On Coupling Constraints in Pessimistic Linear Bilevel Optimization

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Abstract. The literature on pessimistic bilevel optimization with coupling constraints is rather scarce and it has been common sense that these problems are harder to tackle than pessimistic bilevel problems without coupling constraints. In this note, we show that this is not the case. To this end, given a pessimistic problem with coupling constraints, we derive a pessimistic problem without coupling constraints that has the same set of globally optimal solutions. Moreover, our results also show that one can equivalently replace a pessimistic problem with such constraints with an optimistic problem without coupling constraints. This paves the way of both transferring theory and solution techniques from any type of these problems to any other one.

1. Introduction

Bilevel optimization gained significant scientific attention over the last years and decades. One of the main reasons is that this framework allows to model hierarchical decision-making processes in which two agents interact. The leader acts first while anticipating the optimal reaction of the follower, who acts second and who takes the leader's decision into account. For introductions to the field we refer to the books and lecture notes by Dempe (2002), Dempe et al. (2015), and Beck and Schmidt (2021), in which the interested reader can also find many illustrative examples. This field of study dates back to the seminal contributions by von Stackelberg (1934) and von Stackelberg (1952), while the mathematical optimization and operations research communities started to investigate these problems in the 1970s and 1980s; see, e.g., Bracken and McGill (1973), Candler and Norton (1977), and Bialas and Karwan (1984) for the earliest publications. Since then, many scientific advances have been achieved ranging from theoretical studies on, e.g., existence of solutions or optimality conditions over structural insights and reformulations to algorithmic approaches for actually solving these challenging problems.

In this note, we consider two key aspects of the field of bilevel optimization. First, bilevel problems are generally ill-posed in case of multiplicities in the set of optimal reactions of the follower. Usually, this is resolved by fixing the level of cooperation of the follower, i.e., choosing a follower's solution that is in favor of the leader's objective or a solution that is worst for the leader. The former concept leads to the so-called optimistic bilevel problem whereas the latter is referred to as the pessimistic bilevel problem; see, e.g., Dempe (2002) for a general introduction. Second, many contributions in bilevel optimization consider either the case with coupling constraints or without them, where a coupling constraint is an upper-level constraint that explicitly depends on the variables of the follower.

Regarding the first aspect, it is generally believed that the pessimistic bilevel problem is harder to tackle—both in theory and practice. Hence, it got significantly

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fewer attention compared to optimistic bilevel optimization. Nevertheless, important contributions such as regarding optimality conditions (Dempe et al. 2014) or reformulations (Aussel and Svensson 2019) have been made in the last years. With respect to solution methods, we want to particularly highlight the approach by Zeng (2020), which only requires the solution of optimistic bilevel (and maybe further single-level) problems to solve a given pessimistic bilevel problem.

Regarding the aspect of coupling constraints, it has been known for more than 25 years that they may lead to disconnected bilevel feasible sets (Henke et al. 2024), while it is shown by Benson (1989) that the feasible set of a linear bilevel problem without coupling constraints is always connected. The possibility of disconnected feasible sets mainly gained prominence because it allows to model mixed-binary linear problems using purely continuous linear bilevel models, see, e.g., Section 3 in Vicente et al. (1996) and Section 3.1 in Audet et al. (1997). This also played an important role in early NP-hardness proofs of bilevel optimization; see, e.g., Marcotte and Savard (2005). However, both variants of optimistic bilevel optimization with and without coupling constraints are NP-complete; see Buchheim (2023). Hence, there is no difference between the two variants of optimistic bilevel optimization in terms of their computational complexity. Moreover, we showed in a previous paper (Henke et al. 2024) that—although they differ with respect to modeling different types of feasible sets—they do not differ on the level of optimal solutions. More specifically, for every optimistic bilevel problem with coupling constraints, one can derive another optimistic bilevel problem without coupling constraints that has the same set of globally optimal solutions.

In pessimistic bilevel optimization, the literature on problems with coupling constraints is rather scarce; see, e.g., Wiesemann et al. (2013) and Zeng (2020) or the recent survey by Beck et al. (2023). Particularly, some of the main theoretical contributions such as Dempe et al. (2014) and Aussel and Svensson (2019) only consider the case without coupling constraints. Somehow, the common sense seemed to be that pessimistic problems with coupling constraints are much harder to deal with than their variants without coupling constraints. The main contribution of this note is to show that this is not the case for linear bilevel problems. To be more precise, we use the techniques introduced in Zeng (2020) and Henke et al. (2024) to show that for every pessimistic bilevel optimization problem with coupling constraints, we can state a pessimistic bilevel optimization problem without coupling constraints that has the same set of globally optimal solutions. Moreover, we even show that for every pessimistic bilevel optimization problem with coupling constraints we can also derive an optimistic bilevel optimization problem without coupling constraints that has the same set of globally optimal solutions.

2. Problem Statement

In this note, we consider different types of linear bilevel optimization problems. The most basic one is the so-called optimistic bilevel optimization problem without coupling constraints, which reads

$$\min_{x \in X} \quad F_{\mathrm{o}}(x) := c^\top x + \min_y \left\{ d^\top y \colon y \in S(x) \right\}, \tag{1}$$

where $S(x)$ is the set of optimal solutions to the x-parameterized optimization problem

$$\min_y \quad f^\top y \quad \text{s.t.} \quad Cx + Dy \geq b. \tag{2}$$

Here and in what follows, all variables are continuous and we omit the dimensions for better readability. Problem (1) is called optimistic because the leader is able to choose the lower-level variable y among all optimal ones for the follower's problem if there are any multiplicities.

The extended version of this optimistic problem that includes coupling constraints, i.e., upper-level constraints depending on the lower-level variables, can be written as

$$\min_{x \in X} \quad F_{\text{oc}}(x) := c^\top x + \min_y \left\{ d^\top y \colon y \in S(x),\ Ax + By \geq a \right\}, \tag{3}$$

where $S(x)$ is defined as before.

There is also the pessimistic bilevel problem, which we again study in two different versions. In the first one, only the upper-level objective function but not the upper-level constraints depend on the follower's variables. This problem is given by

$$\min_{x \in \bar{X}} \quad F_{\text{p}}(x) := c^\top x + \max_y \left\{ d^\top y \colon y \in S(x) \right\} \tag{4}$$

with

$$\bar{X} := X \cap \{x \colon S(x) \neq \emptyset\}. \tag{5}$$

Here, $S(x)$ is again the same as before.

In the second version, the upper-level constraints depend on y, but we assume that the upper-level objective function does not depend on y anymore. This assumption can be made w.l.o.g. by using the classic epigraph reformulation. The problem then reads

$$\min_{x \in \bar{X}} \quad F_{\text{pc}}(x) := c^\top x \tag{6a}$$

$$\text{s.t.} \quad Ax + By \geq a \quad \text{for all } y \in S(x). \tag{6b}$$

Let us note here that the pessimistic bilevel problems are stated above in a slightly different way compared to what one usually finds in the literature. The difference is that we ensure the non-emptiness of the lower-level's solution set in (5). The problem is that, without this constraint, for the case without coupling constraints, it would be the best for the leader to choose an $x \in X$ for which $S(x) = \emptyset$. The reason is that the inner optimization problem would then be a maximization over the empty set, which formally evaluates to $-\infty$ and which is the best possible outcome for the outer minimization problem. However, in many (even pessimistic) situations, the leader does not want to actually make the follower's problem infeasible. For instance, Dempe et al. (2014) and Aussel and Svensson (2019) consider this pessimistic setting without coupling constraints and without the constraint $S(x) \neq \emptyset$. However, they make the assumption that $S(x) \neq \emptyset$ is satisfied for every $x \in X$, which resolves the situation sketched above. On the other hand, the pessimistic problem with coupling constraints is considered in, e.g., Tahernejad et al. (2020) and Wiesemann et al. (2013). However, both papers do not state any respective constraint or assumption on $S(x)$. This again means that it is possible that the best leader's strategy would be to choose an $x \in X$ so that $S(x) = \emptyset$ holds. This implies that Constraint (6b) vanishes. Although this does not need to be wrong in a formal sense, in our opinion, it is at least questionable if this is what really should be modeled.

For streamlining the presentation of the core ideas of this paper, we make the following standing assumptions.

Standing Assumption. (i) For all $x \in X$, the set $\{y \colon Cx + Dy \geq b\}$ is non-empty and compact.
(ii) The set X is a non-empty polyhedron and all vectors and matrices have rational entries.
(iii) All upper-level objective functions are bounded from below on X.

The first assumption is sufficient to guarantee that $S(x) \neq \emptyset$ holds for all $x \in X$. The latter is also used in Dempe et al. (2014) and Aussel and Svensson (2019). The

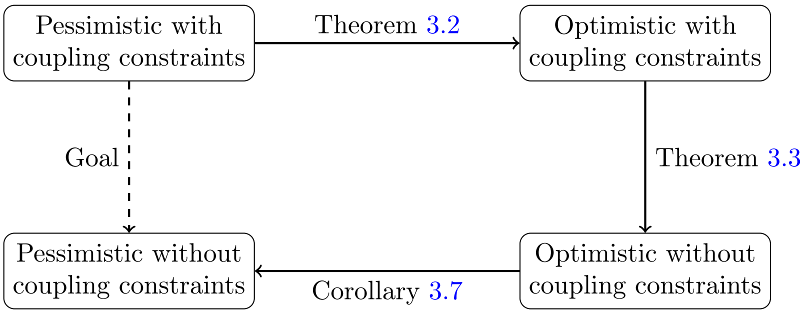

FIGURE 1. Models and Reformulations

second assumption is required for applying the results from Henke et al. (2024) and the third one ensures solvability of the overall problem.

Before we move on to our main results, let us briefly discuss what we consider to be a solution to the stated bilevel problems. In the literature, it is not handled in a unique way if either x or (x, y) is a solution to a bilevel problem. For sure, for the pessimistic bilevel problem (6), the solution can only be in the x-space since the variables y of the follower are connected to a universal quantifier in (6b). In order to be consistent, we also consider the solutions to (1), (3), and (4) to be in the x-space and interpret a respective y, if given at all, only as a certificate for the optimality of x. For a detailed discussion about the representation of solutions of bilevel problems, we refer to Section 2.6 in Goerigk et al. (2025).

3. An Equivalent Reformulation without Coupling Constraints

We now prove that pessimistic bilevel optimization with and without coupling constraints are equivalent on the level of globally optimal solutions. To this end, we derive multiple reformulations and show their equivalence properties. The main proof strategy is given in Figure 1. The top, right, and bottom arc are considered in the respective following sections.

3.1. From Pessimistic to Optimistic Bilevel Optimization with Coupling Constraints.

First, we reformulate the pessimistic bilevel optimization problem with coupling constraints, i.e., Problem (6), as an optimistic bilevel problem with coupling constraints. To this end, we use the results from Section 3.3 in Zeng (2020), in particular Lemma 3. This first requires to re-write the coupling constraints

$$Ax + By \geq a \quad \text{for all } y \in S(x)$$

in a component-wise way. For this, let $A \in \mathbb{R}^{m \times n_x}$, $B \in \mathbb{R}^{m \times n_y}$, and $a \in \mathbb{R}^m$ with m being the number of coupling constraints and with n_x and n_y being the numbers of upper- and lower-level variables, respectively. Hence, we have

$$A_{i\cdot}x + B_{i\cdot}y \geq a_i \quad \text{for all } y \in S(x) \text{ and all } i \in [m] := \{1, \dots, m\}. \tag{7}$$

We are now able to re-phrase Lemma 3 by Zeng (2020) in our notation. For being more self-complete, we also add a simplified proof here.

Lemma 3.1. *Let $x \in X$ be given and consider a fixed $i \in [m]$. Then, x satisfies the i-th coupling constraint in (7) if and only if there exist $\bar{y}$ and*

$$y^i \in \arg\min \left\{B_{i\cdot}y \colon Dy \geq b - Cx,\ f^\top y \leq f^\top \bar{y}\right\}$$

that satisfy

$$D\bar{y} \geq b - Cx, \quad B_{i\cdot}y^i \geq a_i - A_{i\cdot}x.$$

Proof. Let $x \in X$ be given and consider a fixed $i \in [m]$. Then, the i-th coupling constraint in (7) is equivalent to $\min_y\{B_{i\cdot}y \colon y \in S(x)\} \geq a_i - A_{i\cdot}x$, which can be reformulated as $B_{i\cdot}y^i \geq a_i - A_{i\cdot}x$ with $y^i \in \arg\min_y \{B_{i\cdot}y \colon y \in S(x)\}$. Now, let φ denote the optimal-value function of the lower-level problem (2). It follows that x satisfies the i-th coupling constraint if and only if

$$B_{i\cdot}y^i \geq a_i - A_{i\cdot}x \quad \text{with} \quad y^i \in \arg\min_y \left\{B_{i\cdot}y \colon Dy \geq b - Cx,\ f^\top y \leq \varphi(x)\right\}. \tag{8}$$

We now show that the latter is equivalent to the stated conditions in the lemma. First, let us assume that (8) holds. Then, there exists $\bar{y}$ such that $\varphi(x) = f^\top \bar{y}$ and $D\bar{y} \geq b - Cx$ is satisfied. Hence, the conditions of the lemma hold.

Conversely, assume that the conditions of the lemma are satisfied. The feasibility of $\bar{y}$ implies

$$\min_y \left\{B_{i\cdot}y \colon Dy \geq b - Cx,\ f^\top y \leq f^\top \bar{y}\right\} \leq \min_y \left\{B_{i\cdot}y \colon Dy \geq b - Cx,\ f^\top y \leq \varphi(x)\right\}.$$

Hence, (8) is satisfied, which concludes the proof. □

From this lemma, we obtain the following result.

Theorem 3.2. *Let $\mathcal{S}$ be the set of globally optimal solutions to the pessimistic bilevel problem (6) with coupling constraints, i.e., to the problem*

$$\min_{x \in X} \quad F_{\text{pc}}(x) = c^\top x \tag{9a}$$

$$\text{s.t.} \quad Ax + By \geq a \quad \text{for all } y \in S(x). \tag{9b}$$

Moreover, let $\tilde{\mathcal{S}}$ be the set of globally optimal solutions to the optimistic single-leader multi-follower problem

$$\min_{(x,\bar{y}) \in \tilde{X}} \ c^\top x + \min_y \left\{0 \colon y^i \in \tilde{S}^i(x, \bar{y}),\ B_{i\cdot}y^i \geq a_i - A_{i\cdot}x \text{ for all } i \in [m]\right\} \tag{10}$$

with $\tilde{X} = \{(x, \bar{y}) \colon x \in X, D\bar{y} \geq b - Cx\}$, $\tilde{S}^i(x, \bar{y}) = \arg\min_{y'}\{B_{i\cdot}y' \colon Dy' \geq b - Cx, f^\top y' \leq f^\top \bar{y}\}$, and $y = (y^i)_{i=1}^m$. Let $\hat{\mathcal{S}}$ be the set of globally optimal solutions to the optimistic bilevel problem

$$\min_{(x,\bar{y}) \in \tilde{X}} \ c^\top x + \min_y \left\{0 \colon y \in \hat{S}(x, \bar{y}),\ B_{i\cdot}y^i \geq a_i - A_{i\cdot}x \text{ for all } i \in [m]\right\},$$

where $\hat{S}(x, \bar{y})$ denotes the set of optimal solutions to the aggregated lower-level problem

$$\begin{aligned} \min_y \quad & \sum_{i=1}^m B_{i\cdot}y^i \\ \text{s.t.} \quad & Dy^i \geq b - Cx \quad \text{for all } i \in [m], \\ & f^\top y^i \leq f^\top \bar{y} \quad \text{for all } i \in [m]. \end{aligned}$$

Then,

$$\mathcal{S} = \text{proj}_x(\tilde{\mathcal{S}}) = \text{proj}_x(\hat{\mathcal{S}})$$

holds and all optimal objective function values coincide.

Proof. The second identity $\text{proj}_x(\tilde{\mathcal{S}}) = \text{proj}_x(\hat{\mathcal{S}})$ is straightforward because all follower problems in (10) are independent. Hence, we focus on the first identity, i.e., on $\mathcal{S} = \text{proj}_x(\tilde{\mathcal{S}})$. We start by proving $\mathcal{S} \subseteq \text{proj}_x(\tilde{\mathcal{S}})$. To this end, let x be feasible for Problem (9), implying $x \in X$. By applying Lemma 3.1 for all $i \in [m]$, there exist $\bar{y}$ and $y^i \in \arg\min_y\{B_{i\cdot}y \colon Dy \geq b - Cx, f^\top y \leq f^\top \bar{y}\}$ satisfying $D\bar{y} \geq b - Cx$ and $B_{i\cdot}y^i \geq a_i - A_{i\cdot}x$. Note that, formally, we would get a separate $\bar{y}$ for every application of the lemma, i.e., for every $i \in [m]$. However, we can choose the one leading to the smallest right-hand side value $f^\top \bar{y}$. Thus, $y^i \in \tilde{S}^i(x, \bar{y})$ holds, meaning that for the given x, there exists $\bar{y}$ so that $(x, \bar{y})$ is feasible for (10). Since the respective upper-level objective functions coincide and only depend on x, we showed $\mathcal{S} \subseteq \operatorname{proj}_x(\tilde{\mathcal{S}})$. Due to Lemma 3.1 being an if-and-only-if statement, the other direction follows by using the same arguments. □

Hence, we have shown that a pessimistic bilevel problem with coupling constraints can be equivalently re-written as an optimistic bilevel problem with coupling constraints.

3.2. **From Optimistic Bilevel Optimization with to without Coupling Constraints.** To reformulate an optimistic bilevel optimization problem with coupling constraints as an optimistic bilevel optimization problem without coupling constraints, we use the main result of Henke et al. (2024). We restate it in the following for our setting in which the bilevel problems are formulated in the x-space.

Theorem 3.3 (Corollary 2.3 in Henke et al. (2024))**.** *There is a polynomial-sized (in the bit-encoding length of the problem's data) penalty parameter $\kappa > 0$ so that the optimistic bilevel problem (3) with coupling constraints has the same set of globally optimal solutions as the optimistic bilevel problem*

$$\min_{x \in X} \quad c^\top x + \min_{y, \varepsilon} \left\{ d^\top y + \kappa \varepsilon \colon (y, \varepsilon) \in S'(x) \right\}$$

without coupling constraints, where $S'(x)$ is the set of optimal solutions to the x-parameterized lower-level problem

$$\begin{aligned} \min_{y, \varepsilon} \quad & f^\top y \\ \text{s.t.} \quad & Ax + By + \varepsilon e \geq a, \\ & Cx + Dy \geq b, \\ & \varepsilon \geq 0, \end{aligned}$$

where e is the vector of all ones in appropriate dimension. Moreover, both bilevel problems have the same optimal objective function value.

Henke et al. (2024) state as an open question how to compute the penalty parameter κ in polynomial time. This question is answered by Lemma 4 of Lefebvre and Schmidt (2024).

3.3. **From Optimistic to Pessimistic Bilevel Optimization without Coupling Constraints.** In this section, we show how to reformulate an optimistic bilevel problem (1) without coupling constraints as a pessimistic bilevel problem without coupling constraints so that the globally optimal solutions coincide. To this end, we consider the following auxiliary optimistic bilevel problem

$$\min_{(x, \bar{y}) \in \tilde{X}} \quad F_{\text{oa}}(x, \bar{y}) := c^\top x + d^\top \bar{y} + \min_{y, \varepsilon} \left\{ 0 \colon \varepsilon = 0, \, (y, \varepsilon) \in \tilde{S}(x, \bar{y}) \right\} \tag{11}$$

with a single coupling constraint. Again, we use $\tilde{X} = \{(x, \bar{y}) \colon x \in X, D\bar{y} \geq b - Cx\}$ and $\tilde{S}(x, \bar{y})$ denotes the set of optimal points to

$$\min_{y, \varepsilon} \quad f^\top y \tag{12a}$$
$$\text{s.t.} \quad Cx + Dy \geq b, \tag{12b}$$
$$f^\top \bar{y} - f^\top y = \varepsilon, \tag{12c}$$
$$\varepsilon \geq 0. \tag{12d}$$

The main intuition behind this problem is that the leader can choose the most favorable lower-level feasible point $\bar{y}$ in terms of her objective function while the follower computes the non-negative difference between the actual optimal lower-level objective value and the one corresponding to $\bar{y}$ in Constraint (12c). Finally, the coupling constraint $\varepsilon = 0$ ensures that this difference is zero, i.e., the leader's decision $\bar{y}$ is also optimal for the lower-level problem (2).

Lemma 3.4. *For every bilevel feasible point x of the optimistic bilevel problem (1) without coupling constraints, the point $(x, \bar{y})$ with $\bar{y} \in \arg\min_y \{d^\top y \colon y \in S(x)\}$ is also bilevel feasible for the optimistic bilevel problem (11) with the same objective value. Moreover, for every globally optimal point $(x, \bar{y})$ to Problem (11), x is bilevel feasible for (1) with the same objective value.*

Proof. Because x is bilevel feasible for (1), there exists a point $y \in S(x)$ such that $F_{\mathrm{o}}(x) = c^\top x + d^\top y$. Note that the objective function of (12) does not depend on ε. Moreover, for fixed x and any $\bar{y}$ satisfying $Cx + D\bar{y} \geq b$, the inequality $f^\top \bar{y} \geq f^\top y$ holds. Consequently, the optimal objective value of (12) is exactly $f^\top y$. Thus, for a given x and $\bar{y} := y$, we have $(y, 0) \in \tilde{S}(x, \bar{y})$ and $(x, \bar{y})$ is a bilevel feasible point for (11) with $F_{\mathrm{oa}}(x, \bar{y}) = F_{\mathrm{o}}(x)$.

Conversely, because $(x, \bar{y})$ is a globally optimal point for (11), there exists $(y, \varepsilon) \in \tilde{S}(x, \bar{y})$ with $\varepsilon = 0$. Consequently, $f^\top y = f^\top \bar{y}$ holds. Following the same line of arguments as above, we obtain $y \in S(x)$. This implies that x is feasible for (1). We are left to prove that $F_{\mathrm{oa}}(x, \bar{y}) = F_{\mathrm{o}}(x)$ holds. Assume that this is not the case. Hence, $d^\top y \neq d^\top \bar{y}$ needs to hold. If $d^\top y < d^\top \bar{y}$, we could choose $(x, \hat{y})$ with $\hat{y} := y$. This yields $(y, 0) \in \tilde{S}(x, \hat{y})$, which again implies $F_{\mathrm{oa}}(x, \hat{y}) = F_{\mathrm{o}}(x)$. This contradicts the optimality of $(x, \bar{y})$. If $d^\top y > d^\top \bar{y}$, then $\bar{y}$ would be a better lower-level solution than y in terms of the leader's objective function in (1), which, again, is a contradiction. Hence, $F_{\mathrm{oa}}(x, \bar{y}) = F_{\mathrm{o}}(x)$ holds, which ends the proof. □

Using the same proof techniques that lead to Theorem 2.2 of Henke et al. (2024), we move the single coupling constraint $\varepsilon = 0$ of (11) to the leader's objective function. This yields the following lemma.

Lemma 3.5. *There is a polynomial-sized parameter $\kappa > 0$ so that Problem (11) has the same set of globally optimal solutions as the optimistic bilevel problem*

$$\min_{(x,\bar{y}) \in \tilde{X}} \quad F_{\mathrm{o}\kappa}(x, \bar{y}) := c^\top x + d^\top \bar{y} + \min_{y,\varepsilon} \left\{ \kappa\varepsilon \colon (y, \varepsilon) \in \tilde{S}(x, \bar{y}) \right\} \tag{13}$$

without coupling constraints. Here, we again use $\tilde{X} = \{(x, \bar{y}) \colon x \in X, D\bar{y} \geq b - Cx\}$ and $\tilde{S}(x, \bar{y})$ is the set of optimal solutions of (12).

Theorem 3.6. *For any κ, the optimistic bilevel problem (13) without coupling constraints and its pessimistic version*

$$\min_{(x,\bar{y}) \in \tilde{X}} \quad F_{\mathrm{p}\kappa}(x, \bar{y}) := c^\top x + d^\top \bar{y} + \max_{y,\varepsilon} \left\{ \kappa\varepsilon \colon (y, \varepsilon) \in \tilde{S}(x, \bar{y}) \right\} \tag{14}$$

have the same set of feasible and globally optimal solutions.

Proof. Both problems have the same feasible sets. Moreover, for any feasible point $(x, \bar{y})$, the value of ε is uniquely determined. Thus, the inner minimization problem in (13) and the inner maximization problem in (14) have the same value. Thus, $F_{\mathrm{o}\kappa}(x, \bar{y}) = F_{\mathrm{p}\kappa}(x, \bar{y})$ holds. □

Corollary 3.7. *There is a polynomial-sized parameter $\kappa > 0$ so that the optimistic bilevel problem (1) without coupling constraints has the same set of globally optimal solutions as the pessimistic bilevel problem (14) without coupling constraints.*

Proof. The claim follows from Lemmas 3.4 and 3.5 as well as Theorem 3.6. □

To sum up the overall section, we showed that we can reformulate a pessimistic bilevel optimization problem with coupling constraints as one without; see Figure 1. Moreover, note that the resulting problem is of polynomial size in the input size of the originally given one and that this reformulation can be done in polynomial time.

4. Conclusion

We show in this note that—on the level of globally optimal solutions—there is no difference between linear pessimistic bilevel optimization with and without coupling constraints. To be more precise, for a given pessimistic bilevel optimization problem with coupling constraints we can derive another one without coupling constraints having the same global optimizers. Moreover, we even show that we can go from a pessimistic problem with coupling constraints to an optimistic problem without coupling constraints—again having the same global solutions.

It was somehow common sense that having coupling constraints or not makes a significant difference in pessimistic bilevel optimization. It is now shown that this is not the case. In particular, many novel theoretical results or even solution techniques can be gathered for pessimistic problems with coupling constraints by simply studying an equivalent problem without such constraints—or even an optimistic problem.

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