Robust Service Network Design under Travel Time Uncertainty: Formulations and Exact Solutions

Shengnan Shu
Faculty of Business for Science and Technology, School of Management, University of Science and Technology of China, shengnan.shu@connect.polyu.hk

Zhou Xu
Department of Logistics and Maritime Studies, Hong Kong Polytechnic University, lgtzx@polyu.edu.hk

Jin Qi
Department of Industrial Engineering and Decision Analytics, Hong Kong University of Science and Technology, jinqi@ust.hk

We study the continuous-time service network design problem (CTSNDP) under travel time uncertainty, aiming to achieve robust operational efficiency even in the presence of deviations in travel time. Incorporating travel time uncertainty poses a significant challenge, as the time-indexed mixed-integer linear programming (MILP) formulations commonly used for the CTSNDP with deterministic travel times become impractical. To tackle this challenge, we first propose a new consolidation-indexed MILP formulation for the deterministic CTSNDP. This enables us to derive a two-stage robust optimization model and a two-stage robust satisficing model that can produce solutions capable of mitigating the impact of uncertainty in travel time, without precise knowledge of the joint probability distribution of travel times. Exact optimal solutions for these two robust models can be obtained using two tailored column-and-constraint generation algorithms. Our computational results demonstrate the efficiency of both these algorithms, the tractability of the proposed formulations, and the trade-offs involved in achieving robust solutions.

Key words: service network design; continuous time; travel time uncertainty; transportation; robust optimization; robust satisficing; exact algorithm; column-and-constraint generation

1. Introduction

In the transport industry, a substantial proportion of freight is moved by consolidation carriers, such as less-than-truckload motor carriers and express postal service providers. These carriers transport shipments that are small compared to vehicles' capacities, requiring the consolidation of shipments to achieve cost-efficiency. This leads to a service network design problem (SNDP).

The SNDP involves routing shipments from their origins to destinations through a network of terminals, where shipments can be transferred from inbound vehicles to outbound vehicles. Each shipment has a time available for departure from its origin and a time when it is due to arrive at its destination. To transport shipments between terminals, vehicles with limited capacities are needed. At each terminal, when multiple shipments are consolidated, their transportation cannot start until all shipments have arrived and become available at the terminal. Accordingly, a classic SNDP seeks to determine routing and consolidation plans for the shipments, along with the departure schedules of the direct shipping services required for each terminal-to-terminal movement, so that the times when shipments are available and due are met, minimizing the total operational cost.

The classic SNDP and variants of the SNDP have been studied extensively, because of their wide application and theoretical significance, as evidenced by recent review studies such as Crainic and Hewitt (2021), Crainic and Rei (2025), Crainic (2024a,b). However, existing studies primarily focus on *deterministic* variants, assuming that all the problem parameters, such as shipment quantities and travel times, are known in advance. To model deterministic variants, a widely used technique is *discretization*, which discretizes the planning horizon into a number of time points. Using these time points, a deterministic SNDP can be modeled on a time-expanded network, in which each node represents a combination of a time point and a terminal, and each arc connecting two time points represents a shipment's movement between or waiting at terminals during a specific period. The time-expanded network constructed from the discretization can be used effectively to incorporate decisions of the SNDP. Based on this, a mixed-integer linear programming (MILP) formulation can be established. It is *time-indexed*, since the decision variables involved are indexed by the time points of a pair of time-space nodes. The time-indexed MILP formulation can be solved by commercial optimization solvers and received significant attention (Crainic and Hewitt 2021).

However, the time-indexed MILP formulation is only an approximation of the SNDP, where the planning horizon is continuous and vehicles can be dispatched at any time. Achieving high-quality solutions requires fine discretization, which often leads to a large and typically intractable time-indexed MILP formulation of the SNDP. To solve the continuous-time variant of the SNDP (or *CTSNDP* in short), Boland et al. (2017) proposed a Dynamic Discretization Discovery (DDD) algorithm, which iteratively adjusts the discretization level until it reaches an optimal solution. This exact algorithm was later enhanced by Marshall et al. (2021) and Shu et al. (2024).

The formulation and solution algorithms introduced above are only applicable to deterministic variants of the SNDP. By contrast, this paper focuses on the developing an exact solution algorithm for a new variant of the CTSNDP that incorporates uncertainty in travel time. It is an under-explored and challenging task that has significant practical value. Actual travel times for transportation services are often unknown in advance, because of factors such as weather conditions and traffic congestion (Bertsimas et al. 2019, Nikolova and Stier-Moses 2014, Jaillet et al. 2016). This leads to delays and late deliveries, resulting in disruptive impacts, such as penalties and customer dissatisfaction (Lanza et al. 2021).

Despite its significance, uncertainty in travel time has rarely been considered in the design of transportation service networks. Existing studies that incorporate uncertainty in SNDP mainly focus on demand uncertainty, either using a stochastic optimization approach to optimize the average performance (Wang et al. 2019), or a robust optimization approach (Atamtürk and Zhang

2007, Wang and Qi 2020) to optimize the worst-case performance. When departure schedules are involved, existing solution methods all use the time-indexed formulation to incorporate uncertain demands. This is achievable because the planning horizon for these problems can be discretized into a small number of time points, and even with demand uncertainty, all decision variables retain the same time indices as those in the time-indexed formulation of the deterministic SNDP.

Incorporating uncertain travel times for the CTSNDP poses a significant challenge, as the time-indexed formulation of the deterministic SNDP becomes impractical. When travel times are uncertain, the time-indexed formulation needs to incorporate decisions made after travel times are realized. Therefore, in a time-indexed formulation, separate variables for these decisions must be defined with different time indices for different possible realizations of travel times. When the planning horizon is continuous, both the number of possible travel time realizations and the number of possible time indices are infinite, making the solution of such a time-indexed formulation challenging. Even when the planning horizon is discretized into a number of time points, the number of possible travel time realizations can be exponentially large. It remains challenging to solve time-indexed formulation unless it includes only a limited number of time points after the discretization and incorporates a restricted subset of possible travel time realizations, which, however, sacrifices model accuracy and solution quality significantly. Due to this complication, exact solution methods based on the time-indexed formulation for the deterministic CTSNDP, such as the DDD algorithm, cannot be applied to the CTSNDP effectively under travel time uncertainty.

This study tackles the challenge of formulating and solving a robust CTSNDP under travel time uncertainty. It aims to design a transport service network that maintains reliable operational efficiency, even in the presence of travel time deviations and without knowledge of the actual distribution of travel times. We first propose *a new consolidation-indexed MILP formulation for the deterministic CTSNDP*, eliminating the need for time indices. This allows us to derive *tractable MILP formulations for both a two-stage robust optimization model and a two-stage robust satisficing model for the robust CTSNDP*. These robust models will enable decision makers to align their preferences regarding the robustness guarantees of the solutions obtained and the trade-offs involved.

To solve the robust optimization model and the robust satisficing model, we develop two exact algorithms, respectively. They both follow a column-and-constraint generation (C&CG) framework, which has been applied widely to two-stage robust optimization models (see, e.g., Zeng and Zhao 2013, Wang and Qi 2020), but has never been applied to two-stage robust satisficing models before. The critical step in our C&CG algorithms is its solution to a subproblem that finds the worst-case realized travel times for any given first-stage solution, for which we need to analyze and utilize the *optimality properties* of the subproblem. Through extensive computational experiments, we demonstrate the tractability of our proposed models, the effectiveness of our developed algorithms, and the trade-off involved in achieving the robustness guarantees.

Below, we present a literature review in Section 2, and introduce the robust CTSNDP under travel time uncertainty in Section 3. Based on our new formulation for the deterministic CTSNDP in Section 4, we derive the robust optimization model and a robust satisficing model in Section 5, for which we develop and analyze our C&CG algorithms in Section 6. The computational results are discussed in Section 7, followed by a conclusion in Section 8. All notation is summarized in Table 4 of Appendix A and all proofs are presented in Appendix B.

2. Literature Review

Our study is closely related to research into service network design, particularly regarding the integration of travel time uncertainty, as well as studies focused on robust solutions.

Despite its great importance, uncertainty in travel time has rarely been considered in existing studies of SNDP. Of the studies that do consider it, almost all have overlooked delay propagation (e.g., Yao et al. 2014, Zhao et al. 2018, Liang et al. 2019, Lanza et al. 2021), where a delay in one shipment caused by travel time uncertainty triggers delays in other shipments awaiting consolidation at a terminal, leading to consequent delays at other terminals. The exceptions are Demir et al. (2016), Hrušovskỳ et al. (2018), Layeb et al. (2018), which focus on a restricted SNDP where services can only be selected from a small candidate set. In these three studies, along with several others that do not consider consolidation delay propagation (Lanza et al. 2018, 2021), travel time uncertainty is incorporated by adopting a stochastic optimization approach to optimize the average performance. However, this requires knowledge of the joint probability distribution of all travel times, which is often not available. Because of the large number of decision variables and many possible realizations of uncertain factors, solving the stochastic optimization model derived from this approach to optimality is challenging. Therefore, these studies only apply heuristic methods based on simulations or limited travel time samples. Our paper is the first to adopt a robust optimization approach to the CTSNDP under travel time uncertainty, with consideration of consolidation delay propagation and without knowledge of the travel time distribution.

The robust optimization approach only requires a distribution-free uncertainty set that defines the possible realizations of uncertain factors (Bertsimas et al. 2011). It relaxes the need for precise information for distributions, and often leads to an optimization formulation with a tractable reformulation that can be solved efficiently, thereby addressing the critical limitations of the stochastic programming methods currently applied in CTSNDP under travel time uncertainty. In the classic robust optimization approach, the objective is to optimize the worst-case situation over different realizations of uncertain factors. The robust optimization approach has been applied widely to various transportation problems involving time-related uncertainties (Agra et al. 2018, Rodrigues et al. 2019) and network design problems involving demand and cost uncertainties (Atamtürk and Zhang 2007, Wang and Qi 2020, Pessoa and Poss 2015, Altın et al. 2011). This motivates us to explore its application to the CTSNDP under travel time uncertainty.

In a recent study, Long et al. (2023) propose a robust satisficing approach for optimization problems under uncertainty, aiming to find a solution that best achieves a prescribed target of the objective value, minimizing the worst-case normalized violation from the target. Unlike the conventional robust optimization which restricts the possible realizations of uncertain factors to within a pre-specified uncertainty set, the robust satisficing framework allows the nature to choose realization in the whole space. It has also been demonstrated through several applications to have the advantage of improving out-of-sample performance over the classic robust optimization approach (Zhou et al. 2022, Cui et al. 2023). Therefore, while the classic robust optimization approach is often considered conservative because it emphasizes the worst-case objective value, the robust satisficing approach provides a less conservative alternative method to tackle the CTSNDP under uncertainty, with this study serving as an initial exploration.

To apply the robust optimization and robust satisficing approaches, we need to incorporate travel time uncertainty into the CTSNDP formulation. As discussed earlier, this is challenging, since the time-indexed formulations commonly used for deterministic CTSNDP become impractical. Moreover, the MILP formulations presented in Demir et al. (2016), Hrušovskỳ et al. (2018), and Lanza et al. (2024) for the SNDP consist of decision variables with service indices, which are essentially time-indexed, as services are defined by their departure and arrival times. Hewitt and Lehuédé (2023, 2025) use a consolidation-based MILP formulation for the deterministic freight transportation network scheduling problem without time indices. However, defining its decision variables requires time-consuming enumeration of all possible shipment combinations for consolidation on each transportation movement, which can generate a large number of variables and create substantial computational challenges, especially under travel-time uncertainty where delays are allowed and time-window restrictions for feasible consolidations are relaxed. By contrast, our study proposes a new compact formulation of the deterministic CTSNDP without time indices, which always maintains a polynomial number of decision variables and can be extended to incorporate travel time uncertainties while ensuring the tractability of the solution.

3. Problem Description

The robust CTSNDP in this study extends the problem setting of the deterministic CTSNDP by incorporating uncertain travel times. Therefore, we first describe the deterministic CTSNDP in Section 3.1, followed by the robust CTSNDP in Section 3.2.

3.1. Deterministic CTSNDP

Our description of the deterministic CTSNDP is based on the problem setting presented in Boland et al. (2017), but extends it by incorporating shipment holding costs.

Consider a network $\mathcal{D} = (\mathcal{N}, \mathcal{A})$ with a physical node set $\mathcal{N}$ and a directed arc set $\mathcal{A}$, which is referred to as the *flat network*. Each physical node in $\mathcal{N}$ represents a terminal. Each directed arc in $\mathcal{A}$ represents a possible transportation movement from one terminal to another, which can have multiple direct shipping services with different departure times. Consider a commodity set $\mathcal{K}$, where each commodity $k \in \mathcal{K}$ represents a shipment, with its origin denoted by $o^k \in \mathcal{N}$, its destination denoted by $d^k \in \mathcal{N}$, and its shipping quantity denoted by $q^k \in \mathbb{N}_{>0}$. Each commodity $k \in \mathcal{K}$ has an earliest available time $e^k \in \mathbb{N}$ for departure from its origin o^k, and has a due time $l^k \in \mathbb{N}_{>0}$ for arrival at its destination d^k. Each commodity k must be delivered via exactly one shipping path in $\mathcal{D}$ with splitting not allowed, ensuring that it is picked up from the origin o^k after the earliest available time e^k, and delivered to the destination d^k before the due time l^k. To satisfy this, it must use direct shipping services for all arcs along the shipping path, with each service requiring a sufficient number of capacitated vehicles. Along its shipping path, each commodity k can be stored temporarily at any node, allowing consolidation with other commodities for shipping together on various arcs.

In the flat network $\mathcal{D}$, each directed arc $(i, j) \in \mathcal{A}$ is associated with four attributes: (1) a static travel time $\tau_{ij} \in \mathbb{N}_{>0}$ for each transportation movement on this arc; (2) a per-unit-of-flow (travel) cost $c_{ij}^k \in \mathbb{R}_{>0}$ for each commodity $k \in \mathcal{K}$; (3) a capacity $u_{ij} \in \mathbb{N}_{>0}$ per vehicle for (shipping) service from i to j; and (4) a fixed cost $f_{ij} \in \mathbb{R}_{>0}$ per vehicle for (shipping) service from i to j. In addition, both *in-transit* and *in-storage* holding costs are considered here for each commodity. In particular, the in-transit holding costs are incorporated into the flow costs c_{ij}^k for commodities $k \in \mathcal{K}$ and arcs $(i, j) \in \mathcal{A}$. A per-unit-of-demand-and-time in-storage holding cost $h^k \in \mathbb{R}_{\geq 0}$ is incurred when a commodity $k \in \mathcal{K}$ is stored at any node.

With static travel times, the deterministic CTSNDP needs to decide a plan $\mathcal{P}$ for shipping paths and a plan $\mathcal{C}$ to consolidate all the commodities, as well as a schedule $\mathcal{T}$ of departure times for all the required direct shipping services. The objective is to satisfy the delivery requirements for all commodities while minimizing the total cost. (See Example 1 in Appendix A for an illustration of the deterministic CTSNDP and its feasible solution.)

3.2. Robust CTSNDP

We now extend the setting of the deterministic CTSNDP to describe the robust CTSNDP under travel time uncertainty, which we refer to as the robust CTSNDP for short. Suppose that for each arc, the actual travel times of its direct shipping services are uncertain, varying around a nominal value.

As in the stochastic SNDP studied in Lanza et al. (2021), the decision process for the robust CTSNDP has two stages. In the first stage, before actual values of the travel times are realized, the problem needs to determine a routing plan $\mathcal{P}$ and a consolidation plan $\mathcal{C}$. Since $\mathcal{P}$ and $\mathcal{C}$ are crucial to determining the resources for transportation, such as vehicles, which require preparation time, they need to be independent of the realized travel times. Given the first stage solution $(\mathcal{P},\mathcal{C})$, in the second stage, which is after the actual values of the travel times have been realized, the problem needs to determine an actual departure schedule $\mathcal{T}$, which can adapt to the realized travel times. Because of travel time uncertainty, it is costly to satisfy the due time requirements strictly for every possible realization of the travel times. Accordingly, in the second stage of the robust CTSNDP, we relax the due time requirements and impose a penalty g^k for each unit of time by which each commodity k's arrival at its destination is delayed. Thus, after the actual travel times have been realized, both the holding costs and delay penalties are incurred. (See Example 2 in Appendix A for an illustration of how holding costs and delay penalties change for specific routing and consolidation plans when travel times deviate from the nominal values. This highlights the practical need in the CTSNDP to identify routing and consolidation plans that perform well under the worst-case derivations of travel time.)

In the first stage of the robust CTSNDP, both $\mathcal{P}$ and $\mathcal{C}$ need to be determined before the actual travel times are realized. Consequently, travel costs and service costs are determined. By following the *light robustness* approach, proposed by Fischetti and Monaci (2009) and generalized by Schöbel (2014), the first-stage solution $(\mathcal{P},\mathcal{C})$ is required to ensure the existence of a departure schedule $\mathcal{T}$ so that $(\mathcal{P},\mathcal{C})$ and $\mathcal{T}$ form a feasible solution to the deterministic CTSNDP under a nominal scenario, where travel times all take their nominal values. This is also commonly required in practice. Such a first-stage solution $(\mathcal{P},\mathcal{C})$ is referred to as a *nominal timely-implementable first-stage solution.*

The robust CTSNDP needs to identify the first-stage solution $(\mathcal{P},\mathcal{C})$, which must be nominal timely-implementable, and to determine the second-stage solution $\mathcal{T}$, which needs to be adjusted according to the realized travel times. Its objective is to achieve the best performance under the worst-case realization of travel times.

4. A New Formulation for Deterministic CTSNDP

In this section, we propose a new consolidation-indexed MILP model for the deterministic CTSNDP, which does not use time indices in decision variables and constraints. This enables our formulations of two robust models for the CTSNDP under travel time uncertainty in Section 5.

Below, we first introduce a new representation of feasible solutions in Section 4.1, which is defined on a flat network, as opposed to the classical representation defined on a time-expanded network. Based on this new representation, we then derive our consolidation-indexed MILP model for the deterministic CTSNDP in Section 4.2.

4.1. Representation of Feasible Solutions on the Flat Network

A *feasible solution* to the deterministic CTSNDP consists of (i) a routing plan, (ii) a consolidation plan, and (iii) a departure schedule, these being defined as follows. We call a directed path P in the flat network $\mathcal{D}$ a *flat path*, which is represented by its node sequence $(\nu_1, \nu_2, \dots, \nu_{m+1})$ and arc sequence $(a_1, a_2, \dots, a_m)$, with $m \in \mathbb{N}_{>0}$ denoting the total number of its arcs. As in actual practice, the delivery path of each commodity cannot have repeated vertices or arcs, and thus must be an elementary flat path from the origin to the destination of the commodity. Accordingly, a *routing plan* $\mathcal{P}$ is defined as a collection of $|\mathcal{K}|$ elementary flat paths in the flat network $\mathcal{D}$, with each flat path $P^k \in \mathcal{P}$ for $k \in \mathcal{K}$ representing the delivery path of commodity k from its origin o^k to destination d^k, where the node and arc sequences of P^k are denoted by $(\nu_1^k, \nu_2^k, \dots, \nu_{m^k+1}^k)$ and $(a_1^k, a_2^k, \dots, a_{m^k}^k)$, respectively, with $\nu_1^k = o^k$ and $\nu_{m^k+1}^k = d^k$, and with no repeated nodes or arcs.

Given a routing plan $\mathcal{P}$, we need to specify how commodities are consolidated for each arc $\alpha \in \mathcal{A}$ of the flat network $\mathcal{D}$. Let $\mathcal{K}(\mathcal{P}, \alpha) = \{k \in \mathcal{K} \mid \exists a_n^k = \alpha, 1 \le n \le m^k\}$ indicate a subset of commodities whose flat paths in $\mathcal{P}$ pass through arc α. A consolidation on α for $\mathcal{P}$ can be represented by a subset of $\mathcal{K}(\mathcal{P}, \alpha)$, so that commodities in this subset are consolidated to be shipped together on α. Since each commodity cannot be split during transport, in any feasible solution to the deterministic CTSNDP, there are at most $|\mathcal{K}|$ consolidations on each arc α, each of such consolidations representing a direct shipping service with a distinct departure time. We use $C_r^\alpha \subseteq \mathcal{K}(\mathcal{P}, \alpha)$ for $r = 1, 2, \dots, |\mathcal{K}|$ to indicate the r-th consolidation on α for the routing plan $\mathcal{P}$, so that all the commodities in C_r^α are shipped through α together, where C_r^α can be empty. A *consolidation plan* $\mathcal{C}$ for $\mathcal{P}$ can thus be defined as a collection of consolidations $C_r^\alpha \subseteq \mathcal{K}(\mathcal{P}, \alpha)$ for $\alpha \in \mathcal{A}$ and $r \in \{1, 2, \dots, |\mathcal{K}|\}$, where r is referred to as the *consolidation index*. If the consolidations C_r^α for $r = 1, 2, \dots, |\mathcal{K}|$ cover all the commodities $k \in \mathcal{K}(\mathcal{P}, \alpha)$ for each arc $\alpha \in \mathcal{A}$, i.e., $\bigcup_{r=1}^{|\mathcal{K}|} C_r^\alpha = \mathcal{K}(\mathcal{P}, \alpha)$ is satisfied for each $\alpha \in \mathcal{A}$, then such a routing-consolidation pair $(\mathcal{P}, \mathcal{C})$ forms a *flat solution* to the deterministic CTSNDP. (See Example 3 in Appendix A for an illustration of the flat solution.)

Given a flat solution $(\mathcal{P}, \mathcal{C})$, we need to further specify the departure time of each commodity from every node it passes through. Since each flat path in $\mathcal{P}$ is an elementary path, every commodity can depart from the same node at most once. Accordingly, a *departure schedule* $\mathcal{T}$ is defined as a collection of departure times $t_{\nu_n^k}^k$ for $k \in \mathcal{K}$ and $n \in \{1, 2, \dots, m^k\}$, indicating when commodity k departs from node ν_n^k via arc a_n^k of its flat path P^k. Thus, $(\mathcal{P}, \mathcal{C}, \mathcal{T})$ forms a *feasible solution* to the deterministic CTSNDP if the departure schedule $\mathcal{T}$ satisfies conditions: (i) $t_{\nu_n^k}^k \ge e^k$, for $n = 1$; (ii) $t_{\nu_{n+1}^k}^k \ge t_{\nu_n^k}^k + \tau_{a_n^k}$, for $n \in \{1, 2, \dots, m^k - 1\}$; (iii) $t_{\nu_n^k}^k + \tau_{a_n^k} \le l^k$, for $n = m^k$; and (iv) $t_i^k = t_i^{k'}$, for $k \in C_r^{(i,j)}$ and $k' \in C_r^{(i,j)}$ with $(i, j) \in \mathcal{A}$ and $r \in \{1, 2, \cdots, |\mathcal{K}|\}$. Here, for each commodity $k \in \mathcal{K}$, conditions (i) and (iii) together ensure that the departure time from its origin and arrival time at its destination are both within the time window $[e^k, l^k]$, and conditions (ii) result from the travel times of arcs on its flat path. Conditions (iv) ensure that commodities consolidated on the same arc all pass the arc at the same time. A flat solution $(\mathcal{P},\mathcal{C})$ is *timely-implementable*, if there exists such a departure schedule $\mathcal{T}$ that satisfies conditions (i)–(iv).

From a feasible solution $(\mathcal{P},\mathcal{C},\mathcal{T})$, we can obtain holding times H_n^k for nodes ν_n^k with $n = 1,2,\ldots,m^k+1$ on the flat path P^k of each commodity $k \in \mathcal{K}$, with $H_1^k = t^k_{\nu_1^k} - e^k$, $H_n^k = t^k_{\nu_n^k} - (t^k_{\nu_{n-1}^k} + \tau_{a_{n-1}^k})$ for $n \in \{2,\ldots,m^k\}$, and $H^k_{m^k+1} = l^k - (t^k_{\nu^k_{m^k}} + \tau_{a^k_{m^k}})$. Accordingly, we can define $h(\mathcal{P},\mathcal{T})$ to represent the total holding cost, where $h(\mathcal{P},\mathcal{T}) = \sum_{k\in\mathcal{K}}\sum_{n=1}^{m^k+1} h^k q^k H_n^k$, and define $f(\mathcal{P},\mathcal{C})$ to represent the total fixed cost and flow cost, where $f(\mathcal{P},\mathcal{C}) = \sum_{\alpha\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|} f_\alpha \lceil \sum_{k\in C_r^\alpha} q^k/u_\alpha\rceil + \sum_{k\in\mathcal{K}}\sum_{n=1}^{m^k} c^k_{a_n^k} q^k$. Thus, the total cost of solution $(\mathcal{P},\mathcal{C},\mathcal{T})$ equals $f(\mathcal{P},\mathcal{C}) + h(\mathcal{P},\mathcal{T})$.

Without loss of generality, we assume that for each commodity $k \in \mathcal{K}$, the difference $(l^k - e^k)$ between its latest arrival time l^k at the destination and available time e^k at the origin is not less than the length of the shortest-time path from o^k to d^k in the flat network $\mathcal{D}$. This is sufficient to ensure the existence of a feasible solution to the deterministic CTSNDP. The deterministic CTSNDP can thus be formulated as follows, where $\mathbb{S}$ indicates the domain of all the feasible solutions.

$$\text{(Deterministic CTSNDP)} \quad \min_{(\mathcal{P},\mathcal{C},\mathcal{T})\in\mathbb{S}} [f(\mathcal{P},\mathcal{C}) + h(\mathcal{P},\mathcal{T})].$$

4.2. Consolidation-Indexed MILP Model

As described previously, a feasible solution to the deterministic CTSNDP consists of a routing plan $\mathcal{P}$, a consolidation plan $\mathcal{C}$, and a departure schedule $\mathcal{T}$. To represent the routing plan $\mathcal{P}$, we introduce a binary variable x_{ij}^k for each $(i,j)\in\mathcal{A}$ and $k\in\mathcal{K}$, that indicates whether commodity $k\in\mathcal{K}$ passes through arc (i,j). To represent the consolidation plan $\mathcal{C}$, we first introduce a binary variable z_{ijr}^k for each $(i,j)\in\mathcal{A}$, $r\in\{1,2,\cdots,|\mathcal{K}|\}$, and $k\in\mathcal{K}$, indicating whether the r-th consolidation $C_r^{(i,j)}$ on arc (i,j) contains commodity k. We then introduce a non-negative integer variable y_{ijr} for each $(i,j)\in\mathcal{A}$ and $r\in\{1,2,\cdots,|\mathcal{K}|\}$, that indicates the number of vehicles needed by consolidation $C_r^{(i,j)}$ of arc (i,j) to accommodate the commodities in consolidation $C_r^{(i,j)}$. To represent the departure schedule $\mathcal{T}$, we first introduce a non-negative continuous variable v_{ij}^k for each $(i,j)\in\mathcal{A}$ and $k\in\mathcal{K}$, which indicates the time at which commodity k departs from node i when passing through arc (i,j). If commodity k does not pass through arc (i,j), then v_{ij}^k equals 0. We introduce a non-negative continuous variable b_{ijr} for each $(i,j)\in\mathcal{A}$ and $r\in\{1,2,...,|\mathcal{K}|\}$, which represents the time when commodities of the r-th consolidation $C_r^{(i,j)}$ on arc (i,j) depart from node i. We also introduce a non-negative continuous variable w_i^k for $i\in\mathcal{N}$ and $k\in\mathcal{K}$ to represent the holding time for commodity k at terminal i. It equals 0 if commodity k does not pass node i.

Accordingly, the deterministic CTSNDP can be represented by the following compact MILP model, referred to as model DO, where M denotes a sufficiently large constant:

$$\text{[DO]}\ \min \sum_{(i,j)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|} f_{ij}\cdot y_{ijr} + \sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}} (c_{ij}^k q^k)\cdot x_{ij}^k + \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{N}} (h^k q^k)\cdot w_i^k \tag{4.1}$$

$$\text{s.t.} \sum_{(i,j)\in\mathcal{A}} x_{ij}^k - \sum_{(j,i)\in\mathcal{A}} x_{ji}^k = \begin{cases} 1, & i=o^k, \\ -1, & i=d^k, \quad \forall k\in\mathcal{K}, i\in\mathcal{N}, \\ 0, & \text{otherwise}, \end{cases} \tag{4.2}$$

$$u_{ij}y_{ijr} - u_{ij} + 1 \le \sum_{k\in\mathcal{K}} q^k z_{ijr}^k \le u_{ij}y_{ijr}, \quad \forall\ (i,j)\in\mathcal{A}, r\in\{1,2,...,|\mathcal{K}|\}, \tag{4.3}$$

$$\sum_{r=1}^{|\mathcal{K}|} z_{ijr}^k = x_{ij}^k, \quad \forall\ (i,j)\in\mathcal{A}, k\in\mathcal{K}, \tag{4.4}$$

$$\sum_{j:(j,i)\in\mathcal{A}} (v_{ji}^k + \tau_{ji}x_{ji}^k) \le \sum_{j:(i,j)\in\mathcal{A}} v_{ij}^k, \quad \forall\ i\in\mathcal{N}\setminus\{o^k,d^k\}, k\in\mathcal{K}, \tag{4.5}$$

$$\sum_{j:(o^k,j)\in\mathcal{A}} v_{o^kj}^k \ge e^k, \quad \forall\ k\in\mathcal{K}, \tag{4.6}$$

$$\sum_{j:(j,d^k)\in\mathcal{A}} (v_{jd^k}^k + \tau_{jd^k}x_{jd^k}^k) \le l^k, \quad \forall\ k\in\mathcal{K}, \tag{4.7}$$

$$v_{ij}^k \le Mx_{ij}^k, \quad \forall\ (i,j)\in\mathcal{A}, k\in\mathcal{K}, \tag{4.8}$$

$$v_{ij}^k \le b_{ijr} + M(1-z_{ijr}^k), \quad \forall\ (i,j)\in\mathcal{A}, k\in\mathcal{K}, r\in\{1,2,...,|\mathcal{K}|\}, \tag{4.9}$$

$$v_{ij}^k \ge b_{ijr} - M(1-z_{ijr}^k), \quad \forall\ (i,j)\in\mathcal{A}, k\in\mathcal{K}, r\in\{1,2,...,|\mathcal{K}|\}, \tag{4.10}$$

$$w_i^k = \begin{cases} \sum_{j:(i,j)\in\mathcal{A}} v_{ij}^k - e^k, & i=o^k, \\ l^k - \sum_{j:(j,i)\in\mathcal{A}} (v_{ji}^k + \tau_{ji}x_{ji}^k), & i=d^k, \quad \forall\ i\in\mathcal{N}, \forall\ k\in\mathcal{K}, \\ \sum_{j:(i,j)\in\mathcal{A}} v_{ij}^k - \sum_{j:(j,i)\in\mathcal{A}} (v_{ji}^k + \tau_{ji}x_{ji}^k), & \text{otherwise}, \end{cases} \tag{4.11}$$

$$x_{ij}^k \in \{0,1\}, \quad \forall\ (i,j)\in\mathcal{A}, k\in\mathcal{K}, \tag{4.12}$$

$$y_{ijr} \in \mathbb{N}_{\ge 0}, \quad \forall\ (i,j)\in\mathcal{A}, r\in\{1,2,...,|\mathcal{K}|\}, \tag{4.13}$$

$$z_{ijr}^k \in \{0,1\}, \quad \forall\ (i,j)\in\mathcal{A}, k\in\mathcal{K}, r\in\{1,2,...,|\mathcal{K}|\}, \tag{4.14}$$

$$v_{ij}^k \ge 0, \quad \forall\ (i,j)\in\mathcal{A}, k\in\mathcal{K}, \tag{4.15}$$

$$b_{ijr} \ge 0, \quad \forall\ (i,j)\in\mathcal{A}, r\in\{1,2,...,|\mathcal{K}|\}, \tag{4.16}$$

$$w_i^k \ge 0, \quad \forall\ i\in\mathcal{N}, k\in\mathcal{K}. \tag{4.17}$$

The objective function (4.1) indicates the total cost to be minimized, which includes three terms for the total fixed cost, total flow cost, and total holding cost, respectively. Constraints (4.2)–(4.4) are imposed to define the routing and the consolidation plans. Specifically, constraints (4.2) are *flow balance constraints*, ensuring that each commodity travels along one flat path from its origin to its destination. Constraints (4.3) are *capacity constraints*. They ensure that the total quantity of commodities in each consolidation of an arc does not exceed the total capacity of the vehicles assigned to each consolidation of the arc, and imposes the restriction that $y_{ijr} = \lceil (\sum_{k\in\mathcal{K}} q^k z_{ijr}^k)/u_{ij} \rceil$ for every $(i,j)\in\mathcal{A}$, $r\in\{1,2,...,|\mathcal{K}|\}$, which is equal to the number of vehicles needed by consolidation $C_r^{(i,j)}$ of arc (i,j). Constraints (4.4) are *consolidation coverage constraints*, ensuring that for every arc (i,j) on the flat path of commodity $k\in\mathcal{K}$, there exists a consolidation of arc (i,j) that contains k. Constraints (4.5) are imposed to define the departure schedule. Specifically, constraints (4.5)–(4.7) are imposed on commodities' departure times with respect to the travel time of each arc, as well as the earliest available time and the due time of each commodity. Constraints (4.8) ensure that for each commodity, its departure time from each of its unvisited nodes is zero. Constraints (4.9) and (4.10) ensure that for each arc $(i,j)\in\mathcal{A}$, the commodities that are consolidated for shipment together through (i,j) have the same departure time from node i. Constraints (4.11) are imposed to define the holding time for each commodity $k\in\mathcal{K}$ and each node $i\in\mathcal{N}$. Constraints (4.12)-(4.17) define the domains of all the decision variables.

For each feasible solution $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{v},\boldsymbol{b},\boldsymbol{w})$ of model DO, $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ corresponds to a flat solution $(\mathcal{P},\mathcal{C})$, and $\boldsymbol{v}$ corresponds to a departure schedule $\mathcal{T}$ that satisfies conditions (i)–(iv), implying that such $(\mathcal{P},\mathcal{C},\mathcal{T})$ forms a feasible solution to the deterministic CTSNDP. As far as we know, model DO is the first compact MILP model of the deterministic CTSNDP that uses consolidation indices, and thus, we refer to it as the *consolidation-indexed* MILP model of the deterministic CTSNDP.

5. Formulations for Robust CTSNDP

Based on the newly proposed formulation for the deterministic CTSNDP in Section 4, we can formulate the robust CTSNDP by incorporating travel time uncertainty.

First, we formulate the travel time uncertainty for the robust CTSNDP. Suppose that for each arc $\alpha\in\mathcal{A}$, the actual travel time $\tilde{\tau}_\alpha$ for commodities passing through α is determined by $\tilde{\tau}_\alpha=\overline{\tau}_\alpha+\hat{\tau}_\alpha\delta_\alpha$. Here, $\overline{\tau}_\alpha\in\mathbb{N}_{>0}$ is the *nominal* value of $\tilde{\tau}_\alpha$ without any deviations, and $\hat{\tau}_\alpha\in\mathbb{N}$ with $\hat{\tau}_\alpha<\overline{\tau}_\alpha$ is the maximum deviation of $\tilde{\tau}_\alpha$ with respect to the nominal value $\overline{\tau}_\alpha$. The coefficient δ_α is a random variable (with unknown distribution), and its value falls within the range $[-1,1]$. Thus, $\tilde{\tau}_\alpha$ falls within the range $[\overline{\tau}_\alpha-\hat{\tau}_\alpha,\overline{\tau}_\alpha+\hat{\tau}_\alpha]$. Negative deviations in travel times need to be considered because of the holding costs. While we adhere to the classic robust optimization literature (e.g., Bertsimas and Sim 2004) by considering only symmetric supports for travel times, our formulations and solution algorithms can be adapted to accommodate asymmetric supports.

According to the representation in Section 4.1, a consolidation plan for a feasible solution includes $|\mathcal{K}|$ consolidations for each arc $\alpha\in\mathcal{A}$. Let $\tilde{\tau}_{\alpha r}$ for $r\in\{1,2,\ldots,|\mathcal{K}|\}$ indicate the travel time of the service required by the r-th consolidation for $\alpha\in\mathcal{A}$. Let $\mathbb{U}$ indicate the support of the vector $\boldsymbol{\delta}$ of random variables $\delta_{\alpha r}$ for $\alpha\in\mathcal{A}$ and $r\in\{1,2,...,|\mathcal{K}|\}$. We have that

$$\mathbb{U}=\Big\{\boldsymbol{\delta}:\delta_{\alpha r}\in[-1,1],\forall\alpha\in\mathcal{A},r\in\{1,2,...,|\mathcal{K}|\}\Big\}. \tag{5.1}$$

For each realized coefficient value $\boldsymbol{\delta}\in\mathbb{U}$, we use $\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})$ to indicate the vector of the corresponding realized travel times $(\overline{\tau}_\alpha+\hat{\tau}_\alpha\delta_{\alpha r})$ for $\alpha\in\mathcal{A}$ and $r\in\{1,2,...,|\mathcal{K}|\}$, which can be defined as

$$\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})=\Big\{\tilde{\boldsymbol{\tau}}:\tilde{\tau}_{\alpha r}=\overline{\tau}_\alpha+\hat{\tau}_\alpha\delta_{\alpha r},\forall\alpha\in\mathcal{A},r\in\{1,2,...,|\mathcal{K}|\}\Big\}. \tag{5.2}$$

We refer to vector $\boldsymbol{\delta}$ as the *scenario*, and refer to $\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})$ as the realized travel time for $\boldsymbol{\delta}$.

Next, according to the problem description in Section 3.2, the decision process for the robust CTSNDP has two stages. In the first stage, before actual values of the travel times are realized, it needs to determine a routing plan $\mathcal{P}$ and a consolidation plan $\mathcal{C}$, which, according to the definition in Section 4.1, must form a timely-implementable flat solution $(\mathcal{P},\mathcal{C})$. Let $\mathbb{F}$ indicate the domain of all such nominal timely-implementable first-stage solutions.

Given $(\mathcal{P},\mathcal{C})\in\mathbb{F}$, in the second stage, which is after actual travel times $\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})$ with $\boldsymbol{\delta}\in\mathbb{U}$ are realized, the problem needs to determine an actual departure schedule $\mathcal{T}$, which can adapt to the realized travel times. We need to compute the holding costs and delay penalty costs incurred in the second stage. For this, we need to determine an optimal departure schedule $\mathcal{T}=(t^k_{\nu^k_n})_{k\in\mathcal{K},1\le n\le m^k}$, where each $t^k_{\nu^k_n}$ denotes the departure time of commodity k from node ν^k_n on the flat path P^k of $\mathcal{P}$. Specifically, for each commodity $k\in\mathcal{K}$ and arc $a^k_n=(\nu^k_n,\nu^k_{n+1})$ of P^k, since $(\mathcal{P},\mathcal{C})$ is a flat solution, there must exist $r(k,n)\in\{1,2,\dots,|\mathcal{K}|\}$ such that the commodity k is contained in the consolidation $C^{a^k_n}_{r(k,n)}$ of $\mathcal{C}$. Thus, the actual travel time of commodity k on arc a^k_n equals $\tilde{\tau}_{a^k_n,r(k,n)}$. Accordingly, the actual departure schedule $\mathcal{T}$ needs to satisfy conditions (i), (iv), and (5.3) below:

$$t^k_{\nu^k_{n+1}}\ge t^k_{\nu^k_n}+\tilde{\tau}_{a^k_n,r(k,n)},\ \text{for } k\in\mathcal{K},\ n\in\{1,2,\dots,m^k-1\}, \tag{5.3}$$

which are like conditions (ii) with $\tau_{a^k_n}$ replaced by $\tilde{\tau}_{a^k_n,r(k,n)}$. Thus, the domain of such actual departure schedules $\mathcal{T}$ is denoted by $\mathbb{T}(\mathcal{P},\mathcal{C},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$. Let $g(\mathcal{P},\mathcal{T})=\sum_{k\in\mathcal{K}}g^k\cdot\max\{t^k_{\nu^k_{m^k}}+\tilde{\tau}_{a^k_{m^k},r(k,m^k)}-l^k,0\}$, indicating the total delay penalty for an actual departure schedule $\mathcal{T}$ with respect to flat paths in $\mathcal{P}$. Hence, under the realized travel times $\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})$ with $\boldsymbol{\delta}\in\mathbb{U}$, the corresponding second-stage cost, including the holding costs and delay penalties, equals $h(\mathcal{P},\mathcal{T})+g(\mathcal{P},\mathcal{T})$. Its minimum value, $\min_{\mathcal{T}\in\mathbb{T}(\mathcal{P},\mathcal{C},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))}\,[h(\mathcal{P},\mathcal{T})+g(\mathcal{P},\mathcal{T})]\}$, is referred to as the second-stage cost under scenario $\boldsymbol{\delta}$.

The robust CTSNDP aims to identify a robust nominal timely-implementable first-stage solution under travel time uncertainty. In Sections 5.1 and 5.2 below, we adopt two modeling frameworks, namely robust optimization and robust sacrificing, to characterize the robustness of such solutions and derive formulations for two variants of the robust CTSNDP, respectively.

5.1. Variant I: Robust Optimization of CTSNDP

Given an integer $\Gamma\in\mathbb{N}$, known as the *budget of uncertainty*, we can use it to adjust the level of robustness as needed, by introducing the following budgeted uncertainty set $\mathbb{U}(\Gamma)$ of scenarios $\boldsymbol{\delta}$:

$$\mathbb{U}(\Gamma)=\Big\{\boldsymbol{\delta}:\|\boldsymbol{\delta}\|_1\le\Gamma,\delta_{\alpha r}\in[-1,1],\forall\alpha\in\mathcal{A},r\in\{1,2,...,|\mathcal{K}|\}\Big\}, \tag{5.4}$$

where $\|\boldsymbol{\delta}\|_1=\sum_{\alpha\in\mathcal{A},r\in\{1,2,\dots,|\mathcal{K}|\}}|\delta_{\alpha r}|$. It contains all possible $\boldsymbol{\delta}$ such that $\|\boldsymbol{\delta}\|_1$, the total relative deviation of the travel times $\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})$ from their nominal values, does not exceed Γ.

The robust optimization variant of the CTSNDP under travel time uncertainty (or RO-CTSNDP in short) has an objective to minimize the worst-case total two-stage cost with respect to the budgeted uncertainty set $\mathbb{U}(\Gamma)$ on $\boldsymbol{\delta}$. To achieve this, the RO-CTSNDP needs to determine a nominal timely-implementable first-stage solution $(\mathcal{P},\mathcal{C})\in\mathbb{F}$ that minimizes the sum of the first-stage cost (which is independent of the realization of $\boldsymbol{\delta}$) and the worst-case second-stage cost (which is above the budgeted uncertainty set $\mathbb{U}(\Gamma)$ on $\boldsymbol{\delta}$). The RO-CTSNDP becomes more conservative as the budget of uncertainty Γ increases. Accordingly, the RO-CTSNDP can be formulated as follows:

$$[\text{RO-CTSNDP}]\quad \min_{(\mathcal{P},\mathcal{C})\in\mathbb{F}}\{f(\mathcal{P},\mathcal{C})+\max_{\boldsymbol{\delta}\in\mathbb{U}(\Gamma)}\ \min_{\mathcal{T}\in\mathbb{T}(\mathcal{P},\mathcal{C},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))}[h(\mathcal{P},\mathcal{T})+g(\mathcal{P},\mathcal{T})]\}.$$

We observe that $(\boldsymbol{x},\boldsymbol{z})$ of the first-stage decisions needs to ensure the existence of a departure schedule that satisfies the constraints with respect to commodities' earliest available times and due times under the nominal scenario. For this, we need to introduce decision variables $\overline{v}_{ij}^k$ and $\overline{b}_{ijr}$ to indicate commodities' departure times and consolidations' departure times for the nominal scenario, like the variables v_{ij}^k and b_{ijr} of model DO. Moreover, in the second stage, constraints with respect to the commodities' due times are relaxed, but delay penalties are imposed. As a result, we need to introduce an additional non-negative continuous decision variable s^k for each $k\in\mathcal{K}$, indicating the delay of commodity k's arrival at its destination.

Then we can reformulate the RO-CTSNDP as a two-stage MINLP (model RO) below, where $\bar{\mathcal{V}}(\boldsymbol{x},\boldsymbol{z})$ denotes as the domain defined by (4.5)-(4.10) and (4.15)-(4.16) under nominal travel times.

$$[\text{RO}]\ \min \sum_{(i,j)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|} f_{ij}\cdot y_{ijr}+\sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}}(c_{ij}^k q^k)\cdot x_{ij}^k+F_{RP}(\boldsymbol{x},\boldsymbol{z}) \tag{5.5}$$

$$\text{s.t. } (4.2)-(4.4),(4.12)-(4.14), \tag{5.6}$$

$$(\overline{\boldsymbol{v}},\overline{\boldsymbol{b}})\in\bar{\mathcal{V}}(\boldsymbol{x},\boldsymbol{z}). \tag{5.7}$$

Here, $F_{RP}(\boldsymbol{x},\boldsymbol{z})$ indicates the worst-case second-stage cost and can be calculated by the max-min optimization model below, referred to as model RP$(\boldsymbol{x},\boldsymbol{z})$:

$$[\text{RP}(\boldsymbol{x},\boldsymbol{z})]\quad \max_{\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}):\boldsymbol{\delta}\in\mathbb{U}(\Gamma)}\ \min \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{N}}(h^k q^k)\cdot w_i^k+\sum_{k\in\mathcal{K}} g^k\cdot s^k \tag{5.8}$$

$$\text{s.t. } (4.6),(4.8)-(4.10),(4.15)-(4.17), \tag{5.9}$$

$$\sum_{j:(j,i)\in\mathcal{A}}(v_{ji}^k+\sum_{r=1}^{|\mathcal{K}|}\tilde{\tau}_{jir}z_{jir}^k)\le\sum_{j:(i,j)\in\mathcal{A}}v_{ij}^k,\quad \forall\, i\in\mathcal{N}\setminus\{o^k,d^k\},k\in\mathcal{K}, \tag{5.10}$$

$$\sum_{j:(j,d^k)\in\mathcal{A}}(v_{jd^k}^k+\sum_{r=1}^{|\mathcal{K}|}\tilde{\tau}_{jd^kr}z_{jd^kr}^k)\le l^k+s^k,\quad \forall\, k\in\mathcal{K}, \tag{5.11}$$

$$w_i^k\ge\begin{cases}\sum\limits_{j:(i,j)\in\mathcal{A}}v_{ij}^k-e^k, & i=o^k,\\ (l^k+s^k)-\sum\limits_{j:(j,i)\in\mathcal{A}}(v_{ji}^k+\sum\limits_{r=1}^{|\mathcal{K}|}\tilde{\tau}_{jir}z_{jir}^k), & i=d^k,\quad \forall\, i\in\mathcal{N},\forall\, k\in\mathcal{K},\\ \sum\limits_{j:(i,j)\in\mathcal{A}}v_{ij}^k-\sum\limits_{j:(j,i)\in\mathcal{A}}(v_{ji}^k+\sum\limits_{r=1}^{|\mathcal{K}|}\tilde{\tau}_{jir}z_{jir}^k), & \text{otherwise,}\end{cases} \tag{5.12}$$

$$s^k \geq 0, \quad \forall k \in \mathcal{K}. \tag{5.13}$$

The objective (5.5) of model RO is to minimize the sum of the deterministic first-stage cost and the worst-case second-stage cost with respect to the uncertainty set $\mathbb{U}(\Gamma)$ on $\boldsymbol{\delta}$. The first-stage cost includes the fixed costs and the flow costs shown in the first two terms of (5.5). The worst-case second-stage cost is represented by $F_{RP}(\boldsymbol{x},\boldsymbol{z})$. In model RO, the constraints in (5.6) are the same as those of model DO imposed on $(\boldsymbol{x},\boldsymbol{z})$. Constraints in (5.7) are like those in model DO, with τ_{ji} replaced by the nominal travel times $\overline{\tau}_{ji}$. These constraints are imposed to ensure the existence of a feasible departure schedule under the nominal scenario.

The max-min optimization model RP$(\boldsymbol{x},\boldsymbol{z})$ computes the worst-case second-stage cost for $(\boldsymbol{x},\boldsymbol{z})$. Given any $\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})$ with $\boldsymbol{\delta} \in \mathbb{U}(\Gamma)$, the inner minimization problem of RP$(\boldsymbol{x},\boldsymbol{z})$ needs to determine variables $(\boldsymbol{v},\boldsymbol{b},\boldsymbol{w},\boldsymbol{s})$, with the objective of minimizing the second-stage cost equal to the sum of the holding costs and delay penalties as shown in (5.8). Most of the constraints in the inner minimization problem are the same as those of model DO imposed on $(\boldsymbol{v},\boldsymbol{b},\boldsymbol{w})$, except (5.10), (5.11) and (5.12). Compared with constraints (4.5), (4.7), and (4.11) of model DO, constraints (5.10), (5.11), and (5.12) replace $\tau_{ji}x^k_{ji}$ with $\sum_{r=1}^{|\mathcal{K}|} \tilde{\tau}_{jir} z^k_{jir}$ for each $(j,i) \in \mathcal{A}$, as the latter indicates the actual travel time of commodity k on arc (j,i) if k passes through (j,i). Moreover, the decision variable s^k for $k \in \mathcal{K}$ is included in the right-hand sides of constraints (5.11) and (5.12), in order to represent the delay in commodity k's arrival at its destination.

In addition, we define LP$(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$ as the inner minimization problem of RP$(\boldsymbol{x},\boldsymbol{z})$, subject to (5.9)-(5.13), with $F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$ as its optimal objective value, representing the second-stage cost of $(\boldsymbol{x},\boldsymbol{z})$ for scenario $\boldsymbol{\delta} \in \mathbb{U}(\Gamma)$.

5.2. Variant II: Robust Satisficing of CTSNDP

We follow the modeling framework of Long et al. (2023) to establish the robust satisficing variant of the CTSNDP under travel time uncertainty (or RS-CTSNDP in short). Let $\mathcal{Z}_0$ represent the optimal objective value of the deterministic CTSNDP under nominal travel times. Define a prescribed target $\mathcal{Z}$ for the total two-stage cost with $\mathcal{Z} \geq \mathcal{Z}_0$, which signifies a designated cost level or an acceptable loss of optimality relative to the empirical optimization model. The RS-CTSNDP needs to determine a nominal timely-implementable first-stage solution $(\mathcal{P},\mathcal{C}) \in \mathbb{F}$ that best achieves the prescribed target $\mathcal{Z}$, so that the worst-case normalized magnitude of the deviation from the target is minimized. Accordingly, the RS-CTSNDP can be formulated as

$$\min_{(\mathcal{P},\mathcal{C})\in\mathbb{F}} \{\rho \in \mathbb{R}_{\geq 0} : f(\mathcal{P},\mathcal{C}) + \min_{\mathcal{T}\in\mathbb{T}(\mathcal{P},\mathcal{C},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))} [h(\mathcal{P},\mathcal{T}) + g(\mathcal{P},\mathcal{T})] - \mathcal{Z} \leq \rho\|\boldsymbol{\delta}\|_1, \ \forall \boldsymbol{\delta} \in \mathbb{U}\}.$$

Here, the constraints imposed on the first-stage solution $(\mathcal{P},\mathcal{C}) \in \mathbb{F}$ restrict the deviation of the total two-stage cost from the prescribed target $\mathcal{Z}$ to not exceed $\rho\|\boldsymbol{\delta}\|_1$ for every possible scenario $\boldsymbol{\delta}$ in the uncertainty set $\mathbb{U}$. As a result, ρ indicates the worst-case magnitude of the deviation from the prescribed cost target, normalized by the total relative deviation $\|\boldsymbol{\delta}\|_1$ of the travel times. This quantity measures the fragility of a given solution and needs to be minimized to attain robustness.

Similarly, RS-CTSNDP can also be formulated as a two-stage MINLP (model RS) below:

$$[\text{RS}] \ \min \rho \tag{5.14}$$

$$\text{s.t.} \sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}}(c_{ij}^k q^k)\cdot x_{ij}^k + \sum_{(i,j)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|} f_{ij}\cdot y_{ijr} + F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})) - \mathcal{Z} \le \rho\|\boldsymbol{\delta}\|_1, \ \forall\ \boldsymbol{\delta}\in\mathbb{U}, \tag{5.15}$$

$$(4.2)-(4.4),\ (4.12)-(4.14),\ (5.7),\ \rho\ge 0. \tag{5.16}$$

Model RS aims to minimize ρ, which represents the worst-case magnitude of the deviation from the prescribed cost target, normalized by the total relative deviation $\|\boldsymbol{\delta}\|_1$ of the travel times. Here, as defined in Section 5.1, $F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$ is the optimal objective value of model $\text{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$, indicating the second-stage cost of $(\boldsymbol{x},\boldsymbol{z})$ under any given $\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})$ with $\boldsymbol{\delta}$ in the uncertainty set $\mathbb{U}$. In model RS, constraints (5.15) specify that for every possible scenario $\boldsymbol{\delta}$, the deviation of the total two-stage cost from the prescribed target $\mathcal{Z}$ cannot exceed $\rho\|\boldsymbol{\delta}\|_1$. The domain of variable ρ is defined by $\rho\ge 0$ in (5.16). Other constraints in (5.16) are the same as those in model RO, ensuring that $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ forms a nominal timely-implementable first-stage solution.

6. Exact Algorithms for Robust CTSNDP

Following the C&CG framework of Zeng and Zhao (2013), we develop two exact algorithms to solve models RO and RS, respectively. Their critical step in these models is the solution to a subproblem, which aims to compute the worst-case scenario $\boldsymbol{\delta}$ for a given first-stage solution. For model RO, as in existing studies, this subproblem is a max-min problem, which can be reformulated as an MILP using the dual of its inner minimization LP, and solved by an optimization solver. For model RS, this subproblem is more complicated, as it involves max-min fractional optimization and cannot be reformulated as an MILP. To address this, we develop a novel enhanced bisection search procedure.

Below we first illustrate and analyze our C&CG algorithm for model RS in detail, and then discuss our C&CG algorithm for model RO. Their implementations, including acceleration techniques, are illustrated online at `https://github.com/SSN0712/2024.06.26`.

6.1. C&CG Algorithm for Robust Satisficing Model

For any given first-stage solution $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ we define $F_1(\boldsymbol{x},\boldsymbol{y})$ below as its first-stage total cost:

$$F_1(\boldsymbol{x},\boldsymbol{y}) = \sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}}(c_{ij}^k q^k)\cdot x_{ij}^k + \sum_{(i,j)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|} f_{ij}\cdot y_{ijr} \tag{6.1}$$

Let $\mathcal{X}$ denote the domain of $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\overline{\boldsymbol{v}},\overline{\boldsymbol{b}})$ defined by linear constraints (5.6)–(5.7), and $\mathcal{Q}(\boldsymbol{\delta})$ denote the domain of $(\boldsymbol{v},\boldsymbol{b},\boldsymbol{w},\boldsymbol{s})$ defined by linear constraints (5.9)–(5.13) under the realized travel time $\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})$. For any possible scenario $\boldsymbol{\delta}\in\mathbb{U}$, $F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$ computed by $\text{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$ represents the second-stage cost of any first-stage solution $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ under $\boldsymbol{\delta}$. Thus, model RS proposed in Section 5.2 can be rewritten as the following noncompact MILP:

$$
\begin{aligned}
&[\text{RSMILP}]\ \min \rho \\
&\text{s.t. } F_1(\boldsymbol{x},\boldsymbol{y}) + \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{N}}(h^k q^k)\cdot w_i^{k(\boldsymbol{\delta})} + \sum_{k\in\mathcal{K}} g^k \cdot s^{k(\boldsymbol{\delta})} - \mathcal{Z} \le \rho\|\boldsymbol{\delta}\|_1, \quad \forall\, \boldsymbol{\delta}\in\mathbb{U}, && (6.2)\\
&(\boldsymbol{v}^{(\boldsymbol{\delta})},\boldsymbol{b}^{(\boldsymbol{\delta})},\boldsymbol{w}^{(\boldsymbol{\delta})},\boldsymbol{s}^{(\boldsymbol{\delta})}) \in \mathcal{Q}(\boldsymbol{\delta}), \quad \forall\, \boldsymbol{\delta}\in\mathbb{U}, && (6.3)\\
&\rho \ge 0, (\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\overline{\boldsymbol{v}},\overline{\boldsymbol{b}}) \in \mathcal{X}.
\end{aligned}
$$

Here, $(\boldsymbol{v}^{(\boldsymbol{\delta})},\boldsymbol{b}^{(\boldsymbol{\delta})},\boldsymbol{w}^{(\boldsymbol{\delta})},\boldsymbol{s}^{(\boldsymbol{\delta})})$ represents a vector of second-stage decision variables associated with each possible scenario $\boldsymbol{\delta}$ in $\mathbb{U}$. Constraints (6.2) and (6.3) ensure that the deviation of the total two-stage cost from the prescribed target $\mathcal{Z}$ does not exceed $\rho\|\boldsymbol{\delta}\|_1$ for every possible scenario $\boldsymbol{\delta}\in\mathbb{U}$. As a result, solving model RS is reduced to solving the above noncompact MILP model RSMILP.

Model RSMILP can be relaxed by replacing $\mathbb{U}$ in (6.2) and (6.3) with any of its subsets $\Lambda\subseteq\mathbb{U}$. The resulting relaxation is referred to as model RSMILP(Λ), and can be strengthened by appending to Λ more scenarios $\boldsymbol{\delta}\in\mathbb{U}$. When $\Lambda=\mathbb{U}$, models RSMILP(Λ) and RSMILP are equivalent.

6.1.1. Computing the worst-case scenario $\boldsymbol{\delta}$ by enhanced bisection search Consider any given first-stage solution $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$. For any $\boldsymbol{\delta}\in\mathbb{U}$, the ratio $(F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z})/\|\boldsymbol{\delta}\|_1$ represents a *normalized cost deviation* from the prescribed target under scenario $\boldsymbol{\delta}$. Here we slightly abuse the notation to define $\sigma/\|\mathbf{0}\|_1=0$ for $\sigma=0$, $\sigma/\|\mathbf{0}\|_1=+\infty$ for $\sigma>0$, and $\sigma/\|\mathbf{0}\|_1=-\infty$ for $\sigma<0$. Accordingly, constraints (6.2) and (6.3) in model RSMILP imply that the normalized cost deviation with respect to the prescribed target $\mathcal{Z}$ cannot exceed ρ for all $\boldsymbol{\delta}\in\mathbb{U}$.

The maximum value of the normalized cost deviation over all $\boldsymbol{\delta}\in\mathbb{U}$ is defined as the *worst-case normalized cost deviation* of $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$, and the corresponding $\boldsymbol{\delta}$ that leads to the ratio achieving the maximum value is referred to as the *worst-case scenario* for $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$, with respect to model RS. Computing such a worst-case scenario $\boldsymbol{\delta}$ can be formulated as the following *max-min fractional optimization model*, which is referred to as model FO$(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$:

$$
[\text{FO}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})]\ \rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) = \max_{\boldsymbol{\delta}\in\mathbb{U}} \frac{F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z}}{\|\boldsymbol{\delta}\|_1}. \tag{6.4}
$$

where $F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$ is defined by a minimization problem LP$(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$ and $\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ denotes the optimal objective value of model FO$(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$.

Solving the above max-min fractional optimization model FO$(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ to exact optimality is challenging, as it cannot be directly reformulated as an MILP, nor can it be solved directly by the classic bisection search. To tackle this, we develop a novel enhanced bisection search procedure. It starts with a lower bound ρ_l and an upper bound ρ_h on the optimal objective value $\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ of model FO$(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$. In each iteration, it first evaluates whether the middle point $\hat{\rho}=(\rho_l+\rho_u)/2$ is greater than $\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$. If $\hat{\rho}$ is larger, the upper bound ρ_h is decreased to $\hat{\rho}$. Otherwise, the lower bound ρ_l is increased to $\hat{\rho}$. After the obtained lower bound ρ_l is further enhanced, the procedure proceeds to the next iteration unless ρ_l equals $\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$.

Given any guessed value $\hat{\rho}$, consider the following optimization model, which does not involve fractional optimization and whose optimal objective value is denoted by $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho})$:

$$G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho}) = \max_{\boldsymbol{\delta}\in\mathbb{U}} \; F_1(\boldsymbol{x},\boldsymbol{y}) + F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})) - \mathcal{Z} - \hat{\rho}\|\boldsymbol{\delta}\|_1. \tag{6.5}$$

Lemma 1 below indicates that we can determine whether $\hat{\rho}$ is less than, greater than, or equal to $\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ by evaluating the value of $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho})$.

Lemma 1 *If $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho}) > 0$, then $\hat{\rho} < \rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$. Otherwise, if $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho}) \le 0$, then $\hat{\rho} \ge \rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$. If $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho}) = 0$, then $\hat{\rho} = \rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$.*

To solve $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho})$, which is a max-min problem, we need to reformulate it as an MILP. For this, we establish Proposition 1 below, stating an *optimality property of $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho})$ to restrict the domains of variables δ_{ijr}.*

Proposition 1 *There exists an optimal solution to the nonlinear optimization model defined in (6.5) such that (i) $\delta_{ijr} \in \{-1,0,1\}$ for each $(i,j)\in\mathcal{A}$, and that (ii) $\delta_{ijr}\in\{0,1\}$ for each $(i,j)\in\mathcal{A}$ $r\in\{1,2,...,|\mathcal{K}|\}$ with $\sum_{k\in\mathcal{K}} h^k q^k \hat{\tau}_{ij} \le \hat{\rho}$.*

Recall that $F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$ is defined by a linear program $\mathrm{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$. Let β_i^k, γ^k, ψ^k, η_{ij}^k, θ_{ijr}^k, ξ_{ijr}^k, and λ_i^k denote the dual variables associated with its constraints (5.10), (4.6), (5.11), (4.8)-(4.10), and (5.12), respectively. Let Ω indicate the feasible domain of its dual.Accordingly, based on Proposition 1, we can reformulate $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho})$ as an MILP shown in Proposition 2.

Proposition 2 *$G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho})$ defined in (6.5) can be equivalently written as the following MILP:*

$$\begin{aligned}
\max \quad & F_1(\boldsymbol{x},\boldsymbol{y}) - \mathcal{Z} + \sum_{(j,i)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|}\hat{\varphi}_{jir} - \sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}}(M_1 x_{ij}^k)\cdot\eta_{ij}^k + \sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|}[M_1(z_{ijr}^k-1)]\cdot(\theta_{ijr}^k+\xi_{ijr}^k) \\
& + \sum_{k\in\mathcal{K}} e^k\cdot(\gamma^k - \lambda_{o^k}^k) + \sum_{k\in\mathcal{K}} l^k\cdot(\lambda_{d^k}^k - \psi^k) && (6.6)\\
s.t. \quad & (\boldsymbol{\beta},\boldsymbol{\gamma},\boldsymbol{\psi},\boldsymbol{\eta},\boldsymbol{\theta},\boldsymbol{\xi},\boldsymbol{\lambda})\in\Omega, && (6.7)\\
& \hat{\zeta}_{ijr,-1} + \hat{\zeta}_{ijr,1} + \hat{\zeta}_{ijr,0} = 1, \quad \forall\ (i,j)\in\mathcal{A}, r\in\{1,2,...,|\mathcal{K}|\}, && (6.8)\\
& M_2(\hat{\zeta}_{jir,\ell}-1) \le \hat{\varphi}_{jir} - \Big(\sum_{k\in\mathcal{K}_i} z_{jir}^k(\beta_i^k-\lambda_i^k) + \sum_{k\in\mathcal{K}_i^d} z_{jir}^k(\psi^k-\lambda_i^k)\Big)\tilde{\tau}_{jir,\ell} + \hat{\rho}|\ell| \le M_2(1-\hat{\zeta}_{jir,\ell}), \\
& \qquad\qquad\qquad \forall\ (j,i)\in\mathcal{A}, r\in\{1,2,...,|\mathcal{K}|\}, \ell\in\{-1,0,1\}, && (6.9)\\
& \hat{\zeta}_{ijr,-1} = 0, \quad \forall\ (i,j)\in\mathcal{A}(\hat{\rho}), r\in\{1,2,...,|\mathcal{K}|\}, && (6.10)\\
& \hat{\zeta}_{ijr,\ell}\in\{0,1\}, \quad \forall\ (i,j)\in\mathcal{A}, r\in\{1,2,...,|\mathcal{K}|\}, \ell\in\{-1,0,1\}. && (6.11)
\end{aligned}$$

where $\mathcal{K}_i = \{k \in \mathcal{K} : i \neq o^k \text{ and } i \neq d^k\}$, $\mathcal{K}_i^d = \{k \in \mathcal{K} : i = d^k\}$, $\mathcal{A}(\hat{\rho}) = \{(i,j) \in \mathcal{A} : \sum_{k\in\mathcal{K}} h^k q^k \hat{\tau}_{ij} \leq \hat{\rho}\}$, *and* M_2 *is a sufficiently large constant. Given* $\hat{\zeta}_{ijr,-1}$ *and* $\hat{\zeta}_{ijr,1}$ *for* $(i,j)\in\mathcal{A}$ *and* $r\in\{1,...,|\mathcal{K}|\}$ *in the optimal solution of this MILP, the corresponding worst-case scenario* $\boldsymbol{\delta}$ *can be obtained by*

$$\delta_{ijr} = -\hat{\zeta}_{ijr,-1} + \hat{\zeta}_{ijr,1}, \quad \forall\ (i,j)\in\mathcal{A}, r\in\{1,...,|\mathcal{K}|\}. \tag{6.12}$$

Next, consider any given lower bound ρ_l on $\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$. Let $\boldsymbol{\delta}(\rho_l)$ indicate the realization of $\boldsymbol{\delta}$, derived by (6.12) from the optimal solution to model $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\rho_l)$ defined in (6.6)–(6.11). Define ρ_l' below to indicate the normalized cost deviation under $\boldsymbol{\delta}(\rho_l)$:

$$\rho_l' = \left(F_1(\boldsymbol{x},\boldsymbol{y}) + F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}(\rho_l))) - \mathcal{Z}\right)/\|\boldsymbol{\delta}(\rho_l)\|_1. \tag{6.13}$$

Lemma 2 below implies that we can always enhance ρ_l to a better lower bound ρ_l'. Moreover, as shown later in the proof of Theorem 1, this enhancement is essential to guarantee that Algorithm 1 solves model FO$(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ to exact optimality within a finite number of iterations.

Lemma 2 *If* $\rho_l \leq \rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$, *then* ρ_l' *defined in (6.13) satisfies that* $\rho_l \leq \rho_l' \leq \rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$.

Below, we provide a summary of our enhanced bisection search procedure for any given first-stage solution $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ in Algorithm 1, along with its correctness and convergence in Theorem 1.

Algorithm 1 Enhanced Bisection Search Procedure for any Given $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$

1. If $(F_1(\boldsymbol{x},\boldsymbol{y}) + F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\mathbf{0})) - \mathcal{Z}) > 0$, return $+\infty$ as the value of the worst-case normalized cost deviation of $\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$, and $\mathbf{0}$ as the worst-case scenario for $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$.
2. Initialize the values of ρ_l and ρ_h such that $\rho_l \leq \rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}) \leq \rho_h$.
3. Set $\hat{\rho} = (\rho_h + \rho_l)/2$, solve the maximization MILP model (6.6)–(6.11) to compute $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho})$.
4. If $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho}) > 0$, increase ρ_l to $\hat{\rho}$, and if $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho}) < 0$, decrease ρ_h to $\hat{\rho}$. Then go to Step 5. However, if $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho}) = 0$, increase ρ_l to $\hat{\rho}$, derive the scenario $\boldsymbol{\delta}(\rho_l)$ from the optimal solution to the model by (6.12), and then go to Step 6.
5. Enhancement: Solve the MILP model (6.6)–(6.11) to compute $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\rho_l)$, derive the scenario $\boldsymbol{\delta}(\rho_l)$ by (6.12), and compute ρ_l' from $\boldsymbol{\delta}(\rho_l)$ by (6.13). If $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\rho_l) = 0$, then go to Step 6. Otherwise, set ρ_l to ρ_l', and go to Step 3 for the next iteration.
6. Return ρ_l as the worst-case normalized cost deviation $\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$, and return $\boldsymbol{\delta}(\rho_l)$ as the worst-case scenario for $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$.

Theorem 1 *Consider any given feasible first-stage decisions* $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$. *(i) Algorithm 1 is guaranteed to terminate within a finite number of iterations, with the value of* $\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ *and a worst-case scenario* $\boldsymbol{\delta}$ *for* $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ *returned. (ii) Let* $\rho_l^{(0)}$ *and* $\rho_h^{(0)}$ *denote the initial values of* ρ_l *and* ρ_h *assigned* *in Step 3 of Algorithm 1. Then, for any $\epsilon > 0$, after $\lceil \log_2((\rho_h^{(0)} - \rho_l^{(0)})/\epsilon) \rceil$ iterations of Steps 3–6, Algorithm 1 obtains a lower bound ρ_l on $\rho^*(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ and a scenario $\boldsymbol{\delta}(\rho_l) \in \mathbb{U}$, such that $\rho_l \leq \rho^*(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \leq \rho_l + \epsilon$ and $F_1(\boldsymbol{x}, \boldsymbol{y}) + F_{LP}(\boldsymbol{x}, \boldsymbol{z}, \tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}(\rho_l))) - \mathcal{Z} \geq \rho_l \|\boldsymbol{\delta}(\rho_l)\|_1$.*

Theorem 1 indicates that our enhanced bisection search procedure in Algorithm 1 is an exact algorithm that solves model FO$(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ within a finite number of iterations. It also implies that Algorithm 1 solves model FO$(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ to an accuracy $\epsilon > 0$ within $\lceil \log_2((\rho_h^{(0)} - \rho_l^{(0)})/\epsilon) \rceil$ iterations.

6.1.2. RS-C&CG Algorithm In each iteration n, where $n = 1, 2, \cdots$, our C&CG algorithm for model RS (referred to as RS-C&CG algorithm) first solves an optimal solution $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}}, \phi)$ to model RSMILP(Λ) as the master problem for a particular subset Λ of $\mathbb{U}$. It then applies the enhanced bisection search procedure of Algorithm 1 to solve FO$(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ as the subproblem, and obtains the worst-case normalized cost deviation $\rho^*(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$, denoted by $\rho^{(n)}$, as well as the corresponding worst-case scenario, denoted by $\boldsymbol{\delta}^{(n)}$. Since RSMILP$(\Lambda)$ is a relaxation of model RS, the optimal objective value obtained is a lower bound on the optimal objective value of model RS. Since $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ forms a nominal timely-implementable first-stage solution to model RS, $\max\{0, \rho^{(n)}\}$ provides an upper bound on the optimal objective value of model RS. If the lower bound equals the upper bound, model RS is solved to the optimum. The algorithm terminates with an optimal solution given by $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}})$. Otherwise, it appends the identified scenario $\boldsymbol{\delta}^{(n)}$ to Λ. Model RSMILP(Λ) of the master problem is extended and strengthened with new decision variables $(\boldsymbol{v}^{(\boldsymbol{\delta})}, \boldsymbol{b}^{(\boldsymbol{\delta})}, \boldsymbol{w}^{(\boldsymbol{\delta})}, \boldsymbol{s}^{(\boldsymbol{\delta})})$ and their new constraints in (6.2)- (6.3). The algorithm proceeds to the next iteration. Theorem 2 below establishes the correctness and convergence of the algorithm.

Theorem 2 *RS-C&CG returns an optimal solution to model RS in a finite number of iterations.*

6.2. C&CG Algorithm for Robust Optimization Model

Similarly, model RO proposed in Section 5.1 can be rewritten as the following noncompact MILP:

$$
\begin{aligned}
\text{[ROMILP]} \ \min \quad & \sum_{k \in \mathcal{K}} \sum_{(i,j) \in \mathcal{A}} (c_{ij}^k q^k) \cdot x_{ij}^k + \sum_{(i,j) \in \mathcal{A}} \sum_{r=1}^{|\mathcal{K}|} f_{ij} \cdot y_{ijr} + \phi && (6.14) \\
s.t. \quad & \phi \geq \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}} (h^k q^k) \cdot w_i^{k(\boldsymbol{\delta})} + \sum_{k \in \mathcal{K}} g^k \cdot s^{k(\boldsymbol{\delta})}, \quad \forall \ \boldsymbol{\delta} \in \mathbb{U}(\Gamma), && (6.15) \\
& (\boldsymbol{v}^{(\boldsymbol{\delta})}, \boldsymbol{b}^{(\boldsymbol{\delta})}, \boldsymbol{w}^{(\boldsymbol{\delta})}, \boldsymbol{s}^{(\boldsymbol{\delta})}) \in \mathcal{Q}(\boldsymbol{\delta}), \quad \forall \ \boldsymbol{\delta} \in \mathbb{U}(\Gamma), && (6.16) \\
& (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \overline{\boldsymbol{v}}, \overline{\boldsymbol{b}}) \in \mathcal{X}. && (6.17)
\end{aligned}
$$

Here, ϕ is a newly introduced decision variable, and $(\boldsymbol{v}^{(\boldsymbol{\delta})}, \boldsymbol{b}^{(\boldsymbol{\delta})}, \boldsymbol{w}^{(\boldsymbol{\delta})}, \boldsymbol{s}^{(\boldsymbol{\delta})})$ represents a vector of second-stage decision variables associated with each possible scenario $\boldsymbol{\delta}$ in $\mathbb{U}(\Gamma)$. Constraints (6.15) and (6.16) ensure that ϕ equals the worst-case second-stage cost. As a result, solving the min-max-min model RO is reduced to solving the above noncompact MILP model ROMILP.

Model ROMILP can also be relaxed by replacing $\mathbb{U}(\Gamma)$ in constraints (6.15) and (6.16) with any of its subsets $\Lambda \subseteq \mathbb{U}(\Gamma)$. The resulting relaxation is referred to as model ROMILP(Λ). The relaxation can be strengthened by appending more scenarios $\boldsymbol{\delta}$ in $\mathbb{U}(\Gamma)$ to Λ.

Our C&CG algorithm for model RO, referred to as RO-C&CG algorithm, iteratively solves model ROMILP(Λ) to obtain a first-stage solution $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ and appends its worst-case scenario $\boldsymbol{\delta}$ to Λ, until $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ implies an optimal solution. With respect to model RO, a scenario $\boldsymbol{\delta}$ is a worst-case scenario, if the second-stage cost of $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ equals under $\boldsymbol{\delta}$ the worst-case second-stage cost $F_{RP}(\boldsymbol{x}, \boldsymbol{z})$. Both $\boldsymbol{\delta}$ and $F_{RP}(\boldsymbol{x}, \boldsymbol{z})$ can be determined by solving model RP($\boldsymbol{x}, \boldsymbol{z}$) in (5.8)–(5.13).

Our RO-C&CG algorithm for model RO is similar to the RS-C&CG algorithm for model RS, except that it can apply an optimization solver directly to solve model RP($\boldsymbol{x}, \boldsymbol{z}$) as the subproblem. This is because by Proposition 3 below, RP($\boldsymbol{x}, \boldsymbol{z}$) has an equivalent maximization MILP model.

Proposition 3 *The max-min model RP($\boldsymbol{x}, \boldsymbol{z}$) defined by (5.8)–(5.13) for $F_{RP}(\boldsymbol{x}, \boldsymbol{z})$ can be equivalently written as a maximization MILP model.*

7. Computational Experiments

We performed two sets of computational experiments. The first set aimed to assess the performance of our exact algorithms in solving model RO and model RS. The second set aimed to evaluate the performance of solutions obtained from model RO and model RS with different parameters for the uncertainty sets and under different performance criteria. In our implementation of RO-C&CG and RS-C&CG algorithms, we employed the asymmetric representatives strategy widely used in the graph coloring literature (Campêlo et al. 2008, Malaguti et al. 2009) to break solution symmetry and enhance tractability for master problems, and used the Gurobi solver (v.10.0.2) with a single thread to solve the master problems and subproblems. All experiments were conducted on a PC with an Intel(R) Core(TM) i7-8700 CPU at 3.20 GHz and 64 GB RAM. Based on the seven instance classes of the fixed-charge capacitated multi-commodity network design problem in Ghamlouche et al. (2003), we generated 210 test instances of the robust CTSNDP randomly with 30 per class. Their sizes are comparable to those in previous studies on SNDP under uncertainties (Wang and Qi 2020, Lanza et al. 2021). The instances and their generation method, as well as more implementation details for the algorithms are available online at `https://github.com/SSN0712/2024.06.26`.

7.1. Algorithm Performance of RO-C&CG and RS-C&CG

In the first set of experiments, both RO-C&CG and the RS-C&CG were executed with an eight-hour time limit and an optimality tolerance of 0.01% between the best upper and lower bounds. We solved the deterministic model DO based on nominal travel times using the Gurobi solver for all instances considered, and its optimal objective value is denoted as $\mathcal{Z}_0$. The cost target $\mathcal{Z}$ of model RS was set to $\lceil (1+0.05) \cdot \mathcal{Z}_0 \rceil$, while the uncertainty budget Γ of model RO was set to $\lceil 0.05 \cdot |\mathcal{K}| \rceil$.

Table 1 Computational Performance of RO-C&CG and RS-C&CG Algorithms.

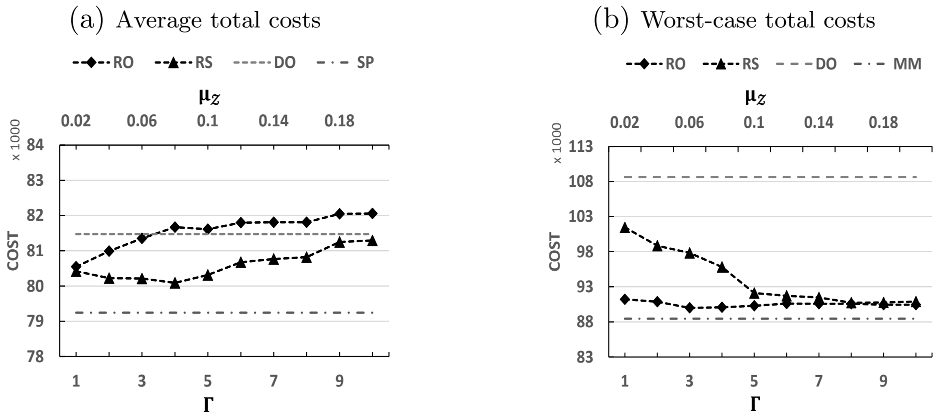
Class	$\lvert\mathcal{N}\rvert$	$\lvert\mathcal{A}\rvert$	$\lvert\mathcal{K}\rvert$	RO-C&CG					RS-C&CG				
				opt%	g%	T	Im%		opt%	g%	T	Im%	
							mean	max				mean	max
R4	10	60	10	100.0	0.00	0.1	6.4	29.7	100.0	0.00	0.6	37.8	100.0
R5	10	60	25	100.0	0.00	3.5	7.2	28.1	100.0	0.00	5.9	46.7	100.0
R6	10	60	50	93.3	0.07	2582.1	6.5	21.5	90.0	1.27	4368.2	46.5	85.5
R7	10	82	10	100.0	0.00	0.2	11.9	29.2	100.0	0.00	0.6	33.3	100.0
R8	10	83	25	100.0	0.00	4.5	11.8	23.1	100.0	0.00	6.5	48.7	85.4
R9	10	83	50	100.0	0.00	1527.2	6.9	11.9	100.0	0.00	43.1	37.4	82.2
R10	20	120	40	100.0	0.00	74.1	9.0	26.7	96.7	0.29	1440.2	52.1	91.9
Mean				99.0	0.01	598.4	8.5	29.7	98.1	0.22	837.9	43.2	100.0

The results are presented in Table 1. For each instance class (which contains 30 instances) and for each C&CG algorithm, we report the percentage of the instances solved to optimality in column opt%, the average optimality gap in column g% (defined as the percentage gap between the best upper and lower bounds found), and the average computational time in CPU seconds in column T. Moreover, we present the mean and maximum of the improvement ratio against the optimal deterministic solution, with respect to the objective value of the corresponding robust model, in column Im%. Consider model RO as an example. Its improvement ratio for each instance is computed by $\frac{C_{DO}-\mathrm{UB}}{C_{DO}}$, where C_{DO} denotes the objective value of model RO achieved by the optimal deterministic solution and UB denotes the best upper bound value found for model RO.

Table 1 shows the effectiveness of our RO-C&CG and RS-C&CG algorithms. Within the time limit, they both solve over 98% of the instances to optimality, and achieve optimality gaps of 0.01% and 0.22% on average, respectively. Columns Im% indicate that compared with the deterministic optimal solutions, the best upper bounds produced by our RO-C&CG and RS-C&CG algorithms significantly improve the objective values with respect to model RO and model RS by 8.5% and 43.2% on average and by 29.7% and 100.0% at maximum, respectively.

Table 1 confirms the computational tractability of both model RO and model RS that we have derived for the robust CTSNDP, underscoring their practical usefulness. Notably, our study presents the first development of a C&CG-based algorithm to effectively solve a two-stage robust satisficing model, encouraging its future extensions to other problems that encompass uncertainties. Additional computational results and detailed analysis are available online at `https://github.com/SSN0712/2024.06.26`. In particular, the results show that compared to the optimal deterministic solutions, the best robust solutions obtained on average require about 5.2% more directed shipping services but 13.3% fewer consolidations, thereby incurring transport costs that are 3.3% higher. Moreover, additional computational results also show that the two robust models for large instances with 100 or more commodities are challenging to solve, highlighting the need for future studies to further improve the solution algorithms.

Figure 1 Performance comparison of solutions from models RO and RS.



7.2. Solution Performance of Models RO and RS

Models RO and RS have distinct objectives for robustness. Following the approach in Atamtürk and Zhang (2007), our second set of experiments evaluates the trade-offs for achieving robustness. This involves comparing first-stage solutions obtained from models RO and RS based on their average and worst-case performances over a set of scenarios. For this, we generated a set Π of scenarios, assuming that each scenario has an equal probability. For each scenario, realized travel times were randomly generated within their supports by uniform distributions. We calculated the average and the worst-case total cost of each first-stage solution over the scenario set Π. For comparison, we solve a stochastic programming model (SP) and a min-max optimization model (MM) defined on Π, aiming to minimize the average total cost and the worst-case total cost over all scenarios in Π, respectively (see Appendix C). Their solutions provide the minimum achievable average total cost and the minimum achievable worst-case total cost over all scenarios in Π, respectively. However, unlike models RO and RS, models SP and MM both presume knowledge of Π.

Due to the tractability of models SP and MM, we set $|\Pi| = 200$ and focused on instance class R7. For each instance in R7, we used RO-C&CG to solve model RO for each uncertainty budget $\Gamma \in \{1, 2, ..., 10\}$, and used RS-C&CG to solve model RS for each cost target $\mathcal{Z} = \lceil (1 + \mu_z) \cdot \mathcal{Z}_0 \rceil$ with $\mu_z \in \{0.02, 0.04, ..., 0.2\}$, where $\mathcal{Z}_0$ is set as the optimal objective value of model DO. We compare them with the minimum achievable average total cost, obtained from model SP, and the minimum achievable worst-case total cost, obtained from model MM, as well as those values attained by optimal nominal solutions. The results are shown in Figures 1(a) and 1(b), where the total cost along the vertical axis is the mean across all the instances in R7.

Figure 1(a) shows that both model RO (with any Γ) and model RS (with any μ_z) can produce solutions with good average performance, within 4% from the minimum achievable average total cost obtained from model SP, while solutions from model RS exhibit better average performance than model RO. Figure 1(b) shows that model RO (with any Γ) and model RS (with $\mu_z > 0.1$) can produce solutions of good worst-case performance, within 5% of the minimum achievable worst-case total cost obtained from model MM, while solutions from model RO exhibit better worst-case performance than model RS. We can see that the impact of Γ on the average and the worst-case performances of model RO is not as significant as the impact of μ_z on these performances of model RS. Moreover, the average and the worst-case performances of robust solutions with appropriate parameters μ_z and Γ surpass those of the optimal nominal solutions, further confirming the need to integrate travel time uncertainty into service network design.

In addition, we conducted further experiments to evaluate the price of robustness, defined as the performance under nominal scenario (Bertsimas and Sim 2004). Results are available online at `https://github.com/SSN0712/2024.06.26`, indicating that the nominal performance of model RS outperforms that of model RO and is better controlled by the cost target parameter.

Our findings confirm the practical usefulness of model RO and model RS. Compared with model DO, they can obtain more robust solutions to save real operational cost. Compared with model SP and model MM, they are computationally more tractable, and do not need the distribution information of travel times. They can achieve comparable average solution performance to that of model SP, and comparable worst-case solution performance to that of model MM, although model RS sometimes requires an appropriate parameter setting to achieve this. Compared with model RS, model RO has better worst-case solution performance, making it useful for conservative decision makers. Compared with model RO, model RS has better average performance, making it useful for decision makers who prioritize average performance but have limited distribution information about travel times. Moreover, the cost target of model RS is a more effective parameter for adjusting the trade-off involved in achieving the robustness guarantees for solutions obtained.

7.3. Computational Performance of Model DO

Although both the consolidation-based formulation proposed by Hewitt and Lehuédé (2023, 2025) and the consolidation-indexed formulation proposed in this study incorporate consolidations, the two models remain fundamentally different. The consolidation-based formulation of Hewitt and Lehuédé (2023, 2025) contains binary variables for all possible shipment combinations that may be consolidated on each transportation movement. By contrast, our consolidation-indexed formulation creates a bounded set of consolidation indices, whose size is no larger than the number of commodities, and then uses binary assignment variables to determine which commodities are included in each indexed consolidation. As a result, the consolidation-based formulation proposed by Hewitt and Lehuédé (2023, 2025) needs to enumerate all possible shipment combinations in advance to define its variables, while our consolidation-indexed model completely avoids such an enumeration. To further illustrate the computational benefits of our approach, we conducted additional numerical experiments to compare the computational performance of the consolidation-based formulation of Hewitt and Lehuédé (2023, 2025) and our consolidation-indexed formulation on two sets of deterministic instances: the deterministic CTSNDP instances generated in this study with explicit commodity availability and due times (including large instances with 100 or more commodities), and the instances of the deterministic Freight Transportation Network Scheduling Problem with predetermined commodity paths (FTNSP) generated by Hewitt and Lehuédé (2025).

For the deterministic CTSNDP instances, we denote the adapted consolidation-based formulation by *Cons-CTSNDP* and our consolidation-indexed formulation by DO. We note that *Cons-CTSNDP* is not the original model of Hewitt and Lehuédé (2023). Their consolidation-based formulation was developed for the scheduled SNDP with predefined commodity paths and holding costs, and therefore cannot be directly applied to the general CTSNDP instances considered in this study. To enable a meaningful comparison, we adapted their consolidation-based modeling idea to the deterministic CTSNDP by incorporating routing decisions and holding costs. Because no predefined path is given for each commodity in this setting, the adapted model must enumerate all timely feasible consolidations on all arcs, where the enumeration for each arc was restricted to commodities that can traverse that arc without violating their time constraints. We note that this formulation adapts the valid inequalities proposed in Hewitt and Lehuédé (2023), but does not include the additional consolidation-pruning rule proposed therein, because such pruning is not valid once travel time uncertainty is incorporated. This comparison also excludes the hybrid formulation of Hewitt and Lehuédé (2023), which combines consolidation-based and time-indexed formulations over different parts of the time horizon. Both *Cons-CTSNDP* and DO are solved using Gurobi (v.10.0.2, single thread) with a 1% optimality tolerance and a one-hour time limit, consistent with standard practice for these instances.

For the deterministic FTNSP instances from Hewitt and Lehuédé (2025), we compare the consolidation-based formulation (*Cons-FINSP*$(\mathcal{C})$) and its dynamic column generation method (*IP-ColGen*) against our consolidation-indexed formulation (FTNSP_DO). Since FTNSP assumes predefined commodity routes and does not include holding costs, we adapt our consolidation-indexed formulation accordingly by removing the variables and constraints associated with routing decisions and holding costs. Following the experimental settings in Hewitt and Lehuédé (2025), we solve FTNSP_DO using Gurobi (v.10.0.2, single thread) with a 1% optimality tolerance and a two-hour time limit.

The implementation details of these formulations, together with the complete computational results, are available online at `https://github.com/SSN0712/2024.06.26`. Tables 2 and 3 summarize the results for the deterministic CTSNDP and FTNSP instances, respectively. The column "Solved (%)" reports the percentage of instances solved to optimality, "Time (s)" reports the average running time over all instances, and "Time for Solved (s)" reports the average running

Table 2 **Computational Results on Deterministic CTSNDP Instances.**

Method	Solved (%)	Time (s)
Cons-CTSNDP	90.28	408.35
DO	93.61	265.45

Table 3 **Computational Results on Deterministic FTNSP Instances of Hewitt and Lehuédé (2025).**

Method	Solved (%)	Time for Solved (s)
Cons-FINSP($\mathcal{C}$)	89.29	354.27
IP-ColGen	91.96	189.37
FTNSP_DO	100.00	10.89

time over only those instances solved to optimality. The reported running times include the time required to enumerate consolidations. The results for the benchmark methods *Cons-FINSP*($\mathcal{C}$) and *IP-ColGen* are taken from Hewitt and Lehuédé (2025).

As shown in Tables 2 and 3, our consolidation-indexed formulation solves more instances to optimality in less average time, thus outperforming the consolidation-based formulation and its column generation method. Thus, although this is not the primary focus of our study, our new consolidation-indexed formulation advances the literature on deterministic freight transportation network scheduling problems.

Moreover, we note that the corresponding MIP model could not be constructed within the prescribed time or memory limit for 4% of the deterministic CTSNDP instances and 11% of the FTNSP instances (involving up to 400 and 750 commodities), due to the need to enumerate consolidations and the resulting large number of consolidation variables. In contrast, our formulation provides feasible solutions for all these instances within the prescribed time and memory limits, with an average optimality gap of less than 1%. The benefit of our formulation becomes more evident when many commodities, for example more than 60, can traverse the same arc within their time windows, since the number of feasible consolidations to be enumerated on each arc grows rapidly with the number of such timely feasible commodities. This enumeration burden becomes even more critical under travel time uncertainty with delay penalties: while time-window constraints can be used to prune consolidations in the deterministic setting, delay penalties allow late arrivals at a cost, thereby enlarging the set of admissible consolidations and increasing the enumeration burden.

8. Conclusions

This paper studies a robust CTSNDP under travel time uncertainty, for which we derive several computationally tractable formulations and effective exact algorithms. It has established a strong foundation for future research: (i) There is great interest in enhancing our exact algorithms. One possible enhancement is to develop tailored branch-and-bound algorithms to solve the MILP models of both the subproblems and master problems involved in our exact algorithm. (ii) As the first attempt at incorporating travel time uncertainty into robust service network design, we have developed robust optimization and robust satisficing models based on polyhedral uncertainty sets. Further exploration of alternative robust optimization approaches, such as the distributionally robust optimization approach, is an area of interest. (iii) Our robust optimization model and robust satisficing models, accompanied by their C&CG algorithms, provide a solid foundation that can be extended to tackle travel time uncertainty in various variations of the CTSNDP, such as the CTSNDP with packing considerations, and other transportation problems.

References

Agra, A., Christiansen, M., Hvattum, L.M., Rodrigues, F., 2018. Robust optimization for a maritime inventory routing problem. Transportation Science 52, 509–525.

Altın, A., Yaman, H., Pınar, M.Ç., 2011. The robust network loading problem under hose demand uncertainty: formulation, polyhedral analysis, and computations. INFORMS Journal on Computing 23, 75–89.

Atamtürk, A., Zhang, M., 2007. Two-stage robust network flow and design under demand uncertainty. Operations Research 55, 662–673.

Bertsimas, D., Brown, D.B., Caramanis, C., 2011. Theory and applications of robust optimization. SIAM Review 53, 464–501.

Bertsimas, D., Delarue, A., Jaillet, P., Martin, S., 2019. Travel time estimation in the age of big data. Operations Research 67, 498–515.

Bertsimas, D., Sim, M., 2004. The price of robustness. Operations Research 52, 35–53.

Boland, N., Hewitt, M., Marshall, L., Savelsbergh, M., 2017. The continuous-time service network design problem. Operations Research 65, 1303–1321.

Campêlo, M., Campos, V.A., Corrêa, R.C., 2008. On the asymmetric representatives formulation for the vertex coloring problem. Discrete Applied Mathematics 156, 1097–1111.

Crainic, T.G., 2024a. Consolidation-based Transportation Planning: The Service Network Design Methodology. CIRRELT.

Crainic, T.G., 2024b. Service Network Design for Consolidation-based Transportation: Advanced Topics. Bureau de Montreal, Université de Montreal.

Crainic, T.G., Hewitt, M., 2021. Service network design, in: Network Design with Applications to Transportation and Logistics. Springer, pp. 347–382.

Crainic, T.G., Rei, W., 2025. 50 years of operations research for the tactical planning of consolidation-based freight transportation. EURO Journal on Transportation and Logistics , 100157.

Cui, Z., Ding, J., Long, D.Z., Zhang, L., 2023. Target-based resource pooling problem. Production and Operations Management 32, 1187–1204.

Demir, E., Burgholzer, W., Hrušovskỳ, M., Arıkan, E., Jammernegg, W., Van Woensel, T., 2016. A green intermodal service network design problem with travel time uncertainty. Transportation Research Part B: Methodological 93, 789–807.

Fischetti, M., Monaci, M., 2009. Light robustness, in: Robust and online large-scale optimization. Springer, pp. 61–84.

Ghamlouche, I., Crainic, T.G., Gendreau, M., 2003. Cycle-based neighbourhoods for fixed-charge capacitated multicommodity network design. Operations Research 51, 655–667.

Hewitt, M., Lehuédé, F., 2023. New formulations for the scheduled service network design problem. Transportation Research Part B: Methodological 172, 117–133.

Hewitt, M., Lehuédé, F., 2025. The freight transportation network scheduling problem: An integer programming-based column generation algorithm. INFORMS Journal on Computing 38, 710–728.

Hrušovskỳ, M., Demir, E., Jammernegg, W., Van Woensel, T., 2018. Hybrid simulation and optimization approach for green intermodal transportation problem with travel time uncertainty. Flexible Services and Manufacturing Journal 30, 486–516.

Jaillet, P., Qi, J., Sim, M., 2016. Routing optimization under uncertainty. Operations Research 64, 186–200.

Lanza, G., Crainic, T.G., Passacantando, M., Scutellà, M.G., 2024. A continuous time physical graph based formulation to scheduled service network design, in: International Conference on Optimization and Decision Science, Springer. pp. 265–275.

Lanza, G., Crainic, T.G., Rei, W., Ricciardi, N., 2018. A study on travel time stochasticity in service network design with quality targets, in: International Conference on Computational Logistics, Springer. pp. 401–416.

Lanza, G., Crainic, T.G., Rei, W., Ricciardi, N., 2021. Scheduled service network design with quality targets and stochastic travel times. European Journal of Operational Research 288, 30–46.

Layeb, S.B., Jaoua, A., Jbira, A., Makhlouf, Y., 2018. A simulation-optimization approach for scheduling in stochastic freight transportation. Computers & Industrial Engineering 126, 99–110.

Liang, J., Wu, J., Qu, Y., Yin, H., Qu, X., Gao, Z., 2019. Robust bus bridging service design under rail transit system disruptions. Transportation Research Part E: Logistics and Transportation Review 132, 97–116.

Long, D.Z., Sim, M., Zhou, M., 2023. Robust satisficing. Operations Research 71, 61–82.

Malaguti, E., Monaci, M., Toth, P., 2009. Models and heuristic algorithms for a weighted vertex coloring problem. Journal of Heuristics 15, 503–526.

Marshall, L., Boland, N., Savelsbergh, M., Hewitt, M., 2021. Interval-based dynamic discretization discovery for solving the continuous-time service network design problem. Transportation Science 55, 29–51.

Nikolova, E., Stier-Moses, N.E., 2014. A mean-risk model for the traffic assignment problem with stochastic travel times. Operations Research 62, 366–382.

Pessoa, A.A., Poss, M., 2015. Robust network design with uncertain outsourcing cost. INFORMS Journal on Computing 27, 507–524.

Rodrigues, F., Agra, A., Christiansen, M., Hvattum, L.M., Requejo, C., 2019. Comparing techniques for modelling uncertainty in a maritime inventory routing problem. European Journal of Operational Research 277, 831–845.

Schöbel, A., 2014. Generalized light robustness and the trade-off between robustness and nominal quality. Mathematical Methods of Operations Research 80, 161–191.

Shu, S., Xu, Z., Baldacci, R., 2024. Incorporating holding costs in continuous-time service network design: New model, relaxation, and exact algorithm. Transportation Science 58, 279–556.

Wang, X., Crainic, T.G., Wallace, S.W., 2019. Stochastic network design for planning scheduled transportation services: The value of deterministic solutions. INFORMS Journal on Computing 31, 153–170.

Wang, Z., Qi, M., 2020. Robust service network design under demand uncertainty. Transportation Science 54, 676–689.

Yao, B., Hu, P., Lu, X., Gao, J., Zhang, M., 2014. Transit network design based on travel time reliability. Transportation Research Part C: Emerging Technologies 43, 233–248.

Zeng, B., Zhao, L., 2013. Solving two-stage robust optimization problems using a column-and-constraint generation method. Operations Research Letters 41, 457–461.

Zhao, Y., Xue, Q., Cao, Z., Zhang, X., 2018. A two-stage chance constrained approach with application to stochastic intermodal service network design problems. Journal of Advanced Transportation 2018, 1–18.

Zhou, M., Sim, M., Lam, S.W., 2022. Advance admission scheduling via resource satisficing. Production and Operations Management 31, 4002–4020.

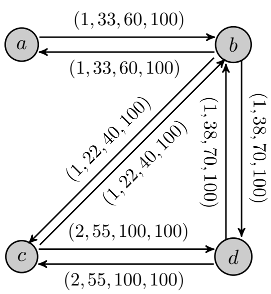
Appendix A: Glossary of Notation and illustrative Examples

Table 4 Glossary of notation.

Notation	Meaning
$\mathcal{D}$	(flat) network $\mathcal{D} = (\mathcal{N}, \mathcal{A})$
$\mathcal{N}$	node set of network $\mathcal{D}$
$\mathcal{A}$	arc set of network $\mathcal{D}$
$\mathcal{K}$	set of commodities
o^k	origin of commodity $k \in \mathcal{K}$
d^k	destination of commodity $k \in \mathcal{K}$
q^k	demand of commodity $k \in \mathcal{K}$
τ_{ij}	travel time of arc $(i,j) \in \mathcal{A}$
c_{ij}^k	per-unit-of-flow cost of arc $(i,j) \in \mathcal{A}$ and commodity $k \in \mathcal{K}$
f_{ij}	fixed cost of arc $(i,j) \in \mathcal{A}$
u_{ij}	capacity of arc $(i,j) \in \mathcal{A}$
e^k	earliest available time of commodity $k \in \mathcal{K}$
l^k	latest arrival time of commodity $k \in \mathcal{K}$
h^k	per-unit-of-demand-and-time (in-storage holding) cost of commodity $k \in \mathcal{K}$ at a terminal
$\mathcal{P} = \{P^k\}_{k\in\mathcal{K}}$	routing plan with P^k representing a path for commodity $k \in \mathcal{K}$
$\mathcal{C} = \{C_r^\alpha\}_{\alpha\in\mathcal{A}, r\in\{1,2,\ldots,\|\mathcal{K}\|\}}$	consolidation plan with C_r^α subset of commodities and with r the consolidation index
$\mathcal{T}$	departure schedule
$f(\mathcal{P},\mathcal{C})$	total fixed cost and flow cost of solution $(\mathcal{P},\mathcal{C},\mathcal{T})$
$h(\mathcal{P},\mathcal{T})$	total holding cost of solution $(\mathcal{P},\mathcal{C},\mathcal{T})$
$g(\mathcal{P},\mathcal{T})$	total delay penalty of solution $(\mathcal{P},\mathcal{C},\mathcal{T})$
$\boldsymbol{\delta}$	vector of random variables $\delta_{\alpha r}$ for $\alpha \in \mathcal{A}$ and $r \in \{1,2,\ldots,\|\mathcal{K}\|\}$
$\tilde{\tau}_{ij}$	uncertain travel time of arc $(i,j) \in \mathcal{A}$
$\tilde{\boldsymbol{\tau}}$	vector of uncertain travel times $\tilde{\tau}_{ij}$
$\bar{\tau}_{ij}$	nominal value of $\tilde{\tau}_{ij}$
$\hat{\tau}_{ij}$	maximum deviation of $\tilde{\tau}_{ij}$ with respect to nominal value $\bar{\tau}_{ij}$
$\mathcal{Z}$	prescribed target of the total two-stage cost
$\mathbb{U}$	support of vector $\boldsymbol{\delta}$
$\mathbb{U}(\Gamma)$	budgeted uncertainty set of vector $\boldsymbol{\delta}$ with Γ denoting a budget of uncertainty
$\mathbb{D}$	domain of all feasible solutions $(\mathcal{P},\mathcal{C},\mathcal{T})$
$\mathbb{F}$	domain of all nominal timely-implementable first-stage solutions
$\mathbb{T}(\mathcal{P},\mathcal{C},\tilde{\boldsymbol{\tau}})$	domain of departure schedule $\mathcal{T}$ with respect to solution $(\mathcal{P},\mathcal{C})$
$\boldsymbol{x}$	decision variables on routing plans
$\boldsymbol{z}$	decision variables on consolidation plans
$\boldsymbol{y}$	decision variables on numbers of vehicles
$\boldsymbol{v}$	decision variables on departure times
$\mathcal{X}$, $\mathcal{Q}$	domains of decision variables
DO	consolidation-indexed formulation for deterministic CTSNDP
RO	robust optimization model for robust CTSNDP
RS	robust satisficing model for robust CTSNDP
RP$(\boldsymbol{x},\boldsymbol{z})$	max-min model defined by (5.8)–(5.13) for the worst-case second-stage cost
LP$(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}})$	inner minimization problem of model RP$(\boldsymbol{x},\boldsymbol{z})$ as a linear program
ROMILP	noncompact MILP defined in (6.14)-(6.17), a reformulation of model RO
RSMILP	noncompact MILP reformulation of model RS, with constraints (6.2)-(6.3) included
FO$(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$	model defined in (6.4) for the worst-case normalized cost deviation of $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$
SP	stochastic programming model in Appendix C to minimize expected total cost
MM	min-max optimization model in Appendix C to minimize worst-case total cost
$F_1(\boldsymbol{x},\boldsymbol{y})$	first-stage cost of solution $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$
$F_{RP}(\boldsymbol{x},\boldsymbol{z})$	worst-case second-stage cost of solution $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$
$F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}})$	optimal objective value of model LP$(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}})$, the minimum cost of $(\boldsymbol{x},\boldsymbol{z})$ under $\tilde{\boldsymbol{\tau}}$
$\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$	optimal objective value of model FO$(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$
$G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho})$	optimal objective value of model defined in (6.5)

Figure 2 **An illustrative example for the CTSNDP. In (a), each arc (i,j) is associated with four values, representing its flow costs c_{ij}^k (same for all commodities k), fixed cost f_{ij}, static travel time τ_{ij}, and vehicle capacity u_{ij}. In the example, arcs (i,j) and (j,i) share the same data. In (c) and (d), each arc represents a direct shipping service required, with four values associated representing the set of commodities consolidated for shipping together, the departure time, the travel time, and the arrival time. In (c), travel times equal τ_{ij}. In (d), travel times deviate from τ_{ij}.**

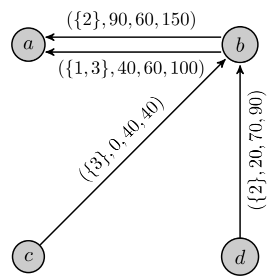
(a) Flat network $\mathcal{D}$



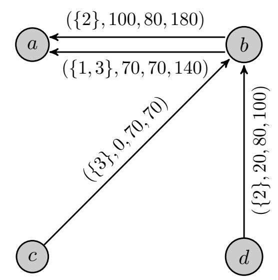
(b) Commodity data

k	o^k	d^k	q^k	e^k	l^k	h^k
1	b	a	25	0	120	0.01
2	d	a	30	20	180	0.01
3	c	a	40	0	180	0.01

(c) A feasible solution under fixed travel times



(d) Adjusting departures for deviated travel times



Example 1 (Deterministic CTSNDP). Consider a deterministic CTSNDP instance defined in Figure 2(a) and Figure 2(b). The feasible solution shown in Figure 2(c) specifies a routing plan $\mathcal{P}$, indicating that commodity 1 is delivered via path (b,a), commodity 2 is delivered via path (d,b,a), and commodity 3 is delivered via path (c,b,a). It also specifies a consolidation plan $\mathcal{C}$, indicating that commodities 1 and 3 are consolidated for a direct shipping service from b to a, and no consolidation is needed for other direct shipping services. With the static travel times τ_{ij}, it also specifies a departure schedule $\mathcal{T}$, indicating that commodity 2 departs from node d at time 20, arrives at node b at 90, departs from node b at time 90 without holding, and arrives at node a at time 150. Commodity 3 departs from node c at time 0, and arrives at node b at time 40. Commodities 1 and 3 depart from node b together at time 40, and arrive together at node a at time 100. Before arriving at destinations, only commodity 1 has a positive waiting time at node b from time 0 to time 40. Accordingly, the total fixed cost, flow cost, and holding cost can be computed as $22+38+33\times 2=126$, $1\times 40+1\times 30+1\times(25+30+40)=165$, $0.01\times 25\times 40=10$, respectively. This results in a total cost of 301.

Example 2 (Under Deviated Travel Times). Consider the CTSNDP instance defined in Figure 2(a) and Figure 2(b). Let the travel times τ_{ij} shown in Figure 2(c) be the nominal values for the travel times. Consider a situation where the actual travel times, as shown in Figure 2(d), deviate from τ_{ij}. In response to the deviations, the departure schedule for the routing and consolidation plan in Figure 2(c) needs to be adjusted as follows (see Figure 2(d) for illustration). Commodity 2 departs from node d at time 20, arrives at node b at time 100, departs from node b at time 100 without holding, and then arrives at node a at time 180. Commodity 3 departs from node c at time 0, and arrives at node b at time 70. Commodities 1 and 3 depart together from node b at time 70, arrive together at node a at time 140. While fixed and flow costs remain unchanged, the holding cost increases to $0.01\times 25\times 70=17.5$ because commodity 1 has a longer holding time at node b. Assuming a unit delay penalty of 5 for each commodity, we have that the delay in delivery of commodity 1 incurs a penalty of $5\times(140-120)=100$. This results in a total cost of 408.5.

Example 3 (Flat Solution). Consider the feasible solution shown in Figure 2(c) to the instance of the deterministic CTSNDP defined in Figure 2(a) and Figure 2(b). For the routing plan $\mathcal{P}$, it consists of paths $P^1=(b,a)$, $P^2=(d,b,a)$ and $P^3=(c,b,a)$. For the consolidation plan $\mathcal{C}$, it consists of consolidations $C_1^{(b,a)}=\{1,3\}$, $C_2^{(b,a)}=\{2\}$, $C_3^{(b,a)}=\emptyset$, $C_1^{(c,b)}=\{3\}$, $C_2^{(c,b)}=C_3^{(c,b)}=\emptyset$, $C_1^{(d,b)}=\{2\}$, $C_2^{(d,b)}=C_3^{(d,b)}=\emptyset$, and $C_r^{\alpha}=\emptyset$ for $\alpha\in\{(a,b),(b,c),(c,d),(d,c),(b,a)\}$ and $r\in\{1,2,3\}$.

Appendix B: Proof of Statements

B.1. Proof of Lemma 1

Recall that we slightly abuse the notation to define that $\sigma/\|\mathbf{0}\|_1=0$ for $\sigma=0$, $\sigma/\|\mathbf{0}\|_1=+\infty$ for $\sigma>0$, and $\sigma/\|\mathbf{0}\|_1=-\infty$ for $\sigma<0$. Consider any $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})\in\mathcal{X}$ and any given $\hat{\rho}$.

First, if $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho})>0$, then according to (6.5), we have that $\max_{\boldsymbol{\delta}\in\mathbb{U}}\{F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z}-\hat{\rho}\|\boldsymbol{\delta}\|_1\}>0$, which implies that there exists a $\boldsymbol{\delta}^*\in\mathbb{U}$ with $F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}^*))-\mathcal{Z}-\hat{\rho}\|\boldsymbol{\delta}^*\|_1>0$. Thus,

$$\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})=\max_{\boldsymbol{\delta}\in\mathbb{U}}[F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z}]/\|\boldsymbol{\delta}\|_1\ge[F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}^*))-\mathcal{Z}]/\|\boldsymbol{\delta}^*\|_1>\hat{\rho}$$

Second, if $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho})<0$, then we have that $\max_{\boldsymbol{\delta}\in\mathbb{U}}\{F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z}-\hat{\rho}\|\boldsymbol{\delta}\|_1\}<0$, which implies that $F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z}-\hat{\rho}\|\boldsymbol{\delta}\|_1<0,\quad\forall\boldsymbol{\delta}\in\mathbb{U}$. Thus, $[F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z}]/\|\boldsymbol{\delta}\|_1<\hat{\rho}$ for all $\boldsymbol{\delta}\in\mathbb{U}$. Therefore, we obtain that

$$\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})=\max_{\boldsymbol{\delta}\in\mathbb{U}}\frac{F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z}}{\|\boldsymbol{\delta}\|_1}<\hat{\rho}.$$

Third, if $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho})=0$, then we obtain that $\max_{\boldsymbol{\delta}\in\mathbb{U}}\{F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z}-\hat{\rho}\|\boldsymbol{\delta}\|_1\}=0$, which implies that there exists a $\boldsymbol{\delta}^*\in\mathbb{U}$ such that $F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}^*))-\mathcal{Z}-\hat{\rho}\|\boldsymbol{\delta}^*\|_1=0$, and $F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z}-\hat{\rho}\|\boldsymbol{\delta}\|_1\le 0,\quad\forall\boldsymbol{\delta}\in\mathbb{U}\setminus\{\boldsymbol{\delta}^*\}$. Thus,

$$\frac{F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}^*))-\mathcal{Z}}{\|\boldsymbol{\delta}^*\|_1}=\hat{\rho},\ \text{and}\quad\frac{F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z}}{\|\boldsymbol{\delta}\|_1}\le\hat{\rho},\ \text{for all}\ \boldsymbol{\delta}\in\mathbb{U}\setminus\{\boldsymbol{\delta}^*\},$$

which implies that

$$\hat{\rho}^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})=\max_{\boldsymbol{\delta}\in\mathbb{U}}\frac{F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z}}{\|\boldsymbol{\delta}\|_1}=\hat{\rho}.$$

Hence, Lemma 1 is proved.

B.2. Proof of Proposition 1

First, we need to establish Lemma 3 below, which indicates that model LP$(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$ always has a feasible solution for each $(\boldsymbol{x},\boldsymbol{z})$ that satisfies constraints (5.6)–(5.7) and for each $\boldsymbol{\delta}\in\mathbb{U}$.

Lemma 3 *For any $(\boldsymbol{x},\boldsymbol{z})$ that satisfies constraints (5.6)–(5.7), and for any $\boldsymbol{\delta}\in\mathbb{U}$, model LP$(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$ always has a feasible solution.*

Proof. Any given $(\boldsymbol{x},\boldsymbol{z})$ that satisfies constraints (5.6)–(5.7) corresponds to a nominal timely-implementable flat solution $(\mathcal{P},\mathcal{C})$. Consider any $\boldsymbol{\delta}\in\mathbb{U}$ with the corresponding realized travel time $\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})$. For such $(\mathcal{P},\mathcal{C})$ and $\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})$, we first show as follows that there exists a departure schedule $\mathcal{T}$ such that conditions (i)–(iv) are satisfied, from which we can then obtain a feasible solution to model LP$(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$.

For the nominal timely-implementable flat solution $(\mathcal{P},\mathcal{C})$, consider each commodity $k\in\mathcal{K}$ and its flat path P^k in $\mathcal{P}$ with an arc sequence denoted by $(a_1^k,...,a_{m^k}^k)$. For each $n\in\{1,2,\dots,m^k\}$, there must exist a consolidation $C_{r_n^k}^{a_n^k}\in\mathcal{C}$ for arc a_n^k with $r_n^k\in\{1,2,\cdots,|\mathcal{K}|\}$ such that $k\in C_{r_n^k}^{a_n^k}$. We can now construct a network $\mathcal{G}_\mathcal{C}=\{\mathcal{N}_\mathcal{C},\mathcal{A}_\mathcal{C}\}$ where each non-empty consolidation $C_r^\alpha\in\mathcal{C}$ corresponds to a node, denoted by $\langle\alpha,r\rangle$, in the node set $\mathcal{N}_\mathcal{C}$, and each pair of consolidations $C_{r_n^k}^{a_n^k}$ and $C_{r_{n+1}^k}^{a_{n+1}^k}$ for $k\in\mathcal{K}$ and $n\in\{1,...,m^k-1\}$ corresponds to an arc $(\langle a_n^k,r_n^k\rangle,\langle a_{n+1}^k,r_{n+1}^k\rangle)$ in the arc set $\mathcal{A}_\mathcal{C}$.

Since the flat solution $(\mathcal{P},\mathcal{C})$ is a nominal timely-implementable first-stage solution, there exists a departure schedule $\mathcal{T}$ which satisfies conditions (i)–(iv) with nominal travel times $\overline{\boldsymbol{\tau}}$. According to $\mathcal{T}$, for each consolidation $C_r^\alpha\in\mathcal{C}$ of arc $\alpha=(\nu,\nu')\in\mathcal{A}$ we can obtain its corresponding departure time from node ν, which is denoted by $t_{\alpha,r}$. For each pair of consolidations $C_{r_n^k}^{a_n^k}$ and $C_{r_{n+1}^k}^{a_{n+1}^k}$ with $k\in\mathcal{K}$ and $n\in\{1,...,m^k-1\}$, the departure time of $C_{r_n^k}^{a_n^k}$ from node ν_n^k plus the nominal value $\overline{\tau}_{a_n^k}$ of travel time of arc a_n^k must be less than or equal to the departure time of $C_{r_{n+1}^k}^{a_{n+1}^k}$ from node ν_{n+1}^k. Thus, by the definition of $\mathcal{G}_\mathcal{C}=\{\mathcal{N}_\mathcal{C},\mathcal{A}_\mathcal{C}\}$, we obtain that $t_{\alpha,r}+\overline{\tau}_\alpha\le t_{\alpha',r'}$, for all $(\langle\alpha,r\rangle,\langle\alpha',r'\rangle)\in\mathcal{A}_\mathcal{C}$. This, together with $\overline{\tau}_\alpha>0$ for all $\alpha\in\mathcal{A}$, implies that $\mathcal{G}_\mathcal{C}$ must be an acyclic network and thus has a topological ordering of nodes in $\mathcal{N}_\mathcal{C}$, denoted by $(\langle\alpha_1,r_1\rangle,\langle\alpha_2,r_2\rangle,\dots,\langle\alpha_{|\mathcal{N}_\mathcal{C}|},r_{|\mathcal{N}_\mathcal{C}|}\rangle)$.

Next, consider each possible realized travel time $\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})$ with any $\boldsymbol{\delta}\in\mathbb{U}$. For $n=1,2,\dots,|\mathcal{N}_\mathcal{G}|$, we can set the departure time of consolidation $C_{r_n}^{\alpha_n}$, denoted by $\hat{t}_{\alpha_n,r_n}$, iteratively as follows: $\hat{t}_{\alpha_1,r_1}=\max_{k\in\mathcal{K}}e^k$ and $\hat{t}_{\alpha_n,r_n}=\hat{t}_{\alpha_{n-1},r_{n-1}}+\max_{(i,j)\in\mathcal{A}}\{\overline{\tau}_{ij}+\hat{\tau}_{ij}\}$ for $n=2,3,\dots,|\mathcal{N}_\mathcal{C}|$. Thus, it can be seen that for each commodity $k\in\mathcal{K}$, $\hat{t}_{\alpha_1^k,r_1^k}\ge\hat{t}_{\alpha_1,r_1}=\max\limits_{k\in\mathcal{K}}e^k\ge e^k$ and $\hat{t}_{\alpha_{n+1}^k,r_{n+1}^k}\ge\hat{t}_{\alpha_n^k,r_n^k}+\max\limits_{(i,j)\in\mathcal{A}}\{\overline{\tau}_{ij}+\hat{\tau}_{ij}\}\ge\hat{t}_{\alpha_n^k,r_n^k}+\tilde{\tau}_{\alpha_n^k}$ for $n=1,\dots,m^k-1$.

Thus, by setting the departure time of commodity k for node ν_n^k to be equal to $\hat{t}_{\alpha_n^k,r_n^k}$, for $n=1,2,\dots,m^k$ and $k\in\mathcal{K}$, we obtain a plan $\hat{\mathcal{T}}$ which satisfies the conditions (i), (ii), and (iv), under the travel time $\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})$. From such a departure schedule $\hat{\mathcal{T}}$, we can obtain the values of variables v_{ij}^k, b_{ijr}, w_i^k, and s^k according to their definitions, which form a feasible solution to model LP$(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$. Hence, Lemma 3 is proved. □

Recall that $F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$ is defined by a linear program $LP(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}$. As in Section 6.1.1, let β_i^k, γ^k, ψ^k, η_{ij}^k, θ_{ijr}^k, ξ_{ijr}^k, and λ_i^k denote the dual variables associated with constraints (5.10), (4.6), (5.11), (4.8)-(4.10), and (5.12), respectively. Let Ω indicate the feasible domain of its dual, which is a convex polyhedron defined by some linear constraints. By Lemma 3 and the strong duality theorem, the optimal objective value of LP$(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}})$ equals that of its dual linear program.

Accordingly, we can replace the LP formulation of $F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$ with its dual to reformulate the model defined in (6.5) for $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho})$ as the following nonlinear optimization model:

$$\begin{aligned}G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho})\,=\,\max\ F_1(\boldsymbol{x},\boldsymbol{y})+\Big[&\sum_{(j,i)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|}\Big(\sum_{k\in\mathcal{K}_i}z_{jir}^k(\beta_i^k-\lambda_i^k)+\sum_{k\in\mathcal{K}_i^d}z_{jir}^k(\psi^k-\lambda_i^k)\Big)\cdot\tilde{\tau}_{jir}\\&-\sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}}(M_1x_{ij}^k)\cdot\eta_{ij}^k+\sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|}[M_1(z_{ijr}^k-1)]\cdot(\theta_{ijr}^k+\xi_{ijr}^k)\\&+\sum_{k\in\mathcal{K}}e^k\cdot(\gamma^k-\lambda_{o^k}^k)+\sum_{k\in\mathcal{K}}l^k\cdot(\lambda_{d^k}^k-\psi^k)\Big]-\mathcal{Z}-\sum_{(i,j)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|}\hat{\rho}|\delta_{ijr}|\end{aligned}\tag{B.1}$$

$$\text{s.t.} \quad (\boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\psi}, \boldsymbol{\eta}, \boldsymbol{\theta}, \boldsymbol{\xi}, \boldsymbol{\lambda}) \in \Omega, \tag{B.2}$$

$$\tilde{\tau}_{ijr} = \overline{\tau}_{ij} + \hat{\tau}_{ij}\delta_{ijr}, \quad \forall\, (i,j) \in \mathcal{A}, r \in \{1, 2, ..., |\mathcal{K}|\}, \tag{B.3}$$

$$-1 \le \delta_{ijr} \le 1, \quad \forall\, (i,j) \in \mathcal{A}, r \in \{1, 2, ..., |\mathcal{K}|\}. \tag{B.4}$$

Next, to further prove Proposition 1, for any given $(\boldsymbol{x}, \boldsymbol{z})$ and $\hat{\rho}$, consider any optimal solution $(\boldsymbol{\beta}^*, \boldsymbol{\gamma}^*, \boldsymbol{\psi}^*, \boldsymbol{\eta}^*, \boldsymbol{\theta}^*, \boldsymbol{\xi}^*, \boldsymbol{\lambda}^*, \tilde{\boldsymbol{\tau}}^*, \boldsymbol{\delta}^*)$ of the optimization model defined in (B.1)–(B.4). By fixing $(\boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\psi}, \boldsymbol{\eta}, \boldsymbol{\theta}, \boldsymbol{\xi}, \boldsymbol{\lambda}) = (\boldsymbol{\beta}^*, \boldsymbol{\gamma}^*, \boldsymbol{\psi}^*, \boldsymbol{\eta}^*, \boldsymbol{\theta}^*, \boldsymbol{\xi}^*, \boldsymbol{\lambda}^*)$, the nonlinear optimization model defined in (B.1)–(B.4) reduces to the following nonlinear model on $\boldsymbol{\delta}$, denoted as model $\mathbf{S}_1$.

$$[\mathbf{S}_1] \; \max \quad \sum_{(j,i)\in\mathcal{A}} \sum_{r=1}^{|\mathcal{K}|} \Big\{ \hat{\tau}_{jir} \Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta^{k*}_i - \lambda^{k*}_i) + \sum_{k\in\mathcal{K}^d_i} z^k_{jir}(\psi^{k*} - \lambda^{k*}_i) \Big) \cdot \delta_{jir} - \hat{\rho} \cdot |\delta_{jir}| \Big\}$$
$$s.t. \quad -1 \le \delta_{ijr} \le 1, \quad \forall\, (i,j) \in \mathcal{A}, r \in \{1, 2, ..., |\mathcal{K}|\}$$

We can see that $\boldsymbol{\delta}^*$ is an optimal solution of model $\mathbf{S}_1$. For any optimal solution $\hat{\boldsymbol{\delta}}$ of model $\mathbf{S}_1$, $(\boldsymbol{\beta}^*, \boldsymbol{\gamma}^*, \boldsymbol{\psi}^*, \boldsymbol{\eta}^*, \boldsymbol{\theta}^*, \boldsymbol{\xi}^*, \boldsymbol{\lambda}^*, \tilde{\boldsymbol{\tau}}(\hat{\boldsymbol{\delta}}), \hat{\boldsymbol{\delta}})$ forms a feasible solution of the optimization model in (B.1)–(B.4), and it has the same objective value as that of $(\boldsymbol{\beta}^*, \boldsymbol{\gamma}^*, \boldsymbol{\psi}^*, \boldsymbol{\eta}^*, \boldsymbol{\theta}^*, \boldsymbol{\xi}^*, \boldsymbol{\lambda}^*, \tilde{\boldsymbol{\tau}}^*, \boldsymbol{\delta}^*)$. Thus, $(\boldsymbol{\beta}^*, \boldsymbol{\gamma}^*, \boldsymbol{\psi}^*, \boldsymbol{\eta}^*, \boldsymbol{\theta}^*, \boldsymbol{\xi}^*, \boldsymbol{\lambda}^*, \tilde{\boldsymbol{\tau}}(\hat{\boldsymbol{\delta}}), \hat{\boldsymbol{\delta}})$ is also an optimal solution to the optimization model in (B.1)–(B.4).

Consider any optimal solution $\hat{\boldsymbol{\delta}}$ to model $\mathbf{S}_1$. Because $\hat{\boldsymbol{\delta}}$ is optimal, it can be seen that for any $(j,i) \in \mathcal{A}$ and $r \in \{1, 2, ..., |\mathcal{K}|\}$, if $\hat{\delta}_{jir} > 0$, then $\hat{\tau}_{jir}\Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta^{k*}_i - \lambda^{k*}_i + \sum_{k\in\mathcal{K}^d_i} z^k_{jir}(\psi^{k*} - \lambda^{k*}_i)\Big) \ge 0$, and that if $\hat{\delta}_{jir} < 0$, then $\hat{\tau}_{jir}\Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta^{k*}_i - \lambda^{k*}_i + \sum_{k\in\mathcal{K}^d_i} z^k_{jir}(\psi^{k*} - \lambda^{k*}_i)\Big) \le 0$. This is because otherwise, $\hat{\boldsymbol{\delta}}$ cannot be an optimal solution to model $\mathbf{S}_1$, as we can increase its objective value by changing the sign of each $\hat{\delta}_{jir}$ with $\hat{\tau}_{jir}\Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta^{k*}_i - \lambda^{k*}_i) + \sum_{k\in\mathcal{K}^d_i} z^k_{jir}(\psi^{k*} - \lambda^{k*}_i)\Big) \cdot \hat{\delta}_{jir} < 0$ to its opposite. Thus, we obtain that $\hat{\tau}_{jir}\Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta^{k*}_i - \lambda^{k*}_i) + \sum_{k\in\mathcal{K}^d_i} z^k_{jir}(\psi^{k*} - \lambda^{k*}_i)\Big) \cdot \hat{\delta}_{jir} \ge 0$ for all $(j,i) \in \mathcal{A}$ and $r \in \{1, 2, ..., |\mathcal{K}|\}$. Accordingly, model $\mathbf{S}_1$ is equivalent to the following maximization LP, denoted as model $\mathbf{S}_2$:

$$[\mathbf{S}_2] \; \max \quad \sum_{(j,i)\in\mathcal{A}} \sum_{r=1}^{|\mathcal{K}|} \Big\{ \Big[\Big| \hat{\tau}_{jir} \Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta^{k*}_i - \lambda^{k*}_i) + \sum_{k\in\mathcal{K}^d_i} z^k_{jir}(\psi^{k*} - \lambda^{k*}_i) \Big) \Big| - \hat{\rho} \Big] \cdot \delta^+_{jir} \Big\}$$
$$s.t. \quad 0 \le \delta^+_{ijr} \le 1, \quad \forall\, (i,j) \in \mathcal{A}, r \in \{1, 2, ..., |\mathcal{K}|\}$$

From any optimal solution $\boldsymbol{\delta}^+$ of model $\mathbf{S}_2$, we can derive an optimal solution of model $\mathbf{S}_1$ by setting $\delta_{jir} = \delta^+_{jir}$ if $\hat{\tau}_{jir}\Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta^{k*}_i - \lambda^{k*}_i + \sum_{k\in\mathcal{K}^d_i} z^k_{jir}(\psi^{k*} - \lambda^{k*}_i)\Big) \ge 0$, and setting $\delta_{jir} = -\delta^+_{jir}$ if $\hat{\tau}_{jir}\Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta^{k*}_i - \lambda^{k*}_i + \sum_{k\in\mathcal{K}^d_i} z^k_{jir}(\psi^{k*} - \lambda^{k*}_i)\Big) < 0$, for each $(j,i) \in \mathcal{A}$ and $r \in \{1, 2, ..., |\mathcal{K}|\}$, so that their objective function values are the same.

For model $\mathbf{S}_2$, its constraint matrix associated with $\delta^+_{ijr} \le 1$ for all $(i,j) \in \mathcal{A}$ and $r \in \{1, 2, ..., |\mathcal{K}|\}$ is totally unimodular, as it contains only one entry of 1 in each column. Thus, the feasible solution region of model $\mathbf{S}_2$ is an integral polytope. There must exist an integral optimal solution to model $\mathbf{S}_2$ with $\delta^+_{ijr} \in \{0, 1\}$ for each $(i,j) \in \mathcal{A}$ and $r \in \{1, 2, ..., |\mathcal{K}|\}$. This implies that there exists an optimal solution $\boldsymbol{\delta}$ to model $\mathbf{S}_1$ with $\delta_{ijr} \in \{-1, 0, 1\}$ for each $(i,j) \in \mathcal{A}$ and $r \in \{1, 2, ..., |\mathcal{K}|\}$. Such a solution $\boldsymbol{\delta}$ must also be optimal for the model in (B.1)–(B.4) and the model in (6.5).

Moreover, consider such an optimal solution $\boldsymbol{\delta}'$ to the model in (6.5) that satisfies $\delta'_{ijr} \in \{-1,0,1\}$ for all $(i,j)\in\mathcal{A}$ and $r\in\{1,2,...,|\mathcal{K}|\}$. Suppose there exist $(i,j)\in\mathcal{A}$ and $r\in\{1,2,\cdots,|\mathcal{K}|\}$ with $\sum_{k\in\mathcal{K}} h^k q^k \hat{\tau}_{ij} \le \hat{\rho}$ and $\delta'_{ijr}=-1$. By changing only the value of δ'_{ijr} from -1 to 0, we can obtain a new scenario in $\mathbb{U}$, which is denoted by $\boldsymbol{\delta}''$. Under $\boldsymbol{\delta}''$, the travel time for the r-th consolidation through arc (i,j) is increased by $\hat{\tau}_{ij}$, resulting in the decrease of the total holding cost by $\sum_{k\in\mathcal{K}} h^k q^k \hat{\tau}_{ij}$ at maximum. This implies that $F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}''))-F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}'))\ge -\sum_{k\in\mathcal{K}} h^k q^k \hat{\tau}_{ij}$. Thus, noting that $\|\boldsymbol{\delta}''\|_1-\|\boldsymbol{\delta}'\|_1=-1$, it can be seen that the difference between the objective value of model in (6.5) under $\boldsymbol{\delta}''$ and that under $\boldsymbol{\delta}'$ is at least $\hat{\rho}-\sum_{k\in\mathcal{K}} h^k q^k \hat{\tau}_{ij}$, which, due to $\sum_{k\in\mathcal{K}} h^k q^k \hat{\tau}_{ij}\le\hat{\rho}$, must be non-negative. Thus, $\boldsymbol{\delta}''$ must also be an optimal solution to the model in (6.5). By repeating this iteratively, we can obtain an optimal solution $\boldsymbol{\delta}$ to the model in (6.5) that satisfies both (i) and (ii) of Proposition 1. The proof is completed.

B.3. Proof of Proposition 2

By Proposition 1, constraints (B.4) can be replaced with $\delta_{ijr}\in\{-1,0,1\}$ for all $(i,j)\in\mathcal{A}$ and $r\in\{1,2,...,|\mathcal{K}|\}$, and with $\delta_{ijr}\in\{0,1\}$ for all $(i,j)\in\mathcal{A}(\hat{\rho})$ and $r\in\{1,2,...,|\mathcal{K}|\}$ where $\mathcal{A}(\hat{\rho})=\{(i,j)\in\mathcal{A}:\sum_{k\in\mathcal{K}} h^k q^k \hat{\tau}_{ij}\le \hat{\rho}\}$. By (B.3) we have that $\tilde{\tau}_{ijr}\in\{\overline{\tau}_{ijr}-\hat{\tau}_{ijr},\overline{\tau}_{ijr},\overline{\tau}_{ijr}+\hat{\tau}_{ijr}\}$, which, together with $\overline{\tau}_{ijr}\in\mathbb{N}_{>0}$, $\hat{\tau}_{ijr}\in\mathbb{N}_0$ and $\overline{\tau}_{ijr}>\hat{\tau}_{ijr}$, implies that $\tilde{\tau}_{ijr}\in\mathbb{N}_{>0}$. Moreover, let a new variable $\hat{\varphi}_{jir}$ represent each nonlinear term $\Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta^k_i-\lambda^k_i)+\sum_{k\in\mathcal{K}^d_i} z^k_{jir}(\psi^k-\lambda^k_i)\Big)\cdot\tilde{\tau}_{jir}-\hat{\rho}|\delta_{ijr}|$. We replace each integer variable δ_{jir} with three new binary variables $\hat{\zeta}_{jir,-1}$, $\hat{\zeta}_{jir,0}$ and $\hat{\zeta}_{jir,1}$, which are used to indicate whether δ_{ijr} equals -1, 0 and 1, respectively. Let $\tilde{\tau}_{ijr,-1}=\overline{\tau}_{ijr}-\hat{\tau}_{ijr}$, $\tilde{\tau}_{ijr,0}=\overline{\tau}_{ijr}$ and $\tilde{\tau}_{ijr,1}=\overline{\tau}_{ijr}+\hat{\tau}_{ijr}$. Accordingly, the linear constraints (6.8)–(6.11) can be derived. Thus, the nonlinear optimization model in (B.1)–(B.4) for $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho})$ can be reformulated to the maximization MILP model shown in Proposition 2. The proof is completed.

B.4. Proof of Lemma 2

Consider any $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})\in\mathcal{X}$. By Lemma 1, if $\rho_l\le\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$, we have that $\max_{\boldsymbol{\delta}\in\mathbb{U}}\{F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z}-\rho_l\|\boldsymbol{\delta}\|_1\}=G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\rho_l)\ge 0$. Note that $\boldsymbol{\delta}(\rho_l)$ indicates a realization of $\boldsymbol{\delta}$ such that $F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}(\rho_l)))-\mathcal{Z}-\rho_l\|\boldsymbol{\delta}(\rho_l)\|_1=\max_{\boldsymbol{\delta}\in\mathbb{U}}\{F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z}-\rho_l\|\boldsymbol{\delta}\|_1\}$. Since $\max_{\boldsymbol{\delta}\in\mathbb{U}}\{F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z}-\rho_l\|\boldsymbol{\delta}\|_1\}\ge 0$, we obtain that $F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}(\rho_l)))-\mathcal{Z}-\rho_l\|\boldsymbol{\delta}(\rho_l)\|_1\ge 0$, which implies that

$$\rho'_l=\frac{F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}(\rho_l)))-\mathcal{Z}}{\|\boldsymbol{\delta}(\rho_l)\|_1}\ge\rho_l.$$

Hence, $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\rho'_l)=\max_{\boldsymbol{\delta}\in\mathbb{U}}\{F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))-\mathcal{Z}-\rho'_l\|\boldsymbol{\delta}\|_1\}\ge F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}(\rho_l)))-\mathcal{Z}-\rho'_l\|\boldsymbol{\delta}(\rho_l)\|_1=0$. Thus, by Lemma 1, we obtain that $\rho'_l\le\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$. Lemma 2 is proved.

B.5. Proof of Theorem 1

To prove statement (i) of Theorem 1, consider each iteration n of Algorithm 1. Let $\rho^{(n)}_l$ denote the value of ρ_l updated in Step 4. Algorithm 1 solves $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\rho^{(n)}_l)$ in Step 5, derives its optimal solution $\boldsymbol{\delta}(\rho^{(n)}_l)$ of $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\rho^{(n)}_l)$ by (6.12), and computes the value of $\rho'^{(n)}_l$ from $\boldsymbol{\delta}(\rho^{(n)}_l)$ by $\rho'^{(n)}_l=(F_1(\boldsymbol{x},\boldsymbol{y})+$

$F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}(\rho_l^{(n)})))-\mathcal{Z})/\|\boldsymbol{\delta}(\rho_l^{(n)})\|_1$. If Algorithm 1 does not terminate at iteration n, then $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\rho_l^{(n)})>0$, which, together with Lemma 1, implies $\rho_l^{(n)}<\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$. Thus, by Lemma 2, we have that

$$\rho_l^{(n)}<\rho_l'^{(n)}\le\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}). \tag{B.5}$$

By the definition of $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\rho_l'^{(n)})$ and $\rho_l'^{(n)}$, we have that

$$G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\rho_l'^{(n)})\ge F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}(\rho_l^{(n)})))-\mathcal{Z}-\rho_l'^{(n)}\|\boldsymbol{\delta}(\rho_l^{(n)})\|_1=0. \tag{B.6}$$

Next, consider each iteration $m\ge n+1$. If Algorithm 1 does not terminate at iteration m, we have that

$$\rho_l^{(m)}\ge\rho_l'^{(n)}, \tag{B.7}$$

$$G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\rho_l^{(m)})=F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}(\rho_l^{(m)})))-\mathcal{Z}-\rho_l^{(m)}\|\boldsymbol{\delta}(\rho_l^{(m)})\|_1>0, \tag{B.8}$$

$$F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}(\rho_l^{(n)})))-\mathcal{Z}-\rho_l^{(m)}\|\boldsymbol{\delta}(\rho_l^{(n)})\|_1\le 0, \tag{B.9}$$

where (B.7) and (B.8) are implied by Step 4 of Algorithm 1 and Lemma 1, and (B.9) is implied by (B.6) and (B.7). By (B.5) and (B.7) we obtain that $\rho_l^{(n)}<\rho_l^{(m)}$. This, together with (B.8) and (B.9), implies that $\boldsymbol{\delta}(\rho_l^{(n)})$ and $\boldsymbol{\delta}(\rho_l^{(m)})$ are not equal.

By Proposition 1, each $\boldsymbol{\delta}$ derived by (6.12) satisfies that $\delta_{ijr}\in\{-1,0,1\}$ for all $(i,j)\in\mathcal{A}$ and $r\in\{1,2,\cdots,|\mathcal{K}|\}$. Therefore, as there are a finite number of such $\boldsymbol{\delta}$, Algorithm 1 must terminate in a finite number of iterations. Moreover, when Algorithm 1 terminates, we have that $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\rho_l)=0$. By Lemma 1, we obtain that Algorithm 1 returns $\rho_l=\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$, and accordingly, $\boldsymbol{\delta}(\rho_l)$ is the corresponding worst-case scenario for $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$. Hence, the first statement of Theorem 1 is proved.

The statement (ii) of Theorem 1 follows directly from the property of bisection search. For any $\epsilon>0$, Algorithm 1 only needs at most $\lceil\log_2((\rho_h^{(0)}-\rho_l^{(0)})/\epsilon)\rceil$ iterations to ensure $\rho_h-\rho_l\le\epsilon$. At each iteration of Algorithm 1, by Lemma 1 and Lemma 2 we note that $\rho_l\le\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})\le\rho_h$ and $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\rho_l)\ge 0$. Thus, after $\lceil\log_2((\rho_h^{(0)}-\rho_l^{(0)})/\epsilon)\rceil$ iterations, we have $\rho_l\le\rho^*(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})\le\rho_h\le\rho_l+\epsilon$. By $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\rho_l)\ge 0$ we also have $(F_1(\boldsymbol{x},\boldsymbol{y})+F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}(\rho_l)))-\mathcal{Z}\ge\rho_l\|\boldsymbol{\delta}(\rho_l)\|_1$. Hence, the statement (ii) of Theorem 1 is also proved.

B.6. Proof of Theorem 2

At each iteration of algorithm RS-C&CG, UB and LB are updated by solving the corresponding master problem and subproblem, while a new worst-case scenario $\boldsymbol{\delta}$ in $\mathbb{U}$ is obtained and added into the scenario subset Λ. algorithm RS-C&CG stops when $UB=LB$.

First, we show that algorithm RS-C&CG returns an optimal solution to model RS if it terminates with $UB=LB$. As model RSMILP(Λ) is a relaxation of model RS, the value of LB, which equals the optimal objective value of model RSMILP(Λ), is a valid lower bound on the optimal objective value of model RS. As UB is the worst-case normalized cost deviation of a first-stage solution $(\hat{\boldsymbol{x}},\hat{\boldsymbol{y}},\hat{\boldsymbol{z}})$, it provides a valid upper bound on the optimal objective value of model RS. Thus, when $UB=LB$, $(\hat{\boldsymbol{x}},\hat{\boldsymbol{y}},\hat{\boldsymbol{z}})$ forms an optimal solution to model RS.

Next, we show that algorithm RS-C&CG must terminate with $UB=LB$ in a finite number of iterations. We note that at each iteration n, if the worst-case scenario $\boldsymbol{\delta}^{(n)}$ identified by the subproblem of algorithm

RS-C&CG is not in the current scenario subset Λ, it will be added to Λ. According to Proposition 1, $\boldsymbol{\delta}^{(n)}$ satisfies that $\delta_{ijr}^{(n)} \in \{-1,0,1\}$ for all $(i,j)\in\mathcal{A}$ and $r\in\{1,2,\cdots,|\mathcal{K}|\}$, and has a finite number of possible values. Therefore, in a finite number of iterations, $\boldsymbol{\delta}^{(n)}$ identified by the subproblem of algorithm RS-C&CG must be included in the current scenario set Λ. In such a situation, both LB and UB must be equal to the optimal objective value of the current master problem, implying that $(\hat{\boldsymbol{x}},\hat{\boldsymbol{y}},\hat{\boldsymbol{z}})$ forms an optimal solution to model RS. This completes the proof of Theorem 2.

B.7. Proof of Proposition 3

Recall that $F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$ is defined by a linear program $LP(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$, which, by Lemma 3, always has a feasible solution. Similar to our reformulation of $G(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\hat{\rho})$, we can replace the LP formulation of $F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$ with its dual, thereby reformulating the inner minimization problem of the max-min model RP$(\boldsymbol{x},\boldsymbol{z})$ of $F_{RP}(\boldsymbol{x},\boldsymbol{z})$ defined in (5.8)–(5.13). The reformulation takes the form of a nonlinear optimization model with a bi-linear objective function, as shown below:

$$
\begin{aligned}
F_{RP}(\boldsymbol{x},\boldsymbol{z}) = \max \quad & \sum_{(j,i)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|}\Big(\sum_{k\in\mathcal{K}_i} z_{jir}^k(\beta_i^k-\lambda_i^k)+\sum_{k\in\mathcal{K}_i^d} z_{jir}^k(\psi^k-\lambda_i^k)\Big)\cdot\tilde{\tau}_{jir} \\
& -\sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}}(M_1x_{ij}^k)\cdot\eta_{ij}^k+\sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|}[M_1(z_{ijr}^k-1)]\cdot(\theta_{ijr}^k+\xi_{ijr}^k) \\
& +\sum_{k\in\mathcal{K}} e^k\cdot(\gamma^k-\lambda_{o^k}^k)+\sum_{k\in\mathcal{K}} l^k\cdot(\lambda_{d^k}^k-\psi^k) && \text{(B.10)}\\
\text{s.t.} \quad & (\boldsymbol{\beta},\boldsymbol{\gamma},\boldsymbol{\psi},\boldsymbol{\eta},\boldsymbol{\theta},\boldsymbol{\xi},\boldsymbol{\lambda})\in\Omega, && \text{(B.11)}\\
& \tilde{\tau}_{ijr}=\overline{\tau}_{ij}+\hat{\tau}_{ij}\delta_{ijr}, \quad \forall\ (i,j)\in\mathcal{A}, r\in\{1,2,...,|\mathcal{K}|\}, && \text{(B.12)}\\
& -1\le\delta_{ijr}\le 1, \quad \forall\ (i,j)\in\mathcal{A}, r\in\{1,2,...,|\mathcal{K}|\}, && \text{(B.13)}\\
& \sum_{(i,j)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|}|\delta_{ijr}|\le\Gamma. && \text{(B.14)}
\end{aligned}
$$

Proposition 4 below indicates that the domain of each variable δ_{ijr} can be restricted to $\{-1,0,1\}$ without changing the optimal objective value of the nonlinear optimization model above.

Proposition 4 *There exists an optimal solution to the nonlinear optimization model defined in (B.10)–(B.14) such that $\delta_{ijr}\in\{-1,0,1\}$ for each $(i,j)\in\mathcal{A}$ and $r\in\{1,2,...,|\mathcal{K}|\}$.*

Proof. For any given $(\boldsymbol{x},\boldsymbol{z})$, consider any optimal solution $(\boldsymbol{\beta}^*,\boldsymbol{\gamma}^*,\boldsymbol{\psi}^*,\boldsymbol{\eta}^*,\boldsymbol{\theta}^*,\boldsymbol{\xi}^*,\boldsymbol{\lambda}^*,\tilde{\boldsymbol{\tau}}^*,\boldsymbol{\delta}^*)$ of the nonlinear optimization model defined in (B.10)–(B.14). By fixing $(\boldsymbol{\beta},\boldsymbol{\gamma},\boldsymbol{\psi},\boldsymbol{\eta},\boldsymbol{\theta},\boldsymbol{\xi},\boldsymbol{\lambda})=(\boldsymbol{\beta}^*,\boldsymbol{\gamma}^*,\boldsymbol{\psi}^*,\boldsymbol{\eta}^*,\boldsymbol{\theta}^*,\boldsymbol{\xi}^*,\boldsymbol{\lambda}^*)$, the model defined in (B.10)–(B.14) reduces to the following nonlinear model on $\boldsymbol{\delta}$, denoted as model $\mathbf{R}_1$.

$$
\begin{aligned}
[\mathbf{R}_1] \quad \max \quad & \sum_{(j,i)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|}\Big\{\hat{\tau}_{jir}\Big(\sum_{k\in\mathcal{K}_i} z_{jir}^k(\beta_i^{k^*}-\lambda_i^{k^*})+\sum_{k\in\mathcal{K}_i^d} z_{jir}^k(\psi^{k^*}-\lambda_i^{k^*})\Big)\cdot\delta_{jir}\Big\}\\
s.t. \quad & -1\le\delta_{ijr}\le 1, \quad \forall\ (i,j)\in\mathcal{A}, r\in\{1,2,...,|\mathcal{K}|\},\\
& \sum_{(i,j)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|}|\delta_{ijr}|\le\Gamma.
\end{aligned}
$$

Accordingly, $\boldsymbol{\delta}^*$ must be an optimal solution to model $\mathbf{R}_1$. Moreover, for any optimal solution $\hat{\boldsymbol{\delta}}$ to model $\mathbf{R}_1$, $(\boldsymbol{\beta}^*, \boldsymbol{\gamma}^*, \boldsymbol{\psi}^*, \boldsymbol{\eta}^*, \boldsymbol{\theta}^*, \boldsymbol{\xi}^*, \boldsymbol{\lambda}^*, \tilde{\boldsymbol{\tau}}(\hat{\boldsymbol{\delta}}), \hat{\boldsymbol{\delta}})$ forms a feasible solution to the nonlinear optimization model defined in (B.10)–(B.14), with the same objective value as that of $(\boldsymbol{\beta}^*, \boldsymbol{\gamma}^*, \boldsymbol{\psi}^*, \boldsymbol{\eta}^*, \boldsymbol{\theta}^*, \boldsymbol{\xi}^*, \boldsymbol{\lambda}^*, \tilde{\boldsymbol{\tau}}^*, \boldsymbol{\delta}^*)$. Thus, $(\boldsymbol{\beta}^*, \boldsymbol{\gamma}^*, \boldsymbol{\psi}^*, \boldsymbol{\eta}^*, \boldsymbol{\theta}^*, \boldsymbol{\xi}^*, \boldsymbol{\lambda}^*, \tilde{\boldsymbol{\tau}}(\hat{\boldsymbol{\delta}}), \hat{\boldsymbol{\delta}})$ is also an optimal solution to the model defined in (B.10)–(B.14).

Consider any optimal solution $\hat{\boldsymbol{\delta}}$ to model $\mathbf{R}_1$. Due to the optimality of $\hat{\boldsymbol{\delta}}$, it can be seen that for any $(j,i) \in \mathcal{A}$ and $r \in \{1,2,...,|\mathcal{K}|\}$, if $\hat{\delta}_{jir} > 0$, then $\hat{\tau}_{jir}\Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta_i^{k*} - \lambda_i^{k*} + \sum_{k\in\mathcal{K}_i^d} z^k_{jir}(\psi^{k*} - \lambda_i^{k*})\Big) \geq 0$, and that if $\hat{\delta}_{jir} < 0$, then $\hat{\tau}_{jir}\Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta_i^{k*} - \lambda_i^{k*} + \sum_{k\in\mathcal{K}_i^d} z^k_{jir}(\psi^{k*} - \lambda_i^{k*})\Big) \leq 0$. This is because otherwise, $\hat{\boldsymbol{\delta}}$ cannot be an optimal solution to model $\mathbf{R}_1$, as we can increase its objective value by changing the sign of each $\hat{\delta}_{jir}$ with $\hat{\tau}_{jir}\Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta_i^{k*} - \lambda_i^{k*}) + \sum_{k\in\mathcal{K}_i^d} z^k_{jir}(\psi^{k*} - \lambda_i^{k*})\Big) \cdot \hat{\delta}_{jir} < 0$ to its opposite. Thus, we obtain that $\hat{\tau}_{jir}\Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta_i^{k*} - \lambda_i^{k*}) + \sum_{k\in\mathcal{K}_i^d} z^k_{jir}(\psi^{k*} - \lambda_i^{k*})\Big) \cdot \hat{\delta}_{jir} \geq 0$ for all $(j,i) \in \mathcal{A}$ and $r \in \{1,2,...,|\mathcal{K}|\}$. Accordingly, model $\mathbf{R}_1$ is equivalent to the following maximization LP, denoted as model $\mathbf{R}_2$:

$$
\begin{aligned}
[\mathbf{R}_2] \quad \max \quad & \sum_{(j,i)\in\mathcal{A}} \sum_{r=1}^{|\mathcal{K}|} \Big\{ \Big| \hat{\tau}_{jir}\Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta_i^{k*} - \lambda_i^{k*} + \sum_{k\in\mathcal{K}_i^d} z^k_{jir}(\psi^{k*} - \lambda_i^{k*})\Big) \Big| \cdot \delta^+_{jir} \Big\} \\
s.t. \quad & \sum_{(i,j)\in\mathcal{A}} \sum_{r=1}^{|\mathcal{K}|} \delta^+_{ijr} \leq \Gamma, \\
& 0 \leq \delta^+_{ijr} \leq 1, \quad \forall\ (i,j) \in \mathcal{A}, r \in \{1,2,...,|\mathcal{K}|\}.
\end{aligned}
$$

From any optimal solution $\boldsymbol{\delta}^+$ to model $\mathbf{R}_2$, we can derive an optimal solution to model $\mathbf{R}_1$ by setting $\delta_{jir} = \delta^+_{jir}$ if $\hat{\tau}_{jir}\Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta_i^{k*} - \lambda_i^{k*} + \sum_{k\in\mathcal{K}_i^d} z^k_{jir}(\psi^{k*} - \lambda_i^{k*})\Big) \geq 0$, and setting $\delta_{jir} = -\delta^+_{jir}$ otherwise, for each $(j,i) \in \mathcal{A}$ and $r \in \{1,2,...,|\mathcal{K}|\}$, so that their objective values are the same.

For model $\mathbf{R}_2$, its constraint matrix associated with $\sum_{(i,j)\in\mathcal{A}} \sum_{r=1}^{|\mathcal{K}|} \delta^+_{ijr} \leq \Gamma$ and $\delta^+_{ijr} \leq 1$ for all $(i,j) \in \mathcal{A}$ and $r \in \{1,2,...,|\mathcal{K}|\}$ is totally unimodular, as it contains two entries of 1 in each column. This implies that with an integral Γ, the feasible solution region of model $\mathbf{R}_2$ is an integral polytope. Thus, there exists an integral optimal solution to model $\mathbf{R}_2$ with $\delta^+_{ijr} \in \{0,1\}$ for each $(i,j) \in \mathcal{A}$ and $r \in \{1,2,...,|\mathcal{K}|\}$. This implies that there exists an optimal solution $\boldsymbol{\delta}$ to model $\mathbf{R}_1$ with $\delta_{ijr} \in \{-1,0,1\}$ for each $(i,j) \in \mathcal{A}$ and $r \in \{1,2,...,|\mathcal{K}|\}$. Therefore, there exists an optimal solution to the nonlinear optimization model defined in (B.10)–(B.14) that satisfies $\delta_{ijr} \in \{-1,0,1\}$ for each $(i,j) \in \mathcal{A}$ and $r \in \{1,2,...,|\mathcal{K}|\}$. Hence, Proposition 4 is proved. □

Continuing Proof of Proposition 3 We can now prove Proposition 3 as follows. By Proposition 4, constraints (B.13) can be replaced with $\delta_{ijr} \in \{-1,0,1\}$ for all $(i,j) \in \mathcal{A}$ and $r \in \{1,2,...,|\mathcal{K}|\}$. By (B.12) we have that $\tilde{\tau}_{ijr} \in \{\overline{\tau}_{ijr} - \hat{\tau}_{ijr}, \overline{\tau}_{ijr}, \overline{\tau}_{ijr} + \hat{\tau}_{ijr}\}$, which, together with $\overline{\tau}_{ijr} \in \mathbb{N}_{>0}$, $\hat{\tau}_{ijr} \in \mathbb{N}_0$ and $\overline{\tau}_{ijr} > \hat{\tau}_{ijr}$, implies that $\tilde{\tau}_{ijr} \in \mathbb{N}_{>0}$. Moreover, we introduce a new variable φ_{jir} to represent each nonlinear term $\Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta_i^k - \lambda_i^k) + \sum_{k\in\mathcal{K}_i^d} z^k_{jir}(\psi^k - \lambda_i^k)\Big) \cdot \tilde{\tau}_{jir}$. We then replace each integer variable δ_{jir} with three new binary variables $\zeta_{jir,-1}$, $\zeta_{jir,0}$ and $\zeta_{jir,1}$, which are used to indicate whether δ_{ijr} equals -1, 0 and 1, respectively. Let $\tilde{\tau}_{ijr,-1} = \overline{\tau}_{ijr} - \hat{\tau}_{ijr}$, $\tilde{\tau}_{ijr,0} = \overline{\tau}_{ijr}$ and $\tilde{\tau}_{ijr,1} = \overline{\tau}_{ijr} + \hat{\tau}_{ijr}$. Accordingly, the following linear constraints can be derived for the newly introduced variables, where M_3 is a sufficiently large constant.

$$
\zeta_{ijr,-1} + \zeta_{ijr,0} + \zeta_{ijr,1} = 1, \quad \forall\ (i,j) \in \mathcal{A}, r \in \{1,2,...,|\mathcal{K}|\}, \tag{B.15}
$$

$$-M_3(1-\zeta_{jir,\ell}) \le \varphi_{jir} - \Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta^k_i - \lambda^k_i) + \sum_{k\in\mathcal{K}^d_i} z^k_{jir}(\psi^k - \lambda^k_i)\Big)\tilde{\tau}_{jir,\ell} \le M_3(1-\zeta_{jir,\ell}),$$
$$\forall\ (j,i)\in\mathcal{A}, r\in\{1,2,...,|\mathcal{K}|\}, \ell = \{-1,0,1\}, \tag{B.16}$$

$$\zeta_{ijr,\ell}\in\{0,1\}, \quad \forall\ (i,j)\in\mathcal{A}, r\in\{1,2,...,|\mathcal{K}|\}, \ell=\{-1,0,1\}. \tag{B.17}$$

Here, constraints (B.15) ensure that exactly one of the three variables $\zeta_{jir,-1}$, $\zeta_{jir,0}$ and $\zeta_{jir,1}$ equals 1, constraints (B.16) ensure that each variable φ_{jir} equals $\Big(\sum_{k\in\mathcal{K}_i} z^k_{jir}(\beta^k_i - \lambda^k_i) + \sum_{k\in\mathcal{K}^d_i} z^k_{jir}(\psi^k - \lambda^k_i)\Big)\cdot\tilde{\tau}_{jir}$, and constraints (B.17) define the domain of the variables $\zeta_{ijr,-1}$, and $\zeta_{ijr,0}$, and $\zeta_{ijr,1}$. Thus, constraints $(B.14)$ can be replaced with the following linear constraint:

$$\sum_{(i,j)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|}(\zeta_{ijr,-1}+\zeta_{ijr,1}) \le \Gamma. \tag{B.18}$$

Therefore, the nonlinear optimization model defined by (B.10)–(B.14) for $F_{RP}(\boldsymbol{x},\boldsymbol{z})$ can be further reformulated to the following maximization MILP model:

$$\begin{aligned} F_{RP}(\boldsymbol{x},\boldsymbol{z}) = \max\ & \sum_{(j,i)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|}\varphi_{jir} - \sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}}(M_1 x^k_{ij})\cdot\eta^k_{ij} + \sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|}[M_1(z^k_{ijr}-1)]\cdot(\theta^k_{ijr}+\xi^k_{ijr}) \\ & + \sum_{k\in\mathcal{K}} e^k\cdot(\gamma^k - \lambda^k_{o^k}) + \sum_{k\in\mathcal{K}} l^k\cdot(\lambda^k_{d^k} - \psi^k) \\ \text{s.t.}\ & (\boldsymbol{\beta},\boldsymbol{\gamma},\boldsymbol{\psi},\boldsymbol{\eta},\boldsymbol{\theta},\boldsymbol{\xi},\boldsymbol{\lambda})\in\Omega,\ (\text{B.15}) - (\text{B.17}) \text{ and } (\text{B.18}). \end{aligned}$$

Hence, Proposition 3 is proved. $\square$

Appendix C: Models SP and MM

Given a set Π of scenarios assumed to be uniformly distributed, we can formulate a stochastic programming model as follows, aiming to minimize the expected total cost over all possible scenarios in Π.

$$[\text{SP}] \quad \min_{(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\overline{\boldsymbol{v}},\overline{\boldsymbol{b}})\in\mathcal{X}} \sum_{\boldsymbol{\delta}\in\Pi}\frac{1}{|\Pi|}\Big(\sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}}(c^k_{ij}q^k)x^k_{ij} + \sum_{(i,j)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|} f_{ij}y_{ijr} + F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))\Big)$$

This utilizes the second-stage cost $F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$, which is defined by a minimization LP, $LP(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$, for our models RO and RS. As a result, model SP is a minimization MILP.

Similarly, given Π we can formulate a min-max optimization model as follows, aiming to minimizing the worst-case total cost over all possible scenarios in Π.

$$[\text{MM}] \quad \min_{(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\overline{\boldsymbol{v}},\overline{\boldsymbol{b}})\in\mathcal{X}} \sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}}(c^k_{ij}q^k)x^k_{ij} + \sum_{(i,j)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|} f_{ij}y_{ijr} + \max_{\boldsymbol{\delta}\in\Pi} F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})).$$

Model MM can be further reformulated as follows:

$$\begin{aligned} \min_{\Phi,\ (\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\overline{\boldsymbol{v}},\overline{\boldsymbol{b}})\in\mathcal{X}}\ & \sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}}(c^k_{ij}q^k)x^k_{ij} + \sum_{(i,j)\in\mathcal{A}}\sum_{r=1}^{|\mathcal{K}|} f_{ij}y_{ijr} + \Phi \\ \text{s.t.}\ & \Phi \ge F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta})),\ \forall\boldsymbol{\delta}\in\Pi. \end{aligned}$$

It can be seen that the above reformulation of model MM is a minimization MILP, because $F_{LP}(\boldsymbol{x},\boldsymbol{z},\tilde{\boldsymbol{\tau}}(\boldsymbol{\delta}))$ is defined by a minimization LP.