OPTIMAL SPORTS LEAGUE REALIGNMENT*

NATHAN BRIXIUS†

Abstract. We consider approaches for optimally organizing competitive sports leagues in light of competitive and logistical considerations. A common objective is to assign teams to divisions so that intradivisional travel is minimized. We present a bilinear programming formulation based on k-way equipartitioning, and show how this formulation can be extended to account for additional constraints and objectives. We show that our formulation and extensions can be solved directly using modern solvers. We present computational results for all major North American professional sports leagues.

Key words. Sports Realignment

AMS subject classifications. 90B80, 90C27, 90C35, 90-XX

1. Introduction.

We consider organizing teams within competitive sports leagues. For teams $t_1, ...t_n \in T$, we define a realignment to be a disjoint partitioning $\{D_1, \ldots, D_K\}$ with $D_i \in \mathcal{P}(T)$. We refer to each D_i as a division. Let $\mathcal{V}$ be the set of all possible realignments. Optimal League Realignment (OLR) can be stated as the problem $\min_{V \in \mathcal{V}} f(V)$ for a cost function f.

We use OLR-D to refer to the case where f is the sum of distances between teams assigned to the same division. Numerous methods for solving OLR-D have been proposed in the literature. Saltzman and Bradford [13] found locally optimal solutions to a linearly constrained nonlinear formulation OLR-D for the National Football League (NFL). When the NFL realigned after the 2002 season, Mitchell [10] showed that an optimal realignment would have improved travel distance by 18% over that selected by the league. Xi and Mitchell [7] used a branch-and-cut-and-price approach to solve a mixed integer linear programming formulation of OLR-D for the NFL. Both heuristic and mixed integer programming methods were employed by [9] to solve OLR-D across several North American professional sports leagues.

The OLR-D formulations [7], [9], [10], [13] all address the core problem of minimizing the sum of total distances between teams assigned to the same division. We show that OLR-D can be formulated and solved directly as a bilinear mixed integer program using modern solvers. Further, we consider extensions of OLR-D incorporating a wide range of constraints and objectives.

In Section 2 we summarize relevant domain knowledge for OLR. In Section 3, we present a equipartitioning-based integer programming formulation of OLR-D, building on the work of [7]. We show how this general bilinear mixed integer programming formulation can accomodate additional realignment considerations not previously considered. In Section 3.3 we show how our bilinear formulation can be modified to address OLR variants. In Section 4, we present computational results.

2. Sports League Realignment.

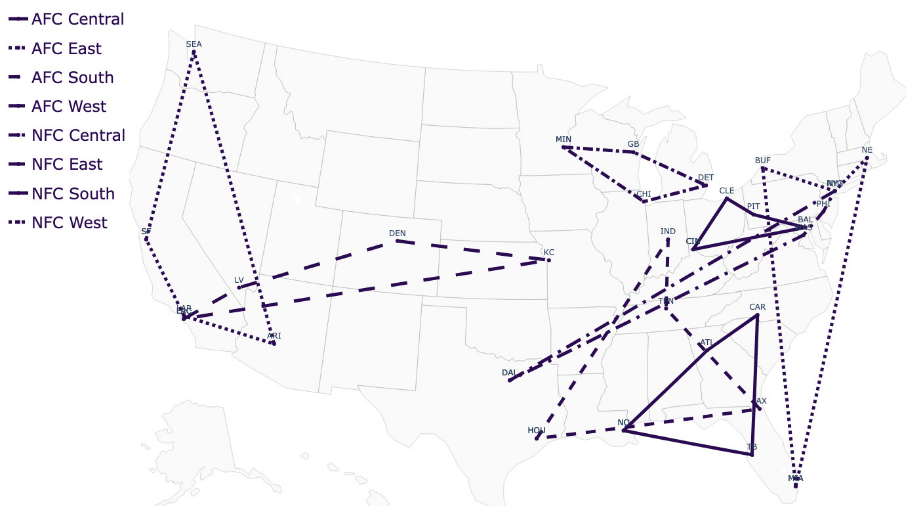
In North American sports leagues such as the NFL, Major League Baseball (MLB), the National Basketball Association (NBA), and National Hockey League (NHL), teams are aligned into hierarchical structures. These alignments influence the scheduling of games, playoff participation, travel costs, and other considerations.

We will frequently use the NFL as an example in what follows. The NFL's 32

*Submitted May 16, 2024.

†Airbnb, Inc., San Francisco, CA (nathan.brixius@airbnb.com).

teams are partitioned into two conferences, further subdivided into four divisions of four teams each. Divisional assignments for the 2023 NFL season are shown in Figure 1, with each division rendered as a tour between team locations. The locations and key attributes of NFL teams for the 2023 season are given in Appendix A.



AFC				NFC			
Central	East	South	West	Central	East	South	West
BAL	BUF	HOU	DEN	CHI	DAL	ATL	ARI
CIN	MIA	IND	KC	DET	NYG	CAR	LAR
CLE	NE	JAX	LAC	GB	PHI	NO	SEA
PIT	NYJ	TEN	LV	MIN	WAS	TB	SF

FIG. 1. *Divisional alignment of National Football League teams, 2023.*

We consider leagues with a two-level hierarchy with conferences C and divisions D. Let $S \in \mathbb{R}^{C \times D}$, with s_{ij} the number of teams to be assigned to division $i \in D$ within $j \in C$. Let $m = |C||D|$ be the total number of divisions.

The NFL, NBA, NHL, and MLB all fit this hierarchy, though with different team and division counts. Therefore the methods herein are applicable to all major North American sports leagues.

3. Realignment Model Formulations. We consider the graph-based formulation of OLR-D presented in [7], with vertices representing teams and edges representing distance between pairs of team locations. OLR-D is equivalent to partitioning G into k subcliques with sizes given by S, referred to by [7] as k-way equipartitioning. We seek the equipartitioning that minimizes total distance within subcliques, and refer to such a partitioning as a realignment.

Let $x_{tij} \in \{0, 1\}$, $t \in T$, $j \in C$, $i \in D$. We let $x_{tij} = 1$ iff team t is assigned to division i in conference j. Letting d_{tu} be the distance between teams t and u, OLR-D is the minimization problem

$$\begin{aligned} \min \quad & \sum_{tuij} d_{tu} x_{tij} x_{uij} \\ \text{s.t.} \quad & \sum_{t} x_{tij} = s_{tij} \quad \forall i \in D, j \in C \\ & \sum_{ij} x_{tij} = 1 \qquad\qquad \forall t \in T \\ & x_{tij} \in \{0,1\}. \end{aligned} \tag{3.1}$$

Observe that (3.1) is a bilinear quadratic integer program with mn variables and $m+n$ constraints. Despite (3.1)'s nonconvexity, in Section 4 we shall see that for typical m and n, optimal solutions are readily obtained by commercial solvers.

It is well known that (3.1) can be reformulated as a mixed integer linear program by introducing an auxiliary variable $z_{tuij} = x_{tij} x_{uij}$. Since $x_{tij} \in \{0,1\}$, we have

$$\begin{aligned} \min \quad & \sum_{tuij} d_{tu} z_{tuij} \\ \text{s.t.} \quad & \sum_{t} x_{tij} = s_{ij} \qquad\qquad \forall i \in D, j \in C \\ & \sum_{ij} x_{tij} = 1 \qquad\qquad\qquad \forall t \in T \\ & z_{tuij} \le x_{tij} \qquad \forall t, u \in T, i \in D, j \in C \\ & x_{tij}, z_{tuij} \in \{0,1\}. \end{aligned} \tag{3.2}$$

This formulation has n^2+n variables and $m+n+mn^2$ constraints. The runtime characteristics of (3.1) and (3.2) are compared in Section 4.

As was noted, formulations equivalent to (3.1) and (3.2) have been considered in previous work. However, there may be additional practical considerations desired to be accounted for in a realignment. We will now show how (3.1) or (3.2) can be augmented to account for such considerations. In what follows, $t, u \in T$ unless otherwise specified.

3.1. Divisional constraints. Competitive and financial considerations may motivate forcing or forbidding certain divisional assignments. For example, the NFL's Dallas Cowboys and New York Giants have been in the same division since 1961 but are geographically distant, a relationship not naturally preserved by solving (3.1). The condition that teams t and u should be assigned to the same division is expressed as

$$x_{tij} = x_{uij}, \quad \forall i \in D, j \in C. \tag{3.3}$$

Teams t and u are disallowed from sharing a division by the relation

$$x_{tij} + x_{uij} \le 1, \quad \forall i \in D, j \in C. \tag{3.4}$$

3.2. Stability. Economic and competitive considerations may make it undesirable for an existing alignment to be changed significantly. We can account for this league stability consideration by introducing a lower bound $c_{\min}$ on the number of teams whose assignments should match a given past realignment. Let $w_{tij} = 1$ if team t was assigned to division i in conference j in a previous realignment, and $w_{tij} = 0$ otherwise. Let $W = \{(t,i,j) \mid w_{tij} = 1\}$. Stability is then enforced by

$$\sum_{(t,i,j)\in W} x_{tij} \geq c_{\min}. \tag{3.5}$$

A related problem is to minimize the number of team reassignments while ensuring a decrease in travel distance. Letting $d_{\max}$ be an upper bound on travel distance, we have the reformulation

$$\begin{aligned}
\min \quad & v \\
\text{s.t.} \quad & \sum_{tuij} d_{tu} z_{tuij} \leq d_{\max} \\
& \sum_{t} x_{tij} = s_{ij} && \forall i \in D, j \in C \\
& \sum_{ij} x_{tij} = 1 && \forall t \in T \\
& v = N - \sum_{(t,i,j)\in W} x_{tij} \\
& x_{tij} \in \{0,1\}.
\end{aligned} \tag{3.6}$$

3.3. Schedule-aware optimization. Team travel distances depend on the locations of the opponents they are scheduled to play. Since teams typically do not play all other teams in a league, scheduling can be an important realignment consideration. Creating optimal schedules is itself an interesting problem with a rich literature, see for example [3], [6], and [11]. Multi-league scenarios were considered more recently in [8].

While it is typical for league schedules to change on a yearly basis, realignment is less frequent. For example, NFL schedules change every season whereas the last divisional realignment occurred in 2002. Therefore realigning to optimize for a particular schedule is not useful in practice. However, certain leagues create schedules with patterns that can be accounted for in realignment.

As of 2023, each NFL team plays seventeen games per season. Each team plays the other three teams in its own division twice per year, four teams from a division in their own conference once, and all four teams in a division in the other conference once. They play two games with other opponents within their conference, and the remaining game against an opponent outside their conference. Let $y_{tj} = 1$ iff team $t \in T$ is assigned to conference $j \in C$, regardless of division. The objective in (3.1) is then replaced by the mean seasonal travel distance

$$\sum_{tuij} d_{tu}(6x_{tij}x_{uij} + 6y_{tj}y_{uj} + 5y_{ti}(1 - y_{ui})) \tag{3.7}$$

and we add the additional structural constraint

$$y_{tj} = \sum_{d \in c} x_{tij}, \quad \forall t \in T, i \in D, j \in C. \tag{3.8}$$

3.4. Competitive Balance. Competitive balance may also be of concern for realignment because divisional assignments frequently influence scheduling and playoff participation. We interpret competitive balance as the problem of realigning such that the overall strength of each division is as equal as possible. Let the estimated strength of team t be p_t, and the strength of division $i \in D$ to be the sum of the strengths of its teams.

A realignment that minimizes maximal deviation between divisional strength is given by

$$\begin{aligned} \min \quad & \bar{r} \\ \text{s.t.} \quad & \sum_t x_{tij} = s_{ij} && \forall i \in D, j \in C \\ & \sum_{ij} x_{tij} = 1 && \forall t \in T \\ & \sum_t p_t x_{tij} = r_{ij} && \forall i \in D, j \in C \\ & \bar{r} \geq r_{ij} - r_{kl} && \forall i, k \in D, j, l \in C \\ & x_{tij} \in \{0, 1\}. \end{aligned} \tag{3.9}$$

Divisional strength and distance can also be considered together. [12] used exact and Tabu search techniques for k-way partitioning, incorporating node-level weights as constraints on the partitioning. Alternatively, we may augment (3.1) with competitive constraints by introducing an upper bound $r_{\max}$ on maximal divisional deviation in strength. The resulting formulation is

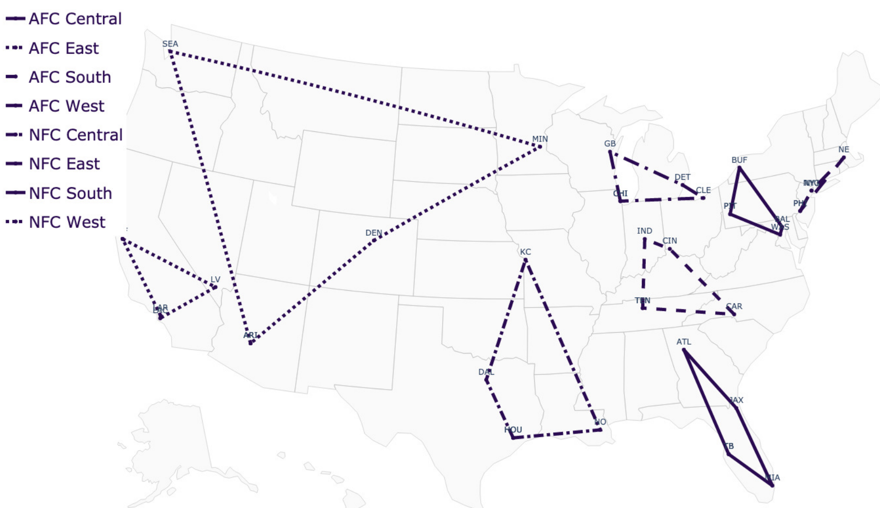
$$\begin{aligned} \min \quad & \sum_{tuij} d_{tu} x_{tij} x_{uij} \\ \text{s.t.} \quad & \sum_t x_{tij} = s_{ij} && \forall i \in D, j \in C \\ & \sum_{ij} x_{tij} = 1 && \forall t \in T \\ & r_{ij} = \sum_t x_{tij} p_t && \forall i \in D, j \in C \\ & \bar{r} \geq r_{ij} - r_{kl} && \forall i, k \in D, j, l \in C \\ & \bar{r} \leq r_{\max} \\ & x_{tij} \in \{0, 1\}. \end{aligned} \tag{3.10}$$

We have implemented a Python package for solving OLR-D and the variants previously described using the Gurobi and SCIP solvers. The source code and data for experiments are provided in [4].

4. Computational results.

We now present computational results for OLR-D and variants. All experiments were performed on a MacBook Pro M2 using Gurobi 11.0 and SCIP 8.0. The source code and data for experiments are provided in [4].

We first consider the bilinear formulation (3.1) for the 2023 NFL season. The current divisional alignment for the NFL is given in Figure 1, with an objective value of 43972.39. The optimal solution value is 28070.55 and the corresponding divisional alignment is given in Figure 2.



AFC				NFC			
Central	East	South	West	Central	East	South	West
ATL	LAC	CAR	NE	DAL	CHI	BAL	ARI
JAX	LAR	CIN	NYG	HOU	CLE	BUF	DEN
MIA	LV	IND	NYJ	KC	DET	PIT	MIN
TB	SF	TEN	PHI	NO	GB	WAS	SEA

FIG. 2. *Optimal NFL divisional alignment, 2023*

We solved the same formulation for the 2003 NFL season, confirming that the solution matches that provided in [9].

In Table 1 we present results for several algorithms applied across leagues and seasons. To obtain smaller instances we further divided leagues into their constituent conferences. The solution value of (3.1) is given in the `Optimal` column. The distances for realignments used by the respective league for the specified season are given in the `Incumbent` column, and the gap from optimal in `Inc. Gap`. Finally, we report the realignment distance for a greedy implementation where we repeatedly select the team with the smallest distance to a previously aligned team.

League	Season	Optimal	Incumbent	Inc. Gap	Greedy	Gr. Gap
MLB	2023	40150.82	58724.44	46.26%	52080.08	29.71%
MLB AL	2023	34923.44	35091.38	0.48%	43723.35	25.20%
MLB NL	2023	23076.48	23633.06	2.41%	26519.78	14.92%
NBA	2023	40057.53	41135.55	2.69%	43227.30	7.91%
NBA East	2023	15419.64	15469.88	0.33%	16311.08	5.78%
NBA West	2023	24637.89	25665.67	4.17%	27011.17	9.63%
NFL	2002	27577.90	50278.91	82.32%	36185.97	31.21%
NFL	2023	28070.55	43972.39	56.65%	30205.15	7.60%
NFL AFC	2023	18881.69	23191.47	22.83%	18881.69	0.00%
NFL NFC	2023	17803.52	20780.92	16.72%	17803.52	0.00%
NHL	2023	105605.67	112554.69	6.58%	139566.37	32.16%
NHL East	2023	36042.84	42595.26	18.18%	41893.10	16.23%
NHL West	2023	69562.83	69959.42	0.57%	81505.71	17.17%

Table 1
Realignment distances by league and approach

We next consider the mixed integer programming formulation (3.2). Since the formulations (3.1) and (3.2) are equivalent, solutions are identical. The runtimes for solving (3.1) and (3.2) using Gurobi are given in Table 2.

In Table 2 we measure the runtime to solve (3.1) for the NFL, NBA, MLB, and NHL. We report runtimes both for the bilinear formulation (3.1) and the mixed integer programming (MIP) formulation (3.2).

League	Season	Bilinear	MIP
MLB	2023	2.92	5.87
NBA	2023	2.97	11.22
NFL	2002	2.53	13.24
NFL	2023	4.60	16.86
NHL	2023	8.25	13.96

Table 2
Gurobi runtimes for (3.1) and (3.2)

As observed in in Table 2, runtime varies by problem instance, due to differences in the distance matrix D. Runtime performance also depends on the solver considerations. To solve (3.1) we used Gurobi 11.0.0, a commercial solver. We also used SCIP 7.0.1, an open source solver. SCIP was unable to solve any of the instances in Table 2 to optimality in 24 hours. When we subdivide leagues into their constituent conferences, both Gurobi and SCIP are able to realign all leagues optimally, as shown in Table 3. Comparative runtimes are directionally consistent with benchmarks for standard MIP test sets, see [5].

League	Season	Conf.	Gurobi	SCIP
MLB	2023	AL	0.256	0.470
MLB	2023	NL	0.053	0.365
NBA	2023	East	0.090	0.409
NBA	2023	West	0.167	1.077
NFL	2023	AFC	0.087	2.025
NFL	2023	NFC	0.134	0.867
NHL	2023	East	0.045	0.176
NHL	2023	West	0.017	0.150

TABLE 3
Gurobi and SCIP runtimes for (3.2)

In Section 3.2 we introduced the concept of stability, where we seek to limit the number of permitted team reassignments. In Figure 3 we show the relationship between optimality gap, runtime, and the number of permitted team reassignments. The number of reassignments is controlled using (3.5).

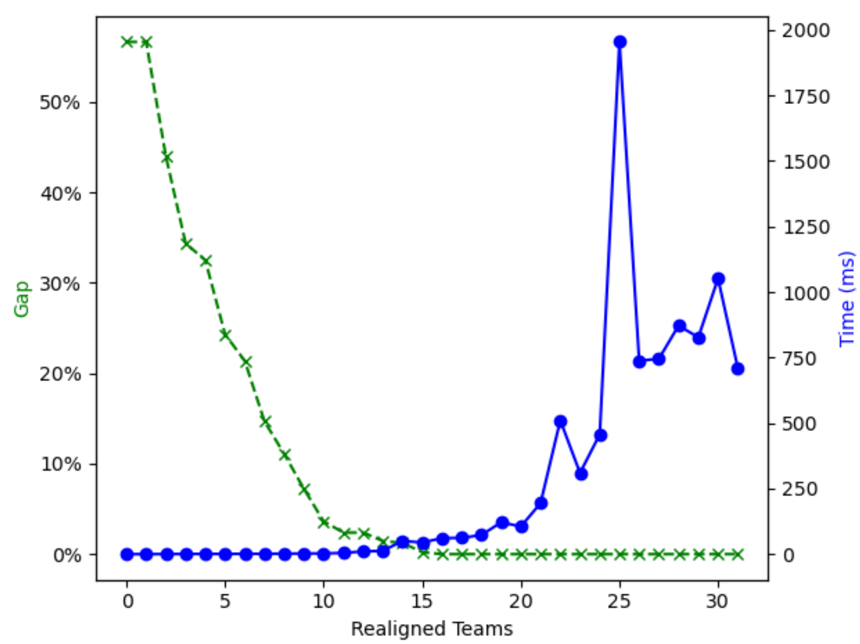


FIG. 3. *Solve time vs. relative gap for stability constraints, NFL 2023*

We next consider the alternative formulation (3.6). The incumbent distance for the 2023 NFL season is 43972.39. The optimal distance is 28070.55. In Figure 4 we repeatedly solve (3.6) for the using intermediate values of d_{max} between incumbent and optimal distances. The number of realigned teams from the incumbent is shown with the optimality gap on the y-axis.

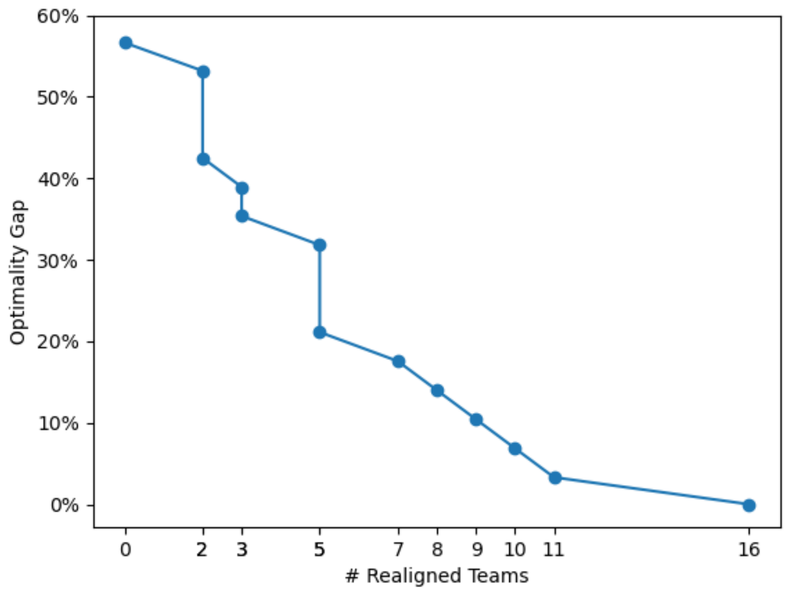


Fig. 4. *Optimality Gap - Max Swaps for NFL teams (2023)*

While we have focused on the NFL, the approaches described apply to other North American professional sports leagues. Optimal realignments for the NBA, MLB, and NHL are given in Appendix B. The incumbent alignments for the NBA and NHL are closer to their respective optimal realignments than the NFL and MLB. The NBA and NHL have geographically defined Eastern and Western conferences, whereas NFL and MLB conference composition relates to historical factors[1].

5. Conclusions.

We have presented several integer programming formulations for Optimal League Realignment. These formulations are able to incorporate practical considerations for realignment, as well as differing objectives. We have shown that modern commercial solvers can solve OLR in its more natural bilinear formulation more quickly than with a mixed integer programming formulation. Further, we have shown that our forumulations apply all major North American sports leagues.

We envision several possibilities for extensions and improvements. Column generation and cutting plane approaches are applicable for k-way equipartitioning, as in [1] and [12]. Another possibility is to devise hybrid variants of the formulations in order to accommodate noncommercial solvers. For example, a heuristic could be used to create an initial partition of divisions and teams, and the resulting subproblems could then be solved and unified.

Appendix A. National Football League Teams, 2023.

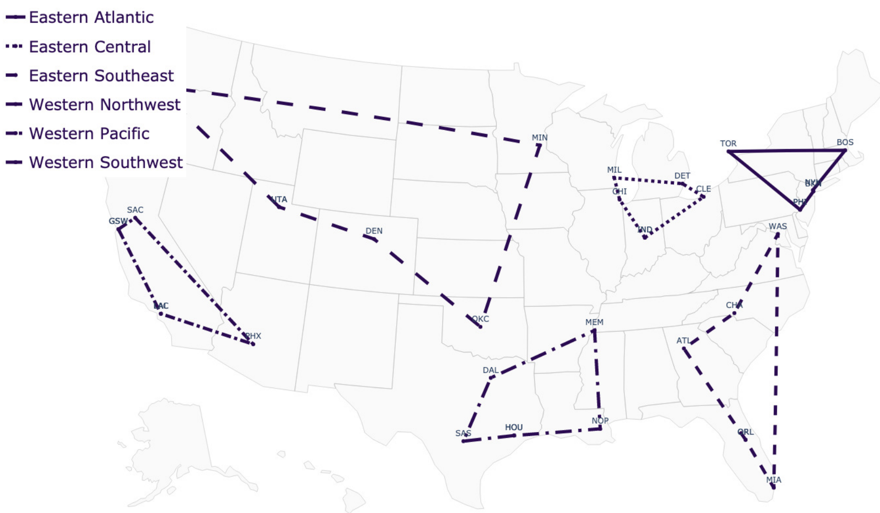
[1] The basis for the modern NFL was formed by a merger with the American Football League in 1970 [14]. Major League Baseball was formed by agreement of the National and American leauges in 1903, and were run semi-independently for decades after, see [2].

Team	Abbr.	Conf.	Div.	Latitude	Longitude
Arizona Cardinals	ARI	NFC	West	33.528	-112.263
Atlanta Falcons	ATL	NFC	South	33.755	-84.401
Baltimore Ravens	BAL	AFC	Central	39.278	-76.622
Buffalo Bills	BUF	AFC	East	42.773	-78.787
Carolina Panthers	CAR	NFC	South	35.225	-80.852
Chicago Bears	CHI	NFC	Central	41.862	-87.617
Cincinnati Bengals	CIN	AFC	Central	39.096	-84.516
Cleveland Browns	CLE	AFC	Central	41.506	-81.699
Dallas Cowboys	DAL	NFC	East	32.748	-97.093
Denver Broncos	DEN	AFC	West	39.743	-105.021
Detroit Lions	DET	NFC	Central	42.339	-83.045
Green Bay Packers	GB	NFC	Central	44.501	-88.062
Houston Texans	HOU	AFC	South	29.684	-95.410
Indianapolis Colts	IND	AFC	South	39.760	-86.163
Jacksonville Jaguars	JAX	AFC	South	30.323	-81.636
Kansas City Chiefs	KC	AFC	West	39.049	-94.484
Las Vegas Raiders	LV	AFC	West	36.090	-115.183
Los Angeles Chargers	LAC	AFC	West	33.864	-118.261
Los Angeles Rams	LAR	NFC	West	34.014	-118.288
Miami Dolphins	MIA	AFC	East	25.958	-80.238
Minnesota Vikings	MIN	NFC	Central	44.974	-93.259
New England Patriots	NE	AFC	East	42.090	-71.264
New Orleans Saints	NO	NFC	South	29.951	-90.081
New York Giants	NYG	NFC	East	40.813	-74.074
New York Jets	NYJ	AFC	East	40.813	-74.074
Philadelphia Eagles	PHI	NFC	East	39.901	-75.167
Pittsburgh Steelers	PIT	AFC	Central	40.446	-80.015
San Francisco 49ers	SF	NFC	West	37.403	-121.970
Seattle Seahawks	SEA	NFC	West	47.595	-122.331
Tampa Bay Buccaneers	TB	NFC	South	27.975	-82.503
Tennessee Titans	TEN	AFC	South	36.166	-86.771
Washington Commanders	WAS	NFC	East	38.907	-76.864

TABLE 4
National Football League Teams and Locations, 2023

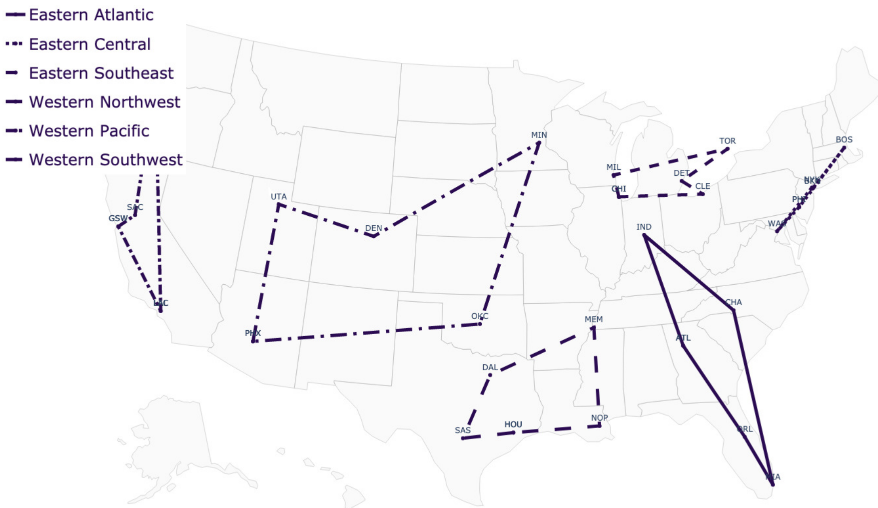
Appendix B. North American Sports League Alignments, 2023.

B.1. National Basketball Association divisional alignments (2023). Incumbent and optimal divisional alignments for the NBA are given in Figures 5 and 6, respectively. The NBA has 30 teams, divided into two conferences and six divisions.



Eastern			Western		
Atlantic	Central	Southeast	Northwest	Pacific	Southwest
BKN	CHI	ATL	DEN	GSW	DAL
BOS	CLE	CHA	MIN	LAC	HOU
NYK	DET	MIA	OKC	LAL	MEM
PHI	IND	ORL	POR	PHX	NOP
TOR	MIL	WAS	UTA	SAC	SAS

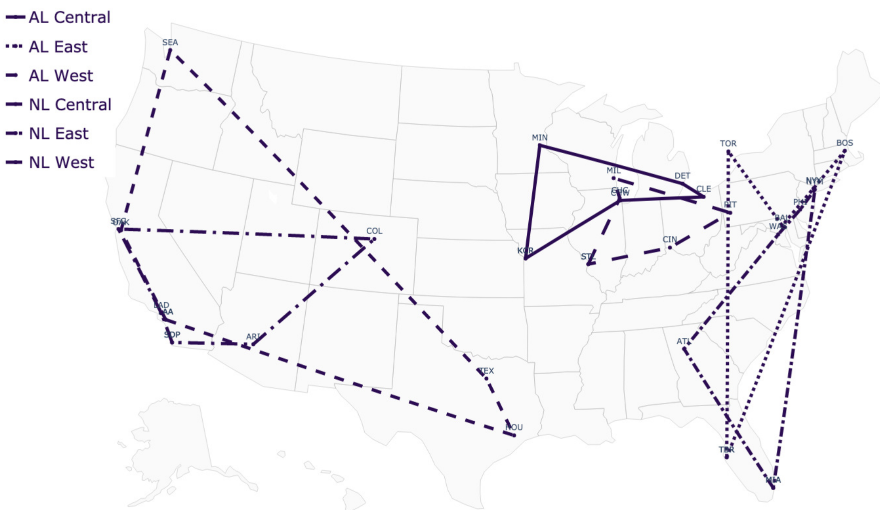
FIG. 5. *NBA team locations and divisional alignment (2023)*



Eastern			Western		
Atlantic	Central	Southeast	Northwest	Pacific	Southwest
ATL	BKN	CHI	DAL	GSW	DEN
CHA	BOS	CLE	HOU	LAC	MIN
IND	NYK	DET	MEM	LAL	OKC
MIA	PHI	MIL	NOP	POR	PHX
ORL	WAS	TOR	SAS	SAC	UTA

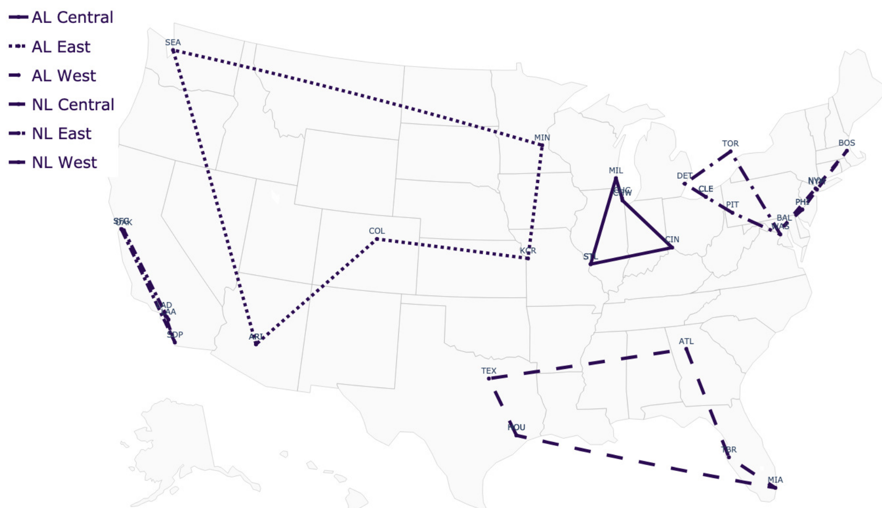
FIG. 6. *Optimal NBA divisional alignment, 2023*

B.2. Major League Baseball divisional alignments (2023). Incumbent and optimal divisional alignments for MLB are given in Figures 7 and 8, respectively. MLB has 30 teams, divided into two leagues and six divisions.



AL			NL		
Central	East	West	Central	East	West
CHW	BAL	HOU	CHC	ATL	ARI
CLE	BOS	LAA	CIN	MIA	COL
DET	NYY	OAK	MIL	NYM	LAD
KCR	TBR	SEA	PIT	PHI	SDP
MIN	TOR	TEX	STL	WAS	SFG

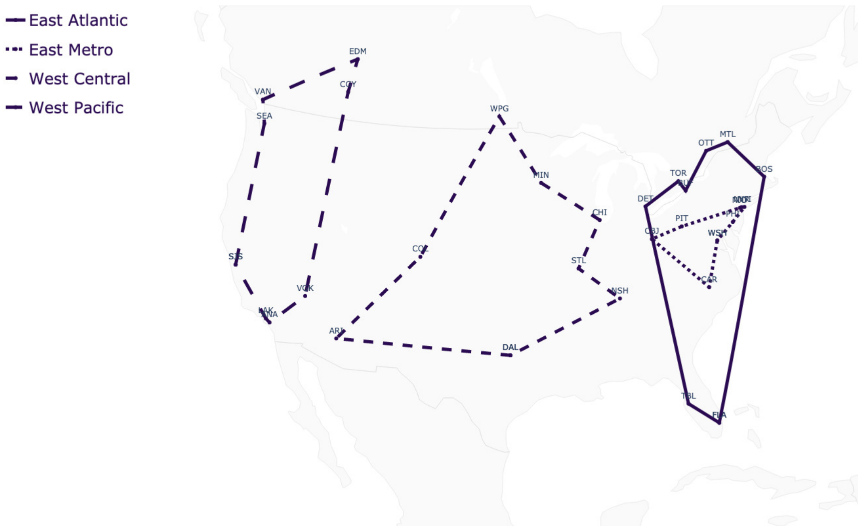
FIG. 7. *MLB team locations and divisional alignment (2023)*



AL			NL		
Central	East	West	Central	East	West
CHC	ARI	BAL	ATL	LAA	CLE
CHW	COL	BOS	HOU	LAD	DET
CIN	KCR	NYM	MIA	OAK	PIT
MIL	MIN	NYY	TBR	SDP	TOR
STL	SEA	PHI	TEX	SFG	WAS

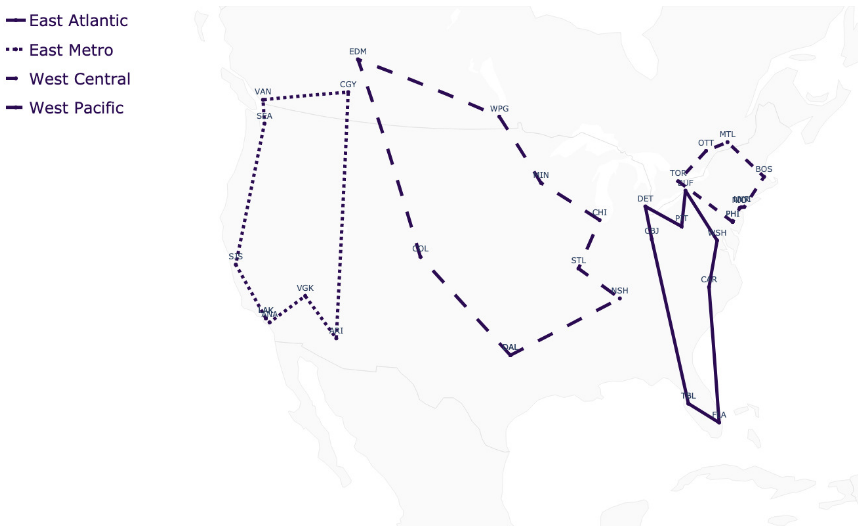
FIG. 8. *Optimal MLB divisional alignment, 2023*

B.3. National Hockey League divisional alignments (2023). Incumbent and optimal divisional alignments for the NHL are given in Figures 9 and 10, respectively. The NHL has 32 teams, divided into two conferences with four divisions each.



East		West	
Atlantic	Metro	Central	Pacific
BOS	CAR	ARI	ANA
BUF	CBJ	CHI	CGY
DET	NJD	COL	EDM
FLA	NYI	DAL	LAK
MTL	NYR	MIN	SEA
OTT	PHI	NSH	SJS
TBL	PIT	STL	VAN
TOR	WSH	WPG	VGK

FIG. 9. *NHL team locations and divisional alignment (2023)*



East		West	
Atlantic	Metro	Central	Pacific
BUF	ANA	BOS	CHI
CAR	ARI	MTL	COL
CBJ	CGY	NJD	DAL
DET	LAK	NYI	EDM
FLA	SEA	NYR	MIN
PIT	SJS	OTT	NSH
TBL	VAN	PHI	STL
WSH	VGK	TOR	WPG

FIG. 10. *Optimal NHL divisional alignment, 2023*

Acknowledgments. We would like to acknowledge Tom Bliss of the NFL for inspiring the investigation of this problem.

REFERENCES

[1] S. M. Al-Ykoob and H. D. Sherali, *A complementary column generation approach for the graph equipartition problem*, Informatica, 31 (2020), pp. 1–20, https://doi.org/10.15388/20-INFOR391.

[2] T. America, *Baseball America 2024 Almanac*, Baseball America, 2024, https://books.google.com/books?id=slMo0AEACAAJ.

[3] J. Birge, *Scheduling a professional sports league*, INFORMS Transactions on Education, 5 (2004).

[4] N. Brixius, *Sports league realignment.* https://github.com/natebrix/sports-realignment, 2024, https://doi.org/10.5281/zenodo.1234567.

[5] A. Gleixner, G. Hendel, G. Gamrath, T. Achterberg, M. Bastubbe, T. Berthold, P. Christophel, K. Jarck, T. Koch, J. Linderoth, M. Lübbecke, H. Mittelmann, T. Ralphs, D. Salvagnin, Y. Shinano, and D. Weninger, *Miplib 2017: data-driven compilation of the 6th mixed-integer programming library*, Mathematical Programming Computation, 13 (2021), pp. 443–490, https://doi.org/10.1007/s12532-020-00194-3, https://link.springer.com/article/10.1007/s12532-020-00194-3.

[6] M. Henz, *Scheduling a major college basketball conference: Revisited*, Operations Research, 49 (2001), pp. 163–168.

[7] X. Ji and J. E. Mitchell, *Finding optimal realignments in sports leagues using a branch-and-cut-and-price approach*, International Journal of Operational Research, 1 (2005), https:

//doi.org/10.1504/IJOR.2005.007436.

[8] M. Li, M. Davari, and D. Goossens, *Multi-league sports scheduling with different leagues sizes*, European Journal of Operational Research, 307 (2023), pp. 313–327, https://doi.org/10.1016/j.ejor.2022.10.010.

[9] B. Macdonald and W. Pulleyblank, *Realignment in the NHL, MLB, NFL, and NBA*, Journal of Quantitative Analysis in Sports, 10 (2014), pp. 1–16, https://doi.org/10.1515/jqas-2013-0070, https://ideas.repec.org/a/bpj/jqsprt/v10y2014i2p16n4.html.

[10] J. Mitchell, *Realignment in the national football league: Did they do it right?*, Naval Research Logistics, 50 (2003), pp. 863–701, https://doi.org/10.1002/nav.10084.10.1002/nav.10084.

[11] G. Nemhauser and M. Trick, *Scheduling a major college basketball conference*, Operations Research, 46 (1998), pp. 1–8.

[12] D. Recalde, D. Severín, R. Torres, and P. Vaca, *An exact approach for the balanced k-way partitioning problem with weight constraints and its application to sports team realignment*, Journal of Combinatorial Optimization, 36 (2018).

[13] R. M. Saltzman and R. M. Bradford, *Optimal realignments of the teams in the national football league*, European Journal of Operational Research, 93 (1996), pp. 469–475, https://doi.org/10.1137/140951758.

[14] T. Schramm, *Here's how it happened*, 1966, https://vault.si.com/vault/1966/06/20/heres-how-it-happened (accessed 2024-05-14).