Data-driven robust menu planning for food services: Reducing food waste by using leftovers

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Abstract

With food waste levels of about 30%, mostly caused by overproduction, reducing food waste poses an important challenge in the food service sector. As food is prepared in advance rather than on demand, there is a significant risk that meals or meal components remain uneaten. Flexible meal planning can promote the reuse of these leftovers and, thereby, reduce the sector's environmental impact. We propose an innovative menu planning model, incorporating not only the traditional meal planning decisions, determining the meals offered per day and the ingredients to purchase, but also decisions related to the storage of surplus meals and their use on the following days. Demand uncertainty is a crucial aspect of this problem. Given that demand is generated based on human preferences, which are hard to model, we use a data-driven adjustable robust optimization approach and show how this can be solved efficiently. Our model then aims to provide a menu with the least environmental impact or cost while aligning with customer preferences. In this setting, we prove that demand uncertainty is the main driver of waste. By applying our model to a real-life case study of a food service provider, we demonstrate that reusing leftovers is a viable strategy for reducing purchasing costs by 4% and the environmental impacts of waste by 11-19%. Our findings suggest, furthermore, that food service providers should prioritize the production of the cheapest meals to minimize purchasing costs, while vegetarian/plant-based meals should be prioritized to reduce environmental impacts. Further analysis of additional case settings shows that increasing order frequency and making early purchases during the planning horizon allows for more flexibility in responding to fluctuating demand, thereby reducing waste. These findings offer actionable insights into promoting a circular (bio)economy in industries managing perishable products under uncertain demand.

1 Introduction

Food waste poses an important challenge for present-day society, contributing significantly to economic, environmental, and social impacts, including handling costs, water usage, and food scarcity (Lins et al., 2021). The issue is particularly pressing in the final stages of the food chain due to the accumulated investments and value-added processes from earlier stages (FAO 2019). Despite this high impact, food services often struggle to reduce food waste due to high levels of uncertainty and the very short shelf-life of prepared food, resulting in waste levels of about 20-30% of the initially purchased food (Eriksson et al., 2017). At the aggregated level, this can quickly lead to a sizeable contribution, with a case study in the Netherlands revealing that food services wasted just over 55 million kilograms of food, valued at almost 650 million euro (Klerx, 2023) and accounting for approximately 110,000 tons of CO_2 emissions (van Beek, 2023).

One of the primary causes of food waste in the food service sector is overproduction due to the inability of the industry to operate on a make-to-order basis, resulting in leftovers in the form of surplus meals or meal components (Tomaszewska et al., 2021), amounting to more than half of the total food-service waste (Eriksson et al., 2017). As processed food, these leftovers are often less versatile than raw ingredients and pose a higher risk of spoilage (Musa Aamir and Hasan, 2018). To address this issue, the repurposing of meals and meal components should be encouraged, e.g., by using leftover bread for bread pudding. This strategy aligns with the principles of food valorization, which considers reuse for food purposes the most preferable intervention if food waste prevention is not feasible. By enhancing resource efficiency through effective waste management, this approach facilitates the transition from a linear to a circular economy (Papargyropoulou et al., 2014), an important research direction for Operations Management (Agrawal et al., 2019).

Successful food valorization requires the consideration of product composition, quality, and perishability to ensure safe consumption of the repurposed products (Wang and Ng, 2019). In this context, ingredient purchasing decisions are important when balancing cost-effectiveness with spoilage risks. Strategic ordering strategies and effective inventory management are, thus, essential for food waste minimization (Swink et al., 2022).

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Consequently, food services must optimize purchasing, production, and inventory decisions simultaneously while considering possible valorization options.

Menu planning models are valuable tools in this context, which enable the optimization of these decisions while holding the potential to support meal component valorization when inventory and reuse decisions are incorporated. Adaptable recipes allowing ingredient substitution, including the option to repurpose surplus meal components as ingredients in other meals, can, for example, prevent unnecessary waste by adjusting future production to the realized demand. Furthermore, reuse can decrease costs as it reduces the required ingredient quantities (Lee, 2012). However, uncertainty in customer behavior often makes demand unpredictable (Filimonau et al., 2020), which drives leftover waste (Gonçalves et al., 2023). As such, accounting for demand fluctuations is crucial to developing effective waste-reduction strategies in the food service sector, and presents a key research area in operations management to promote the circular economy (Agrawal et al., 2019).

This paper proposes a robust optimization model for designing menu plans in the food service sector, using flexible recipes while incorporating inventory and food reuse decisions. By using a multi-stage adjustable robust optimization approach (ARO), we address demand uncertainty via the use of a data-driven uncertainty set which is constructed with historical data from a real-life case study. Applying our model to a real-life case study of a food service provider allows us to derive important insights to improve operational and tactical decision-making within food services and other industries handling short-life-cycle products, such as the fashion industry, where circular approaches gain in importance (Long and Gui, 2024). The contributions of this paper are, thus, threefold:

(i) Unlike traditional models, our study addresses demand uncertainty in menu planning through the integration of flexible recipes, inventory management and the reuse of leftovers, thereby advancing the menu planning literature. In this way, we provide a more realistic and practical approach to menu planning in uncertain environments, promoting both sustainability and adaptability.

(ii) We demonstrate an effective approach to managing demand uncertainty using multi-stage ARO, being the first to apply this approach to menu planning problems, to the best of our knowledge. In this context, our analysis proves that an optimal solution to the multi-stage ARO problem can be obtained by restricting the data-driven uncertainty set to only a few scenarios, significantly reducing computational complexity without sacrificing optimality. This methodological contribution extends beyond menu planning, offering insights for other applications of robust optimization in domains with similar uncertainty structures. We then employ a rolling horizon approach to obtain solutions, dynamically adjusting menu planning decisions in response to demand fluctuations to efficiently repurpose surplus meal components.

(iii) Using our model, we effectively balance economic and environmental objectives, providing actionable insights to address the challenges of food waste management in the food service sector. We show that reusing leftovers is both economically and environmentally advantageous, underscoring the importance of integrating reuse strategies into food service operations. The most environmentally beneficial components are those that can be widely repurposed or whose ingredients can be largely substituted, leading to a higher diversity of reused components under environmental objectives compared to economic objectives, where reuse is only beneficial when it directly reduces purchasing costs. Additionally, we identify that weekly meal variety, such as the prevention of similar meals on consecutive days, significantly increases both costs and environmental impacts, whereas imposing daily variety requirements, like at least one vegetarian meal per day, has a negligible impact on our indicators, making it a more effective way to incorporate diversity in menu planning. Furthermore, we show that increasing order frequency and placing orders earlier in the planning horizon substantially reduces waste by enabling greater adaptability to fluctuating demand.

The remainder of the paper is structured as follows. Section 2 provides an overview of the related literature, while Section 3 delineates the problem at hand. Section 4 outlines the proposed model and its robust formulation, with the solution methods being explained in Section 5. Section 6 describes the chosen case study, and the numerical experiments are presented in Section 7. Our findings are then discussed in Section 8, before Section 9 concludes the paper.

2 Literature review

This paper contributes to two important streams of literature: (i) the literature on menu planning models, which is presented in Section 2.1 and relates to the practical application considered in our study, and (ii) the field of Adjustable Robust Optimization related to the integration of uncertainty in optimization approaches, which is presented in Section 2.2.

2.1 Menu planning

Menu planning has long been a subject of interest in academic research (Lancaster, 1992), employing diverse methodologies such as Data Envelopment Analysis (see, e.g., Reynolds and Taylor, 2011; Kanellopoulos et al., 2020) and optimization models (see, e.g., van Wonderen et al., 2023; Moreira et al., 2022). Traditionally, these models emphasize nutritional quality, but recently, environmental concerns have gained increasing attention (van Dooren, 2018).

Studies addressing environmental concerns tend to embed these within the objective (see, e.g., Eustachio Colombo et al., 2019; Stern et al., 2023) rather than modeling reduction strategies, such as reuse or recycling, explicitly in the decision making. This trend is also evident in menu planning studies that consider food waste. For instance, Stern et al. (2023) propose an integer linear optimization model for school menus where the primary objective is to maximize plate consumption, thereby minimizing waste, while also using nutritional quality, environmental impact, and cost objectives. Also optimizing school meals, Eustachio Colombo et al. (2020) develop menus with minimal deviation from existing ones, considering nutritional value and greenhouse gas emissions while measuring different food waste categories before and after implementation of the designed menu, including plate, kitchen, and serving waste, as well as surplus meal waste. Concentrating on households, van Rooijen et al. (2024) optimize for food waste, as well as costs, greenhouse gas emissions, and nutritional value of weekly menu plans using a mixed-integer optimization model, considering package sizes. Focusing on food banks instead, Buisman et al. (2019) develop a strategic-tactical mixed-integer linear optimization model in the context of food donation acceptance, considering ingredient procurement for meal production to align with customer demand while quantifying ingredient waste. Despite these advancements in menu planning models, inventory management remains an underexplored aspect, particularly when it comes to its role in reducing food waste, with Buisman et al. (2019) being one of the few to incorporate inventory decisions. This is a significant oversight because poor inventory management is related to several causes of food waste, such as overstocking and spoilage (Papargyropoulou et al., 2014; Gonçalves et al., 2023), so that improving inventory management is identified as a key area in the field of operations management when aiming to reduce food waste (Akkaş and Gaur, 2022). In addition, incorporating repurposing strategies for surplus products can further mitigate waste (Betz et al., 2015), providing strategies for improvement within the model. To this end, we incorporate inventory and repurposing decisions in addition to the commonly used purchasing, production, and/or serving decisions made in menu planning studies. While using surplus products to create new products has been studied in the retail sector (Lee and Tongarlak, 2017), our research is the first to apply this approach in the food service sector.

Another significant gap in the literature is the lack of consideration for uncertainty. Most menu planning studies operate in deterministic settings, such as school catering, where the number of meals required is known in advance. However, demand fluctuates in many other real-world settings, like restaurants or buffets, making it essential to account for demand uncertainty since unanticipated variations in demand can lead to overproduction and, consequently, surplus or leftover waste (Akkaş and Gaur, 2022). Although some studies address uncertainty in optimization models for the food service industry, they often lack the flexibility needed to adapt to dynamic demand conditions. For instance, Sel et al. (2017) propose a stochastic mixed-integer linear optimization model to improve production and distribution scheduling within a closed-loop catering supply chain, considering the costs of food waste due to overproduction under demand uncertainty. However, a limitation of their approach is the assumption of fixed menus, which restricts flexibility in responding to varying customer preferences, thereby reducing the potential for mitigating food waste. Optimizing inventory and production decisions for catering companies, Aka and Akyüz (2021) use fuzzy parameters to account for demand, production, and cost uncertainties while considering costs associated with waste. They introduce a bit more flexibility in their menu by focusing on meals derived from two food groups but their model lacks strategies for reusing surplus food, leaving unused ingredients as waste. Vasilakakis and Giannikos (2023) develop a multi-objective mixed-integer optimization model with goal programming to construct restaurant meal plans, using a rather conservative robust optimization approach that lacks adaptivity while addressing food's nutrient content uncertainty rather than demand uncertainty. Furthermore, they leave food waste unaddressed and optimize for costs only.

Flexible recipes with ingredient substitution can offer a promising solution for better managing demand uncertainty. These recipes allow dynamic adjustments of purchasing and production decisions based on fluctuating demand, reducing the risk of surplus food. However, the use of such recipes is limited in menu planning models. Most articles typically focus on meal components that combine into a meal rather than on (flexible) recipes that transform ingredients into individual meal components. While Crama et al. (2004) incorporate multiple recipes per product, their mixed binary nonlinear optimization model is designed for a factory rather than a food service, and focuses solely on minimizing purchasing costs, without accounting for environmental impacts or waste. Additionally, it excludes inventory management entirely, let alone reuse decisions.

In conclusion, our study stands out from the existing ones by proposing an approach for menu planning with waste considerations using flexible recipes while considering uncertain demand, offering a data-driven, robust approach to sustainable, circular, and adaptable meal planning.

2.2 Adjustable robust optimization

Robust optimization (RO) and stochastic optimization (SO) are the two main streams of research in the field of Operations Research aimed at managing uncertainty. SO requires knowledge about the probability distribution, which is hard to estimate in contexts where demand is directly linked to customer behavior (Filimonau et al., 2020). In addition, the resulting formulation results into optimization problems that are computationally challenging (Ben-Tal and Nemirovski, 2000; Bertsmas and den Hertog, 2022). As such, the focus in this study will be on RO, which does not require probability distributions but finds solutions safeguarded against scenarios within a given set, called the uncertainty set (Bertsmas and den Hertog, 2022).

The RO paradigm includes two main modeling approaches. In static RO, it is assumed that the decision variables are independent of the uncertain parameter, and decisions can be made before these uncertain parameters materialize. However, in many real-world settings, some decisions depend on the actual realization of uncertain parameters, while others can be made independently. Adjustable robust optimization (ARO) is a more suitable approach in such cases. ARO distinguishes the decisions into independent variables, the "here-and-now" decisions, and dependent variables, the "wait-and-see" variables (Ben-Tal et al., 2004). ARO formulations can be two-stage or multi-stage, where a multi-stage formulation is suitable for problems spanning multiple periods. Within the domain of multi-stage ARO, several approximation methods, such as affine decision rules (Lappas and Gounaris, 2016; Santos et al., 2020), finite adaptability approximations (Bertsimas and Caramanis, 2010; Feng et al., 2019), and finite scenarios (Marandi and van Houtum, 2020; Borumand et al., 2024) have been applied to overcome the computational challenges inherent to ARO due to its NP-hardness (Yanıkoğlu et al., 2019). Researchers have also started exploring data-driven methods for constructing uncertainty sets to better reflect real-world data patterns (Zhang et al., 2022; Goerigk and Khosravi, 2023; Wang et al., 2024; Suyal and Sharma, 2024; Neofytou et al., 2025). We refer interested readers to the tutorials in Bertsimas and Thiele (2006) and Bertsimas et al. (2018).

ARO has shown strong potential in dealing with uncertainties in many practical problems, including logistics (Marandi and van Houtum, 2020; Borumand et al., 2024), inventory management (Ben-Tal et al., 2005; Santos et al., 2020; Kang et al., 2023, 2024), machine learning (Hooshmand et al., 2025), vehicle routing (Jaillet et al., 2016; Lei et al., 2016), portfolio optimization (Fliedner and Liesiö, 2016), electrical power production (Jiang et al., 2014; Zugno et al., 2016), and scheduling (Lappas and Gounaris, 2016; Feng et al., 2019; Wang et al., 2023). For more applications of ARO and its solution approaches, we refer the interested reader to the review of Yanıkoğlu et al. (2019). In the food industry, ARO applications remain limited, with existing research primarily focused on food aid distribution (Dang et al., 2023; de Moor et al., 2024), and waste recovery problems (Wang and Ng, 2019; Xiong et al., 2021).

Being the first to apply ARO to solve menu planning problems, our work contributes to the data-driven multi-stage ARO literature methodologically by demonstrating that an optimal solution can be obtained for a class of data-driven uncertainty sets by restricting these sets to a small subset of scenarios, thereby significantly improving computational tractability.

3 Problem setting

In this research, we study a menu planning problem under demand uncertainty for a food service provider serving a diverse set of customers with different meal preferences. As a result, offering a variety of meals is essential to meet customer demands. These meals are organized into groups, e.g., "main" or "dessert", with interchangeable options within each group. For example, for the "main" meal group the meals "curry rice" or "lasagna" can be served.

Meals are prepared by combining several meal components on a plate, with each component belonging to a specific category. Within these categories, individual components are interchangeable as long as the required proportions are maintained. For example, the "curry rice" meal may require 50% of the "curry" category, for which either the meal components "katsu curry" or "korma curry" can be plated.

Like in the plating process, the production of meal components requires various ingredient categories in specific proportions. These categories include substitutable ingredients that are purchased, supplied from inventory, or even derived from meal components. For instance, the "katsu curry" meal component may require 30% of the "stock" category, regardless of whether it is the ingredient "vegetable stock" or "beef stock". The concept of interchangeable inputs of meal components, meals, and meal groups is visualized in Figure 1.

Using these flexible recipes, a menu plan is created for a time horizon consisting of multiple periods (for example, a time horizon of 1 working week, with 5 time periods of a day). In every period, the food service provider needs to decide on (i) which ingredients need to be purchased, (ii) which meal components need to be produced, (iii) which ingredients and meal components need to be stored in inventory for (re)use in the following periods, and (iv) which meal components need to be plated for serving (shown in Figure 2). Leftover meal components are stored in inventory for reuse until they reach the end of their shelf life, at which point they become waste. We assume that inventory cannot be retained at the end of the time horizon, as many food service providers operate exclusively on working days, and storing inventory over weekends is generally undesirable.

Figure 1: Schematic exemplary illustration of input categories for meal component production and plating. Black arrows indicate that products belong to another product's input category, and when dotted, the lines also indicate substitutability. Grey arrows indicate that the recipes of the corresponding meals are not fully shown.

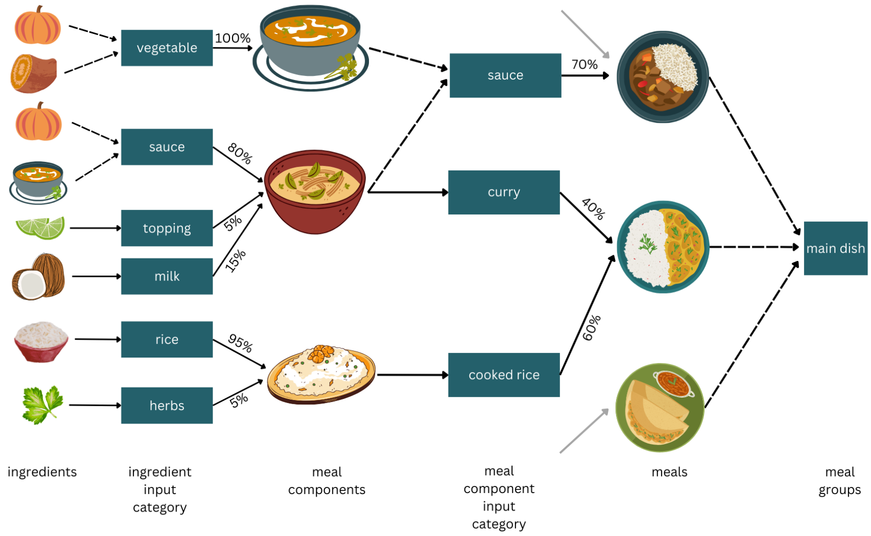
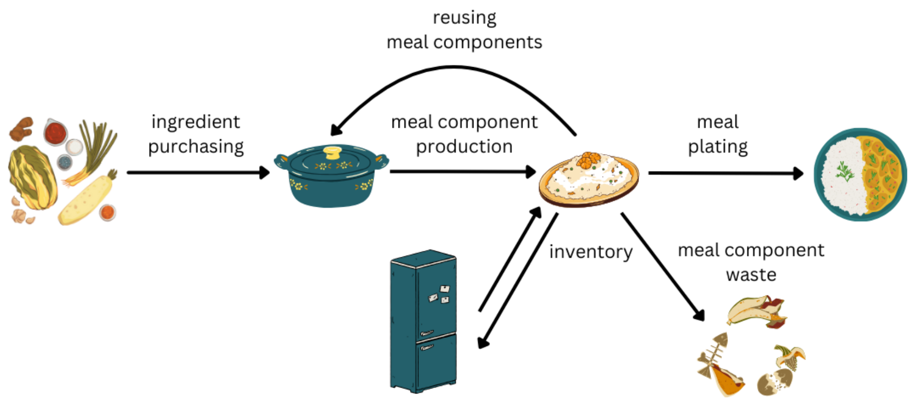


Figure 2: Overview of the menu planning process per period.



4 Model formulation

Based on the description provided in Section 3, we first present the mixed-integer linear optimization model of the deterministic problem in Section 4.1, before extending this to a multi-stage adjustable robust formulation in Section 4.2 to account for uncertainty.

4.1 Deterministic formulation

In this section, we introduce the notation of the deterministic model in Table 1. We present the objectives of our model in Section 4.1.1 and the constraints in Section 4.1.2.

Table 1: Nomenclature

Sets	
$\mathcal{R}$	Set of meal components, indexed by r
$\mathcal{E}$	Set of ingredients, indexed by i ($\mathcal{E} \subset \mathcal{R}$)
$\mathcal{T}$	Set of time periods, indexed by $t = 1, \dots, T$
$\mathcal{S}$	Set of shelf lives, indexed by $s = 1, \dots, S$

$\mathcal{P}$	Set of meals, indexed by p
$\mathcal{G}$	Set of meal groups, indexed by g
$\mathcal{C}_r$	Set of input categories of ingredients required to produce meal component r, indexed by c
$\mathcal{L}_p$	Set of input categories of meal components required for plating the meal components into meal p, indexed by l
$\mathcal{Z}$	Set of specifications of meals, indexed by z
Parameters	
δ_{tg}	Demand quantity for meal group g in period t [kg]
ρ_{cr}	Proportion of ingredients from input category c required to produce meal component r
σ_{lp}	Proportion of meal components from input category l required for plating the meal components into meal p
ζ_{icr}	Parameter indicating whether (1=yes, 0=no) ingredient or meal component i belongs to input category c, required to produce meal component r
α_{rlp}	Parameter indicating whether (1=yes, 0=no) meal component r belongs to input category l, required for plating the meal component into meal p
β_{pg}	Parameter indicating whether (1=yes, 0=no) meal p belongs to meal group g
θ_{rs}	Parameter indicating whether (1=yes, 0=no) meal component r can be produced with shelf life s
τ	The minimum number of periods within which a meal component can be served only once
Θ	Total meal component production capacity per period [kg]
Λ_{rs}	Inventory at the beginning of the first period of meal component r with shelf life s [kg]
υ_t	Purchasing costs per time period t [€/kg]
γ	Ordering costs per period an order is made [€]
ϵ_r	Environmental impact of wasting meal component r [impact/kg]
M	Big number
ϕ_{rz}	Parameter indicating whether (1=yes, 0=no) meal component r has specification z
Ψ_z	Minimum number of meals served per day, with specification z
Φ_z	Maximum number of meals served per day, with specification z
Ξ	Maximum number of periods in which an order can be placed
Decision variables	
$B_{tir} \in \mathbb{R}_{\geq}$	Purchased quantity of ingredient i for production of meal component r at the start of time period t [kg]
$X_{trs} \in \mathbb{R}_{\geq}$	Production quantity of meal component r during time period t with shelf life s [kg]
$H_t \in \{0,1\}$	$\begin{cases} 1, & \text{if an order for ingredients is made at the start of period } t \\ 0, & \text{otherwise} \end{cases}$
$W_{tr} \in \mathbb{R}_{\geq}$	Waste quantity of meal component r at the end of time period t [kg]
$I_{trs} \in \mathbb{R}_{\geq}$	Inventory quantity of meal component r at the beginning of time period t with shelf life s [kg]
$V_{trsr'} \in \mathbb{R}_{\geq}$	Reused quantity of meal component r with shelf life s during time period t to produce meal component r' [kg]
$N_{trsp} \in \mathbb{R}_{\geq}$	Plated quantity of meal component r with shelf life s during time period t into meal p [kg]
$K_{tpr} \in \{0,1\}$	$\begin{cases} 1, & \text{if meal component } r \text{ is used for meal } p \text{ during period } t \\ 0, & \text{otherwise} \end{cases}$
$J_{tp} \in \{0,1\}$	$\begin{cases} 1, & \text{if meal } p \text{ is served during period } t \\ 0, & \text{otherwise} \end{cases}$
$O_{tpz} \in \{0,1\}$	$\begin{cases} 1, & \text{if meal } p \text{ is served and all meal components used have specification } z \text{ during period } t \\ 0, & \text{otherwise} \end{cases}$
$Z_{tpz} \in \{0,1\}$	$\begin{cases} 1, & \text{if all meal components used for meal } p \text{ have specification } z \text{ during period } t \\ 0, & \text{otherwise} \end{cases}$
$A_{tpg} \in \mathbb{R}_{\geq}$	Serving quantity of meal p during time period t to satisfy demand for meal group g [kg]

4.1.1 Objectives

We optimize the menu plan using economic and environmental objectives. The economic objective is defined by the sum of the purchasing and ordering costs (1a), while the environmental objective measures the negative impact of wasted products. In this context, four environmental indicators relevant to food production have been selected based on the analysis in Van Mierlo et al. (2017): climate change, resource use of fossils, and land and water use. The total environmental impact for each indicator is then calculated by multiplying the amount of wasted products by the associated environmental impact (1b).

$$\min\left\{\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{E}}\sum_{r\in\mathcal{R}}\upsilon_t B_{tir}+\sum_{t\in\mathcal{T}}\gamma H_t\right\} \tag{1a}$$

$$\min\left\{\sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}}\epsilon_r W_{tr}\right\} \tag{1b}$$

4.1.2 Constraints

This section describes the model's constraints, organized into four main groups: mass balance, demand, meal variety, and ordering constraints.

Mass balance constraints Mass balance constraints are essential to regulating flows through the system at the production and plating stages.

Mass balances at production stage Starting with the production stage, we ensure that inputs (including purchased ingredients as well as reused meal components) are proportional to the generated outputs while allowing flexibility in the input selection by grouping inputs into broader categories (1c). Upon production, shelf lives are product-dependent but not ingredient-dependent (i.e. $\sum_{s\in\mathcal{S}}\theta_{rs}=1$). Moreover, purchases can only occur during the same period in which an order is placed (1d), and total production must not exceed capacity (1e).

$$\sum_{i\in\mathcal{E}}\zeta_{rci}B_{tir}+\sum_{i\in\mathcal{R}}\sum_{s\in\mathcal{S}}\zeta_{rci}V_{tisr}=\rho_{cr}\sum_{s\in\mathcal{S}}\theta_{rs}X_{trs} \qquad \forall t\in\mathcal{T}, r\in\mathcal{R}, c\in\mathcal{C}_r \tag{1c}$$

$$\sum_{i\in\mathcal{E}}\sum_{r\in\mathcal{R}}B_{tir}\leq MH_t \qquad \forall t\in\mathcal{T} \tag{1d}$$

$$\sum_{r\in\mathcal{R}}\sum_{s\in\mathcal{S}}\theta_{rs}X_{trs}\leq\Theta \qquad \forall t\in\mathcal{T} \tag{1e}$$

Mass balances at plating stage At the plating stage, meal components can serve multiple purposes: they can be plated directly, (re)used as ingredients for other meal components, or stored into inventory (1f). However, if components are not plated and have an insufficient shelf life to be stored for another period, they must be discarded (1g). This restriction also applies in the final period, where unused components cannot be stored and should be discarded (1h). We fix the inventory at the beginning of the first period (1i). Additionally, there is no inventory with the highest shelf life S, as stocking is done at the end of a period, inevitably reducing shelf life by one period (1j). Next to meal components, ingredients can also be stored in inventory. To facilitate this methodologically in the model, ingredients should be classified as meal components such that they can be 'produced' and consequently stored (see Figure 2 for the procedure). To prevent redundant flows, we prohibit the reuse of stored products within the same period of their production (1l), and also prohibit stored ingredients from being used to produce the same ingredient (1k).

$$\theta_{rs}X_{trs} + I_{trs} = \sum_{p\in\mathcal{P}}\sum_{l\in\mathcal{L}_p}\alpha_{rlp}N_{trsp} + \sum_{r'\in\mathcal{R}}\sum_{c\in\mathcal{C}_{r'}}\zeta_{r'cr}V_{trsr'} + I_{t+1,r,s-1} \qquad \forall t\in\{1,\dots,T-1\}, r\in\mathcal{R}, s\in\{2,\dots,S\} \tag{1f}$$

$$\theta_{rs}X_{trs} + I_{trs} = \sum_{p\in\mathcal{P}}\sum_{l\in\mathcal{L}_p}\alpha_{rlp}N_{trsp} + \sum_{r'\in\mathcal{R}}\sum_{c\in\mathcal{C}_{r'}}\zeta_{r'cr}V_{trsr'} + W_{tr} \qquad \forall t\in\{1,\dots,T-1\}, r\in\mathcal{R}, s=1 \tag{1g}$$

$$\sum_{s\in\mathcal{S}}(\theta_{rs}X_{trs} + I_{trs}) = \sum_{p\in\mathcal{P}}\sum_{l\in\mathcal{L}_p}\sum_{s\in\mathcal{S}}\alpha_{rlp}N_{trsp} + \sum_{r'\in\mathcal{R}}\sum_{c\in\mathcal{C}_{r'}}\sum_{s\in\mathcal{S}}\zeta_{r'cr}V_{trsr'} + W_{tr} \qquad \forall t=T, r\in\mathcal{R} \tag{1h}$$

$$I_{trs} = \Lambda_{rs} \qquad \forall t=1, r\in\mathcal{R}, s\in\mathcal{S} \tag{1i}$$

$$I_{trs} = 0 \qquad \forall t\in\{2,\dots,T\}, r\in\mathcal{R}, s=S \tag{1j}$$

$$V_{tisi} = 0 \qquad \forall t\in\mathcal{T}, i\in\mathcal{E}, s\in\mathcal{S} \tag{1k}$$

$$\theta_{rs}V_{tisr} = 0 \qquad \forall t\in\mathcal{T}, i\in\mathcal{R}, r\in\mathcal{R}, s\in\mathcal{S} \tag{1l}$$

Demand constraints When meals are selected to be plated, we must ensure that the meal components from each input category match the required proportions for the meal (1m). The assembled meal can then be served to customers to satisfy the demand (1n).

$$\sum_{r\in\mathcal{R}}\sum_{s\in\mathcal{S}}\alpha_{rlp}N_{trsp} = \sigma_{lp}\sum_{g\in\mathcal{G}}\beta_{pg}A_{tpg} \qquad \forall t\in\mathcal{T}, p\in\mathcal{P}, l\in\mathcal{L}_p \tag{1m}$$

$$\sum_{p\in\mathcal{P}}\beta_{pg}A_{tpg} = \delta_{tg} \qquad \forall t\in\mathcal{T}, g\in\mathcal{G} \tag{1n}$$

Meal variety constraints The meal variety constraints introduce meal diversity on a weekly and daily basis.

Weekly meal variety To avoid repetition of meals, we ensure that each meal component is plated in only one period within a specified number of periods (1o), using logical constraints to link the plating decisions (1p).

$$\sum_{p\in\mathcal{P}}\sum_{t-\tau\leq t'<t}K_{t'pr} \leq 1 \qquad \forall t\in\{\tau+1,\dots,T\}, r\in\mathcal{R} \tag{1o}$$

$$\frac{1}{M}K_{tpr} \leq \sum_{l\in\mathcal{L}_p}\sum_{s\in\mathcal{S}}\alpha_{rlp}N_{trsp} \leq \qquad \forall t\in\mathcal{T}, r\in\mathcal{R}, p\in\mathcal{P} \tag{1p}$$

Daily meal variety To accommodate diverse customer preferences, we ensure that meals aligned with their dietary preferences are available at every period. We specify the minimum and maximum number of meal components meeting certain dietary specifications, such as vegetarian or non-vegetarian, that are served daily (1q).

$$\Psi_z \leq \sum_{p\in\mathcal{P}}O_{tpz} \leq \Phi_z \qquad \forall t\in\mathcal{T}, z\in\mathcal{Z} \tag{1q}$$

Then, we introduce a couple of logical constraints to connect the relevant decisions in (1r) - (1t). First, we define a meal as "served" if meal components are used for that meal ($J_{tp} = 1$) (1r). Furthermore, we ensure that a meal inherits the dietary specification only if all of its components meet that specification ($Z_{tpz} = 1$) (1s). By combining these rules, we ensure that a meal with a specification can only be considered served if the meal itself is served ($J_{tp} = 1$), and all its components meet the same specification ($Z_{tpz} = 1$) (1t). Note that in this constraint, instead of $\frac{1}{3}$, any value in the interval $(0, \frac{1}{2})$ would suffice.

$$MJ_{tp} \geq \sum_{r\in\mathcal{R}}K_{tpr} \geq J_{tp} \qquad \forall t\in\mathcal{T}, p\in\mathcal{P} \tag{1r}$$

$$M(1-Z_{tpz}) \geq \sum_{r\in\mathcal{R}}(K_{tpr} - \phi_{rz}K_{tpr}) \geq 1 - Z_{tpz} \qquad \forall t\in\mathcal{T}, p\in\mathcal{P}, z\in\mathcal{Z} \tag{1s}$$

$$\frac{2}{3}O_{tpz} \leq \frac{1}{2}(Z_{tpz} + J_{tp}) \leq \frac{2}{3} + O_{tpz} \qquad \forall t\in\mathcal{T}, p\in\mathcal{P}, z\in\mathcal{Z} \tag{1t}$$

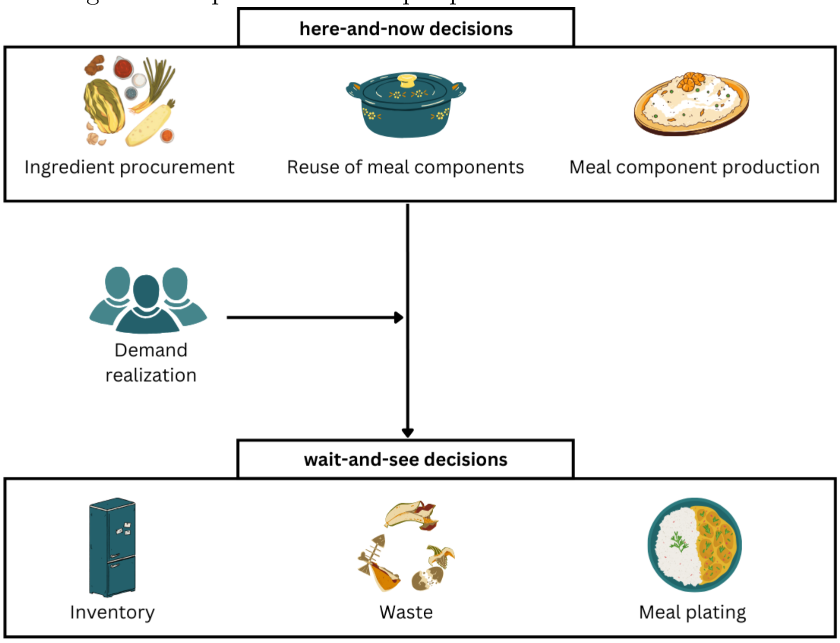
Ordering constraints We limit the total number of purchase orders across the entire time horizon.(1u).

$$\sum_{t \in \mathcal{T}} H_t \leq \Xi \tag{1u}$$

4.2 Multi-stage adjustable robust formulation

Uncertainty in customer numbers and behavior makes the demand for food service providers unpredictable, as outlined in Section 1. This unpredictability is the primary cause of leftover food waste, as in perfect information situations, good inventory management can reduce this waste to zero (as we prove later in Proposition 1 in Section 5). Therefore, accounting for this variability is essential in menu planning. However, as deterministic formulations cannot account for uncertainty, we use the robust optimization (RO) methodology, which safeguards solutions against scenarios from a predefined set, known as an uncertainty set. We define an uncertainty set for the demand parameter, capturing the possible range of demand values. Since menu planning decisions must be made step-by-step over time as new demand information emerges, we use a specific RO approach called multi-stage adjustable robust optimization (ARO), which offers flexibility in adjusting decisions as more information becomes available. In this section, we describe the multi-stage ARO formulation of the problem and visualize the sequence of events in Figure 3.

Figure 3: Sequence of events per period in the ARO model.



Before demand is fully known, purchasing, production, and reuse decisions must be made in the first period. The decision variables related to these decisions are the here-and-now decisions, as these have to be made in advance without any information about the demand. Thus, B_{1ir}, V_{1isr}, X_{1rs}, and H_1 become the here-and-now decision variables. After the demand is realized, we can make the so-called wait-and-see decisions that depend on the realized demand, which we denote by $\delta_{[t]}$, indicating the vector with the demands up to and including period t. Wait-and-see decisions include meal plating, demand satisfaction, inventory, and waste quantities. Additionally, as we move into future periods, purchasing, production, and reuse decisions depend also on the realized demand in the previous periods. This means that these decisions also fall under the wait-and-see category. Thus, the wait-and-see decisions are $B_{tir}(\delta_{[t]})$, $V_{trsr'}(\delta_{[t]})$, $X_{trs}(\delta_{[t]})$, $H_t(\delta_{[t]})$, for $t = 2, \ldots, T$, and $I_{trs}(\delta_{[t]})$, $W_{tr}(\delta_{[t]})$, $N_{trsp}(\delta_{[t]})$, $K_{tpr}(\delta_{[t]})$, $J_{tp}(\delta_{[t]})$, $O_{tpz}(\delta_{[t]})$, $Z_{tpz}(\delta_{[t]})$, and $A_{tpg}(\delta_{[t]})$ for $t = 1, \ldots, T$. The complete formulation of the ARO model can be found in Appendix A.

As the literature outlines, the computational complexity of a multi-stage ARO model depends not only on the problem but also on how the uncertainty set is constructed. In this paper, we use the method of Yazdani et al. (2023) to construct the data-driven uncertainty set based on historical data. Their approach's advantage is that it produces an uncertainty set that is the union of simple sets. In this paper, we restrict our uncertainty set to a union of boxes to ensure a balance between computational efficiency and level of conservativeness.

5 Solution approach

This section explains how one can solve the multi-stage ARO menu planning problem described in Section 4.2. In this context, we first show that our multi-stage ARO problem can be solved using a finite scenario approach before explaining how we run a simulation model with a rolling horizon method to obtain the dynamic menu over the time horizon.

5.1 Exact reformulation

The goal of this section is to show that our multi-stage ARO problem can be solved exactly using a finite scenario (FS) approach, where we limit our analysis to a finite number of scenarios from the uncertainty set (see, e.g., Yanıkoğlu et al., 2019). This approach generally results in a computationally tractable problem at the cost of potentially missing the true optimal solution. However, in some cases, like those considered by Marandi and van Houtum (2020), limiting ourselves to only a few scenarios still leads to the optimal solution. In this section we analyze whether we can select a specific scenario set that would provide us with the true optimal solution.

To start our analysis, let us consider the multi-stage ARO menu planning problem (Section 4.2) with a minimization objective and a box uncertainty set D, where $\delta^L \leq \delta \leq \delta^U$. Let $\mathcal{T} = 1$ and $\Lambda = 0$. We drop the index t in the formulation for ease of notation. (1i) and (1j) imply that I can be excluded from the decision variables when $\mathcal{T} = 1$ and $\Lambda = 0$. Similarly, (1l) allows us to omit V. Let x be the collection of all here-and-now decision variables, where $x = (B_{ir}, V_{isr}, X_{rs}, H)$, and y be the collection of all wait-and-see variables, where $y = (W_r(\delta), N_{rsp}(\delta), K_{pr}(\delta), J_p(\delta), O_{pz}(\delta), Z_{pz}(\delta), A_{pg}(\delta))$. Then, we can write our ARO problem as

$$\min_{x\in\mathcal{X}} \max_{\delta\in D} \min_{y\in\mathcal{Y}(x,\delta)} P(x, y, \delta), \tag{2}$$

where $\mathcal{X}$ is the set of feasible decisions for the here-and-now variables, and $\mathcal{Y}(x, \delta)$ is the set of feasible solutions for the wait-and-see variables, given the value of x and demand δ, and where $P(x, y, \delta)$ is either the economic (1a) or environmental objective function (1b). For a given x, let $g(x) := \max_{\delta\in D} \min_{y\in\mathcal{Y}(x,\delta)} P(x, y, \delta)$ and $f(x, \delta) := \min_{y\in\mathcal{Y}(x,\delta)} P(x, y, \delta)$.

We first show that for the ARO problem (2), the total purchasing quantity cannot be lower than the maximum aggregated possible demand δ^U.

Lemma 1. *Given any feasible x, if $f(y, \delta)$ is feasible for any $\delta \in D$, then*

$$\sum_{r\in\mathcal{R}} \sum_{i\in\mathcal{E}} B_{ir} \geq \sum_{g\in\mathcal{G}} \delta_g^U. \tag{3}$$

Proof. All proofs are presented in Appendix B ☐

Lemma 1 shows that to have a feasible menu, the total purchasing quantity cannot be lower than the aggregated demand for the scenario δ^U. In the following theorem, we prove that, for the economic objective, the worst-case demand scenario corresponds to the highest demand scenario, as this would result in the largest purchasing quantity and, consequently, the highest purchasing costs, meaning that we can solve the problem with the economic objective by limiting the uncertainty set to only δ^U.

Theorem 1. *Considering the economic objective function, an optimal here-and-now solution x to the ARO problem can be obtained by solving the deterministic problem* (1) *with the demand scenario δ^U.*

Theorem 1 asserts that the optimal here-and-now solution when minimizing the economic objective can be obtained by merely solving the deterministic problem (1). In the next proposition, we show that the generated waste is zero for any deterministic problem with the economic objective.

Proposition 1. *In a deterministic problem with the economic objective, waste is zero.*

After analyzing the economic objective, let us now consider the environmental objective. Interestingly, in contrast to the economic objective, we prove that when minimizing the environmental impact of waste, the optimal solution is obtained by limiting the uncertainty set to only two scenarios.

Theorem 2. *For any feasible x,*

$$\max_{\delta\in D} \min_{y\in Y(x,\delta)} E(W, \delta) = \max_{\delta\in\bar{D}} \min_{y\in Y(x,\delta)} E(W, \delta), \tag{4}$$

where $\bar{D} = \{\delta^L, \delta^U\}$.

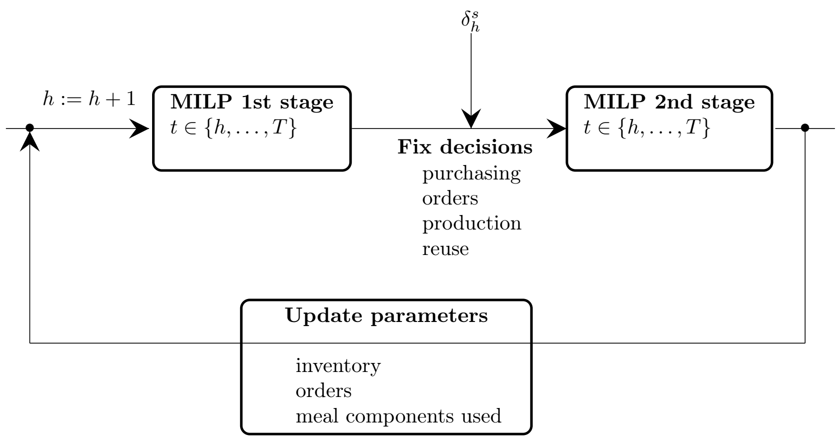
Theorem 2 states that the highest waste occurs when large quantities of meal components are produced to ensure that any potential demand can be satisfied, but the actual demand turns out to be low. Therefore, for robust menu planning, it is sufficient to consider only the lowest and highest demand vectors. This analysis assumes no initial inventory, though most results hold even with starting inventory. An exception is Proposition 1, as, in a deterministic setting, waste may occur even without purchases if initial inventory exceeds demand.

In this section, we showed how to solve the multi-stage ARO problem with a box uncertainty set, which can also be applied to several other problem classes, as we describe in Appendix C. Moreover, as recent research has shown that the uncertainty set can be a union of multiple boxes (see, e.g., Zhang et al., 2022; Goerigk and Khosravi, 2023), we show that our solution method (selecting only the relevant extreme demand scenarios as proven in Theorem 1 and 2) is also valid for such uncertainty sets using the following argument: Let us assume that $D = \bigcup_{k=1,...,K} D^k$, where D^k is the box with the lower bound δ^{L^k} and the upper bound δ^{U^k}. In this case, similar to the above analysis, to solve the multi-stage ARO problem with the economic objective, we can restrict the uncertainty set to $\{\delta^{U^k}, k = 1, ..., K\}$ and for the environmental objective, we can restrict the uncertainty set to $\bigcup_{k=1,...,K}\{\delta^{L^k}, \delta^{U^k}\}$.

5.2 Rolling horizon

We then use a rolling horizon approach to obtain solutions for our multi-stage menu planning ARO problem (see Section 2.2) while capturing the uncertain nature of the demand. This approach allows us to generate solutions for each period, continuously adapting to observed demand as it materializes in each period. With a standard ARO approach, only "here-and-now" decisions can be fixed, but by using the rolling horizon, we can convert "wait-and-see" decisions in the following period into "here-and-now" ones, allowing us to fix both types of solutions. This approach also reflects the practical approach in food services, where plating and serving decisions are made after customer arrival. Figure 4 illustrates the schematic overview of our rolling horizon implementation, where the planning is solved for all h periods in the planning horizon, from h till T. Each period consists of two sequential solution phases: the first-stage and second-stage optimizations. In the first-stage optimization, the here-and-now decisions concern purchasing, order, production, and reuse decisions (B_{hir}, V_{hisr}, X_{hrs}, and H_h). After this stage, the (random) demand δ_h is revealed. Then, in the second-stage optimization, the model is re-solved for the same period h but now the purchasing, order, production, and reuse decisions (the here-and-now decisions in the first stage) are fixed to the solutions found in the first-stage optimization. The wait-and-see decisions of the current time period ($N_{hrsp}(\delta_h^s)$, $I_{trs}(\delta_{[t]})$, $W_{hrs}(\delta_h^s)$, $A_{phg}(\delta_h^s)$, $K_{hpr}(\delta_h^s)$, $J_{hp}(\delta_h^s)$, $O_{tpz}(\delta_h^s)$, $Z_{hpz}(\delta_h^s)$, and $A_{hpg}(\delta_h^s)$) are now treated as here-and-now decisions in the second-stage optimization, meaning that they now only respond to the revealed demand δ_h. The solutions for these wait-and-see decisions are saved after the second stage, and the same procedure repeats for the next period. The inventory quantities available in the next period ($I_{h+1,r,s}$) are used as starting inventories for the next optimization. Moreover, ordering and plating decisions (H_h, K_{hpr}) are saved to ensure the satisfaction of constraints that cover cumulative sums of decision variables across multiple periods (e.g., (1u), which limits the total orders over the entire time horizon to a specified maximum).

Figure 4: Rolling horizon approach.



6 Case study

To illustrate and test our model while allowing us to gain practical insights, we use data from a real-life case study, which is based on a Dutch food service provider. A selection of their recipes has been made, including seven meal groups ($\mathcal{G}$) with 30 meals ($\mathcal{P}$) in total, composed out of 56 meal components ($\mathcal{R}$). Of these components, 27 could be reused as ingredients for other meal components. The recipes were made using 125 ingredients ($\mathcal{E}$). The time horizon ($\mathcal{T}$) consists of 5 time periods in days, which defines the range of shelf life ($\mathcal{S}$) from 1 to 5 days, and we assume zero inventory at the beginning of the planning horizon.

Recipe and demand data were collected directly from the food service provider and used to construct a box uncertainty set with a single cluster. Shelf life information was sourced from the Dutch Nutrition Centre (Stichting Voedingscentrum Nederland), while ingredient purchasing costs were determined based on the prices of similar ingredients sourced from Dutch wholesalers and retailers. To assess environmental impacts, data were sourced from the Agribalyse 3.01 database, using the (adapted) Environmental Foodprint (EF), version 3.0, methodology, adhering to the European Life Cycle Analysis calculation guidelines (AGRIBALYSE, 2022), with the functional unit standardised to 1 kg of product for all impact assessments. Data will be made available on request.

Case settings

Waste is caused by uncertain demand (see Proposition 1). Therefore, we developed several case settings based on the real-life case study to investigate strategies to reduce waste. To study the effects of allowing meal component repurposing, we developed two main case settings that adapt to the real-life case study: one with and one without the reuse of meal components. The settings are described in the following.

- **Status quo - no reuse (SQ-NR):** This case setting represents a common case setting in current food services, with restrictions on weekly and daily meal variety and limited ordering moments. Reuse of meal components is not allowed in this setting.
- **Status quo - reuse (SQ-R):** Building on the SQ-NR setting, this case setting allows for the reuse of meal components in the preparation of other meal components.

Relaxed case settings

Building on the SQ-R setting, we want to study the impact of more flexibility in terms of meal choices and order placements. To this end, we developed several specific relaxed case settings by altering parameters to evaluate the impact of these restrictions on the solution structure. The relaxed case settings are described in the following:

- **Status quo - reuse with no weekly variety restrictions (SQ-R-NWV):** In this case setting, the restriction on weekly meal variety is slightly relaxed, allowing meals to be served on consecutive days.
- **Status quo - reuse with no daily variety restrictions (SQ-R-NDV):** Here, the restriction on daily meal variety is removed, meaning no minimum or maximum number of meals needs to be served for any meal specification.
- **Status quo - reuse with no maximum order moments (SQ-R-NMO):** This case setting lifts the restriction on the maximum number of order moments, allowing the maximum number of orders to be equal to the entire time horizon length (5 days in this case).
- **Status quo - reuse with no restrictions (SQ-R-U):** Both variety and order constraints are lifted, allowing for unrestricted reuse of meal components.

The parameter settings of all case settings are provided in Table 2.

Table 2: Parameter settings of the case study

Parameter	SQ-NR	SQ-R	SQ-R-NWV	SQ-R-NDV	SQ-R-NMO	SQ-R-U
Weekly meal variety (τ)	1	1	2	1	1	2
Daily meal variety (z = non-vegetarian)	$\Psi_z = 1$	$\Psi_z = 1$	$\Psi_z = 1$	–	$\Psi_z = 1$	–
Maximum number of order time periods (Ξ)	3	3	3	3	5	5
Reuse (yes/no)	no	yes	yes	yes	yes	yes

7 Results

Using Julia, version 1.8.2-linux-x86_64, and the Gurobi solver, version 10.0.1-GCCcore-11.3.0, we obtain results for the different case settings, minimizing a single objective at a time for the economic cost (CO) objective, and environmental climate change (CC), land use (LU), resource use of fossils (RF), and water use (WU) objectives. When optimizing for the environmental objectives, alternative optimal solutions may arise, as only impacts from wasted products are accounted for, in which case we choose the best alternative solution in terms of the economic objective. We consistently apply 200 randomly generated demand scenarios across all case settings and average the results. This section presents the outcomes of the case settings developed from the real-life case study, where Section 7.1 presents the setting reflecting a standard scenario in food services, and Section 7.2 presents a setting that incorporates meal component reuse. Subsequently, Section 7.3 presents the results of the relaxed case settings in which meal variety and order restrictions have been relaxed.

7.1 Status quo - no reuse (SQ-NR)

Examining the differences in objective values for the status quo setting without reuse, Table 3 illustrates the percentage change in key performance indicators (KPIs) across various objectives, relative to the situation in which each respective KPI/objective is optimized. The differences between optimizing for economic objectives versus environmental objectives are significant. While cost optimization leads to substantially higher environmental impacts (181-658%), optimizing for environmental objectives results in more modest cost increases relative to the cost-optimized scenario (149-181%).

Table 3: Average values of key performance indicators in the SQ-NR case setting

Objective	Costs (%)	Climate change (%)	Land use (%)	Resource use of fossils (%)	Water use (%)
CO	-	556%	658%	181%	188%
CC	149%	-	118%	111%	117%
LU	181%	129%	-	152%	136%
RF	157%	134%	164%	-	130%
WU	155%	149%	161%	121%	-

Looking at general trends for all objectives in the no-reuse setting, Table 4 highlights that both waste and inventory are predominantly composed of meal components. Since orders can be placed relatively regularly (three times per working week), ingredients can be purchased when needed, reducing the need for storage and minimizing the risk of waste.

Focusing on the objective of cost optimization, we observe that purchasing quantities are minimized under this objective. The model prioritizes meals with low cost (such as yogurt with muesli, fruit-infused water or vegetables with a dip) thereby reducing purchasing expenses. As some meals weigh less than their alternatives (such as a flatbread compared to a club sandwich, or a rice roll compared to a couscous salad), and therefore require lower purchasing quantities and thus costs, serving quantities are lowest under this objective (301 kg). However, this strategy reduces meal variety, evidenced by the lowest number of unique meal types served per week among all optimization objectives (16 as opposed to 33-35 meals). Additionally, as some non-vegetarian meals tend to be cheaper than their alternatives in our dataset (such as hamburgers with fries, fish and chips, chicken stock, and hotdogs), their proportion is highest in the cost-optimized scenario (31%). This choice contributes significantly to the high environmental impacts observed under this objective.

Table 4: Average values of several indicators in the SQ-NR case setting

Objective	Purchased food wasted (%)	Meal component waste (%)	Meal component inventory (%)	Non-vegetarian production (%)	Unique meal types/week	Inventory quantity [kg]	Purchased quantity [kg]	Serving quantity [kg]
CO	35%	92%	52%	31%	16	924	461	301
CC	30%	85%	59%	11%	33	532	476	333
LU	33%	85%	63%	14%	33	565	500	335
RF	29%	89%	59%	16%	35	522	469	334
WU	29%	88%	57%	14%	34	524	472	334

Under environmental objectives, waste proportions are slightly lower (29-33%) than under the cost objective (35%), reducing overall waste impact. Moreover, the model prioritizes preventing meal component waste over ingredient waste, as the former generally has a higher environmental impact due to additional processing. Analyzing the types of meal components wasted most, we observe two primary patterns. First, as expected, components with relatively low environmental impact, (e.g., stock, soup, and fries), are more frequently wasted. Second, meal variety constraints can force the production of high-impact options, increasing the risk of waste. Another significant driver of waste is the finite planning horizon, which necessitates placing orders earlier in the week to minimize the risk of leftovers. As a result, inventory levels are lower in the earlier periods and tend to accumulate only in the final periods, leading to waste predominantly occurring at the end of the planning horizon.

Overall, when reuse is prohibited, the optimal economic menu plan prioritizes meals with the lowest purchasing costs. In contrast, the most environmentally friendly menu plan focuses on minimizing the environmental footprint of meal components while maintaining low inventories to reduce the risk of waste.

7.2 Status quo - reuse (SQ-R)

Investigating the effect of allowing the reuse of meal components, we observe that in the SQ-R case setting 8% of the meal components are reused under the economic objective and 11-16% under the environmental objectives, which, as shown in Table 5, improves all objectives with 4-19% compared to the case when reuse is prohibited. Furthermore, in many cases, the values of the non-optimized key performance indicators improve as well. For example, an 18% decrease in cost compared to the no-reuse setting is obtained when land use is optimized. These results show that reuse can simultaneously reduce waste and costs.

Table 5: Average values of the key performance indicators in the SQ-R case setting

Objective	Costs (%)	Climate change (%)	Land use (%)	Resource use of fossils (%)	Water use (%)
CO	-4%	0%	0%	+7%	-3%
CC	-3%	-16%	-14%	-18%	-18%
LU	-18%	-15%	-16%	-31%	-26%
RF	-1%	0%	0%	-19%	-16%
WU	+2%	-10%	-14%	-17%	-11%

Costs are improved with 4% under the cost objective because the purchasing costs of some reused meal components are cheaper than the ingredients they replace (such as snack vegetables compared to soup vegetables or fruit salad compared to fresh fruit). Looking at the results in more detail in Table 6, we observe that this resulted in a reduction in the number of unique meals by 8% and ingredients by 7%. Although this limited diversity of unique reused components reduces overall meal variety, it almost eliminates ingredient waste as leftover ingredients, stocked in larger quantities per item, can be reused in the same meals. Nevertheless, the increase in meal component waste offsets this benefit, resulting in no net reduction in overall waste quantities.

Table 6: Differences in average waste quantities and serving numbers in the SQ-R case setting compared to the SQ-NR case setting (%)

Objective	Waste (kg)	Ingredient waste (kg)	Ingredient inventory (kg)	Meal component waste (kg)	Unique meals	Unique ingredients
CO	0%	-98%	-14%	+9%	-8%	-7%
CC	-12%	+5%	-15%	-15%	+19%	+8%
LU	-23%	+3%	-5%	-27%	+12%	+13%
RF	-8%	+25%	-8%	-12%	+18%	+4%
WU	-8%	+55%	-16%	-17%	+14%	+6%

Under environmental objectives, however, waste is significantly reduced (8–23%), leading to greater improvements in objective values compared to cost minimization, ranging from 11-19%. Reuse reduces meal component waste in two ways. First, by repurposing components to meet the demands of other meal groups, thereby increasing serving potential. Second, by extending their shelf life when the remaining shelf life of the meal component is shorter than that of the meal component it can be incorporated into. For example, if tomato sauce is available in inventory with a remaining shelf life of one day, but it is reused to prepare chili con carne, which has a longer shelf life, waste can be avoided. This reduction in meal component waste offsets the increase in ingredient waste, which is much lower in quantity than meal component waste. Since the types of reused components vary every period, a broader range of meal components is reused, enhancing variety. Additionally, reuse often requires purchasing complementary ingredients to make a full meal component, further expanding the variety of ingredients used. Since meal components can be produced and stored for future use, this replaces ingredient stocking.

Another observation is that a higher proportion of meal components produced are reusable, i.e., comes from the set of 27 meal components that can be reused as ingredients for other meal components, ranging from 34–56% in the reuse case, compared to 27–44% in the status quo without reuse (SQ-NR). Note that the same set of meal components are available in the SQ-NR case setting, but reuse of these components is prohibited. A similar trend is seen in inventory, where 49–61% of the meal components in inventory is reusable under reuse, versus only 31–50% in the setting without reuse. This shows that meal components that do not necessarily have the best attributes in terms of costs or impact, become more advantageous to produce and store when they can be reused.

We also observe that, under environmental objectives, meal components with greater flexibility for reuse are the most beneficial. This includes components that can be repurposed into many different meals or those whose ingredients can be widely substituted with reused components. This trend is reflected in the number of unique reused meal components, which is significantly higher under the environmental objectives (11-15 meal components) compared to the economic objective (3 meal components). Similarly, the number of unique meal components incorporating reused ingredients is 18–21 meal components under environmental objectives, versus just 7 for the economic objective. In addition, meal components for which a large proportion of their ingredients can be replaced with reused meal components are particularly advantageous, environmentally speaking. For example, incorporating soups or vegetable salads into new soups, using vegetable salads or fries as substitutes for side dishes in full meals, and repurposing fruit-, milk-, or bread-based products into puddings.

7.3 Relaxed case settings

We evaluate the impact of restrictions on the optimal solutions under reuse by comparing the cases with relaxed restrictions to the restricted reuse case (SQ-R). We see in Table 7 that relaxing weekly variety (SQ-R-NWV) significantly reduces objective values, ranging from -10 to -46%, while relaxing the daily variety restriction (SQ-R-NDV), has a negligible impact (changing ≤ 1 %) compared to the SQ-R case setting. Comparing the case setting with a relaxed maximum order frequency restriction (SQ-R-NMO) to the SQ-R case setting, we also see that it does not impact the solution under the economic objective as the optimal number of orders remains three per week because each additional order increases costs due to fixed order costs. The environmental impacts, on the other hand, are reduced significantly (with 28-48%) because orders can be made every period, offering more flexibility in responding to the realized demand, and thus, waste can be prevented better, and ingredient waste eliminated. When all meal variety and order frequency restrictions are relaxed (SQ-R-U), substantial improvements in objective values occur compared to the SQ-R case setting, especially for environmental goals, with reductions of 78-85%, and cost reductions of 11%. Additionally, the proportion of purchased food that ends up as waste drops significantly to only 8-9 % under environmental objectives. The reuse proportion decreases to 9% and 11% because, with fewer restrictions, particularly more frequent orders, the need for reuse to manage demand uncertainty is reduced.

Table 7: Average objective values of the relaxed case settings compared to the SQ-R case setting optimized for that objective.

Relaxed case settings	CO	CC	LU	RF	WU
SQ-R-NWV	-10%	-14%	-46%	-28%	-39%
SQ-R-NDV	-1%	0%	-1%	0%	0%
SQ-R-NMO	-1%	-28%	-37%	-48%	-42%
SQ-R-U	-11%	-80%	-85%	-78%	-80%

8 Discussion

The results of our comprehensive analysis reveal that reusing leftover meal components provides significant economic and environmental advantages, simultaneously benefiting both businesses and the environment. From an economic perspective, we show, in this context, that reuse reduces costs when repurposed components are cheaper than the ingredients they substitute, aligning with Lee and Tongarlak (2017), who found that reusing leftovers in retail settings reduces purchasing costs. From an environmental perspective, we observe that greater flexibility in reusing meal components reduces meal component waste and thus the corresponding environmental impacts. This aligns with Engström and Carlsson-Kanyama (2004), who found that fixed menus restrict opportunities for repurposing leftovers. Moreover, higher levels of flexibility can encourage the use of a diverse range of unique meal components, thus challenging the notion that waste reduction compromises variety, and contrasts with recommendations from Stichting tegen Voedselverspilling (2024), a Dutch non-profit organization dedicated to reducing food waste. To further decrease environmental impacts, the production of low-impact meals, such as vegetarian or plant-based options, should be prioritized, as also indicated by van Rooijen et al. (2024).

Investigating meal variety specifications in more detail, our analysis also finds that variety imposed on a daily basis, like at least one meat-based meal per day, has a more limited effect on costs and environmental impact than enforcing variety throughout the week, such as preventing the same meal from being served two days in a row. Thus, if food services want to increase meal variety, we recommend enforcing this on a daily rather than a weekly basis. Moreover, as not every consumer dines at a food service every day of the week, and reuse can increase variety, we argue that waste minimization does not have to impact the dining experience negatively.

Investigating different order policies, we observe moreover that increasing order frequency is another key strategy for reducing waste impact, as this allows for more flexibility in responding to demand fluctuations. Food service providers could, thus, explore options for more frequent deliveries from suppliers or consider sourcing from nearby supermarkets, requiring collaboration throughout the chain, as discussed by Yetkin Özbük and Coşkun (2020). If frequent deliveries are not feasible, placing orders early in the planning horizon reduces stocking requirements and increases opportunities for (re)using leftovers.

Overall, the findings from our research highlight that it is important that food service providers invest in repurposing strategies, introduce recipe alterations that allow for substitutability, and train staff to implement these changes effectively. To allow for more flexibility in using leftovers, we further recommend describing meals on menus without specifying ingredients. Additionally, to reuse meals safely, we advise reducing displayed portions by increasing restocking frequency and adopting front cooking, because food safety regulations mandate the disposal of presented meal components at the end of the day. Safe reuse also requires swift cooling, appropriate storage, and rapid reheating. Nevertheless, more supportive policy guidance on safe leftover handling can help food services in adopting reuse strategies, and policymakers can further support waste reduction by integrating reuse strategies into public catering and offering training programs to encourage widespread adoption.

To increase reuse potential, future research should explore optimal display strategies based on demand forecasts that satisfy consumer demands while minimizing waste. Additionally, future studies could investigate how flexible menu planning could be integrated into smart systems, such as smart fridges and waste-measuring bins, enabling food services to use our insights in practice.

9 Conclusions

Using a data-driven multi-stage robust optimization model to design sustainable menu plans, this study investigates the potential of reuse in food services from economic and environmental perspectives. A key novelty in this work is the integration of flexible recipes that allow for ingredient substitution, which helps respond to demand uncertainty and contributes to the existing literature on menu planning models by explicitly incorporating repurposing strategies. Considering various relevant case settings, we validate our model's applicability through a real-life case study conducted at a food service provider. The findings from our study offer valuable managerial insights, demonstrating that reuse of leftovers can significantly reduce both costs and environmental impacts. Furthermore, our results highlight that increased reuse flexibility and purchasing frequency improve adaptability to demand fluctuations, without compromising meal variety. In addition, we find that the prevention of meal component over ingredient waste is more effective, as the former typically has a greater impact due to additional processing.

From a methodological perspective, we contribute to the existing literature by proving that uncertain demand is a key driver of food waste caused by overproduction, and that accounting for this variability in menu planning is essential. Furthermore, we improve computational efficiency while maintaining solution quality by showing that optimal solutions can be obtained by considering only a limited set of demand scenarios. Our theorems further demonstrate that the number of scenarios required for optimization depends on the interplay between problem feasibility and worst-case coverage. In minimization problems where feasibility and the objective function are negatively correlated with each other as the uncertain parameter changes, a single extreme scenario may suffice to ensure both feasibility and worst-case coverage, whereas for problems where feasibility and the objective function are positively correlated upon changes in the uncertain parameter, we may require the two extreme scenarios.

The impact of our methodological insights extends beyond menu planning to areas like inventory control, disaster response logistics, and energy grid management, where uncertainty impacts decisions and objectives in a similar fashion. Similarly, our managerial insights extend to other industries that produce products from multiple components. Repurposing strategies, central to our research, offer valuable pathways for reducing waste and enhancing resource efficiency in these sectors. Especially with the rising interest in circular economies, our research holds potential for industries where components might be replaced by other, perhaps even recycled or reused components.

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References

Agrawal, V. V., Atasu, A., and Van Wassenhove, L. N. (2019). OM Forum—New Opportunities for Operations Management Research in Sustainability. *Manufacturing & Service Operations Management*, 21(1):1–12.

Aka, S. and Akyüz, G. (2021). An inventory and production model with fuzzy parameters for the food sector. *Sustainable Production and Consumption*, 26:627–637.

Akkaş, A. and Gaur, V. (2022). OM Forum—Reducing Food Waste: An Operations Management Research Agenda. *Manufacturing & Service Operations Management*, 24(3):1261–1275.

Ben-Tal, A., Golany, B., Nemirovski, A., and Vial, J.-P. (2005). Retailer-supplier flexible commitments contracts: A robust optimization approach. *Manufacturing and Service Operations Management*, 7(3):248 – 271.

Ben-Tal, A., Goryashko, A., Guslitzer, E., and Nemirovski, A. (2004). Adjustable robust solutions of uncertain linear programs. *Mathematical programming*, 99(2):351–376.

Ben-Tal, A. and Nemirovski, A. (2000). Robust solutions of linear programming problems contaminated with uncertain data. *Mathematical programming*, 88:411–424.

Bertsimas, D. and Caramanis, C. (2010). Finite adaptability in multistage linear optimization. *IEEE Transactions on Automatic Control*, 55(12):2751–2766.

Bertsimas, D., Gupta, V., and Kallus, N. (2018). Data-driven robust optimization. *Mathematical Programming*, 167:235–292.

Bertsimas, D. and Thiele, A. (2006). *Robust and Data-Driven Optimization: Modern Decision Making Under Uncertainty*, chapter 4, pages 95–122.

Bertsmas, D. and den Hertog, D. (2022). *Robust and Adaptive Optimization*. Dynamic Ideas LLC.

Betz, A., Buchli, J., Göbel, C., and Müller, C. (2015). Food waste in the swiss food service industry – magnitude and potential for reduction. *Waste Management*, 35:218–226.

Borumand, A., Marandi, A., Nookabadi, A. S., and Atan, Z. (2024). An oracle-based algorithm for robust planning of production routing problems in closed-loop supply chains of beverage glass bottles. *Omega*, 122:102939.

Buisman, M. E., Haijema, R., Akkerman, R., and Bloemhof, J. M. (2019). Donation management for menu planning at soup kitchens. *European Journal of Operational Research*, 272(1):324–338.

Crama, Y., Pascual J, R., and Torres, A. (2004). Optimal procurement decisions in the presence of total quantity discounts and alternative product recipes. *European Journal of Operational Research*, 159(2):364–378.

Dang, D.-C., Currie, C. S., Onggo, B. S., Chaerani, D., and Achmad, A. L. H. (2023). Budget allocation of food procurement for natural disaster response. *European Journal of Operational Research*, 311(2):754 – 768.

de Moor, D., Wagenaar, J., Poos, R., den Hertog, D., and Fleuren, H. (2024). A robust approach to food aid supply chains. *European Journal of Operational Research*, 318(1):269 – 285.

Engström, R. and Carlsson-Kanyama, A. (2004). Food losses in food service institutions examples from sweden. *Food Policy*, 29(3):203–213.

Eriksson, M., Osowski, C. P., Malefors, C., Björkman, J., and Eriksson, E. (2017). Quantification of food waste in public catering services - a case study from a swedish municipality. *Waste management*, 61:415–422.

Eustachio Colombo, P., Patterson, E., Lindroos, A. K., Parlesak, A., and Elinder, L. S. (2020). Sustainable and acceptable school meals through optimization analysis: an intervention study. *Nutrition journal*, 19:1–15.

Eustachio Colombo, P., Patterson, E., Schäfer Elinder, L., Lindroos, A. K., Sonesson, U., Darmon, N., and Parlesak, A. (2019). Optimizing school food supply: integrating environmental, health, economic, and cultural dimensions of diet sustainability with linear programming. *International Journal of Environmental Research and Public Health*, 16(17):3019.

Feng, W., Zhang, Y., Rong, G., and Feng, Y. (2019). Finite Adaptability in Data-Driven Robust Optimization for Production Scheduling: A Case Study of the Ethylene Plant. *Industrial and Engineering Chemistry Research*, 58(16):6505–6518.

Filimonau, V., Todorova, E., Mzembe, A., Sauer, L., and Yankholmes, A. (2020). A comparative study of food waste management in full service restaurants of the United Kingdom and the Netherlands. *Journal of Cleaner Production*, 258:120775.

Fliedner, T. and Liesiö, J. (2016). Adjustable robustness for multi-attribute project portfolio selection. *European Journal of Operational Research*, 252(3):931 – 946.

Goerigk, M. and Khosravi, M. (2023). Optimal scenario reduction for one-and two-stage robust optimization with discrete uncertainty in the objective. *European Journal of Operational Research*, 310(2):529–551.

Gonçalves, C., Saraiva, S., Nunes, F. M., and Saraiva, C. (2023). Food waste in public food service sector—surplus and leftovers. *Resources*.

Hooshmand, F., Seilsepour, F., and MirHassani, S. (2025). Adjustable robust optimization approach for svm under uncertainty. *Omega*, 131.

Jaillet, P., Qi, J., and Sim, M. (2016). Routing optimization under uncertainty. *Operations Research*, 64(1):186 – 200.

Jiang, R., Zhang, M., Li, G., and Guan, Y. (2014). Two-stage network constrained robust unit commitment problem. *European Journal of Operational Research*, 234(3):751 – 762.

Kanellopoulos, A., Gerdessen, J. C., Ivancic, A., Geleijnse, J. M., Bloemhof-Ruwaard, J. M., and van't Veer, P. (2020). Designing healthier and acceptable diets using data envelopment analysis. *Public Health Nutrition*, 23(13):2290–2302.

Kang, Z., Marandi, A., Basten, R. J., and de Kok, T. (2023). Robust spare parts inventory management. Available at SSRN.

Kang, Z., Marandi, A., Basten, R. J., Stip, J., and de Kok, T. (2024). Robust spare parts inventory management with emergency shipments. Available at SSRN.

Klerx, J. (2023). Horeca verspilt 5,6 miljoen kilogram minder voedsel - maar nog altijd voor bijna 650 miljoen euro (The food service industry wastes 5.6 million kilograms less food - but still worth almost 650 million euros). Retrieved from `https://www.rabobank.nl/kennis/d011381929-horeca-verspilt-5-6-miljoen-kilogram-minder-voedsel-maar-nog-altijd-voor-bijna-650-miljoen-euro`.

Lancaster, L. M. (1992). The history of the application of mathematical programming to menu planning. *European Journal of Operational Research*, 57(3):339–347.

Lappas, N. H. and Gounaris, C. E. (2016). Multi-stage adjustable robust optimization for process scheduling under uncertainty. *AIChE Journal*, 62(5):1646–1667.

Lee, D. (2012). Turning Waste into By-Product. *Manufacturing & Service Operations Management*, 14(1):115–127.

Lee, D. and Tongarlak, M. H. (2017). Converting retail food waste into by-product. *European Journal of Operational Research*, 257(3):944–956.

Lei, C., Lin, W.-H., and Miao, L. (2016). A two-stage robust optimization approach for the mobile facility fleet sizing and routing problem under uncertainty. *Computers and Operations Research*, 67:75 – 89.

Lins, M., Zandonadi, R. P., Raposo, A., and Ginani, V. (2021). Food Waste on Foodservice: An Overview through the Perspective of Sustainable Dimensions. *Foods*, 10.

Long, X. and Gui, L. (2024). Waste Not Want Not? The Environmental Implications of Quick Response and Upcycling. *Manufacturing & Service Operations Management*, 26(2):612–631.

Marandi, A. and van Houtum, G.-J. (2020). Robust location-transportation problems with integer-valued demand. Available at Optimization Online.

Moreira, R. P. C., Marcelino, C. G., Martins, F. V. C., Wanner, E. F., Jimenez-Fernandez, S., and Salcedo-Sanz, S. (2022). A Multi-objective Approach for the Menu Planning Problem: A Brazilian Case Study. In Pereira, A. I., Košir, A., Fernandes, F. P., Pacheco, M. F., Teixeira, J. P., and Lopes, R. P., editors, *Optimization, Learning Algorithms and Applications*, pages 275–290, Cham. Springer International Publishing.

Musa Aamir, Huzaifa Ahmad, Q. J. and Hasan, S. M. (2018). Waste Not, Want Not: A Case Study on Food Waste in Restaurants of Lahore, Pakistan. *Journal of Food Products Marketing*, 24(5):591–610.

Neofytou, A., Liu, B., and Akartunalı, K. (2025). Compact uncertainty sets for robust optimization based on bootstrapped dirichlet process mixture model. *Optimization*, pages 1–44.

Papargyropoulou, E., Lozano, R., K. Steinberger, J., Wright, N., and bin Ujang, Z. (2014). The food waste hierarchy as a framework for the management of food surplus and food waste. *Journal of Cleaner Production*, 76:106–115.

Reynolds, D. and Taylor, J. (2011). Validating a DEA-based menu analysis model using structural equation modeling. *International Journal of Hospitality Management*, 30(3):584–587.

Santos, M. C., Agra, A., and Poss, M. (2020). Robust inventory theory with perishable products. *Annals of Operations Research*, 289(2):473–494.

Sel, , Pınarbaşı, M., Soysal, M., and Çimen, M. (2017). A green model for the catering industry under demand uncertainty. *Journal of Cleaner Production*, 167:459–472.

Stern, A. L., Levine, S., Richardson, S. A., Blackstone, N. T., Economos, C., and Griffin, T. S. (2023). Improving school lunch menus with multi-objective optimisation: nutrition, cost, consumption and environmental impacts. *Public Health Nutrition*, 26(8):1715–1727.

Stichting tegen Voedselverspilling (2024). Leaflet Verspillingsvrije Catering (Leaflet food waste free food services). Retrieved from `https://samentegenvoedselverspilling.nl/kennisbank/kb-tips-voor-horecaondernemers`.

Suyal, M. and Sharma, S. (2024). A review on analysis of k-means clustering machine learning algorithm based on unsupervised learning. *Journal of Artificial Intelligence and Systems*, 6:8–95.

Swink, M., Hu, K., and Zhao, X. (2022). Analytics applications, limitations, and opportunities in restaurant supply chains. *Production and Operations Management*, 31(10):3710–3726.

Tomaszewska, M., Bilska, B., and Kołożyn-Krajewska, D. (2021). Food waste in catering establishments – an analysis of causes and consequences. *European Journal of Sustainable Development*, 10:365–365.

van Beek, S. (2023). Horeca verspilt 5,6 miljoen minder voedsel (The food service industry wastes 5.6 million less food). Retrieved from `https://www.foodclicks.nl/nieuws/horeca-verspilt-56-miljoen-minder-voedsel/`.

van Dooren, C. (2018). A review of the use of linear programming to optimize diets, nutritiously, economically and environmentally. *Frontiers in Nutrition*, 5.

Van Mierlo, K., Rohmer, S., and Gerdessen, J. C. (2017). A model for composing meat replacers: Reducing the environmental impact of our food consumption pattern while retaining its nutritional value. *Journal of Cleaner Production*, 165:930–950.

van Rooijen, M., Gerdessen, J., Claassen, G., and de Leeuw, S. (2024). Optimizing household food waste: The impact of meal planning, package sizes, and performance indicators. *Resources, Conservation and Recycling*, 205:107559.

van Wonderen, D., Melse-Boonstra, A., and Gerdessen, J. C. (2023). Iron bioavailability should be considered when modeling omnivorous, vegetarian, and vegan diets. *The Journal of Nutrition*, 153(7):2125–2132.

Vasilakakis, K. and Giannikos, I. (2023). A robust optimization model for determining optimal diets for food and beverage (f & b) items. *International Journal of Supply and Operations Management*, 10(4):485 – 500.

Wang, I., Becker, C., Van Parys, B., and Stellato, B. (2024). Mean robust optimization. *Mathematical Programming*.

Wang, S. and Ng, T. S. (2019). Robustness of resource recovery systems under feedstock uncertainty. *Production and Operations Management*, 28(3):628–649.

Wang, Y., Zhang, Y., Zhou, M., and Tang, J. (2023). Feature-driven robust surgery scheduling. *Production and Operations Management*, 32(6):1921–1938.

Xiong, J., Wang, S., and Ng, T. S. (2021). Robust bilevel resource recovery planning. *Production and Operations Management*, 30(9):2962–2992.

Yanıkoğlu, I., Gorissen, B. L., and den Hertog, D. (2019). A survey of adjustable robust optimization. *European Journal of Operational Research*, 277(3):799–813.

Yazdani, A., Marandi, A., Basten, R. J., and de Kok, T. (2023). A clustering-based uncertainty set for robust optimization. Available at Optimization Online.

Yetkin Özbük, R. M. and Coşkun, A. (2020). Factors affecting food waste at the downstream entities of the supply chain: A critical review. *Journal of Cleaner Production*, 244:118628.

Zhang, C., Wang, Z., and Wang, X. (2022). Machine learning-based data-driven robust optimization approach under uncertainty. *Journal of Process Control*, 115:1–11.

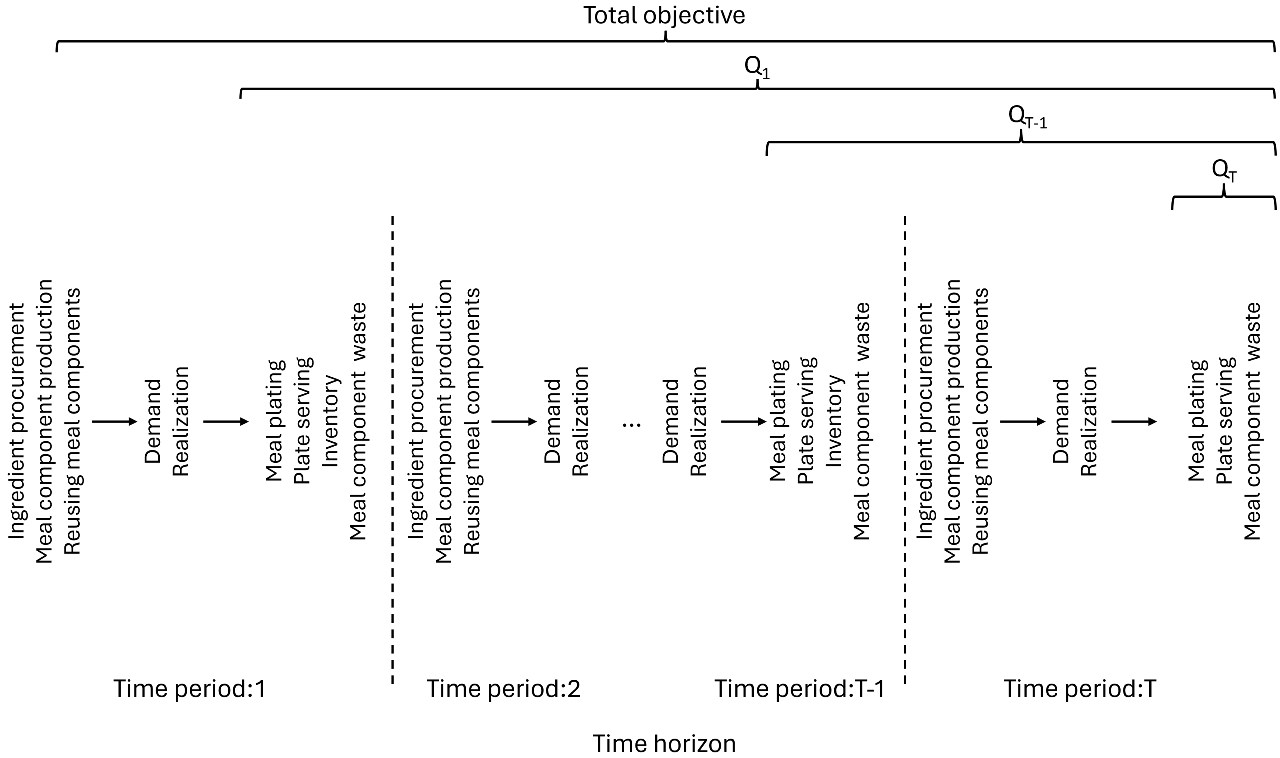
Zugno, M., Morales, J. M., and Madsen, H. (2016). Commitment and dispatch of heat and power units via affinely adjustable robust optimization. *Computers and Operations Research*, 75:191 – 201.

Appendices

This paper includes 3 appendices. A presents the adjustable robust formulation. Appendix B provides the proofs for the theorems in Section 5.1. Appendix C discusses the broader applicability of the theorems introduced in Section 5.1.

A Adjustable robust formulation

Figure 5: Events occurrence schema



This appendix provides the multi-stage adjustable robust formulation, with Figure 5 presenting the sequence of events. Let us consider either the economic (1a) or environmental objective function (1b). Let x be the collection of all here-and-now decision variables, i.e., $x = (B_{1ir}, V_{1isr}, I_{1rs}, X_{1rs}, H_1)$, y_t be the collection of all wait-and-see variables made at period t, i.e.,

$$y_t = (W_{tr}, N_{trsp}, K_{tpr}, J_{tp}, O_{tpz}, Z_{tpz}, A_{tpg}, B_{t+1,i,r}, V_{t+1,i,sr}, I_{t+1,r,s}, X_{t+1,r,s}, H_{t+1})$$

and $y_{[t]}$ be the collection of all wait-and-see variables made until the beginning of period t, i.e.,

$$y_{[t]} = (W_{\tilde{t}r}, N_{\tilde{t}rsp}, K_{\tilde{t}pr}, J_{\tilde{t}p}, O_{\tilde{t}pz}, Z_{\tilde{t}pz}, A_{\tilde{t}pg}, B_{\tilde{t}+1,i,r}, V_{\tilde{t}+1,i,sr}, I_{\tilde{t}+1,r,s}, X_{\tilde{t}+1,r,s}, H_{\tilde{t}+1})_{\tilde{t}=1,\ldots,t}.$$

Let $y_{[0]}$ be the vector of all zeros.

We denote by $P_0(x)$ the part of the objective function depending only on x and by $P_t(y_t)$ the part of the objective function dependent on $y_t, t = 1, \dots, T$. Given this notation, we can write the ARO problem as

$$
\begin{aligned}
Q_0 \quad &= \min_{x} \quad P_0(x) + \max_{\delta_{[1]} \in D_1} Q_1(x, y_{[0]}, \delta_{[1]}) \\
&\text{s.t.} \quad \sum_{i\in\mathcal{E}} \zeta_{rci} B_{1ir} + \sum_{i\in\mathcal{R}} \sum_{s\in\mathcal{S}} \zeta_{rci} V_{1isr} - \rho_{cr} \sum_{s\in\mathcal{S}} \theta_{rs} X_{1rs} = 0, && \forall r \in \mathcal{R}, c \in \mathcal{C}_r, \\
&\qquad \sum_{i\in\mathcal{E}} \sum_{r\in\mathcal{R}} B_{1ir} - MH_1 \leq 0, \\
&\qquad \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \theta_{rs} X_{1rs} \leq \Theta, \\
&\qquad I_{1rs} = \Lambda_{rs}, && \forall r \in \mathcal{R}, s \in \mathcal{S}, \\
&\qquad V_{1isi} = 0, && \forall i \in \mathcal{E}, s \in \mathcal{S}, \\
&\qquad \theta_{rs} V_{1isr} = 0, && \forall i \in \mathcal{R}, r \in \mathcal{R}, s \in \mathcal{S},
\end{aligned}
$$

where, for any $t \in \{1, 2, \dots, T-1\}$, we have

$$
\begin{aligned}
&Q_t(x, y_{[t-1]}, \delta_{[t]}) = \\
&\min_{y_t} \quad P_t(y_t) + \max_{\delta_{t+1} \in D_t} Q_{t+1}(x, y_{[t]}, \delta_{[t+1]}) \\
&\text{s.t.} \quad \sum_{p \in \mathcal{P}} \sum_{l \in \mathcal{L}_p} \alpha_{rlp} N_{trsp}(\delta_{[t]}) + I_{t+1,r,s-1}(\delta_{[t]}) = \\
&\qquad\qquad \theta_{rs} X_{trs}(\delta_{[t]}) + I_{trs}(\delta_{[t]}) - \sum_{r' \in \mathcal{R}} \sum_{c \in \mathcal{C}_{r'}} \zeta_{r'cr} V_{t,r,s,r'}(\delta_{[t]}), && \forall r \in \mathcal{R}, s \in \{2, \ldots, S\}, \\
&\quad \sum_{p \in \mathcal{P}} \sum_{l \in \mathcal{L}_p} \alpha_{rlp} N_{trsp}(\delta_{[t]}) + W_{tr}(\delta_{[t]}) = \\
&\qquad\qquad \theta_{rs} X_{trs}(\delta_{[t]}) + I_{trs}(\delta_{[t]}) - \sum_{r' \in \mathcal{R}} \sum_{c \in \mathcal{C}_{r'}} \zeta_{r'cr} V_{trsr'}(\delta_{[t]}), && \forall r \in \mathcal{R}, s = 1, \\
&\quad I_{trS}(\delta_{[t]}) = 0, && \forall r \in \mathcal{R}, \\
&\quad V_{t+1,i,s,i}(\delta_{[t]}) = 0, && \forall i \in \mathcal{E}, s \in \mathcal{S}, \\
&\quad \theta_{rs} V_{t+1,i,s,r}(\delta_{[t]}) = 0, && \forall i \in \mathcal{R}, r \in \mathcal{R}, s \in \mathcal{S}, \\
&\quad \sum_{r \in \mathcal{R}} \sum_{s \in \mathcal{S}} \alpha_{rlp} N_{trsp}(\delta_{[t]}) - \sigma_{lp} \sum_{g \in \mathcal{G}} \beta_{pg} A_{tpg}(\delta_{[t]}) = 0, && \forall p \in \mathcal{P}, l \in \mathcal{L}_p, \\
&\quad \sum_{p \in \mathcal{P}} \beta_{pg} A_{tpg}(\delta_{[t]}) = \delta_{tg}, && \forall g \in \mathcal{G}, \\
&\quad K_{tpr}(\delta_{[t]}) \leq 1 - \sum_{\substack{t' \in \mathcal{T}, \\ t-\tau+1 \leq t' \leq t}} K_{t'pr}(\delta_{[t]}), && \text{if } t \geq \tau, \\
&\quad \frac{1}{M} K_{tpr}(\delta_{[t]}) \leq \sum_{l \in \mathcal{L}_p} \sum_{s \in \mathcal{S}} \alpha_{rlp} N_{trsp}(\delta_{[t]}) \leq M K_{tpr}(\delta_{[t]}), && \forall r \in \mathcal{R}, p \in \mathcal{P}, \\
&\quad \Psi_z \leq \sum_{p \in \mathcal{P}} O_{tpz}(\delta_{[t]}) \leq \Phi_z, && \forall z \in \mathcal{Z}, \\
&\quad M J_{tp}(\delta_{[t]}) \geq \sum_{r \in \mathcal{R}} K_{tpr}(\delta_{[t]}) \geq J_{tp}(\delta_{[t]}), && \forall p \in \mathcal{P}, \\
&\quad M(1 - Z_{tpz}(\delta_{[t]})) \geq \sum_{r \in \mathcal{R}} (K_{tpr}(\delta_{[t]}) - \phi_{rz} K_{tpr}(\delta_{[t]})) \geq 1 - Z_{tpz}(\delta_{[t]}), && \forall p \in \mathcal{P}, z \in \mathcal{Z}, \\
&\quad \frac{2}{3} O_{tpz}(\delta_{[t]}) \leq \frac{1}{2} (Z_{tpz}(\delta_{[t]}) + J_{tp}(\delta_{[t]})) \leq \frac{2}{3} + O_{tpz}(\delta_{[t]}), && \forall p \in \mathcal{P}, z \in \mathcal{Z}, \\
&\quad H_{t+1}(\delta_{[t]}) \leq \Xi - \sum_{\tilde{t}=1}^{t} H_{\tilde{t}}(\delta_{[t]}), \\
&\quad \sum_{i \in \mathcal{E}} \zeta_{rci} B_{t+1,i,r}(\delta_{[t]}) + \sum_{i \in \mathcal{R}} \sum_{s \in \mathcal{S}} \zeta_{rci} V_{t+1,i,s,r}(\delta_{[t]}) - \rho_{cr} \sum_{s \in \mathcal{S}} \theta_{rs} X_{t+1,r,s}(\delta_{[t]}) = 0, && \forall r \in \mathcal{R}, c \in \mathcal{C}_r, \\
&\quad \sum_{i \in \mathcal{E}} \sum_{r \in \mathcal{R}} B_{t+1,i,r}(\delta_{[t]}) \leq M H_{t+1}(\delta_{[t]}), \\
&\quad \sum_{r \in \mathcal{R}} \sum_{s \in \mathcal{S}} \theta_{rs} X_{t+1,r,s}(\delta_{[t]}) \leq \Theta,
\end{aligned}
$$

and

$$
\begin{aligned}
&Q_T(x, y_{[T-1]}, \delta_{[T]}) = \\
&\quad \min_{y_T} \quad P(y_T) \\
&\quad \text{s.t.} \quad \sum_{s\in\mathcal{S}}(\theta_{rs}X_{Trs}(\delta_{[T]}) + I_{Trs}(\delta_{[T]})) - \sum_{r'\in\mathcal{R}}\sum_{c\in\mathcal{C}_{r'}}\zeta_{r'cr}V_{T,r,s,r'}(\delta_{[T]}) = \\
&\qquad\qquad \sum_{s\in\mathcal{S}}\sum_{p\in\mathcal{P}}\sum_{l\in\mathcal{L}_p}\alpha_{rlp}N_{Trsp}(\delta_{[T]}) + W_{Tr}(\delta_{[T]}), && \forall r\in\mathcal{R}, \\
&\qquad I_{TrS}(\delta_{[T]}) = 0, && \forall r\in\mathcal{R}, \\
&\qquad \sum_{r\in\mathcal{R}}\sum_{s\in\mathcal{S}}\alpha_{rlp}N_{Trsp}(\delta_{[T]}) - \sigma_{lp}\sum_{g\in\mathcal{G}}\beta_{pg}A_{Tpg}(\delta_{[T]}) = 0, && \forall p\in\mathcal{P}, l\in\mathcal{L}_p, \\
&\qquad \sum_{p\in\mathcal{P}}\beta_{pg}A_{Tpg}(\delta_{[T]}) = \delta_{Tg}, && \forall g\in\mathcal{G}, \\
&\qquad K_{t'pr}(\delta_{[T]}) \le 1 - \sum_{\substack{t'\in\mathcal{T}, \\ T-\tau+1\le t'\le T}} K_{t'pr}(\delta_{[T]}), && \text{if } T\ge\tau, \\
&\qquad \frac{1}{M}K_{tpr}(\delta_{[T]}) \le \sum_{l\in\mathcal{L}_p}\sum_{s\in\mathcal{S}}\alpha_{rlp}N_{Trsp}(\delta_{[T]}) \le MK_{Tpr}(\delta_{[T]}), && \forall r\in\mathcal{R}, p\in\mathcal{P}, \\
&\qquad \Psi_z \le \sum_{p\in\mathcal{P}}O_{Tpz}(\delta_{[T]}) \le \Phi_z, && \forall z\in\mathcal{Z}, \\
&\qquad MJ_{Tp}(\delta_{[T]}) \ge \sum_{r\in\mathcal{R}}K_{Tpr}(\delta_{[T]}) \ge J_{Tp}(\delta_{[T]}), && \forall p\in\mathcal{P}, \\
&\qquad M(1 - Z_{Tpz}(\delta_{[T]})) \ge \sum_{r\in\mathcal{R}}(K_{Tpr}(\delta_{[T]}) - \phi_{rz}K_{Tpr}(\delta_{[T]})) \ge 1 - Z_{Tpz}(\delta_{[T]}), && \forall p\in\mathcal{P}, z\in\mathcal{Z}, \\
&\qquad \frac{2}{3}O_{Tpz}(\delta_{[T]}) \le \frac{1}{2}(Z_{Tpz}(\delta_{[T]}) + J_{Tp}(\delta_{[T]})) \le \frac{2}{3} + O_{Tpz}(\delta_{[T]}), && \forall p\in\mathcal{P}, z\in\mathcal{Z}, \\
&\qquad H_T(\delta_{[T]}) \le \Xi - \sum_{\tilde{t}=1}^{T-1}H_{\tilde{t}}(\delta_{[T]}).
\end{aligned}
$$

B Methodological analysis of exact formulation

Proof of Lemma 1. Aggregating (1c) gives

$$\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{E}}B_{ir} = \sum_{r\in\mathcal{R}}\sum_{s\in\mathcal{S}}X_{rs}, \tag{8}$$

and for any $\delta \in D$, (1h) can be simplified to

$$\sum_{r\in\mathcal{R}}\sum_{s\in\mathcal{S}}X_{rs} = \sum_{p\in\mathcal{P}}\sum_{r\in\mathcal{R}}\sum_{s\in\mathcal{S}}N_{rsp}(\delta) + \sum_{r\in\mathcal{R}}W_r(\delta). \tag{9}$$

Combining (8) and (9), we can conclude that for any $\delta \in D$,

$$\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{E}}B_{ir} = \sum_{p\in\mathcal{P}}\sum_{r\in\mathcal{R}}\sum_{s\in\mathcal{S}}N_{rsp}(\delta) + \sum_{r\in\mathcal{R}}W_r(\delta). \tag{10}$$

Furthermore, for any $\delta \in D$, (1m) can be aggregated and simplified to

$$\sum_{p\in\mathcal{P}}\sum_{r\in\mathcal{R}}\sum_{s\in\mathcal{S}}N_{rsp}(\delta) = \sum_{p\in\mathcal{P}}\sum_{g\in\mathcal{G}}A_{pg}(\delta) \tag{11}$$

and (1n) to

$$\sum_{g\in\mathcal{G}}\delta_g = \sum_{g\in\mathcal{G}}\sum_{p\in\mathcal{P}}A_{pg}(\delta). \tag{12}$$

Combining (11) and (12), we can conclude that

$$\sum_{p\in\mathcal{P}}\sum_{r\in\mathcal{R}}\sum_{s\in\mathcal{S}}N_{rsp}(\delta) = \sum_{g\in\mathcal{G}}\delta_g. \tag{13}$$

Combining (10) and (13), we can conclude that for any $\delta \in D$,

$$\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{E}} B_{ir} = \sum_{p\in\mathcal{P}} \delta_g + \sum_{r\in\mathcal{R}} W_r(\delta). \tag{14}$$

The non-negativity of $W_r(\delta)$ implies that

$$\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{E}} B_{ir} \geq \sum_{g\in\mathcal{G}} \delta_g \quad \forall \delta \in D. \tag{15}$$

Since the inner problem $f(x,\delta)$ is feasible, (3) holds. □

Proof of Theorem 1. For the economic objective, the inner problem $f(x,\delta)$ is a feasibility problem because the objective function does not depend on the wait-and-see variables. For a given x, if there is a demand $\delta \in D$, such that the inner problem is infeasible, then $f(x,\delta) = +\infty$. Therefore, $g(x) = +\infty$. Since we minimize the economic objective, such x cannot be optimal. Therefore, for any optimal x, the inner problem $f(x,\delta)$ is feasible for any $\delta \in D$. From the proof of Lemma 1, we know

$$\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{E}} B_{ir} \geq \sum_{g\in\mathcal{G}} \delta_g \quad \forall \delta \in D.$$

Since the economic objective minimizes the ordering cost, and higher demand results in a higher economic objective, the worst-case occurs at δ^U. □

Proof of Proposition 1. With the economic objective, ordering costs are minimized. Hence, purchasing quantities are minimized. By contradiction, if the waste is not zero, similar to the proof of Lemma 1, the purchasing quantities are strictly more than the aggregated demand. Therefore, we can reduce the waste by ordering less, so that the aggregated optimal ordering quantities equal the aggregated demand, resulting in zero waste. □

Proof of Theorem 2. Since x is feasible, we know from Lemma 1 that

$$\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{E}} B_{ir} \geq \sum_{p\in\mathcal{P}} \delta_g^U. \tag{16}$$

Therefore, for any $\delta \in D$, we have

$$\sum_{p\in\mathcal{P}} \delta_g + \sum_{r\in\mathcal{R}} W_r(\delta) \geq \sum_{p\in\mathcal{P}} \delta_g^U,$$

implying that

$$\sum_{r\in\mathcal{R}} W_r(\delta) \geq \sum_{p\in\mathcal{P}} \left(\delta_g^U - \delta_g\right).$$

Hence, for any $\delta \in D$, the optimal total waste equals $\sum_{p\in\mathcal{P}} \left(\delta_g^U - \delta_g\right)$, and is thus decreasing in δ. Therefore, its maximum is obtained at δ^L. Thus, we can reduce the uncertainty set to scenarios where (16) holds and contains δ^L, and $\bar{D}$ is one of such sets.

□

C Generalizability of the theoretical results

Our analysis reveals that when minimizing the environmental impact of waste, the optimal solution requires considering only two scenarios, whereas, for cost minimization, a single scenario suffices. The intuition behind these theorems is that the solutions obtained for the selected scenarios should both be feasible for all scenarios and contain the worst-case scenario among all scenarios, and the number of required scenarios depends on how the uncertain parameter interacts with the decision variables and the objective function.

For minimization problems where feasibility is negatively correlated with the objective function upon changes in the uncertain parameter, meaning that an increase in the uncertain parameter decreases the number of feasible decisions but increases the objective value, or a decrease in the uncertain parameter increases feasibility while decreasing the objective value, a single extreme scenario ensures worst-case coverage. For example, consider an optimization problem of minimizing transportation distances under uncertain customer numbers. In this problem, an increase in customers decreases feasibility of serving them but increases total distances, thereby increasing the objective function. Likewise, in a lost sales minimization problem with minimum production constraints affected by uncertain employee sickness, an increase in employee sickness decreases the number of feasible solutions for meeting minimum production requirements but increases the number of lost sales.

On the other hand, for minimization problems where feasibility is positively correlated with the objective upon changes in the uncertain parameter, where an increase of the uncertain parameter decreases the number of feasible decisions but also decreases the objective function value, or a decrease in the uncertain parameter increases both, we may require the two extreme scenarios. This principle holds, for instance, for problems minimizing inventory quantities under uncertain demand that needs to be satisfied, as a decrease in demand increases the number of feasible solutions and also increases the objective as inventory quantities increase.