MULTISTAGE STOCHASTIC FACILITY LOCATION UNDER FACILITY DISRUPTION UNCERTAINTY

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ABSTRACT

We consider a multistage variant of the classical stochastic capacitated facility location problem under facility disruption uncertainty. Two solution algorithms for this problem class are presented: (1) stochastic dual dynamic integer programming (SDDiP), the state-of-the-art algorithm for solving multistage stochastic integer programs, and (2) shadow price approximation (SPA), an algorithm utilizing trained parameters of the linear value function approximation to minimize an upper bound on the optimal objective value. Numerical investigations demonstrate SPA consistently outperforming SDDiP across all instances from an additional dataset adapted from renowned library ORLib.

***K*eywords** Multistage Stochastic Facility Location; Approximate Dynamic Programming; Shadow Price Approximation; Stochastic Dual Dynamic Integer Programming

1 Introduction

Facility location problem (FLP) is a well-studied problem in operations research aiming to optimally locate facilities in order to provide services to and satisfy demands of customers. There have been many variants of the classic FLP to respond to the diverse needs of different industries and firms; among them are location models in a continuous space (1), stochastic location models (2), location of hubs (3), healthcare facilities (4), public sector facilities (5), and location for humanitarian logistics (6).

In this article, we consider facility location decisions accounting for facility disruptions. Incidents like power outages, industrial accidents, problems with the transportation infrastructure, and natural catastrophes disrupt facilities and may cause facility failures (7). The impact of these events and the probability of their occurrences are difficult to estimate due to lack of high-quality historical data (8). Facility location decisions are long-term and difficult to rectify. Thus, it is important to take into account future disruption uncertainties to assure that facility location decisions are sufficiently robust to avert significant costs in the future.

Although there exist a stream of literature covering this subject - reliable facility location models - they only cover single- and two-stage models (9). Some disruptions may occur multiple times across the planning horizon and some facilities need to be re-opened when disruptions are overcome. To the best of our knowledge, the *multistage* case of facility location under facility disruptions has only yet been explored in (10). We present the multistage stochastic facility location problem under facility disruptions as multistage stochastic mixed-integer program (MSIP). On solution methodology aspect, this problem class has only been recently tackled due to its complexity and difficulty to solve. Stochastic dual dynamic integer programming (SDDiP) is the current state-of-the-art to solve MSIP problems

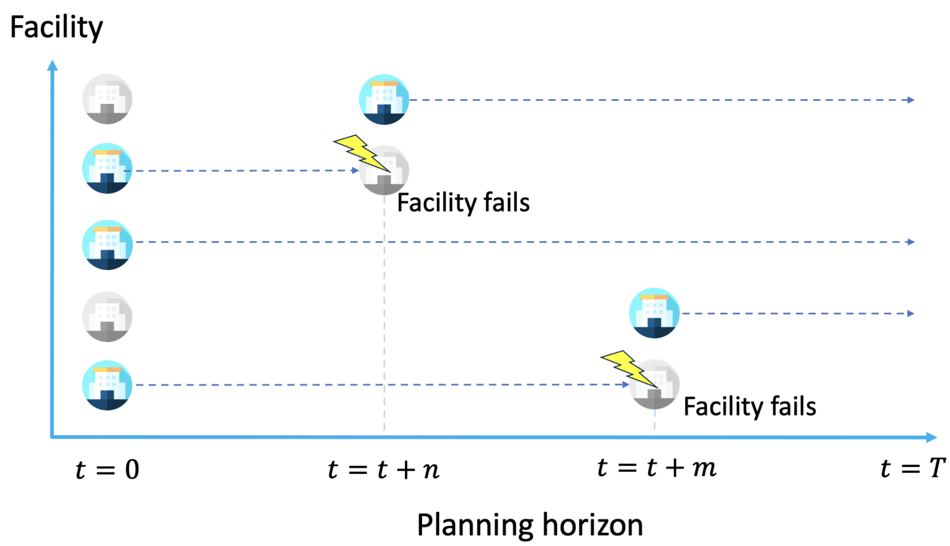

Figure 1: Timeline of a multistage stochastic facility location under facility disruption uncertainty.

presented in (11). It requires MSIP to be reformulated as a dynamic program and can obtain an optimal policy that is computationally tractable.

A novel technique to solve this problem is the shadow price approximation (SPA). SPA trains the parameters of a linear value function approximation by minimizing an upper bound on the optimal objective value.

A comparable methodology to SPA is the parametric cost function approximation (CFA) presented in (12). Although both algorithms rely on tuning parameters, SPA and CFA work with different model formulations. While SPA chooses parameters of a linear value function approximation thereby working with a decomposed form of the decision problem, parametric CFA chooses parameters of a *lookahead* model.

We present numerical results of these two approximate dynamic programming algorithms, SDDiP and SPA, to solve an instance of the MSFLP.

To summarize, this paper strives to accomplish three main objectives. Firstly, it introduces a broader class of problems, expanding the scope of FLP as presented in (10), which mainly addresses a specific scenario. Secondly, it aims to enrich the available datasets for problem benchmarking. These additional datasets are derived from the esteemed ORLib repository (13) but are tailored to encompass multiple stages and stochastic elements. Lastly, the paper delves into the calibration of SPA, a facet not explored in depth in (10), providing further insights into this aspect.

2 Formulation of Multistage Stochastic FLP under Facility Disruption

We begin by introducing the multistage stochastic facility location problem under facility discruption as a multistage stochastic integer program and present its reformulation as a dynamic program with binary state variables.

2.1 Multistage Stochastic Integer and Dynamic Programming Formulations

The multistage stochastic mixed-integer problem (MSIP) is given by

$$\min_{\mathbf{u_1},\mathbf{y_1}\in\mathcal{X}_1} \Big\{ v_1^\top \mathbf{u_1} + w_1^\top \mathbf{y_1} + \mathbb{E}_{\boldsymbol{\xi}_{[2,T]}|\xi_{[1,1]}} \Big[\min_{\mathbf{u_2},\mathbf{y_2}\in\mathcal{X}_2(\mathbf{u_1},\boldsymbol{\xi}_2)} \Big\{ v_2^\top \mathbf{u_2} + w_2^\top \mathbf{y_2} + \dots + \mathbb{E}_{\boldsymbol{\xi}_{[T,T]}|\xi_{[1,T-1]}} \Big[\min_{\mathbf{u_T},\mathbf{y_T}\in\mathcal{X}_T(\mathbf{u_{1:T-1}},\boldsymbol{\xi}_T)} \Big\{ v_T^\top \mathbf{u_T} + w_T^\top \mathbf{y_T} \Big\} \Big] \dots \Big\} \Big] \Big\}, \tag{1}$$

where $\mathbf{u}_t$ is the state variable and $\mathbf{y}_t$ is a local variable appearing only at stage $t \in T$, $v_t^\top$ and $w_t^\top$ are the corresponding cost at time $t \in 1, \dots, T$, and $\mathcal{X}_t$ is the feasible set. The stochastic data process $(\boldsymbol{\xi}_1, \dots, \boldsymbol{\xi}_T)$ is modeled where $\boldsymbol{\xi}_1$ is deterministic and $\boldsymbol{\xi}_2, \dots, \boldsymbol{\xi}_T$ will be revealed gradually in time.

We can now write down the dynamic programming (DP) reformulation of (1). The optimal value function at stage t, $V_t(\mathbf{u}_{t-1}, \boldsymbol{\xi}_t)$, is the optimal expected objective value given state $(\mathbf{u}_{t-1}, \boldsymbol{\xi}_t)$, and assuming that optimal action will be taken at each stage t.

$$V_t(\mathbf{u_{t-1}}, \boldsymbol{\xi}_t) = \min_{\mathbf{u}_t, \mathbf{y}_t} \{v_t^\top \mathbf{u}_t + w_t^\top \mathbf{y}_t + \mathcal{V}_{t+1}(\mathbf{u}_t, \boldsymbol{\xi}_t) : B_t \mathbf{u}_{t-1} + A_t \mathbf{u}_t + C_t \mathbf{y}_t = b_t\}, \tag{2}$$

for $t = 1, \ldots, T$ where $\mathcal{V}_{t+1}(\mathbf{u}_t, \boldsymbol{\xi}_t)$ is the expected value cost-to-go function,

$$\mathcal{V}_{t+1}(\mathbf{u}_t) := \mathbb{E}[V_{t+1}(\mathbf{u}_t, \boldsymbol{\xi}_{t+1}) | \boldsymbol{\xi}_t]. \tag{3}$$

If we assume $\boldsymbol{\xi}_t$ to be Markovian, i.e. it follows the Markov property or the *memoryless* property, the expectation depends only on $\boldsymbol{\xi}_t$ rather than the whole history of the data process, with $\mathcal{V}_T \equiv 0$. Finally, let us define the optimal policy as

$$\pi^*(\mathbf{u}_{t-1}, \boldsymbol{\xi}_t) = \arg\min_{\mathbf{u}_t, \mathbf{y}_t} \{v_t^\top \mathbf{u}_t + w_t^\top \mathbf{y}_t + \mathcal{V}_{t+1}(\mathbf{u}_t, \boldsymbol{\xi}_t) : B_t \mathbf{u}_{t-1} + A_t \mathbf{u}_t + C_t \mathbf{y}_t = b_t\} \tag{4}$$

for $t = 1, \ldots, T$ in set $\mathbf{\Pi}$ as the *policy* which specifies the decision to make for all possible states regardless of which state at stage t.

2.2 Multistage Stochastic FLP under Facility Disruption (MSFLPD)

Following the formulations above, we propose the multistage stochastic facility location model under facility disruption uncertainty as a multi-stage, discrete-time, stochastic-dynamic optimization problem, as follows

$$\min \quad \sum_{t}^{T} (\sum_{i=1}^{F} f_{it} y_{it} + \sum_{i=1}^{F} \sum_{j=1}^{B} d_{ij} x_{ijt}) \tag{5}$$

$$\text{s.t.} \quad u_{it} = u_{it-1}(1 - \xi_{it}) + y_{it}, \quad \forall i = 1, \ldots, F, t = 1, \ldots, T \tag{6}$$

$$\sum_{j}^{B} x_{ijt} \leq C_i u_{it}, \quad \forall i = 1, \ldots, F, t = 1, \ldots, T \tag{7}$$

$$\sum_{i}^{F} x_{ijt} = D_{jt}, \quad \forall j = 1, \ldots, B, t = 1, \ldots, T \tag{8}$$

$$u_{it}, y_{it} \in \{0, 1\}, \quad \forall i = 1, \ldots, F, j = 1, \ldots, B, t = 1, \ldots, T \tag{9}$$

$$x_{ijt} \in \mathbb{Z}, \quad \forall i = 1, \ldots, F, j = 1, \ldots, B, t = 1, \ldots, T. \tag{10}$$

We define binary variable $y_{it} \in \{0, 1\}$ to denote facility opening decision and integer variable x_{ijt} to denote service level as the local variables. We also define binary variable $u_{it} \in \{0, 1\}$ as our state variable to track the state of facility $i \in F$ at stage $t \in T$. We denote $u_{it} = 1$ if facility $i \in \{1, ..., F\}$ is open at stage $t \in \{1, ..., T\}$, and $u_{it} = 0$ otherwise. In addition, we observe a realization of the random variable $\xi_{it} \in \{0, 1\}$ such that $\xi_{it} = 1$ if the facility i at stage t is disrupted and $\xi_{it} = 0$ otherwise.

The objective (5) minimizes of the total cost which includes the fixed cost of opening facility $i \in \{1, ..., F\}$ at stage $t \in \{1, ..., T\}$ and the transportation cost d_{ij} from demand point $j \in \{1, ..., B\}$ to facility $i \in \{1, ..., F\}$. The local variable decisions and transition of the state variable are governed by a series of constraints. Constraints (6) keep track of the state of facility $i \in \{1, ..., F\}$ as affected by random variable ξ_{it} across each stage. Constraints (7) impose assigning of capacity C_i if and only if facility $i \in \{1, ..., F\}$ is open. Constraints (8) impose that the total demand D_{jt} of customer j must be fully satisfied. Finally, constraints (9) and (10) show the decision variable domains.

3 Solution Methods

We present two solution techniques to solve MSFLPD - stochastic dual dynamic integer programming (SDDiP) and shadow price approximation (SPA).

3.1 Stochastic Dual Dynamic Integer Programming (SDDiP)

SDDiP is an extension of the celebrated stochastic dual dynamic programming (SDDP) method to solve MSIP problems (11). Unlike SDDP, SDDiP uses Lagrangian relaxation to derive tight cuts of the cost-to-go functions of stochastic mixed-integer problems. SDDiP guarantees to find the exact optimal solution of any MSIP if the time-coupling decision variables that define the state of the dynamic program are binary variables. This bodes well with MSFLPD's binary state variables u_{it} - state of facility $i \in \{1, ..., F\}$ at stage $t \in \{1, ..., T\}$.

Each iteration of SDDiP begins with sampling a subset of scenarios of the stochastic process. Then, SDDiP proceeds by undertaking two key steps - forward simulation and backward pass.

In the forward simulation, the algorithm draws a sample of random realization from the stochastic process, $(\boldsymbol{\xi}_2^\omega, ..., \boldsymbol{\xi}_T^\omega)$, for $\omega \in \Omega$, and then solves the subproblems at each stage t of the dynamic program using the latest cutting plane approximation of the cost-to-go function. Particularly, in every iteration k, the subproblem at stage t is of the form:

$$\begin{aligned} \bar{V}_t^k(\mathbf{u_{t-1}}^k, \boldsymbol{\xi}_t^\omega) := \min_{u_{it}, x_{ijt}, y_{it}} \quad & \sum_{i=1}^{F}(f_i y_{it} + \sum_{j=1}^{B} x_{ijt} d_{ij}) + \bar{\mathcal{V}}_{t+1}^k(\mathbf{u_t}) \\ \text{s.t.} \quad & u_{it} = u_{i,t-1}(1-\xi_{it}) + y_{it}, \quad \forall\, i = 1, \dots, F, t = 1, \dots, T, \\ & \text{Eqs.} \quad (7)-(10), \end{aligned} \tag{11}$$

for $t = 1, \dots, T$, where $\boldsymbol{\xi}_t^\omega$ is the ω uncertainty realization at stage t. Each forward simulation, generates a sequence of sample decisions, $((u_{it}^k, x_{ijt}^k, y_{it}^k))_{t=1}^T$ that are made based on the realized uncertainties of the sampled scenario $(\boldsymbol{\xi}_2^\omega, ..., \boldsymbol{\xi}_T^\omega)$.

The function $\bar{\mathcal{V}}_{t+1}^k(\mathbf{u_t})$ is defined as a set of cutting planes (cuts) whose minimum approximates the true expected cost-to-go function $\mathcal{V}_{t+1}^k(\mathbf{u_t})$ from below. The function can be expressed as a linear program which is given by

$$\begin{aligned} \bar{\mathcal{V}}_{t+1}^k(\mathbf{u_t}) := \quad \min \quad & \theta_t \\ \text{s.t.} \quad & \theta_t \geq L_t \\ & \theta_t \geq N_{t+1}^{-1} \sum_{j=1}^{N_{t+1}} (\alpha_{t+1}^{lj} + (\beta_{t+1}^{lj})^\top \mathbf{u_t}) \qquad \forall l \leq k-1. \end{aligned} \tag{12}$$

The backward pass begins from the final stage T. Given the solution $\mathbf{u_{T-1}^k}$ from iteration k and an uncertainty realization from $\{\boldsymbol{\xi}_T^j, 1 \leq j \leq N_T\}$, let $P_T^{kj}(\mathbf{u_{t-1}^k}, \boldsymbol{\xi}_t^\omega, \bar{\mathcal{V}}_t^k)$ be a relaxation of the forward problem $\bar{V}_T^k(\mathbf{u_{T-1}^k}, \boldsymbol{\xi}_T^j, \bar{\mathcal{V}}_T^{k+1})$. Solving $P_T^{kj}(\mathbf{u_{t-1}^k}, \boldsymbol{\xi}_t^\omega, \bar{\mathcal{V}}_t^k)$ for each j produces a cut defined by $\theta(\alpha_T^{lj}, \beta_T^{lj})$ which is valid for the value function $V_T(\mathbf{u}_{T-1}, \boldsymbol{\xi}_T^j)$. Then, the cuts $\theta(\alpha_T^{lj}, \beta_T^{lj})$ are aggregated obtaining (12) which is valid for expected cost-to-go function $\mathcal{V}_{T-1}^k(\mathbf{u}_{T-1})$. Furthermore, the lower approximation of the expected cost-to-go function is updated from $\bar{\mathcal{V}}_{T-1}^k(\mathbf{u}_t)$ to $\bar{\mathcal{V}}_{T-1}^{k+1}(\mathbf{u}_t)$. Backward pass then proceeds to stage $T-1$. As the first stage computation is completed, and having solved a lower approximation of the original problem, the optimal solution value of the first stage problem $t = 1$ is a valid lower bound of the original problem.

SDDiP introduces a family of valid cuts, called Lagrangian cuts, which are able to obtain strong duality for mixed integer programs. Like SDDP, SDDiP is also sampling-based algorithm and exhibits favorable scalability on solving large-scale problems. The limiting assumption of SDDiP, as with SDDP, is that the random process has to be stagewise independent. Some techniques to incorporate stagewise dependency are presented in (18).

3.2 Shadow Price Approximation (SPA)

SPA is an approximation algorithm proposed in (10). The main idea of SPA is to train the slope of a linear value function approximation by minimizing an upper bound on the optimal objective value that can be obtained via Monte Carlo simulation. These slopes are similar to the notion of shadow prices of the non-anticipatory constraints that connect successive time periods - hence, the name of the algorithm.

Choosing the slope vector that minimizes an upper bound on the optimal objective value is effectively a nonconvex, stochastic optimization problem that is known to be computationally intractable. Thus, SPA is cast as a policy search strategy that can be supported by any method that is suitable for unconstrained (derivative-free) global optimization.

Instead of developing a stochastic model, SPA only needs access to independent time series, which can also be real data. SPA thereby interacts directly with a simulation model of the resulting policy unlike many other approximate dynamic programming techniques that rely on some form of backwards recursion. SPA can be easily integrated with gradient-based and stochastic search methods which are widely used in machine learning and global optimization.

The SPA algorithm, like SDDiP, undertakes two primary steps - forward simulation and shadow price updating. The algorithm proceeds as follows:

Step 1. Initialize iteration $n = 0$ and shadow prices $\lambda_t^{u,0} = 0$, for every $t = 1, \ldots, T$.

Step 2. At each iteration n, we randomly select a scenario path $(\hat{\xi}_1^n, \ldots, \hat{\xi}_T^n)$.

[**Forward simulation**]

Step 2.1. Solve the dynamic programming recursion of the form

$$\begin{aligned} \bar{V}_t^n(u_{it-1}^n, \xi_{it}^n; \lambda_{it}^{u,n-1}) = \min_{u_{it}^n, x_{ijt}^n, y_{it}^n} \quad & \sum_{i=1}^{F}(f_i y_{it} + \sum_{j=1}^{B} d_{ij} x_{ijt}) + \underbrace{\sum_{i=1}^{F} \lambda_{it}^{u,n-1} u_i}_{\text{Shadow Price}} \\ \text{s.t.} \quad & u_{it} = u_{i,t-1}(1 - \xi_{it}) + y_{it}, \quad \forall\, i = 1, \ldots, F, t = 1, \ldots, T, \\ & \text{Eqs.} \quad (7) - (10), \end{aligned} \tag{13}$$

as a sequence of mixed-integer problems that encodes information on the cost-to-go only via the slope vector of the linear value function approximation (the shadow prices).

Step 2.2. Obtain realized upper bound $\hat{\bar{z}}^n$ as follows,

$$\hat{\bar{z}}^n(\lambda_{it}^{u,n-1}) = \sum_{t \in T} \left[\sum_{i=1}^{F} (f_{it} y_{it}(\bar{\pi}^n(u_{it-1}^n, \hat{\xi}_{it}^n; \lambda_{it}^{u,n-1})) + \sum_{i=1}^{F} \sum_{j=1}^{B} d_{ij} x_{ijt}(\bar{\pi}^n(u_{it-1}^n, \hat{\xi}_{it}^n; \lambda_{it}^{u,n-1})) \right], \tag{14}$$

a realization of the total cost for given slope vectors $\lambda_{it}^{u,n-1}$.

[*Shadow price updating*]

Step 3. With the obtained realized upper bound $\hat{\bar{z}}^n$ and parameters $\lambda_{it}^{u,n-1}$, the shadow price is updated using a generic update function $U^n(\cdot, \cdot)$, supported by a chosen global optimization method, that returns a new set of trial shadow prices,

$$\lambda_{it}^{u,n} \leftarrow U^n(\hat{\bar{z}}^n, \lambda_{it}^{u,n-1}). \tag{15}$$

Various choices of global optimization methods include, but are not limited to, stochastic gradient descent (14), covariance matrix adaptation - evolutionary strategy (CMA-ES) (15), and Bayesian optimization (16).

Step 4. At the end of iteration N, SPA returns a slope vector λ_{it}^{u*} that approximately minimizes $\bar{z}$

$$\lambda_{it}^{u*} \sim \arg\min_{\lambda_{it}^{uN}} \hat{\bar{z}}(\lambda_{it}^{uN}). \tag{16}$$

Step 5. To obtain an approximate upper bound of the optimal objective value, we simulate the optimal policy $\bar{z}(\lambda_{it}^{u*})$

$$\bar{z}(\lambda_{it}^{u*}) = S^{-1} \sum_{s=1}^{S} \hat{\bar{z}}^n(\lambda_{it}^{u*}). \tag{17}$$

Whether or not the chosen global optimization methods is able to find a good linear approximation depends on the initial choice of parameters, the search region, as well as the number of iterations, and is likely to be problem-specific. In the worst case, no improvement over the (initial) greedy policy, with $\lambda_t^{u,0} = 0$, for every $t = 1, \ldots, T$, is possible. In the best case, the method selects the best linear approximation which is optimal if the true cost-to-go function is linear in the region of the state space that can be reached by the optimal policy.

As we will see below, a linear approximation can produce near-optimal results for the chosen problem. In which way this result extends to other problems remains subject of future work.

4 Numerical Results

We present the results of employing both SPA and SDDiP to solve Problem (5)-(10). The case instance chosen, with parameters shown in Table 1, is a multi-stage adaptation of the capacitated facility location problem, drawing its test instances from (13), a renowned repository of test datasets spanning various facility location and other OR problems. To include a greater depth of robustness in our model's testing and the deployed solution methodologies, different levels of disruptions were synthesized. These disruptions are denoted by $S = \{Low, Medium, High\}$ levels. Additionally, in a bid to intricately gauge the algorithms' performance, we designated three incremental time limit criteria, represented

Table 1: Parameters used for the numerical investigation (adapted from (13)).

Parameters	**Value**
T - planning horizons	36, 52, 100
F - set of candidate facilities	16
B - set of customers	50
C_{it} - capacity	50
D_{jt} - demand	taken from (13)
f_{it} - fixed cost of opening a facility	taken from (13)
d_{ijt} - transportation cost	taken from (13)
ξ_{it} - facility failure (uncertainty)	Randomly generated [$High$-60%, $Medium$-40%, Low-20%]

Table 2: Numerical investigation of MSFLPD using SDDiP and SPA.

Time Limit Criterion $(\times 10^3)$							
Instance CAP44-A							
Stages (T)	Runtime	S	SDDiP-LB	SDDiP-UB	SPA-UB	SDDiP-Gap	SPA-Gap
36	*300s*	High	138994.0	139445.4	139395.9	0.32%	0.28%
		Medium	133262.4	134092.1	133518.0	0.62%	0.19%
		Low	127614.4	128619.3	127614.8	0.78%	0.00%
	900s	High	138994.0	139170.1	139083.6	0.13%	0.06%
		Medium	133262.3	134043.7	133518.0	0.58%	0.19%
		Low	127614.4	128595.6	127614.8	0.77%	0.00%
	1800s	High	138994.0	139088.0	139020.6	0.07%	0.02%
		Medium	133262.4	133763.6	133518.0	0.38%	0.19%
		Low	127614.4	128421.1	127614.8	0.63%	0.00%
52	*300s*	High	109354.0	110594.0	109857.0	1.13%	0.45%
		Medium	100611.2	102174.4	101551.2	1.54%	0.93%
		Low	92297.0	93850.3	92925.7	1.67%	0.68%
	900s	High	109354.0	110254.3	109857.0	0.82%	0.45%
		Medium	100611.2	102422.7	101482.7	1.78%	0.86%
		Low	92297.0	93978.7	92925.7	1.81%	0.68%
	1800s	High	109354.0	110340.3	109857.0	0.90%	0.46%
		Medium	100611.2	102175.9	101449.5	1.54%	0.83%
		Low	92297.0	93955.6	92925.7	1.78%	0.68%
100	*300s*	High	89966.6	94229.2	93537.2	4.63%	3.89%
		Medium	73972.8	78148.8	77173.9	5.49%	4.24%
		Low	57089.1	63360.7	60764.9	10.41%	6.40%
	900s	High	89966.6	93848.1	93391.8	4.22%	3.76%
		Medium	73972.8	78306.5	76756.4	5.69%	3.69%
		Low	57089.1	62813.4	60764.9	9.55%	6.24%
	1800s	High	89966.6	93678.5	93411.5	4.04%	3.74%
		Medium	73972.8	78308.7	76887.7	5.69%	3.86%
		Low	57089.1	62813.3	60722.8	9.55%	6.17%

as $\mathcal{T}$ = {300s, 900s, 1800s}, to serve as benchmarks. SPA resorts to CMA-ES (15) for its outer loop and both methods use Gurobi 10 to solve the mixed-integer subproblems.

The findings from our expansive numerical exploration of MSFLPD via SDDiP and SPA are consolidated in Table 2. Distinct columns are reserved for various metrics, including SDDiP lower bounds (SDDiP-LB), SDDiP upper bound (SDDiP-UB), and SPA upper bound (SPA-UB). The percentage gaps delineated represent the %-difference between SDDiP lower bound and SDDiP upper bound (SDDiP Gap), and the %-difference between SDDiP lower bound and SPA upper bound (SPA Gap). From our analysis, SPA emerges as a more effective solution, consistently identifying superior policies for MSFLPD across all case instances. This observation persists even when strengthened *Benders* cuts are employed for SDDiP - these are valid, finite cuts wherein a generic Benders cut is augmented by solving a specific mixed integer program subproblem, where the solution mirrors a basic optimal LP dual solution. Such results underscore the inherent complexity and challenge posed by MSFLPD, especially when considering the risk facility failure.

5 Conclusion

In this paper, we present a *multi-stage* variant of the facility location problem under facility disruption (MSFLPD), a new broader class of FLP. We model the problem as a multi-stage stochastic mixed-integer program, and discuss two methods to solve the problem, stochastic dual dynamic integer programming (SDDiP) and shadow price approximation (SPA). While SDDiP is an exact solution approach, it is also difficult to implement, whereas SPA is an approximation method that is relatively easy to apply for anyone familiar with machine learning. We also present additional datasets for benchmarking as adapted from the renowned ORLib respository. We conduct extensive numerical experiments to compare the performance of the two methods in solving the problem. We find that, across all problem instances, SPA emerges as the more effective solution, consistently identifying superior policies for MSFLPD than SDDiP. These results underscore the inherent complexity and challenge posed by MSFLPD, especially when considering facility disruption risk.

References

[1] Hunagund, I. B., Pillai, V. M., & Kempaiah, U. N. (2021). A survey on discrete space and continuous space facility layout problems. Journal of Facilities Management.

[2] Correia, I., & Saldanha-da-Gama, F. (2019). Facility location under uncertainty. In Location science (pp. 185-213). Springer, Cham.

[3] Alumur, S. A., Campbell, J. F., Contreras, I., Kara, B. Y., Marianov, V., & O'Kelly, M. E. (2021). Perspectives on modeling hub location problems. European Journal of Operational Research, 291(1), 1-17.

[4] Ahmadi-Javid, A., Seyedi, P., & Syam, S. S. (2017). A survey of healthcare facility location. Computers & Operations Research, 79, 223-263.

[5] Haase, K., Knörr, L., Krohn, R., Müller, S., & Wagner, M. (2019). Facility location in the public sector. In Location Science (pp. 745-764). Springer, Cham.

[6] Trivedi, A., & Singh, A. (2018). Facility location in humanitarian relief: a review. International Journal of Emergency Management, 14(3), 213-232.

[7] Cheng, C., Adulyasak, Y., & Rousseau, L. M. (2021). Robust facility location under disruptions. INFORMS Journal on Optimization, 3(3), 298-314.

[8] Cui, T., Ouyang, Y., & Shen, Z. J. M. (2010). Reliable facility location design under the risk of disruptions. Operations research, 58(4-part-1), 998-1011.

[9] Snyder, L. V., Atan, Z., Peng, P., Rong, Y., Schmitt, A. J., & Sinsoysal, B. (2016). OR/MS models for supply chain disruptions: A review. Iie Transactions, 48(2), 89-109

[10] Seranilla, B. K. D., & Löhndorf, N. (2023). Optimizing vaccine distribution in developing countries under natural disaster risk. Naval Research Logistics.

[11] Zou, J., Ahmed, S., & Sun, X. A. (2019). Stochastic dual dynamic integer programming. Mathematical Programming, 175(1), 461-502

[12] Powell, W., & Ghadimi, S. (2022). The Parametric Cost Function Approximation: A new approach for multistage stochastic programming. arXiv preprint arXiv:2201.00258.

[13] Beasley, John E. "OR-Library: distributing test problems by electronic mail." Journal of the operational research society 41.11 (1990): 1069-1072.

[14] Bottou, L., & Bousquet, O. (2011). 13 the tradeoffs of large-scale learning. Optimization for machine learning, 351.

[15] Hansen, N., Akimoto, Y., & Baudis, P. (2022) "CMA-ES/pycma on Github" URL https://doi.org/10.5281/zenodo.2559634.

[16] Nogueira, F. (2014). Bayesian Optimization: Open source constrained global optimization tool for Python. URL https://github.com/fmfn/BayesianOptimization.

[17] Pereira, M. V., & Pinto, L. M. (1991). Multi-stage stochastic optimization applied to energy planning. Mathematical programming, 52(1), 359-375.

[18] Löhndorf, N., & Shapiro, A. (2019). Modeling time-dependent randomness in stochastic dual dynamic programming. European Journal of Operational Research, 273(2), 650-661.