Freight consolidation through carrier collaboration - A cooperative game

Fabio Mercurio[a,*], Loe Schlicher[a,**], Sonja U.K. Rohmer[b,**], Tom Van Woensel[a]

[a] *Department of Industrial Engineering and Innovation Sciences, School of Industrial Engineering, Eindhoven University of Technology, P.O. Box 513, Atlas, 5600MB Eindhoven, The Netherlands*
[b] *Department of Logistics and Operations Management, HEC Montréal, 3000 Chemin de la Côte-Ste-Catherine, QC H3T 2A7 Montréal , Canada*

Abstract

Reducing inefficient truck movements, this research investigates the potential of freight consolidation through carrier collaboration. Considering the financial benefits of consolidation and the additional cost arising from collaboration, we propose a cooperative game to explore under which circumstances carriers can collaborate. We show that stable cost allocations are not always possible, affecting stability and thus hindering collaboration, though stability is guaranteed under certain conditions. Numerical experiments indicate, however, that sustainable collaboration is likely outside of the extreme cases.

Keywords: Transportation; Cooperative Game Theory; Freight consolidation; Capacity sharing; Stable allocations

1. Introduction

The rise in e-commerce, generally associated with smaller individual shipment sizes, has led to an increase in less-than-truckload (LTL) shipping, requiring higher levels of coordination between shipments in order to keep costs low (Investopedia, 2022b). However, within the dynamic e-commerce environment, it is often difficult to achieve such high levels of coordination between orders of a carrier due to short delivery timelines, resulting in inefficient truck movements that do not make good use of the available capacities. Freight consolidation through carrier collaboration offers a promising avenue, in this context, to achieve overall higher shipping volumes without significantly increasing delivery times (Investopedia, 2022a). Although examples of this type of collaboration already exist (see, e.g., CT Logistics (2024); Hub Group (2024)), large-scale implementation is still hampered by a lack of trust as well as other practical challenges that prevent data and capacity sharing between companies (Basso et al., 2019; Karam et al., 2021). Emerging technologies, such as Blockchain or the Internet of Things, are helping to alleviate some of these challenges by paving the way for improved data sharing, real-time tracking, and secure and transparent data exchange (Hribernik et al., 2020; Ferrell et al., 2020; DHL, 2022). Investing in these new technologies can, however, be expensive and thus might hinder possible collaborations between carriers.

This paper aims to address this issue by investigating under which circumstances carriers are interested in collaboration, considering both the financial benefits of consolidation and the additional costs arising from collaboration. For this purpose, we study a setting where multiple carriers must deliver to the same final location, passing via several intermediate locations (e.g., a hub). In this setting, the carriers can choose to operate independently, solely paying a cost for every unit of goods transported, or to collaborate, using spare capacity to consolidate shipments

[*] Corresponding author
[**] We would like to stress that the authors contributed equally to the development of this paper
Email addresses: f.mercurio@tue.nl (Fabio Mercurio), l.p.j.schlicher@tue.nl (Loe Schlicher), sonja.rohmer@hec.ca (Sonja U.K. Rohmer), t.v.woensel@tue.nl (Tom Van Woensel)

with other players and achieve better resource utilization. Despite its potential benefits from an operational perspective, this consolidation may also trigger additional costs as carriers need to invest in new and often expensive technologies (e.g., secure and transparent data sharing and/or real-time tracking), pay for insurance, and purchase handling equipment to facilitate the transfer of goods between collaborators. We refer to all these costs as *transfer*, which include both a variable and a fixed component, where the former depends on the number of units transferred. At the same time, the latter presents a more general investment cost that is independent of the amount of goods shared. Formulating this situation as a cooperative game, we then investigate under which type of cost structures carrier collaboration is stable. We do this by studying the core of our game, which presents the set of all allocations that divides the joint costs amongst the carriers so that no group of carriers has reasons to leave the collaboration. In this context, we demonstrate that collaboration is not always stable, so some carriers might prefer to leave the collaboration even if the joint cost reduction is positive (i.e., if the benefits of consolidation outweigh the initial investment costs). At the same time, however, we show that collaboration is stable if fixed transfer costs are very low, very high, or symmetric. In addition, our extensive numerical experiments indicate that collaboration is likely to be sustainable outside of these extreme cases, although exceptions might exist.

1.1. Some related literature

Given its potential for practice, collaboration in transportation systems has received quite some attention within the scientific literature (see, e.g., the literature reviews of Pan et al. (2019) and Ferrell et al. (2020)). This includes the discussion on how parties should compensate each other financially in case of collaboration, which is usually addressed using cooperative game theory (Guajardo and Rönnqvist, 2016). In this context, it is common for studies to investigate the core of the considered cooperative game, corresponding to a specific transport situation in which different parties collaborate. Examples of such situations are the assignment of demands to routes owned by collaborators (Hu et al., 2013; Agarwal and Ergun, 2010), the sharing of vehicle capacity as well as the creation of routes satisfying the joint delivery planning (Lozano et al., 2013; Özener and Ergun, 2008; Gansterer and Hartl, 2018, 2020). Based on the properties of the considered situation, some studies focus then on proving core non-emptiness for specific subclasses (e.g., Agarwal and Ergun (2008); Markakis and Saberi (2003); van Zon et al. (2021)). In contrast, others present a numerical investigation of the core and its behavior under different properties (e.g., Lozano et al. (2013); Lai et al. (2022)). Our work combines these two by proving core non-emptiness for several subclasses while studying its properties numerically outside of these subclasses.

Outside of the transportation context, the setting in our paper has the most overlap with commodity flow games (see, e.g., Agarwal and Ergun (2008), and Markakis and Saberi (2003)). In these games, there is an underlying network in which players own capacity on the arcs and a set of commodities, for which the transfer of a single unit leads to a player-specific profit. Players can collaborate by sharing arc capacity and transporting only the most profitable commodities through the network. Our work differs in this aspect as we assume that players need to transport *all* orders. Moreover, instead of maximizing profit, we minimize the cost associated to transport and transfers, by accounting for a fixed cost that needs to be paid whenever orders are transfered between parties. This fixed cost can break the collaboration, whereas collaboration is guaranteed in commodity flow games, which do not incur such fixed costs.

1.2. Outline of the paper

The remainder of the paper is organized as follows. In §2, we introduce the transfer and transport situation addressed in our paper before introducing the corresponding cooperative game in §3. In §4, we then explore sufficient conditions that lead to core non-emptiness, while §5 provides the results of a numerical experiment, exploring core non-emptiness based on multiple parameters. Finally, concluding remarks are provided in §6.

2. The transfer and transport situation

We consider a transportation environment with a set $N \subseteq \mathbb{N}$ of carriers, in which each carrier $i \in N$ needs to transport a specific volume $D_i \in \mathbb{R}_{\geq 0}$ between two points, by passing through a set of points (e.g., a hub location, or a distribution or consolidation center). In this context, we assume that all carriers travel via the same starting, intermediate, and final points. In practice, these points do not need to be precisely the same but may be sufficiently close (e.g., in an urban context, they might be located in the same neighborhood or district of a city). We denote the set of points by $P_f = \{1, 2, \ldots, n, n+1\}$ with 1 representing the starting point and $n+1$ the final point, while the remaining points are intermediate points.

In addition, we assume that each carrier $i \in N$ has capacity $Q_i^p \in \mathbb{R}_{\geq 0}$ available to transport demand from point $p \in P_f \backslash \{n+1\}$ to $p+1$, and the cost of transporting one unit of volume equals $c_i^p \in \mathbb{R}_{\geq 0}$. For notational convenience, we let $P = P_f \backslash \{n+1\}$ be the set of points where transfers can occur since there is no need to transfer volume once the destination has been reached. Moreover, we assume carriers always have sufficient capacity to serve their own volume (i.e., $D_i \leq Q_i^p$ for all $i \in N$ and all $p \in P$). Yet, instead of transporting their own volume, carriers may *transfer* all or part of their volume at any point $p \in P$ to another carrier, depending on available capacity. Transferring between parties, however, incurs costs which need to be taken into account. We distinguish between two types of transfer costs, namely a variable cost s_{ij}^p and a fixed cost t_{ij}^p. In this case, the variable cost refers to the handling costs when transferring the goods from one truck to another, while the fixed cost presents an investment cost, e.g., for secure and transparent data sharing and/or real-time tracking. The amount of volume transferred from carrier $i \in N$ to carrier $j \in N$ at point $p \in P$ is denoted by $x_{ij}^p \in \mathbb{R}_{\geq 0}$, whereas the amount of volume that carrier $i \in N$ decides to keep and transport to the next point by itself is denoted by $x_{ii}^p \in \mathbb{R}_{\geq 0}$. The costs associated with this internal transfer also consist of a variable component $s_{ii}^p \in \mathbb{R}_{\geq 0}$ and a fixed component $t_{ii}^p \in \mathbb{R}_{\geq 0}$. Figure 1 presents a stylized representation of the situation.

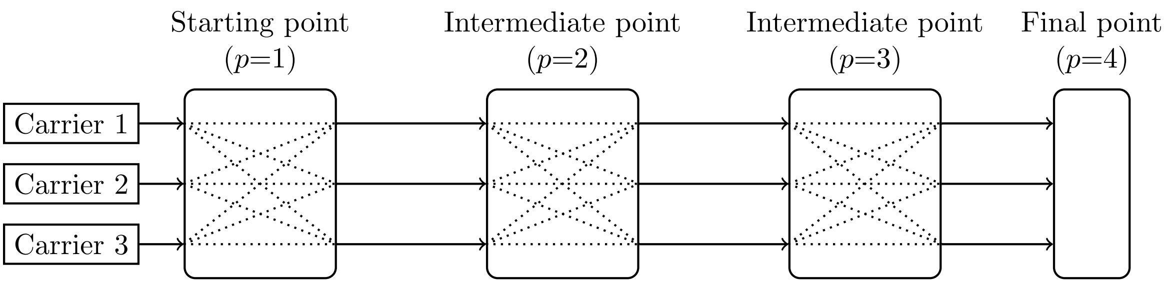

Figure 1: Representation with three carriers, three transfer points, and the final point, with $n = 3$. Note that transfers are not considered at the final point.

It should be noted that our model is also able to deal with situations where some carriers enter their volume later in the network (e.g., at point 2), which can be integrated by setting transportation costs and internal variable and fixed transfer costs of these carriers equal to zero up until their point of entry in the system, while external variable and fixed transfer costs should be sufficiently high in order to not be considered.

The carriers aim to jointly determine how to *transport* and *transfer* the entire volume such that the associated cost is minimized. This optimization problem, which we will refer to as Problem (1), can be formulated as the following mixed integer linear programming problem:

$$\min \sum_{p\in P}\sum_{i\in N}\Big(c_i^p\sum_{j\in N}x_{ji}^p+\sum_{j\in N}(s_{ji}^p x_{ji}^p+t_{ji}^p z_{ji}^p)\Big) \tag{1a}$$

$$\text{s.t.} \sum_{j\in N} x_{ji}^p \le Q_i^p \qquad \forall i\in N \quad \forall p\in P \tag{1b}$$

$$\sum_{j\in N} x_{ji}^p = \sum_{j\in N} x_{ij}^{p+1} \qquad \forall i\in N \quad \forall p\in P\backslash\{n\} \tag{1c}$$

$$\sum_{j\in N} x_{ij}^1 = D_i \qquad \forall i\in N \tag{1d}$$

$$x_{ij}^p \le M z_{ij}^p \qquad \forall i\in N\ \forall j\in N\ \forall p\in P \tag{1e}$$

$$x_{ij}^p \in \mathbb{R}_{\ge 0} \qquad \forall i\in N\ \forall j\in N\ \forall p\in P \tag{1f}$$

$$z_{ij}^p \in \{0,1\} \qquad \forall i\in N\ \forall j\in N\ \forall p\in P \tag{1g}$$

In this formulation, the objective function (1a) aims to minimize the total cost, while constraints (1b) represent the available capacity per carrier to transport demand between points. Constraints (1c) ensure that the entire volume transported by a carrier is transferred to the next intermediate point, and constraints (1d) enforce that the entire volume of each carrier is assigned at the starting point. The big-M constraints (1e) ensure that a fixed transfer cost is paid whenever some positive volume is transfered between carriers, with $M \ge \sum_{i\in N} D_i$ for all $i, j \in N$ and $p \in P$. Finally, all variables are non-negative, and z_{ij}^p is binary.

We refer to our setting as a *transfer and transport situation* $\theta = (N, P, D, Q, c, s, t)$ with $D = (D_i)_{i\in N}$, $Q = (Q_i^p)_{i\in N, p\in P}$, $c = (c_i^p)_{i\in N,p\in P}$, $s = (s_{ij}^p)_{i\in N,j\in N,p\in P}$ and $t = (t_{ij}^p)_{i\in N,j\in N,p\in P}$, and denote the set of all transfer and transport situations by Θ.
Table 1 presents an overview of all the parameters of θ and variables used.

Table 1: Sets, parameters, and variables

Sets	
N	Set of carriers
P	Set of points
Parameters	
D_i	Demand that carrier i has to deliver
c_i^p	Cost of transporting a unit of demand for carrier i from point p to point $p+1$
Q_i^p	Transport capacity of carrier i from point p to point $p+1$
s_{ij}^p	Variable costs for transferring from carrier i to carrier j at point p
t_{ij}^p	Fixed costs for transferring from carrier i to j at point p
M	Big-M (i.e., a sufficiently large number)
Variables	
x_{ij}^p	Volume transferred from carrier i to j at point p
z_{ij}^p	1 if there is a positive volume transferred from carrier i to j at point p, 0 otherwise

Relatively to Problem (1), since there is no cost incentive to transport or transfer more than needed through the network, it is possible to relax equality constraints (1c) and (1d) of Problem (1) without changing its optimal value. We describe this formally in Observation 1.

Observation 1. *In Problem (1), "=" constraints (1c) can be relaxed into "$\le$" constraints and "=" constraints (1d) can be relaxed into "$\ge$" constraints, without changing its optimal value.*

From now onwards, when referring to Problem (1), we refer to the optimization problem in which constraints (1c) and (1d) are replaced by inequality constraints (as in Observation 1).

In the following, we present an illustrative example of our setting, for which the parameters do not necessarily represent reality but are selected to keep calculations simple and easy to follow.

Example 1. Consider a transfer and transport situation $\theta \in \Theta$ with $N = \{1, 2, 3\}$, $P = \{1, 2\}$, and the remaining parameters being equivalent to the values shown in Table 2.

Table 2: Parameters of the problem of Example 1.

i	1	2	3
D_i	6	4	3

(a) Carriers' demands

Q_i^p	i =1	2	3
p=1	7	8	7
2	7	6	7

(b) Carriers' capacities

c_i^p	$i = 1$	2	3
$p = 1$	6	1	9
2	9	2	4

(c) Carriers' costs

t_{ij}^1, s_{ij}^1	j =1	2	3
i =1	0,0	4,2	6,1
2	9,4	0,0	7,1
3	5,4	5,3	0,0

(d) Fixed and variable costs on the first point

t_{ij}^2, s_{ij}^2	j =1	2	3
i =1	0,0	6,4	9,4
2	5,4	0,0	5,3
3	9,3	5,2	0,0

(e) Fixed and variable costs on the second point.

Using a standard solver package for solving integer linear programming problems, we can find an optimal solution for this problem with an objective value of 120. We present this solution in Table 3 and illustrate it visually in Figure 2.

Table 3: Optimal solution of the problem in Example 1.

x_{ij}^1	j =1	2	3
i =1	2	4	0
2	0	4	0
3	0	0	3

z_{ij}^1	j =1	2	3
i =1	1	1	0
2	0	1	0
3	0	0	1

x_{ij}^2	j =1	2	3
i =1	2	0	0
2	0	6	2
3	0	0	3

z_{ij}^2	j =1	2	3
i =1	1	0	0
2	0	1	1
3	0	0	1

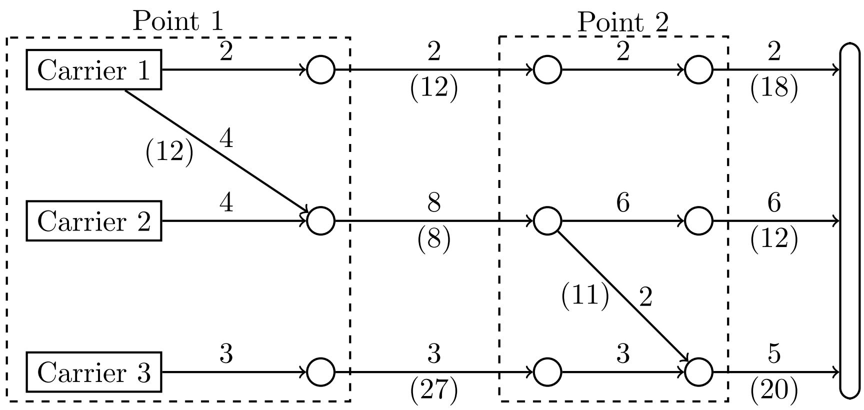

Figure 2: Optimal solution graph of Example 1. The numbers without brackets represent the volume flow, and the numbers within brackets represent the costs related to that flow, which are omitted if there is no cost.

If the carriers decide to transport their volume individually, carrier 1 ends up with a cost of 90 (i.e., $6 \cdot (6 + 9)$), carrier 2 with a cost of 12 (i.e., $4 \cdot (1 + 2)$) and carrier 3 with a cost of 39 (i.e., $3 \cdot (9 + 4)$). Carriers can thus benefit from collaborating (i.e., transferring their volume to others), as $90 + 12 + 39 > 120$. △

This example demonstrates that carriers can benefit from coordinating transport. However, to sustain such a collaboration, it is important that the associated costs (e.g., the 120 of Example 1) are allocated in a stable way (i.e., in a way that sustains the collaboration). In the upcoming section, we use cooperative game theory to study this cost allocation problem.

3. The cooperative game

We now associate to each transfer and transport situation $\theta \in \Theta$ a transferable utility cooperative game (N, C^θ). We call this game the Cooperative Transporting and Transferring (CTT) game. In this game, N denotes the set of carriers, which we will now refer to as players, and $C^\theta : 2^N \to \mathbb{R}$ denotes the characteristic cost function associating a real value to any coalition $S \subseteq N$, representing a subset of players. This cost function $C^\theta(S)$ reflects the optimal cost value that can be achieved when the players in S decide to cooperate, i.e., the optimal value of Problem (1) restricted to the players participating in coalition S. Defining $C^\theta(\varnothing) = 0$, and coalition $S \subseteq N, S \neq \varnothing$, we then obtain the following formulation based on the logic of Problem (1) and the result of Observation 1:

$$C^\theta(S) = \min \sum_{p\in P}\sum_{i\in S}\Big(c_i^p \sum_{j\in S} x_{ji}^p + \sum_{j\in S}(s_{ji}^p x_{ji}^p + t_{ji}^p z_{ji}^p)\Big) \tag{2a}$$

$$\text{s.t.} \sum_{j\in S} x_{ji}^p \leq Q_i^p \qquad \forall i \in S \quad \forall p \in P \tag{2b}$$

$$\sum_{j\in S} x_{ji}^p \leq \sum_{j\in S} x_{ij}^{p+1} \qquad \forall i \in S \quad \forall p \in P\backslash\{n\} \tag{2c}$$

$$\sum_{j\in S} x_{ij}^1 \geq D_i \qquad \forall i \in S \tag{2d}$$

$$x_{ij}^p \leq M z_{ij}^p \qquad \forall i \in S\ \forall j \in S\ \forall p \in P \tag{2e}$$

$$x_{ij}^p \in \mathbb{R}_+ \qquad \forall i \in S\ \forall j \in S\ \forall p \in P \tag{2f}$$

$$z_{ij}^p \in \{0,1\} \qquad \forall i \in S\ \forall j \in S\ \forall p \in P \tag{2g}$$

The central question in this paper is whether $C^\theta(N)$ can be allocated in a stable way amongst the players. In the literature, it is common to address this question by investigating the core of the associated game, as the core represents the set of allocations for which no individual player, nor a group of players (i.e., coalition) has incentives to break from the grand coalition (Shapley et al., 1965). For our CTT game, the core is defined as:

$$\mathscr{C}(N, C^\theta) = \Big\{u \in \mathbb{R}^N \mid \sum_{i\in N} u_i = C^\theta(N), \quad \sum_{i\in S} u_i \leq C^\theta(S)\ \forall\, S \subseteq N,\ S \neq \varnothing\Big\}. \tag{3}$$

Example 2 shows a CTT game with an allocation in the core.

Example 2. Reconsidering the setting of Example 1, we know the value of $C^\theta(\{1\})$, $C^\theta(\{2\})$, $C^\theta(\{3\})$, and $C^\theta(N)$ from the example. To determine the values of $C^\theta(\{1,2\})$, $C^\theta(\{1,3\})$, and $C^\theta(\{2,3\})$, we solve the associated optimization problem (see Problem (2)). The values of the characteristic cost function are presented in Table 4, with the corresponding solutions reported in Appendix A. We obtain $u = (80, 5, 35) \in \mathscr{C}(N, c^{\theta)})$ and so the core is non-empty.

Table 4: Coalitional costs for the problem of Example 1.

S	1	2	3	1,2	1,3	2,3	N
$C^\theta(S)$	90	12	39	86	129	42	120

△

However, not all CTT games have a non-empty core, which we demonstrate in Example 3.

Example 3. Consider a transfer and transport situation $\theta \in \Theta$ with $N = \{1, 2, 3\}$, $P = \{1, 2\}$, and the other parameter settings as shown in Table 5.

Table 5: Parameters of the problem of Example 3

i	1	2	3
D_i	3	3	6

(a) Carriers' demands

Q_i^p	1	2	3
$l=1$	7	7	9
2	6	5	9

(b) Carriers' capacities

c_i^p	1	2	3
1	9	7	2
2	2	8	6

(c) Carriers' costs

t_{ij}^1, s_{ij}^1	$j=1$	2	3
$i=1$	0,0	9,4	5,1
2	7,2	0,0	7,2
3	8,1	8,2	0,0

(d) Fixed and variable costs on the first point

t_{ij}^2, s_{ij}^2	$j=1$	2	3
$i=1$	0,0	7,2	8,4
2	5,1	0,0	8,4
3	4,1	7,2	0,0

(e) Fixed and variable costs on the second point.

S	1	2	3	1,2	1,3	2,3	N
$C^\theta(S)$	33	45	48	68	66	85	110

Table 6: Coalitional values for the empty core example

The coalitional values for this example are reported in Table 6, while the solutions of the associated optimization problem per coalition are reported in Appendix B.
Suppose now that $\mathscr{C}(N, C^\theta) \neq \emptyset$, and $u \in \mathscr{C}(N, C^\theta)$. Then by stability and efficiency, we know that $u_i \geq C^\theta(N) - C^\theta(N \backslash \{i\})$ for all $i \in N$. Hence, $u_1 \geq c(N) - c(\{2,3\}) = 110 - 85 = 25$, $u_2 \geq c(N) - c(\{1,3\}) = 110 - 66 = 44$ and $u_3 \geq c(N) - c(\{1,2\}) = 110 - 68 = 42$. Thus $u_1 + u_2 + u_3 \geq 25 + 44 + 42 = 111 > 110$, which is a contradiction. Therefore, the core is empty. △

Example 3 thus illustrates that a stable allocation of the joint costs is not always possible, which may hinder potential collaboration.

4. Sufficient conditions for core non-emptiness

Given that the core of our CTT game can be empty, we now focus on identifying sufficient conditions for which the core of CTT games is non-empty.

4.1. No fixed transfer costs

We start with a special class of transfer and transport situations $\Theta_{NF} \subset \Theta$, namely the ones without fixed transfer costs, i.e., $t_{ij}^p = 0$ for all $i, j \in N$ and $p \in P$. Note that for any $\theta \in \Theta_{NF}$, constraints (2e) and variables $(z_{ij}^p)_{i,j \in N, p \in P}$ are redundant. It turns out that any CTT game (N, C^θ) with $\theta \in \Theta_{NF}$ can be recognized as a so-called linear production game. In a linear production game, there is a group of players $N \subseteq \mathbb{N}$ with each player $i \in N$ owning a vector $b^i = (b_j^i)_{j=1}^{|R|} \in \mathbb{R}^{|R|}$ of resources, with R the set of resources. The resources are used to produce a set K of products, with $p_k \in \mathbb{R}$ being the price of product $k \in K$, and matrix $A \in \mathbb{R}^{|R| \times |K|}$ representing the number and type of resources needed to produce specific products. Instead of working independently, players can bundle their resources and make products together. If a coalition $S \subseteq N$ decides to bundle resources, the profit is given by

$$v^{LP}(S) = \max \sum_{k \in K} p_k y_k \quad s.t. \quad \sum_{k \in K} a_{jk} y_k \leq \sum_{i \in S} b_j^i \text{ for all } j \in R \text{ and } y \in \mathbb{R}_{\geq 0}^{|K|} \tag{4}$$

Proposition 1. *Every CTT game (N, C^θ) with $\theta \in \Theta_{NF}$ is a linear production game.*

Proof. First, we show that our game $(N, C^{\theta_{NF}})$ is equivalent to game (N, c') with the following formulation for all $S \subseteq N$,

$$c'(S) = \min \sum_{p\in P}\sum_{i\in N}\left(c_i^p \sum_{j\in N} x_{ji}^p + \sum_{j\in N} s_{ji}^p x_{ji}^p\right) \tag{5a}$$

$$\text{s.t.} \sum_{j\in N} x_{ji}^p \leq Q_i^p I_S(i) \qquad \forall i \in N \quad \forall p \in P \tag{5b}$$

$$\sum_{j\in N} x_{ji}^p \leq \sum_{j\in N} x_{ij}^{p+1} \qquad \forall i \in N \quad \forall p \in P\backslash\{n\} \tag{5c}$$

$$\sum_{j\in N} x_{ij}^1 \geq D_i I_S(i) \qquad \forall i \in N \tag{5d}$$

$$x_{ij}^p \in \mathbb{R}_+ \qquad \forall i \in N\, \forall j \in N\, \forall p \in P, \tag{5e}$$

where $I_S(i)$ is an indicator function, i.e., $I_S(i) = 1$ if $i \in S$ and 0 otherwise. To show equivalence, we first prove that for the optimization problem of coalition $S \subseteq N$ of game (N, c') we have $x_{ij}^p = 0$ if $i \notin S$ or $j \notin S$ for all $i, j \in N$ and all $p \in P$. From constraints (5b), we learn that $\sum_{j\in N} x_{ji}^p \leq 0$ for all $i \notin S$ and all $p \in P$. Combined with the fact that all variables are non-negative, we can thus conclude that $x_{ij}^p = 0$ if $j \notin S$ for all $i \in N$ and all $p \in P$. Next, to show that $x_{ij}^p = 0$ if $i \notin S$ for all $j \in N$ and all $p \in P$, we use that the optimization problem of coalition S in game (N, c') is a special instance of Problem (1), with $D_i = 0$ for all $i \notin S$ and $Q_i^p = 0$ for all $i \notin S$ and all $p \in P$. Since the optimization problem is a special instance of Problem (1), we can apply Observation 1, implying that inequalities (5c) and (5d) can be considered as equality constraints. Using this, we will first show that $x_{ij}^1 = 0$ if $i \notin S$ for all $j \in N$ and subsequently that $x_{ij}^p = 0$ if $i \notin S$ for all $j \in N$ and all $p \in P\backslash\{1\}$. By formulating constraints (5d) as equality constraints, we learn that $x_{ij}^1 = 0$ if $i \notin S$ for all $j \in N$. Similarly, by reformulating constraints (5c) as equality constraints, knowing that $x_{ji}^{p-1} = 0$ if $i \notin S$ for all $j \in N$ and all $p \in P\backslash\{1\}$, we observe immediately that $x_{ij}^p = 0$ if $i \notin S$ for all $j \in N$ and all $p \in P\backslash\{1\}$.
Hence, $x_{ij}^p = 0$ if $i \notin S$ or $j \notin S$ for all $i, j \in N$. Consequently, there is no need to consider these variables and we can thus replace all N's by S's in the optimization problem of $c'(S)$. As this renders the indicator function obsolete, the optimization problem of $c'(S)$ coincides with the one of $c^\theta(S)$. That is, game $(N, c^{\theta_{NF}})$ and (N, c') are equivalent.

Next, we observe that game (N, c') can be recognized as a linear production game if (i) the objective function is multiplied with a minus sign, (ii) the minimization problem is replaced by a maximization problem, (iii) the constraints are presented in standard form, and (iv) the following resource vectors $(b^i)_{i\in N}$ with $b^i = ((b_{j,p}^{i,1})_{j\in N, p\in P}, (b_{j,p}^{i,2})_{j\in N, p\in P\backslash\{n\}}, (b_j^{i,3})_{j\in N})$ for all $i \in N$ and

$$b_{j,p}^{i,1} = \begin{cases} Q_i^p & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \quad \text{for all } j \in N \text{ and all } p \in P$$

$$b_{j,p}^{i,2} = 0 \text{ for all } j \in N \text{ and all } p \in P\backslash\{n\}$$

$$b_j^{i,3} = \begin{cases} -D_i & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases} \quad \text{for all } j \in N$$

are considered. Note, vector $(b_{j,p}^{i,1})_{j\in N, p\in P}$ refers, in this case, to constraints (5b), while $(b_{j,p}^{i,2})_{j\in N, p\in P\backslash\{n\}}$ refers to constraints (5c), and $(b_j^{i,3})_{j\in N}$ refers to constraints (5d) for all $i \in N$. Consequently, game (N, c') is a linear production game. Moreover, as game (N, C_{NF}^θ) and game (N, c') are equivalent, game (N, C_{NF}^θ) is also a linear production game, which concludes this proof. □

We want to stress that this result is not a direct consequence of the fact that the optimization problem of a coalition is a linear programming problem, like the linear programming problem in a linear production game. For instance, in a linear production game, the size of the objective does not depend on the size of the coalition, while it does in CTT games.

Given that all CTT games (N, C^{θ}) with $\theta \in \Theta_{NF}$ are linear production games, we can now apply properties of linear production games within our context. More precisely, from Owen (1975), we know that the Owen set is a subset of the core of linear production games corresponding to the set of optimal solutions of the dual of the grand-coalition problem. The dual of the grand-coalition problem for our CTT game reads,

$$\max \sum_{i \in N} (D_i \eta_i) - \sum_{i \in N} \sum_{p \in P} (Q_i^p \gamma_i^p) \tag{6a}$$

$$\text{s.t. } \varphi_j^{p-1} - \varphi_i^p - \gamma_i^p \leq c_i^p + s_{ji}^p \qquad \forall i \in N \, \forall j \in N \, \forall p \in P \backslash \{1, n\} \tag{6b}$$

$$\varphi_j^{n-1} - \gamma_i^n \leq c_i^n + s_{ji}^n \qquad \forall i \in N \, \forall j \in N \tag{6c}$$

$$\eta_j - \varphi_i^1 - \gamma_i^1 \leq c_i^1 + s_{ji}^1 \qquad \forall i \in N \, \forall j \in N \tag{6d}$$

$$\gamma_i^p, \varphi_i^p, \eta_i \in \mathbb{R}_+ \qquad \forall i \in N \, \forall p \in P \tag{6e}$$

where any optimal solution $(D_i \eta_i - \sum_{p \in P} (Q_i^p \gamma_i^p))_{i \in N}$ with η_i, γ_i for $i \in N$ derived from solving the dual formulation in Problem (6), presents a core element of (N, c^{θ}) with $\theta \in \Theta_{NF}$. We can interpret η_i as the price a carrier i needs to pay for each unit of demand shipped, γ_i as a discount received for the capacity provided by carrier i to support the coalition, while φ_i^p represents how price and discounts are transferred among carriers given their costs and capacities.

4.2. Unlimited fixed transfer cost

In addition to games without fixed transfer costs, we also explore a class of transfer and transport situations $\theta_{UTC} \subset \Theta$ for which $t_{ij}^p > D_i \cdot \sum_{p \in P} (c_i^p + s_{ii}^p) + \sum_{p \in P} t_{ii}^p$ for all $i, j \in N$ with $i \neq j$ and all $p \in P$, i.e., where the fixed transfer costs are extremely high compared to the transport costs. Because these transfer costs are so high, there is no reason for carriers to transfer units amongst each other. As a consequence, every carrier is transporting its own demand. This also leads to a CTT game for which the core is non-empty.

Proposition 2. *Let $\theta \in \Theta_{UTC}$. Then, the corresponding CTT game has a non-empty core.*

Proof. Let the cost for a carrier to transport its own volume, i.e., $D_i \sum_{p \in P} (c_i^p + s_{ii}^p) + \sum_{p \in P} t_{ii}^p$, be less than any t_{ij}^p. In that case, transferring some units is more expensive than dispatching the whole demand on the carrier-owned route. This leads to an optimal solution $x_{ii}^p = D_i$ and $x_{ij}^p = 0$ for all $i, j \in N$ with $i \neq j$ and all $p \in P$. Therefore, according to Problem (2),

$$C^{\theta}(S) = \sum_{i \in S} (D_i \sum_{p \in P} c_i^p) = \sum_{i \in S} C(\{i\})$$

It is evident that $(u_i)_{i \in N} = (C(\{i\}))_{i \in N}$ is a core allocation according to (3). □

We want to stress that a similar argument can be used to argue that a CTT game with extremely high variable transfer costs has a non-empty core.

4.3. Unlimited capacity with equal fixed and variable transfer costs

Finally, we discuss a class of transfer and transport situations $\Theta_{UCETC} \subset \Theta$ for which $Q_i^p > \sum_{i \in N} D_i$ for all $p \in P$ and $t_{ij}^p = t_{i'j'}^p$ and $s_{ij}^p = s_{i'j'}^p$ for all $i, i', j, j' \in N$ and all $p \in P$. This describes a situation where due to the lack of capacity restrictions, in combination with the equal (fixed and variable) transfer costs, the entire volume will follow the same transfer policy after consolidation at point 1.

Proposition 3. *Let $\theta \in \Theta_{UCETC}$. Then, the corresponding CTT game has a non-empty core.*

Proof. Let $S \subseteq N$, while, for notational convenience, we denote the fixed transfer cost by $\bar{t}$ and the variable transfer cost by $\bar{s}$. Then,

$$c^\theta(S) = (|S|+|P|-1)\bar{t} + \left(\sum_{i\in S} D_i\right)\left(\bar{s}|P|+\sum_{p\in P}\min_{i\in S} c_i^p,\right)$$

which results from the fact that the entire volume is transferred to one carrier at point 1 and stays consolidated until the final point. Note that this consolidated flow might (of course) be transported by different carriers depending on the transport costs between points. We will show that the core is non-empty by proving that vector $u = (u_i)_{i\in N}$ is in the core, with

$$u_i = \left(1 + \frac{|P|-1}{|N|}\right)\bar{t} + D_i\left(\bar{s}|P|+\sum_{p\in P}\min_{i\in N} c_i^p\right).$$

First, we can observe that x is efficient since

$$\sum_{i\in N} u_i = \sum_{i\in N}\left(\left(1 + \frac{|P|-1}{|N|}\right)\bar{t} + D_i\left(\bar{s}|P|+\sum_{p\in P}\min_{i\in N} c_i^p\right)\right) = c^\theta(N).$$

Next, we can observe for $S \subseteq N$

$$\begin{aligned}
\sum_{i\in S} u_i &= \sum_{i\in S}\left(\left(1 + \frac{|P|-1}{|N|}\right)\bar{t} + D_i\left(\bar{s}|P|+\sum_{p\in P}\min_{i\in N} c_i^p\right)\right) \\
&= |S|\bar{t} + \frac{|S|}{|N|}|(P|-1)\bar{t} + \left(\sum_{i\in S} D_i\right)\left(\bar{s}\cdot|P|+\sum_{p\in P}\min_{i\in N} c_i^p\right) \\
&\leq |S|\bar{t} + (|P|-1)\bar{t} + \left(\sum_{i\in S} D_i\right)\left(\bar{s}|P|+\sum_{p\in P}\min_{i\in S} c_i^p\right) \\
&= c^\theta(S),
\end{aligned} \tag{7}$$

where the inequality follows from $\frac{|S|}{|N|} \leq 1$ and $\min_{i\in N} c_i^p \leq \min_{i\in S} c_i^p$ for all $p \in P$. □

In this case, we have not only seen that the core is non-empty, but we have found a core allocation based on the optimal path of each carrier and a division of the fixed costs.

5. Numerical experiments

In the previous section, we investigated sufficient conditions for core non-emptiness. In particular, we showed that the core is non-empty when the fixed transfer costs are very low, very high, or symmetric. In this section we will investigate how much we can generalize core non-emptiness when these conditions are not met. In the following, we first explain how we construct the instances used for our experiments, specifying the parameters and the related transfer and transport situations before presenting the results of these experiments in §5.2.

5.1. Instance design

For our experiments, we randomly generate instances with three, four, or five carriers (i.e., $|N| \in \{3,4,5\}$) that need to transport their volume via three, four, or five points (i.e., $|P| \in \{3,4,5\}$). In this context, we assume that carriers can operate in two types of markets:

- **A symmetric market**, where all carriers need to transport demand volume drawn from the same probability distribution; in our case, we set $D_i \sim \mathrm{D} = \mathrm{U}[80, 120]$ for all $i \in N$. In this case, all carriers have, on average, the same presence in the market.

- **A market with a dominant carrier**, where one carrier $i^* \in N$, referred to as the *dominant carrier* transports on average around 50% of the total demand volume, so that $D_{i^*} \sim (|N|-1) \cdot \mathrm{D}$, while the demand volume of all other carriers follows the probability distribution D as proposed in the symmetric market. In this way, we can study the impact of an asymmetric market on core non-emptiness.

Moreover, to observe the impact of capacity on core non-emptiness, we consider instances that range from settings where each carrier has sufficient capacity to serve all the demand to settings where each carrier has little or no space to serve extra demand. As such, to obtain the capacity Q_i^p of carrier $i \in N$ between points $p \in P$ and $p+1$, we first draw a number from the demand distribution D and subsequently multiply this with a capacity ratio, which is cap-r $\in \{1.1, 1.25, 1.5, 2, 2.5, 3, 6\}$. A low capacity ratio of cap-r $= 1.1$ would reflect a setting where each carrier has little or no space to transport additional volumes, while a capacity ratio of cap-r $= 6$ would reflect the case where each carrier can serve the entire volume in the system. To avoid infeasibility, we set the generated demand of a carrier as a lower bound, so that $Q_i^p \sim \max\{D_i, \mathrm{D} \cdot \text{cap-r}\}$ for all $i \in N$ and $p \in P$.

To investigate the impact of different cost structures on core non-emptiness, we consider variable transfer costs as a reference and propose two ratio parameters that relate transportation costs with variable transfer costs and fixed transfer costs with both demand and variable transfer costs. To generate the variable costs of transferring one unit of volume from i to j at point $p \in P$, we draw a number $s_{ij}^p \sim \mathrm{S} = \mathrm{U}[80, 120]$ for all $i, j \in N$ and $p \in P$. Next, to generate c_i^p, which represents the cost of transporting one unit of volume of carrier $i \in N$ from $p \in P$ to $p+1$, we draw a number from S and multiply this value with a ratio trans-r $\in \{1, 10, 100\}$, so that $c_i^p \sim \text{trans-r} \cdot \mathrm{S}$ for all $i \in N$ and all $p \in P$. Differently from the transportation costs, we want to scale the fixed transfer costs to both the variable transfer costs and the demands. Thus, to generate t_{ij}^p, i.e., the fixed transfer costs of transferring volume from i to j at point $p \in P$, we multiply a number in the interval of the variable transfer cost (i.e., $s_{ij}^p \sim \mathrm{S}$) with a number in the interval of the demand within a symmetric market (i.e., $D_i \sim \mathrm{D}$) and subsequently multiply this with a ratio fix-r $\in \{0.1, 1, 10, 100, 1000, 10000, 1000000\}$, so that $t_{ij}^p \sim \text{fix-r} \cdot \mathrm{S} \cdot \mathrm{D}$ for all $i, j \in N$ and all $p \in P$. Furthermore, we consider two special cases for the fixed transfer costs while referring to the one we have described so far as the "Standard" scenario:

- **No internal fixed transfer cost**, i.e., the setting with $t_{ii}^p = 0$ for all $i \in N$ and $p \in P$, while all other fixed transfer costs are generated in the same way as described earlier. This describes the case where no extra investment is needed for internal transfering. We call this scenario "No internal".

- **A group discount**, which reflects a setting where carriers already collaborate and, thus, the fixed transfer costs associated with them might be lower. To account for this, we randomly divide the group of carriers N into two subgroups N_1, N_2, with $N_1 \cup N_2 = N$ and $|N_1| = \lceil |N|/2 \rceil$. Within the two groups, N_1 and N_2, we reduce the fixed transfer cost between each pair of carriers, t_{ij}^p with 50%. Formally, for all $i, j \in N_1$, all $i, j \in N_2$ and all $p \in P$, we set $t_{ij}^p \sim 1/2 \cdot \text{fix-r} \cdot \mathrm{S} \cdot \mathrm{D}$. We assume that all other fixed transfer costs are generated in the same way as before. We call this scenario "Group discount".

Table 7 presents an overview of the configurations used to generate the instances for our experiments. We create one instance for *each* possible combination of parameters in Table 7.

Table 7: Parameter configurations used for our experiments.

Experimental Parameter	*Values*
Number of carriers	3, 4, 5
Number of points	3, 4, 5
Demand distribution	Symmetric, Big carrier
Fixed cost ratio (fix-r)	0.1, 1, 10, 100, 1000, 10000, 100000
Transportation cost ratio (trans-r)	1, 10, 100
Internal fixed costs scenario	Standard, No internal, Group
Capacity ratio (cap-r)	1.1, 1.25, 1.5, 2, 2.5, 3, 6

5.2. Results

In Figure 3, we present in white bars the frequency of instances for which $C(N) < \sum_{i \in N} C(\{i\})$, which we refer to as "real collaborations", and in the yellow bars we present the percentage of these real-collaboration instances for which the core is non-empty. Based on this, we observe that the majority of games with real collaboration have a non-empty core, with the highest percentage (99.9%) observed for instances with a fix-r ratio equal to 0.1 and the lowest percentage (73.55%) observed for a capacity ratio of 2.5. These numbers indicate that stable collaborations are still very likely, even without very low, very high, or symmetric fixed transfer costs. At the same time, the impact of the number of carriers, the number of points, the transportation cost ratio, and the market type seems to be minor in comparison to the impact of the capacity, the fixed transfer costs as well as the considered fixed transfer cost scenarios on both real collaborations as well as the frequency of non-empty cores.

Investigating the impact of capacity in more detail, we first observe a decrease in core non-emptiness (from 99.75% to 73.55%) as capacity increases. This is followed by an increase, which peaks at 94.5% for the case where each carrier has sufficient capacity to serve all demand (cap-r = 6). These numbers indicate that the more extreme cases provide more stable conditions for real collaboration, while the range in between creates conditions in which a stable collaboration is less likely. The underlying reasoning for this drop might be that some coalitions can transport their volume over the network without suffering from capacity constraints, while others do not. That is, some coalitions find an optimal transport solution for which none of the capacity constraints are binding, while for other coalitions some of these capacity constraints are binding. Consequently, the coalitions without binding capacity constraints benefit more from the collaboration than the others, which can break stability. This effect disappears when capacity becomes sufficiently large (cap-r = 6) or very tight (cap-r = 1.1).

Focusing on different transfer cost settings, we observe that the absence of internal fixed transfer costs considerably reduces the frequency of real collaborations (22.41% of the cases instead of 99.36% and 98.94%), in comparison to the "Standard" and "Group discount" scenarios. This relatively low frequency of real collaborations follows from the fact that transporting volume individually (which is not a real collaboration) is often cheaper. Despite real collaborations being sparse whenever no internal fixed transfer costs are considered, the core is often non-empty in case of real collaborations with higher probabilities than in the "Standard" and "Group discount" scenarios (99.66% instead of 87.68% and 86.25%, respectively). Looking in more detail at the fixed costs ratio, we observe a decrease in the frequency of core non-emptiness as well as real collaborations as fix-r increases, i.e., the core is non-empty in 99.9% of the cases for fix-r = 0.1 and drops to 80.93% for fix-r = $100,000$, and real collaborations drop from 88.71% for fix-r = 0.1 to 65.7% for fix-r = 1000. This is, however, not true if we only focus on the fixed cost scenario "No internal". This is illustrated in Figure 4, where we present the results separated by the considered fixed cost ratio scenarios, i.e., "Standard", "Group" and "No internal", while using a more detailed scale of fix-r (i.e., we reran our experiments with a more detailed scale of fix-r ranging between 0.1 and 100). From Figure 4, we learn that the decrease

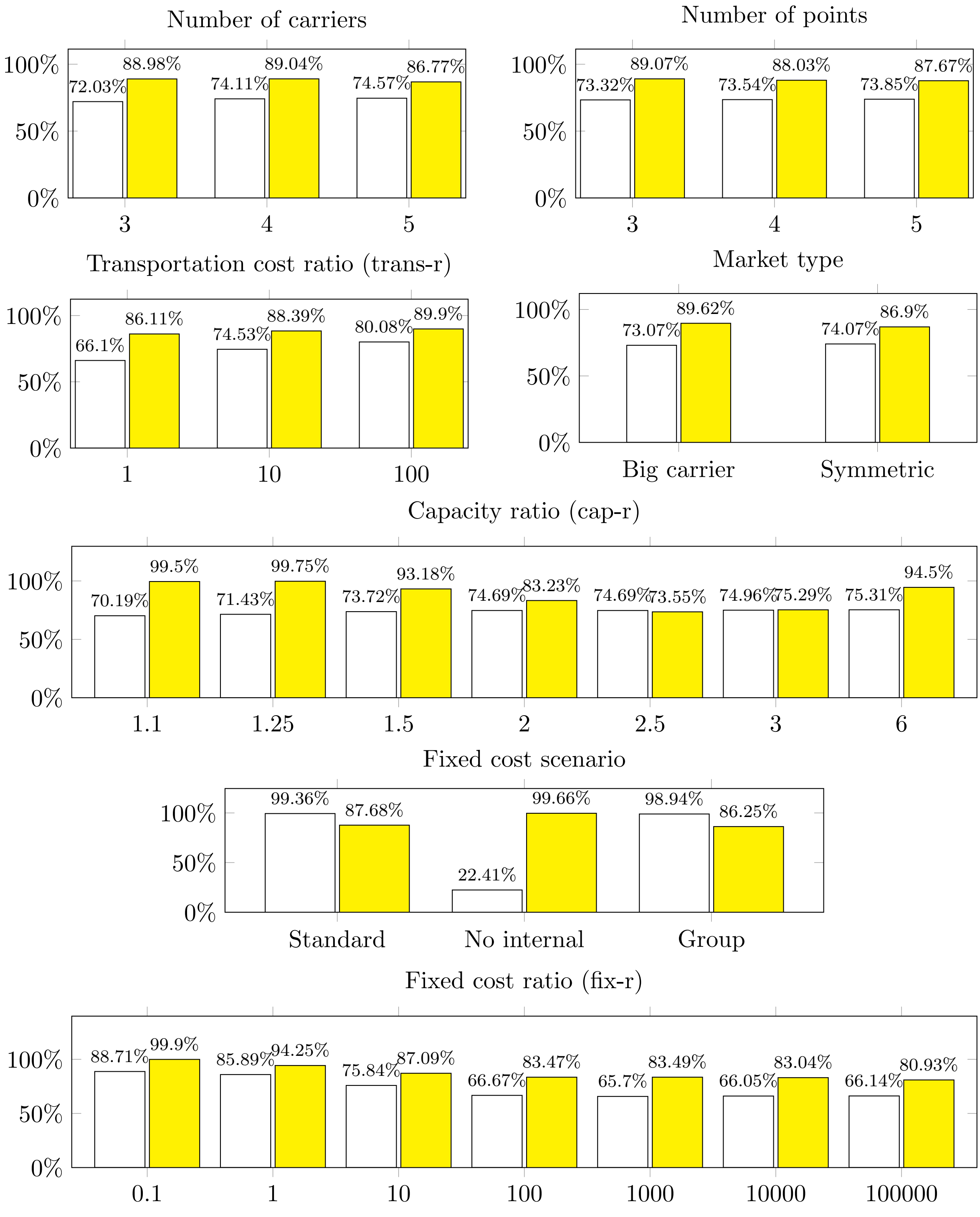

Figure 3: Non-additive games (white bar) and non-empty core (yellow bar) frequencies in the first experiment.

in core non-emptiness is only caused by the fixed cost scenario "Standard" and "Group", while the frequency of core non-emptiness for the fixed cost scenario "No Internal" is always above 95% and relatively stable.

In summary, the numerical experiments indicate that collaboration is very likely, even without very low, very high, or symmetric fixed transfer costs. However, exceptions exist, most often due to partial capacity restrictions.

Figure 4a: Fixed cost ratio (fix-r) with internal fixed cost scenario “Standard”

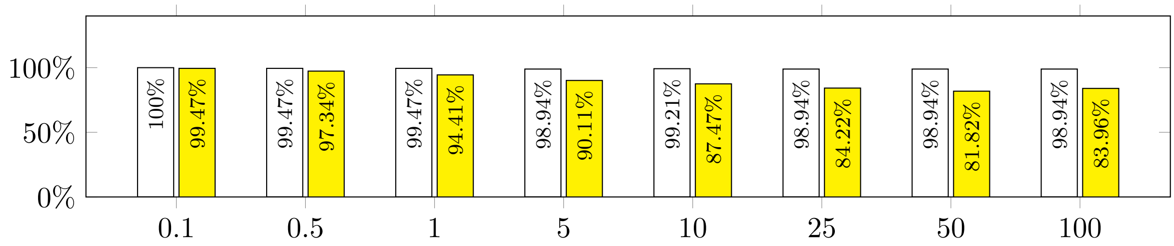

Figure 4b: Fixed cost ratio (fix-r) with internal fixed cost scenario “No internal”

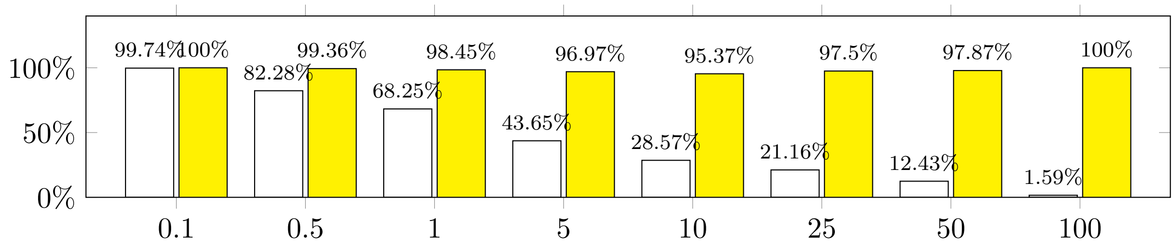

Figure 4c: Fixed cost ratio (fix-r) with internal fixed cost scenario “Group”

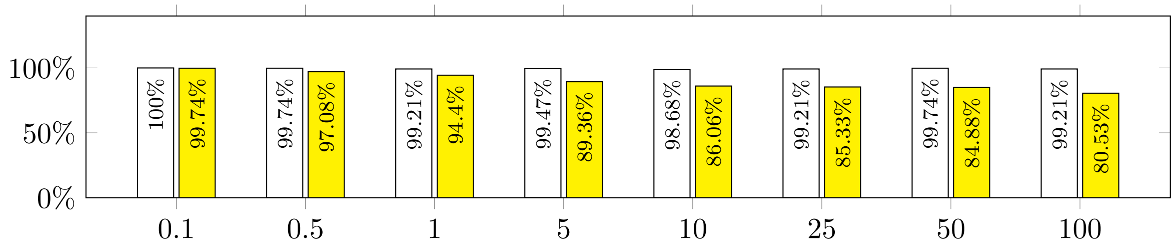

Figure 4: Comparison of the impact of fix-r between the three discount scenarios.

6. Concluding remarks

In this paper, we studied a setting where multiple carriers deliver to the same final location and thus may decide to collaborate, sharing the costs of shipping and transferring, including fixed costs arising from potential investments. In this context, we show that a stable allocation of the joint costs is not always possible, which affects stability and thus may hinder collaboration. However, stability is guaranteed when fixed transfer costs are very low, very high, or symmetric. Via numerical experiments, we discover furthermore that collaboration is very likely to be sustainable outside of these extreme cases, although exceptions may exist, most likely related to capacity restrictions.

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Fabio Mercurio: Conceptualization; Data curation; Formal analysis; Investigation; Methodology; Project administration; Resources; Software; Validation; Visualization; Writing - original draft; Writing - review & editing.
Loe Schlicher: Conceptualization; Data curation; Formal analysis; Funding acquisition; Investigation; Methodology; Project administration; Resources; Supervision; Validation; Visualization; Writing - original draft; Writing - review & editing.
Sonja U.K. Rohmer: Conceptualization; Data curation; Formal analysis; Funding acquisition;

Investigation; Methodology; Project administration; Resources; Supervision; Validation; Visualization; Writing - original draft; Writing - review & editing..
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References

Agarwal, R. and Ergun, Ö. (2008). Mechanism design for a multicommodity flow game in service network alliances. *Operations Research Letters*, 36(5):520–524.

Agarwal, R. and Ergun, Ö. (2010). Network design and allocation mechanisms for carrier alliances in liner shipping. *Operations research*, 58(6):1726–1742.

Basso, F., D'Amours, S., Rönnqvist, M., and Weintraub, A. (2019). A survey on obstacles and difficulties of practical implementation of horizontal collaboration in logistics. *International Transactions in Operational Research*, 26(3):775–793.

CT Logistics (2024). Ct transaver® program. https://www.ctlogistics.com/3pl/shipping-co-op. (Accessed on Feb 12, 2024).

DHL (2022). The logistics trend radar. https://www.dhl.com/nl-en/home/insights-and-innovation/insights/logistics-trend-radar.html. (Accessed on Jan 24, 2024).

Ferrell, W., Ellis, K., Kaminsky, P., and Rainwater, C. (2020). Horizontal collaboration: opportunities for improved logistics planning. *International Journal of Production Research*, 58(14):4267–4284.

Gansterer, M. and Hartl, R. F. (2018). Collaborative vehicle routing: a survey. *European Journal of Operational Research*, 268(1):1–12.

Gansterer, M. and Hartl, R. F. (2020). Shared resources in collaborative vehicle routing. *Top*, 28(1):1–20.

Guajardo, M. and Rönnqvist, M. (2016). A review on cost allocation methods in collaborative transportation. *International transactions in operational research*, 23(3):371–392.

Hribernik, M., Zero, K., Kummer, S., and Herold, D. M. (2020). City logistics: Towards a blockchain decision framework for collaborative parcel deliveries in micro-hubs. *Transportation Research Interdisciplinary Perspectives*, 8:100274.

Hu, X., Caldentey, R., and Vulcano, G. (2013). Revenue sharing in airline alliances. *Management Science*, 59(5):1177–1195.

Hub Group (2024). Retail consolidation. https://www.hubgroup.com/logistics-management/consolidation-fulfillment/retail-consolidation/. (Accessed on Feb 12, 2024).

Investopedia (2022a). Assembly service: What it is, how it works. https://www.investopedia.com/terms/a/assembly-service.asp. (Accessed on Feb 12, 2024).

Investopedia (2022b). Less-than-truckload definition and shipping service basics. https://www.investopedia.com/terms/l/lessthantruckload.asp#:~:text=Less%2Dthan%2Dtruckload%2C%20also%20known%20as%20less%2Dthan,between%20150%20and%2015%2C000%20pounds. (Accessed on Feb 12, 2024).

Karam, A., Reinau, K. H., and Østergaard, C. R. (2021). Horizontal collaboration in the freight transport sector: Barrier and decision-making frameworks. *European Transport Research Review*, 13:1–22.

Lai, M., Cai, X., and Hall, N. G. (2022). Cost allocation for less-than-truckload collaboration via shipper consortium. *Transportation Science*, 56(3):585–611.

Lozano, S., Moreno, P., Adenso-Díaz, B., and Algaba, E. (2013). Cooperative game theory approach to allocating benefits of horizontal cooperation. *European Journal of Operational Research*, 229(2):444–452.

Markakis, E. and Saberi, A. (2003). On the core of the multicommodity flow game. In *Proceedings of the 4th ACM conference on Electronic commerce*, pages 93–97.

Owen, G. (1975). On the core of linear production games. *Mathematical programming*, 9(1):358–370.

Özener, O. Ö. and Ergun, Ö. (2008). Allocating costs in a collaborative transportation procurement network. *Transportation Science*, 42(2):146–165.

Pan, S., Trentesaux, D., Ballot, E., and Huang, G. Q. (2019). Horizontal collaborative transport: survey of solutions and practical implementation issues. *International Journal of Production Research*, 57(15-16):5340–5361.

Shapley, L. S. et al. (1965). *On balanced sets and cores.* Rand Corporation.

van Zon, M., Spliet, R., and van den Heuvel, W. (2021). The joint network vehicle routing game. *Transportation Science*, 55(1):179–195.

Appendix A. Coalitional solutions of Example 2

We repeat in Table A.8 the parameters of the CTT game of Example 2, which are the ones of Example 1.

Table A.8: Parameters of the problem of Examples 1 and 2.

i	1	2	3
D_i	6	4	3

(a) Carriers' demands

Q_i^p	i =1	2	3
p=1	7	8	7
2	7	6	7

(b) Carriers' capacities

c_i^p	$i=1$	2	3
$p=1$	6	1	9
2	9	2	4

(c) Carriers' costs

t_{ij}^1, s_{ij}^1	j =1	2	3
i =1	0,0	4,2	6,1
2	9,4	0,0	7,1
3	5,4	5,3	0,0

(d) Fixed and variable costs on first point

t_{ij}^2, s_{ij}^2	j =1	2	3
i =1	0,0	6,4	9,4
2	5,4	0,0	5,3
3	9,3	5,2	0,0

(e) Fixed and variable costs on second point.

In Table 4, we represent the coalitional costs of the cooperative game in Example 2. We describe here the solutions leading to those costs. We know that $C(\{i\}) = \sum_{p \in P} ((s_{ii}^p + c_i^p)D_i + t_{ii}^p)$.

Therefore,
$C(\{1\}) = (0+6) \cdot 6 + 0 + (0+9) \cdot 6 + 0 = 90$,
$C(\{2\}) = (0+1) \cdot 4 + 0 + (0+2) \cdot 4 + 0 = 12$, and
$C(\{3\}) = (0+9) \cdot 3 + 0 + (0+4) \cdot 3 + 0 = 39$.

For coalition $S = \{1, 2\}$:
$x^1_{11} = 4,\ x^1_{12} = 2,\ x^1_{21} = 0,\ x^1_{22} = 4,$
$x^2_{11} = 4,\ x^2_{12} = 0,\ x^2_{21} = 0,\ x^2_{22} = 6,$
$z^1_{11} = 1,\ z^1_{12} = 1,\ z^1_{21} = 0,\ z^1_{22} = 1,$
$z^2_{11} = 1,\ z^2_{12} = 0,\ z^2_{21} = 0,\ z^2_{22} = 1.$
So
$\sum_{p\in P}\sum_{i\in S}\sum_{j\in S}\left((c^p_i + s^p_{ji})x^p_{ji} + t^p_{ji}z^p_{ji}\right) = (s^1_{12}\cdot x^1_{12} + t^1_{12}) + (c^1_1 * \sum_{j\in S} x^1_{j1}) + (c^1_2 * \sum_{j\in S} x^1_{j2}) + (c^2_1 * \sum_{j\in S} x^2_{j1}) + (c^2_2 * \sum_{j\in S} x^2_{j2}) = (2\cdot 2 + 4) + (6\cdot 4) + (1\cdot 6) + (9\cdot 4) + (2\cdot 6) = 86.$

For coalition $S = \{1, 3\}$:
$x^1_{11} = 6,\ x^1_{13} = 0,\ x^1_{31} = 0,\ x^1_{33} = 3,$
$x^2_{11} = 6,\ x^2_{13} = 0,\ x^2_{31} = 0,\ x^2_{33} = 3,$
$z^1_{11} = 1,\ z^1_{13} = 0,\ z^1_{31} = 0,\ z^1_{33} = 1,$
$z^2_{11} = 1,\ z^2_{13} = 0,\ z^2_{31} = 0,\ z^2_{33} = 1.$
So
$\sum_{p\in P}\sum_{i\in S}\sum_{j\in S}\left((c^p_i + s^p_{ji})x^p_{ji} + t^p_{ji}z^p_{ji}\right) = (c^1_1\cdot \sum_{j\in S} x^1_{j1}) + (c^1_3\cdot \sum_{j\in S} x^1_{j3}) + (c^2_1\cdot \sum_{j\in S} x^2_{j1}) + (c^2_3\cdot \sum_{j\in S} x^2_{j3}) = (6\cdot 6) + (9\cdot 3) + (9\cdot 6) + (4\cdot 3) = 129.$

For coalition $S = \{2, 3\}$:
$x^1_{22} = 4,\ x^1_{23} = 0,\ x^1_{32} = 2,\ x^1_{33} = 1,$
$x^2_{22} = 6,\ x^2_{23} = 0,\ x^2_{32} = 0,\ x^2_{33} = 1,$
$z^1_{22} = 1,\ z^1_{23} = 0,\ z^1_{32} = 1,\ z^1_{33} = 1,$
$z^2_{22} = 1,\ x^2_{23} = 0,\ z^2_{32} = 0,\ z^2_{33} = 1.$
So
$\sum_{p\in P}\sum_{i\in S}\sum_{j\in S}\left((c^p_i + s^p_{ji})x^p_{ji} + t^p_{ji}z^p_{ji}\right) = (s^1_{32}\cdot x^1_{32} + t^1_{32}) + (c^1_2\cdot \sum_{j\in S} x^1_{j2}) + (c^1_3\cdot \sum_{j\in S} x^1_{j3}) + (c^2_2\cdot \sum_{j\in S} x^2_{j2}) + (c^2_3\cdot \sum_{j\in S} x^2_{j3}) = (3\cdot 2 + 5) + (1\cdot 6) + (9\cdot 1) + (2\cdot 6) + (4\cdot 1) = 42.$

For coalition $S = \{1, 2, 3\}$:
$x^1_{11} = 2,\ x^1_{12} = 4,\ x^1_{13} = 0,\ x^1_{21} = 0,\ x^1_{22} = 4,\ x^1_{23} = 0,\ x^1_{31} = 0,\ x^1_{32} = 0,\ x^1_{33} = 3,$
$x^2_{11} = 2,\ x^2_{12} = 0,\ x^2_{13} = 0,\ x^2_{21} = 0,\ x^2_{22} = 6,\ x^2_{23} = 2,\ x^2_{31} = 0,\ x^2_{32} = 0,\ x^2_{33} = 3,$
$z^1_{11} = 1,\ z^1_{12} = 1,\ z^1_{13} = 0,\ z^1_{21} = 0,\ z^1_{22} = 1,\ z^1_{23} = 0,\ z^1_{31} = 0,\ z^1_{32} = 0,\ z^1_{33} = 1,$
$z^2_{11} = 1,\ z^2_{12} = 0,\ z^2_{13} = 0,\ z^2_{21} = 0,\ z^2_{22} = 1,\ z^2_{23} = 1,\ z^2_{31} = 0,\ z^2_{32} = 0,\ z^2_{33} = 1.$
So
$\sum_{p\in P}\sum_{i\in S}\sum_{j\in S}\left((c^p_i + s^p_{ji})x^p_{ji} + t^p_{ji}z^p_{ji}\right) = (s^1_{12}\cdot x^1_{12} + t^1_{12}) + (s^2_{23}\cdot x^2_{23} + t^2_{23}) + (c^1_1\cdot \sum_{j\in S} x^1_{j1}) + (c^1_2\cdot \sum_{j\in S} x^1_{j2}) + (c^1_3\cdot \sum_{j\in S} x^1_{j3}) + (c^2_1\cdot \sum_{j\in S} x^2_{j1}) + (c^2_2\cdot \sum_{j\in S} x^2_{j2}) + (c^2_3\cdot \sum_{j\in S} x^2_{j3}) = (2\cdot 4 + 4) + (3\cdot 2 + 5) + (6\cdot 2) + (1\cdot 8) + (9\cdot 3) + (9\cdot 2) + (2\cdot 6) + (4\cdot 5) = 120.$

Appendix B. Coalitional solutions of Example 3

We repeat in Table B.9 the parameters of the CTT game of Example 3.
In Table 6 we represent the coalitional costs of the cooperative game in Example 3. We describe here the solutions leading to those costs.
We know that $C(\{i\}) = \sum_{p\in P}((s^p_{ii} + c^p_i)D_i + t^p_{ii})$. Therefore,
$C(\{1\}) = (9\cdot 3) + (2\cdot 3) = 33,$

Table B.9: Parameters of the problem of Example 3

i	1	2	3
D_i	3	3	6

(a) Carriers' demands

Q_i^p	1	2	3
$l=1$	7	7	9
2	6	5	9

(b) Carriers' capacities

c_i^p	1	2	3
1	9	7	2
2	2	8	6

(c) Carriers' costs

t_{ij}^1, s_{ij}^1	$j=1$	2	3
$i=1$	0,0	9,4	5,1
2	7,2	0,0	7,2
3	8,1	8,2	0,0

(d) Fixed and variable costs on first point

t_{ij}^2, s_{ij}^2	$j=1$	2	3
$i=1$	0,0	7,2	8,4
2	5,1	0,0	8,4
3	4,1	7,2	0,0

(e) Fixed and variable costs on second point.

$C(\{2\}) = (7 \cdot 3) + (8 \cdot 3) = 45$, and
$C(\{3\}) = (2 \cdot 6) + (6 \cdot 6) = 48$.

For coalition $S = \{1,2\}$:
$x_{11}^1 = 3,\ x_{12}^1 = 0,\ x_{21}^1 = 0,\ x_{22}^1 = 3,$
$x_{11}^2 = 3,\ x_{12}^2 = 0,\ x_{21}^2 = 3,\ x_{22}^2 = 0,$
$z_{11}^1 = 1,\ z_{12}^1 = 0,\ z_{21}^1 = 0,\ z_{22}^1 = 1,$
$z_{11}^2 = 1,\ z_{12}^2 = 0,\ z_{21}^2 = 1,\ z_{22}^2 = 0.$
So
$\sum_{p\in P}\sum_{i\in S}\sum_{j\in S}\left((c_i^p + s_{ji}^p)x_{ji}^p + t_{ji}^p z_{ji}^p\right) = (s_{21}^2 \cdot x_{21}^2 + t_{21}^2) + (c_1^1 \cdot \sum_{j\in S} x_{j1}^1) + (c_2^1 \cdot \sum_{j\in S} x_{j2}^1) + (c_1^2 \cdot \sum_{j\in S} x_{j1}^2) =$
$(1 \cdot 3 + 5) + (9 \cdot 3) + (7 \cdot 3) + (2 \cdot 6) = 68.$

For coalition $S = \{1,3\}$:
$x_{11}^1 = 0,\ x_{13}^1 = 3,\ x_{31}^1 = 0,\ x_{33}^1 = 6,$
$x_{11}^2 = 0,\ x_{13}^2 = 0,\ x_{31}^2 = 6,\ x_{33}^2 = 3,$
$z_{11}^1 = 0,\ z_{13}^1 = 1,\ z_{31}^1 = 0,\ z_{33}^1 = 1,$
$z_{11}^2 = 0,\ z_{13}^2 = 0,\ z_{31}^2 = 1,\ z_{33}^2 = 1.$
So
$\sum_{p\in P}\sum_{i\in S}\sum_{j\in S}\left((c_i^p + s_{ji}^p)x_{ji}^p + t_{ji}^p z_{ji}^p\right) = (s_{13}^1 \cdot x_{13}^1 + t_{13}^1) + (s_{31}^2 \cdot x_{31}^2 + t_{31}^2) + (c_3^1 \cdot \sum_{j\in S} x_{j3}^1) + (c_1^2 \cdot \sum_{j\in S} x_{j1}^2) + (c_3^2 \cdot \sum_{j\in S} x_{j3}^2) = (1 \cdot 3 + 5) + (1 \cdot 6 + 4) + (2 \cdot 9) + (2 \cdot 6) + (6 \cdot 3) = 66.$

For coalition $S = \{2,3\}$:
$x_{22}^1 = 0,\ x_{23}^1 = 3,\ x_{32}^1 = 0,\ x_{33}^1 = 6,$
$x_{22}^2 = 0,\ x_{23}^2 = 0,\ x_{32}^2 = 0,\ x_{33}^2 = 9,$
$z_{22}^1 = 0,\ z_{23}^1 = 1,\ z_{32}^1 = 0,\ z_{33}^1 = 1,$
$z_{22}^2 = 0,\ x_{23}^2 = 0,\ z_{32}^2 = 0,\ z_{33}^2 = 1.$
So
$\sum_{p\in P}\sum_{i\in S}\sum_{j\in S}\left((c_i^p + s_{ji}^p)x_{ji}^p + t_{ji}^p z_{ji}^p\right) = (s_{23}^1 \cdot x_{23}^1 + t_{23}^1) + (c_3^1 \cdot \sum_{j\in S} x_{j3}^1) + (c_3^2 \cdot \sum_{j\in S} x_{j3}^2) = (2 \cdot 3 + 7) +$
$(2 \cdot 9) + (6 \cdot 9) = 85.$

For coalition $S = \{1,2,3\}$:
$x_{11}^1 = 0,\ x_{12}^1 = 0,\ x_{13}^1 = 3,\ x_{21}^1 = 0,\ x_{22}^1 = 3,\ x_{23}^1 = 0,\ x_{31}^1 = 0,\ x_{32}^1 = 0,\ x_{33}^1 = 6,$
$x_{11}^2 = 0,\ x_{12}^2 = 0,\ x_{13}^2 = 0,\ x_{21}^2 = 3,\ x_{22}^2 = 0,\ x_{23}^2 = 0,\ x_{31}^2 = 3,\ x_{32}^2 = 0,\ x_{33}^2 = 6,$
$z_{11}^1 = 0,\ z_{12}^1 = 0,\ z_{13}^1 = 1,\ z_{21}^1 = 0,\ z_{22}^1 = 1,\ z_{23}^1 = 0,\ z_{31}^1 = 0,\ z_{32}^1 = 0,\ z_{33}^1 = 1,$
$z_{11}^2 = 0,\ z_{12}^2 = 0,\ z_{13}^2 = 0,\ z_{21}^2 = 1,\ z_{22}^2 = 0,\ z_{23}^2 = 0,\ z_{31}^2 = 1,\ z_{32}^2 = 0,\ z_{33}^2 = 1.$

So $\sum_{p\in P}\sum_{i\in S}\sum_{j\in S}\Big((c_i^p + s_{ji}^p)x_{ji}^p + t_{ji}^p z_{ji}^p\Big) = (s_{13}^1 \cdot x_{13}^1 + t_{13}^1) + (s_{21}^2 \cdot x_{21}^2 + t_{21}^2) + (s_{31}^2 \cdot x_{31}^2 + t_{31}^2) + (c_2^1 \cdot \sum_{j\in S} x_{j2}^1) + (c_3^1 \cdot \sum_{j\in S} x_{j3}^1) + (c_1^2 \cdot \sum_{j\in S} x_{j1}^2) + (c_3^2 \cdot \sum_{j\in S} x_{j3}^2) = (1\cdot 3+5) + (1\cdot 3+5) + (1\cdot 3+4) + (7\cdot 3) + (2\cdot 9) + (2\cdot 6) + (6\cdot 6) = 110.$