An enhanced mathematical model for optimal simultaneous preventive maintenance scheduling and workshop planning

Gabrijela Obradović · Ann-Brith Strömberg · Kristian Lundberg

Abstract For a system to stay operational, maintenance of its components is required and to maximize the operational readiness of a system, preventive maintenance planning is essential. There are two stakeholders—a system operator and a maintenance workshop—and a contract regulating their joint activities. Each contract leads to a bi-objective optimization problem. Components that require maintenance are taken out from operating systems and sent to the maintenance workshop, which should perform all maintenance activities on time in order to satisfy the contract. Upon being maintained, the components are sent back and available to be used in the operating systems. The ability of the workshop to fulfill the contract is highly dependent on its capacity. Our modeling of this system–of–systems includes problem structuring of the planning of preventive maintenance for the operating systems, the maintenance workshop scheduling as well as the stocks of damaged and repaired components.

The mixed-integer linear optimization (MILP) model we present is partly based on an optimization model of a preventive maintenance scheduling problem with interval costs over a finite and discretized time horizon, which we generalize and extend with a non-preemptive flow of components through the workshop and the stocks of (damaged and repaired) components. Our results measure the effect of the workshop capacity on the level of component avail-

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ability as well as on the utilization rate. We also analyze the maximum possible reduction of the costs of two stakeholders for increased levels of the workshop capacity. The resulting modeling can be utilized as a decision aiding tool, and help improve the decision making processes for the stakeholders.

Keywords System Maintenance · Workshop Scheduling · Mixed-Integer Linear Optimization Model · Optimization of Contracting Forms · Simultaneous Scheduling · Bi-Objective Optimization · Decision making support

1 Introduction

The demand for integrated collaboration in supply chains and production has been increasing in the dynamic business world. Companies strive to optimize their operations and remain competitive, and one way to achieve that is to foster integrated cooperation. By doing so, organizations streamline their supply chains, enhance production efficiency and effectively respond to changing market demands.

Maintenance is performed in order for a system to remain in/get restored to its operational state (Swanson, 2001). Maintaining a system typically means repairing, replacing, overhauling, inspecting, servicing, adjusting, or testing the system and/or its components, so that there are no interruptions of the system's planned operations. The outcome of an effective maintenance planning is a reduced risk of failure (Papakostas et al., 2010) and an optimal use of the system's life (and of lives of its components). *Preventive maintenance* (PM) is planned and performed after a specified period of time, or when a specified system has been used for a certain period of time, in order to reduce the probability of failure of the system. *Corrective maintenance* (CM), on the other hand, is performed after a failure has occurred as a corrective measure to restore the system into an operational state. CM typically comes with a higher cost, since it is often associated with unplanned disruptions in the operations. While we consider PM scheduling, CM is implicitly accounted for by an additional cost that is increasing with the time between PM occasions, and reflects the increasing risk of having to perform CM. See Yu and Strömberg (2021) for a model that uses failure time distributions to model such additional costs.

We consider a setting with two stakeholders, one being the *system operator* and the other being the *maintenance workshop*. The system operator is performing the operations (considering, e.g., train traffic, the system operator would be operating the trains according to a given timetable), while the maintenance workshop is performing the repairs of components. The repaired components are sent back to the system operator, thus creating a circular flow. The two stakeholders can work independently or they can share their information and cooperate towards a common goal. We consider the latter case, in which the stakeholders are integrated. Each stakeholder has one (or several) objective(s) they wish to optimize. The collaboration between the two stakeholders is typically governed by a contract. In this work, we model and study an 'availability of repaired components' contract type (see Sec. 3).

In (Zhang et al., 2023), a model for scheduling maintenance of military aircraft fleets under limited resources has been proposed. It has been done by using a heuristic approach that optimizes maintenance intervals and prioritizes tasks based on safety and mission criticality. Verhoeff and Verhagen (2023) propose a MILP model that solves the flight and maintenance planning problem using component substitution scheduling while being aligned with overall aircraft flight and maintenance planning. Mohammad Hadian et al. (2023) propose a simulation-based optimization approach to integrate maintenance planning and safety stock determination in deteriorating manufacturing systems; the approach involves the use of simulation to model the systems' deterioration processes, when optimizing the maintenance policy and safety stock level jointly to minimize cost and maximize system availability. Another example of integrating maintenance scheduling and planning for large-scale asset fleets, with the aim of optimizing maintenance activities while minimizing downtime and costs, is presented in (Schulze Spüntrup et al., 2018). Examples of integrated scheduling and planning for a multi-year planning horizon for heterogeneous fleets while incorporating uncertainty are presented in (van der Weide et al., 2022; Deng and Santos, 2022). In our previous work (see Obradović (2021); Obradović et al. (2022)), we investigated the effect of different contracting forms on the efficiency of maintenance activities and the flow of components through the system–of–systems, as well as the availability of the systems over time. First, we presented a model with individual components' flow (Obradović, 2021) and then we presented a model with an aggregated flow of components of the same component type (Obradović et al., 2022). We now introduce the modeling of jobs[1] combined with a non-preemptive model for scheduling the repair of components in the maintenance workshop. Moreover, we present a new formulation of an 'availability of repaired components' contract between the stakeholders. We formulate a multi-criteria optimization model representing: (i) the scheduling of the PM occasions for the components of the system(s), and (ii) the scheduling of the repair activities in the maintenance workshop.

The main contribution of this work is a mathematical model of the integration and simultaneous scheduling of replacement and non-preemptive repair of components used in multiple systems using job scheduling. Moreover, a mathematical model of an availability contracting form between the stakeholders and its analysis via a bi-objective optimization problem is presented, which may be utilized to aid and improve decision making processes.

The remainder of the article is as follows. In Section 2, we describe the multi-system PMSPIC[2] (MS-PMSPIC), the structure of the maintenance workshop, the stock dynamics modeling, and their integration with the operational demand on the systems. We define the optimization objectives corresponding to the two stakeholders; one to the system maintenance and one to the

[1] Every action taken in the maintenance workshop is considered as a job

[2] Part of the model presented in this article, corresponding to the scheduling of PM activities, is based on the *preventive maintenance scheduling problem with interval costs* (PMSPIC) model presented in (Gustavsson et al., 2014).

maintenance workshop. In Section 3, we present the multi-objective modeling. Results are presented in Section 4, and in Section 5 we present conclusions and ideas for future research.

2 Mathematical model

A number of systems are operating to meet a common production demand. Each system is assigned to a (predefined) operating schedule, resulting in time-windows at which the systems' components may undergo maintenance. As systems operate their components degrade and, eventually, they require maintenance (e.g., service, replacement, or repair). During a maintenance occasion, (one or several) components are taken out of the system, sent to the maintenance workshop where they are repaired, and returned back to the stock of repaired components, where they are available to be used again (in any of the systems). The components sent for repair are, if possible, instantly replaced by currently available components on the stock of repaired components (i.e., if the stock is not empty). Thus, there exists a circulating flow of (individual) components, that are used and degraded, replaced, repaired, and then made available for a system to use them again. The illustration of the system–of–systems is presented in Figure 1, and it is modeled with the aim to keep the operating systems operational, if possible, such that the capacity of the maintenance workshop is not exceeded. A difference compared to our previous work (Obradović, 2021) is that, due to the fact that modeling flow of individual components lead to a computational intractability of the model for larger instance sizes, we do not track individual components. We enhance the model presented in Obradović et al. (2022) such that so-called *jobs* are introduced in the maintenance workshop. After a component is demounted from a system and once it is about to be processed in the maintenance workshop, it is assigned a new 'job id'.

To enable a so-called time-indexed modeling (see, e.g., van den Akker et al. (2000)) the time is discretized. The number of times a components will undergo repair is correlated with the length of the planning period.

We start by formally defining in Section 2.1 the MS-PMSPIC—which models the replacement scheduling for the components of the systems considered—along with a mixed-binary linear optimization (MBLP) formulation. Further, in Section 2.2, we present a model for the maintenance workshop scheduling using mixed-integer linear optimization (MILP). The two systems (i.e., scheduling problems) are further integrated, in Section 2.3, through the dynamics of the stocks of (damaged and repaired) components. We conclude this section with a summary of the complete MILP model.

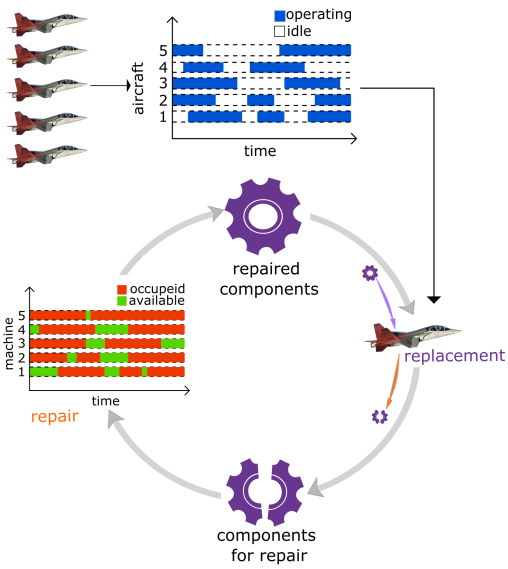


Fig. 1: Illustration of the problem for an application with a system of aircraft.

2.1 The Multi-System Preventive Maintenance Scheduling Problem with Interval Costs

The *multi-system preventive maintenance scheduling problem with interval costs* (MS-PMSPIC) is defined in (Obradović et al., 2022); cf. (Gustavsson et al., 2014). As the problem definition is unchanged, we present the definition and a brief summary of the model (for detailed modeling and description of the MS-PMSPIC, see Obradović et al. (2022), section 2.1).

Definition 1 (MS-PMSPIC) Consider K systems $k \in \mathcal{K} := \{1, \ldots, K\}$ with component types $i \in \mathcal{I} := \{1, \ldots, I\}$ and $\mathcal{J}_i := \{1, \ldots, J_i\}$ denoting the set of individual components of type i, and a set $\mathcal{T} := \{1, \ldots, T\}$ of time steps at which maintenance of the systems can be performed, where T represents the planning horizon. A PM schedule consists of a set of replacement times in $\mathcal{T}$ for each system k and component type i. A maintenance occasion for system k at time t generates the maintenance occasion cost d_t^k. If PM of a component of type i in system k is scheduled at the times $s \in \mathcal{T} \cup \{0\}$ and $t \in \{s+1, \ldots, T+1\}$, but not in the (possibly empty) time interval $\{s+1, \ldots, t-1\}$, then the maintenance interval, denoted (s, t), generates the interval cost c_{st}^i. For each component type $i \in \mathcal{I}$ no maintenance interval should be longer than $\bar{t}_i$. Find a PM schedule that satisfies the MS-PMSPIC feasibility problem.

We next present the (binary linear) feasibility model for the MS-PMSPIC, which together with an assigned objective function (defined in Sec. 3) comprises a binary linear optimization problem. With the decision variables being defined as

$$x_{st}^{ik} = \begin{cases} 1, & \text{if a component of type } i \text{ in system } k \text{ receives} \\ & \text{PM at times } s \text{ and } t, \text{ but not in-between,} \\ 0, & \text{otherwise,} \end{cases} \qquad \begin{array}{l} i \in \mathcal{I},\ k \in \mathcal{K}, \\ 0 \leq s < t \leq T+1, \end{array}$$

$$z_t^k = \begin{cases} 1, & \text{if maintenance of system } k \text{ occurs at time } t, \\ 0, & \text{otherwise,} \end{cases} \qquad k \in \mathcal{K},\ t \in \mathcal{T},$$

the feasible set of the MS-PMSPIC is defined by

$$\sum_{r=1}^{T+1} x_{0r}^{ik} = 1, \qquad i \in \mathcal{I},\ k \in \mathcal{K}, \tag{1a}$$

$$\sum_{s=0}^{t-1} x_{st}^{ik} = \sum_{r=t+1}^{T+1} x_{tr}^{ik}, \qquad i \in \mathcal{I},\ t \in \mathcal{T},\ k \in \mathcal{K}, \tag{1b}$$

$$\sum_{s=0}^{t-1} x_{st}^{ik} \leq z_t^k, \qquad i \in \mathcal{I},\ t \in \mathcal{T},\ k \in \mathcal{K}, \tag{1c}$$

$$x_{st}^{ik} = 0, \qquad \bar{t}_i \leq s + \bar{t}_i < t \leq T+1,\ i \in \mathcal{I},\ k \in \mathcal{K}. \tag{1d}$$

The description of the constraints is given Obradović et al. (2022), Sec. 2.1.

When planning for a PM activity, the operational schedules for the systems provide time windows during which maintenance may be scheduled (i.e., PM may not be planned nor performed while a system is operating). This was modeled with a parameter $\overline{z}_t^k$, defined for each $t \in \mathcal{T}$ and all $k \in \mathcal{K}$, which specifies whether PM is allowed to be scheduled for system k at time t or not. This parameter constitutes an input to the MS-PMSPIC model (see Obradović et al. (2022), Sec. 2.1.), and it provides upper bounds on the variables representing maintenance occasions. This is modeled as:

$$z_t^k \leq \overline{z}_t^k, \qquad t \in \mathcal{T},\ k \in \mathcal{K}. \tag{2}$$

2.2 Maintenance workshop scheduling

Components that require maintenance are sent to the *maintenance workshop*. The workshop consists of a number of (identical) repair lines/machines which are used for repair. A repair line has a capacity (per time step) of one unit. The repair of any component requires one repair line per time step throughout a predefined and consecutive (i.e., preemption[3] is not allowed) number of time steps. At the time when a component of type $i \in \mathcal{I}$ is taken out of a system it

[3] Preemption means activity splitting. If preemption is not allowed, an activity may not be interrupted and continued at a later point in time.

is assigned a job from the ordered set $\mathcal{N}_i := \{1, \ldots, N_i\}$. Once a component arrives at the workshop it is instantaneously available to be repaired, but it may have to wait if there is not enough spare capacity in the workshop at the given time step(s). Upon being repaired, a component is returned to the stock of repaired components (i.e., to the system operator). A maintenance workshop schedule specifies times at which components that arrive to the workshop should start maintenance, and on which repair line.

This problem is identified as a non-preemptive *identical parallel machines scheduling problem* (IPMSP); see (Brucker and Knust, 2012, Ch. 1.2.2). *Machine scheduling problems* (MSP) are special cases of the *resource-constrained project scheduling problem* (RCPSP) (Brucker and Knust, 2012, Ch. 1.1), which is one of the basic complex scheduling problems. Typically, the purpose of this type of problem is to schedule activities (preemptively or non-preemptively) over a planning period, while the resource capacities are respected and an objective function (or a few) is optimized.

Definition 2 (IPMSP) Consider a set $\mathcal{L} := \{1, \ldots, L\}$ of identical component repair machines and the (individual) components $j \in \mathcal{J}_i$ of types $i \in \mathcal{I}$ that arrive at the workshop. Repair times $p^i > 0$ are defined for each component type $i \in \mathcal{I}$. Each individual component is assigned a job id $n \in \mathcal{N}_i$ once it is to be processed in the maintenance workshop. At most $L \geq 1$ machines can operate simultaneously. A solution to this problem is a feasible maintenance workshop schedule.

The preemptive IPMSP with a (weighted) sum objective is polynomially solvable (Lawler et al., 1993, Ch. 8.0), whereas its version with a minimax, i.e., makespan, objective is NP-hard (Brucker and Knust, 2012, Ch. 2.1). According to Soper and Strusevich (2022), finding an optimal non-preemptive schedule on parallel machines with a makespan objective is NP-hard even for the case of two identical machines. Our workshop scheduling problem is a non-preemptive IPMSP with an objective (see Section 3) that is neither a (weighted) sum nor a makespan; hence we cannot with certainty conclude its computational complexity.

To model our non-preemptive IPMSP as a MILP, we define for each $n \in \mathcal{N}_i$, $i \in \mathcal{I}$, $l \in \mathcal{L}$, and $t \in \mathcal{T}$, the variables

$$u_t^{inl} = \begin{cases} 1, & \text{if a component of type } i \text{ starts maintenance at time } t \text{ as job } n \text{ in} \\ & \text{machine } l, \\ 0, & \text{otherwise,} \end{cases}$$

and, for each $t \in \mathcal{T}$, the number $\ell_t \in \mathbb{Z}_+$ of active parallel machines at time t, and model the constraints

$$\sum_{i\in\mathcal{I}} \sum_{n\in\mathcal{N}_i} \sum_{s=t-p^i+1}^{t} u_s^{inl} \leq 1, \qquad t \in \mathcal{T},\ l \in \mathcal{L}, \tag{3a}$$

$$\sum_{l\in\mathcal{L}} \sum_{t\in\mathcal{T}} u_t^{inl} \leq 1, \qquad n \in \mathcal{N}_i,\ i \in \mathcal{I}, \tag{3b}$$

$$\sum_{l\in\mathcal{L}}\sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}_i}\sum_{s=t-p^i+1}^{t} u_s^{inl} = \ell_t, \qquad t\in\mathcal{T}. \tag{3c}$$

The constraints (3a) state that each machine $l \in \mathcal{L}$ can process at most one job at each time step $t \in \mathcal{T}$, and that any job $n \in \mathcal{N}_i$ that starts processing in a machine l at a certain time step t, will occupy the machine until it is finished, at p^i time steps later, i.e., the scheduling is non-preemptive. The constraints (3b) make sure that each job $n \in \mathcal{N}_i$ is assigned to component type i at most once over all repair lines $l \in \mathcal{L}$ and all time steps $t \in \mathcal{T}$. To enable decision support for capacity investments in the maintenance workshop, we vary the capacity in the workshop (L). The constraints (3c) define the loading ℓ_t (i.e., the number of repair lines occupied) of the maintenance workshop at time step t. The constraints (3a) and (3c) together imply that the workshop loading ℓ_t cannot exceed the maximal number, L, of repair lines in the workshop. Note that there may be (many) equally good and symmetric solutions with respect to jobs. For that reason, we may activate the symmetry breaking provided by the solver[4].

IPMSP and the MS-PMSPIC are connected via the stocks of components. Thus, we further introduce the modeling of the stock of (damaged and repaired) components.

2.3 Stock dynamics

Upon taking a component out of a system, the component is instantly (i.e., there is no waiting time) sent to the stock of damaged components. Until the scheduled repair, the components remains on the stock. In between the stock of damaged components and the maintenance workshop, there is a transport time $\delta_a^i \geq 0$. Once the component is repaired, it is sent to the stock of repaired components with a transport time $\delta_b^i \geq 0$. Until the component is to be placed into a(ny) system again, it remains on the stock. The transport times are assumed to be integer.

Let us introduce the variables required for modeling the stock dynamics, for all $i \in \mathcal{I}$:

- a_t^i (b_t^i) : the number of individuals of component type i on the stock of damaged (repaired) components at time $t \in \mathcal{T} \cup \{0\}$;
- α_t^{ink} : 1 if an individual of component type i is taken out of a system $k \in \mathcal{K}$ at time $t \in \mathcal{T}$ and allocated to job $n \in \mathcal{N}_i$; 0 otherwise;
- β_t^i : the number of individuals of component of type i placed in any of the systems $k \in \mathcal{K}$ at time $t \in \mathcal{T}$.

[4] The Gurobi solver eliminates symmetries in the model, whence we do not need to include such constraints in the model.

The stock of damaged components is then defined, for all $i \in \mathcal{I}$, using following constraints:

$$\sum_{n \in \mathcal{N}_i} \alpha_t^{ink} = \sum_{s=0}^{t-1} x_{st}^{ik}, \qquad k \in \mathcal{K},\ t \in \mathcal{T}, \tag{4a}$$

$$\sum_{t \in \mathcal{T}} \sum_{k \in \mathcal{K}} \alpha_t^{ink} \leq 1, \qquad n \in \mathcal{N}_i, \tag{4b}$$

$$a_t^i - a_{t-1}^i = \sum_{n \in \mathcal{N}_i} \left(\sum_{k \in \mathcal{K}} \alpha_t^{ink} - \sum_{l \in \mathcal{L}} u_{t+\delta_a^i}^{inl} \right), \quad t \in \{1-\delta_a^i, \ldots, T+1\}, \tag{4c}$$

$$a_t^i \geq 0, \qquad t \in \{1-\delta_a^i, \ldots, T+1\}, \tag{4d}$$

The constraints (4a) connect the variables x_{st}^{ik} from model (1) of the MS-PMSPIC with the stock od damaged components: when a component of type i is taken out of a system $k \in \mathcal{K}$ at time t, the value of α_t^{ink} becomes 1 and the component is allocated to (exactly one) job n; otherwise $\alpha_t^{ink} = 0$ holds. The constraints (4b) make sure that a job n is allocated to at most one component i throughout the whole planning period $\mathcal{T}$ and over all systems $k \in \mathcal{K}$. The constraints (4c) define the level of components of type i on the stock of damaged components at time t; the number of components of type $i \in \mathcal{I}$ at time t depends on the number of the components (of the same type) in the previous time step $t-1$, whether components are taken out of any system k and placed on the stock at time step t, and whether they start maintenance at time step $t+\delta_a^i$. The constraints (4d) ensure that the stock level for component type i is non-negative at every time step t.

Due to the initialization (see constraints (6)), a_0^i and α_t^i, $t \in \{1-\delta_a^i, \ldots, 0\}$ are fixed and represent input data to our model.

The stock of repaired components is modeled, for all $i \in \mathcal{I}$, as

$$\beta_t^i = \sum_{k \in \mathcal{K}} \sum_{r=t+1}^{T+1} x_{tr}^{ik}, \qquad t \in \mathcal{T}, \tag{5a}$$

$$b_t^i = b_{t-1}^i - \beta_t^i + \sum_{n \in \mathcal{N}_i} \sum_{l \in \mathcal{L}} u_{t-\delta_b^i-p^i}^{inl}, \qquad t \in \mathcal{T} \cup \{T+1\}, \tag{5b}$$

$$b_t^i \geq \underline{b}^i, \qquad t \in \mathcal{T}. \tag{5c}$$

The constraints (5a) connect the stock of repaired components with model (1) of the MS-PMSPIC. When a component of type i is placed into a system k at time t, β_t^i takes value 1 (if there is $m \geq 0$ components that are simultaneously placed into m systems at time step t, then β_t^i takes value m). The stock level for component type $i \in \mathcal{I}$ at time t is defined in (5b); it depends on the level in the previous time step $t-1$, the number of components taken out of the stock of repaired components and placed in one of the systems k at time t, and the number of components arriving at the stock of repaired components at time t. The constraints (5c) ensure that the level of repaired components of type i may never go below the lower limit $\underline{b}^i \geq 0$ on the stock of repaired components.

Due to the initialization (see constraints (6)), b_0^i, β_0^i, and u_t^i, $t \in \{1 - \delta_b^i - p^i, \ldots, 0\}$ are fixed and represent input data to our model.

2.4 Boundary conditions

To model the boundaries (i.e., the beginning and the end of the planning period), we introduce the following constraints:

$$J_i = \sum_{k\in\mathcal{K}} \sum_{r\in\overline{T}} x_{0r}^{ik} + \bar{a}_0^i + \bar{b}_0^i + \sum_{r=-\delta_b^i-p^i+1}^{0} \sum_{l\in\mathcal{L}} \sum_{n\in\mathcal{N}_i} \bar{u}_r^{inl}, \qquad i \in \mathcal{I}, \tag{6}$$

$$b_t^i \geq b_0^i - \mu^i, \qquad i \in \mathcal{I},\ t \in \{T + 2 - \bar{s}, \ldots, T + 1\}, \tag{7}$$

We assign fixed values (randomized, or according to the systems' states at the respective time points) to the variables $a^i_{-\delta_a^i} := \bar{a}^i_{-\delta_a^i}$ and $b_0^i := \bar{b}_0^i \geq \underline{b}^i \geq 0$, such that the equalities (6) are satisfied, where the set $\overline{T}$ is defined as $\{\min_{i\in\mathcal{I}}\{1 - p^i - \delta_b^i\}, \ldots, \max_{i\in\mathcal{I}}\{p^i, \delta_a^i\}\}$. The resulting initial values (at $t = 0$) of the variables u_r^{inl}, α_t^{ink}, and a_t^i, for relevant indices are used to initialize the model at $t = 0$. In order to stabilize the level of the repaired components at $T + 1$ (i.e., the end of the planning period), the constraints (7) are defined, where $\bar{s} \geq 1$ is the number of time steps at the end of the planning period during which the tolerance level[5] $\mu^i \geq 0$ is applied to component type $i \in \mathcal{I}$.

Alternatively, one may impose a high penalty on the stock level at the end (or, for the last $\bar{s}$ time steps) of the planning period. This approach will show a similar effect as the constraints (7), but being less constraining.

2.5 The complete model of the system–of–systems

To summarize, the set of feasible solutions to the complete maintenance scheduling problem is modeled by (1)–(7). In addition, variables x_{st}^{ik}, z_t^k, u_t^{inl}, and α_t^{in} are required to be binary while a_t^i, b_t^i, β_t^i, and ℓ_t are to be non-negative and integer, for all relevant indices.

3 Definition of contracts and optimization objectives

The feasibility problem is defined in Section 2 and we next include the optimization objective functions. We define the costs for the two stakeholders and an 'availability of components' contract between the stakeholders. The problem is identified as a bi-objective optimization problem.

[5] If μ^i is set to zero, the requirement on the stock will probably increase over a few planning periods.

3.1 A bi-objective optimization problem

Costs. The two stakeholders face different costs. Namely, the system operator has to pay for the PM activities and the maintenance workshop has to pay penalties whenever not fulfilling the availability contract requirements, i.e., it faces costs for going below a predefined lower limit on the stock of available components; see Table 1.

Cost	Notation	Stakeholder
Preventive maintenance	$C^{\mathrm{PM}}(x, z)$	System operator
Availability penalty	$C^{\mathrm{AV}}(y)$	Maintenance workshop

Table 1: Costs faced by the two stakeholders

The preventive maintenance cost is defined as in Obradović et al. (2022), Sec. 3.2:

$$C^{\mathrm{PM}}(x, z) := \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} d_t z_t^k + \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}} \sum_{t=1}^{T+1} \sum_{s=0}^{t-1} c_{st}^i x_{st}^{ik}, \tag{8}$$

and it represents the total sum of the set-up and interval costs.

The availability penalty cost is defined as

$$C^{\mathrm{AV}}(y) := \sum_{i \in \mathcal{I}} c_i^{\mathrm{AV}} \sum_{t \in \mathcal{T}} y_t^i, \tag{9}$$

where there is a non-negative cost c_i^{AV} for every unit y_t^i of number of components at time step t that go below a certain limit $\underline{\underline{b}}^i \geq 0$ on the stock of available components.

Bi-objective optimization problem. Whether the operational schedule is disturbed (and to which extent) or not, is partly affected by the availability of components on the stock of repaired components. In the event of an unexpected failure, the damaged component can be replaced with no/minimal interruptions in the planned operations, as long as there is a component of the same type available. Also, in order to enable the planning of an efficient PM schedule, it is essential that there is (almost) always enough spare parts on the stock.

For the availability contract type, we model the bi-objective optimization problem as to

$$\underset{x,y,z,u,a,\alpha,b,\beta,\ell}{\text{minimize}} \quad \left[C^{\mathrm{PM}}(x, z),\ C^{\mathrm{AV}}(y)\right], \tag{10a}$$

$$\text{subject to} \quad (1)\text{–}(7), \tag{10b}$$

$$y_t^i \geq \underline{\underline{b}}^i - b_t^i, \qquad i \in \mathcal{I},\ t \in \mathcal{T}, \tag{10c}$$

$$y_t^i,\ b_t^i \geq 0, \qquad i \in \mathcal{I},\ t \in \mathcal{T}, \tag{10d}$$

where the inequalities (10c)–(10d) define y_t^i as the measure of how much the stock level b_t^i for component type i at time t falls below the lower limit $\underline{\underline{b}}^i$ on the stock of available components. The objective (10a) is to minimize the maintenance cost (8) subject to the stock level not going below $\underline{b}^i$, and the penalty (9) for the stock level going below $\underline{\underline{b}}^i$.

3.2 Complexity analysis

We next analyze the computational complexity of the model (10). By defining special cases of the single-objective minimization problems (1)–(7), (8) and (1)–(7), (9), (10c), (10d) both problems are reduced to the PMSPIC, which is an NP-hard problem. Thereby, the bi-objective problem (10), after (any) scalarization (Ehrgott, 2005, Sec. 8.3), is reduced to an NP-hard problem.

Theorem 1 (Complexity) *The complete model of the system–of–systems* (1)*–*(7)*, with either of the objective functions* (8) *or* (9)*, binary requirements on the variables x_{st}^{ik}, z_t^k, u_t^{inl}, and α_t^{in}, and non-negativity and integer requirements on the variables y_t^i, a_t^i, b_t^i, β_t^i, and ℓ_t, for all relevant values of the indices, is NP-hard.*

Proof Consider the constraints (1)–(7), with the relevant binary, non-negativity, and integer requirements on the variables. Assume that the capacity of the maintenance workshop equals the total number of jobs, i.e., that $L = \sum_{i\in\mathcal{I}} N_i$ holds. Moreover, assume that for each component type $i \in \mathcal{I}$, the number of individual components fulfills $J_i \geq K \left\lceil \frac{T+1}{1+\delta_a^i+p^i+\delta_b^i} \right\rceil$ (where 1 in the denominator represents the shortest time a component spends in a system k and $\delta_a^i + p^i + \delta_b^i$ is the shortest possible turn–around time for component type i), that $\bar{b}_0^i = J_i - K$, and that $\underline{\underline{b}}^i = 0$. Then, each repair job can always be instantly performed in the workshop and there will always be a (repaired) component in stock for replacement.

The problem (1)–(7), with the objective to minimize (8), is hence reduced to the minimization of (8) subject to (1), which separates into one instance of the PMSPIC for each of the systems $k \in \mathcal{K}$. As stated in Definition 1 and the reasoning thereafter, the PMSPIC is NP-hard; see also (Gustavsson et al., 2014). Therefore, there exists an instance of the problem of minimizing (8) which is NP-hard.

Now, consider the problem (1)–(7), (10c), (10d) with the objective to minimize (9). Since $\underline{\underline{b}}^i = 0$, the problem separates over minimizing (9) subject to $y_t^i \geq 0$ for all relevant i and t, and minimizing a zero objective subject to (1)–(7) and $b_t^i \geq 0$ for all relevant i and t. Minimization of (9) subject to $y_t^i \geq 0$ for all relevant i and t results in $y_t^i = 0$, for all relevant i and t (for all non-negative cost coefficients) and there are no constraints involving y (or any of other) variables. For the former problem, setting all y variables to zero will be an optimal solution, such that the optimal value of (9) will equal zero (that is, $C^{\mathrm{AV}}(y^*) = 0$ for any optimal solution $(x^*, z^*, u^*, \ldots, y^*)$ since $y^* = 0$ is optimal whenever $\underline{b}^i = 0$). The latter problem is reduced to the problem of minimizing (8) subject to (1)–(7), with the costs $c^i_{st} = 0$ and $d_t = 0$, for all relevant indices. Hence, there exists an instance of the problem of minimizing (9) which is NP-hard.

We conclude that the bi-objective problem (10) is NP-hard. □

Based on this theorem, we conclude that the bi-objective MILP (10) is a computationally demanding problem.

4 Application: Implementation, tests, and results

The industrial application we present comes from a collaboration with the Swedish aerospace and defence company Saab AB. For the purpose of assessing contracting forms between the stakeholders and aiding the decision making processes, the instance sizes are considered to be reasonable from a practical application point of view and the data sets used are based on knowledge mediated from the industrial partner. All data that is used is normalized.

The implementation is made using Julia (2012) and JuMP (Dunning et al., 2017), and the computations are performed by Gurobi (2020) on a laptop computer with a 2.4 GHz Intel Core i5 processor and 8 GB of RAM memory. The computer used has one processor with four cores, supporting in total maximum of eight threads.

4.1 The main test instances and multi-objective settings

We consider $K = 10$ systems, each of which has $I = 5$ component types and $J_i = 25$ (individual) components of each type $i \in \mathcal{I}$. The operational and maintenance related differences of the component types are reflected by their respective repair times in the maintenance workshop. The same holds for the randomly chosen (within the same order of magnitude) due dates. Moreover, different component types are also assigned differently structured interval costs which are increasing with the time between two (consecutive) maintenance occasions. This cost structure reflects an increasing risk of unexpected failures, whence CM. The planning horizon is $T = 40$ time steps and the workshop capacity is limited to either $L = 25$ or $L = 30$ parallel machines. The maximal number of jobs[6] used for component type i during a planning period $\mathcal{T}$ is $N_i := J_i \left\lceil \frac{T}{p^i + \delta^i_b + \delta^i_a + \underline{t}_i} \right\rceil$, where $\underline{t}_i$ (:= 1) equals the minimum number of time steps each individual component can spend in any of the systems. The processing times take values $p^i \in \{3, 4, 5\}$, $i \in \mathcal{I}$ and the transport times between the

[6] The number of variables in the model is (approximately) proportional to the maximal number of all jobs, i.e., $N := \sum_{i \in \mathcal{I}} N_i$; hence the problem size grows with number of jobs.

stocks and the maintenance workshop are $\delta_a^i = 2$ and $\delta_b^i = 1$ for $i \in \mathcal{I}$. Lower limits on the stock of repaired components are $\underline{b}^i = \underline{\underline{b}}^i = 1$ for $i \in \mathcal{I}$. The maintenance costs are $d_t = 5$, for all $t \in \mathcal{T}$ and c_{st}^i is varied: the smallest value (for the maintenance interval length of 1) is 5 for all $i \in \mathcal{I}$ while the cost for the largest allowed maintenance interval length varies for different component types $i \in \mathcal{I}$ and is in the range of $20 : 50$. Availability penalty costs take values $c_i^{AV} \in \{5, \dots, 10\}$. The stabilizing tolerance levels in (7) are chosen as $\mu^i := \{2, 5\}$ and the number of time steps during which they are applied is $\bar{s} := 1$. The input to the model (i.e., the timetable for the systems' operations) is obtained as a result of the model in (Gavranis and Kozanidis, 2015) applied to the set $\mathcal{K}$ of systems over the planning period $\mathcal{T}$.

4.2 Computational tests and results

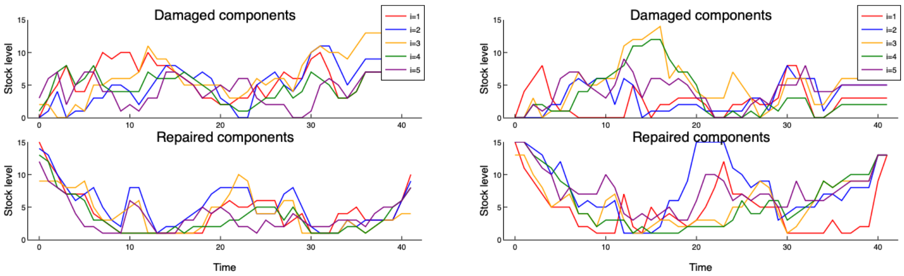
4.2.1 The case without a contract between the stakeholders

We first consider the case without a contract between the stakeholders, that is, the model considered is constituted by the formulas (1)–(8), and where the preventive maintenance cost (8) is minimized. The instance sizes and solution times are presented in Table 2. We observe how the computing times grow with increasing instance sizes (in all cases except when there is added capacity in the workshop, i.e., an increased value of L, and the other parameters are unchanged). In this section, we analyze in more detail two of the instances that differ in the workshop capacity, which is varied between $L = 25$ in the first instance and $L = 30$ in the second.

Instance size					# rows	# variables	# binary variables		gap	solution	presolve
I	J_i	K	T	L			original	after presolve	[%]	time [s]	time [s]
3	20	8	40	20	51 763	213 857	213 144	98 804	0.00	4.31	1.60
3	25	8	40	20	58 103	256 977	256 264	127 332	0.82	19.00	1.62
5	25	10	40	25	117 989	533 109	531 950	261 785	0.61	245.00	4.36
5	25	10	40	30	126 937	598 107	596 950	298 051	1.61	110.84	3.91
5	25	10	60	30	210 925	1 248 073	1 246 400	756 087	6.02	331.00	12.52
5	35	10	60	30	282 630	1 703 483	1 701 800	1 022 664	1.39	1679.00	18.09
5	35	5	100	30	370 818	3 680 363	3 677 650	2 798 247	5.91	4056.85	60.92

Table 2: Instance sizes and computing times. The lower limit on the stock of available components is $\underline{b}^i = 1$, $i \in \mathcal{I}$, for all instances considered.

Figure 2 shows the levels of the stocks of components over the planning period for the two instances considered. In the beginning of the planning period the stocks of repaired components possess fairly high levels and, due to the stabilization constraints (7), similar (up to the tolerance μ^i) stock levels are maintained at the end of the planning horizon. By employing the stabilizing constraints, the good state of the stocks of repaired components at the end of each planning period is maintained. As can be expected, the observed levels of the stocks of repaired components are higher when the workshop capacity is higher, i.e., for $L = 30$.



(a) $L = 25$ (b) $L = 30$

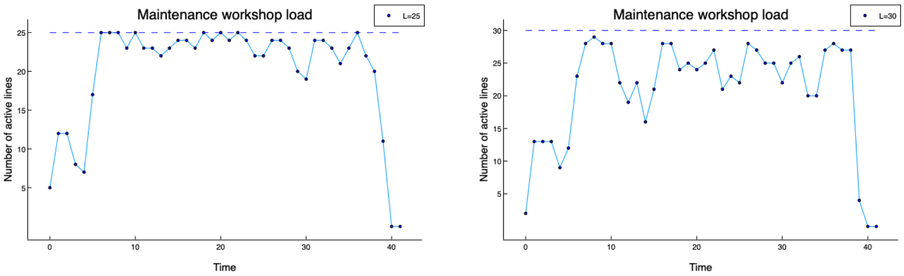
Fig. 2: Resulting stock levels over time when maintenance costs are minimized

Figure 3 shows the loading of the maintenance workshop over the planning period for the two instances considered. Due to higher levels on the stock of the repaired components in the beginning of the planning period (see Figure 2), the demand for repairing the components is lower for the first few time steps. The same effect can be observed in the maintenance workshop schedules (see Figure 4). We can see that the capacity in the maintenance workshop (i.e., number of repair lines) is less constraining in the case of $L = 30$ than for $L = 25$. However, in the case of $L = 25$, the workshop loading is almost at the full capacity during most time steps. A constantly high loading in the workshop would leads to an increased risk of the whole system–of–systems being inoperational (a lack of capacity of component repairs will eventually lead to an inability to do replacements of components in the systems).

Figure 4 shows the maintenance workshop schedules[7] for the two instances. As discussed above, the workshop schedules more jobs per repair line in the case of $L = 25$ as compared to the case of $L = 30$ repair lines, due to smaller number of repair lines available at each time step. As a result, there is less possibility for scheduling repairs in the workshop (represented by white space in the maintenance workshop schedule) when $L = 25$ as compared to an increased workshop capacity, when $L = 30$.

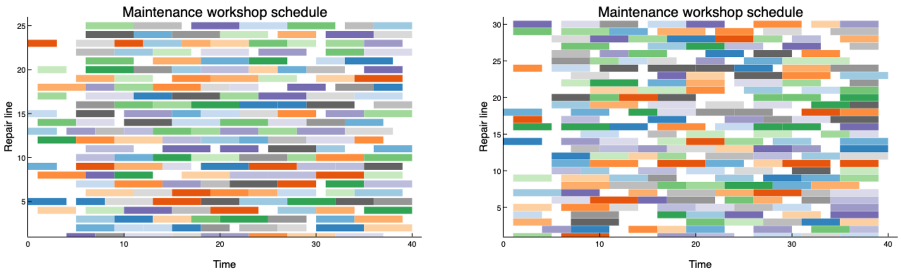
Figure 5 shows the average proportion of the component life span that is unutilized (i.e., a measure of how long before the end of the component's life span a component undergoes a maintenance activity) over the whole planning period. The unutilized life can be viewed as wasted life, which implies a loss

[7] In Figure 4 each job is assigned a color, but jobs are unique and do not repeat, even though colors do.



(a) $L = 25$ (b) $L = 30$

Fig. 3: Maintenance workshop loading over time when maintenance costs are minimized



(a) $L = 25$ (b) $L = 30$

Fig. 4: Maintenance workshop schedule when maintenance costs are minimized

of invested resources. With a higher capacity in the workshop ($L = 30$) we observe that there is more unutilized component life. This is justified by the fact that the workshop possesses a higher capacity for components' repair, which means that components can (and will) be repaired more often, on average. The average utilization when $L = 25$ is 73 %; it decreases to 71 % for $L = 30$. Hence, there is a trade-off between decreasing the risk of component failure and increasing its utilization.

4.2.2 The case of an availability of components contract between the stakeholders

We next consider an availability contract between the stakeholders, that is, the bi-objective mathematical model studied is constituted by the constraints and objectives in (1)–(10).

As we have a bi-objective optimization problem (10), we are interested in finding Pareto optimal solutions; see e.g., (Ehrgott, 2005, Ch. 2.1). A solution is called Pareto optimal if none of the objective functions can be improved in

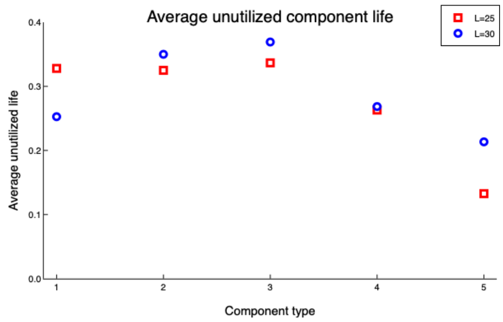
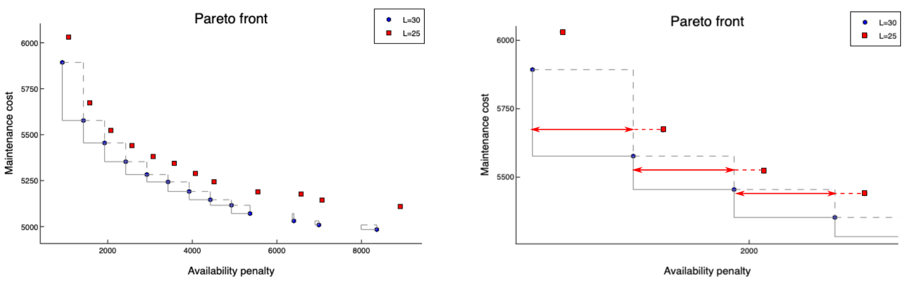


Fig. 5: Average unutilized (i.e., wasted) component life when maintenance costs are minimized for $L = 25$ and $L = 30$ repair lines

value without degrading at least one of the other objectives' values. In order to find (the set of all) Pareto optimal solutions, we utilize the ϵ-*constraint method* (Mavrotas, 2009), which (in the bi-objective case) iteratively optimizes one objective function and constrains the other. In this application, we minimize the maintenance cost objective and include an ϵ-constraint on the availability penalty objective, with equidistant values of ϵ.



(a) Computed points on the Pareto fronts for the workshop capacity $L \in \{25, 30\}$ and areas of uncertainty of the optimal objective values for $L = 30$

(b) Uncertainty interval of the availability penalty reduction, for an increased workshop capacity; a zoom-in

Fig. 6: Illustration of the possible reduction of the availability penalty for an increased maintenance workshop capacity from $L = 25$ to $L = 30$ repair lines. The Pareto front represents the availability penalty objective (9) vs. the maintenance cost objective (8). Parameter values: $(I, J_i, K, T, \underline{b}^i, \underline{\underline{b}}^i) = (5, 15, 10, 40, 5, 5)$; $L \in \{25, 30\}$; $\epsilon = 500$ in the ϵ-constraint method.

Figure 6(a) shows the points found on the Pareto front in the bi-objective problem, with the objectives (8) and (9), for $L \in \{25, 30\}$. An area of uncertainty is defined (by an open rectangular set) for each point on the Pareto front corresponding to $L = 30$; it represents the area between two computed points on the Pareto front and which may contain additional Pareto points. For $L = 25$ ($L = 30$), the availability penalty (defined in (9) as the sum over the planning period of penalties for going below a certain level) is in the interval $[1073, 8917]$ ($[925, 8362]$) while the total maintenance cost is in the interval $[5109, 6031]$ ($[4984, 5893]$). In order to reduce the availability penalty (i.e., keeping the stock of available components above or reducing the distance to $\underline{\underline{b}}^i$, $i \in \mathcal{I}$), one could perform less PM activities on the system operator's side, which would yield higher maintenance costs. On the other hand, performing PM regularly (i.e., before the component's life span, at which the risk for failure increases) increases the demand for repaired components, which leads to an increased difficulty to maintain high levels on the stock of repaired components (i.e., the availability penalties likely increases). We set $\underline{\underline{b}}^i_t = 5$, $i \in \mathcal{I}$, and, according to (10), penalize for going below $\underline{\underline{b}}^i_t$ on the stock of available components.[8]

As we have two stakeholders whose objective functions are both minimized, we can analyze improvements in values of those two objectives when the capacity of the maintenance workshop is increased. Figure 6(b) illustrates the uncertainty intervals (red arrows) for the reduction of the availability penalty when the number of repair lines in the maintenance workshop is increased from $L = 25$ to $L = 30$. A reduction of the availability penalties corresponds to an increased capability to maintain the desired level on the stock of repaired components. For each red point on the Pareto front (corresponding to $L = 25$) there is an uncertainty interval for the reduction of the availability penalty for the same (fixed) maintenance cost level. Increasing the number of repair lines to $L = 30$ leads to the availability penalty being in the corresponding interval.

Table 3 lists the possible reductions in maintenance cost and availability penalty when the maintenance workshop capacity is increased from $L = 25$ to $L = 30$ repair lines. The numbers are illustrated in Figure 6(a), where the weakly Pareto optimal points[9] are removed. For each point on the Pareto front for $L = 25$, we define a maximal possible reduction of maintenance cost and an availability reduction interval, when increasing the number of repair lines to $L = 30$. The availability reduction interval is defined by the smallest and largest possible reductions of the availability penalty when L is increased. Note that the largest possible reduction in the availability penalty increases as the maintenance cost decreases.

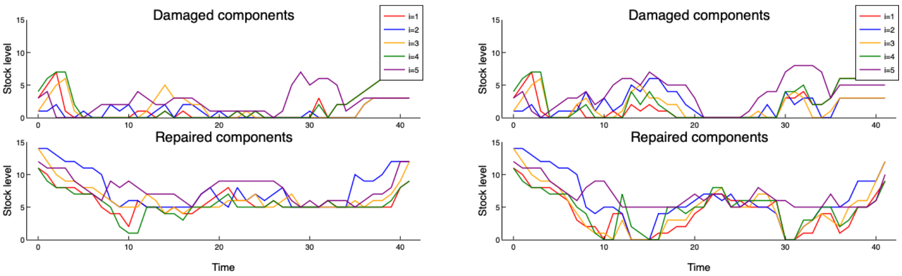
We will now take a closer look at the two extreme solutions on the Pareto front for $L = 30$: the left-most point (minimizing the availability penalty) having a maintenance cost of 5227 and an availability penalty of 962, and the right-most point (minimizing the maintenance cost) with the maintenance cost of 4821 and the availability penalty of 11359. Figure 7 shows the levels

[8] When the contract in place is the availability contract (10), the constraints (5c) are removed, while penalties for the stock going below a certain level are included.

[9] A Pareto point is *weakly Pareto optimal* if it does not improve all objective functions in the multi-objective optimization setting.

Maintenance cost ($L = 25$)	Maximum possible reduction of maintenance cost ($L = 25 \Rightarrow L = 30$)	Availability penalty ($L = 25$)	Availability penalty reduction interval ($L = 25 \Rightarrow L = 30$)
6 031	454	1 073	[148 , 148]
5 673	218	1 573	[148 , 648]
5 523	170	2 073	[148 , 648]
5 441	158	2 573	[148 , 648]
5 381	138	3 073	[148 , 1 148]
5 344	153	3 573	[148 , 1 148]
5 289	143	4 073	[148 , 1 648]
5 244	128	4 514	[89 , 1 589]
5 189	118	5 550	[185 , 1 125]
5 177	146	6 573	[172 , 2 648]
5 144	135	7 067	[74 , 2 664]
5 109	125	8 917	[555 , 3 992]

Table 3: Maximum possible reduction of maintenance cost and reduction of availability penalty intervals for each Pareto point corresponding to $L = 25$ when the workshop capacity is increased to $L = 30$.



(a) Left-most point on the Pareto front (minimal availability penalty)

(b) Right-most point on the Pareto front (minimal maintenance cost)

Fig. 7: Resulting stock levels over time for $L = 30$.

on the stocks of damaged and repaired components over the planning horizon when the workshop capacity is $L = 30$. Prioritizing the minimization of availability penalties (i.e., the left-most point on the Pareto front) leads to overall higher levels on the stock of available components. In return, the maintenance intervals grow longer and the maintenance costs increase.

Figure 8 shows a significant difference in the utilization of components lives for the two extreme Pareto optimal points. In the left-most point (minimizing the availability penalty) the average utilization is about 79%, while in the right-most point (minimizing the maintenance cost) it is around 73%. This is explained by the fact that when prioritizing the minimization of the maintenance cost (i.e., the right part of the Pareto front), components are replaced before the end of their life is reached, thus reducing the average component utilization. On the other hand, prioritizing the minimization of the availability

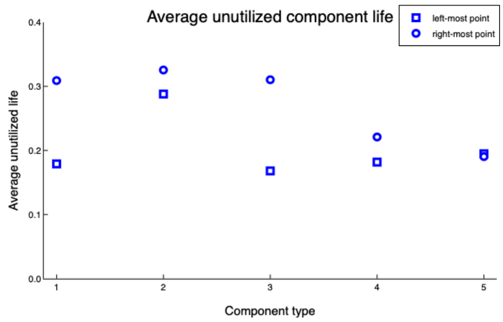


Fig. 8: Average unutilized (i.e., wasted) component life over the planning horizon for the left-most (minimal availability penalty) and right-most (minimal maintenance cost) Pareto point for $L = 30$.

penalty (i.e., the left part of the Pareto front) leads to longer intervals between two maintenance occasions and, thereby, higher utilization rates.

5 Conclusions and Future Research

We present a mixed-integer optimization (MILP) model for PM scheduling and non-preemptive maintenance planning. The model is, in a bi-objective setting, further utilized to optimize the availability contract. Our results measure the interplay between the workshop capacity and the level of component availability, as well as the corresponding cost trade-off between the stakeholders. We also analyze how the average component utilization rate depends on the workshop capacity as well as on the different optimization objectives.

This model allows an investigation of the availability contract. Further, the model can be utilized as a planning tool when the maintenance workshop and the system operator are integrated; both with an availability contract and without one. As a result of this work, we can find the threshold levels for having spare capacity in the workshop, we can asses the stock level dynamics, as well as the risk levels of potential corrective maintenance costs based on the utilization rate. Moreover, this modeling can be used as a decision support tool for the stakeholders.

We define and analyze one form of an availability contract between the stakeholders. Currently, one way to reduce the risk for unexpected failures (i.e., the need for CM) is to not allow too large maintenance intervals. Hence, another interesting research question would be to incorporate the corrective maintenance operations and costs in our modelling. As we use a fairly simple model for the maintenance workshop, another improvement of this work would be to use more complex model of the maintenance workshop (e.g., a flow-shop model), which would make the whole model more representative for a real scenario of the system–of–systems. Another future goal is to develop a tailored and more efficient solution method, to enable the solution of larger instances.

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Statements and Declarations

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