Quantum Annealing for the two-level Facility Location Problem

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Abstract. This study explores the effectiveness of quantum approaches in addressing combinatorial optimization problems, arising in the logistics domain. In particular, we concentrate on the two-level Facility Location Problem, which is known to be NP-hard and therefore unable to be solved in a polynomial amount of time. Due to the difficulties in addressing these problems, we explore the potential of quantum annealing techniques to solve the Quantum Unconstrained Binary Optimization formulation, using the D-Wave solver. Furthermore, given that this formulation is still underperforming for large instances, we propose a method to preprocess the logistic network. This method has been developed with the intention of reducing the size of the logistic network, thus allowing for improved system performance as the size of the instances increases.
We demonstrate the efficacy of our proposed solution approach through the execution of computational experiments. The objective of these experiments is to validate the performance of quantum annealing with our preprocessing network techniques.

Keywords: Facility Location Problem, Logistics, QUBO, Quantum Algorithms, Quantum Annealing, D-Wave

1 Introduction

In logistics and supply chain management, finding optimal solutions for site allocation is crucial to improve the efficiency and profitability of business operations. Optimizing these processes ensures efficient distribution of resources and optimal management of goods flows, which are central aspects for companies involved in trade and industry. The Facility Location Problem (FLP) is an optimization problem concerned with determining the optimal locations for new plants or facilities in order to minimize total cost and maximize operational efficiency. Within the domain of FLPs, there exist several variants, each underscoring different objectives and constraints. The two-levels location problem is a significant subfield of FLPs. It involves selecting the optimal sites for two-tier logistics facilities, such as central distribution centers (CDCs) and regional distribution centers (RDCs), to maximize operational efficiency and reduce overall costs. CDCs serve as major hubs for large-scale product distribution, aggregating goods from production facilities. RDCs provide local distribution, reaching end customers directly with efficiency and speed. However, logistics location problems are notoriously complex and classified as NP-hard [7], meaning that finding an optimal solution requires exponential computational time. Consequently, addressing the increasing complexity and challenges requires the adoption of innovative approaches. In this context, the use of quantum techniques in operations research offers a promising prospect. This rapidly evolving field opens new horizons for solving combinatorial optimization problems more efficiently than traditional computational methods. The utilization of quantum techniques for optimization presents numerous benefits, such as the capacity to handle large problems and the potential to obtain more accurate and robust solutions. However, there are also challenges to be addressed, such as the implementation of efficient and scalable quantum algorithms. An emerging methodology used in solving optimization problems with quantum techniques is the Quantum Unconstrained Binary Optimization (QUBO) formulation. The QUBO formulation is a type of combinatorial optimization problem that aims to minimize or maximize an unconstrained quadratic objective function involving only binary variables. This function is defined as a combination of linear and quadratic terms of the binary variables, where the coefficients of these terms represent the importance and interaction of the variables in the problem. This formulation is essential in the field of quantum computing because it allows direct mapping of the QUBO problem onto a physical system of interacting spins using the Ising Hamiltonian [5]. This mapping is accomplished by assigning each QUBO variable a spin, while the terms of the QUBO objective function correspond to the couplings between the spins. Solving the QUBO problem involves finding the spin state that minimizes the energy associated with the Ising Hamiltonian. The connection between Ising's Hamiltonian and quantum computers is crucial because it can be implemented directly on quantum hardware. Solving the associated problem is equivalent to finding the fundamental state of the quantum system, which corresponds to the optimal solution of the QUBO problem. This formulation is particularly used for solving problems with quantum annealers. Quantum Annealing (QA) is an optimization technique that exploits the properties of quantum mechanics to search for the global minimum of the objective function. In particular, QA uses two quantum phenomena: superposition, which allows the system to explore many solutions simultaneously and quantum tunneling, which allows it to overcome energy barriers that might trap a classical algorithm in a local minimum. The adiabatic evolution process starts with a quantum system in a simple fundamental state and slowly evolves it toward the fundamental state of the QUBO problem, which is the optimal solution.

The objective of this work is to implement a hybrid classical-quantum algorithm using a quantum annealer for the two-stage FLP problem. This approach combines classical optimization techniques with QA, exploiting the advantages of both methodologies. In our approach, we incorporate a Preprocessing Procedure of Logistic Network (PPLN) that aims to reduce the complexity of the network in the FLP problem by minimizing the number of variables involved. This step is essential to prevent the exponential increase in variables that occurs after the problem is transformed into QUBO. Subsequently, both QUBO versions were considered: one without PPLN and the other with PPLN applied and then transformed into QUBO (PQUBO). Both versions were solved using SA and QA.

Organisation of paper: This paper is structured as follows. In Section 2, we review the scientific literature related to two-level FLPs and quantum resultant approaches for combinatorial optimization problems. We also describe the main scientific contributions of this work. Section 3 presents the problem description, the mathematical formulation and the proposed PPLN. Section 4 describes the proposed solution approach for the problem. Section 5 presents the computational tests and analysis of the results. Section 6 summarizes the concluding remarks and prospects for future studies.

2 Related works

We briefly review the most interesting papers that illustrate the state of the art regarding the two-level optimization problem in logistics network planning and the quantum approaches to solving optimization problems. Additionally, we describe the main scientific contributions of this work.

Two-level FLP: Aardal et al. (1996) [1] propose a novel method for solving two-level structure location problems using a cutting plane approach. They introduce an extended multi-commodity flow formulation based on the analysis of the single-level flow formulation. Chardaire et al. (1999) [4] present an formulation of the two-level plant location problem and proposes algorithms to test the proposed approach. Results indicate that such a method may be promising. Bumb (2001) [3] presents an approximation algorithm for the two-level uncommodity structure localization problem. The author demonstrates that the algorithm achieves a minimum of 47% of the optimal expected value, regardless of the value of a random variable. Zhang (2006) [14] considers the approximation of the multi-level structure location problem, focusing mainly on the two-level problem and variants with “soft” capabilities. An algorithm based on the quasi-greedy approach is proposed to approximate the problem and other variants, obtaining significantly better results than previous ones. Addis et al. (2012) [2] present an experimental analysis of a two-level FLP with practical applications in the design of telecommunications networks. The authors discuss the effective use of reformulation techniques through discretization and Dantzig-Wolfe decomposition. A column generation-based optimization algorithm, enhanced with price stabilization and improvement techniques, was developed. Experimental analysis demonstrates that this algorithm is significantly more effective than a general solver in optimizing instances lacking a specific structure. Ramshani et al. (2019) [13] propose a two-level FLP with single assignment under outage probability to determine the optimal locations of two types of facilities and the optimal assignment of primary and backup routes for customers. The paper presents two formulations of the problem and develops a tabu search algorithm (TS) and a heuristic method based on route subset selection (RSS) to solve them. The study demonstrates that TS and RSS are superior to the commercial solver Gurobi in terms of reducing solution time, particularly for larger problems. Additionally, a sensitivity analysis was conducted on the number of routes used in RSS, revealing that more precise route selection can significantly enhance solution quality. Karatas and Dasci (2020) [9] examined the problem of locating and sizing two-tier facilities to maximize expected demand coverage. The lower-level facilities serve as the primary points of contact for customers, while the higher-level facilities function as centers that offer services to these contact points. The paper presents numerous numerical experiments to evaluate the performance of the proposed mixed integer linear programming (MILP) models.

The application of quantum approaches to combinatorial optimization problems: Grover et al. (2019) [8] provide a method for formulating and solving QUBO. The aim is to address the challenge of resolving complex optimization problems through the use of quantum computing techniques. The approach entails a detailed guide on formulating QUBO problems, utilizing specialized solvers and applying combinatorial optimization techniques. Results demonstrate the effectiveness of QUBO methods in solving a wide range of optimization problems, with practical applications in fields such as logistics, finance and operations research. Montanez-Barrera et al. (2023) [11] introduce a novel approach, termed Unbalanced penalization, for encoding inequality constraints of combinatorial optimization problems into QUBO penalizations. The approach eliminates the need for additional slack variables, making it suitable for both gate-based quantum computers and QAs. By employing this method, they demonstrate superior solutions in terms of both quality and quantity compared to the approach with slack variables. Results consistently show that the unbalanced penalization method outperforms the slack variable approach in solving the Bin Packing problem with a larger number of items. Montanez-Barrera et al. (2023) [12] focus on the Traveling Salesman Problem (TSP). Utilizing the unbalanced penalization method, they compare performance with the slack variable approach across a wide range of solvers, including the D-Wave Advantage QPU, the hybrid D-Wave solver and CPLEX. Results indicate that the unbalanced penalization approach surpasses the slack variable method and enables finding valid solutions for larger TSP instances. Ding et al. (2021) [6] implement a QA algorithm to solve logistic network design problems. The approach combines QA with classical simulation and is tested on 12 logistics network design problems. Malviya et al. (2023) [10] suggest optimizing the location of distribution centers for package delivery through logistics network optimization using QA. They use a hybrid quantum-classical approach with sampling of the QUBO problem using the Kerberos sampler.

Contribution of the paper: the objective of this work is to propose a quantum approach for solving the two-level FLP. To the best of our knowledge, no other work has ever addressed the resolution of this type of problem, but exclusively the resolution of the single-level variant. Furthermore, we propose a PPLN to decrease the problem size and improve the resolution performance.

3 Problem Definition

3.1 Characteristics of the problem

Consider an optimization problem involving the allocation of CDCs in a context where production facilities (IPs) must serve these CDCs, which in turn must serve RDCs. Let I be the set of production facilities, C the set of potential central distribution centers and R the set of regional distribution centers. Fig. 1 proposes the representation of this problem.

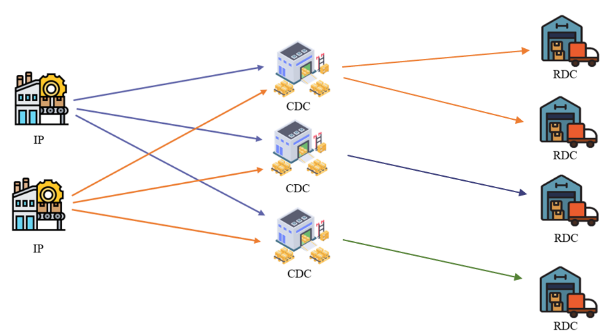

Fig. 1. A multi-level localization problem.

The problem can be represented by a graph $G = (V, E)$, where V is the set of nodes including IPs, CDCs and RDCs and E is the set of arcs connecting IPs to CDCs and CDCs to RDCs.
Let a_{ij} be the cost depending on distance between each IP $i \in I$ and each CDC $j \in C$, b_{jr} be the cost depending on distance between each CDC $j \in C$ and each RDC $r \in R$ and c_j be the activation costs associated with each CDC $j \in C$. Let p_i be defined as the maximum amount of goods that an $i \in I$ can send to the CDCs, d_r be the demand required by each RDC r and q_j be the storage capacity of the goods for each CDC j. Let N_j be the set of IPs that can serve CDC j and T_r be the set of CDCs that can serve RDC r. It is assumed that each RDC must be served by only one CDC.
The aforementioned notations definitions are summarized in Table 1.

Notation	Description
I	Set of IPs
C	Set of potential CDCs
R	Set of RDCs
V	Set of nodes including IPs, CDCs and RDCs
E	Set of arcs connecting IPs to CDCs and CDCs to RDCs
a_{ij}	Cost depending on distance between each $i \in I$ and each $j \in C$
b_{jr}	Cost depending on distance between each $j \in C$ and each $r \in R$
c_j	Activation costs associated with each $j \in C$
p_i	Maximum amount of goods that an $i \in I$ can send to the CDCs
d_r	Demand required by each $r \in R$
q_j	Storage capacity of the goods for each $j \in C$
N_j	Set of IPs that can serve $j \in C$
T_r	Set of CDCs that can serve $r \in R$

Table 1. Notations for the Multi-level Localization Problem.

3.2 Mathematical formulation

To model this problem, the following binary variables are introduced:

$$x_{ij} = \begin{cases} 1, & \text{if IP } i \text{ serves CDC } j \\ 0, & \text{otherwise} \end{cases}$$

$$y_{jr} = \begin{cases} 1, & \text{if CDC } j \text{ serves RDC } r \\ 0, & \text{otherwise} \end{cases}$$

$$z_j = \begin{cases} 1, & \text{if CDC } j \text{ is activated} \\ 0, & \text{otherwise} \end{cases}$$

The problem is mathematically formulated as follows.

$$\min \quad \sum_{j \in C} \sum_{i \in N_j} a_{ij} x_{ij} + \sum_{r \in R} \sum_{j \in T_r} b_{jr} y_{jr} + \sum_{j \in C} c_j z_j \tag{1}$$

$$\text{s.t.} \sum_{j \in T_r} y_{jr} = 1 \quad \forall r \in R \tag{2}$$

$$\sum_{r \in R} d_r y_{jr} \leq \sum_{i \in N_j} p_i x_{ij} \quad \forall j \in C \tag{3}$$

$$\sum_{i \in N_j} p_i x_{ij} \leq q_j z_j \quad \forall j \in C \tag{4}$$

$$x_{ij}, y_{jr}, z_j \in \{0, 1\} \quad \forall i \in I, \forall j \in C, \forall r \in R \tag{5}$$

The objective function (1) aims to minimize the distance between logistics sites and the total cost associated with the allocation of CDCs. It is expressed as the sum of three components:

- **first term:** corresponds to the sum of the cost depending on distances between IPs and the CDCs they serve;
- **second term:** represents the sum of the cost depending on distances between the CDCs and the RDCs they serve;
- **third term:** corresponds to the sum of the activation costs of the activated CDCs.

Constraints (2) ensure that each RDC must be served by exactly one activated CDC. Constraints (3) ensure that for each CDC j, the total quantity demanded by RDCs served by j cannot be greater to the quantity of goods sent by IPs to j. Constraints (4) assure that the total load sent by IPs to CDC j must conform to the storage capacity of that CDC, if it is active. Constraints (5) define the domain of the variables.

3.3 Preprocessing Procedure of Logistic Network

The PPLN aims to reduce the size of the set C before transforming the problem into the QUBO formulation. As a result, fewer qubits are required to represent the problem, significantly improving the efficiency and feasibility of the solution, considering the limited availability of qubits in quantum computers. During preprocessing, we propose an algorithm that involves the following steps:

1. **Creation of set** A_j: that represents the set of all RDCs covered by CDC j;
2. **Calculation of the values** β **and** γ: the values β and γ, represent the minimum and maximum number of CDCs required to guarantee the capacity constraints, respectively. These values are determined by formulas (6) and (7), respectively:

$$\beta = \left\lceil \frac{|R|}{\left\lfloor \frac{\max\{\sum_{i \in I} p_i, \max\{q_j, j \in C\}\}}{\min\{d_r, r \in R\}} \right\rfloor} \right\rceil \tag{6}$$

$$\gamma = \left\lceil \frac{|R|}{\left\lfloor \frac{\min\{\sum_{i \in I} p_i, \min\{q_j, j \in C\}\}}{\min\{d_r, r \in R\}} \right\rfloor} \right\rceil \tag{7}$$

3. **Creation of the *for* loop**: we generate a *for* loop in which the value c varies from β to γ and for each iteration of the loop:

(a) **Combination Creation**: we generate all c CDC combinations covering all RDCs;

(b) **Sorting combinations**: we sort the combinations in ascending order according to the value

$$\sum_{j \in \text{comb}} c_j + \sum_{i \in I} \sum_{j \in \text{comb}} a_{ij} + \sum_{r \in R} \sum_{j \in \text{comb}} b_{jr} \tag{8}$$

(c) **Verification of combinations**: we check the ordered combinations to ensure that they guarantee the capacity constraints.

(d) **Selection of valid combination**: the first combination that satisfies all capacity constraints is selected as the new set C and the loop is break.

Thus, the PPLN identifies a suboptimal configuration of CDCs that is significantly smaller in size than the initial configuration and meets all capacity constraints.
Fig. 2 shows a flowchart of the PPLN steps to reduce the set C.

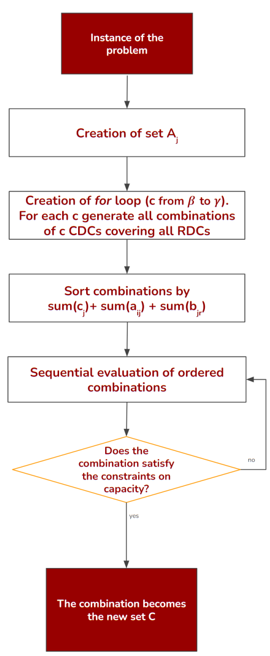

Fig. 2. Flowchart of the PPLN to reduce the set C.

4 Solution approach

In this work, we adopt a structured, multi-step methodological approach to address the two-level FLP. Our methodology began with the conversion of the MILP formulation of the FLP into a QUBO formulation. However, this transformation has limited scalability due to the large number of binary variables involved. In theory, when a sufficient number of qubits are available, it will be possible to effectively solve these problems. Currently, the complexity introduced by the QUBO transformation may negate the benefits of using quantum methods for solving optimization problems. To address this challenge, we have incorporated an additional preprocessing step into our approach. Our method, explained in detail in section 3.3, aims to streamline the problem and reduce the number of variables to control their excessive growth. We then apply both, the QUBO and PQUBO version, to solve the problem using SA and QA.
We now describe the solution approach in detail:

1. **PPLN**: we propose a method of reducing the logistic network by decreasing the cardinality of the set C;

2. **Resolution with Classic Solver**: we solve both unpreprocessed and preprocessed problems using the Gurobi solver to obtain optimal benchmark solutions;

3. **Translation into QUBO formulation**: we use the dimod api library to translate the problems into the QUBO formulation;

4. **Resolution with SA**: we apply the SA algorithm to solve the problems translated into the QUBO formulation;

5. **Resolution with QA**: we use QA of D-Wave solver to solve the problems.

The employed method is presented in detail in Fig. 3, outlining the key steps we followed to conduct the implementation of the problem.

5 Computational study

This section presents the results of our computational experiments. We solve the model with Gurobi ver. 11 and the implementation is done entirely in Python. All computations are performed on an 4Ghz processor and 32GB of RAM.

5.1 Generation of instances

The following section outlines the methodology employed in the generation of the instances. Experimental tests are conducted within a 100x100 unit square region. Logistics site coordinates are randomly generated within this region using

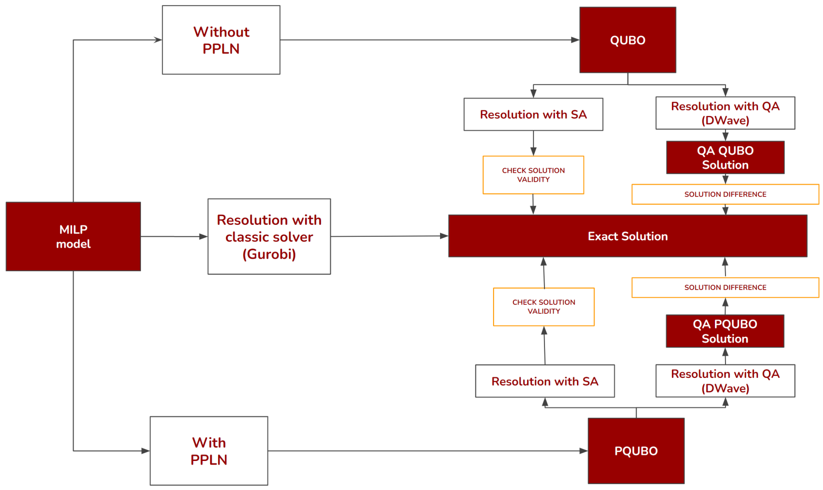

Fig. 3. Problem solving methodology.

Cartesian coordinates. Additionally, randomly generated values were assigned to p_i, d_r and q_j. The activation cost of CDCs is calculated based on their storage capacity, q_j. A realistic principle is adopted: the higher the storage capacity of the CDC, the higher its activation cost. To determine the coverage distance ρ, we calculate the maximum value among all distances, represented by d_M, i.e., this is the maximum distance within the considered logistic system, and we then calculate the minimum operational distance between nodes in the network (excluding zero), represented by d_m. The coverage distance ρ is obtained as a weighted combination of d_M and d_m, defined as $0.8\ d_M + (1 - 0.8)\ d_m$. This value represents the maximum distance that a logistics site can effectively cover another site, both for the coverage of IPs to CDCs and for the coverage of CDCs to RDCs. This reflects the concept that CDCs must be served by IPs and RDCs must be served by CDCs. The coverage distance ρ is obtained to generate two sets: N_c and T_r.
Tables 2 summarizes the parameter setting used for our computational results.

5.2 Numerical results

In order to assess the performance of our proposed solution approach, we evaluate the results obtained along two dimensions: problem size and solution quality. We initially solve the two-level FLP using the classical solver Gurobi. The problem is solved in both its MILP and QUBO formulations. Table 3 summarizes the results obtained in terms of problem size. The first four columns show the instance name and cardinality of the sets $|I|$, $|C|$ and $|R|$. The fifth and sixth columns present the number of variables of the MILP problem solved with Gurobi and the problem size after the QUBO transformation (in terms of the number

Notations	Description	Value
I	Set of IPs	
C	Set of potential CDCs	
R	Set of RDCs	
N_j	Set of IPs that can serve $j \in C$	
T_r	Set of CDCs that can serve $r \in R$	
a_{ij}	Distances between $i \in I$ and $j \in C$	Manhattan Metric
b_{jr}	Distances between $j \in C$ and $r \in R$	Manhattan Metric
p_i	Max goods $i \in I$ can send to CDCs	Random in range (80, 100)
d_r	Demand required by each $r \in R$	Random in range (5, 40)
q_j	Storage capacity of the good for $j \in C$	Random in range (100, 500)
c_j	Activation costs associated with $j \in C$	$1000 + 0,5 \cdot q_j$
ρ	Coverage Distance	$0,8 \cdot d_{\max} + (1 - 0,8) \cdot d_{\min}$

Table 2. Instance Characteristics.

of bits and the number of links between bits), respectively.

Test	|I|	|C|	|R|	Number of Variables (MILP)	Problem Size (QUBO)
I1	2	4	6	36	(96,637)
I2	3	6	9	78	(183,1772)
I3	4	8	12	136	(275,3306)
I4	5	10	15	210	(381,5478)
I5	6	12	18	300	(520,8885)
I6	7	14	21	406	(666,13131)
I7	8	16	24	528	(818,18205)
I8	9	18	27	666	(997,24615)
I9	10	20	30	820	(1189,32406)
I10	15	30	45	1830	(2402, 97864)
I11	20	40	60	3240	(3993, 215775)
I12	30	60	90	7260	(8442, 687135)

Table 3. Comparison of problem size MILP and QUBO formulation.

The results in Table 3 clearly demonstrate that the transformation from the MILP formulation to the QUBO formulation of the analyzed problem results in a significant increase in the number of variables as the instance size increases. Therefore, to limit this rapid growth of the number of variables, we propose the PPLN described in Section 3.3.

As shown by the results in Table 4, this method is highly effective in significantly reducing the number of variables after the PQUBO transformation. In particular, it can be seen that the percentage reduction in terms of the number of bits (indicator of the number of variables) with the use of our MNLP compared to when not using it is 87,1% on average. The significance of this outcome is substantiated by Fig. 4, which illustrates the growth in the number of variables as the size of the instance increases. We compare the increase in the number of variables in the MILP model after transformation to QUBO and when transformed to PQUBO. It can be observed that the growth in number of variables is effectively mitigated by the implementation of the proposed PPLN.

Test	|I|	|C|	|R|	QUBO		PQUBO	
				Problem Size	**Objective Function**	**Problem Size**	**Objective Function**
I1	2	4	6	(96,637)	1782	(26,185)	1782
I2	3	6	9	(183,1772)	1864	(31,285)	1864
I3	4	8	12	(275,3306)	1856	(35,385)	1856
I4	5	10	15	(381,5478)	3157	(77,978)	3217
I5	6	12	18	(520,8885)	2399	(44,666)	2399
I6	7	14	21	(666,13131)	2630	(48,818)	3091
I7	8	16	24	(818,18205)	3479	(104,1996)	3709
I8	9	18	27	(997,24615)	3810	(112,2367)	3888
I9	10	20	30	(1189,32406)	3729	(120,277)	4269
I10	15	30	45	(2402, 97864)	5395	(243, 8175)	5500
I11	20	40	60	(3993, 215775)	-	(403, 17620)	7315
I12	30	60	90	(8442, 687135)	-	(710, 45855)	9298
I13	40	80	120	-	-	-	-

Table 4. Comparison of the problem size and objective function for QUBO and PQUBO formulations.

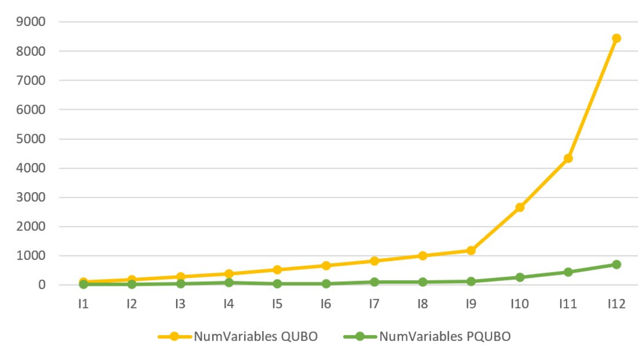

Fig. 4. Comparison of the growth in problem size between QUBO and PQUBO formulations.

The Table 4 also shows the objective functions obtained without and with the PPLN. The solution obtained by solving the PQUBO formulation with Gurobi is optimal for four instances (I1, I2, I3 and I5). Generally, the average gap between the two solutions is 4% across all instances. For instances I6 and I9, the optimality gap of the objective function is not low. This occurs because our method identifies the solution by sorting combinations in a manner that sums all activation costs and all distances to and from CDCs in the combination. Although this value may not precisely correspond to the actual activation costs and distances, it still provides a useful estimate for identifying the most promising solutions. Furthermore, the method does not search for a superior solution with a greater number of CDCs, since the objective of PPLN is to minimize the number of CDCs in the network.

In summary, the results obtained indicate that our primary goal in using MNLP, i.e., to significantly reduce the size of the problem, has been achieved with a reduction of 87,1%, as highlighted above. Regarding the analysis of the quality of the solutions, there is generally an average gap of 4% in the objective function over all instances between the optimal solution obtained with and without MNLP.

While MNLP can be employed to identify a solution for a greater number of instances (I11 and I12), it is less effective for larger instances, such as I13. Consequently, we have opted to implement our model on quantum hardware, given that it is theoretically possible that as quantum hardware improves, it will be able to solve these instances.

The subsequent step is to employ the SA in order to resolve our problem. The SA algorithm is applied to the translated problems in QUBO and PQUBO formulation to verify the correctness of the different conversions without encountering the potential errors of current quantum hardware. The results are presented in Table 5.

				WITHOUT PPLN			WITH PPLN	
Test	**|I|**	**|C|**	**|R|**	**Optimal Objective Function**	**Objective Function**	**Time** [seconds]	**Objective Function**	**Time** [seconds]
I1	2	4	6	1782	2793	305,990	1782	75,554
I2	3	6	9	1864	None	625,221	1864	101,307
I3	4	8	12	1856	None	1049,372	1856	120,963
I4	5	10	15	3157	None	1519,322	3795	279,254
I5	6	12	18	2399	None	1615,810	None	177,691

Table 5. Comparison of objective function and iteration time for SA with and without PPLN.

Table 5 shows the objective function generated when the MILP formulation is solved with Gurobi without the PPLN (Optimal Objective Function), the objective function generated with and without the PPLN using the SA method (with a number of iterations equal to 100000), and the resolution time for SA. As shown in Table 5, with the SA algorithm to which our PPLN is applied, we obtain the optimal solution for instances I1, I2, I3 and I5, and an admissible solution for the other instances. It should be noted that the analysis is limited to the first five instances, because for larger instances, the SA does not converge to solution. The "None" designation in the tables indicates that no solution is found for the instances. The results demonstrate the efficacy of the various con-

version methodologies by solving the problem with the SA algorithm.
In a final step, experiments are conducted using D-Wave's QA with and without PPLN.
Table 6 presents the objective function results obtained by solving the MILP

Test	|I|	|C|	|R|	WITHOUT PPLN			WITH PPLN	
				Optimal Objective Function	**Objective Function**	**Time** [seconds]	**Objective Function**	**Time** [seconds]
I1	2	4	6	1782	2803	354,53	1782	17,01
I2	3	6	9	1864	None	354,42	1864	12,77
I3	4	8	12	1856	None	354,94	1856	67,70
I4	5	10	15	3157	None	354,94	3471	293,05
I5	6	12	18	2399	None	354,94	2399	148,65
I6	7	14	21	2630	None	354,94	3135	292,43
I7	8	16	24	3479	None	536,31	4161	539,03
I8	9	18	27	3810	None	532,85	None	543,60

Table 6. Comparison of objective function and iteration time for QA with and without PPLN.

formulation with Gurobi without the use of the PPLN (Optimal Objective Function), the objective function generated with and without the PPLN using the QA method, as well as the resolution time for QA. As indicated in Table 6, the application of the PPLN before the use of the QA algorithm resulted in the optimal solution for instances I1, I2, I3, and I5, while an admissible solution was obtained for the remaining instances with an average objective gap of 4% across all instances. It is important to note that the analysis is limited to the initial eight instances, as the QA algorithm is unable to reach a solution within the 500 second time limit for larger instances. In such cases, the result is denoted as "None" in the table.
Experiments carried out on actual quantum hardware provide evidence that the solution derived from the current availability of qubits is effective, thereby confirming the viability of the proposed methodology. It is essential to recognize that quantum hardware is still in its early stages of development. It is thus the primary objective of this study to demonstrate the capacity of contemporary quantum computers to address the issues delineated in the domain of logistics.

6 Conclusions

In our work, we developed and implemented a MILP model for the two-level FLP. This involved optimal site selection for two-level logistics facilities such as CDCs and RDCs. Our main goal is to integrate a preprocessing method of the logistics data network to optimize the efficiency and efficacy of QA algorithms. We, therefore, propose the MNLP with the goal of reducing the complexity of the problem, increasing the probability of obtaining solutions for large instances,

and, as a result, effectively implementing the problem using QA. Our numerical tests confirmed that the our solution method leads to a significant reduction in problem size, greatly improving performance in terms of execution time and ability to converge to the optimal solution. In particular, the use of the PPLN enabled the SA and QA algorithms to converge to optimal solutions for a larger number of problem instances, with significantly lower execution time than scenarios without PPLN. The solutions proposed by QA were found to be closer to the optimal solution identified by Gurobi, demonstrating a significant improvement in the effectiveness of the optimal value search.
As for future developments, we intend to further expand our approach to even larger logistics networks and more complex contexts, such as global supply chain management and distribution in urban environments.

Declarations

Conflicts of interest/Competing interests: The authors have no conflicts of interest to declare that are relevant to the content of this article.
Code availability: The implemented codes are available on request from the authors.

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