\mathbb{P} \in \mathcal{P}_0(\mathcal{Z})$ to define $\tilde{\boldsymbol{z}}$ as a multivariate random variable with support $\mathcal{Z}$ and distribution $\mathbb{P}$. We use $\mathbb{E}_{\mathbb{P}}[\tilde{v}]$ to denote the expectation of a random variable, $\tilde{v} \sim \mathbb{P}$, over its distribution. Finally, $\mathbf{0}$ ($\mathbf{1}$) denotes the vector of all zeros (ones) and $\boldsymbol{e}_i$ denotes the ith basis vector. The dimensions of these vectors should be clear from the context. All the proofs appear in Appendix A.

2. Workforce management with crowdsourced delivery

We are examining a delivery workforce management platform that operates over a planning horizon of T time periods. Before the planning horizon, the platform creates I different work shifts to offer to crowdsourced couriers and makes the employment decision of contracted couriers for each shift. During the operational stage, if the contracted couriers cannot handle all the delivery jobs, the platform hires ad-hoc couriers based on their bidding for available jobs. As the arrivals of packages and the ad-hoc couriers' availability and bidding behaviors are unknown before the planning horizon, it is challenging for the platform to make the employment decision of contracted couriers, such that all packages can be delivered on time with a minimal expected cost.

Problem description

For each work shift, $i \in [I]$, we define $\mathcal{S}_i \subseteq [T]$ as the set of periods within the time horizon covered by the shift. For each period $t \in [T]$, we define the vector $\boldsymbol{a}_t \in \{0,1\}^I$, where $a_{ti} = 1$ if $t \in \mathcal{S}_i$, and $a_{ti} = 0$, otherwise, for all $i \in [I]$. Without loss of generality, we assume that a sufficient number of couriers have registered to work for each shift because the platform can release the shifts several days or weeks earlier than the start of the planning horizon. We define $\boldsymbol{x} \in \mathcal{X} \subseteq \mathbb{Z}_+^I$ as the employment decision, where x_i represents the number of contracted couriers hired to work at shift $i \in [I]$. The feasible set $\mathcal{X}$ can encapsulate the detailed constraints associated with individual contracted couriers. Then, the number of contracted workers at time $t \in [T]$ is $\boldsymbol{a}_t^\top \boldsymbol{x}$. The compensation to the contracted courier in the ith, $i \in [I]$ work shift is w_i. Hence, the total compensation for hiring contracted workers would be $\boldsymbol{w}^\top \boldsymbol{x}$.

Before the beginning of each period, $t \in [T]$, the set of packages has arrived and must be delivered by the end of the tth period. The delivery workforce management platform would solve a VRP, which lexicographically minimizes the number of couriers and the total traveling distance while adhering to delivery time windows and couriers' capacity constraints. Hence, the solutions to the VRP provide the number of jobs $J_t \in \mathbb{Z}_+$, where each job is a set of packages to be delivered by one courier by the end of the time period. As the employment decision, $\boldsymbol{x}$ is made before customer orders are realized, there may exist situations where the hired contracted couriers cannot complete all the delivery tasks. Specifically, at the beginning of period t, there would be potential $(J_t - \boldsymbol{a}_t^\top \boldsymbol{x})^+$ jobs that the contracted couriers could not fulfill.

To provide high-quality service to customers, the platform guarantees that all packages realized at the beginning of a period will be delivered by the end of that period. Hence, the platform considers another matching mechanism by hiring ad-hoc couriers. Specifically, the platform releases all the J_t jobs to ad-hoc couriers who would bid for these jobs. In particular, the ad-hoc couriers specify their desired compensations for each job from a given list of N possible payments, $p_1, p_2, \ldots, p_N > 0$, with $p_n \leq p_{n+1}$. Let K_t denote the number of ad-hoc couriers participating in bidding at period t. We define the corresponding bidding set $\mathcal{B}_t$ as the set of tuples $(k, j, n) \in [K_t] \times [J_t] \times [N]$, with each tuple (k, j, n) representing courier k has bidden for job j for payment p_n. For convenience, we also define the projection of $\mathcal{B}_t$ on the tuples $(k, j) \in [K_t] \times [J_t]$ as follows

$$\bar{\mathcal{B}}_t := \{(k, j) \in [K_t] \times [J_t] \ \mid \ \exists n \in [N] : (k, j, n) \in \mathcal{B}_t\}.$$

Assumption 1. *We assume that p_N is high enough so that we can always find someone to take up any delivery job for that price. Consequently, we can assume that every job $j \in [J_t]$ can be assigned to an unique bidder $k_j \in [K_t]$, so that $(k_j, j) \in \bar{\mathcal{B}}_t$ for all $j \in [J_t]$ and $|\{k_j \mid j \in [J_t]\}| = J_t$.*

To ensure Assumption 1 always holds in practice, we can temporarily assign each job $j \in [J_t]$ to a dummy courier at payment p_N. Whenever a dummy courier is assigned, we can always replace it with someone willing to deliver the corresponding job at that price. If no ad-hoc courier is available to take the job, we can outsource the job to a third-party logistics company at payment p_N. In an extreme case, neither ad-hoc couriers nor third-party logistics companies are available; we can consider a penalty cost p_N occurs, as p_N is assumed to be sufficiently high. In fact, this extreme case can also be regarded as where a dummy courier takes up the job with payment p_N.

After the bids have been gathered, the platform would assign jobs to ad-hoc couriers. For a realized bidding set $\mathcal{B}_t$, we can obtain the information, J_t, K_t, and $\bar{\mathcal{B}}_t$. Subsequently, the planner decides the employment of ad-hoc couriers with the minimum crowdsourcing cost, $f_t(\boldsymbol{x}, \mathcal{B}_t)$, where

$$
\begin{aligned}
f_t(\boldsymbol{x}, \mathcal{B}_t) = \min \quad & \sum_{(k,j,n)\in\mathcal{B}_t} p_n s_{kj} \\
\text{s.t.} \quad & \sum_{k:(k,j)\in\bar{\mathcal{B}}_t} s_{kj} \le 1 && \forall j \in [J_t], \\
& \sum_{j:(k,j)\in\bar{\mathcal{B}}_t} s_{kj} \le 1 && \forall k \in [K_t], \\
& \sum_{(k,j)\in\bar{\mathcal{B}}_t} s_{kj} \ge J_t - \boldsymbol{a}_t^\top \boldsymbol{x} \\
& s_{kj} \ge 0 && \forall (k,j) \in \bar{\mathcal{B}}_t,
\end{aligned}
\tag{1}
$$

in which the first set of constraints ensures that job $j \in [J_t]$ is assigned to at most one ad-hoc courier, and the second set of constraints requires that each ad-courier $k \in [K_t]$ is assigned to at most one job. The third set of constraints requires that all jobs be assigned to contracted or ad-hoc couriers. Since this is a network flow optimization problem, if $\boldsymbol{a}_t^\top \boldsymbol{x} \in \mathbb{Z}$, then there exist binary optimal solutions for the decision variables s_{kj}, $(k,j) \in \bar{\mathcal{B}}_t$ such that $s_{kj} = 1$ if courier k's bid for job j is accepted, and $s_{kj} = 0$ otherwise. All the remaining jobs are consequently assigned to contracted couriers.

Note that the considered problem applies to both the one-to-many and many-to-many logistics systems, as we do not explicitly solve the routing problem. This is because various VRPs have been well-studied in the literature, and many algorithms can be used by platforms to tackle the associated VRP and decide the number of jobs and the delivery routes. When applied to the one-to-many logistics system, both contracted and ad-hoc couriers start their jobs from the depot. After delivering all the packages included in a job, the ad-hoc courier can directly leave the platform and does not need to return to the depot. For a contracted courier, returning to the depot or not depends on the shift s/he is hired. If the current time period is the last one in a shift, the courier can leave the system without returning to the depot; otherwise, s/he must return to the depot to perform a job in the subsequent period. In a many-to-many delivery system, at the beginning of a time period, couriers directly go to customers' sites to pick up packages and then deliver them to corresponding destinations. After finishing the last delivery in an assigned job, the ad-hoc courier leaves the platform. For a contracted courier, if the current time period is the last one in a shift, s/he can directly leave the system; otherwise, s/he continues to perform another job in the subsequent period by going from the last customer in the current period to the first customer assigned in the next period.

Empirical model

During the planning horizon, we do not know the future arrivals of packages and how ad-hoc couriers would bid for the jobs. We denote $(\tilde{\mathcal{B}}_1,\dots,\tilde{\mathcal{B}}_T)$ as the joint random bidding sets for all time periods, and its true distribution $\mathbb{Q}^\star$, $(\tilde{\mathcal{B}}_1,\dots,\tilde{\mathcal{B}}_T)\sim\mathbb{Q}^\star$ is unobservable to the decision-maker. Hence, it is impossible to determine the true optimum solution in the following *ideal optimization problem*.

$$\begin{aligned} Z^\star = \min\ & \boldsymbol{w}^\top\boldsymbol{x} + \mathbb{E}_{\mathbb{Q}^\star}\left[\sum_{t\in[T]} f_t(\boldsymbol{x},\tilde{\mathcal{B}}_t)\right] \\ \text{s.t.}\ & \boldsymbol{x}\in\mathcal{X}. \end{aligned} \tag{2}$$

Nevertheless, we have access to Ω historical sample path records of the bidding sets $\mathcal{B}_t^\omega$, for all $\omega\in[\Omega]$, $t\in[T]$. Accordingly, we denote $\hat{\mathbb{Q}}$ as the empirical distribution such that

$$\mathbb{E}_{\hat{\mathbb{Q}}}\left[\tilde{\mathcal{B}}_t = \mathcal{B}_t^\omega \quad \forall t\in[T]\right] = \frac{1}{\Omega} \qquad \forall\omega\in[\Omega].$$

To obtain the decisions for the contracted couriers, $\boldsymbol{x}$, we can solve the *empirical optimization problem* that minimizes the average cost for the planning horizon as follows

$$\begin{aligned} \hat{Z} = \min\ & \boldsymbol{w}^\top\boldsymbol{x} + \mathbb{E}_{\hat{\mathbb{Q}}}\left[\sum_{t\in[T]} f_t(\boldsymbol{x},\tilde{\mathcal{B}}_t)\right] \\ \text{s.t.}\ & \boldsymbol{x}\in\mathcal{X}, \end{aligned} \tag{3}$$

which is equivalent to the following linear optimization problem,

$$\begin{aligned} \hat{Z} = \min\ & \boldsymbol{w}^\top\boldsymbol{x} + \frac{1}{\Omega}\sum_{\omega\in[\Omega]}\sum_{t\in[T]}\sum_{(k,j,n)\in\mathcal{B}_t^\omega} p_n s_{tkj}^\omega \\ \text{s.t.}\ & \sum_{k:(k,j)\in\bar{\mathcal{B}}_t^\omega} s_{tkj}^\omega \le 1 && \forall j\in[J_t^\omega], t\in[T], \omega\in[\Omega], \\ & \sum_{j:(k,j)\in\bar{\mathcal{B}}_t^\omega} s_{tkj}^\omega \le 1 && \forall k\in[K_t^\omega], t\in[T], \omega\in[\Omega], \\ & \sum_{(k,j)\in\bar{\mathcal{B}}_t^\omega} s_{tkj}^\omega \ge J_t^\omega - \boldsymbol{a}_t^\top\boldsymbol{x} && \forall t\in[T], \omega\in[\Omega], \\ & s_{tkj}^\omega \ge 0 && \forall (k,j)\in\bar{\mathcal{B}}_t^\omega, t\in[T], \omega\in[\Omega], \\ & \boldsymbol{x}\in\mathcal{X}. \end{aligned} \tag{4}$$

While our work does not address the correlation between the bidding set $\mathcal{B}_t$ and the employment decision $\boldsymbol{x}$, we acknowledge their potential relationship. For instance, the availability of jobs for ad-hoc couriers to bid on may be influenced by the number of contracted couriers, while other factors like the utility of ad-hoc couriers could also impact the bidding set $\mathcal{B}_t$. Despite these potential correlations, accurately characterizing the relationship between $\boldsymbol{x}$ and $\mathcal{B}_t$ presents a challenge due to the lack of comprehensive literature on the subject. Indeed, any attempt to precisely model this interaction is susceptible to modeling errors, further supporting the use of robust optimization methods to mitigate such errors. Additionally, although modeling decision-dependent uncertainties might be a viable option, it could render our problem intractable, as single-stage robust models with decision-dependent uncertainties are already computationally cumbersome (Nohadani and Sharma 2018, Chen et al. 2023), not to mention that we consider a two-stage model.

3. Basic reduced information model

Apart from being a large-scale linear optimization problem, we are unable to extend Problem (4) to a data-driven robust optimization model due to the difficulties of characterizing the statistics associated with the random bidding sets, $(\tilde{\mathcal{B}}_1, \ldots, \tilde{\mathcal{B}}_T)$. As such, we propose a reduced information model that evaluates an upper bound of the crowdsourcing cost function. Specifically, After the bids have been gathered at the tth period, we solve the following assignment problem,

$$\begin{aligned} \min \ & \sum_{(k,j,n)\in\mathcal{B}_t} p_n s_{kj} \\ \text{s.t. } & \sum_{k:(k,j)\in\bar{\mathcal{B}}_t} s_{kj} = 1 \ \forall j \in [J_t], \\ & \sum_{j:(k,j)\in\bar{\mathcal{B}}_t} s_{kj} \le 1 \ \ \forall k \in [K_t], \\ & s_{kj} \ge 0 \qquad\qquad \forall (k,j) \in \bar{\mathcal{B}}_t, \end{aligned} \tag{5}$$

and obtain its optimum binary solution, $s^*_{kj} \in \{0,1\}$, $(k,j) \in \bar{\mathcal{B}}_t$. Observe that under Assumption 1, Problem (5) is a feasible assignment problem. Subsequently, we determine the *basic reduced information vector*, $\boldsymbol{z}_t \in \mathbb{R}^N$ where

$$z_{tn} = \sum_{(k,j):(k,j,n)\in\mathcal{B}_t} s^*_{kj} \qquad \forall n \in [N]. \tag{6}$$

Speaking intuitively, z_{tn} is the maximum number of ad-hoc couriers, based on the optimal assignment solution of Problem (5), who could be assigned for the jobs for payment p_n. Namely, for payment p_n, we only care about the total number of ad-hoc couriers that can be dispatched, and the *reduced information* is the job preference a courier has specified for this payment.

THEOREM 1. *The crowdsourcing cost function with basic reduced information,*

$$\begin{aligned} g_t(\boldsymbol{x}, \boldsymbol{z}_t) = \min \ & \boldsymbol{p}^\top \boldsymbol{y} \\ \text{s.t. } & \boldsymbol{1}^\top \boldsymbol{y} \ge \boldsymbol{1}^\top \boldsymbol{z}_t - \boldsymbol{a}_t^\top \boldsymbol{x} \\ & \boldsymbol{0} \le \boldsymbol{y} \le \boldsymbol{z}_t, \end{aligned} \tag{7}$$

is an upper bound of the crowdsourcing cost function, i.e.,

$$f_t(\boldsymbol{x}, \mathcal{B}_t) \le g_t(\boldsymbol{x}, \boldsymbol{z}_t).$$

Moreover, the bound is tight if there exists an optimal binary solution of Problem (1) *such that* $s_{kj} \leq s^*_{kj}$ *for all* $(k,j) \in \bar{\mathcal{B}}_t$.

The proof is straightforward. We can see that $g_t(\boldsymbol{x}, \boldsymbol{z}_t)$ corresponds to an assignment, where the solution $\boldsymbol{s}^*$ is used to find the cheapest $\mathbf{1}^\top \boldsymbol{z}_t - \boldsymbol{a}_t^\top \boldsymbol{x}$ bidders. Such an assignment is a feasible solution to Problem (1); thus, $g_t(\boldsymbol{x}, \boldsymbol{z}_t)$ is an upper bound of $f_t(\boldsymbol{x}, \mathcal{B}_t)$.

The bidding scheme mandated by the platform influences the accuracy of the reduced information model in evaluating the crowdsourcing cost function. In a single payment value bidding, each ad-hoc courier, $k \in [K_t]$ can bid for any number of jobs but for one payment value $r_k \in \{p_1, \ldots, p_N\}$ for any job assigned by the platform.

Theorem 2 **(Single payment value bidding)**. *Under the single payment value bidding scheme, the basic reduced information model evaluates the crowdsourcing cost exactly, i.e.,* $g_t(\boldsymbol{x}, \boldsymbol{z}_t) = f_t(\boldsymbol{x}, \mathcal{B}_t)$ *for any number of assignments* $J_t - \boldsymbol{a}_t^\top \boldsymbol{x} \in \{0, 1, \ldots, J_t\}$.

In the proof of Theorem 2, we first show that $g_t(\boldsymbol{x}, \boldsymbol{z}_t)$ is connected to a minimum-cost network flow problem. From discrete convex analysis, the minimum-cost network flow problem is an M-convex problem (Murota 1998, Chen and Li 2021), which possesses the property that its local minimum equals the global minimum. We then prove $g_t(\boldsymbol{x}, \boldsymbol{z}_t) = f_t(\boldsymbol{x}, \mathcal{B}_t)$ by mathematical induction from $\boldsymbol{a}_t^\top \boldsymbol{x} = 0$ to $\boldsymbol{a}_t^\top \boldsymbol{x} = J_t$, and each step of induction would utilize the aforementioned property of the M-convex problem.

Given the historical information $\mathcal{B}_t^\omega$, for $\omega \in [\Omega]$, $t \in [T]$, by solving Problem (5), we can obtain the corresponding basic reduced information vector, $\boldsymbol{z}_t^\omega$, which is used to evaluate the upper bound of the crowdsourcing cost function. The basic reduced information allows us to characterize the underlying random variables associated with the delivery workforce management problem. For each period, $t \in [T]$, the random variable $\tilde{\boldsymbol{z}}_t$ represents the random basic reduced information vector associated with the bidding sets. For convenience, we define $\tilde{\boldsymbol{z}} := (\tilde{\boldsymbol{z}}_t)_{t \in [T]}$ and its support set is

$$\mathcal{Z} := \{(\boldsymbol{z}_t)_{t \in [T]} | \boldsymbol{z}_t \in \mathcal{Z}_t \ \forall t \in [T]\},$$

$$\mathcal{Z}_t = \{\boldsymbol{z} \in \mathbb{R}^N_+ \mid \mathbf{1}^\top \boldsymbol{z} \leq \bar{z}_t\},$$

where $\bar{z}_t$ is the maximum number of jobs that would ever arrive at the time period t. We denote the Ω historical realizations of the random variables by $\boldsymbol{z}_t^\omega \in \mathcal{Z}_t$, $\omega \in [\Omega]$, $t \in [T]$. Correspondingly, we define the empirical distribution $\hat{\mathbb{P}} \in \mathcal{P}_0(\mathcal{Z})$, $\tilde{\boldsymbol{z}} \sim \hat{\mathbb{P}}$ such that for all $\omega \in [\Omega]$, $\hat{\mathbb{P}}[\tilde{\boldsymbol{z}}_t = \boldsymbol{z}_t^\omega \quad \forall t \in [T]] = \frac{1}{\Omega}$. Specifically, with the basic reduced information, we solve the following empirical optimization problem,

$$\begin{aligned} \bar{Z}_0 = \min \ & \boldsymbol{w}^\top \boldsymbol{x} + \mathbb{E}_{\hat{\mathbb{P}}}\left[\sum_{t \in [T]} g_t(\boldsymbol{x}, \tilde{\boldsymbol{z}}_t)\right] \\ \text{s.t.} \ & \boldsymbol{x} \in \mathcal{X}. \end{aligned} \tag{8}$$

which is an upper bound to the empirical optimization problem (3), and it is equivalent to the following linear optimization problem,

$$
\begin{aligned}
\bar{Z}_0 = \min\ & \boldsymbol{w}^\top \boldsymbol{x} + \frac{1}{\Omega} \sum_{\omega\in[\Omega]} \sum_{t\in[T]} \boldsymbol{p}^\top \boldsymbol{y}_t^\omega \\
\text{s.t.}\ & \mathbf{1}^\top \boldsymbol{y}_t^\omega \geq \mathbf{1}^\top \boldsymbol{z}_t^\omega - \boldsymbol{a}_t^\top \boldsymbol{x} && \forall t\in[T], \omega\in[\Omega], \\
& \mathbf{0} \leq \boldsymbol{y}_t^\omega \leq \boldsymbol{z}_t^\omega && \forall t\in[T], \omega\in[\Omega], \\
& \boldsymbol{x}\in\mathcal{X}.
\end{aligned}
$$

For a bidding platform where multiple payment values are allowed, the basic reduced information model may not be exact. Unfortunately, the relative performance gap can be unbounded. Consider an instance with $N = 3$, $J_t = K_t = 2$, $\boldsymbol{p} = (\epsilon, 1, 3)$, for some small $\epsilon > 0$ and $\mathcal{B}_t = \{(1,1,1),(1,2,2),(2,1,2),(2,2,3)\}$. Hence, $\boldsymbol{z}_t = (0,2,0)$. For $\boldsymbol{a}_t^\top \boldsymbol{x} = 1$, observe that $f_t(\boldsymbol{x},\mathcal{B}_t) = \epsilon$, while $g_t(\boldsymbol{x},\boldsymbol{z}_t) = 1$, implying that the relative performance gap, $g_t(\boldsymbol{x},\boldsymbol{z}_t)/f_t(\boldsymbol{x},\mathcal{B}_t) = 1/\epsilon$ can be arbitrarily large. Nevertheless, we can narrow the gap through a more general reduced information model, which will be discussed in the next section.

4. Generalized reduced information model

We can further improve the basic reduced information model to obtain a tighter bound on the crowdsourcing cost function. The introduction of the generalized reduced information model is inspired by the infimal convolution (Chen and Sim 2009), a technique in convex analysis. The key idea is to split the number of contracted couriers into multiple integer numbers, and each number corresponds to a breakpoint and a crowdsourcing cost function. Then, we can show that the convex combination of all the crowdsourcing cost functions would be a tighter bound of the function $f_t(\boldsymbol{x},\mathcal{B}_t)$.

To do so, for each period $t\in[T]$, we consider L_t breakpoints $u_{t\ell}\in[0,\bar{z}_t]\cap\mathbb{Z}_+$, $\ell\in[L_t]$, with $u_{t1}=0$. Subsequently, we derive an approximation of the crowdsourcing cost function that would be tight if $\boldsymbol{a}_t^\top\boldsymbol{x} = u_{t\ell}$ for some $\ell\in[L_t]$. In particular, given the bidding information $\mathcal{B}_t$, $t\in[T]$, we determine the set of generalized reduced information vectors, $\boldsymbol{z}_t^\ell\in\mathbb{R}^N$, $\ell\in[L_t]$ where

$$
z_{tn}^\ell = \sum_{(k,j):(k,j,n)\in\mathcal{B}_t} s_{kj}^{t\ell} \qquad \forall n\in[N], \ell\in[L_t] \tag{9}
$$

and

$$
\begin{aligned}
\boldsymbol{s}^{t\ell} \in \arg\min \quad & \sum_{(k,j,n)\in\mathcal{B}_t} p_n s_{kj} \\
\text{s.t.} \quad & \sum_{k:(k,j)\in\bar{\mathcal{B}}_t} s_{kj} \le 1 && \forall j \in [J_t], \\
& \sum_{j:(k,j)\in\bar{\mathcal{B}}_t} s_{kj} \le 1 && \forall k \in [K_t], \\
& \sum_{(k,j)\in\bar{\mathcal{B}}_t} s_{kj} \ge J_t - u_{t\ell} \\
& s_{kj} \ge 0 && \forall (k,j) \in \bar{\mathcal{B}}_t.
\end{aligned} \tag{10}
$$

Observe that since $u_{t1} = 0$, $\boldsymbol{z}_t^1$ corresponds to the basic reduced information vector. We also note that whenever $u_{t\ell} \ge J_t$, we have $\boldsymbol{z}_t^\ell = \boldsymbol{0}$.

We now extend to a generalized reduced information model by first characterizing the underlying random variable. As a generalization, we define $\tilde{\boldsymbol{z}} := (\tilde{\boldsymbol{z}}_t^\ell)_{t\in[T],\ell\in[L_t]}$, where $\tilde{\boldsymbol{z}}_t^\ell$ represents the random reduced information vector associated with the breakpoint $\ell \in [L_t]$ at the tth period. We consider breakpoints separable support sets

$$
\mathcal{Z} := \left\{ (\boldsymbol{z}_t^\ell)_{t\in[T],\ell\in[L_t]} \middle| \boldsymbol{z}_t^\ell \in \mathcal{Z}_t^\ell \ \forall t \in [T], \ell \in [L_t] \right\},
$$

where

$$
\mathcal{Z}_t^\ell = \{ \boldsymbol{z} \in \mathbb{R}_+^N \mid \boldsymbol{1}^\top \boldsymbol{z} \le \bar{z}_{t\ell} \},
$$

and $\bar{z}_{t\ell} = \bar{z}_t - u_{t\ell} \ge 1$. Hence, noting that $u_1 = 0$, with $L = 1$, the random variable $\tilde{\boldsymbol{z}}$ is a generalization over the random basic reduced information vector. Correspondingly, the empirical distribution is $\hat{\mathbb{P}} \in \mathcal{P}_0(\mathcal{Z})$, $\tilde{\boldsymbol{z}} \sim \hat{\mathbb{P}}$ such that for all $\omega \in [\Omega]$,

$$
\hat{\mathbb{P}} \left[\tilde{\boldsymbol{z}}_t^\ell = \boldsymbol{z}_t^{\ell\omega} \ \ \forall t \in [T], \ell \in [L_t] \right] = \frac{1}{\Omega},
$$

where $\boldsymbol{z}_t^{\ell\omega} \in \mathcal{Z}_t^\ell$, $\ell \in [L_t]$ is the historical realization of the generalized reduced information associated with the bidding set $\mathcal{B}_t^\omega$ at $t \in [T]$.

THEOREM 3. *For any $\boldsymbol{x} \in \mathcal{X}$, $\boldsymbol{\eta}, \boldsymbol{\gamma} \in \mathbb{R}_+^{L_t}$ such that $\boldsymbol{1}^\top \boldsymbol{\gamma} = 1$ and $\boldsymbol{1}^\top \boldsymbol{\eta} = \boldsymbol{a}_t^\top \boldsymbol{x}$, we have*

$$
f_t(\boldsymbol{x}, \mathcal{B}_t) \le \sum_{\ell\in[L_t]} h_t(\gamma_\ell, \eta_\ell - u_{t\ell}\gamma_\ell, \boldsymbol{z}_t^\ell), \tag{11}
$$

where

$$
\begin{aligned}
h_t(\gamma, \eta, \boldsymbol{z}) = \min \ & \boldsymbol{p}^\top \boldsymbol{y} \\
\text{s.t.} \ & \boldsymbol{1}^\top \boldsymbol{y} \ge \boldsymbol{1}^\top \boldsymbol{z}\gamma - \eta \\
& \boldsymbol{0} \le \boldsymbol{y} \le \boldsymbol{z}\gamma.
\end{aligned} \tag{12}
$$

Moreover,

$$\begin{aligned}\mathbb{E}_{\hat{\mathbb{P}}}\left[g_t(\boldsymbol{x}, \tilde{\boldsymbol{z}}_t^1)\right] \geq \min\ & \mathbb{E}_{\hat{\mathbb{P}}}\left[\sum_{\ell \in [L_t]} h_t(\gamma_\ell, \eta_\ell - u_{t\ell}\gamma_\ell, \tilde{\boldsymbol{z}}_t^\ell)\right] \\ \text{s.t.}\ & \mathbf{1}^\top \boldsymbol{\gamma} = 1 \\ & \mathbf{1}^\top \boldsymbol{\eta} = \boldsymbol{a}_t^\top \boldsymbol{x} \\ & \boldsymbol{\eta}, \boldsymbol{\gamma} \in \mathbb{R}_+^{L_t}.\end{aligned} \tag{13}$$

In the proof of Theorem 3, we first construct function $h_t(\gamma_\ell, \eta_\ell - u_{t\ell}\gamma_\ell, \boldsymbol{z}_t^\ell)$ and then rewrite it in the form of $\gamma_\ell h_t(1, \eta_\ell/\gamma_\ell - u_{t\ell}, \boldsymbol{z}_t^\ell)$. Subsequently, we prove that $h_t(1, \eta - u_{t\ell}, \boldsymbol{z}_t^\ell) \geq \bar{f}_t(\eta, \mathcal{B}_t)$, where $\bar{f}_t(\eta, \mathcal{B}_t)$ corresponds to the exact crowdsourcing cost when η contracted couriers are available. We next prove that $f_t(\boldsymbol{x}, \mathcal{B}_t)$ is upper bounded by the convex combination of functions $\bar{f}_t(\eta_\ell/\gamma_\ell, \mathcal{B}_t)$, where the coefficient of each function is γ_ℓ. Then we conclude that $f_t(\boldsymbol{x}, \mathcal{B}_t)$ is also upper bounded by the convex combination of functions $h_t(1, \eta_\ell/\gamma_\ell - u_{t\ell}, \boldsymbol{z}_t^\ell)$, each with a coefficient of γ_ℓ. Up to now, the relationship in (11) is proved. Since the basic reduced information model is a special case of the generalized information model, the bound (13) naturally holds.

PROPOSITION 1. *Let $\mathcal{L} \subseteq [L_t]$ be a subset of breakpoints and*

$$\begin{aligned}\bar{h}_t(\boldsymbol{x}, \mathcal{L}) = \min\ & \sum_{\ell \in \mathcal{L}} h_t(\gamma_\ell, \eta_\ell - u_{t\ell}\gamma_\ell, \boldsymbol{z}_t^\ell) \\ \text{s.t.}\ & \mathbf{1}^\top \boldsymbol{\gamma} = 1 \\ & \mathbf{1}^\top \boldsymbol{\eta} = \boldsymbol{a}_t^\top \boldsymbol{x} \\ & \boldsymbol{\eta}, \boldsymbol{\gamma} \in \mathbb{R}_+^{|\mathcal{L}|},\end{aligned}$$

then we have the following results:

1. *Monotonicity: for any $\mathcal{L} \subseteq \mathcal{L}' \subseteq [L_t]$,*

$$\bar{h}_t(\boldsymbol{x}, \mathcal{L}') \leq \bar{h}_t(\boldsymbol{x}, \mathcal{L}).$$

2. *Tightness: for any $\boldsymbol{x} \in \mathcal{X}$, if $u_{t\ell^*} = \boldsymbol{a}_t^\top \boldsymbol{x}$ for some $\ell^* \in \mathcal{L}$, then the approximation of the crowdsourcing cost function would be tight, i.e.,*

$$f_t(\boldsymbol{x}, \mathcal{B}_t) = \bar{h}(\boldsymbol{x}, \mathcal{L}).$$

The monotonicity property holds naturally because the optimal solution to the optimization problem $\bar{h}(\boldsymbol{x}, \mathcal{L})$ is a feasible solution to problem $\bar{h}(\boldsymbol{x}, \mathcal{L}')$ if $\mathcal{L} \subseteq \mathcal{L}'$. The function $\bar{h}_t(\gamma_\ell, \eta_\ell - u_{t\ell}\gamma_\ell, \boldsymbol{z}_t^\ell)$ is exactly the same as the function $f_t(\boldsymbol{x}, \mathcal{B}_t)$ by setting $\gamma_{\ell^*} = 1$, $\eta_{\ell^*} = u_{t\ell^*} = \boldsymbol{a}_t^\top \boldsymbol{x}$. Thus, Proposition 1 holds. Although this proposition is straightforward, it has important implications. Specifically, our approximation with the generalized reduced information model would be tight if the set of breakpoints successfully includes the value of $\boldsymbol{a}_t^\top \boldsymbol{x}$. Since the possible values of L_t are finite, belonging to the set $\{0, 1, \cdots, \bar{z}_t\}$, the approximation would be exact if we let $L_t = \bar{z}_t$. This is because $\boldsymbol{a}_t^\top \boldsymbol{x} < \bar{z}_t$ and the value of $\boldsymbol{a}_t^\top \boldsymbol{x}$ can be included in the set of breakpoints in this case.

REMARK 1. Notice that Proposition 1 holds for any bidding set $\mathcal{B}_t$ and reduced information vectors $\boldsymbol{z}_t^\ell, \ell \in [L_t]$; thus, taking expectation with respect to the empirical distribution $\mathbb{P}$ on both sides would yield the same results.

In the same vein, we can also extend the generalized reduced information model, which solves the following empirical optimization problem,

$$\begin{aligned} Z_0 = \min\ & \boldsymbol{w}^\top \boldsymbol{x} + \mathbb{E}_{\hat{\mathbb{P}}}\left[\sum_{t\in[T]}\sum_{\ell\in[L_t]} h_t(\gamma_{t\ell}, \eta_{t\ell} - u_{t\ell}\gamma_{t\ell}, \tilde{\boldsymbol{z}}_t^\ell)\right] \\ \text{s.t.}\ & \boldsymbol{1}^\top \boldsymbol{\eta}_t = \boldsymbol{a}_t^\top \boldsymbol{x} && \forall t \in [T], \\ & \boldsymbol{1}^\top \boldsymbol{\gamma}_t = 1 && \forall t \in [T], \\ & \boldsymbol{\eta}_t, \boldsymbol{\gamma}_t \in \mathbb{R}_+^{L_t} && \forall t \in [T], \\ & \boldsymbol{x} \in \mathcal{X}, \end{aligned} \tag{14}$$

or equivalently

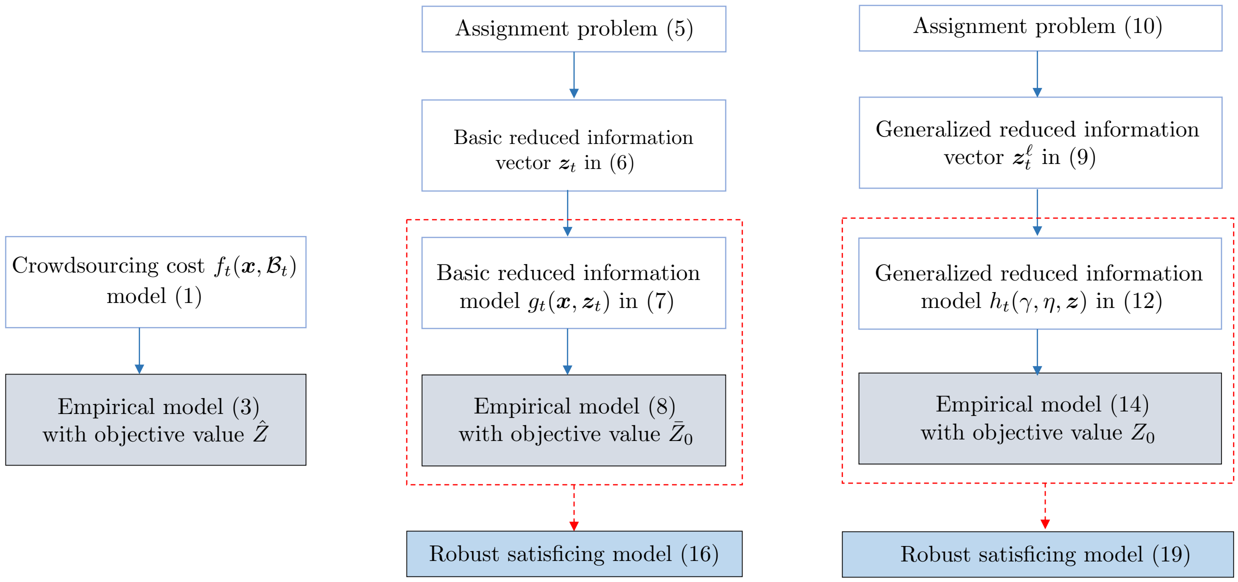
$$\begin{aligned} Z_0 = \min\ & \boldsymbol{w}^\top \boldsymbol{x} + \frac{1}{\Omega}\sum_{\omega\in[\Omega]}\sum_{t\in[T]}\sum_{\ell\in[L_t]} \boldsymbol{p}^\top \boldsymbol{y}_t^{\ell\omega} \\ \text{s.t.}\ & \boldsymbol{1}^\top \boldsymbol{y}_t^{\ell\omega} \geq \boldsymbol{1}^\top \boldsymbol{z}_t^{\ell\omega}\gamma_{t\ell} - \eta_{t\ell} + u_{t\ell}\gamma_{t\ell} && \forall t \in [T], \omega \in [\Omega], \ell \in [L_t], \\ & \boldsymbol{0} \leq \boldsymbol{y}_t^{\ell\omega} \leq \boldsymbol{z}_t^{\ell\omega}\gamma_{t\ell} && \forall t \in [T], \omega \in [\Omega], \ell \in [L_t], \\ & \boldsymbol{1}^\top \boldsymbol{\eta}_t = \boldsymbol{a}_t^\top \boldsymbol{x} && \forall t \in [T], \\ & \boldsymbol{1}^\top \boldsymbol{\gamma}_t = 1 && \forall t \in [T], \\ & \boldsymbol{\eta}_t, \boldsymbol{\gamma}_t \in \mathbb{R}_+^{L_t} && \forall t \in [T], \\ & \boldsymbol{x} \in \mathcal{X}. \end{aligned} \tag{15}$$

Moreover, as the result of Theorem 3, it provides an improvement over the basic information model of Problem (8), *i.e.*, $\hat{Z} \leq Z_0 \leq \bar{Z}_0$. The approximation of the empirical average crowdsourcing cost is exact if there exists an optimal solution $\boldsymbol{x}$ of Problem (4) such that

$$\boldsymbol{a}_t^\top \boldsymbol{x} \in \{u_{t\ell} | \ell \in [L_t]\} \qquad \forall t \in [T].$$

Figure 1 summarizes the relationships among models and shows how we construct the empirical and robust models progressively. Specifically, the exact crowdsourcing cost $f_t(\boldsymbol{x}, \mathcal{B}_t)$ is evaluated in model (1), and its corresponding empirical model for making the employment decisions of contracted couriers, $\boldsymbol{x}$, is presented in (3). To extend Problem (3) to a data-driven robust model, we propose two reduced information models. In particular, after solving the assignment problem (5) and obtaining its optimal solution, we determine the basic reduced information vector $\boldsymbol{z}_t$ according to Equation (6). Subsequently, we construct the basic reduced information model (7) and its corresponding empirical model (8). Based on model (7) and the empirical model's objective value $\bar{Z}_0$, a robust satisficing model is built, which will be detailed in the next section. Similarly, we can obtain the generalized reduced information model and its corresponding empirical and robust models.

Figure 1 The relationships among models



5. Data-driven robust models

As we do not know how future uncertainty would evolve, the solution to the empirical optimization problem may not necessarily perform as well when the actual uncertainty is realized in the future. This overfitting phenomenon is known as the *optimizer's curse* (Smith and Winkler 2006). The reduced information models allow us to characterize the distribution of the underlying random variable and formulate robust models to mitigate the uncertainty associated with the data-driven empirical optimization problem. This section will show how to develop data-driven robust optimization models using the reduced information models. Our numerical tests based on Solomon's data sets for VRPs in Section 6 show that the basic reduced information model performs quite well. Hence, because of its simplicity, we will first focus on the basic reduced information approach for solving the robust delivery workforce management problem under uncertainty. We will also discuss extending the result to the generalized reduced information model.

Robust model with basic reduced information

To build a data-driven robust model, we first focus on the basic reduced information model where the true distribution of the random basic reduced information vector, $\tilde{z} \sim \mathbb{P}^\star$, $\mathbb{P}^\star \in \mathcal{P}_0(\mathcal{Z})$ is unobservable to the decision-maker. Because the empirical distribution, $\hat{\mathbb{P}}$, is not the true distribution, $\mathbb{P}^\star$, the empirical optimization model would often yield inferior solutions in out-of-sample performance evaluations. To overcome this issue, Mohajerin Esfahani and Kuhn (2018) and Gao and Kleywegt (2023) have proposed a data-driven robust optimization model with an ambiguity set that is characterized by a type-1 Wasserstein metric $\Delta(\mathbb{P}, \hat{\mathbb{P}})$, which evaluates the statistical distance between a candidate distribution $\mathbb{P}$ from the empirical distribution, $\hat{\mathbb{P}}$. In particular, suppose

$\mathbb{P}^{\Omega}$ denotes the distribution that governs the distribution of the independent samples $\tilde{\boldsymbol{z}}^1,\ldots,\tilde{\boldsymbol{z}}^{\Omega}$ drawn from $\mathbb{P}^\star$ for which the empirical distribution $\hat{\mathbb{P}}$ is constructed. Then under some light-tall distribution assumption, $\mathbb{P}^{\Omega}\left[\Delta(\mathbb{P}^\star,\hat{\mathbb{P}})>r\right]$ would diminish rapidly to zero as the statistical distance r increases (Fournier and Guillin 2015). The robust optimization problem for the basic reduced information model is as follows,

$$\begin{aligned}\bar{Z}_r = \min\ & \boldsymbol{w}^\top \boldsymbol{x} + \sup_{\substack{\mathbb{P}\in\mathcal{P}_0(\mathcal{Z}):\\ \Delta(\mathbb{P},\hat{\mathbb{P}})\le r}} \mathbb{E}_{\mathbb{P}}\left[\sum_{t\in[T]} g_t(\boldsymbol{x},\tilde{\boldsymbol{z}}_t)\right]\\ \text{s.t. }\ & \boldsymbol{x}\in\mathcal{X},\end{aligned}$$

where in practice, the size parameter r is determined via cross-validation to achieve better out-of-sample performance than the non-robust optimization model.

Instead of restricting the distribution to within the vicinity of the empirical distribution, in addressing the issues of robustness in data-driven optimization problems, Long et al. (2022) propose a robust satisficing model specified by a target $\tau \ge \bar{Z}_0$. In particular, the robust satisficing model for the problem is as follows

$$\begin{aligned}\kappa_\tau = \min\ & k\\ \text{s.t. }\ & \boldsymbol{w}^\top \boldsymbol{x} + \mathbb{E}_{\mathbb{P}}\left[\sum_{t\in[T]} g_t(\boldsymbol{x},\tilde{\boldsymbol{z}}_t)\right] \le \tau + k\Delta(\mathbb{P},\hat{\mathbb{P}})\ \forall \mathbb{P}\in\mathcal{P}_0(\mathcal{Z}) \\ & \boldsymbol{x}\in\mathcal{X}, k\ge 0.\end{aligned} \tag{16}$$

To obtain a tractable model for our robust satisficing problem, we propose the following *time additive Wasserstein metric*,

$$\Delta(\mathbb{P},\hat{\mathbb{P}}) := \inf_{\mathbb{Q}\in\mathcal{P}_0(\mathcal{Z}^2)} \left\{ \mathbb{E}_{\mathbb{Q}}\left[\sum_{t\in[T]} \frac{1}{\bar{z}_t}||\tilde{\boldsymbol{z}}_t - \tilde{\boldsymbol{\xi}}_t||_\infty\right] \;\middle|\; (\tilde{\boldsymbol{z}},\tilde{\boldsymbol{\xi}})\sim\mathbb{Q}, \tilde{\boldsymbol{z}}\sim\mathbb{P}, \tilde{\boldsymbol{\xi}}\sim\hat{\mathbb{P}} \right\},$$

where $\|\cdot\|_\infty$ is the L_∞-norm.

Note that the robust optimization model specifies the size parameter r of the ambiguity set, whereas the robust satisficing model specifies the target τ. To enhance out-of-sample performance, these parameters are typically determined via cross-validation, especially in problems with limited data, such as our crowdsourced delivery problem. We prefer the robust satisficing paradigm because the target τ provides a balanced trade-off between optimality and robustness. This allows decision-makers to adjust τ relative to the optimal objective of the empirical model within a defined range based on their preferences. Additionally, the search space for setting τ in the robust satisficing model is more intuitive and interpretable compared to adjusting the size parameter r in the robust optimization model.

The model (16) considers all possible distributions in $\mathbb{P} \in \mathcal{P}_0(\mathcal{Z})$ while simultaneously controlling the level of expected target violation under any distribution relative to the distance of this distribution from the empirical distribution $\hat{\mathbb{P}}$. Since the basic reduced information model is a conservative approximation of the actual problem, the objective of the robust satisficing problem κ_τ can be associated with how well the actual expected cost, when evaluated on the true unobservable distribution, would not excessively exceed the threshold τ. As the value of κ_τ decreases, a smaller magnitude of expected target violation could arise under any distribution. More discussions about the robust models, especially understanding them from the perspective of worst-case scenarios, are provided in Appendix B.

Proposition 2. *Under the time additive Wasserstein metric, the robust satisficing problem* (16) *is equivalent to the following robust optimization problem*

$$\begin{aligned} \kappa_\tau = \min\ & k \\ \text{s.t.}\ & \boldsymbol{w}^\top \boldsymbol{x} + \frac{1}{\Omega}\sum_{\omega\in[\Omega]}\sum_{t\in[T]} \sup_{\boldsymbol{z}_t \in \mathcal{Z}_t} \left\{ g_t(\boldsymbol{x},\boldsymbol{z}_t) - \frac{k}{\bar{z}_t}\|\boldsymbol{z}_t - \boldsymbol{z}_t^\omega\|_\infty \right\} \le \tau \\ & \boldsymbol{x} \in \mathcal{X}, k \ge 0. \end{aligned} \tag{17}$$

Note that the basic reduced crowdsourcing cost function, $g_t(\boldsymbol{x},\boldsymbol{z}_t)$ for $t \in [T]$ is determined by solving the linear optimization problem (7) after the realization of $\boldsymbol{z}_t$. Hence, its optimal recourse $\boldsymbol{y}_t$ is a mapping of $\boldsymbol{z}_t$, and we can model Problem (17) as an adaptive robust optimization problem as follows:

$$\begin{aligned} \min\ & k \\ \text{s.t.}\ & \boldsymbol{w}^\top \boldsymbol{x} + \frac{1}{\Omega}\sum_{\omega\in[\Omega]}\sum_{t\in[T]} \sup_{\boldsymbol{z}_t \in \mathcal{Z}_t} \left\{ \boldsymbol{p}^\top \boldsymbol{y}_t(\boldsymbol{z}_t) - \frac{k}{\bar{z}_t}\|\boldsymbol{z}_t - \boldsymbol{z}_t^\omega\|_\infty \right\} \le \tau \\ & \mathbf{1}^\top \boldsymbol{y}_t(\boldsymbol{z}_t) \ge \mathbf{1}^\top \boldsymbol{z}_t - \boldsymbol{a}_t^\top \boldsymbol{x} && \forall \boldsymbol{z}_t \in \mathcal{Z}_t, t \in [T], \\ & \mathbf{0} \le \boldsymbol{y}_t(\boldsymbol{z}_t) \le \boldsymbol{z}_t && \forall \boldsymbol{z}_t \in \mathcal{Z}_t, t \in [T], \\ & \boldsymbol{y}_t : \mathbb{R}^N \to \mathbb{R}^N && \forall t \in [T], \\ & \boldsymbol{x} \in \mathcal{X}, k \ge 0. \end{aligned}$$

Adaptive robust optimization problems are generally computationally challenging problems. Hence, these problems are often solved approximately by replacing the recourse decision $\boldsymbol{y}_t$ with linear decision rules or affine recourse adaptations (Ben-Tal et al. 2004, Chen et al. 2020). However, because the second stage optimization problem (7) does not have complete recourse, such an approximation may not obtain a feasible solution of the robust satisficing problem (Long et al. 2022) for any reasonable chosen target, $\tau > \bar{Z}_0$. Hence, it is surprising that we can model the robust satisficing problem (17) exactly as the following deterministic optimization problem.

THEOREM 4. *The robust satisficing problem* (17) *is equivalent to the following deterministic optimization problem*

$$
\begin{aligned}
\kappa_\tau = \min\ & k \\
\text{s.t.}\ & \boldsymbol{w}^\top \boldsymbol{x} + \frac{1}{\Omega} \sum_{\omega \in [\Omega]} \sum_{t \in [T]} r_t^\omega \le \tau, \\
& r_t^\omega \ge \bar{z}_t \phi_{tn}^\omega + (\boldsymbol{\theta}_{tn}^\omega)^\top \boldsymbol{z}_t^\omega - \boldsymbol{a}_t^\top \boldsymbol{x} p_n && \forall \omega \in [\Omega], t \in [T], n \in [N], \\
& \bar{z}_t \|\boldsymbol{\theta}_{tn}^\omega\|_1 \le k && \forall \omega \in [\Omega], t \in [T], n \in [N], \\
& \mathbf{1}\phi_{tn}^\omega + \boldsymbol{\theta}_{tn}^\omega \ge \boldsymbol{q}_n && \forall \omega \in [\Omega], t \in [T], n \in [N], \\
& \phi_{tn}^\omega \ge 0, \boldsymbol{\theta}_{tn}^\omega \in \mathbb{R}^N && \forall \omega \in [\Omega], t \in [T], n \in [N], \\
& r_t^\omega \ge 0 && \forall \omega \in [\Omega], t \in [T], \\
& \boldsymbol{x} \in \mathcal{X}, k \ge 0,
\end{aligned}
\tag{18}
$$

where the vector $\boldsymbol{q}_n \in \mathbb{R}^N$, $n \in [N]$ *has elements*

$$
q_{nm} = \begin{cases} p_m & \text{if } m \in [n-1] \\ p_n & \text{otherwise} \end{cases} \qquad \forall m \in [N].
$$

If the empirical optimization problem (8) *is solvable, then the robust satisficing problem is also feasible for all* $\tau \ge \bar{Z}_0$, *and* $\kappa_\tau \in [0, \bar{\kappa}]$, *where*

$$
\bar{\kappa} = (\mathbf{1}^\top \boldsymbol{p}) \max_{t \in [T]} \{\bar{z}_t\}.
$$

We note that although Problem (7) has *relatively complete recourse*, the lack of *complete recourse* condition still makes it challenging to solve the adaptive robust optimization problem. A prevalent strategy to tackle the data-driven adaptive robust optimization problems with the Wasserstein distance metric is finding approximation solutions using the affine recourse adaptations (Chen et al. 2020). However, to ensure that such an approximation also encapsulates the solution of the empirical optimal model, the robust optimization problem must satisfy the condition of complete recourse. This requirement is more demanding than the relatively complete recourse condition (Long et al. 2022). For example, Bertsimas et al. (2019) show that approximations can still result in infeasibilities, even when the relatively complete recourse condition is met. Fortunately, our reduced information models manage to circumvent the demand for the complete recourse condition by exploiting their inherent structures to derive exact solutions without using approximation methods.

The proof of Theorem 4 mainly utilizes the duality results in robust optimization, coupled with an investigation into the structure of our problem. Specifically, the feasible region of $g_t(\boldsymbol{x}, \boldsymbol{z}_t)$ is the intersection of a box region and a hyperplane; thus, the number of extreme points is finite. Whereas it is computationally prohibited to traverse all the extreme points to find the optimal solution because the time complexity of the traversal algorithm is $O(2^N)$. However, by delving into the duality of $g_t(\boldsymbol{x}, \boldsymbol{z}_t)$, we will only need to evaluate a few affine functions with a time complexity of $O(N)$. This pivotal observation facilitates the derivation of the exact reformulation as presented in (18).

Although the basic robust satisficing model is feasible for any $\tau \geq \bar{Z}_0$, since $\bar{Z}_0 \geq \hat{Z}$, it would not be feasible for $\tau \in [\hat{Z}, \bar{Z}_0)$, which can be an issue if $\bar{Z}_0$ is significantly larger than $\hat{Z}$. To reduce the conservativeness, we have to consider solving the generalized reduced information model that would achieve $Z_0 = \hat{Z}$. A computationally viable approach is to use the optimal solution of Problem (4), $\boldsymbol{x}$ and consider $L_t = 2$ breakpoints with $u_{t2} = \boldsymbol{a}_t^\top \boldsymbol{x}$ $\forall t \in [T]$. Based on Theorem 3, we have $Z_0 = \hat{Z}$. Next, we show how to extend the robust satisficing model to incorporate the generalized reduced information.

Robust model with generalized reduced information

We now extend to the generalized reduced information model with $\tilde{\boldsymbol{z}} = (\tilde{\boldsymbol{z}}_t^\ell)_{t\in[T],\ell\in[L_t]}$. To obtain a computationally tractable model, we consider the following $\boldsymbol{\gamma}$*-weighted time-additive* Wasserstein metric,

$$\Delta_{\boldsymbol{\gamma}}(\mathbb{P}, \hat{\mathbb{P}}) := \inf_{\mathbb{Q}\in\mathcal{P}_0(\mathcal{Z}^2)} \left\{ \mathbb{E}_{\mathbb{Q}} \left[\sum_{t\in[T]} \sum_{\ell\in[L_t]} \frac{1}{\bar{z}_{t\ell}} \gamma_{t\ell} || \tilde{\boldsymbol{z}}_t^\ell - \tilde{\boldsymbol{\xi}}_t^\ell ||_\infty \right] \;\middle|\; (\tilde{\boldsymbol{z}}, \tilde{\boldsymbol{\xi}}) \sim \mathbb{Q}, \tilde{\boldsymbol{z}} \sim \mathbb{P}, \tilde{\boldsymbol{\xi}} \sim \hat{\mathbb{P}} \right\}.$$

Accordingly, we propose the following robust satisficing model associated with the generalized reduced information model,

$$\begin{aligned}
\kappa_\tau = \min\ & k \\
\text{s.t.}\ & \boldsymbol{w}^\top \boldsymbol{x} + \mathbb{E}_{\mathbb{P}} \left[\sum_{t\in[T]} \sum_{\ell\in[L_t]} h_t(\gamma_{t\ell}, \eta_{t\ell} - u_{t\ell}\gamma_{t\ell}, \tilde{\boldsymbol{z}}_t^\ell) \right] \leq \tau + k\Delta_{\boldsymbol{\gamma}}(\mathbb{P}, \hat{\mathbb{P}}) \ \forall \mathbb{P} \in \mathcal{P}_0(\mathcal{Z}), \\
& \mathbf{1}^\top \boldsymbol{\eta}_t = \boldsymbol{a}_t^\top \boldsymbol{x} && \forall t \in [T], \\
& \mathbf{1}^\top \boldsymbol{\gamma}_t = 1 && \forall t \in [T], \\
& \boldsymbol{\eta}_t, \boldsymbol{\gamma}_t \in \mathbb{R}_+^{L_t} && \forall t \in [T], \\
& \boldsymbol{x} \in \mathcal{X}.
\end{aligned} \tag{19}$$

Observe that when $\eta_{t1} = \boldsymbol{a}_t^\top \boldsymbol{x}$, and $\gamma_{t1} = 1$, Problem (19) will recover the solution of the basic reduced information robust satisficing problem (16). Hence, Problem (19) has a lower objective value, leading to lower probabilities of violating the target at various levels.

Unlike the basic reduced information model, we cannot solve the generalized robust satisficing model as a single convex optimization problem even if $\mathcal{X}$ is a convex set. Nevertheless, the following result shows we can solve the general robust model via a binary search on a bounded interval of k.

THEOREM 5. *The robust satisficing problem* (19) *is equivalent to the following deterministic optimization problem*

$$\begin{aligned} \kappa_\tau = \min\ & k \\ \text{s.t.}\ & \rho(k) \le \tau \\ & k \ge 0, \end{aligned} \tag{20}$$

where

$$\begin{aligned} \rho(k) = \min\ & \boldsymbol{w}^\top \boldsymbol{x} + \frac{1}{\Omega} \sum_{\omega \in [\Omega]} \sum_{t \in [T]} \sum_{\ell \in [L_t]} r_t^{\ell\omega} \\ \text{s.t.}\ & r_t^{\ell\omega} \ge \alpha_{tn}^{\ell\omega}(k)\gamma_{t\ell} - (\eta_{t\ell} - u_{t\ell}\gamma_{t\ell})p_n \quad \forall \omega \in [\Omega], t \in [T], n \in [N], \ell \in [L_t], \\ & \mathbf{1}^\top \boldsymbol{\eta}_t = \boldsymbol{a}_t^\top \boldsymbol{x} \quad \forall t \in [T], \\ & \mathbf{1}^\top \boldsymbol{\gamma}_t = 1 \quad \forall t \in [T], \\ & \boldsymbol{\eta}_t \in \mathbb{R}^{L_t}, \boldsymbol{\gamma}_t \in \mathbb{R}_+^{L_t} \quad \forall t \in [T], \\ & r_t^{\ell\omega} \ge 0 \quad \forall \omega \in [\Omega], t \in [T], \ell \in [L_t], \\ & \boldsymbol{x} \in \mathcal{X}, \end{aligned} \tag{21}$$

and

$$\begin{aligned} \alpha_{tn}^{\ell\omega}(k) = \max\ & \boldsymbol{q}_n^\top \boldsymbol{z} - \tfrac{k}{\bar{z}_{t\ell}} \|\boldsymbol{z} - \boldsymbol{z}_t^{\ell\omega}\|_\infty \\ \text{s.t}\ & \mathbf{1}^\top \boldsymbol{z} \le \bar{z}_{t\ell} \\ & \boldsymbol{z} \ge \mathbf{0}. \end{aligned}$$

Moreover, if the empirical optimization problem (15) *is solvable, then for all* $\tau \ge Z_0$*, the robust satisficing problem is also feasible, and* $\kappa_\tau \in [0, \bar{\kappa}]$ *for the same* $\bar{\kappa}$ *defined in Theorem 4.*

Observe that if $\mathcal{X}$ is a polyhedron, then Problem (21) would be a linear optimization problem. Consequently, using a binary search, we can solve the generalized robust satisficing model by solving a modest number of convex optimization problems.

Finally, we conclude this subsection by discussing why we use a Robust Satisficing (RS) approach rather than other robust optimization methods. It is important to note that we are addressing a problem where data is collected over time, resulting in a limited number of samples. For instance, a year's worth of data provides only 52 data points for weekly analysis. Therefore, we are dealing with a situation characterized by a limited sample size, compounded by the challenges of generating future samples and the potential for a non-stationary environment. Consequently, solving the empirical optimization problem alone may not lead to improved out-of-sample performance. Our goal is to achieve solution robustness to enhance out-of-sample performance beyond what empirical optimization models can offer.

Data-driven Robust Optimization (RO) and RS are two approaches that can consistently improve empirical optimization models because they can replicate the solutions of empirical optimization models for some choice of their hyperparameters, such as the size of the uncertainty set for RO being set at zero and the target for RS being set at the empirical optimal value. These hyperparameters can be determined by cross-validation to ensure consistent performance of the empirical optimization models. In contrast, moment-based Distributionally Robust Optimization (DRO) models do not replicate the solutions of the empirical optimization models, and their performance relative to empirical optimization models depends on the underlying problem. In our numerical study, the DRO approach fails to improve over the data-driven approach. To summarize, while the family of solutions generated by RO and RS can be the same, we focus on the data-driven RS model because it provides consistent performance and clear interpretability via its target-based hyperparameter setting.

Statistical justification

When utilizing the empirical model, the platform derives an objective value, yet its attainability in future applications cannot be guaranteed. The *Optimizer's curse*, as described by Smith and Winkler (2006), suggests that such an objective is often unachievable in out-of-sample tests. To mitigate this, we propose two robust satisficing models in this paper. When platform planners adopt these models with a cost target τ for determining couriers' employment, they may want to know the probability of τ being exceeded in out-of-samples. Thus, we provide upper bounds on this probability in this section. While these bounds may not directly inform the target setting or the model's performance, they offer theoretical assurance about the reliability of achieving the target. These bounds, in turn, boost the planners' confidence in adopting robust satisficing models to secure a certain level of expected out-of-sample performance.

THEOREM 6. *Consider the random bidding sets* $(\tilde{\mathcal{B}}_1, \dots, \tilde{\mathcal{B}}_T) \sim \mathbb{Q}^\star$ *which generate the random variable associated with the generalized reduced information* $\tilde{\boldsymbol{z}} \sim \mathbb{P}^\star$. *Let* $\mathbb{P}^\Omega$ *be the distribution that governs the distribution of independent samples* $\tilde{\boldsymbol{z}}^\omega$, $\omega \in \Omega$ *drawn from* $\mathbb{P}^\star$. *The following holds for the optimal solution to Problem* (19)*:*

1. **Confidence guarantee of Fournier and Guillin (2015).** *For any* $\tau \geq Z_0$,
$$\mathbb{P}^\Omega \left[\boldsymbol{w}^\top \boldsymbol{x} + \mathbb{E}_{\mathbb{Q}^\star} \left[\sum_{t \in [T]} f_t(\boldsymbol{x}, \tilde{\mathcal{B}}_t) \right] > \tau + T r \kappa_\tau \right] \leq \begin{cases} c_1 \exp(-c_2 \Omega r^{NL}) & \forall r \in [0,1] \\ 0 & \forall r > 1, \end{cases}$$
for some positive c_1 *and* c_2 *that depend on* $\mathbb{E}_{\mathbb{P}^\star}[\exp(||\tilde{\boldsymbol{z}}||_\infty)]$ *and* NL, $L = \sum_{t \in [T]} L_t$.
2. **Confidence guarantee for significant target shortfalls.** *For given* $\tau \geq Z_0$,
$$\mathbb{P}^\Omega \left[\boldsymbol{w}^\top \boldsymbol{x} + \mathbb{E}_{\mathbb{Q}^\star} \left[\sum_{t \in [T]} f_t(\boldsymbol{x}, \tilde{\mathcal{B}}_t) \right] > \tau + T r \kappa_\tau \right] \leq \begin{cases} \exp\left(-2\Omega \left(r - \mu\right)^2\right) & \forall r \in [\mu, 1] \\ 0 & \forall r > 1, \end{cases}$$
where
$$\mu := \mathbb{E}_{(\tilde{\boldsymbol{z}}, \tilde{\boldsymbol{\xi}}) \sim \mathbb{P}^\star \times \mathbb{P}^\star} \left[\frac{1}{T} \sum_{t \in [T]} \max_{\ell \in [L_t]} \left\{ \frac{1}{\bar{z}_{t\ell}} \| \tilde{\boldsymbol{z}}_t^\ell - \tilde{\boldsymbol{\xi}}_t^\ell \|_\infty \right\} \right],$$
noting that $\mu \leq 1$.

We present two probability bounds in this theorem. For the confidence bound of Fournier and Guillin (2015) to dominate the second bound for $r \in [0, \mu]$, we would require enough samples such that $c_1 \exp(-c_2 \Omega \mu^{NL}) < 1$. However, since data is collected over time in the workforce management problem, we expect the number of data samples available to be far fewer than necessary for the confidence guarantee to be practically useful; if the planning horizon is seven days, the weekly demand data may only provide 52 samples per year. Hence, in practical situations, we expect the deviation between in-sample and out-of-sample performance to be significant.

Nevertheless, for justifying our robust satisficing model, the second probability bound provides the assurance that, for a fixed number of samples Ω, the probability of target shortfalls exceeding $Tr\kappa_\tau$ decreases exponentially in $(r-\mu)^2$ and diminishes to 0 when $r > 1$. Although this bound is not useful when $r < \mu$, it is indeed a better bound than the first one, especially when μ is a small number. This also means that greater shortfalls may occur but with an exponentially decreasing probability. Minimizing the violation probability is consistent with the objective of the robust satisficing model, which aims to achieve the lowest possible value of κ_τ. Notably, we provide a simple expression specific to the robust satisficing model rather than using the results of Fournier and Guillin (2015). More importantly, unlike the confidence guarantee of Fournier and Guillin (2015), the second bound is independent of the number of breakpoints in the generalized reduced information model. Additionally, introducing more breakpoints can result in a lower value of κ_τ and a reduced violation probability bound, further motivating the use of the robust satisficing model constructed from the $\boldsymbol{\gamma}$-weighted time-additive Wasserstein metric.

Theorem 6 provides a useful theoretical framework for the robust satisficing model. However, to set the target, a practical implementation may require additional techniques, such as cross-validation on the target parameter, to optimize out-of-sample performance.

6. Numerical studies

In this section, we first evaluate the performance of the basic and generalized reduced information models across multiple payment values. We subsequently compare the robust satisficing model and the empirical model using simulated data. The data and source code for reproducing the results can be found in the electronic companion.

Evaluation of the reduced information models using Solomon's data sets

In Section 3, we demonstrate through a pathological example that the basic reduced information model could have an arbitrarily large relative performance gap with respect to the true model when multiple payment values are allowed. However, in practical delivery problems, the performance gap is usually acceptable. To further investigate the impact of multiple payment values, we conduct experiments using Solomon's data sets (Solomon 1987) for the VRP with time windows, where hierarchical objectives are considered, with the primary aim being to minimize the number of vehicles and then the travel distance. The solutions are available at `https://sun.aei.polsl.pl/ /~zjc/best-solutions-solomon.html`. We only consider the C1, R1, and RC1 data sets, as these contain instances where more than ten vehicles are used. We do not consider the R2, C2, and RC2 data sets because they only involve a small number of vehicles (2, 3, or 4). Thus, there are 29 instances in total.

For each instance, we consider a list of $N = 5$ possible payments, with $p_n = \lfloor 0.8d_{min} + (n - 1)\frac{(1.2d_{max} - 0.8d_{min})}{(N-1)} \rfloor$, where d_{min} and d_{max} represent the minimal and maximal route lengths, respectively. Note that the route length does not include the travel distance from the last customer to the depot. We set $K_t = 2J_t$ and consider four cases for bidding jobs. In each case, every ad-hoc courier could provide $N' \in \{2, \ldots, 5\}$ payment values. For each courier $j \in [J_t]$, we randomly generate their coordinates within the minimal and maximal coordinates of all nodes, then calculate the travel distances required to complete each job, which is the sum of the distance between a courier's current location and the depot, and the route length associated with a job. Subsequently, we sort the travel distances of each courier in non-decreasing order, denoting the resulting list as D_j. For the first $\lfloor \frac{|J_t|}{N'} \rfloor$ jobs in D_j, we calculate the average travel distance $\bar{d}$ and set the courier's payment for each of these jobs as the one that has the minimal absolute deviation from $\bar{d}$. We then follow a similar procedure to set the payment for the second $\lfloor \frac{|J_t|}{N'} \rfloor$ jobs until N' payments are specified. We acknowledge that this bidding scheme is simplistic and intended for illustrative purposes only. More complex bidding schemes can be developed by incorporating other influential factors, such as the courier's destination after finishing a job and the properties of the jobs (*e.g.*, the number of customers included in a job and package weights).

Subsequently, we solve Problems (1) and (7) for $\boldsymbol{a}_t^\top \boldsymbol{x} \in \{0, \ldots, J_t - 1\}$. For a combination of $(N', \boldsymbol{a}_t^\top \boldsymbol{x})$, we randomly generate 20 instances, where each instance differentiates from the other in the payment values provided by a courier and the sets of bidding jobs under each payment. The average percentage gaps are reported in Table 1, where gaps are calculated by $\frac{g_t(\boldsymbol{x}, \boldsymbol{z}_t) - f_t(\boldsymbol{x}, \mathcal{B}_t)}{f_t(\boldsymbol{x}, \mathcal{B}_t)}$. Moreover, we also present the results under $N' = 1$ to give some intuition of Theorem 2. To provide insights into the differences among payment values, we give the price ratio between the maximal value p_5 and the minimal p_1 in the table.

Table 1 indicates that the price ratio p_5/p_1 ranges from 2.02 to 9.71, relatively high values compared to the average earnings of Amazon Flex drivers, which is stated to be between 18 to 25 dollars per hour, resulting in a ratio of around 1.39. Despite this, our approximation model performs well, with average performance gaps of less than 5% observed in 101 out of 116 cases where multiple payment values are allowed. In 35 cases, the average gaps are less than 1%. When the single payment value bidding scheme is employed, the reduced information model generates

Table 1 Average gaps (in %) between $g_t(\boldsymbol{x}, \boldsymbol{z}_t)$ and $f_t(\boldsymbol{x}, \mathcal{B}_t)$ under multiple payment values

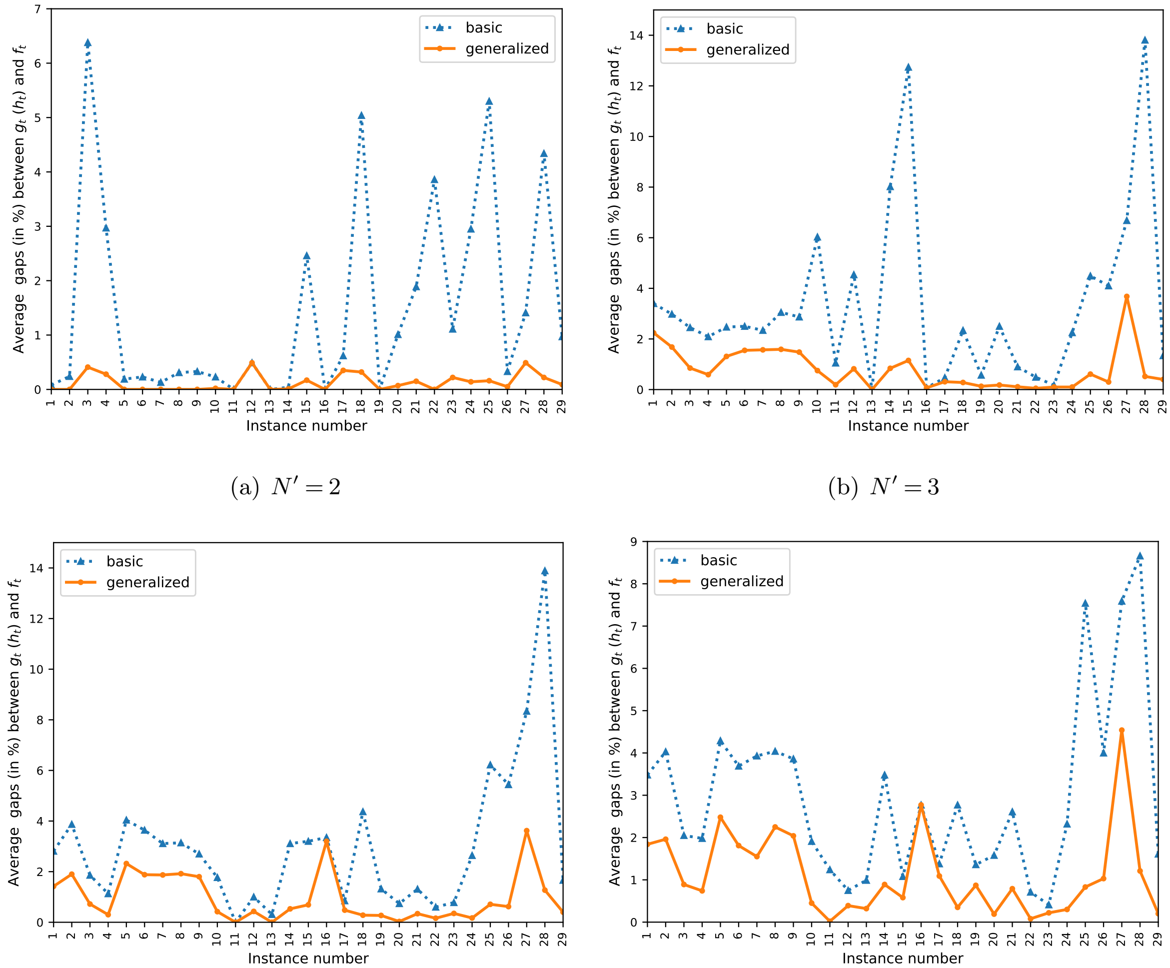
Number	Instance	p_5/p_1	$N'=1$	$N'=2$	$N'=3$	$N'=4$	$N'=5$	Number	Instance	p_5/p_1	$N'=1$	$N'=2$	$N'=3$	$N'=4$	$N'=5$
1	C101	2.81	0.00	0.08	3.40	2.81	3.48	16	R107	3.00	0.00	0.00	0.07	3.35	2.77
2	C102	2.81	0.00	0.24	2.98	3.87	4.03	17	R108	2.02	0.00	0.62	0.47	0.85	1.38
3	C103	4.25	0.00	6.38	2.45	1.86	2.05	18	R109	2.42	0.00	5.04	2.34	4.37	2.77
4	C104	4.25	0.00	2.97	2.09	1.13	1.98	19	R110	2.05	0.00	0.00	0.57	1.32	1.36
5	C105	2.81	0.00	0.19	2.47	4.04	4.29	20	R111	2.67	0.00	1.01	2.50	0.74	1.58
6	C106	2.81	0.00	0.23	2.50	3.64	3.69	21	R112	2.02	0.00	1.89	0.90	1.31	2.61
7	C107	2.81	0.00	0.13	2.34	3.11	3.93	22	RC101	4.07	0.00	3.86	0.49	0.60	0.71
8	C108	2.81	0.00	0.31	3.05	3.14	4.04	23	RC102	2.62	0.00	1.11	0.17	0.78	0.41
9	C109	2.81	0.00	0.33	2.87	2.70	3.86	24	RC103	3.08	0.00	2.95	2.25	2.64	2.32
10	R101	6.42	0.00	0.23	6.03	1.77	1.91	25	RC104	2.53	0.00	5.30	4.49	6.22	7.54
11	R102	9.71	0.00	0.00	1.05	0.00	1.24	26	RC105	3.67	0.00	0.33	4.10	5.44	4.00
12	R103	5.70	0.00	0.49	4.54	1.00	0.75	27	RC106	2.87	0.00	1.41	6.68	8.34	7.59
13	R104	2.09	0.00	0.00	0.00	0.31	0.99	28	RC107	3.46	0.00	4.34	13.81	13.89	8.66
14	R105	4.23	0.00	0.04	8.02	3.11	3.48	29	RC108	2.69	0.00	0.97	1.34	1.66	1.61
15	R106	3.48	0.00	2.46	12.74	3.20	1.08								

exact solutions, as illustrated in columns $N' = 1$. Based on these results, we conclude that our basic reduced information model can provide good approximations for Problem (1) when applied to practical delivery problems.

To provide a comparative analysis of the performance of the generalized information model, we conduct further experiments using the same instances using $L = 2$ breakpoints, where $u_0 = 0$ and $u_2 = \lfloor J_t/2 \rfloor$. Figure 2 plots the average percentage gaps between the basic/generalized reduced information model and the crowdsourcing cost function. The results reveal that the generalized model provides tighter bounds than the basic model and sometimes significantly improves the bounds. For example, when $N' = 3$, the average gap between the basic model and the crowdsourcing cost function is approximately 14% in instance RC107 (*i.e.*, instance number 28); however, the generalized model reduces the gap to 0.5%. Moreover, we observe that the generalized model produces solutions with performance gaps of less than 5% for all instances. When $N' = 2$, the average gap of the generalized model is less than 0.5% for all the instances. Therefore, we can conclude that the generalized reduced information model can provide better approximations for Problem (1) and be useful in tackling practical delivery problems. Finally, we note that similar results are obtained under an alternative bidding scheme. We provide the detailed experimental settings and results in Appendix C.

Evaluation of robust satisficing models via a simulated crowdsourced delivery system This section uses a simulated crowdsourced delivery system to evaluate the robust satisficing models. Suppose we have $K = 60$ ad-hoc couriers in total performing delivery tasks within an area $\mathcal{A} = [0, 40] \times [0, 40] \subseteq \mathbb{R}^2$. For each courier $k \in [K]$ at the beginning of the time period $t \in [T]$, her/his current location is denoted as $l_{kt} \in \mathcal{A}$. The planning horizon consists of $T = 8$ time periods, representing a typical workday of 8 hours. There are a total of 26 work shifts, each lasting between 1 to 4 periods. The compensation for each contracted courier in the ith work shift is given by

Figure 2 **Average gaps (in %) between the reduced information models and the crowdsourcing cost function under a distance-based bidding scheme**



(a) $N' = 2$

(b) $N' = 3$

(c) $N' = 4$

(d) $N' = 5$

$w_i = 20\sigma_i \times 0.9^{\sigma_i - 1}$, where the basic compensation for each period is 20 and σ_i is the number of periods that shift i covers. A discount factor of 0.9 is used to distinguish shifts; otherwise, hiring a person for a shift of a certain number of periods is equivalent to hiring that person for multiple continuous shifts, each lasting for one period. It is also reasonable in real-world applications that shorter shift has higher compensation per hour since it is more flexible.

Simulation process. At the beginning of the time period t, we randomly select K_t couriers out of $[K]$, where K_t is sampled from a normal distribution with the mean and the standard deviation being $K/2$. The value of K_t is then rounded to the nearest integer and truncated to the interval $[0, K]$. We assume the other couriers in $[K]$ are occupied by their own affairs and will appear in a uniformly random location in $\mathcal{A}$ at the beginning of the time period $t+1$. There are J_t jobs to be performed within the time period t, which are independent of the couriers and the previous decisions made by the platform. Since our problem does not explicitly solve the VRP, for simplicity, we assume each job includes exactly one delivery. The lower and upper bounds of J_t are set to $J_{min} = 10$ and $J_{max} = 50$, respectively. We first randomly generate the value of J_t from a normal distribution with the mean and the standard deviation being $(J_{min} + J_{max})/2$. We then round the value to the nearest integer and truncate it to the interval $[J_{min}, J_{max}]$. Each delivery job is associated with an origin l_j^O and a destination l_j^D, which are uniformly randomly distributed in $\mathcal{A}$.

The platform provides couriers with $N = 5$ possible bidding prices, *i.e.*, $\{p_1, p_2, \ldots, p_5\} = \{20, 30, \cdots, 60\}$. The bidding price of each courier $k \in [K_t]$ for job $j \in [J_t]$ is set as the smallest value in $\{p_1, \ldots, p_N\}$ that is higher than or equal to the courier's expected payment of taking that job, which is calculated by

$$U_{kjt} = dist(l_{kt}, l_j^O) + dist(l_j^O, l_j^D) + \epsilon_{kt}, \tag{22}$$

where $dist$ is the Euclidean distance and ϵ_{kt} represents the courier's idiosyncratic noise at time period t such that $(\epsilon_{k1}, \ldots, \epsilon_{kT}) \sim \mathcal{N}(\mathbf{0}, \Sigma_k)$, where $\Sigma_k = 5\boldsymbol{I}$. Whenever $K_t < J_t$, *i.e.*, the available couriers cannot take all the jobs, we assume a third-party delivery company could provide $J_t - K_t$ couriers, each of them would take the job with the highest price p_N. To reflect the fluctuation of price in different time intervals, we let the couriers' expected payments increase and decrease by 20% in periods $t \in \{1, 8\}$ and $t \in \{4, 5\}$ to simulate the peak and off-peak periods, respectively.

At the beginning of the time period t, each hired ad-hoc courier k would travel through the path $l_{kt} \to l_j^O \to l_j^D$ to finish the assigned job. Consequently, the courier's location becomes $l_{kt+1} = l_j^D$ at the beginning of the time period $t+1$. If a courier provides her/his biddings but is not hired by the platform (due to $K_t > J_t$), we assume that the courier's location stays unchanged until the start of the next period. Note that we can also randomly generate locations in $\mathcal{A}$ for these couriers, which will not affect the solutions' performance. To evaluate the performance of the proposed models, we generate 7 training samples and 200 testing samples for each instance.

Choice of breakpoints. To compare the performance of the RS model and the empirical model (EMP) associated with the generalized reduced information model, we begin by solving the assignment problem to obtain the generalized reduced information vectors $\boldsymbol{z}_t^{\ell\omega}$, for all $t \in [T]$, $\ell \in [L_t]$, and $\omega \in [\Omega]$. We then solve the empirical model (15) and obtain its objective value Z_0. Next, we consider a target ratio r and set $\tau = rZ_0$. Using a binary search algorithm, we solve problem (20) to produce the solution of the robust satisficing model (19). When solving the generalized reduced information model, we consider three cases for setting the breakpoints in each period:

- $L_t = 1$ with $u_{t1} = 0$. In this case, the generalized reduced information model coincides with the basic reduced information model (7).

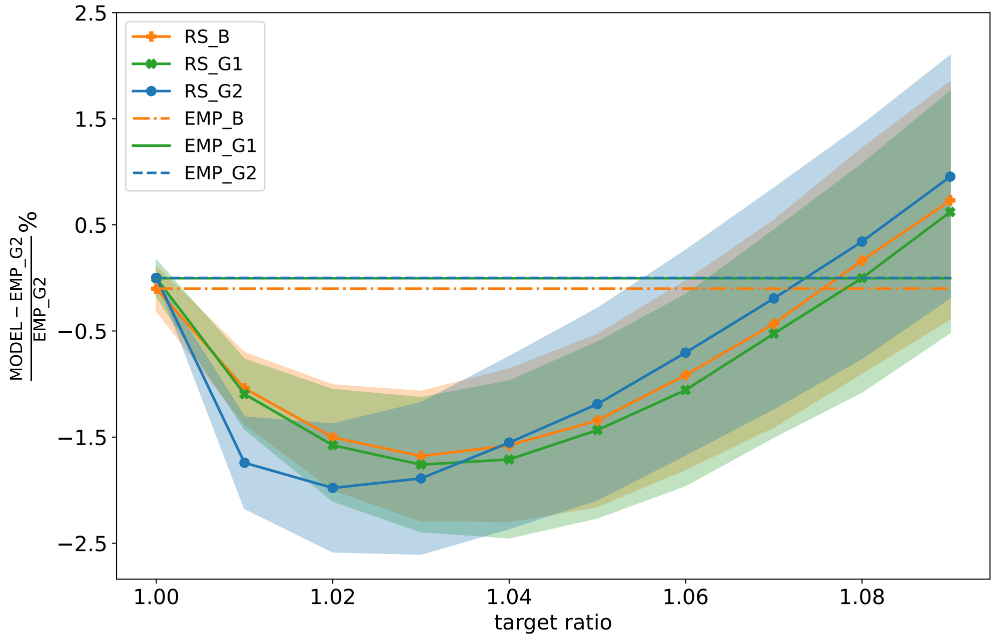
- $L_t = 2$ with $u_{t1} = 0$ and $u_{t2} = \lfloor 0.5 \sum_{\omega \in [\Omega]} J_t^{\omega}/\Omega \rfloor$, where J_t^{ω} is the number of jobs in period $t \in [T]$ under training sample $\omega \in [\Omega]$.
- $L_t = 2$ with $u_{t1} = 0$ and $u_{t2} = \boldsymbol{a}_t^{\top}\hat{\boldsymbol{x}}$, where $\hat{\boldsymbol{x}}$ is the solution to the empirical optimization problem (3) with reformulation in (4). Note that if we choose this breakpoint, the generalized reduced information model (15) solves exactly the empirical optimization problem (3).

For ease of notation, we use symbols B, G1, and G2 to denote the above three settings of breakpoints and append them to EMP and RS to represent the empirical and robust satisficing models associated with the breakpoints under these settings. Finally, we evaluate the performance of both RS and EMP models in testing samples by solving problem (1) and obtain the second-stage crowdsourcing costs. To accelerate the resolution of models, we relax the feasible set $\mathcal{X}$ of the decision variable $\boldsymbol{x}$ from $\mathbb{Z}_+^I$ to $\mathbb{R}_+^I$. Our preliminary results show that the differences in the out-of-sample performance of integer solutions (round real-number solutions to the nearest integers) and real-number solutions are negligible. Thus, we conduct the following experiments by setting $\boldsymbol{x} \in \mathbb{R}_+^I$.

Comparison of out-of-sample performance. To assess the out-of-sample performance of the RS and empirical models, we generate 60 instances. Figure 3 presents the average performance along with the 95% confidence intervals (CIs), with the CIs depicted as shaded areas. The vertical axis of the figure represents the value (in %) of the difference in test objective between MODEL and EMP_G2, divided by the test objective of EMP_G2, denoted by $\frac{\text{MODEL}-\text{EMP\_G2}}{\text{EMP\_G2}}$ for short, where MODEL $\in$ {RS_B, RS_G1, RS_G2, EMP_B, EMP_G1, EMP_G2}. We observe that the average performance of EMP_B, EMP_G1, and EMP_G2 are comparable in the out-of-sample test, and RS models produce better average out-of-sample performance than the corresponding EMP models when the target is set slightly higher than the objective of EMP models. Moreover, RS_G2 exhibits superior performance to RS_B and RS_G1. Furthermore, when the target ratio varies between 1.01 and 1.05, the upper bounds of the CIs for the RS models' performance also outperform the corresponding EMP models. In addition, we conduct experiments to explore the out-of-sample performance of the moment-based DRO problem for the basic reduced information model. The detailed analysis is provided in Appendix C. The comparison results show that the DRO model performs worse than its corresponding empirical model, further verifying the superiority of the RS framework for our problem.

We note that the random bidding sets could also be correlated in the sense that each courier's remaining capacity or location follows a complex Markov decision process. Preliminary experiments were performed, where the covariance matrix, Σ_k, was assumed to deviate from the identity matrix, denoting the case where noises across different time periods were correlated. Nonetheless, the results did not show any distinct variations and patterns compared to the case with independent noises.

Figure 3 **Comparison of out-of-sample performance (mean values and 95% CIs in shaded areas) between the robust satisficing and the empirical models**

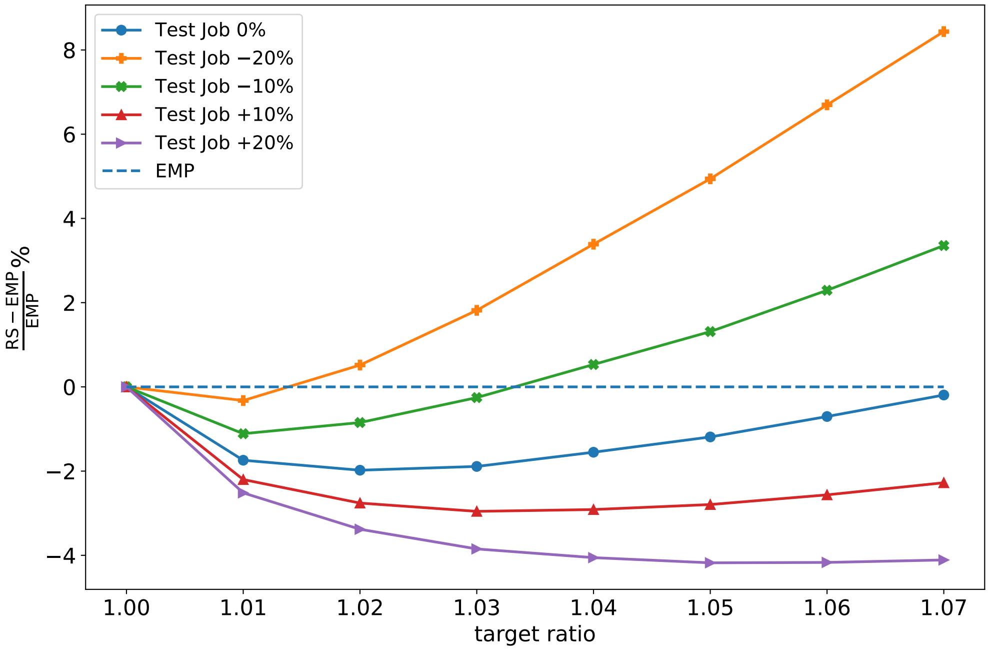


We can expect these results because the simulation process is sufficiently complex and has some inherent randomness. To conclude, while we can introduce correlations into the bidding set, neither theoretical nor experimental results suggest that they would significantly impact the outcomes.

To further evaluate the performance of the RS model, we introduce more uncertainty in the number of jobs in the testing samples. Specifically, we set the low and upper bounds to $J_{min} = \lfloor 10(1-\zeta) \rfloor$ and $J_{max} = \lfloor 50(1+\zeta) \rfloor$, where $\zeta \in \{-20\%, -10\%, 0\%, +10\%, +20\%\}$ represents the percentage variations of job numbers, and generate J_t following the previous approach. We evaluate the out-of-sample performance of the RS_G2 and the EMP_G2 models and report the average comparison over 50 random instances in Figure 4. When the testing samples have a more significant variation in job numbers, the benefit of using the RS_G2 model becomes more apparent. When the variation is minor, the RS_G2 model with a small target performs better than the EMP_G2 model. Thus, we advise that decision-makers adopt a modestly conservative target in the RS model, say, 1.02, which would be advantageous across almost all the scenarios, including low and high levels of uncertainty and risk. Moreover, if a greater variation in the job numbers is expected in the future, then a conservative target in the robust satisficing model could lead to a more significant improvement in the actual performance.

Investigation on different schemes of ad-couriers' expected payments. In practice, the ad-couriers' expected payment schemes could be quite complex, influenced by several factors besides the distances. In this section, we investigate four different settings of the expected payments. The

Figure 4 **Comparison of out-of-sample performance between the robust satisficing and the empirical models with different deviations of job numbers in testing samples**

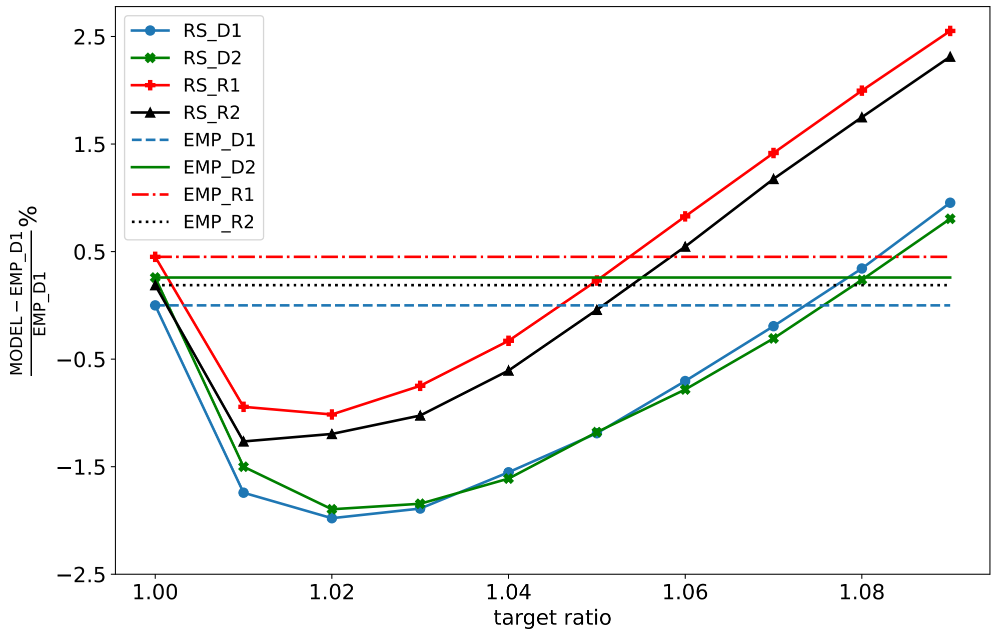


first two are based on distances. The other two are completely random, representing the case of couriers' unknown preferences, as some studies have shown that couriers may have various or even completely different preferences even though the covariates are the same (Rechavi and Toch 2022).

- D1: The first one is aligned with the setting in the previous experiments. Namely, the expected payment for courier k to take job j at the time period t is calculated using Equation (22).
- D2: The second one is more complex, introducing a coefficient to the distance function, *i.e.*, $U_{kjt} = \alpha_k \left(dist(l_{kt}, l_j^O) + dist(l_j^O, l_j^D)\right) + \epsilon_{kt}$. The coefficient α_k is sampled from the uniform distribution $\text{Uni}[0.5, 1.5]$, which represents the courier's expected payment per unit distance.
- R1: The third one is a random scheme, representing that the couriers' bidding behaviors are completely random. The expected payment is calculated by $U_{kjt} = \varrho_{kjt} + \epsilon_{kt}$, where ϱ_{kjt} is drawn from $\text{Uni}[p_1, p_N]$.
- R2: The last one uses the same formula as in the case R1. However, we draw ϱ_{kjt} from a two-point distribution between p_1 and p_N, each with a probability of 0.5. This scheme represents a more extreme case than case R1.

The patterns observed in Figure 5 across various preference settings are consistent with those observed in the earlier section. This consistency suggests that, despite variations and intricacies in the setting of couriers' bidding behaviors, our robust satisficing model can consistently outperform the empirical model. Moreover, a modestly conservative target in the RS model, say, a value of 1.02,

Figure 5 **Comparison of out-of-sample performance among different settings of couriers' preferences**



would be sufficient to guarantee satisfactory out-of-sample performance. Thus, in practical applications, stakeholders operating the crowdsourcing delivery platforms can confidently deploy our model, anticipating good outcomes regardless of the specific nuances of their operational settings and the distinct personal preferences of couriers.

In sum, our experiments using Solomon's data sets suggest that the performance gap between the reduced information models and the true model is usually acceptable for practical delivery problems. More importantly, through simulated data, we show that the robust satisficing model can produce better out-of-sample performance than the empirical model when the target is set slightly higher than the objective of the empirical model.

7. Conclusion

To offer affordable and reliable delivery services, e-commerce platforms and local businesses are increasingly turning to crowdsourced delivery resources. However, the uncertainties surrounding ad-hoc couriers' availability and job bidding behavior have presented a challenge to the management of the workforce. In this study, we propose a robust satisficing framework that accounts for the uncertainty of ad-hoc couriers and the cost associated with hiring them when necessary. Our framework provides a practical tool for decision-making that enables platforms to effectively manage their workforce resources and balance their cost objectives with service quality requirements. Future work in this area could explore the integration of predictive analytics into the robust satisficing framework to provide better insights into future demand and couriers' availability, which could further enhance the platform's decision-making capability.

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Appendix A Proofs

A.1 Proof of Theorem 1

Observe that the value of $g_t(\boldsymbol{x}, \boldsymbol{z}_t)$ corresponds to the allocation of $J_t - \boldsymbol{a}_t^\top \boldsymbol{x}$ ad-hoc couriers using the solution $\boldsymbol{s}^*$, which has J_t assigned couriers, and then removing $\boldsymbol{a}_t^\top \boldsymbol{x}$ of the most expensive bidders. Such an assignment is a feasible solution to Problem (1). Hence,

$$f_t(\boldsymbol{x}, \mathcal{B}_t) \leq g_t(\boldsymbol{x}, \boldsymbol{z}_t).$$

Now suppose there exists an optimal solution of Problem (1) such that $\boldsymbol{s} \leq \boldsymbol{s}^*$. Then we can construct the solution,

$$y_n = \sum_{(k,j):(k,j,n)\in\mathcal{B}_t} s_{kj} \qquad \forall n \in [N],$$

which is feasible in Problem (7) and its objective

$$\boldsymbol{p}^\top \boldsymbol{y} = \sum_{(k,j,n)\in\mathcal{B}_t} p_n s_{kj}$$

coincides with the optimum objective of Problem (1). Hence, in this case, we also have $f_t(\boldsymbol{x}, \mathcal{B}_t) \geq g_t(\boldsymbol{x}, \boldsymbol{z}_t)$. □

A.2 Proof of Theorem 2

We first define the set,

$$\mathcal{F} := \left\{ \mathcal{K} \subseteq [K_t] \;\middle|\; \begin{array}{l} \forall k \in \mathcal{K}, \exists j_k \in [J_t] : \\ (k, j_k) \in \bar{\mathcal{B}}_t, \\ |\mathcal{K}| = |\{j_k | k \in \mathcal{K}\}| \end{array} \right\},$$

so that each element in $\mathcal{F}$ is a set of couriers in which every courier in the set could be assigned to a unique job.

Given a set of selected couriers $\mathcal{K} \in \mathcal{F}$, we consider the following assignment problem that minimizes the total cost,

$$\begin{aligned} G(\mathcal{K}) = \min \quad & \sum_{(k,j,n)\in\mathcal{B}_t, k\in\mathcal{K}} p_n s_{kj} \\ \text{s.t.} \quad & \sum_{k:(k,j)\in\bar{\mathcal{B}}_t, k\in\mathcal{K}} s_{kj} \leq 1 \;\; \forall j \in [J_t], \\ & \sum_{j:(k,j)\in\bar{\mathcal{B}}_t} s_{kj} = 1 \qquad \forall k \in \mathcal{K}, \\ & \sum_{j:(k,j)\in\bar{\mathcal{B}}_t} s_{kj} = 0 \qquad \forall k \in [K_t]\setminus\mathcal{K}, \\ & s_{kj} \geq 0 \qquad\qquad \forall (k,j) \in \bar{\mathcal{B}}_t, \end{aligned}$$

where the second collection of constraints specifies that any courier in $\mathcal{K}$ must be assigned with one job, and the third collection of constraints ensures that any courier outside $\mathcal{K}$ cannot be assigned with any job. The above assignment problem is a minimum-cost network flow optimization problem. From discrete convex analysis, the minimum-cost network flow problem is an M-convex problem, *i.e.*, $G: \mathcal{F} \to \mathbb{R}$ is an M-convex function (see example 2.3 in Murota 1998 or section 4.1 in Chen and Li 2021). A feasible solution $\mathcal{K}$ is a *local minimum* in the sense that we cannot replace any courier $k_1 \in \mathcal{K}$ with a courier $k_2 \in [K_t] \backslash \mathcal{K}$ such that $r_{k_2} < r_{k_1}$ and $(\mathcal{K} \cup \{k_2\}) \backslash \{k_1\} \in \mathcal{F}$. By Theorem 4.6 in Murota (1998), any local minimum of the M-convex function is also a global minimum. Note that whenever referring to the local/global minimum of G, we assume the domain of G is contained in a hyperplane $\{\mathcal{K} \in \mathcal{F} : |\mathcal{K}| = \eta\}$ for some $\eta \in \{0, 1, \ldots, J_t\}$.

Next, we consider a sequence of optimal solutions for the following problems,

$$
\begin{aligned}
f^{\eta} = \min \quad & \sum_{(k,j,n)\in\mathcal{B}_t} p_n s_{kj} \\
\text{s.t.} \quad & \sum_{k:(k,j)\in\bar{\mathcal{B}}_t} s_{kj} \le 1 \ \forall j \in [J_t], \\
& \sum_{j:(k,j)\in\bar{\mathcal{B}}_t} s_{kj} \le 1 \ \forall k \in [K_t], \\
& \sum_{(k,j)\in\bar{\mathcal{B}}_t} s_{kj} \ge \eta \\
& s_{kj} \ge 0 \qquad\qquad \forall (k,j) \in \bar{\mathcal{B}}_t,
\end{aligned}
$$

for $\eta \in \{0, 1, \ldots, J_t\}$, and we use $\boldsymbol{s}^{\eta}$ to denote the corresponding optimal solution. Obviously, we have $\min_{\mathcal{K}\in\mathcal{F},|\mathcal{F}|=\eta} G(\mathcal{K}) \ge f^{\eta}$. For a given $\eta \in [J_t]$, let $\mathcal{K}_{\eta}$ be the corresponding set of selected couriers,

$$
\mathcal{K}_{\eta} = \{k \in [K_t] \ \mid \ \exists j \in [J_t] : s^{\eta}_{kj} = 1\},
$$

and each courier, $k \in \mathcal{K}_{\eta}$ is assigned to the job $j_k \in [J_t]$ so that $s^{\eta}_{kj_k} = 1$. Observe that $\mathcal{K}_{\eta} \in \mathcal{F}$, $|\mathcal{K}_{\eta}| = \eta$, and the total payment, $\sum_{k\in\mathcal{K}_{\eta}} r_k$ does not depend on how the η jobs are being assigned to the couriers in $\mathcal{K}_{\eta}$. Hence,

$$
f^{\eta} = \min_{\mathcal{K}\in\mathcal{F},|\mathcal{K}|=\eta} G(\mathcal{K}) = G(\mathcal{K}_{\eta}).
$$

Now let $\mathcal{K}_{\eta-1} = \mathcal{K}_{\eta} \backslash \{k_0\}$ for some $k_0 \in \arg\max_{k\in\mathcal{K}_{\eta}} \{r_k\}$. We claim the set $\mathcal{K}_{\eta-1}$ is also a local minimum of G (in domain $\{\mathcal{K} \in \mathcal{F} : |\mathcal{K}| = \eta - 1\}$). Suppose this is not the case, we can replace some courier $k_1 \in \mathcal{K}_{\eta-1}$ with a courier $k_2 \in [K_t] \backslash \mathcal{K}_{\eta-1}$ such that $r_{k_2} < r_{k_1}$ and $\bar{\mathcal{K}} \cup \{k_2\} \in \mathcal{F}$, where $\bar{\mathcal{K}} = \mathcal{K}_{\eta} \backslash \{k_0, k_1\}$. Observe that $k_2 \ne k_0$ since $r_{k_0} \ge r_k$ for all $k \in \mathcal{K}_{\eta-1}$. To arrive at the contradiction that $\mathcal{K}_{\eta-1}$ is not a local minimum, we consider an assignment solution, $\bar{\boldsymbol{s}}$ for couriers in $\bar{\mathcal{K}}$ that based on $\boldsymbol{s}^{\eta}$ as follows

$$
\bar{s}_{kj} = \begin{cases} s^{\eta}_{kj} & \text{if } k \notin \{k_0, k_1\} \\ 0 & \text{otherwise} \end{cases} \qquad \forall (k,j) \in \bar{\mathcal{B}}_t.
$$

Since the assignment problem is a network flow problem, we can construct the residual network associated with the solution $\bar{\boldsymbol{s}}$ (see, *e.g.*, Ahuja et al. 1988). Moreover, because $\bar{\mathcal{K}} \cup \{k_2\} \in \mathcal{F}$, we can find a feasible assignment solution using a max-flow algorithm by sending a unit of residual flow from node k_2, along an augmenting path on the residual network associated with $\bar{\boldsymbol{s}}$ (refer to augmenting path algorithm for max-flow in Ahuja et al. 1988), that will terminate at one of the unassigned jobs, $\ell \in [J_t]\backslash\bar{\mathcal{J}}$ where $\bar{\mathcal{J}}$ is the set of jobs that assigned to couriers in $\bar{\mathcal{K}}$ based on $\bar{\boldsymbol{s}}$. If $\ell \neq j_{k_0}$, then we would have $\bar{\mathcal{K}} \cup \{k_0, k_2\} \in \mathcal{F}$. However, it contradicts that $\mathcal{K}_\eta$ is the local minimum since we can replace $k_1 \in \mathcal{K}_\eta$ with k_2 to achieve a lower total payment. On the other hand, if $\ell = j_{k_0} \neq j_{k_1}$, then $\bar{\mathcal{K}} \cup \{k_1, k_2\} \in \mathcal{F}$. However, this will also contradict that $\mathcal{K}_\eta$ is local minimum; since $r_{k_0} \geq r_{k_1} > r_{k_2}$, we can replace $k_0 \in \mathcal{K}_\eta$ with k_2 to achieve a lower total payment. Therefore, by contradiction, we must have $\bar{\mathcal{K}} \cup \{k_2\} \notin \mathcal{F}$, implying that the set $\mathcal{K}_{\eta-1}$ is local minimum. Consequently, it is also global minimum and we have $f^{\eta-1} = \min_{\mathcal{K}\in\mathcal{F},|\mathcal{K}|=\eta-1} G(\mathcal{K}) = G(\mathcal{K}_{\eta-1})$.

Notice that the assignment optimization problem in G only uses couriers in $\mathcal{K}_{\eta-1}$ which rules out the courier with the highest cost from $\mathcal{K}_\eta$. Therefore, if $G(\mathcal{K}_\eta)$ equals the optimal objective value of the basic reduced information model (7) with $\mathbf{1}^\top \boldsymbol{z}_t - \boldsymbol{a}_t^\top \boldsymbol{x} = \eta$ assignments, then $G(\mathcal{K}_{\eta-1})$ must equal the optimal objective value of the basic reduced information model with $\eta - 1$ assignments. Also, from previous analysis, if $\mathcal{K}_\eta \in \arg\min_{\mathcal{K}\in\mathcal{F},|\mathcal{K}|=\eta} G(\mathcal{K})$ and $f^\eta = G(\mathcal{K}_\eta)$, then $\mathcal{K}_{\eta-1} \in \arg\min_{\mathcal{K}\in\mathcal{F},|\mathcal{K}|=\eta-1} G(\mathcal{K})$ and $f^{\eta-1} = G(\mathcal{K}_{\eta-1})$. Obviously, $G(\mathcal{K}_{J_t})$ equals the optimal objective value of the basic reduced information model with J_t assignments, $\mathcal{K}_{J_t} \in \arg\min_{\mathcal{K}\in\mathcal{F},|\mathcal{K}|=J_t} G(\mathcal{K})$ and $f^{J_t} = G(\mathcal{K}_{J_t})$. By mathematical induction, the basic reduced information model evaluates the crowdsourcing cost exactly for any number of assignments $\eta \in \{0, 1, \ldots, J_t\}$. □

A.3 Proof of Theorem 3

Observe that since $\eta_\ell \geq 0$, we have $h_t(0, \eta_\ell, z_t^\ell) = 0$. Moreover, for all $\gamma_\ell > 0$, we have

$$\begin{aligned} h_t(\gamma_\ell, \eta_\ell - u_{t\ell}\gamma_\ell, \boldsymbol{z}_t^\ell) &= \min\left\{ \boldsymbol{p}^\top \boldsymbol{y} \;\middle|\; \begin{array}{l} \mathbf{1}^\top \boldsymbol{y} \geq \mathbf{1}^\top \boldsymbol{z}_t^\ell \gamma_\ell - (\eta_\ell - u_{t\ell}\gamma_\ell) \\ \mathbf{0} \leq \boldsymbol{y} \leq \boldsymbol{z}_t^\ell \gamma_\ell \end{array} \right\} \\ &= \gamma_\ell \min\left\{ \boldsymbol{p}^\top \boldsymbol{y} \;\middle|\; \begin{array}{l} \mathbf{1}^\top \boldsymbol{y} \geq \mathbf{1}^\top \boldsymbol{z}_t^\ell - (\eta_\ell/\gamma_\ell - u_{t\ell}) \\ \mathbf{0} \leq \boldsymbol{y} \leq \boldsymbol{z}_t^\ell \end{array} \right\} \\ &= \gamma_\ell h_t(1, \eta_\ell/\gamma_\ell - u_{t\ell}, \boldsymbol{z}_t^\ell). \end{aligned}$$

Moreover, following the same argument in the proof of Theorem 1, observe that for $\eta \geq u_{t\ell}$ the value of the function $h_t(1, \eta - u_{t\ell}, \boldsymbol{z}_t^\ell)$ corresponds to the allocation of $J_t - \eta$ ad-hoc couriers using the solution $\boldsymbol{s}^{t\ell}$, which has $J_t - u_{t\ell}$ assigned couriers, and removing $\eta - u_{t\ell}$ of the most expensive bidders, with at most one partial removal if η is fractional. However, when $\eta < u_{t\ell}$, we have $h_t(1, \eta - u_{t\ell}, \boldsymbol{z}_t^\ell) = \infty$, since the underlying minimization problem would be infeasible. Hence, we have

$$h_t(1, \eta - u_{t\ell}, \boldsymbol{z}_t^\ell) \geq \bar{f}_t(\eta, \mathcal{B}_t),$$

where

$$\begin{aligned} \bar{f}_t(\eta, \mathcal{B}_t) = \min \ & \sum_{(k,j,n)\in\mathcal{B}_t} p_n s_{kj} \\ \text{s.t.} \ & \sum_{k:(k,j)\in\bar{\mathcal{B}}_t} s_{kj} \leq 1 && \forall j \in [J_t], \\ & \sum_{j:(k,j)\in\bar{\mathcal{B}}_t} s_{kj} \leq 1 && \forall k \in [K_t], \\ & \sum_{(k,j)\in\bar{\mathcal{B}}_t} s_{kj} \geq J_t - \eta \\ & s_{kj} \geq 0 && \forall (k,j) \in \bar{\mathcal{B}}_t. \end{aligned}$$

Therefore, any feasible solution to Problem (11)

$$\begin{aligned} & \sum_{\ell\in[L_t]} h_t(\gamma_\ell, \eta_\ell - u_{t\ell}\gamma_\ell, \boldsymbol{z}_t^\ell) \\ = & \sum_{\ell\in[L_t]:\gamma_\ell>0} \gamma_\ell h_t(1, \eta_\ell/\gamma_\ell - u_{t\ell}, \boldsymbol{z}_t^\ell) \\ \geq & \sum_{\ell\in[L_t]:\gamma_\ell>0} \gamma_\ell \bar{f}_t(\eta_\ell/\gamma_\ell, \mathcal{B}_t) \\ \geq & \ \bar{f}_t(\mathbf{1}^\top \boldsymbol{\eta}, \mathcal{B}_t) \\ = & \ \bar{f}_t(\boldsymbol{a}_t^\top \boldsymbol{x}, \mathcal{B}_t) \\ = & \ f_t(\boldsymbol{x}, \mathcal{B}_t) \end{aligned}$$

where the last inequality is due to the function $\bar{f}_t(\eta, \mathcal{B}_t)$ being convex in η, $\boldsymbol{\gamma} \geq \mathbf{0}$ and $\mathbf{1}^\top \boldsymbol{\gamma} = 1$.

To show the bound (13), it suffices to note that

$$h_t(1, \boldsymbol{a}_t^\top \boldsymbol{x} - u_{t1}, \boldsymbol{z}_t^1) = h_t(1, \boldsymbol{a}_t^\top \boldsymbol{x}, \boldsymbol{z}_t^1) = g_t(\boldsymbol{x}, \boldsymbol{z}_t^1).$$

□

A.4 Proof of Proposition 1

The monotonicity property holds naturally because the optimal solution to the optimization problem $\bar{h}(\boldsymbol{x}, \mathcal{L})$ is a feasible solution to problem $\bar{h}(\boldsymbol{x}, \mathcal{L}')$ if $\mathcal{L} \subseteq \mathcal{L}'$.

Let $\gamma_{\ell^*} = 1$, $\boldsymbol{a}_t^\top \boldsymbol{x} = u_{t\ell^*}$ for some $\ell^* \in \mathcal{L}$, and $\gamma_\ell = \eta_\ell = 0$ for $\ell \in \mathcal{L}, \ell \neq \ell^*$, $\eta_{\ell^*} = \boldsymbol{a}_t^\top \boldsymbol{x}$, which would be an optimal solution to the minimization problem since

$$\begin{aligned} f_t(\boldsymbol{x}, \mathcal{B}_t) & \leq \sum_{\ell\in\mathcal{L}} h_t(\gamma_\ell, \eta_\ell - u_{t\ell}\gamma_\ell, \boldsymbol{z}_t^\ell) \\ & = h_t(\gamma_{\ell^*}, \eta_{\ell^*} - u_{t\ell^*}\gamma_{\ell^*}, \boldsymbol{z}_t^{\ell^*}) \\ & = h_t(1, 0, \boldsymbol{z}_t^{\ell^*}) \\ & = f_t(\boldsymbol{x}, \mathcal{B}_t), \end{aligned}$$

where the first inequality is by Theorem 3 and the final equality follows from the same argument in Theorem 1, since the optimal solution of Problem (1) is the same as $\boldsymbol{s}^{\ell^*}$ in which the reduced information $\boldsymbol{z}_t^{\ell^*}$ is derived. □

A.5 Proof of Proposition 2

Based on the definition of the Wasserstein metric, we can rewrite the first group of constraints in Problem (16) as

$$\boldsymbol{w}^\top \boldsymbol{x} + \mathbb{E}_{\mathbb{Q}}\left[\sum_{t\in[T]}\left(g_t(\boldsymbol{x},\tilde{\boldsymbol{z}}_t) - \frac{k}{\bar{z}_t}||\tilde{\boldsymbol{z}}_t - \tilde{\boldsymbol{\xi}}_t||_\infty\right)\right] \le \tau \qquad \forall \mathbb{Q}\in\mathcal{P}_0(\mathcal{Z}^2) : (\tilde{\boldsymbol{z}},\tilde{\boldsymbol{\xi}})\sim\mathbb{Q}, \tilde{\boldsymbol{\xi}}\sim\hat{\mathbb{P}},$$

or equivalently

$$\boldsymbol{w}^\top \boldsymbol{x} + \frac{1}{\Omega}\sum_{\omega\in[\Omega]}\mathbb{E}_{\mathbb{P}^\omega}\left[\sum_{t\in[T]}\left(g_t(\boldsymbol{x},\tilde{\boldsymbol{z}}_t) - \frac{k}{\bar{z}_t}||\tilde{\boldsymbol{z}}_t - \boldsymbol{z}_t^\omega||_\infty\right)\right] \le \tau \qquad \forall \mathbb{P}^\omega\in\mathcal{P}_0(\mathcal{Z}).$$

Since $\mathcal{P}_0(\mathcal{Z})$ contains all Dirac distributions whose unit mass concentrates on any $\boldsymbol{z}\in\mathcal{Z}$, we can also express this as a robust constraint as follows,

$$\begin{aligned} &\boldsymbol{w}^\top \boldsymbol{x} + \frac{1}{\Omega}\sum_{\omega\in[\Omega]}\sup_{\boldsymbol{z}\in\mathcal{Z}}\left\{\sum_{t\in[T]}\left(g_t(\boldsymbol{x},\boldsymbol{z}_t) - \frac{k}{\bar{z}_t}||\boldsymbol{z}_t - \boldsymbol{z}_t^\omega||_\infty\right)\right\} \le \tau \\ \iff &\boldsymbol{w}^\top \boldsymbol{x} + \frac{1}{\Omega}\sum_{\omega\in[\Omega]}\sum_{t\in[T]}\sup_{\boldsymbol{z}_t\in\mathcal{Z}_t}\left\{g_t(\boldsymbol{x},\boldsymbol{z}_t) - \frac{k}{\bar{z}_t}||\boldsymbol{z}_t - \boldsymbol{z}_t^\omega||_\infty\right\} \le \tau. \end{aligned}$$

□

A.6 Proof of Theorem 4

From Proposition 2, we can express the robust satisficing model as

$$\begin{aligned} \kappa_\tau = \min\, & k \\ \text{s.t. } & \boldsymbol{w}^\top\boldsymbol{x} + \frac{1}{\Omega}\sum_{\omega\in[\Omega]}\sum_{t\in[T]} r_t^\omega \le \tau \\ & r_t^\omega \ge \sup_{\boldsymbol{z}_t\in\mathcal{Z}_t}\left\{g_t(\boldsymbol{x},\boldsymbol{z}_t) - \frac{k}{\bar{z}_t}\|\boldsymbol{z}_t - \boldsymbol{z}_t^\omega\|_\infty\right\} \quad \forall\omega\in[\Omega], t\in[T], \\ & \boldsymbol{x}\in\mathcal{X}, k\ge 0. \end{aligned} \tag{A.1}$$

Next we begin to reformulate $g_t(\boldsymbol{x},\boldsymbol{z}_t)$. The feasible region of $g_t(\boldsymbol{x},\boldsymbol{z}_t)$ is the intersection of a box region and a hyperplane; therefore, the number of extreme points is finite. Even so, it is impractical to traverse all the extreme points to find the optimal solution because the time complexity of the traversal algorithm is $O(2^N)$. By leveraging the special structure of the duality of $g_t(\boldsymbol{x},\boldsymbol{z}_t)$, we will only need to evaluate a few affine functions with a time complexity of $O(N)$.

Recall that p_n is non-decreasing in $n \in [N]$, and let $p_0 = 0$. By strong duality of linear optimization, we have

$$\begin{aligned} g_t(\boldsymbol{x}, \boldsymbol{z}_t) &= \min_{\boldsymbol{y}_t} \left\{ \boldsymbol{p}^\top \boldsymbol{y}_t \mid \mathbf{1}^\top \boldsymbol{y}_t \geq \mathbf{1}^\top \boldsymbol{z}_t - \boldsymbol{a}_t^\top \boldsymbol{x}, \mathbf{0} \leq \boldsymbol{y}_t \leq \boldsymbol{z}_t \right\} \\ &= \max_{\lambda \geq 0} \min_{\boldsymbol{y}_t} \left\{ (\boldsymbol{p} - \lambda \mathbf{1})^\top \boldsymbol{y}_t + \lambda \left(\mathbf{1}^\top \boldsymbol{z}_t - \boldsymbol{a}_t^\top \boldsymbol{x} \right) \mid \mathbf{0} \leq \boldsymbol{y}_t \leq \boldsymbol{z}_t \right\}. \end{aligned}$$

A notable observation is that the optimal value of the inner minimization problem

$$\min_{\mathbf{0} \leq \boldsymbol{y}_t \leq \boldsymbol{z}_t} (\boldsymbol{p} - \lambda \mathbf{1})^\top \boldsymbol{y}_t + \lambda \left(\mathbf{1}^\top \boldsymbol{z}_t - \boldsymbol{a}_t^\top \boldsymbol{x} \right)$$

is indeed a *piecewise linear* function of λ with breaks being $p_1, p_2, \ldots, p_N$. More specifically, when $\lambda \in [p_{n-1}, p_n], n \in [N]$, we have

$$\begin{aligned} &\max_{\lambda \in [p_{n-1}, p_n]} \min_{\boldsymbol{y}_t} \left\{ (\boldsymbol{p} - \lambda \mathbf{1})^\top \boldsymbol{y}_t + \lambda \left(\mathbf{1}^\top \boldsymbol{z}_t - \boldsymbol{a}_t^\top \boldsymbol{x} \right) \mid \mathbf{0} \leq \boldsymbol{y}_t \leq \boldsymbol{z}_t \right\} \\ =& \max_{\lambda \in [p_{n-1}, p_n]} \min_{\boldsymbol{y}_t} \left\{ \sum_{m \in [N]} (p_m - \lambda) y_{tm} + \lambda \left(\mathbf{1}^\top \boldsymbol{z}_t - \boldsymbol{a}_t^\top \boldsymbol{x} \right) \mid \mathbf{0} \leq \boldsymbol{y}_t \leq \boldsymbol{z}_t \right\} \\ =& \max_{\lambda \in [p_{n-1}, p_n]} \sum_{m \in [n-1]} (p_m - \lambda) z_{tm} + \lambda \left(\mathbf{1}^\top \boldsymbol{z}_t - \boldsymbol{a}_t^\top \boldsymbol{x} \right) \\ =& \max_{\lambda \in \{p_{n-1}, p_n\}} \sum_{m \in [n-1]} (p_m - \lambda) z_{tm} + \lambda \left(\mathbf{1}^\top \boldsymbol{z}_t - \boldsymbol{a}_t^\top \boldsymbol{x} \right) \\ =& \max \left\{ \boldsymbol{q}_{n-1}^\top \boldsymbol{z}_t - p_{n-1}(\boldsymbol{a}_t^\top \boldsymbol{x}), \boldsymbol{q}_n^\top \boldsymbol{z}_t - p_n(\boldsymbol{a}_t^\top \boldsymbol{x}) \right\}, \end{aligned}$$

and when $\lambda \in [p_N, +\infty)$, the optimality is obtained at $\boldsymbol{y}_t = \boldsymbol{z}_t$,

$$\begin{aligned} &\max_{\lambda \in [p_N, +\infty)} \min_{\boldsymbol{y}_t} \left\{ (\boldsymbol{p} - \lambda \mathbf{1})^\top \boldsymbol{y}_t + \lambda \left(\mathbf{1}^\top \boldsymbol{z}_t - \boldsymbol{a}_t^\top \boldsymbol{x} \right) \mid \mathbf{0} \leq \boldsymbol{y}_t \leq \boldsymbol{z}_t \right\} \\ =& \max_{\lambda \in [p_N, +\infty)} \boldsymbol{p}^\top \boldsymbol{z}_t - \lambda(\boldsymbol{a}_t^\top \boldsymbol{x}) \\ =& \boldsymbol{p}^\top \boldsymbol{z}_t - p_N(\boldsymbol{a}_t^\top \boldsymbol{x}) \\ =& \boldsymbol{q}_N^\top \boldsymbol{z}_t - p_N(\boldsymbol{a}_t^\top \boldsymbol{x}). \end{aligned}$$

Therefore, when evaluating $g_t(\boldsymbol{x}, \boldsymbol{z}_t)$, we only need to consider the maximum of $N+1$ affine functions $\{\boldsymbol{q}_n^\top \boldsymbol{z}_t - p_n(\boldsymbol{a}_t^\top \boldsymbol{x})\}_{n \in [N] \cup \{0\}}$, *i.e.*,

$$\begin{aligned} g_t(\boldsymbol{x}, \boldsymbol{z}_t) &= \max_{n \in [N] \cup \{0\}} \left\{ \boldsymbol{q}_n^\top \boldsymbol{z}_t - p_n(\boldsymbol{a}_t^\top \boldsymbol{x}) \right\} \\ &= \max \left\{ 0, \max_{n \in [N]} \left\{ \boldsymbol{q}_n^\top \boldsymbol{z}_t - p_n(\boldsymbol{a}_t^\top \boldsymbol{x}) \right\} \right\} \end{aligned}$$

Hence, for any $\omega \in [\Omega]$, $t \in [T]$, the constraint (A.1) is equivalently expressed as

$$\begin{aligned} & r_t^\omega \geq \sup_{\boldsymbol{z}_t \in \mathcal{Z}_t} \left\{ g_t(\boldsymbol{x}, \boldsymbol{z}_t) - k \|\boldsymbol{z}_t - \boldsymbol{z}_t^\omega\|_\infty / \bar{z}_t \right\} \\ \iff & \begin{cases} r_t^\omega \geq \boldsymbol{q}_n^\top \boldsymbol{z}_t - \boldsymbol{a}_t^\top \boldsymbol{x} p_n - k \|\boldsymbol{z}_t - \boldsymbol{z}_t^\omega\|_\infty / \bar{z}_t & \forall \boldsymbol{z}_t \in \mathcal{Z}_t, n \in [N] \\ r_t^\omega \geq -k \|\boldsymbol{z}_t - \boldsymbol{z}_t^\omega\|_\infty / \bar{z}_t & \forall \boldsymbol{z}_t \in \mathcal{Z}_t. \end{cases} \end{aligned} \tag{A.2}$$

Observe that since $\boldsymbol{z}_t^\omega \in \mathcal{Z}_t$, we have

$$r_t^\omega \geq -k\|\boldsymbol{z}_t - \boldsymbol{z}_t^\omega\|_\infty/\bar{z}_t \quad \forall \boldsymbol{z}_t \in \mathcal{Z}_t \quad \iff \quad r_t^\omega \geq 0.$$

We next reformulate the constraint (A.2) using the duality theorem. For any $\boldsymbol{q} \in \mathbb{R}^N$, we have

$$\begin{aligned}
&\sup_{\boldsymbol{z}_t \in \mathcal{Z}_t} \left\{\boldsymbol{q}^\top \boldsymbol{z}_t - k\|\boldsymbol{z}_t - \boldsymbol{z}_t^\omega\|_\infty/\bar{z}_t\right\} \\
=& \sup_{\substack{\boldsymbol{z}_t \geq \boldsymbol{0} \\ \boldsymbol{1}^\top \boldsymbol{z}_t \leq \bar{z}_t}} \left\{\boldsymbol{q}^\top \boldsymbol{z}_t - k\left\|\boldsymbol{z}_t - \boldsymbol{z}_t^\omega\right\|_\infty/\bar{z}_t\right\} \\
=& \sup_{\substack{\boldsymbol{z}_t \geq \boldsymbol{0} \\ \boldsymbol{1}^\top \boldsymbol{z}_t \leq \bar{z}_t}} \inf_{\|\boldsymbol{\theta}\|_1 \leq k/\bar{z}_t} \left\{\boldsymbol{q}^\top \boldsymbol{z}_t - \boldsymbol{\theta}^\top\left(\boldsymbol{z}_t - \boldsymbol{z}_t^\omega\right)\right\} \\
=& \inf_{\|\boldsymbol{\theta}\|_1 \leq k/\bar{z}_t} \sup_{\substack{\boldsymbol{z}_t \geq \boldsymbol{0} \\ \boldsymbol{1}^\top \boldsymbol{z}_t \leq \bar{z}_t}} \left\{\boldsymbol{q}^\top \boldsymbol{z}_t - \boldsymbol{\theta}^\top\left(\boldsymbol{z}_t - \boldsymbol{z}_t^\omega\right)\right\} \\
=& \inf_{\|\boldsymbol{\theta}\|_1 \leq k/\bar{z}_t} \sup_{\substack{\boldsymbol{z}_t \geq \boldsymbol{0} \\ \boldsymbol{1}^\top \boldsymbol{z}_t \leq \bar{z}_t}} \left\{\left(\boldsymbol{q} - \boldsymbol{\theta}\right)^\top \boldsymbol{z}_t + \boldsymbol{\theta}^\top \boldsymbol{z}_t^\omega\right\} \\
=& \inf_{\substack{\|\boldsymbol{\theta}\|_1 \leq k/\bar{z}_t \\ \boldsymbol{1}\phi + \boldsymbol{\theta} \geq \boldsymbol{q}, \phi \geq 0}} \left\{\bar{z}_t \phi + \boldsymbol{\theta}^\top \boldsymbol{z}_t^\omega\right\}.
\end{aligned}$$

Therefore, for all $t \in [T], \omega \in [\Omega], n \in [N]$,

$$\begin{aligned}
& r_t^\omega \geq \boldsymbol{q}_{tn}^\top \boldsymbol{z}_t - \boldsymbol{a}_t^\top \boldsymbol{x} p_n - k\|\boldsymbol{z}_t - \boldsymbol{z}_t^\omega\|_\infty/\bar{z}_t \ \forall \boldsymbol{z}_t \in \mathcal{Z}_t \\
\iff & \begin{cases}
r_t^\omega \geq \bar{z}_t \phi_{tn}^\omega + \left(\boldsymbol{\theta}_{tn}^\omega\right)^\top \boldsymbol{z}_t^\omega - \boldsymbol{a}_t^\top \boldsymbol{x} p_n \\
\bar{z}_t \left\|\boldsymbol{\theta}_{tn}^\omega\right\|_1 \leq k \\
\boldsymbol{1}\phi_{tn}^\omega + \boldsymbol{\theta}_{tn}^\omega \geq \boldsymbol{q}_{tn} \\
\text{for some } \phi_{tn}^\omega \in \mathbb{R}_+, \boldsymbol{\theta}_{tn}^\omega \in \mathbb{R}^N.
\end{cases}
\end{aligned}$$

Up to now, we can get the deterministic optimization problem (18) in Therem 4. Finally, to show feasibility for $\tau \geq \bar{Z}_0$, we consider a restricted feasible set of Problem (18) with $\phi_{tn}^\omega = 0$ and $\boldsymbol{q}_n = \boldsymbol{\theta}_{tn}^\omega$ for all $n \in [N], \omega \in [\Omega], t \in [T]$. Hence, we can express the restricted feasible set as

$$\begin{aligned}
\mathcal{Q} &= \left\{ \boldsymbol{x} \in \mathcal{X} \;\middle|\; \begin{array}{l}
\exists r_t^\omega \geq 0 \ \forall \omega \in [\Omega], t \in [T]: \\
\boldsymbol{w}^\top \boldsymbol{x} + \dfrac{1}{\Omega} \displaystyle\sum_{\omega \in [\Omega]} \sum_{t \in [T]} r_t^\omega \leq \tau \\
r_t^\omega \geq \boldsymbol{q}_n^\top \boldsymbol{z}_t^\omega - \boldsymbol{a}_t^\top \boldsymbol{x} p_n \qquad \forall \omega \in [\Omega], t \in [T], n \in [N]
\end{array} \right\} \\
&= \left\{ \boldsymbol{x} \in \mathcal{X} \;\middle|\; \boldsymbol{w}^\top \boldsymbol{x} + \frac{1}{\Omega} \sum_{\omega \in [\Omega]} \sum_{t \in [T]} \max\left\{0, \max_{n \in [N]} \left\{\boldsymbol{q}_n^\top \boldsymbol{z}_t^\omega - \boldsymbol{a}_t^\top \boldsymbol{x} p_n\right\}\right\} \leq \tau \right\} \\
&= \left\{ \boldsymbol{x} \in \mathcal{X} \;\middle|\; \boldsymbol{w}^\top \boldsymbol{x} + \mathbb{E}_{\hat{\mathbb{P}}}\left[\sum_{t \in [T]} g_t(\boldsymbol{x}, \tilde{\boldsymbol{z}}_t)\right] \leq \tau \right\},
\end{aligned}$$

so that any $\boldsymbol{x} \in \mathcal{Q}$ would be feasible in Problem (18). Hence, if the empirical optimization problem (8) is solvable, then its solution would be feasible in the robust satisficing problem for all $\tau \geq \bar{Z}_0$. Moreover, when $k \geq \kappa_\tau \geq \bar{z}_t \|\boldsymbol{q}_n\|_1$,

$$\begin{aligned}
&\boldsymbol{q}_n^\top \boldsymbol{z} - k\|\boldsymbol{z} - \boldsymbol{z}_t^\omega\|_\infty/\bar{z}_t \\
=\ &\boldsymbol{q}_n^\top \boldsymbol{z}_t^\omega + \boldsymbol{q}_n^\top(\boldsymbol{z} - \boldsymbol{z}_t^\omega) - k\|\boldsymbol{z} - \boldsymbol{z}_t^\omega\|_\infty/\bar{z}_t \\
\leq\ &\boldsymbol{q}_n^\top \boldsymbol{z}_t^\omega + \|\boldsymbol{q}_n\|_1\|\boldsymbol{z} - \boldsymbol{z}_t^\omega\|_\infty - k\|\boldsymbol{z} - \boldsymbol{z}_t^\omega\|_\infty/\bar{z}_t \\
\leq\ &\boldsymbol{q}_n^\top \boldsymbol{z}_t^\omega.
\end{aligned}$$

On the other hand,

$$\sup_{\boldsymbol{z}_t \in \mathcal{Z}_t} \{\boldsymbol{q}_n^\top \boldsymbol{z}_t - k\|\boldsymbol{z}_t - \boldsymbol{z}_t^\omega\|_\infty/\bar{z}_t\} \geq \boldsymbol{q}_n^\top \boldsymbol{z}_t^\omega$$

since $\boldsymbol{z}_t^\omega \in \mathcal{Z}_t$. Therefore, when $k \geq \kappa_\tau$,

$$\begin{aligned}
&r_t^\omega \geq \sup_{\boldsymbol{z}_t \in \mathcal{Z}_t} \{g_t(\boldsymbol{x}, \boldsymbol{z}_t) - k\|\boldsymbol{z}_t - \boldsymbol{z}_t^\omega\|_\infty/\bar{z}_t\} \\
\Longleftrightarrow\ &\begin{cases} r_t^\omega \geq \boldsymbol{q}_n^\top \boldsymbol{z}_t - \boldsymbol{a}_t^\top \boldsymbol{x} p_n - k\|\boldsymbol{z}_t - \boldsymbol{z}_t^\omega\|_\infty/\bar{z}_t \ \forall \boldsymbol{z}_t \in \mathcal{Z}_t, n \in [N] \\ r_t^\omega \geq 0 \end{cases} \\
\Longleftrightarrow\ &\begin{cases} r_t^\omega \geq \boldsymbol{q}_n^\top \boldsymbol{z}_t^\omega - \boldsymbol{a}_t^\top \boldsymbol{x} p_n \ \forall n \in [N] \\ r_t^\omega \geq 0 \end{cases} \\
\Longleftrightarrow\ &r_t^\omega \geq g_t(\boldsymbol{x}, \tilde{\boldsymbol{z}}_t^\omega),
\end{aligned}$$

indicating the robust satisficing problem (17) coincides with the empirical optimization problem (8), thus the κ_τ can never be larger than $\bar{\kappa}$ when for any $\tau \geq \bar{Z}_0$. □

A.7 Proof of Theorem 5

Observe that

$$\boldsymbol{w}^\top \boldsymbol{x} + \mathbb{E}_{\mathbb{P}}\left[\sum_{t\in[T]}\sum_{\ell\in[L_t]} h_t(\gamma_{t\ell}, \eta_{t\ell} - u_{t\ell}\gamma_{t\ell}, \tilde{\boldsymbol{z}}_t^\ell)\right] \leq \tau + k\Delta_{\boldsymbol{\gamma}}(\mathbb{P}, \hat{\mathbb{P}}) \quad \forall \mathbb{P} \in \mathcal{P}_0(\mathcal{Z})$$

is equivalent to

$$\boldsymbol{w}^\top \boldsymbol{x} + \frac{1}{\Omega}\sum_{\omega\in[\Omega]} \mathbb{E}_{\mathbb{P}^\omega}\left[\sum_{t\in[T]}\sum_{\ell\in[L_t]}\left(h_t(\gamma_{t\ell}, \eta_{t\ell} - u_{t\ell}\gamma_{t\ell}, \tilde{\boldsymbol{z}}_t^\ell) - \frac{k\gamma_{t\ell}}{\bar{z}_{t\ell}}||\tilde{\boldsymbol{z}}_t^\ell - \boldsymbol{z}_t^{\ell\omega}||_\infty\right)\right] \leq \tau \ \ \forall \mathbb{P}^\omega \in \mathcal{P}_0(\mathcal{Z}).$$

Since $\mathcal{P}_0(\mathcal{Z})$ contains all Dirac distributions whose unit mass concentrates on any $\boldsymbol{z} \in \mathcal{Z}$, and given that the support set is breakpoints separable, we have the following equivalent constraint,

$$\boldsymbol{w}^\top \boldsymbol{x} + \frac{1}{\Omega}\sum_{\omega\in[\Omega]}\sum_{t\in[T]}\sum_{\ell\in[L_t]}\sup_{\boldsymbol{z}_t^\ell \in \mathcal{Z}_t^\ell}\left\{h_t(\gamma_{t\ell}, \eta_{t\ell} - u_{t\ell}\gamma_{t\ell}, \boldsymbol{z}_t^\ell) - \frac{k\gamma_{t\ell}}{\bar{z}_{t\ell}}||\boldsymbol{z}_t^\ell - \boldsymbol{z}_t^{\ell\omega}||_\infty\right\} \leq \tau.$$

Therefore Problem (19) has the following equivalent formulation

$$
\begin{aligned}
&\min\ k\\
&\text{s.t. } \boldsymbol{w}^\top \boldsymbol{x} + \frac{1}{\Omega}\sum_{\omega\in[\Omega]}\sum_{t\in[T]}\sum_{\ell\in[L_t]} r_t^{\ell\omega} \le \tau\\
&\quad r_t^{\ell\omega} \ge \sup_{\boldsymbol{z}\in\mathcal{Z}_t^\ell}\left\{h_t(\gamma_{t\ell},\eta_{t\ell}-u_{t\ell}\gamma_{t\ell},\boldsymbol{z}_t^\ell) - k\gamma_{t\ell}\|\boldsymbol{z}-\boldsymbol{z}_t^{\ell\omega}\|_\infty/\bar{z}_{t\ell}\right\} && \forall \omega\in[\Omega], t\in[T], n\in[N], \ell\in[L_t],\\
&\quad \mathbf{1}^\top \boldsymbol{\eta}_t = \boldsymbol{a}_t^\top \boldsymbol{x} && \forall t\in[T],\\
&\quad \mathbf{1}^\top \boldsymbol{\gamma}_t = 1 && \forall t\in[T],\\
&\quad \boldsymbol{\eta}_t\in\mathbb{R}^{L_t}, \boldsymbol{\gamma}_t\in\mathbb{R}_+^{L_t} && \forall t\in[T],\\
&\quad \boldsymbol{x}\in\mathcal{X}.
\end{aligned}
$$

Consequently, following the similar analysis in the proof of Theorem 4, we have

$$
\begin{aligned}
& r_t^{\ell\omega} \ge \sup_{\boldsymbol{z}_t^\ell\in\mathcal{Z}_t^\ell}\left\{h_t(\gamma_{t\ell},\eta_{t\ell}-u_{t\ell}\gamma_{t\ell},\boldsymbol{z}_t^\ell) - k\gamma_{t\ell}\|\boldsymbol{z}_t^\ell-\boldsymbol{z}_t^{\ell\omega}\|_\infty/\bar{z}_{t\ell}\right\}\\
\iff & \begin{cases} r_t^{\ell\omega} \ge \sup_{\boldsymbol{z}\in\mathcal{Z}_t^\ell}\left\{\boldsymbol{q}_n^\top\boldsymbol{z} - k\|\boldsymbol{z}-\boldsymbol{z}_t^\omega\|_\infty/\bar{z}_{t\ell}\right\}\gamma_{t\ell} - (\eta_{t\ell}-u_{t\ell}\gamma_{t\ell})\,p_n \ \forall n\in[N]\\ r_t^{\ell\omega} \ge \sup_{\boldsymbol{z}\in\mathcal{Z}_t^\ell}\left\{-k\|\boldsymbol{z}-\boldsymbol{z}_t^{\ell\omega}\|_\infty/\bar{z}_{t\ell}\right\}\gamma_{t\ell}\end{cases}\\
\iff & \begin{cases} r_t^{\ell\omega} \ge \alpha_{tn}^{\ell\omega}(k)\gamma_{t\ell} - (\eta_{t\ell}-u_{t\ell}\gamma_{t\ell})\,p_n & \forall n\in[N]\\ r_t^{\ell\omega}\ge 0,\end{cases}
\end{aligned}
$$

where $\alpha_{tn}^{\ell\omega}(k) = \sup_{\boldsymbol{z}\in\mathcal{Z}_t^\ell}\left\{\boldsymbol{q}_n^\top\boldsymbol{z} - k\|\boldsymbol{z}-\boldsymbol{z}_t^\omega\|_\infty/\bar{z}_{t\ell}\right\}$. Up to now, we can obtain the deterministic optimization problem (20) in Theorem 5.

Now we begin to show the feasibility of the robust satisficing model for $\tau \ge Z_0$. Observe that since $\boldsymbol{z}_t^{\ell\omega}\in\mathcal{Z}_t^\ell$, we have

$$
\alpha_{tn}^{\ell\omega}(k) \ge \boldsymbol{q}_n^\top \boldsymbol{z}_t^{\ell\omega}.
$$

However, we note that if $k \ge \bar{z}_{t\ell}\|\boldsymbol{q}_n\|_1$, then

$$
\begin{aligned}
&\boldsymbol{q}_n^\top\boldsymbol{z} - k\|\boldsymbol{z}-\boldsymbol{z}_t^\omega\|_\infty/\bar{z}_{t\ell}\\
=\ & \boldsymbol{q}_n^\top\boldsymbol{z}_t^{\ell\omega} + \boldsymbol{q}_n^\top(\boldsymbol{z}-\boldsymbol{z}_t^\omega) - k\|\boldsymbol{z}-\boldsymbol{z}_t^\omega\|_\infty/\bar{z}_{t\ell}\\
\le\ & \boldsymbol{q}_n^\top\boldsymbol{z}_t^{\ell\omega} + \|\boldsymbol{q}_n\|_1\|\boldsymbol{z}-\boldsymbol{z}_t^\omega\|_\infty - k\|\boldsymbol{z}-\boldsymbol{z}_t^\omega\|_\infty/\bar{z}_{t\ell}\\
\le\ & \boldsymbol{q}_n^\top\boldsymbol{z}_t^{\ell\omega},
\end{aligned}
$$

which implies $\alpha_{tn}^{\ell\omega}(k) = \boldsymbol{q}_n^\top\boldsymbol{z}_t^{\ell\omega}$. Hence, if $k \ge \bar{\kappa} \ge \bar{z}_{t\ell}(\mathbf{1}^\top\boldsymbol{p}) \ge \bar{z}_{t\ell}\|\boldsymbol{q}_n\|_1$ for all $t\in[T], \ell\in[L_t]$, then the following holds

$$
\begin{aligned}
& \begin{cases} r_t^{\ell\omega} \ge \alpha_{tn}^{\ell\omega}(k)\gamma_{t\ell} - (\eta_{t\ell}-u_{t\ell}\gamma_{t\ell})\,p_n & \forall n\in[N]\\ r_t^{\ell\omega}\ge 0\end{cases}\\
\iff & \begin{cases} r_t^{\ell\omega} \ge \boldsymbol{q}_n^\top\boldsymbol{z}_t^{\ell\omega}\gamma_{t\ell} - (\eta_{t\ell}-u_{t\ell}\gamma_{t\ell})\,p_n & \forall n\in[N]\\ r_t^{\ell\omega}\ge 0\end{cases}\\
\iff & r_t^{\ell\omega} \ge h_t(\gamma_{t\ell},\eta_{t\ell}-u_{t\ell}\gamma_{t\ell},\boldsymbol{z}_t^{\ell\omega}).
\end{aligned}
$$

Hence, when $k \geq \bar{\kappa}$, the empirical optimization problem (15) is the same as Problem (21) so that $\rho(k) = Z_0$. Therefore, the robust satisficing problem is feasible for $\tau \geq Z_0$, and κ_τ would not exceed $\bar{\kappa}$. □

A.8 Proof of Theorem 6

For any optimal solution $\boldsymbol{x}, \boldsymbol{\gamma}, \boldsymbol{\eta}, \kappa_\tau$ to Problem (19) given τ, we have from Theorem 3,

$$\mathbb{E}_{\mathbb{Q}^\star}\left[\sum_{t\in[T]} f_t(\boldsymbol{x}, \tilde{\mathcal{B}}_t)\right] \leq \mathbb{E}_{\mathbb{P}^\star}\left[\sum_{t\in[T]}\sum_{\ell\in[L_t]} h_t(\gamma_{t\ell}, \eta_{t\ell} - u_{t\ell}\gamma_{t\ell}, \tilde{\boldsymbol{z}}_t^\ell)\right].$$

Hence, for any $r \geq 0$

$$\begin{aligned}
&\mathbb{P}^\Omega\left[\boldsymbol{w}^\top\boldsymbol{x} + \mathbb{E}_{\mathbb{Q}^\star}\left[\sum_{t\in[T]} f_t(\boldsymbol{x}, \tilde{\mathcal{B}}_t)\right] > \tau + Tr\kappa_\tau\right] \\
\leq\ &\mathbb{P}^\Omega\left[\boldsymbol{w}^\top\boldsymbol{x} + \mathbb{E}_{\mathbb{Q}^\star}\left[\sum_{t\in[T]}\sum_{\ell\in[L_t]} h_t(\gamma_{t\ell}, \eta_{t\ell} - u_{t\ell}\gamma_{t\ell}, \tilde{\boldsymbol{z}}_t^\ell)\right] > \tau + Tr\kappa_\tau\right] \\
\leq\ &\mathbb{P}^\Omega\left[\Delta_{\boldsymbol{\gamma}}(\mathbb{P}^\star, \hat{\mathbb{P}}) > Tr\right] \leq \mathbb{P}^\Omega\left[\bar{\Delta}(\mathbb{P}^\star, \hat{\mathbb{P}}) > Tr\right],
\end{aligned}$$

where

$$\bar{\Delta}(\mathbb{P}, \hat{\mathbb{P}}) := \inf_{\mathbb{Q}\in\mathcal{P}_0(\mathcal{Z}^2)}\left\{\mathbb{E}_{\mathbb{Q}}\left[\sum_{t\in[T]}\max_{\ell\in[L_t]}\left\{\frac{1}{\bar{z}_{t\ell}}\|\tilde{\boldsymbol{z}}_t^\ell - \tilde{\boldsymbol{\xi}}_t^\ell\|_\infty\right\}\right] \;\middle|\; (\tilde{\boldsymbol{z}}, \tilde{\boldsymbol{\xi}}) \sim \mathbb{Q}, \tilde{\boldsymbol{z}} \sim \mathbb{P}, \tilde{\boldsymbol{\xi}} \sim \hat{\mathbb{P}}\right\},$$

noting that $\Delta_{\boldsymbol{\gamma}}(\mathbb{P}^\star, \hat{\mathbb{P}}) \leq \bar{\Delta}(\mathbb{P}^\star, \hat{\mathbb{P}})$ is due to $\mathbf{1}^\top\boldsymbol{\gamma}_t = 1$ and $\boldsymbol{\gamma}_t \geq \mathbf{0}$. By the definition of $\bar{\Delta}$, we have

$$\mathbb{P}^\Omega\left[\bar{\Delta}(\mathbb{P}^\star, \hat{\mathbb{P}}) > Tr\right] \leq \mathbb{P}^\Omega\left[\frac{1}{\Omega}\sum_{\omega\in[\Omega]}\mathbb{E}_{\tilde{\boldsymbol{z}}\sim\mathbb{P}^\star}\left[\sum_{t\in[T]}\max_{\ell\in[L_t]}\left\{\frac{1}{\bar{z}_{t\ell}}\|\tilde{\boldsymbol{z}}_t^\ell - \tilde{\boldsymbol{z}}_t^{\ell\omega}\|_\infty\right\}\right] > Tr\right].$$

Since $\tilde{\boldsymbol{z}}_t^\ell, \tilde{\boldsymbol{z}}_t^{\ell\omega} \in \mathcal{Z}_t$ almost everywhere, we have

$$\begin{aligned}
\|\tilde{\boldsymbol{z}}_t^\ell - \tilde{\boldsymbol{z}}_t^{\ell\omega}\|_\infty &\leq \max_{\substack{\boldsymbol{x},\boldsymbol{y}\geq\mathbf{0}\\ \mathbf{1}^\top\boldsymbol{x}\leq\bar{z}_{t\ell}, \mathbf{1}^\top\boldsymbol{y}\leq\bar{z}_{t\ell}}}\max_{n\in[N]}\{|\boldsymbol{e}_n^\top(\boldsymbol{x} - \boldsymbol{y})|\} \\
&\leq \max_{\substack{\boldsymbol{x},\boldsymbol{y}\geq\mathbf{0}\\ \|\boldsymbol{x}\|_1\leq\bar{z}_{t\ell}, \|\boldsymbol{y}\|_1\leq\bar{z}_{t\ell}}}\max_{n\in[N]}\left\{\max\{\boldsymbol{e}_n^\top\boldsymbol{x}, \boldsymbol{e}_n^\top\boldsymbol{y}\}\right\} \qquad (\text{since } \boldsymbol{x}, \boldsymbol{y} \geq \mathbf{0}) \\
&\leq \max_{n\in[N]}\left\{\max\left\{\max_{\|\boldsymbol{x}\|_1\leq\bar{z}_{t\ell}}\{\boldsymbol{e}_n^\top\boldsymbol{x}\}, \max_{\|\boldsymbol{y}\|_1\leq\bar{z}_{t\ell}}\{\boldsymbol{e}_n^\top\boldsymbol{y}\}\right\}\right\} \\
&= \max_{n\in[N]}\{\max\{\bar{z}_{t\ell}\|\boldsymbol{e}_n\|_\infty, \bar{z}_{t\ell}\|\boldsymbol{e}_n\|_\infty\}\} = \bar{z}_{t\ell},
\end{aligned}$$

where $\boldsymbol{e}_n$ is the nth unit basis vector. Therefore,

$$\mathbb{P}^\Omega\left[\frac{1}{\Omega}\sum_{\omega\in[\Omega]}\mathbb{E}_{\tilde{\boldsymbol{z}}\sim\mathbb{P}^\star}\left[\sum_{t\in[T]}\max_{\ell\in[L_t]}\left\{\frac{1}{\bar{z}_{t\ell}}\|\tilde{\boldsymbol{z}}_t^\ell - \tilde{\boldsymbol{z}}_t^{\ell\omega}\|_\infty\right\}\right] \leq T\right] = 1.$$

Consequently, when $r > 1$, we must have

$$\mathbb{P}^{\Omega}\left[\bar{\Delta}(\mathbb{P}^{\star},\hat{\mathbb{P}}) > Tr\right] = 0.$$

1. We now complete the proof for the first probability bound. Before proving this result, we need to recap the result on the probability bound of the Wasserstein distance in Fournier and Guillin (2015) as follows.

Suppose the actual data-generating distribution $\mathbb{P}^{\star}$, $\tilde{\boldsymbol{z}} \sim \mathbb{P}^{\star}$ is a light-tailed distribution such that

$$\mathbb{E}_{\mathbb{P}^{\star}}[\exp(\|\tilde{\boldsymbol{z}}\|^{\alpha})] < \infty \tag{A.3}$$

for some $\alpha > 1$ and $\mathbb{P}^{\Omega}$ is the distribution that governs the distribution of independent samples $\hat{\boldsymbol{z}}_1, \ldots, \hat{\boldsymbol{z}}_{\Omega}$ drawn from $\mathbb{P}^{\star}$, which constitutes the empirical distribution $\hat{\mathbb{P}}$. Then for any $R \in (0,1)$,

$$\mathbb{P}^{\Omega}\left[\Delta_1(\mathbb{P}^{\star},\hat{\mathbb{P}}) > R\right] \le c_1 \exp(-c_2 \Omega R^{\max\{n_z,2\}}) \tag{A.4}$$

for some positive constants, c_1 and c_2 that only depend on α, $\mathbb{E}_{\mathbb{P}^{\star}}[\exp(\|\tilde{\boldsymbol{z}}\|^{\alpha})]$, n_z being the dimension of $\tilde{\boldsymbol{z}}$, and Δ_1 is the (type-1) Wasserstein metric defined by

$$\Delta_1(\mathbb{P},\hat{\mathbb{P}}) := \inf_{\mathbb{Q}\in\mathcal{P}_0(\mathcal{Z}^2)} \left\{ \mathbb{E}_{\mathbb{Q}}\left[||\tilde{\boldsymbol{z}} - \tilde{\boldsymbol{\xi}}|| \right] \;\middle|\; (\tilde{\boldsymbol{z}},\tilde{\boldsymbol{\xi}}) \sim \mathbb{Q}, \tilde{\boldsymbol{z}} \sim \mathbb{P}, \tilde{\boldsymbol{\xi}} \sim \hat{\mathbb{P}} \right\}.$$

This result serves as a critical step to build the target attainment confidence guarantee since our result eventually relies on the probability bound of the Wasserstein distance. It remains to derive an upper bound for $\mathbb{P}^{\Omega}\left[\bar{\Delta}(\mathbb{P}^{\star},\hat{\mathbb{P}}) > Tr\right]$. Since $\tilde{\boldsymbol{z}} \in \mathcal{Z}$ almost everywhere, it is easy to see

$$\|\tilde{\boldsymbol{z}}\|_{\infty} = \max_{\substack{t\in[T],\ell\in[L_t]\\ n\in[N]}} z_{tn}^{\ell} \le \max_{t\in[T],\ell\in[L_t]} \bar{z}_{t\ell} = \bar{z} \quad a.e.,$$

then

$$\mathbb{E}_{\mathbb{P}^{\star}}[\exp(\|\tilde{\boldsymbol{z}}\|_{\infty}^{\alpha})] \le \exp(\bar{z}^{\alpha}) < \infty$$

for any $\alpha \ge 1$. We can choose $\alpha = 1$ to meet the assumption in inequality (A.3). Since $\min_{t\in[T]} \bar{z}_{t\ell} \ge 1$, we have

$$T\|\tilde{\boldsymbol{z}} - \tilde{\boldsymbol{\xi}}\|_{\infty} \ge \sum_{t\in[T]} \max_{\ell\in[L_t]} \left[\frac{1}{\bar{z}_{t\ell}} ||\tilde{\boldsymbol{z}}_t^{\ell} - \tilde{\boldsymbol{\xi}}_t^{\ell}||_{\infty} \right] \quad a.e.,$$

therefore

$$T\Delta_1(\mathbb{P}^{\star},\hat{\mathbb{P}}) \ge \bar{\Delta}(\mathbb{P}^{\star},\hat{\mathbb{P}}) \implies \mathbb{P}^{\Omega}\left[\bar{\Delta}(\mathbb{P}^{\star},\hat{\mathbb{P}}) > Tr\right] \le \mathbb{P}^{\Omega}\left[\Delta_1(\mathbb{P}^{\star},\hat{\mathbb{P}}) > r\right] \quad \forall r \in [0,1].$$

Further notice the dimension of $\tilde{\boldsymbol{z}}$ is $N(\sum_{t\in[T]} L_t) \ge 2$, the following probability bound hold by inequality (A.4)

$$\mathbb{P}^{\Omega}\left[\boldsymbol{w}^{\top}\boldsymbol{x} + \mathbb{E}_{\mathbb{Q}^{\star}}\left[\sum_{t\in[T]} f_t(\boldsymbol{x},\tilde{\mathcal{B}}_t) \right] > \tau + Tr\kappa_{\tau} \right] \le c_1 \exp(-c_2 \Omega r^{NL}) \quad \forall r \in [0,1],$$

where $L = \sum_{t\in[T]} L_t$, c_1 and c_2 are constants that depend on $\mathbb{E}_{\mathbb{P}^\star}[\exp(||\tilde{\boldsymbol{z}}||_\infty)]$ and NL.

2. We next prove the result for the second probability bound. For ease of expression, we can define the random variables

$$\tilde{\nu}^\omega := \mathbb{E}_{\tilde{\boldsymbol{z}}\sim\mathbb{P}^\star}\left[\sum_{t\in[T]}\max_{\ell\in[L_t]}\left\{\frac{1}{\bar{z}_{t\ell}}\|\tilde{\boldsymbol{z}}_t^\ell - \tilde{\boldsymbol{z}}_t^{\ell\omega}\|_\infty\right\}\right] \quad \forall\omega\in[\Omega],$$

then $\tilde{\nu}^1, \tilde{\nu}^2, \ldots, \tilde{\nu}^\Omega$ are independent random variables and

$$0 \le \mathbb{E}_{\mathbb{P}^\Omega}\left[\tilde{\nu}^\omega\right] = \mu \le \operatorname{ess\,sup}\tilde{\nu}^\omega \le \frac{1}{T}\sum_{t\in[T]}\frac{\bar{z}_{t\ell}}{\bar{z}_{t\ell}} = 1 \quad \forall\omega\in[\Omega].$$

We have the following for any $r\in[\mu,1]$

$$\begin{aligned}
&\mathbb{P}^\Omega\left[\frac{1}{\Omega}\sum_{\omega\in[\Omega]}\mathbb{E}_{\tilde{\boldsymbol{z}}\sim\mathbb{P}^\star}\left[\sum_{t\in[T]}\max_{\ell\in[L_t]}\left\{\frac{1}{\bar{z}_{t\ell}}\|\tilde{\boldsymbol{z}}_t^\ell - \tilde{\boldsymbol{z}}_t^{\ell\omega}\|_\infty\right\}\right] \ge Tr\right] \\
&= \mathbb{P}^\Omega\left[\frac{1}{\Omega}\sum_{\omega\in[\Omega]}\tilde{\nu}^\omega \ge r\right] \\
&\le \inf_{\theta\ge 0}\left\{\exp\left(-\theta r\right)\mathbb{E}_{\mathbb{P}^\Omega}\left[\exp\left(\theta\sum_{\omega\in[\Omega]}\tilde{\nu}^\omega/\Omega\right)\right]\right\} \\
&= \inf_{\theta\ge 0}\left\{\exp\left(-\theta r\right)\mathbb{E}_{\mathbb{P}^\Omega}\left[\prod_{\omega\in[\Omega]}\exp\left(\theta\tilde{\nu}^\omega/\Omega\right)\right]\right\} \\
&= \inf_{\theta\ge 0}\left\{\exp\left(-\theta r\right)\left(\prod_{\omega\in[\Omega]}\mathbb{E}_{\mathbb{P}^\Omega}\left[\exp\left(\theta\tilde{\nu}^\omega/\Omega\right)\right]\right)\right\} \\
&\le \inf_{\theta\ge 0}\left\{\exp\left(-\theta r\right)\left(\prod_{\omega\in[\Omega]}\exp\left(\frac{\theta\mathbb{E}_{\mathbb{P}^\Omega}\left[\tilde{\nu}^\omega\right]}{\Omega} + \frac{\theta^2}{8\Omega^2}\right)\right)\right\} \\
&= \inf_{\theta\ge 0}\left\{\exp\left(\frac{\theta^2}{8\Omega} - \theta r + \frac{\theta\sum_{\omega\in[\Omega]}\mathbb{E}_{\mathbb{P}^\Omega}\left[\tilde{\nu}^\omega\right]}{\Omega}\right)\right\} \\
&\le \inf_{\theta\ge 0}\left\{\exp\left(\frac{\theta^2}{8\Omega} - \theta(r-\mu)\right)\right\} \\
&= \exp\left(-2\Omega\left(r-\mu\right)^2\right),
\end{aligned}$$

where the first equation holds by the definition of $\tilde{\nu}^\omega$, the second inequality holds by Markov inequality, the third equation holds trivially, the fourth equation holds since $\tilde{\nu}^1, \tilde{\nu}^2, \ldots, \tilde{\nu}^\Omega$ are independent, the fifth inequality holds by Hoeffding's lemma, the sixth equation and the seventh inequality hold trivially, and the last equation holds by calculating the minimum of the quadratic function. □

Appendix B On the worst-case scenarios of robust models

To promote the utilization of robust optimization methods in practical situations, one may argue that it is necessary to identify the worst-case scenario so that the decision-maker can comprehensively understand the decision. However, the fact is that the worst-case scenario may not provide direct insight into the optimal decision. A similar opinion is also found in Bertsimas and den Hertog (2022)'s book *Robust and Adaptive Optimization* (Page 26): "*At first glance, RO optimizes for the worst-case scenario. However, this statement is confusing and even incorrect.*"

To explain the perspective stated above, we first consider the classical distributionally free robust optimization, where, conceptually, minimizing the worst-case objective can be viewed from a game-theoretic standpoint. Specifically, the decision-maker adjusts variable $\boldsymbol{x} \in \mathcal{X}$, while an 'adversarial nature' influences input vector $\boldsymbol{z} \in \mathcal{Z}$. The decision-maker seeks to minimize the cost function $f(\boldsymbol{x}, \boldsymbol{z})$, while the adversarial nature endeavors to maximize this cost, opposing the decision-maker.

Mathematically, the decision-maker aims to minimize the worst-case cost imposed by the adversary:

$$Z_D = \inf_{\boldsymbol{x} \in \mathcal{X}} \sup_{\boldsymbol{z} \in \mathcal{Z}} f(\boldsymbol{x}, \boldsymbol{z}).$$

Here, the decision-maker takes the first move, and the adversarial nature responds to provide the worst outcome for the decision-maker. It is important to note that the worst-case scenarios obtained from this perspective may not be unique.

It is also a common narrative to obtain the worst-case scenario as follows:

$$Z_A = \sup_{\boldsymbol{z} \in \mathcal{Z}} \inf_{\boldsymbol{x} \in \mathcal{X}} f(\boldsymbol{x}, \boldsymbol{z}).$$

In this perspective, the adversary acts first, seeking to maximize the disadvantage for the decision-maker. The saddle point worst-case scenario, $\bar{\boldsymbol{z}}$ obtained in this way, could be uniquely determined. However, this perspective, where the adversary acts first, may seem somewhat contrived.

Suppose an equilibrium exists, *i.e.*,

$$Z_D = \inf_{\boldsymbol{x} \in \mathcal{X}} \sup_{\boldsymbol{z} \in \mathcal{Z}} f(\boldsymbol{x}, \boldsymbol{z}) = f(\bar{\boldsymbol{x}}, \bar{\boldsymbol{z}}) = \sup_{\boldsymbol{z} \in \mathcal{Z}} \inf_{\boldsymbol{x} \in \mathcal{X}} f(\boldsymbol{x}, \boldsymbol{z}) = Z_A.$$

The equilibrium associated with minimizing the worst-case objective centers on a specific worst-case scenario, $\bar{\boldsymbol{z}}$, that arises in the equilibrium, provided that the equilibrium exists. Even if it exists, a prevalent misunderstanding is that this particular scenario offers direct insight into obtaining the optimal decision $\boldsymbol{x}$ that minimizes the worst-case objective. Whereas, the reality is that even if $\bar{\boldsymbol{z}}$ is known, the solution $\boldsymbol{x}$ derived from the set $\arg\min_{\boldsymbol{x} \in \mathcal{X}} f(\boldsymbol{x}, \bar{\boldsymbol{z}})$ does not necessarily match the equilibrium solution $\bar{\boldsymbol{x}}$. We illustrate this in the following example:

Consider the biaffine function $f(x,z)=xz$, and the sets $\mathcal{X}=[-1,1]$, $\mathcal{Z}=[-1,1]$. We have

$$Z_D = \min_{x\in[-1,1]}\left\{\max_{z\in[-1,1]} xz\right\} = \min_{x\in[-1,1]}\{\max\{-x,x\}\} = \min_{x\in[-1,1]}|x| = 0.$$

Likewise,

$$Z_A = \max_{z\in[-1,1]}\left\{\min_{x\in[-1,1]} xz\right\} = \max_{z\in[-1,1]}\{\min\{-z,z\}\} = \max_{z\in[-1,1]} -|z| = 0.$$

Therefore, the saddle point is uniquely determined at $\bar{z}=\bar{x}=0$. In contrast, there are infinite solutions that satisfy $\max_{z\in[-1,1]}\bar{x}z$ or $\min_{x\in[-1,1]}x\bar{z}$. Hence, it is our opinion that deriving the worst-case scenarios in robust decision models is an artificial problem, offering limited utility and insight that could be used in practical applications. Therefore, we respectfully disagree that comprehending the worst-case scenarios would help business managers make informed decisions in practical situations.

Furthermore, when there is a discrepancy between the decision-maker's and adversary's evaluations, marked by $Z_D > Z_A$, it highlights a fundamental misalignment in their perspectives, suggesting that achieving equilibrium might not be feasible. In this study, we consider a distributionally robust model, which can be expressed as the following problem:

$$Z_D = \inf_{\boldsymbol{x}\in\mathcal{X}} \sup_{\mathbb{P}\in\mathcal{F}} g(\boldsymbol{x},\mathbb{P}).$$

For the given optimal solution $\bar{\boldsymbol{x}}$, we can find a discrete worst-case distribution obtained from the classical robust optimization representation of the distributionally robust optimization problem. However, this worst-case distribution may not necessarily be the saddle-point worst-case distribution. To obtain the saddle-point worst-case distribution, we need to solve the adversarial problem as follows:

$$Z_A = \sup_{\mathbb{P}\in\mathcal{F}} \inf_{\boldsymbol{x}\in\mathcal{X}} g(\boldsymbol{x},\mathbb{P}).$$

We would like to emphasize that determining the saddle point worst-case distribution in the adversarial problem is generally challenging. While there have been recent analyses of special cases in Chen et al. (2024), finding the saddle point distribution remains a complex task in most scenarios. It is important to emphasize that the adversarial problem is a contrived problem meant for the adversarial nature. Its insights may not be relevant to the decision-maker because it is not a reflection of reality; different robust models would yield different saddle-point worst-case distributions. Furthermore, it only makes sense to consider the saddle point worst-case distribution when an equilibrium, such that $Z_A = Z_D$, exists. However, in our context, since $\mathcal{X}\subseteq\mathbb{Z}_+^I$ is not a convex set, we cannot apply Sion's minimax result and therefore do not expect equilibrium to exist.

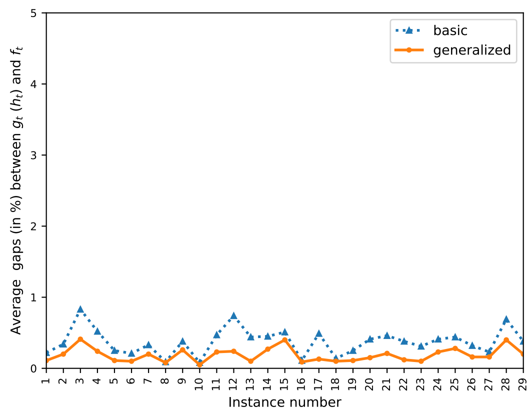
Appendix C Supplementary numerical results and discussions

This section provides supplementary numerical results and discussions.

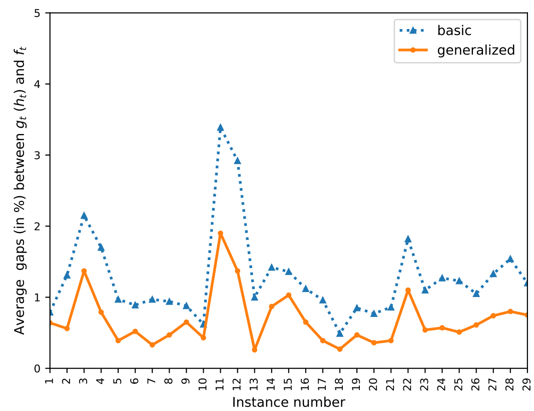
Evaluation of the reduced information models under a random bidding scheme

In this section, we evaluate the performance of both reduced information models under a random bidding scheme, representing the case of couriers' unknown preferences, as some studies have shown that couriers may have various or even completely different preferences even though the covariates are the same (Rechavi and Toch 2022). We set $K_t = J_t$. For each courier $k \in [K_t]$, we first randomly choose N' bidding prices from the set $\{p_1, p_2, \ldots, p_N\}$. Subsequently, we generate a random binary number for each job $j \in [J_t]$, with a value of 0 denoting that courier k is not interested in job j. In this case, we set the bidding price of courier k for job j to p_N. If the random variable equals 1, we randomly choose a price from the N' selected prices for job j. Other parameters' settings are the same as those in Section 6. Figure C1 plots the average percentage gaps between the reduced information models and the crowdsourcing cost function.

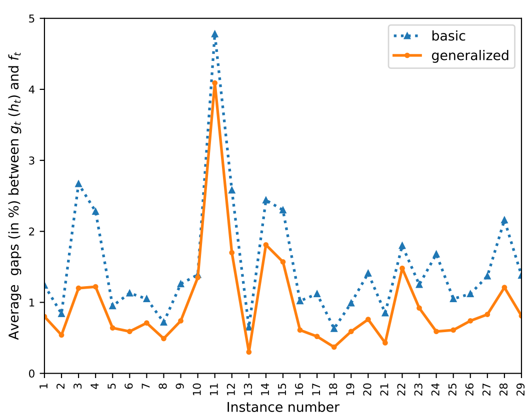
Figure C1 **Average gaps (in %) between the reduced information models and the crowdsourcing cost function under a random bidding scheme**



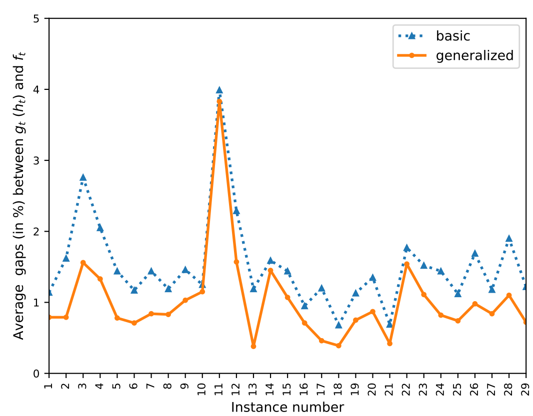
(a) $N' = 2$



(b) $N' = 3$



(c) $N' = 4$



(d) $N' = 5$

Figure C1 shows that the average gaps are within 5% for all the cases. In particular, when $N' = 2$, *i.e.*, two payment values are allowed, both reduced information models can provide satisfying approximation results, controlling the gaps to be smaller than 1%. The generalized model can always provide tighter bounds than the basic model, regardless of the value of N'. The results confirm again the effectiveness of the reduced information models, especially the generalized model, for approximating Problem (1) and tackling practical delivery problems.

Comparison with a moment-based distributionally robust optimization model

We consider a moment-based distributionally robust optimization (DRO) problem for the basic reduced information model, which is as follows,

$$\begin{aligned} Z_{\mathcal{F}} = \min\ & \boldsymbol{w}^\top \boldsymbol{x} + \sup_{\mathbb{P}\in\mathcal{F}} \mathbb{E}_{\mathbb{P}} \left[\sum_{t\in[T]} g_t(\boldsymbol{x}, \tilde{\boldsymbol{z}}_t) \right] \\ \text{s.t.}\ & \boldsymbol{x} \in \mathcal{X}, \end{aligned}$$

where the ambiguity set $\mathcal{F}$ is defined as

$$\mathcal{F} = \left\{ \mathbb{P} \in \mathcal{P}(\mathbb{R}^{TN} \times \mathbb{R}^{TN}) \;\middle|\; \begin{array}{l} (\tilde{\boldsymbol{z}}, \tilde{\boldsymbol{u}}) \in \mathbb{P} \\ \mathbb{E}_{\mathbb{P}}[\tilde{\boldsymbol{z}}] = \boldsymbol{\mu} \\ \mathbb{E}_{\mathbb{P}}[\tilde{\boldsymbol{u}}] = \boldsymbol{\sigma} \\ \mathbb{P}[(\tilde{\boldsymbol{z}}, \tilde{\boldsymbol{u}}) \in \mathcal{Z}_{\text{lift}}] = 1 \end{array} \right\}.$$

Parameters $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ are the component-wise means and the mean absolute deviations (MAD) of the random variables $\tilde{\boldsymbol{z}} = (\tilde{\boldsymbol{z}}_t)_{t\in[T]}$, respectively. These parameters can be directly estimated after obtaining the reduced information vectors from historical data. The auxiliary random variables $\tilde{\boldsymbol{u}}$ are introduced such that the terms inside the expectation constraints in the ambiguity set $\mathcal{F}$ are all linear, and the nonlinear term associated with the MAD is transferred to the support set $\mathcal{Z}_{\text{lift}}$, which is written as

$$\mathcal{Z}_{\text{lift}} = \left\{ (\boldsymbol{z}, \boldsymbol{u}) \in \mathbb{R}^{TN} \times \mathbb{R}^{TN} \;\middle|\; \begin{array}{ll} \boldsymbol{z} \geq 0,\ \boldsymbol{u} \geq 0 & \\ \boldsymbol{1}^\top \boldsymbol{z}_t \leq \bar{z}_t & \forall t \in [T] \\ |z_{tn} - \mu_{tn}| \leq u_{tn} & \forall t \in [T], n \in [N] \end{array} \right\}.$$

To solve this two-stage robust optimization problem, we utilize the robust stochastic optimization framework from Chen et al. (2020), where the linear decision rule (LDR) is used, *i.e.*, the recourse decision $\boldsymbol{y}$ should be defined as an affine function of $\boldsymbol{z}$ and $\boldsymbol{u}$. This framework is associated with an algebraic modeling package called RSOME (Chen and Xiong 2023), which can facilitate the solving process by employing only a few lines of code. We note that the moment-based DRO problem is solved approximately using the LDR, whereas our robust satisficing models can be exactly solved by addressing a few convex optimization problems.

Figure C2 **Comparison of out-of-sample performance between the empirical and the moment-based DRO models**

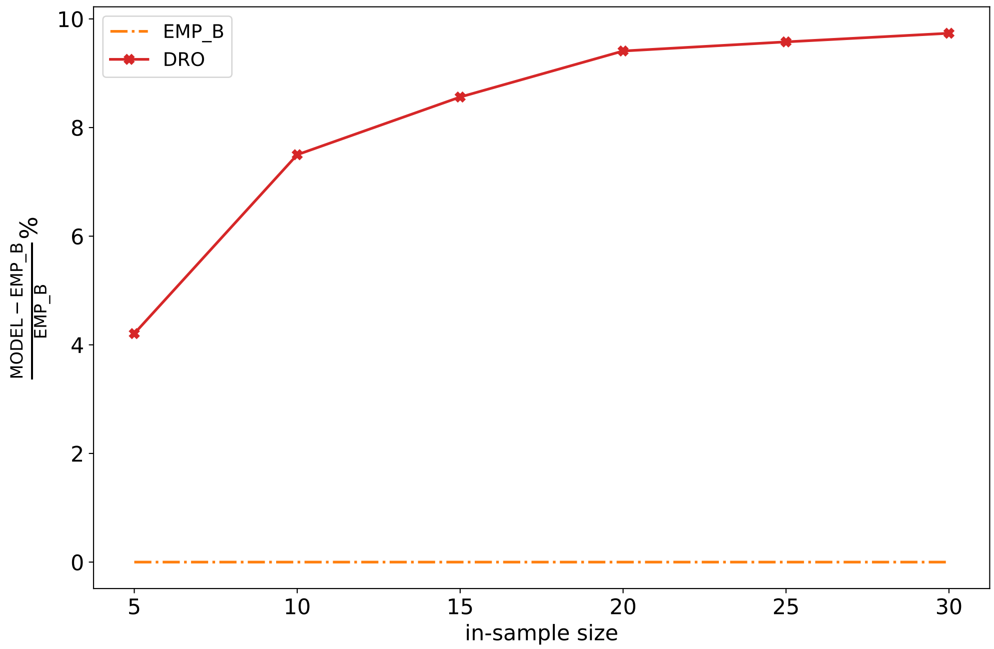


Figure C2 presents a comparison of the out-of-sample performance between the empirical and moment-based DRO models. All the experimental settings are consistent with those of Figure 3 except the in-sample size, which ranges from 5 to 30. The results indicate that the empirical model consistently outperforms the DRO model. Moreover, as the in-sample size increases, the performance gap between the two models widens. The numerical results in Section 6 demonstrate that the robust satisficing model for the basic reduced information model exhibits superior out-of-sample performance compared to its empirical counterpart model. Combining this finding with the results presented here, we can further confirm the superiority of the robust satisficing framework in managing crowdsourcing delivery platforms' workforce.

Selection of the target parameter in the robust satisficing model

We finally discuss how to select the target ratio parameter α for real-world crowdsourced delivery problems with limited datasets based on the insights from this paper. Below is a brief overview of the process: First, the dataset is partitioned into training and testing subsets. Parameters are then evaluated using the training set to identify the best-performing ones, which are subsequently assessed on the testing set. Various cross-validation techniques, like the k-fold cross-validation, can be used to implement this process. In the context of the robust satisficing problem, the parameter of interest is the target ratio α. An important aspect of cross-validation is determining an appropriate search range for this parameter. Our simulation results indicate that restricting the search range for α to $[1.01, 1.05]$ can yield decent performance.

Both data-driven robust optimization and robust satisficing approaches rely on cross-validation to fine-tune parameters in practical scenarios. Compared to the data-driven robust optimization where the radius of the Wasserstein ball is the tuning parameter, the target ratio α is more tangible and intuitive. This parameter represents an acceptable trade-off between optimality and robustness, offering a clear interpretation of the cost incurred to accommodate greater uncertainty. Consequently, decision-makers can adjust α within the recommended range for their own preference.