On a Tractable Single-Level Reformulation of a Multilevel Model of the European Entry-Exit Gas Market with Market Power

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Abstract. We propose a framework that allows to quantitatively analyze the interplay of the different agents involved in gas trade and transport in the context of the European entry-exit system. While previous contributions focus on the case of perfectly competitive buyers and sellers of gas, our novel framework considers the mathematically more challenging case of a strategic and monopolistic gas seller. We present a multilevel framework that is suitable to capture the sequential nature of the decisions taken. We then derive sufficient conditions that allow for reformulating the challenging four-level model as a computationally tractable single-level reformulation. We prove the correctness of this reformulation and use it for solving several test instances to illustrate the applicability of our approach.

1. Introduction

We analyze the organization of gas trade and transport in the context of the European gas market design. In Europe, the so-called "entry-exit system" aims at largely separating gas trade and transport. Agents have to take an involved sequence of actions, which first lead to gas trading independently of grid restrictions and then transport of the traded volumes through the network. In particular, the transmission system operator (TSO) first announces so-called "technical capacities", which are capacities available to the buyers and sellers at the different entry and exit nodes. These technical capacities must be chosen such that any balanced set of quantities less than the technical capacity can be transported through the network. In a second step, suppliers and consumers "book" capacities at the nodes to secure access to the network, trade gas at the marketplace (being capacity-constrained by their bookings) and finally "nominate" the quantities bought and sold at the market for transport. Finally, the gas is routed through the network by the TSO, where feasibility is guaranteed by the ex ante restrictions on technical capacities. For the specific institutional rules we refer to [40] and [31].

It is important to notice that the entry-exit system hinders the efficient use of existing network capacities due to the restrictive requirements on technical capacities. This was not harmful when there was abundant network capacity available, as it has been the case for European gas markets. However, in the context of the green transition, pipeline-bound hydrogen-transport will gain importance. Future hydrogen networks will be build from parts of the existing natural gas networks, which implies that for both, the gas and the hydrogen network, transmission constraints will become an issue. Thus, a comprehensive understanding of the limitations and the efficiency losses of the entry-exit system is crucial to prepare the right regulatory decisions for the envisioned hydrogen gas market and networks.

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The current EU gas market design has already been analyzed in recent scientific contributions that focus on important aspects of properly organizing those markets. A recent strand of literature provides setups that allow to quantitatively assess gas trade and transport in the context of the entry-exit system. Due to the sequential decision-making structure outlined above, this involves multilevel approaches, which, in general, are difficult to tackle as it is known that even bilevel problems are hard to solve; see, e.g., [20, 30] for general hardness results. Recent articles, see, e.g., [6, 18], provide models and computational approaches under the assumption of perfectly competitive behavior of all agents acting on gas markets. In this case, modeling of booking and nomination can be jointly reformulated as decisions taken on the lower level of a bilevel model, whereas all decisions taken by the TSO are modeled in the resulting upper level. The obtained bilevel problem can then be finally reformulated as a single-level optimization problem.

However, market power and strategic behavior of gas suppliers play a crucial role in gas markets; see, e.g., [51]. It is the goal of our analysis to provide and solve a computationally tractable framework, which allows to analyze the interaction of the TSO, price-taking gas buyers, and a strategic monopolistic gas seller in the context of the European entry-exit system.

In more detail, as a starting point of our modeling approach, we consider the sequential decisions taken by the TSO (choice of technical capacities and routing of gas transport), a single strategic and profit-maximizing gas seller (booking and nomination) and gas buyers, which are assumed to be price takers (booking and nomination). The objectives of the seller and the buyers differ in this case. Unlike in a setup under the assumption of perfectly competitive agents, see, e.g., [6, 18, 23], the integration of seller and buyer decisions in a single optimization problem is not possible in this case. The resulting interaction of (i) the TSO while setting technical capacities for entry and exit nodes and routing the gas as well as of (ii) the seller and buyers while making their booking and nomination decisions in the context of the entry-exit system thus results in four different levels. The entire model is formulated in Section 2. As this cannot be solved directly, we derive a computationally solvable single-level reformulation of the overall problem in Section 3. Based on this reformulation, we can solve test instances and conduct performance checks; see Section 4.

Our work is related to several strands of literature. Given the relevance of gas markets in general and given the importance of strategic behavior especially on the supply side, there is a large and well-developed body of literature, which considers many different aspects of strategic supply decisions in this context. There are both highly stylized models, which allow for an analytical solution of the considered problem, as well as larger, more realistic setups, which have to be solved by appropriate computational methods. Prominent examples of stylized and, thus, analytically tractable frameworks are [13, 26, 28, 29, 34, 38, 39, 49]. Computational approaches, which often rely on formulations as complementarity problems are given by [2, 4, 5, 7–10, 14–16, 25, 27, 35, 36, 42, 45, 50]. Many of those contributions do consider some kind of network constraints, ranging from simple linear network flow problems to static but nonlinear gas flow models, which take into account the influence of pressure gradients; e.g., the so-called Weymouth equation [48].

All above-mentioned articles typically assume that congestion management of scarce network capacities is organized in an efficient and centralized manner. Hence, in contrast to our approach, those contributions do not explicitly model and analyze the specific rules of congestion management, which involve a sequence of decisions taken by the TSO, suppliers, and consumers.

Another strand of literature does consider the specific rules of congestion management in the context of the European entry-exit system. Those contributions are typically based on illustrative examples, which discuss the different aspects and implications of the market rules in detail. Those more policy-oriented contributions, however, do not aim at providing computational setups, which would allow to quantify the impact of specific institutional details; see, e.g., [3, 17, 19, 24, 47].

Very recently, several contributions provided setups, which indeed allow for a computational and thus quantitative analysis of the different specific aspects of the entry-exit system. Most of them are based on the model presented in [18]. In, [6], the authors exploit several reformulations to obtain an equivalent single-level model that they solve for different institutional setups of the entry-exit system. Going one step further, load uncertainty has been tackled in [23] by using chance-constrained modeling. For tree-structured networks, highly efficient solution techniques have been developed in [44] and a first step towards addressing active network elements such as compressors are given in [41] for a certain part of the four-level model presented in [18]. All those contributions provide important computational setups that allow to analyze different important aspects of the entry-exit system. In contrast to our contribution, however, they all focus on the case of perfectly competitive behavior of gas suppliers. In our work, we explicitly consider strategic behavior on the supply side.

The remainder of this paper is structured as follows. In Section 2, we present the four-level model with market-power aspects by including a monopolistic gas seller. Afterward, we study the market levels in more detail in Section 3 to obtain a single-level reformulation. Numerical results based on this single-level reformulation are presented and discussed in Section 4 before we close with some concluding remarks in Section 5.

2. The Four-Level Model with a Monopolistic Gas Selling Firm

It is our scope to analyze the sequential decisions of the TSO, who chooses technical capacities and routes gas, together with the decisions of booking and nomination made by firms. We consider the case of strategic booking and nomination decisions of a monopolistic gas seller. The fundamental timing of our setup, in principle, corresponds to the one introduced in [18]. However, as we consider strategic booking and nomination decisions of the gas selling firm, we have to disaggregate seller and buyer decisions into different levels. Formally, this sequential interaction can be described by the following four-level situation:

(i) Specification of technical capacities and booking price floors by the TSO.
(ii) Booking of capacity rights, day-ahead nominations, and setting of gas prices by the gas selling firm.
(iii) Booking of capacity rights and day-ahead nominations by gas buying firms.
(iv) Cost-optimal transport of the realized nominations by the TSO.

Similar to the four-level model proposed in [18], all decisions of the gas sellers and gas buyers are modeled in levels (ii) and (iii). However, in [18], all gas traders book capacities at level (ii) and they nominate quantities in level (iii). In the model we propose, the levels are split according to the different market participants, i.e., the gas seller and the gas buyers. The monopolistic gas selling firm decides on booked and nominated quantities as well as on gas prices at level (ii). The gas buying firms decide on their bookings and nominations at level (iii) based on these gas prices.

We also consider different pricing regimes by introducing price groups. The gas selling firm sets a price for each price group so that all gas buyers in one price group pay the same price. This allows to analyze and compare different pricing regimes like, e.g., nodal pricing, smaller or larger price groups, or a network-wide uniform price.

In the following, we state the multilevel model of the entry-exit market with a monopolistic gas selling firm. We model the gas network as directed and weakly connected graph $G = (V, E)$. The node set of this graph is partitioned into entry nodes V_+, at which the gas selling firm is located, exit nodes V_-, at which the consumers are located, and the remaining inner nodes V_0. We consider a finite set of time periods T and denote the optimal value function of level ℓ with φ^ℓ.

2.1. **Level (i): Specification of Technical Capacities and Booking Price Floors by the TSO.** The player of level (i) is the transmission system operator (TSO), who specifies technical capacities $q^{\mathrm{TC}} := (q_u^{\mathrm{TC}})_{u \in V_+ \cup V_-}$ and booking price floors $\underline{\pi}^{\mathrm{book}} := (\underline{\pi}_u^{\mathrm{book}})_{u \in V_+ \cup V_-}$ for all entry and exit nodes. By doing so, the TSO has to ensure that the bookings and nominations that realize in levels (ii) and (iii) can be transported through the network. The optimization problem of the TSO in level (i) reads

$$\max_{q^{\mathrm{TC}}, \underline{\pi}^{\mathrm{book}}} \quad \sum_{t \in T} \left(\sum_{u \in V_-} \int_0^{q_{u,t}^{\mathrm{nom}}} P_{u,t}(s) \, \mathrm{d}s - \sum_{u \in V_+} c_u^{\mathrm{var}} q_{u,t}^{\mathrm{nom}} \right) - \varphi^4(q^{\mathrm{nom}}) - C \tag{1a}$$

$$\text{s.t.} \quad 0 \le q_u^{\mathrm{TC}}, \ 0 \le \underline{\pi}_u^{\mathrm{book}}, \quad u \in V_+ \cup V_-, \tag{1b}$$

$$\sum_{u \in V_+ \cup V_-} \underline{\pi}_u^{\mathrm{book}} q_u^{\mathrm{book}} = \varphi^4(q^{\mathrm{nom}}) + C, \tag{1c}$$

$$\left(q_+^{\mathrm{book}}, q_+^{\mathrm{nom}}, \pi^{\mathrm{nom}}\right) \text{ solves (2)}, \tag{1d}$$

$$(p, q) \text{ solves (4)}. \tag{1e}$$

Constraint (1c) models that the transport costs $\varphi^4(q^{\mathrm{nom}})$ arising in level (iv) plus additionally given network costs $C \ge 0$ have to be recovered. Note that we consider the optimistic setting, i.e., in case of multiple optimal solutions of the lower levels, the TSO chooses the welfare-maximizing solution. As mentioned above, in our setup, the TSO only has to ensure that the realized nominations from levels (ii) and (iii) have to be physically feasible. For further discussion of more robust settings see, e.g., [6, 23].

2.2. **Level (ii): Booking of Capacity Rights, Day-Ahead Nominations, and Setting of Gas Prices by the Gas Selling Firm.** The player of level (ii) is the monopolistic gas selling firm, which is located at the entry nodes V_+ and has variable production costs $c_u^{\mathrm{var}} > 0$ for all $u \in V_+$. This firm has to take into account the technical capacities q^{TC} as well as booking price floors $\underline{\pi}_u^{\mathrm{book}}$ at all entry and exit nodes $u \in V_+ \cup V_-$, which are given from level (i). It maximizes its profit by booking input capacities $q_+^{\mathrm{book}} := (q_u^{\mathrm{book}})_{u \in V_+}$, choosing nominations $q_+^{\mathrm{nom}} := (q_{u,t}^{\mathrm{nom}})_{u \in V_+, t \in T}$, and setting gas prices π^{nom} at the exit nodes V_-, to which the consumers react as price takers.

We assume that the exit nodes are partitioned into different price groups and that the monopolistic firm sets a gas price being the same for all consumers in each price group. This allows us to consider and compare different pricing regimes. In the literature, this is typically referred to as third-degree price discrimination; see, e.g., [1] for a reference. Let $V_1, \dots, V_k$ be a partition of the exit nodes V_- into k price groups. We use the notation $[k] := \{1, \dots, k\}$ and denote the gas prices by $(\pi_{i,t}^{\mathrm{nom}})_{i \in [k], t \in T}$.

Note that in level (ii), the monopolistic firm decides on bookings q_+^{book} and nominations q_+^{nom} as well as on gas prices π^{nom}, while anticipating the bookings q_-^{book} and nominations q_-^{nom} of the gas buying firms, which are chosen in level (iii).

The monopolistic firm's optimization problem is given by

$$\max_{q_+^{\text{book}}, q_+^{\text{nom}}, \pi^{\text{nom}}} \quad \sum_{t \in T} \left(\sum_{i=1}^{k} \pi_{i,t}^{\text{nom}} \sum_{u \in V_i} q_{u,t}^{\text{nom}} - \sum_{u \in V_+} c_u^{\text{var}} q_{u,t}^{\text{nom}} \right) - \sum_{u \in V_+} \underline{\pi}_u^{\text{book}} q_u^{\text{book}} \tag{2a}$$

$$\text{s.t.} \quad q_u^{\text{book}} \le q_u^{\text{TC}}, \quad u \in V_+, \tag{2b}$$

$$0 \le q_{u,t}^{\text{nom}} \le q_u^{\text{book}}, \quad u \in V_+, \ t \in T, \tag{2c}$$

$$\sum_{u \in V_-} q_{u,t}^{\text{nom}} - \sum_{u \in V_+} q_{u,t}^{\text{nom}} = 0, \quad t \in T, \tag{2d}$$

$$\left(q_-^{\text{book}}, q_-^{\text{nom}}\right) \text{ solves (3)}. \tag{2e}$$

The monopolistic firm's profit (2a) is the difference between the aggregated revenues from selling gas and the costs of the nominated and booked quantities. Constraints (2b)–(2d) model the restriction by technical capacities, compliance with booked capacities, and market clearing.[1] Constraint (2e) models the reactions of the consumers to the market prices $\pi_{i,t}^{\text{nom}}$ set by the monopolistic firm. The existence of a solution of Problem (2) can be ensured under Assumptions 2.1 and 2.2; see Lemma 3.2 below.

In the context of such a multilevel market model, a common concern is strategic over-booking, which means that a firm or consumer books a capacity that is larger than its maximal nomination in order to prevent its competitors from using this capacity. Note that in our setup, the gas selling firm has no incentive to over-book, i.e., we have $q_u^{\text{book}} = \max_{t \in T}\{q_{u,t}^{\text{nom}}\}$ for all entry nodes $u \in V_+$.

2.3. **Level (iii): Booking of Capacity Rights and Day-Ahead Nominations by Gas Buying Firms.** The players of level (iii) are the gas buying firms, which are located at the exit nodes V_-. They observe the gas prices π^{nom}, booking prices $\underline{\pi}_u^{\text{book}}$, and technical capacities q_u^{TC} given from levels (i) and (ii), and react to these prices by booking capacities $q_-^{\text{book}} := (q_u^{\text{book}})_{u \in V_-}$ and by deciding on their nominations $q_-^{\text{nom}} := (q_{u,t}^{\text{nom}})_{u \in V_-, t \in T}$. Demand at the exit nodes is modeled by inverse demand functions $P_{u,t}$, for which we make the following assumption.

Assumption 2.1. *All inverse demand functions are linear and strictly decreasing. Moreover, the intercepts are the same within each price group, i.e.,*

$$P_{u,t}(q_{u,t}^{\text{nom}}) = a_{i,t} - b_{u,t} q_{u,t}^{\text{nom}}$$

holds with $a_{i,t}, b_{u,t} > 0$ for all nodes $u \in V_i$ in price group $i \in [k]$ and all time periods $t \in T$.

Note that the specification of demand depending linearly on market prices (inducing quadratic consumer surplus in the objective functions) is very common in the entire literature providing computational approaches for solving equilibrium problems in energy markets; see, e.g., [9, 14, 49] for some examples. Intercepts in our setup can vary with time periods t. The assumption of equal intercepts $a_{i,t}$ for all nodes in a price group is worthwhile to note. This does restrict the flexibility of our framework. The assumption is required, however, to guarantee concavity of the maximization problem solved by the seller; see Lemma 3.2 and Problem (13) below.

Regarding symmetry of consumption at different nodes of the same price group, observe that slopes $b_{u,t}$ of demand can be different at each node and for each time

[1] Observe that the monopolistic seller thus faces capacity constraints both regarding its own output at the entry nodes as well as its sales at the different exit nodes. Related problems in which monopolistic sellers face different kinds of capacity constraints are prominent in the recent literature; see, e.g., [46].

period. This allows for different consumption levels at the different nodes within the same price group in our setup.

All gas buyers are assumed to be price takers as, e.g., in [18]. Given the prices for bookings ($\underline{\pi}_u^{\text{book}}$) and nominations ($\pi_{i,t}^{\text{nom}}$), the booking and nomination decisions of the gas buyers at each exit node $u \in V_i$ in price group $i \in [k]$ can thus be obtained as the solution of the optimization problem

$$\max_{q_u^{\text{book}}, q_u^{\text{nom}}} \quad \varphi_u(q_u^{\text{book}}, q_u^{\text{nom}}) \tag{3a}$$

$$\text{s.t.} \quad q_u^{\text{book}} \le q_u^{\text{TC}}, \tag{3b}$$

$$0 \le q_{u,t}^{\text{nom}} \le q_u^{\text{book}}, \quad t \in T, \tag{3c}$$

with

$$\varphi_u(q_u^{\text{book}}, q_u^{\text{nom}}) := \sum_{t \in T} \left(\int_0^{q_{u,t}^{\text{nom}}} P_{u,t}(s) \, \mathrm{d}s - \pi_{i,t}^{\text{nom}} q_{u,t}^{\text{nom}} \right) - \underline{\pi}_u^{\text{book}} q_u^{\text{book}}.$$

Here, we use the notation $q_u^{\text{nom}} := (q_{u,t}^{\text{nom}})_{t \in T}$. The existence and uniqueness of a solution of these problems under Assumption 2.1 is discussed in Lemma 3.1 below.

We make the assumption that all network fees are collected from the seller. For perfectly competitive sellers and buyers it is irrelevant for the resulting market equilibrium and the tax incidence who nominally pays a tax; see, e.g., [33]. In our setup, which considers the interplay of gas transport with gas trade and a strategic seller, this equivalence does not always hold. Nevertheless, this assumption is crucial in the next step, because it allows us to eliminate $\underline{q}_{-}^{\text{book}}$ and to obtain the reformulated problem (13).

Assumption 2.2. *For all exit nodes $u \in V_-$ it holds $\underline{\pi}_u^{\text{book}} = 0$ and $q_u^{\text{TC}} > 0$.*

2.4. **Level (iv): Cost-Optimal Transport of the Realized Nominations by the TSO.** The player of level (iv) is the TSO again, who minimizes transport costs resulting from the nominations of the gas seller and the gas buyers. For the modeling of gas physics, we use the nonlinear flow model introduced in [44]. The optimization problem of the TSO in level (iv) is given by

$$\min_{p,q} \quad \sum_{t \in T} c_t^{\text{trans}} (p, q; q^{\text{nom}}) \tag{4a}$$

$$\text{s.t.} \quad (p, q) \in \mathcal{F}(q^{\text{nom}}), \text{i.e., } (p, q) \text{ fulfills (6)–(9)}, \tag{4b}$$

where the transport costs are defined (with a slight abuse of notation) as

$$c_t^{\text{trans}} (p, q; q^{\text{nom}}) = \sum_{e=(u,v) \in E} c_t^{\text{trans}} \left| p_{u,t}^2 - p_{v,t}^2 \right| . \tag{5}$$

The feasible set $\mathcal{F}(q^{\text{nom}})$ models all existing restrictions on the gas pressures $p := (p_{u,t})_{u \in V, t \in T}$ and gas mass flows $q := (q_{e,t})_{e \in E, t \in T}$. These are flow bounds

$$-\infty < q_e^- \le q_{e,t} \le q_e^+ < \infty, \quad e \in E, \ t \in T, \tag{6}$$

balance of flows

$$\sum_{e \in \delta^{\text{out}}(u)} q_{e,t} - \sum_{e \in \delta^{\text{in}}(u)} q_{e,t} = q_{u,t}^{\text{nom}}, \quad u \in V_+, \ t \in T, \tag{7a}$$

$$\sum_{e \in \delta^{\text{out}}(u)} q_{e,t} - \sum_{e \in \delta^{\text{in}}(u)} q_{e,t} = -q_{u,t}^{\text{nom}}, \quad u \in V_-, \ t \in T, \tag{7b}$$

$$\sum_{e \in \delta^{\text{out}}(u)} q_{e,t} - \sum_{e \in \delta^{\text{in}}(u)} q_{e,t} = 0, \quad u \in V_0, \ t \in T, \tag{7c}$$

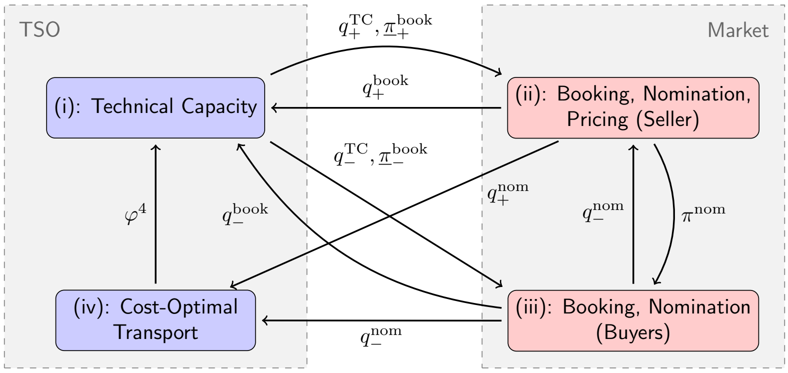


FIGURE 1. Illustration of dependencies between the four levels.

as well as pressure loss constraints and pressure bounds

$$p_{u,t}^2 - p_{v,t}^2 = \Lambda_e q_{e,t}|q_{e,t}|, \quad e = (u,v) \in E, \ t \in T, \tag{8}$$

$$0 < p_u^- \le p_{u,t} \le p_u^+ \le \infty, \quad u \in V, \ t \in T. \tag{9}$$

Here, $\delta^{\text{in}}(u)$ and $\delta^{\text{out}}(u)$ represent in- and outgoing arcs at node u, $\Lambda_e > 0$ is an arc specific constant and $c_t^{\text{trans}} > 0$ represents the transport costs in time period t.

In order to computationally solve instances of this problem, we replace the absolute values in the objective function, see (4a) and (5), and in Constraint (8) of Problem (4) in our implementation as described in the following remark.

Remark 2.3. *We model the absolute values $|q_e|$ in Constraint (8) of Problem (4) by splitting q_e into positive and negative parts by adding the constraints $q_{e,t} = q_{e,t}^{\text{pos}} - q_{e,t}^{\text{neg}}$ and $q_{e,t}^{\text{pos}} q_{e,t}^{\text{neg}} = 0$; see, e.g., [21]. Moreover, we replace the pressure variables by variables for squared pressure $\tilde{p} := p^2$. Thus, we replace Constraint (8) by*

$$\tilde{p}_{u,t} - \tilde{p}_{v,t} = \Lambda_e q_e \left(q_{e,t}^{\text{pos}} + q_{e,t}^{\text{neg}}\right).$$

We also replace the transport costs in the objective function, see (4a) and (5), of Problem (4) by

$$c_t^{\text{trans}} \left|p_{u,t}^2 - p_{v,t}^2\right| = c_t^{\text{trans}} \Lambda_e \left(q_{e,t}^{\text{pos}} + q_{e,t}^{\text{neg}}\right)^2.$$

Bounds for the new flow variables $q_{e,t}^{\text{pos}}$ and $q_{e,t}^{\text{neg}}$ are given by $\max\{0, q_e^-\} \le q_{e,t}^{\text{pos}} \le \max\{0, q_e^+\}$ as well as $\max\{0, -q_e^+\} \le q_{e,t}^{\text{neg}} \le \max\{0, -q_e^-\}$ and bounds for the new pressure variables $\tilde{p}$ are given by $(p_u^-)^2 \le \tilde{p}_{u,t} \le (p_u^+)^2$.

Figure 1 illustrates the dependencies between the four levels.

3. Single-Level Reformulation

We now provide a reformulation of the four-level model into a single-level optimization problem. To this end, we begin by integrating levels (i) and (iv) in one problem in Section 3.1. Then, we reformulate levels (ii) and (iii) as one concave maximization problem in Section 3.2, which can be replaced by its KKT conditions. In Section 3.3 we combine those results to obtain a single-level optimization problem, whose solutions correspond to the solutions of the four-level model described in Section 2.

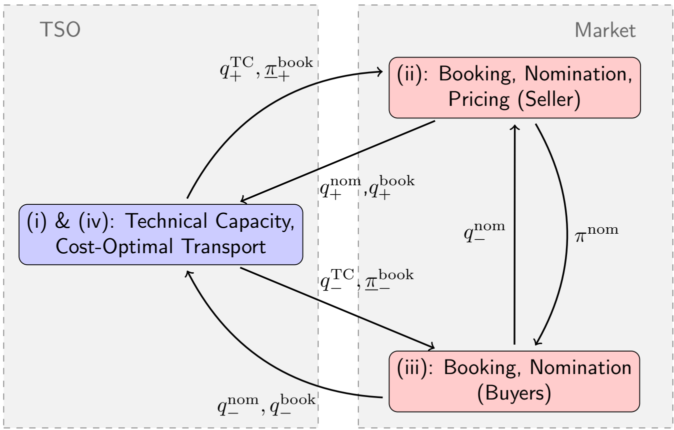


FIGURE 2. Illustration of dependencies between the three levels.

3.1. **Aggregation of Level (i) and Level (iv).** It follows from Theorem 7 of [18], see also Problem (13) in [6], that the TSO levels (i) and (iv) can be aggregated in the following way:

$$\max_{q^{\mathrm{TC}}, \underline{\pi}^{\mathrm{book}}, p, q} \quad \sum_{t \in T} \left(\sum_{u \in V_-} \int_0^{q_{u,t}^{\mathrm{nom}}} P_{u,t}(s)\, \mathrm{d}s - \sum_{u \in V_+} c_u^{\mathrm{var}} q_{u,t}^{\mathrm{nom}} \right) - \sum_{t \in T} c_t^{\mathrm{trans}}\left(p, q; q^{\mathrm{nom}}\right) - C \tag{10a}$$

$$\text{s.t.} \quad 0 \le q_u^{\mathrm{TC}}, \ 0 \le \underline{\pi}_u^{\mathrm{book}}, \quad u \in V_+ \cup V_-, \tag{10b}$$

$$\sum_{u \in V_+ \cup V_-} \underline{\pi}_u^{\mathrm{book}} q_u^{\mathrm{book}} = \sum_{t \in T} c_t^{\mathrm{trans}}\left(p, q; q^{\mathrm{nom}}\right) + C, \tag{10c}$$

$$(p, q) \in \mathcal{F}\left(q^{\mathrm{nom}}\right), \tag{10d}$$

$$\left(q_+^{\mathrm{book}}, q_+^{\mathrm{nom}}, \pi^{\mathrm{nom}}\right) \text{ solves } (2). \tag{10e}$$

Figure 2 illustrates the dependencies between the three levels.

3.2. **Aggregation of Level (ii) and Level (iii).** To aggregate level (ii) and (iii) into one optimization problem, we use that the solutions of level (iii) can be computed explicitly.

Lemma 3.1. *Suppose Assumption 2.1 holds. Then, Problem (3) has an optimal solution for all entry nodes $u \in V_i$ in all price groups $i \in [k]$. The optimal consumers' nominations are unique and given by*

$$q_{u,t}^{\mathrm{nom}} = \max\left\{0, \min\left\{\frac{a_{i,t} - \pi_{i,t}^{\mathrm{nom}}}{b_{u,t}}, \ q_u^{\mathrm{book}}\right\}\right\} \tag{11}$$

for all $t \in T$. For all exit nodes $u \in V_-$ with $\underline{\pi}_u^{\mathrm{book}} > 0$, the optimal consumers' bookings are also unique and given by $q_u^{\mathrm{book}} = \max_{t \in T}\{q_{u,t}^{\mathrm{nom}}\}$. For all exit nodes $u \in V_-$ with $\underline{\pi}_u^{\mathrm{book}} = 0$, the optimal consumers' bookings are from the interval $[\max_{t \in T}\{q_{u,t}^{\mathrm{nom}}\}, q_u^{\mathrm{TC}}]$.

Proof. Consider an arbitrary price group $i \in [k]$ and an entry node $u \in V_i$. Then, the existence of a solution follows from the theorem of Weierstraß. The feasible set of Problem (3) is convex and the objective function is concave and strictly concave in $q_{u,t}^{\mathrm{nom}}$. Thus, the optimal nominations are unique due to Lemma 1 of [32]. The formula for these optimal nominations follows from fixing q_u^{book}, computing the unconstrained global maximum of the quadratic and strictly concave objective function w.r.t. $q^{\mathrm{nom}}_{u,t}$ and projecting it onto the feasible interval $[0, q^{\mathrm{book}}_u]$. The formula for the optimal bookings follows from the fact that the objective function is strictly decreasing in q^{book}_u in case $\underline{\pi}^{\mathrm{book}}_u > 0$ and from the feasibility constraints in case $\underline{\pi}^{\mathrm{book}}_u = 0$. □

From now on, we impose Assumption 2.2, i.e., that the gas buyers do not have to pay anything for their booked capacity. In this case $q^{\mathrm{book}}_u = q^{\mathrm{TC}}_u$ is always an optimal choice according to Lemma 3.1 and the optimal nominations are given by

$$q^{\mathrm{nom}}_{u,t} = \max\left\{0, \min\left\{\frac{a_{i,t} - \pi^{\mathrm{nom}}_{i,t}}{b_{u,t}}, q^{\mathrm{TC}}_u\right\}\right\} \quad u \in V_i,\ i \in [k],\ t \in T. \tag{12}$$

Using this, we can show that Problem (2) is equivalent to the problem

$$\max_{\substack{q^{\mathrm{book}}_+, q^{\mathrm{nom}}_+,\\ \pi^{\mathrm{nom}}, q^{\mathrm{nom}}_-}} \quad \psi(q^{\mathrm{book}}_+, q^{\mathrm{nom}}_+, \pi^{\mathrm{nom}}, q^{\mathrm{nom}}_-) \tag{13a}$$

$$\text{s.t.} \quad q^{\mathrm{book}}_u \le q^{\mathrm{TC}}_u, \quad u \in V_+, \qquad [\pi^{\mathrm{book}}_u] \tag{13b}$$

$$0 \le q^{\mathrm{nom}}_{u,t} \le q^{\mathrm{book}}_u, \quad u \in V_+,\ t \in T, \qquad [\gamma^-_{u,t}, \gamma^+_{u,t}] \tag{13c}$$

$$\sum_{i=1}^{k} \sum_{u \in V_i} q^{\mathrm{nom}}_{u,t} - \sum_{u \in V_+} q^{\mathrm{nom}}_{u,t} = 0, \quad t \in T, \qquad [\lambda_t] \tag{13d}$$

$$0 \le q^{\mathrm{nom}}_{u,t} \le q^{\mathrm{TC}}_u, \quad u \in V_-,\ t \in T, \qquad [\gamma^-_{u,t}, \gamma^+_{u,t}] \tag{13e}$$

$$P_{u,t}(q^{\mathrm{nom}}_{u,t}) - \pi^{\mathrm{nom}}_{i,t} \ge 0, \quad u \in V_i,\ i \in [k],\ t \in T, \qquad [\alpha_{u,t}] \tag{13f}$$

with objective function

$$\begin{aligned} &\psi(q^{\mathrm{book}}_+, q^{\mathrm{nom}}_+, \pi^{\mathrm{nom}}, q^{\mathrm{nom}}_-) \\ := &\sum_{t \in T} \left(\sum_{i=1}^{k} \sum_{u \in V_i} P_{u,t}(q^{\mathrm{nom}}_{u,t}) q^{\mathrm{nom}}_{u,t} - \left(P_{u,t}(q^{\mathrm{nom}}_{u,t}) - \pi^{\mathrm{nom}}_{i,t}\right) q^{\mathrm{TC}}_u \right) \\ &- \sum_{t \in T} \left(\sum_{u \in V_+} c^{\mathrm{var}}_u q^{\mathrm{nom}}_{u,t} \right) - \sum_{u \in V_+} \underline{\pi}^{\mathrm{book}}_u q^{\mathrm{book}}_u. \end{aligned}$$

Note that Problem (13) is a concave maximization problem with linear constraints, and therefore, its KKT conditions are necessary and sufficient for global solutions. We use this later to obtain the single-level reformulation of the entire problem described in Section 2. For this reason, the dual variables corresponding to the constraints are already indicated in brackets in Problem (13).

Lemma 3.2. *Suppose Assumptions 2.1 and 2.2 hold. Then, the following is true:*

(a) *If $(q^{\mathrm{book}}_+, q^{\mathrm{nom}}_+, \pi^{\mathrm{nom}})$ is feasible for Problem (2), then $(q^{\mathrm{book}}_+, q^{\mathrm{nom}}_+, \tilde{\pi}^{\mathrm{nom}}, q^{\mathrm{nom}}_-)$ with q^{nom}_- as defined in (12) and*

$$\tilde{\pi}^{\mathrm{nom}}_{i,t} = \min\{\pi^{\mathrm{nom}}_{i,t}, a_{i,t}\} \quad i \in [k],\ t \in T,$$

is feasible for Problem (13) with the same objective function value.

(b) *If $(q^{\mathrm{book}}_+, q^{\mathrm{nom}}_+, \pi^{\mathrm{nom}}, q^{\mathrm{nom}}_-)$ is a solution of Problem (13), then $(q^{\mathrm{book}}_+, q^{\mathrm{nom}}_+, \pi^{\mathrm{nom}})$ is feasible for Problem (2) with the same objective function value.*

(c) *Problems (2) and (13) both attain a solution and the optimal objective function value of both problems is the same.*

Proof. We begin the proof with a closer look at the modified part of the objective function of Problem (13). For all $i \in [k]$, a slight reformulation shows

$$\sum_{u \in V_i} \left(P_{u,t}(q_{u,t}^{\text{nom}}) q_{u,t}^{\text{nom}} - \left(P_{u,t}(q_{u,t}^{\text{nom}}) - \pi_{i,t}^{\text{nom}} \right) q_u^{\text{TC}} \right)$$
$$= \pi_{i,t}^{\text{nom}} \sum_{u \in V_i} \underbrace{q_{u,t}^{\text{nom}}}_{\geq 0} + \sum_{u \in V_i} \underbrace{\left(P_{u,t}(q_{u,t}^{\text{nom}}) - \pi_{i,t}^{\text{nom}} \right)}_{\geq 0} \underbrace{\left(q_{u,t}^{\text{nom}} - q_u^{\text{TC}} \right)}_{\leq 0}. \tag{14}$$

The signs indicated in the second sum hold for all feasible points of Problem (13).

(a) Every price $\pi_{i,t}^{\text{nom}} \geq a_{i,t}$ implies $q_{u,t}^{\text{nom}} = 0$ for all $u \in V_i$ by Lemma 3.1. The gas seller is thus indifferent between prices $\pi_{i,t}^{\text{nom}} \geq a_{i,t}$ and we can switch from π^{nom} to $\tilde{\pi}^{\text{nom}}$ in Problem (2) without affecting feasibility or the value of the objective function.

Then, the feasibility of $(q_+^{\text{book}}, q_+^{\text{nom}}, \tilde{\pi}^{\text{nom}})$ for Problem (2) and the formula for q_-^{nom} from (12) immediately show that $(q_+^{\text{book}}, q_+^{\text{nom}}, \tilde{\pi}^{\text{nom}}, q_-^{\text{nom}})$ is feasible for Problem (13). Furthermore, observation (14) together with (12) shows that both objective functions attain the same value, because due to the modified prices $\tilde{\pi}^{\text{nom}}$ and Assumption 2.2 for all $u \in V_i$, one of the two factors in the second sum is zero.

(b) The constraints (13e) and (13f) are equivalent to

$$0 \leq q_{u,t}^{\text{nom}} \leq \min \left\{ q_u^{\text{TC}}, \frac{a_{i,t} - \pi_{i,t}^{\text{nom}}}{b_{u,t}} \right\} \quad u \in V_i, \ i \in [k], \ t \in T. \tag{15}$$

Now, assume that there exists a time period t and a price group i with

$$0 \leq q_{u,t}^{\text{nom}} < \min \left\{ q_u^{\text{TC}}, \frac{a_{i,t} - \pi_{i,t}^{\text{nom}}}{b_{u,t}} \right\} \leq \frac{a_{i,t} - \pi_{i,t}^{\text{nom}}}{b_{u,t}} \tag{16}$$

for at least one node $u \in V_i$ and thus $a_{i,t} - \pi_{i,t}^{\text{nom}} > 0$ holds. If we increase the corresponding price $\pi_{i,t}^{\text{nom}}$ to $\pi_{i,t}^{\text{nom}} + \varepsilon$ with $\varepsilon > 0$, in all possibly existing nodes $u \in V_i$ with

$$q_{u,t}^{\text{nom}} = \frac{a_{i,t} - \pi_{i,t}^{\text{nom}}}{b_{u,t}} > 0,$$

we have to decrease the nomination to $q_{u,t}^{\text{nom}} - \varepsilon / b_{u,t}$ to retain feasibility. However, since we know that there exists at least one node $u \in V_i$ satisfying (16), in which we can increase $q_{u,t}^{\text{nom}}$ slightly, for $\varepsilon > 0$ sufficiently small it is possible to adjust $q_{u,t}^{\text{nom}}$, $u \in V_i$, such that we retain feasibility for the higher price $\pi_{i,t}^{\text{nom}} + \varepsilon$, see (15), and do not change $\sum_{u \in V_i} q_{u,t}^{\text{nom}}$. This process can be repeated until

$$q_{u,t}^{\text{nom}} = \min \left\{ q_u^{\text{TC}}, \frac{a_{i,t} - \pi_{i,t}^{\text{nom}}}{b_{u,t}} \right\}$$

holds for all $u \in V_i$. Since we did not change the sum $\sum_{u \in V_i} q_{u,t}^{\text{nom}}$ and ensured that the bounds from (15) are still satisfied, the modified point remains feasible for Problem (13). However, the modified point has a better objective function value, because in (14) we increased $\pi_{i,t}^{\text{nom}}$ by $\varepsilon > 0$ and we increased the second sum from a negative value to zero. This shows that a solution of Problem (13) has to satisfy (12) and, consequently, $(q_+^{\text{book}}, q_+^{\text{nom}}, \pi^{\text{nom}})$ is feasible for Problem (2).

As discussed in (a), in points satisfying (12) the objective function values of both problems coincide.

(c) Problem (13) has a solution according to the theorem of Weierstraß, because its feasible set is a nonempty polytope and its objective function is continuous. Then, using (b), we know that Problem (2) has a feasible point with the

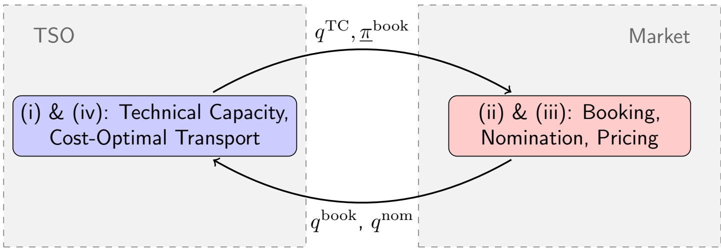


FIGURE 3. Illustration of dependencies between the two levels.

same objective function value. If this would not be a solution of Problem (2), there would exist a feasible point of Problem (2) with a larger objective function value. To this point, according to (a), a feasible point of Problem (13) corresponds with the same larger objective function value. This is a contradiction to having started the discussion with a solution of Problem (13). □

We can thus solve levels (ii) and (iii) by solving Problem (13). Figure 3 illustrates the dependencies between the remaining two levels.

Note that, in Problem (13), the optimal nominations q_-^{nom} and the optimal prices π^{nom} are uniquely determined and, in case the booking costs $\underline{\pi}^{\mathrm{book}}$ are positive, the optimal bookings q_+^{book} are uniquely determined by the nominations q_+^{nom}. However, the optimal nominations q_+^{nom} are not necessarily unique. For this reason, we would like to recall that we consider an optimistic setting, i.e., in case of multiple solutions of Problem (13) the TSO chooses the welfare-optimal one in the level above.

The KKT conditions for the concave maximization Problem (13) consist of the stationarity conditions

$$-\underline{\pi}_u^{\mathrm{book}} - \pi_u^{\mathrm{book}} + \sum_{t\in T} \gamma_{u,t}^+ = 0, \quad u \in V_+, \tag{17a}$$

$$-c_u^{\mathrm{var}} + \gamma_{u,t}^- - \gamma_{u,t}^+ + \lambda_t = 0, \quad u \in V_+,\ t \in T, \tag{17b}$$

$$\sum_{u\in V_i} q_u^{\mathrm{TC}} - \sum_{u\in V_i} \alpha_{u,t} = 0, \quad i \in [k],\ t \in T, \tag{17c}$$

$$\begin{aligned} P_{u,t}(q_{u,t}^{\mathrm{nom}}) + b_{u,t}\left(q_u^{\mathrm{TC}} - q_{u,t}^{\mathrm{nom}} - \alpha_{u,t}\right)& \\ + \gamma_{u,t}^- - \gamma_{u,t}^+ - \lambda_t = 0,& \quad u \in V_i,\ i \in [k],\ t \in T, \end{aligned} \tag{17d}$$

primal feasibility conditions

$$q_u^{\mathrm{book}} \le q_u^{\mathrm{TC}}, \quad u \in V_+, \tag{18a}$$

$$0 \le q_{u,t}^{\mathrm{nom}} \le q_u^{\mathrm{book}}, \quad u \in V_+,\ t \in T, \tag{18b}$$

$$\sum_{i=1}^{k} \sum_{u\in V_i} q_{u,t}^{\mathrm{nom}} - \sum_{u\in V_+} q_{u,t}^{\mathrm{nom}} = 0, \quad t \in T, \tag{18c}$$

$$0 \le q_{u,t}^{\mathrm{nom}} \le q_u^{\mathrm{TC}}, \quad u \in V_-,\ t \in T, \tag{18d}$$

$$P_{u,t}(q_{u,t}^{\mathrm{nom}}) - \pi_{i,t}^{\mathrm{nom}} \ge 0, \quad u \in V_i,\ i \in [k],\ t \in T, \tag{18e}$$

dual feasibility conditions

$$\pi_u^{\mathrm{book}} \ge 0, \quad u \in V_+, \tag{19a}$$

$$\gamma_{u,t}^-, \gamma_{u,t}^+ \ge 0, \quad u \in V_- \cup V_+,\ t \in T, \tag{19b}$$

$$\alpha_{u,t} \ge 0, \quad u \in V_-,\ t \in T, \tag{19c}$$

and the complementary conditions

$$\pi_u^{\text{book}}\left(q_u^{\text{TC}} - q_u^{\text{book}}\right) = 0, \quad u \in V_+, \tag{20a}$$

$$\gamma_{u,t}^- q_{u,t}^{\text{nom}} = 0, \quad u \in V_+ \cup V_-,\ t \in T, \tag{20b}$$

$$\gamma_{u,t}^+\left(q_u^{\text{book}} - q_{u,t}^{\text{nom}}\right) = 0, \quad u \in V_+,\ t \in T, \tag{20c}$$

$$\gamma_{u,t}^+\left(q_u^{\text{TC}} - q_{u,t}^{\text{nom}}\right) = 0, \quad u \in V_-,\ t \in T, \tag{20d}$$

$$\alpha_{u,t}\left(P_{u,t}(q_{u,t}^{\text{nom}}) - \pi_{i,t}^{\text{nom}}\right) = 0, \quad u \in V_i,\ i \in [k],\ t \in T. \tag{20e}$$

3.3. **Single-Level Optimization Problem.** Combining the results from Sections 3.1 and 3.2, we obtain the following single-level reformulation of the four-level problem described in Section 2 by including the KKT conditions of Problem (13) into the combined TSO Problem (10):

$$\max_{\substack{q^{\text{TC}},\underline{\pi}^{\text{book}},p,q,\\ q_+^{\text{book}},q^{\text{nom}},\pi^{\text{nom}},\\ \pi^{\text{book}},\gamma^-,\gamma^+,\lambda,\alpha}} \quad \sum_{t\in T}\left(\sum_{u\in V_-}\int_0^{q_{u,t}^{\text{nom}}} P_{u,t}(s)\,\mathrm{d}s - \sum_{u\in V_+} c_u^{\text{var}} q_{u,t}^{\text{nom}}\right)$$
$$- \sum_{t\in T}\sum_{(u,v)\in E} c_t^{\text{trans}}\left|p_{u,t}^2 - p_{v,t}^2\right| - C \tag{21a}$$

$$\text{s.t.} \quad 0 \le q_u^{\text{TC}},\ 0 \le \underline{\pi}_u^{\text{book}}, \quad u \in V_+ \cup V_-, \tag{21b}$$

$$\sum_{u\in V_+} \underline{\pi}_u^{\text{book}} q_u^{\text{book}} = \sum_{t\in T}\sum_{(u,v)\in E} c_t^{\text{trans}}\left|p_{u,t}^2 - p_{v,t}^2\right| + C, \tag{21c}$$

$$(p,q) \in \mathcal{F}(q^{\text{nom}}),\ \text{i.e., } (p,q) \text{ fulfills (6)–(9)}, \tag{21d}$$

$$(q_+^{\text{book}}, q^{\text{nom}}, \pi^{\text{nom}}, \pi^{\text{book}}, \gamma^-, \gamma^+, \lambda, \alpha) \text{ fulfills (17)–(20)}. \tag{21e}$$

Recall that we could eliminate the booked quantities q_-^{book} of the gas buyers by choosing them as $q_-^{\text{book}} = q_-^{\text{TC}}$; see Lemma 3.1 together with Assumption 2.2. The lemma below shows that we can do something similar with the technical capacities q_+^{TC} of the entry nodes in Problem (21).

Lemma 3.3. *If* (q_+^{TC}, x) *with*

$$x := (q_-^{\text{TC}}, \underline{\pi}^{\text{book}}, p, q, q_+^{\text{book}}, q^{\text{nom}}, \pi^{\text{nom}}, \pi^{\text{book}}, \gamma^-, \gamma^+, \lambda, \alpha)$$

is feasible for Problem (21)*, then* (q_+^{book}, x) *is also feasible for Problem* (21) *with the same objective function value.*

Proof. The technical capacities q_+^{TC} of the entry nodes do not occur in the objective function and only in the constraints (21b), (18a), and (20a). It follows that choosing $q_+^{\text{TC}} = q_+^{\text{book}}$ retains feasibility and does not change the objective function value. ☐

According to the previous result, we can always choose $q_+^{\text{TC}} = q_+^{\text{book}}$ and use this to eliminate q_+^{TC} from Problem (21). The dual variable $\pi^{\text{book}} \ge 0$ can then also be eliminated, which allows for the following simplifications: Constraints (18a), (19a), (20a), and the first half of (21b) are automatically satisfied and can be omitted. The stationarity equation (17a) turns into the inequality

$$-\underline{\pi}_u^{\text{book}} + \sum_{t\in T}\gamma_{u,t}^+ \ge 0.$$

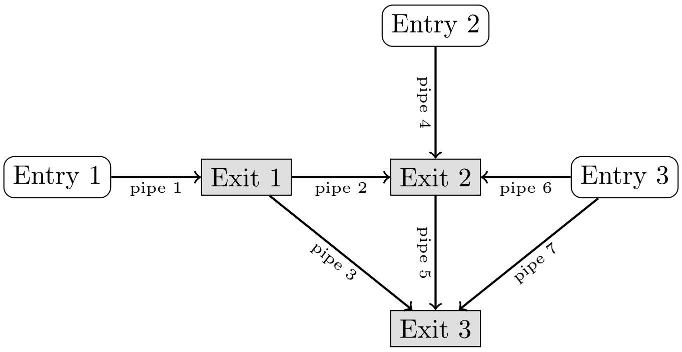


FIGURE 4. The 6-node network. The arcs illustrate the direction of positive flow.

After solving Problem (21), the technical capacities of the entry nodes $u \in V_+$ can be recovered as

$$q_u^{\mathrm{TC}} \begin{cases} = q_u^{\mathrm{book}}, & \text{if } \sum_{t \in T} \gamma_{u,t}^{+} > \underline{\pi}_u^{\mathrm{book}}, \\ \geq q_u^{\mathrm{book}}, & \text{if } \sum_{t \in T} \gamma_{u,t}^{+} = \underline{\pi}_u^{\mathrm{book}}. \end{cases} \tag{22}$$

4. Numerical Results

The reformulation derived in Section 3 allows to computationally solve instances of our original problem. There, a TSO determines technical capacities as well as controls the gas transport in the network and the gas selling firm books and nominates capacities to supply its customers; see Section 2. We solved several test instances by using the reformulation (21), where we removed the variables q_{-}^{book} according to Lemma 3.3. We implemented the modeling of gas physics and transport costs as described in Remark 2.3. In the subsequent Section 4.1, we provide technical details regarding calibration, software and hardware used, and a description of the test instances. We then discuss the results for specific test instances in Section 4.2. Finally, for several different test instances, we analyze the numerical performance of our approach in Section 4.3.

4.1. **Computational Setup and Test Instances.** All optimization problems have been implemented in Python 3.11.1 using Pyomo 6.6.1; see [22]. They have been solved with ANTIGONE 40.1.0; see [37]. The computations have been carried out on a ThinkPad T460s laptop running Windows 10 Enterprise with an Intel Core i7-6600U CPU 2.60 GHz processor. We provide the solver ANTIGONE with bounds for all variables. An overview of the variable bounds as well as the proofs of their correctness can be found in Appendix A; see Table 9.

To numerically test our approach and to also provide an economic interpretation of our results (see Section 4.2), we consider the test instance provided in [6]. It contains the 6-node network depicted in Figure 4. More specifically, the network consists of 3 entry nodes and 3 exit nodes and 7 arcs (pipe $1, \dots,$ pipe 7). The pipes' data for the identical pipes $e \in E$ is given by length L_e, diameter D_e, roughness k_e, and flow bounds $q_e^{\pm}$:

$$L_e = 350\,\mathrm{km}, \quad D_e = 0.5\,\mathrm{m}, \quad k_e = 0.1\,\mathrm{mm}, \quad q_e^{\pm} = \pm 435\,\mathrm{kg\,s^{-1}}.$$

The pressure bounds are given by

$$p_u^- = 40\,\mathrm{bar}, \quad p_u^+ = 65\,\mathrm{bar}, \quad u \in V_+,$$
$$p_u^- = 40\,\mathrm{bar}, \quad p_u^+ = 50\,\mathrm{bar}, \quad u \in V_-.$$

TABLE 1. Parameters of test instances: Variable costs c_u^{var} (in EUR/(1000 N^3/h)) at entries and slopes b_u (in EUR/(1000 $\text{N}^3/\text{h})^2$) and intercepts a_t (in EUR/(1000 N^3/h)) of inverse demand at exits.

Entries		Exits					
					a_t		
	c_u^{var}		b_u	t	6-nodes 70%	6-nodes 100%	6-nodes 140%
Entry 1	63	Exit 1	4.5	1	1330.00	1900.00	2660.00
Entry 2	57	Exit 2	5.0	2	726.83	1038.33	1453.66
Entry 3	71	Exit 3	20.0	3	1225.00	1750.00	2450.00
				4	2030.00	2900.00	4060.00

TABLE 2. Overview of analyzed pricing regimes in the 6-node network.

perfect price discr.:	$V_1 = \{\text{Exit } 1\}$, $V_2 = \{\text{Exit } 2\}$, $V_3 = \{\text{Exit } 3\}$
price groups:	$V_1 = \{\text{Exit } 1, \text{Exit } 2\}$, $V_2 = \{\text{Exit } 3\}$
uniform price:	$V_1 = \{\text{Exit } 1, \text{Exit } 2, \text{Exit } 3\}$

The pressure loss coefficient Λ_e is computed as described in [6] using the formula $\Lambda_e = (\lambda_e c^2 L_e)/((0.25\pi)^2 D_e^5)$. We set the transport cost coefficients to $c^{\text{trans}} = 1$ and the additional network costs to $C = 0$. We consider four time periods. The economic data for the gas selling and buying firms are given in Table 1. To comply with our setup, the intercepts of the inverse demand functions depend only on the time periods, and do not differ between exit nodes. The slopes b_u, however, are different at each exit node, they are assumed not to depend on the time periods in our test instances.

To vary the degree of network congestion, we consider different levels of inverse demand denoted by the scenarios 6-nodes 70%, 6-nodes 100%, and 6-nodes 140%. In these scenarios, the physical properties of the network remain unchanged and the intercepts of the inverse demand functions a_t (see Table 1) at the exit nodes are scaled according to the percentage value in the name of the test instance. In principle, larger demand is likely to create more intensive use of the network and, thus, more congestion.

Depending on the specific organization of the market, the seller might be able to charge different prices at the different exit nodes of the network. Our framework is suitable to determine pricing strategies for any degree of price discrimination. We consider three different configurations: (i) perfect price discrimination with possibly different prices at all exit nodes, (ii) a uniform price, where prices at the exit nodes have to coincide, and (iii) price groups, which is an intermediate scenario; see Table 2. We describe and discuss the results for the 6-nodes instances in Section 4.2.

Additional Test Instances. We consider further instances with 6, 8, 9, and 11 nodes to test the numerical performance of our algorithm in more detail; see Section 4.3. For the instance 6-nodes 50%, the intercepts are given by $a = (950, 519.17, 875, 1450)$. The network of the instances 6-nodes with pipe 8 70%, 6-nodes with pipe 8 100%, and 6-nodes with pipe 8 140% is the network given in Figure 4 with an additional pipe from Entry 2 to Exit 1. The inverse demand functions are the same as in the corresponding instances without the additional pipe; see Table 1.

For the 8-nodes instance, we consider a network consisting of 3 entry nodes, 5 exit nodes, and 10 arcs. The economic data differs only in the demand functions of the

TABLE 3. Intercepts for the instances with $|T| \in \{8, 12, 16\}$.

	6-nodes (with pipe 8)						
t	50%	70%	100%	140%	8-nodes	9-nodes	11-nodes
1	950.00	1330.00	1900.00	2660.00	1660.00	1530.00	4040.07
2	519.17	726.83	1038.33	1453.66	830.00	826.00	3686.97
3	875.00	1225.00	1750.00	2450.00	1530.00	1350.00	3542.87
4	1450.00	2030.00	2900.00	4060.00	2690.00	1530.00	2907.10
5	1045.00	1463.00	2090.00	2926.00	1000.00	1683.00	2378.13
6	571.08	799.51	1142.16	1599.03	450.00	908.60	2399.77
7	787.50	1102.50	1575.00	2205.00	2000.00	1215.00	2558.60
8	1305.00	1827.00	2610.00	3654.00	1700.00	1377.00	2448.57
9	855.00	1197.00	1710.00	2394.00	1494.00	1377.00	1929.73
10	467.25	654.15	934.50	1308.30	747.00	743.40	3189.77
11	962.50	1347.50	1925.00	2695.00	1683.00	1485.00	3216.93
12	1595.00	2233.00	3190.00	4466.00	2959.00	1683.00	3604.10
13	1045.00	1463.00	2090.00	2926.00	1000.00	1683.00	2378.13
14	467.25	654.15	934.50	1308.30	747.00	743.40	3189.77
15	962.50	1347.50	1925.00	2695.00	1683.00	1485.00	3216.93
16	1305.00	1827.00	2610.00	3654.00	1700.00	1377.00	2448.57

gas buyers. The intercepts are given by $a = (1660, 830, 1530, 2690)$ for the four time periods and the slopes are given by $b = (4.5, 4.8, 5, 20, 10)$ for the five exit nodes.

The network of the 9-nodes instance consists of 4 entry nodes, 5 exit nodes, and 13 arcs. The parameters of the inverse demand functions are given by the intercepts $a = (1530, 826, 1350, 1530)$ and the slopes $b = (4.5, 5, 20, 6, 4)$. Moreover, the variable cost at the additional entry node is given by $c^{\text{var}}_{\text{Entry4}} = 60$.

For the configuration price groups of the instances 8-nodes and 9-nodes, the three price groups are given by $V_1 = \{\text{Exit } 1, \text{Exit } 2\}$, $V_2 = \{\text{Exit } 3, \text{Exit } 4\}$, and $V_3 = \{\text{Exit } 5\}$.

The network of the 11-nodes instance consists of three entry and three exit nodes as well as of five inner nodes and 11 arcs with pipe lengths varying from 80 to 210 km. The variable costs at the entry nodes are given by $c^{\text{var}} = (274.8, 270.4, 250.3)$, the intercepts are given by $a = (3756.64, 2562.33, 2312.3, 3336.93)$, and the slopes are given by $b = (16.6, 13.8, 20.7)$.

All other parameters for the described instances are set according to the data given for the 6-nodes instance. For the analysis of the numerical performance, all test instances are also solved for the cases of 8, 12, and 16 time periods. The intercepts for the additional time periods are given in Table 3. We describe and discuss the results for the analysis of numerical performance in Section 4.3.

4.2. **Economic Interpretation.** Our main computational results are provided in Table 4, which contains the values for welfare, the supplier's profits, and the consumer surplus for the different levels of demand and the different pricing regimes. Figure 5 shows detailed nominations and technical capacities, which we obtain in the different cases of our 6-nodes instances. The nominations in the four time periods at the different entry and exit nodes are depicted as blue columns. The bookings of the supplier are depicted by a dotted red line and the technical capacities, which limit the available bookable capacities, are depicted in gray. In some cases, this technical capacity is not binding; see (22). The cases, in which the technical capacities are not binding for the nominations of the consumers can be determined similarly. If $P_{u,t}(q^{\text{nom}}_{u,t}) = \pi^{\text{nom}}_{i,t}$ and $\gamma^{+}_{u,t} = 0$ holds for all $t \in T$, the technical capacity at exit node $u \in V_{-}$ can be chosen

TABLE 4. Comparison of welfare, supplier's profit, and consumer surplus for different levels of demand and different pricing regimes. Top: 6-nodes 70%, middle: 6-nodes 100%, bottom: 6-nodes 140%.

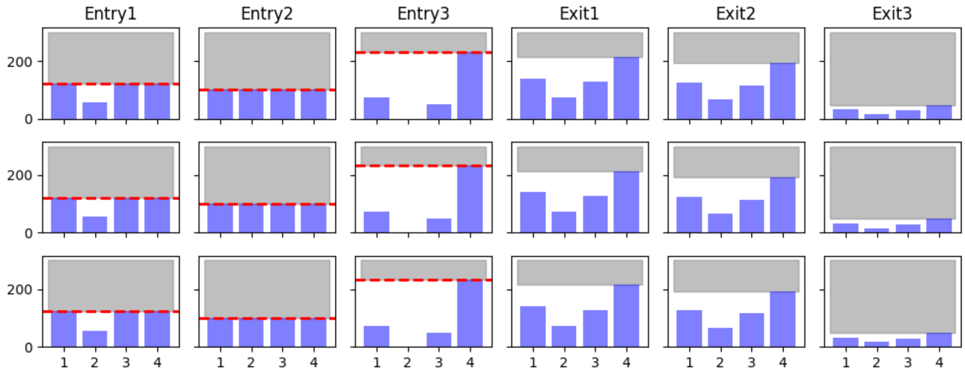
configuration	welfare	supplier profit	consumer surplus
perfect price discr.	1254099.74	836066.49	418033.25
price groups	1254099.74	836066.49	418033.25
uniform price	1254099.74	836066.49	418033.25
perfect price discr.	2366860.28	1694455.94	672404.34
price groups	2354966.62	1686683.62	668283.00
network price	2349913.42	1690616.30	659297.12
perfect price discr.	3904386.19	3016880.31	887505.87
price groups	3836285.03	3009675.22	826609.81
network price	3826532.93	3009261.07	817271.86

arbitrarily with $q_u^{\mathrm{TC}} \geq \max_{t \in T} q_{u,t}^{\mathrm{nom}}$. Whenever a technical capacity is binding in a solution, it is depicted as a gray line. Otherwise, it is illustrated by a gray rectangle, meaning that the technical capacity for this node could be set to any level above the maximal nomination in the optimal solution. We do not depict the bookings of the consumers, because they can be chosen from the set $[\max_{t \in T} q_{u,t}^{\mathrm{nom}}, q_u^{\mathrm{TC}}]$ and thus either coincide with a binding technical capacity or are not uniquely determined.

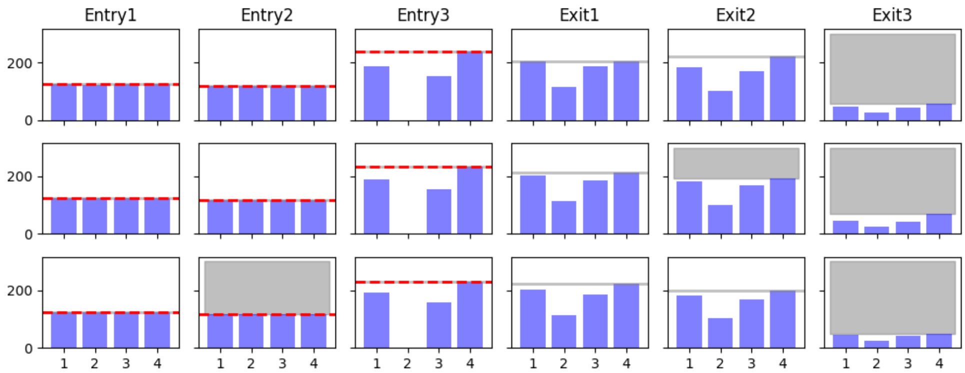
Note that we do not have any uniqueness results—neither for the lower-level nor for the upper-level problem of our bilevel reformulation. For the lower-level problem, this aspect is addressed by choosing the optimistic variant of bilevel optimization as it is discussed in Section 3.2. However, it can still be the case that the corresponding (optimistic) upper-level solution is not uniquely determined as well. Hence, in what follows, we (as usual) discuss the results obtained by solving the derived single-level reformulation but note here that there might be other welfare-optimal solutions as well.

First of all, observe that both welfare and profits are significantly larger in case of higher inverse demand. Moreover, in case of relatively scarce network capacities, different pricing regimes apparently have a larger impact on welfare, profits, and consumer surplus. This becomes most apparent in case of the 6-nodes 70% case, where technical capacities at the exit nodes are never binding (see Figure 5a) and the supplier can choose the unconstrained optimum of setting the same price at all exit nodes. For higher levels of demand (instances 6-nodes 100% and 6-nodes 140%), technical capacities are binding at the exit nodes more often. More specifically, technical capacities are binding at node Exit 1 in all 6-nodes 100% and 6-nodes 140% test instances; see Figures 5b and 5c.

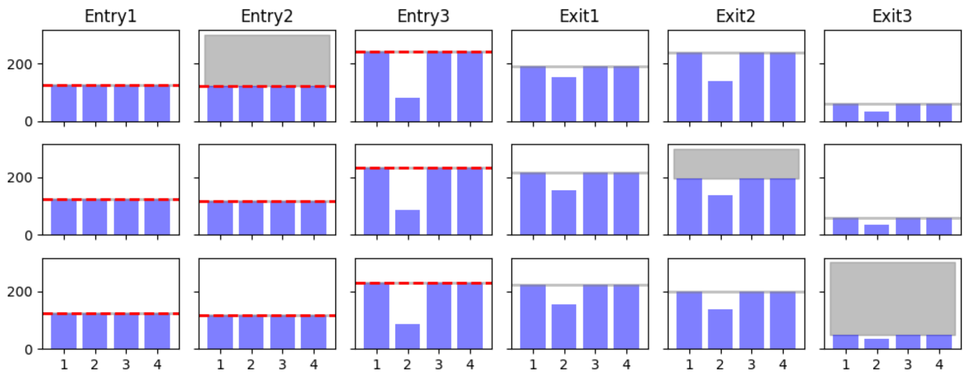
In all these cases, the different pricing regimes indeed deliver different results as the supplier engages in price discrimination among nodes if possible. Interestingly, the possibility of the supplier to choose different prices at the different exit nodes in our results tends to increase both consumer surplus and supplier profits. Apparently, differentiated prices at the different exit nodes allow for a more appropriate representation of the network-congestion situation, which allows for a more favourable choice of technical capacities on the upper level. It is worthwhile to note that this only provides a general intuition of the results. Due to the nonconvex network model considered, see (8), and the nonconvexities induced by the multilevel structure itself, see Section 2, we also obtain exceptions to this general intuition; in the present test



(a) 6-nodes 70%



(b) 6-nodes 100%



(c) 6-nodes 140%

FIGURE 5. Illustration of results for the instances 6-nodes 70%, 6-nodes 100%, and 6-nodes 140%: technical capacities (gray), bookings (red), and nominations (blue) in case of perfect price discrimination (top), price groups (middle), and uniform price (bottom)

scenarios the sole exception being the 6-nodes 100%, where supplier profits are lowest in case of price groups.

4.3. **Numerical Performance.** To analyze the numerical performance, we solved additional instances with up to 11 nodes and up to 16 time periods. The instances are described in Section 4.1. We limit the run time to 6 hours. The run times as well as the number of variables, constraints, and nonlinear terms are depicted in Tables 5–8.

As expected, the complexity increases with the number of time periods considered. For the instances with 4 time periods, 26 out of 30 are solved to global optimality within 10 minutes, and all instances are solved to global optimality in the run time limit of 6 hours. For 8 time periods, 28 instances are solved in the run time limit, compared to 24 for 12 time periods, and 23 for 16 time periods.

The run times also increase with the size of the considered network; see Table 5. For the instances with 4 time periods, the average run time for the instances with 6 nodes is 4.61 s, whereas the average run time for the instances with 8 or more nodes is 1358.32 s. The additional pipe in the instances 6-nodes with pipe 8 also increases complexity. The number of variables, constraints, and nonlinear terms is higher in the 6-nodes with pipe 8 instances as for the 6-nodes instances; see Tables 5–8. This also results in longer run times. The average run time for the 6-nodes instances is 2.51 s compared to 7.54 s for the 6-nodes with pipe 8 instances in the case with 4 time periods.

Also, the instances that are less constrained by the physical constraints have longer run times. This can be observed, i.e., by comparing the run times for the uniform price for the instances 6-nodes 50% (15.88 s) and 6-nodes 140% (0.39 s). This effect can also be observed for the instances 6-nodes with pipe 8 70% (40.88 s) and 6-nodes with pipe 8 140% (2.56 s).

The size of the price groups also affect the complexity and therefore the run times. Consider, i.e., the case with 4 time periods. Here, the average run time for the 6-nodes instances with perfect price discrimination is 0.58 s. In these instances, the three price groups consist of one exit node each. For the case with price groups, i.e., one group of two exit nodes and one group with one exit node, the average run time increases to 1.50 s and to 5.16 s for the case with a uniform price, i.e., all three exit nodes are in one price group. This effect is especially pronounced for the larger test instances with 8, 9, and 11 nodes. For the 11-nodes instance, the run time increases by 686.95 % from perfect price discrimination to price groups and by 138.12 % from price groups to uniform price.

Overall, we see that the instances get very hard to solve already for moderate network sizes and time periods.

5. Conclusion

We provide a multilevel setup that allows to quantitatively analyze the interplay of the TSO with gas sellers and gas buyers in the entry-exit system, a market design which is currently used to organize trading and transport of pipeline-bound natural gas in Europe. With the planned onset of pipeline-bound hydrogen transport (see, e.g., [11, 12, 43]), the entry-exit system might gain further relevance also in the context of zero-carbon energy supply chains. Moreover, the potential re-dedication of natural gas pipelines to be integrated into a new hydrogen network might also induce tighter network constraints for the natural gas network. Hence, a careful assessment of the current market design's efficiency to appropriately use physical transport capacities is very important.

Our work crucially builds on the setup provided in [18]. The latter, however, considers perfectly competitive decisions of buyers and sellers. To the best of our

Table 5. Data for instances with 4 time periods: run times, number of variables, constraints, and nonlinear terms. Top: 6-nodes, middle: 6-nodes with pipe 8, bottom: 8-nodes, 9-nodes, 11-nodes

	run time (s)					
instance	perf. pr. discr.	pr. groups	uniform pr.	var.	constr.	nl. terms
50%	0.53	4.64	15.88	173–177	273–277	243–255
70%	0.64	0.50	3.83	173–177	273–277	243–255
100%	0.42	0.42	0.53	173–177	273–277	243–255
140%	0.74	0.42	0.39	173–177	273–277	243–255
70%	1.54	16.36	40.88	189–193	305–309	267–279
100%	0.67	0.58	4.30	189–193	305–309	267–279
140%	0.48	0.51	2.56	189–193	305–309	267–279
8-nodes	18.37	37.58	65.97	251–159	413–421	359–397
9-nodes	3.17	6516.17	3824.21	297–305	482–490	428–448
11-nodes	18.82	1311.66	19 428.91	225–229	325–329	295–307

Table 6. Data for instances with 8 time periods: run times, number of variables, constraints, and nonlinear terms. Top: 6-nodes, middle: 6-nodes with pipe 8, bottom: 8-nodes, 9-nodes, 11-nodes

	run time (s)					
instance	perf. pr. discr.	pr. groups	uniform pr.	var.	constr.	nl. terms
50%	0.70	81.78	416.86	337–345	459–549	483–507
70%	0.93	0.83	11.19	337–345	459–549	483–507
100%	4.54	1.74	0.77	337–345	459–549	483–507
140%	0.74	5.65	0.67	337–345	459–549	483–507
70%	1.10	0.91	20 262.77	369–377	605–613	531–555
100%	1.08	0.80	795.93	369–377	605–613	531–555
140%	0.83	0.84	16.45	369–377	605–613	531–555
8-nodes	114.69	302.44	6114.98	491–507	821–837	715–755
9-nodes	1.60	—	10 689.91	581–597	958–974	852–892
11-nodes	935.02	—	83.69	441–449	645–653	587–611

knowledge we are the first ones to propose a framework that allows to analyze strategic seller decisions in this specific context. This is of high practical relevance as gas selling takes place in environments with only few players and substantial market power; see [51]. Introducing a strategic monopolistic seller in our model requires to disaggregate buyer and seller decisions into separate levels. This leads to an overall model with significantly increased complexity and the derivation of a single-level reformulation is much more challenging compared to the case of perfectly competitive markets, which have been discussed in the recent literature. Hence, one of our main contributions is that we provide assumptions under which we can formally show that we can derive a tractable single-level reformulation. Based on our reformulation we are then able to solve test instances from the literature. We illustrate the applicability as well as the computational capabilities and limits of our approach using several test instances.

Table 7. Data for instances with 12 time periods: run times, number of variables, constraints, and nonlinear terms. Top: 6-nodes, middle: 6-nodes with pipe 8, bottom: 8-nodes, 9-nodes, 11-nodes

	run time (s)					
instance	perf. pr. discr.	pr. groups	uniform pr.	var.	constr.	nl. terms
50%	1.02	112.05	156.97	501–513	809–821	723–759
70%	1.12	1.04	29.92	501–513	809–821	723–759
100%	0.97	1.12	0.98	501–513	809–821	723–759
140%	1.13	1.05	0.71	501–513	809–821	723–759
70%	1.34	6207.56	1106.88	549–561	905–917	795–831
100%	1.17	1.26	14 914.76	549–561	905–917	795–831
140%	1.10	1.26	41.99	549–561	905–917	795–831
8-nodes	114.84	—	—	731–755	1229–1253	1071–1131
9-nodes	3.21	—	—	865–889	1434–1458	1276–1336
11-nodes	13 562.67	—	—	657–669	965–977	879–915

Table 8. Data for instances with 16 time periods: run times, number of variables, constraints, and nonlinear terms. Top: 6-nodes, middle: 6-nodes with pipe 8, bottom: 8-nodes, 9-nodes, 11-nodes

	run time (s)					
instance	perf. pr. discr.	pr. groups	uniform pr.	var.	constr.	nl. terms
50%	3.30	82.3	375.87	665–681	1077–1093	963–1011
70%	1.95	1.64	69.97	665–681	1077–1093	963–1011
100%	1.50	2.17	1.54	665–681	1077–1093	963–1011
140%	1.40	1.92	1.10	665–681	1077–1093	963–1011
70%	1.71	15 419.72	—	729–745	1205–1221	1059–1107
100%	1.65	1.54	21 219.03	729–745	1205–1221	1059–1107
140%	1.95	1.81	120.59	729–745	1205–1221	1059–1107
8-nodes	3999.60	—	—	971–1003	1637–1669	1427–1507
9-nodes	3.77	—	—	1149–1181	1910–1942	1700–1780
11-nodes	—	—	2695.89	873–889	1285–1301	1171–1219

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Appendix A. Finite Variable Bounds

We present all finite variable bounds in Table 9 and prove their correctness in what follows.

Bounds for q_u^{book}, $u \in V_+$, and q_u^{TC}, $u \in V_-$: For all nodes $u \in V_+ \cup V_-$, we have

$$q_{u,t}^{\text{nom}} \leq q_u^{\text{book}} \leq q_u^{\text{TC}}, \quad t \in T;$$

TABLE 9. Variable bounds. For a variable x, we denote the upper (lower) bound of x by $x^+(x^-)$.

variable	index set	lower bound	upper bound
q_u^{book}	$u \in V_+$	0	$\max_{t\in T}\{(q_{u,t}^{\text{nom}})^+\}$
q_u^{TC}	$u \in V_-$	0	$\max_{t\in T}\{(q_{u,t}^{\text{nom}})^+\}$
$\underline{\pi}_u^{\text{book}}$	$u \in V_+$	0	$\sum_{t\in T}(\gamma_{u,t}^+)^+$
$p_{u,t}$	$u \in V,\ t \in T$	p^-	p^+
$q_{e,t}$	$e \in E,\ t \in T$	q_e^-	q_e^+
$q_{u,t}^{\text{nom}}$	$u \in V_+,\ t \in T$	0	$\sum_{u\in V_-}(q_{u,t}^{\text{nom}})^+$
$q_{u,t}^{\text{nom}}$	$u \in V_-,\ t \in T$	0	$a_{i,t}/b_{u,t}$
$\pi_{i,t}^{\text{nom}}$	$i \in [k],\ t \in T$	0	$a_{i,t}$
$\gamma_{u,t}^-$	$u \in V_+,\ t \in T$	0	$\max\{0, c_u^{\text{var}} - \lambda_t^-\}$
$\gamma_{u,t}^+$	$u \in V_+,\ t \in T$	0	$\max\{0, -c_u^{\text{var}} + \lambda_t^+\}$
$\gamma_{u,t}^-, \gamma_{u,t}^+$	$u \in V_-,\ t \in T$	0	see (24)
λ_t	$t \in T$	$\min_{u\in V_+}\{c_u^{\text{var}}\}$	$\max_{u\in V_-}\{a_{i,t} + b_{u,t}(q_u^{\text{TC}})^+\}$
$\alpha_{u,t}$	$u \in V_-,\ t \in T$	0	$\sum_{u\in V_i}(q_u^{\text{TC}})^+$

see Constraints (18b) and (18d). The bound then follows from the bounds on $q_{u,t}^{\text{nom}}$ for $u \in V_+ \cup V_-$ using the observation that it is never beneficial to choose q_u^{book} or q_u^{TC} larger than necessary.

Bounds for $\underline{\pi}_u^{\text{book}}$: We use the stationarity conditions (17a) together with (21b) and (19a) to obtain

$$0 \le \underline{\pi}_u^{\text{book}} = -\pi_u^{\text{book}} + \sum_{t\in T}\gamma_{u,t}^+ \le \sum_{t\in T}\gamma_{u,t}^+ \le \sum_{t\in T}(\gamma_{u,t}^+)^+ .$$

Bounds for $p_{u,t}$ and $q_{e,t}$: These are directly given in (9) and (6).

Bounds for $q_{u,t}^{\text{nom}}, u \in V_+$: The lower bound is given in (18b) and the upper bound follows from the lower bound together with the market-clearing condition (18c):

$$q_{u,t}^{\text{nom}} \le \sum_{u\in V_-} q_{u,t}^{\text{nom}} \le \sum_{u\in V_-}(q_{u,t}^{\text{nom}})^+ .$$

Bounds for $q_{u,t}^{\text{nom}}, u \in V_-$: The lower bound is given in (18d) and the upper bound follows from (18e) together with the lower bound on $\pi_{i,t}^{\text{nom}}$:

$$q_{u,t}^{\text{nom}} \le \frac{a_{i,t} - \pi_{i,t}^{\text{nom}}}{b_{u,t}} \le \frac{a_{i,t}}{b_{u,t}}.$$

Bounds for $\pi_{i,t}^{\text{nom}}$: The upper bound follows from Lemma 3.2. The lower bound follows from the fact that $\pi_{i,t}^{\text{nom}} < 0$ can only be optimal in Problem (2) if $\sum_{n\in V_i} q_{u,t}^{\text{nom}} = 0$ holds, in which case the gas seller can also choose $\pi_{i,t}^{\text{nom}} = 0$ without changing the objective value.

Bounds for $\gamma_{u,t}^-, \gamma_{u,t}^+, u \in V_+$: We start by showing that if Problem (21) has a feasible point, it also has a feasible point with the same objective function satisfying $\gamma_{u,t}^-\gamma_{u,t}^+ = 0$ for all $t \in T$ and $u \in V_+$. To this end, consider a feasible point of Problem (21) and an entry node $u \in V_+$. If $q_u^{\text{book}} > 0$, then for this node $\gamma_{u,t}^-\gamma_{u,t}^+ = 0$, $t \in T$, follows immediately from (20b) and (20c). Now, suppose that $q_u^{\text{book}} = 0$ and $\gamma_{u,t}^-, \gamma_{u,t}^+ > 0$ holds. Then, we re-define

$$\tilde{\gamma}_{u,t}^- := \max\{\gamma_{u,t}^- - \gamma_{u,t}^+, 0\} \geq 0, \tag{23a}$$

$$\tilde{\gamma}_{u,t}^+ := -\min\{\gamma_{u,t}^- - \gamma_{u,t}^+, 0\} \geq 0, \tag{23b}$$

$$\underline{\tilde{\pi}}_u^{\text{book}} := 0 \tag{23c}$$

for all $t \in T$. This partially modified point is still feasible, because the Constraints (19b), (17a) (in its inequality version after eliminating $\pi_u^{\text{book}} \geq 0$), (17b), (21b) are still satisfied and Constraint (21c) still holds due to $q_u^{\text{book}} = 0$. Moreover, none of the modified variables occurs in the objective function (21a).

It thus suffices to consider feasible points with $\gamma_{u,t}^- \gamma_{u,t}^+ = 0$ for all $t \in T$ and $u \in V_+$. Then, we can use the stationarity condition (17b) together with the bounds on λ_t and obtain

$$\gamma_{u,t}^- = c_u^{\text{var}} - \lambda_t \leq c_u^{\text{var}} - \lambda_t^-, \quad \gamma_{u,t}^+ \leq 0,$$

in the case of $\gamma_{u,t}^+ = 0$ and

$$\gamma_{u,t}^- \leq 0, \quad \gamma_{u,t}^+ = -c_u^{\text{var}} + \lambda_t \leq -c_u^{\text{var}} + \lambda_t^+$$

in the case of $\gamma_{u,t}^- = 0$. We take the maximum of the two possible cases to obtain the upper bound and use the lower bound from (19b).

Bounds for $\gamma_{u,t}^-, \gamma_{u,t}^+, u \in V_-$: By Assumption 2.2, we have $q_u^{\text{TC}} > 0$ and thus (20b) and (20d) imply $\gamma_{u,t}^- \gamma_{u,t}^+ = 0$ for all $t \in T$, $u \in V_-$.

For $\gamma_{u,t}^+ = 0$, Equation (17d) together with (18d), (21b), and the upper bounds on λ_t and $\alpha_{u,t}$ implies

$$\begin{aligned} &\gamma_{u,t}^- = -a_{i,t} + 2b_{u,t}q_{u,t}^{\text{nom}} - b_{u,t}q_u^{\text{TC}} + \lambda_t + b_{u,t}\alpha_{u,t} \leq a_{i,t} + \lambda_t^+ + b_{u,t}\alpha_{u,t}^+, \\ &\gamma_{u,t}^+ \leq 0. \end{aligned}$$

For $\gamma_{u,t}^- = 0$, Equation (17d) together with (18d), (19c), the upper bound on q_u^{TC}, and the lower bound on λ_t implies

$$\begin{aligned} &\gamma_{u,t}^- \leq 0, \\ &\gamma_{u,t}^+ = a_{i,t} - 2b_{u,t}q_{u,t}^{\text{nom}} + b_{u,t}q_u^{\text{TC}} - \lambda_t - b_{u,t}\alpha_{u,t} \leq a_{i,t} + b_{u,t}\left(q_u^{\text{TC}}\right)^+ - \lambda_t^-. \end{aligned}$$

The lower bounds for $\gamma_{u,t}^+, \gamma_{u,t}^-$ are given in (19b) and as upper bounds we use the maximum of the two possible cases:

$$\left(\gamma_{u,t}^-\right)^+ := \max\{0, a_{i,t} + \lambda_t^+ + b_{u,t}\alpha_{u,t}^+\}, \tag{24a}$$

$$\left(\gamma_{u,t}^+\right)^+ := \max\{0, a_{i,t} + b_{u,t}\left(q_u^{\text{TC}}\right)^+ - \lambda_t^-\}. \tag{24b}$$

Bounds for λ_t: Next, we have to make the assumption that there is no time period in which no trade takes place at all. In realistic settings, this extreme situation typically does not occur so we are convinced that the assumption is reasonable.

Assumption A.1. *For all time periods $t \in T$, there exists an entry node $u \in V_+$ with a positive nomination $q_{u,t}^{\text{nom}} > 0$.*

Suppose that Assumption A.1 holds. Consider a time period $t \in T$ and an entry node $u \in V_+$ such that $q_{u,t}^{\text{nom}} > 0$ holds and thus $\gamma_{u,t}^- = 0$ follows from (20b). Then, we can use (17b) together with (19b) to obtain

$$\lambda_t = \gamma_{u,t}^+ + c_u^{\text{var}} \geq c_u^{\text{var}} \geq \min_{v \in V_+} \{c_v^{\text{var}}\}.$$

Due to the market-clearing condition (18c), there also exists an exit node $u \in V_-$ with $q_{u,t}^{\text{nom}} > 0$ and, thus, $\gamma_{u,t}^- = 0$ follows from (20b). Using (17d) together with (18d), (19b), and (19c), we then obtain

$$\begin{aligned}\lambda_t &= (a_{i,t} - 2b_{u,t}q_{u,t}^{\text{nom}} + b_{u,t}q_u^{\text{TC}}) - \gamma_{u,t}^+ - \alpha_{u,t}b_{u,t} \\ &\leq a_{i,t} + b_{u,t}\left(q_u^{\text{TC}}\right)^+ \leq \max_{v \in V_-}\{a_{i,t} + b_{v,t}\left(q_v^{\text{TC}}\right)^+\}.\end{aligned}$$

Bounds for $\alpha_{u,t}$: For $u \in V_i$, $i \in [k]$, we have

$$0 \leq \alpha_{u,t} \leq \sum_{v \in V_i} \alpha_{v,t} = \sum_{v \in V_i} q_v^{\text{TC}} \leq \sum_{v \in V_i} \left(q_v^{\text{TC}}\right)^+.$$

The first inequality holds due to (19c) and the equality holds by (17c).

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