Identifying Regions Vulnerable to Obstetric Unit Closures using Facility Location Modeling with Patient Behavior

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Abstract

Limited geographic access to obstetric care prevents some pregnant people from receiving timely and risk-appropriate services. This challenge is especially acute in rural areas, where rural residents often travel far distances to obstetric care. Furthermore, obstetric access is worsening due to the growing number of closures of rural hospitals' obstetric units, often due to financial concerns. In response, government organizations have initiated programs to prevent rural hospital closures, but there are limited tools to identify where best to allocate these resources. In this work, we propose the Maximum Choice-based Expectation Facility Location Problem (MCE-FLP), a generalization of the maximum capture facility location problem, to forecast which obstetric unit closures would have the worst impact on expected travel distance to care. In this problem, we model patients' obstetric care-seeking behavior using a multinomial logit (MNL) discrete choice model, which we show is more accurate than assuming patients seek their nearest obstetric unit. We show that the MCE-FLP's objective function is not convex or concave in general, and thus the state-of-the-art maximum capture solution methods are not applicable. We present two linear reformulations of the MCE-FLP and design branch-and-cut (B&C) approaches using analytical solutions to obtain the cuts. In our case study, we apply the MCE-FLP to the obstetric care system in the state of Georgia, and we find that the projected worst-case closures are isolated from other obstetric units and would impact above-average rates of marginalized groups. In this work, we propose a novel generalization of the maximum capture problem, which maximizes an expected value that is dependent on patient/consumer choice, and we demonstrate that this problem can be applied to determine worst-case obstetric closures that should be prevented to maintain critical access to care.

Keywords. Choice-based facility location; Healthcare access; Random utility maximization; Multinomial logit model; Branch-and-cut

1 Introduction

The United States' maternal mortality rate, 32.6 deaths per 100,000 live births, has increased by 27.7% from 2018 and is the highest rate among high-income countries (Chen *et al.* 2025). An estimated 80% of these pregnancy-related deaths are preventable (Trost *et al.* 2022). Moreover, there are significant disparities, as rural women are twice as likely to die due to pregnancy compared with their urban peers (Harrington *et al.* 2023). This disparity is partially explained by a lack of geographic access to hospital-based obstetric care, which consists of healthcare services before, during, and after pregnancy (Kilpatrick *et al.* 2019).

The widespread trend of obstetric unit closures across the United States has decreased access to obstetric care, particularly in rural areas. Between 2011 and 2021, 25% of rural obstetric hospitals closed their obstetric units (Chartis Group 2023). These closures are most common in the most rural counties (Hung *et al.* 2017), potentially worsening already high rates of adverse maternal outcomes (Merkt *et al.* 2021, Hansen *et al.* 2022). As a result, travel distances to obstetric care have increased, which is associated with delayed initiation and infrequent prenatal care, higher rates of adverse maternal outcomes, and more NICU admissions (Kennedy *et al.* 2022, Deng *et al.* 2024, Minion *et al.* 2022). Although many hospital administrators believe that obstetric care is an essential community service, low delivery volume, difficulty in staffing, and low reimbursement rates can cause obstetric care facilities to be of poor quality and unprofitable, which may lead to their closures (Kozhimannil *et al.* 2022, Hung *et al.* 2016). A survey of rural hospital administrators found that 41.7% of rural obstetric hospitals reported having fewer births than they required for financial viability, and 29.9% reported having fewer births than they required for clinical safety (Kozhimannil *et al.* 2022). Thus many more obstetric units may be at risk of closure.

The causes of obstetric unit closures and their consequences on perinatal health outcomes are well-studied (Carroll *et al.* 2022, Fischer *et al.* 2024, Lorch *et al.* 2013, Sullivan *et al.* 2021).

However, there has been little work on forecasting the impacts of potential obstetric unit closures and strategically preventing these closures. As policymakers introduce hospital support programs, such as Health Resources and Services Administration (HRSA)'s Rural Maternity and Obstetrics Management Strategies Program (Coombs 2022), it is critical to identify the best use of these programs' resources to support obstetric care. When forecasting the impacts of closures, it is important to understand how patients seek obstetric care. Previous studies examining where patients seek labor & delivery (L&D) care have found that between 11% and 51% of patients *bypass* their closest obstetric hospital. That is, they deliver at a farther obstetric hospital, even when their closest obstetric hospital provides the services that they need (Thorsen *et al.* 2023, Carrel *et al.* 2023).

The goal of this work is to identify which obstetric units are essential for ensuring access to obstetric care within a region. In this article, we consider the following research question: "Which obstetric unit closures would have the worst-case impacts on access to care, when considering patient care-seeking behavior?" To answer this question, we develop a new framework for integrating care-seeking behavior as a random utility maximization model into facility location design decisions. In our setting, this framework represents an adversary that can close obstetric units to maximize the expected travel distance to obstetric care while considering patient behavior. More generally, this framework can be used to design facility locations to maximize the expectation of some function with respect to the induced patient behavior (or consumer choice).

1.1. Facility Location Background

Facility location is a classical operations research problem in which a decision maker seeks to optimally locate facilities that serve consumers (Daskin 2013). These problems arise in a wide variety of contexts, including healthcare services (Cho *et al.* 2014), distribution networks (Li *et al.* 2013), and humanitarian logistics (Boonmee *et al.* 2017). Classical facility location models assume that consumers will be served by their closest facility. However, this assumption is often violated to some degree in reality. Accordingly, there has been growing attention on how consumers' choices of which facilities to patronize should affect facility location decisions.

One approach for representing consumer behavior is the random utility maximization framework, in which consumers are assumed to have random utilities associated with various facilities and choose to patronize facilities that maximize their utility (McFadden 1972). Benati and Hansen (2002) were the first to integrate a random utility maximization model representing consumer behavior into the facility location problem. They describe a setting where a decision maker chooses to locate facilities in a competitive market to maximize their market share, which they refer to as the "maximum capture" model. They integrate a multinomial logit (MNL) model, a discrete choice random utility maximization model, which provides a simple structure for the probability of a consumer choosing a specific facility. The maximum capture model laid the foundation for integrating consumer choice into the facility location problem, and most of the literature on this topic has focused on this objective in competitive settings (Haase 2009, Haase and Müller 2015, Freire *et al.* 2016, Mai and Lodi 2020). However, there are other non-competitive contexts in which incorporating choice could be critical to facility location decisions. For example, in primary education, decision makers may want to minimize students' travel costs (Haase *et al.* 2019). In disaster relief, decision makers may want to minimize the response time of locating distribution centers while ensuring coverage (Duran *et al.* 2011). In healthcare, decision makers may want to maximize the quality of care that patients receive, which depends on where patients seek care. In this article, we explore how patients' obstetric care-seeking behavior may affect our understanding of which facilities are critical to providing access.

1.2. Contributions

Altogether, this article makes contributions in three aspects: model, algorithm, and application.

In our modeling:

1. We introduce a generalization of the maximum capture problem, which we refer to as the Maximum Choice-based Expectation Facility Location Problem (MCE-FLP). In the MCE-FLP, we consider the facility location problem in which the objective is to maximize an expectation of a function with respect to the probability distribution of consumers' choice of facility. We illustrate how the maximum capture problem is a special case of the MCE-FLP.

2. We examine the MCE-FLP framework when consumer choice is modeled using a MNL discrete choice model. We show that, under the MNL model, the MCE-FLP objective function is not concave or convex in general, unlike the maximum capture objective function.

Thus, the state-of-the-art solution methods for the maximum capture problem that take advantage of concavity do not extend to the MCE-FLP. However, we show that the MCE-FLP has a quasiconcave objective function and can be represented as a mixed-integer linear fractional program (MILFP).

3. We present two linear reformulations of the MCE-FLP. The first formulation is a probability-based linear model in which the induced choice probabilities are directly represented as decision variables, as done by Haase (2009) for the maximum capture problem. The second formulation is a linear fractional-based formulation. We use a variable substitution method (Yue *et al.* 2013) to transform this formulation into a mixed-integer linear program (MILP). We demonstrate that both of these representations lend themselves to decomposition by consumer zone.

In our algorithm:

4. We design a branch-and-cut (B&C) approach for each linear reformulation based on the integer L-shaped method (Laporte and Louveaux 1993). We decompose our problem into a main problem in which the decision maker selects the facilities. Then, given the selection of the facilities, we represent each consumer zone's choice of facility as a subproblem. We exploit structures of these subproblems to derive the optimal dual variables analytically which are then used to generate optimality cuts. We provide a numerical study comparing the linear reformulations, B&C, and B&C with analytical solution cuts approaches. We demonstrate that solving the MILP formulations directly is the fastest approach for small problems, and the B&C approaches with analytical solution cuts are the fastest approaches for the largest problems. Specifically, for the largest test instances in our study, the B&C approach with analytically-generated cuts can solve twice as many instances as an approach that solves the MILP formulations directly.

In our application:

5. We apply the MCE-FLP in a case study examining access to obstetric care in the state of Georgia. We fit a simple MNL model to birth certificate records in Georgia and find that this MNL model better approximates patient obstetric care-seeking behavior compared with a

model that assumes patients seek care at their closest obstetric unit. Using this simple MNL model, we consider which simultaneous obstetric unit closures would have the worst impact on access to obstetric care, which could inform policymakers on where to focus their effort and resources to prevent closures. We find that the MCE-FLP projected worst-case obstetric unit closures would affect above-average populations of already marginalized groups, and that these obstetric units are not frequently bypassed, indicating their critical role in providing access to obstetric care.

1.3. Organization of the article

The remainder of this article is organized as follows: In Section 2, we provide some important background on integrating consumer behavior with random utilities into the facility location problem. In Section 3, we formally define the MCE-FLP. In Section 4, we analyze the properties of the MCE-FLP and propose linear reformulations, and in Section 5, we develop corresponding B&C solution methods. In Section 6, we compare the performance of these solution methods in computational experiments, and in Section 7, we present our case study of the state of Georgia. Finally, in Section 8, we summarize our findings and discuss limitations and opportunities for future research.

2 Literature Review

In this article, we seek to forecast the impacts of potential obstetric unit closures based on patients' care-seeking behavior represented as discrete choice model, which is a mathematical tool for describing how individuals make choices among a discrete set of alternatives based on the features of those alternatives. These models are used in a variety of contexts, including transportation (Schwanen and Mokhtarian 2005, Ben-Akiva and Bierlaire 1999), healthcare (Prosser *et al.* 2023), and marketing (Berry 1994). Given the importance of accurately representing consumer behavior, discrete choice modeling approaches are being integrated into optimization frameworks for a variety of problems, including revenue management (Talluri and van Ryzin 2004) and assortment (Rusmevichientong *et al.* 2010). One such problem, and the focus of this article, is the facility location problem.

Many optimization models incorporating choice use the MNL model because of its simple structure. The first maximum capture facility location approach under the MNL model was introduced by Benati and Hansen (2002). The challenge in this approach is that the resulting mathematical program yields a nonlinear formulation. Multiple solution methods have been proposed for this nonlinear formulation. A subset of these approaches reformulate the resulting problem into MILP models, which are convenient to solve (Benati and Hansen 2002, Haase 2009, Zhang *et al.* 2012, Aros-Vera *et al.* 2013). These linear reformulation approaches were compared by Haase and Müller (2014), who found that the most efficient is the Haase (2009) approach.

A more recent stream of literature on the maximum capture problem under the MNL model has taken advantage of the concavity of the objective function by proposing algorithms that use an outer-approximation to solve this nonlinear facility location problem. Ljubić and Moreno (2018) propose a B&C algorithm that combines outer-approximation and submodular cuts, and Mai and Lodi (2020) propose a multicut outer-approximation cutting plane approach. The problem can also be solved by a convex mixed-integer nonlinear program (MINLP) solver (e.g., Bonami *et al.* (2008)). These two solution approaches (Ljubić and Moreno 2018, Mai and Lodi 2020) are regarded as state-of-the-art for the maximum capture problem under the MNL model.

Another stream of research has focused on solving variations of the maximum capture problem under other choice models. For example, Lin and Tian (2021) propose an efficient generalized Benders decomposition algorithm to solve a variant where consumers make choices based on the gravity rule, and the decision maker can choose zone-specialized attractiveness to maximize profit. Legault and Frejinger (2024) propose a partial decomposition approach to solve the maximum capture problem where utility is approximated by sampling realizations of its random terms to avoid the limitations of the MNL model. Dam *et al.* (2022) formalize the maximum capture problem under the generalized extreme value family. Their objective function's continuous relaxation is neither convex nor concave, and consequently, the approaches of Ljubić and Moreno (2018) and Mai and Lodi (2020) cannot be applied. Instead, they develop a greedy heuristic.

There is growing recognition of the importance of patient choice in healthcare planning and operations. For example, Feldman *et al.* (2014) incorporate patient preference and behavior in designing appointment schedules. Ata *et al.* (2021) incorporate patients' strategic choices in kidney allocation policy. Zhang *et al.* (2012), Haase and Müller (2015), and Bravo *et al.* (2025) use discrete choice models in the location planning of preventive healthcare facilities to maximize capture. In contrast to these articles, we consider the expected travel distance to facilities instead of capture because of the known association between negative health outcomes (e.g., adverse pregnancy outcomes, delayed initiation of prenatal care) and travel distance to obstetric hospitals.

In this article, we examine a choice-based facility location problem motivated by access to obstetric care. Our work lies at the intersection of two streams of research on obstetric care: that which examines where pregnant people seek obstetric care (Gudayu 2022, Harris *et al.* 2024, Koller *et al.* 2024) and that which considers the location planning of obstetric units (Chouksey *et al.* 2022, Karakaya and Meral 2022, Galvão *et al.* 2002). From an application perspective Hwang *et al.* (2022) is the most closely related work, which incorporating patient preference into a facility location problem for the Korean perinatal care system. Their objective coverage-based, to reduce medically underserved areas, and they use linear reformulation techniques and solve a MILP. In contrast to this article, we propose a generalizable facility location that maximizes some choice-based expectation, and we propose an efficient decomposition algorithm to solve our problem.

3 The Maximum Choice-Based Expectation Facility Location Problem

3.1. Problem Setting

We now formally introduce the Maximum Choice-based Expectation Facility Location Problem (MCE-FLP). We consider a problem in which a decision maker is choosing to add or remove obstetric units from hospitals, which provide services to patients in a given region. We assume that the patients who seek care from the available obstetric units can be partitioned into communities (i.e., consumer zones) $\mathcal{C} = \{1, ..., C\}$, and that $b_c \in \mathbb{R}$ is the demand for services originating from patients located in community $c \in \mathcal{C}$. In our context, b_c is the number of births originating from community $c \in \mathcal{C}$. The decision maker then needs to decide where to locate obstetric units among a set of candidate hospital locations, $\mathcal{H} = \{1, ..., H\}$.

To capture patient behavior, we assume that patients seek obstetric care according to a random utility maximization model. Each community of patients $c \in \mathcal{C}$ associates a random utility, u_{ch}, with each obstetric hospital $h \in \mathcal{H}$. The random utility maximization model assumes that patients make choices that maximize their utility. This framework allows us to compute the probability that a patient from community c seeks care at an obstetric hospital h as $p_{ch} = P(u_{ch} \geq u_{ch'}, \forall h' \in \mathcal{H})$.

Given the patients' random utilities, the decision maker can then choose which subset of candidate hospitals $X \subseteq \mathcal{H}$ should be selected to provide obstetric care (i.e., to be obstetric hospitals) so as to maximize the expectation of an objective of interest with respect to the patients' choice probabilities. We allow that other restrictions, such as the total number of hospitals offering obstetric services, are captured by a feasible set $\mathcal{X} \subseteq P(\mathcal{H})$ (where $P(\mathcal{H})$ denotes the power set of hospitals). For example, if exactly r obstetric units are allowed to be open, then $\mathcal{X} = \{X \subseteq \mathcal{H} : |X| = r\}$. We represent our objective of interest as a random variable Z, which takes on the value z_{ch} with probability $p_{ch}(X)$ given that the subset of hospitals selected is given by X. We refer to this as the MCE-FLP:

$$\max_{X \in \mathcal{X}} \quad \mathbb{E}^X[Z] = \sum_{c \in C} b_c \sum_{h \in X} z_{ch} p_{ch}(X) \tag{1}$$

where $\mathbb{E}^X[Z]$ represents the expectation of our objective Z taken with respect to the choice probabilities, $p_{ch}(X)$, when the subset of candidate hospitals, X, is selected.

3.2. The Multinomial Logit Random Utility Maximization Model

A random utility maximization model assumes that patients from community $c \in C$ receive some random utility, u_{ch}, from seeking care at a hospital $h \in \mathcal{H}$. A general form used to represent the random utility is that the random utility u_{ch} is the sum of two parts, i.e., $u_{ch} = v_{ch} + \epsilon_{ch}$. Here, v_{ch} refers to the deterministic part of the utility and represents observed attributes of a community of patients c and hospital h. Meanwhile, ϵ_{ch} is an error term to represent utility attributed to unobserved attributes. In much of the maximum capture facility location literature, the multinomial logit (MNL) model is used because of its simple specification of the choice probability. In the MNL model, the unknown component, ϵ_{ch}, is assumed to follow the Gumbel distribution. Throughout the remainder of this article, we will focus on the MCE-FLP framework in (1) under an MNL model.

Under an MNL model of patient choice, $p_{ch}(X)$ can be computed as

$$p_{ch}(X) = \frac{\exp(v_{ch})}{\sum_{h' \in X} \exp(v_{ch'}) + \alpha_c^0}$$

where $v_{ch} = (\beta^*)^T a_{ch}$ is the utility associated with a community of patients $c \in C$ and hospital $h \in X$, β^* are the parameters of the MNL model, and a_{ch} is an attribute of the community-hospital pair. Going forward, we will denote $\alpha_{ch} = \exp(v_{ch})$. In a competitive facility location context, α_c^0 represents the utility associated with a competitor firm's facilities. For example, if $Y \subset \mathcal{H}$ is the set of locations where the competitor has located facilities, then $\alpha_c^0 = \sum_{k \in Y} exp(v_{ck})$. Alternatively, in our healthcare context, α_c^0 represents the utility of "no-choice" (e.g., foregoing labor in an obstetric hospital). In all contexts, α_c^0 represents the utility of fixed alternatives and is a constant, and p_c^0 is the associated choice probability. We assume that our decision maker is only concerned with the choice-based expectation corresponding to the patients captured by hospitals, $X \subseteq \mathcal{H}$.

Thus, specifying the MCE-FLP in (1) with an MNL model of patient choice is represented as:

$$\max_{X \in \mathcal{X}} \quad \mathbb{E}^X[Z] = \sum_{c \in C} b_c \sum_{h \in X} z_{ch} \frac{\alpha_{ch}}{\sum_{h' \in X} \alpha_{ch'} + \alpha_c^0}. \tag{2}$$

3.3. Examples of MCE-FLP

We now provide some examples of the MCE-FLP.

1. **Maximum Capture.** The MCE-FLP is a generalization of the maximum capture problem, where a decision maker seeks to locate facilities to maximize their expected market share under a consumer choice model. In the maximum capture case, $z_{ch} = 1$, and the problem can be written as

$$\max_{X \in \mathcal{X}} \sum_{c \in C} b_c \sum_{h \in X} p_{ch}(X). \tag{3}$$

   The problem in (3) under an MNL model of choice was introduced by Benati and Hansen (2002).

2. **Maximum Service Level.** Another example of the MCE-FLP is the case where a decision maker seeks to maximize the expected level of care provided by a patient's chosen hospital. In this case, each hospital location provides some level of service, ℓ_h, where higher levels of services correspond with higher quality of service. Thus, this problem can be written as

$$\max_{X \in \mathcal{X}} \sum_{c \in C} b_c \sum_{h \in X} \ell_h p_{ch}(X).$$

   In this case, the decision maker is incentivized to both locate hospitals at locations that are capable of higher levels of service and at locations that capture larger proportions of patients.

3. **Maximum Distance.** A final example of the MCE-FLP, and the context we explore in this article, is the case where a decision maker seeks to maximize the expected distance traveled by the captured patients, a p-median problem. In this case, $z_{ch} = d_{ch}$, where d_{ch} is the distance between community of patients $c \in C$ and hospital $h \in \mathcal{H}$. Thus, this problem can be written as

$$\max_{X \in \mathcal{X}} \sum_{c \in C} b_c \sum_{h \in X} d_{ch} p_{ch}(X). \tag{4}$$

We explore this context as an adversarial version of a p-median problem with choice in detail in our case study. In this problem, an adversarial decision maker's objective is to maximize the expected population-weighted distance traveled by closing obstetric units. We use this problem to identify which obstetric unit closures would have the worst-case impact on patients' travel distances, which could inform obstetric unit closure prevention policies.

4 Modeling and Analysis

In this section, we introduce mathematical models to solve the MCE-FLP under a MNL choice model. We analyze these models and derive solution algorithms presented in §5. First, we show that the MCE-FLP can be formulated as a mixed-integer programming model:

$$\max_{x} \quad \sum_{c \in C} b_c \sum_{h \in \mathcal{H}} z_{ch} p_{ch}(x) \tag{5a}$$

$$\text{s.t.} \quad \sum_{h \in \mathcal{H}} x_h = r, \tag{5b}$$

$$x_h \in \{0, 1\}, \quad \forall h \in \mathcal{H}, \tag{5c}$$

where $x_h = 1$ if an obstetric unit is located at hospital $h \in \mathcal{H}$ and 0 otherwise, and $p_{ch}(x)$ represents probability that a patient in community $c \in C$ seeks obstetric care at hospital $h \in \mathcal{H}$ under the available obstetric hospitals dictated by x. Constraint (5b) states that the decision maker will place exactly r hospitals.

Under the MNL model, we can represent the patient choice probability as

$$p_{ch}(x) = \frac{\alpha_{ch} x_h}{\sum_{h' \in \mathcal{H}} \alpha_{ch'} x_h + \alpha_c^0}.$$

Thus, the MCE-FLP under MNL can be formulated as a mixed-integer nonlinear program (MINLP):

$$\max_{x} \quad \sum_{c \in C} b_c \frac{\sum_{h \in \mathcal{H}} z_{ch} \alpha_{ch} x_h}{\sum_{h' \in \mathcal{H}} \alpha_{ch'} x_{h'} + \alpha_c^0} \tag{6a}$$

$$\text{s.t.} \quad \sum_{h \in \mathcal{H}} x_h = r, \tag{6b}$$

$$x_h \in \{0, 1\}, \quad \forall h \in \mathcal{H}. \tag{6c}$$

4.1. Analysis of the MCE-FLP

The maximum capture problem's objective function is a special case of MCE-FLP in which $z_{ch} = 1$ for all $c \in C$ and $h \in \mathcal{H}$. Under this condition, the continuous relaxation of the objective function (6a) is concave (Benati and Hansen 2002). This property has served as the foundation for many of the state-of-the-art methods for solving the maximum capture problem (Ljubić and Moreno 2018, Mai and Lodi 2020). However, we show that this property does not hold for our objective function (6a) in general. All proofs are deferred to Appendix A for ease of reading.

Proposition 1. *The continuous relaxation of the MCE-FLP objective function* (6a) *is not convex or concave in general.*

This result indicates that solution methods for the maximum capture problem that take advantage of the concavity of the objective are generally not valid for the MCE-FLP. However, these approaches may apply in special cases. For instance, if $z_{ck} = z_{cw}\ \forall k, w \in \mathcal{H}$, then the MCE-FLP objective function (6a) becomes a scaled maximum capture objective, and all maximum capture solution methods would correctly produce an optimal solution.

Although the MCE-FLP's objective function (6a) is not convex or concave, it is a sum of linear fractional functions, which are quasilinear (Bazaraa *et al.* 2006). We elaborate how this property can be used to reformulate the problem into a mixed-integer linear program (MILP) in §4.2.2.

4.2. Mixed-Integer Linear Programming Reformulations

We now demonstrate how (6) can be reformulated into a MILP using two approaches. In §4.2.1, we adapt the most efficient maximum capture linear reformulation from Haase (2009), which explicitly formulates choice probabilities as decision variables. We refer to this as the probability-based linear reformulation. In §4.2.2, we present a linear reformulation using the linear fractional structure of the MCE-FLP objective function. We refer to this as the linear fractional-based reformulation.

4.2.1. Probability-Based Linear Reformulation The early linear formulations of the maximum capture problem are *probability-based formulations*. That is, these formulations use continuous decision variables p to reflect consumer choice probabilities and use logic-based constraints to relate these choice probabilities to the available facilities dictated by binary decision variables x.

For example, Haase (2009) present a linear reformulation of the maximum capture problem (3). Using our notation, this Haase (2009) reformulation has the objective function,

$$\max_{x,p} \quad \sum_{c \in C} b_c \sum_{h \in \mathcal{H}} p_{ch}. \tag{7}$$

We adapt the Haase (2009) linear reformulation from the maximum capture to the MCE-FLP (6). To do so, we first update the objective function to

$$\max_{x,p} \quad \sum_{c \in C} b_c \sum_{h \in \mathcal{H}} z_{ch} p_{ch}. \tag{8}$$

The Haase (2009) formulation also has a set of constraints that enforce that values of p_{ch} are correctly evaluated for a set of open facilities (e.g., hospitals) $x_h \; \forall h \in \mathcal{H}$. Freire *et al.* (2016) show that one of these constraints (9) holds at equality when we consider the maximum capture objective function (7) and when $x_h = 1$, ensuring that p_{ch} is correctly evaluated,

$$p_{ch} \leq \frac{\alpha_{ch}}{\alpha_c^0} p_c^0 \tag{9}$$

$$= \frac{\alpha_{ch}}{\alpha_c^0} \frac{\alpha_c^0}{\sum\limits_{h' \in \mathcal{H}} \alpha_{ch'} x_{h'} + \alpha_c^0} = \frac{\alpha_{ch}}{\sum\limits_{h' \in \mathcal{H}} \alpha_{ch'} x_{h'} + \alpha_c^0}. \tag{10}$$

With the MCE-FLP objective function (8), we are no longer strictly maximizing all p_{ch}. Thus, when $x_h = 1$, (9) is not necessarily held at equality. Therefore, to solve the MCE-FLP with the Haase (2009) reformulation, we add a constraint (11) from the Aros-Vera *et al.* (2013) linear reformulation to enforce that p_{ch} and p_c^0 take on the correct values when $x_h = 1$,

$$p_c^0 \leq \frac{\alpha_c^0}{\alpha_{ch}} p_{ch} + (1 - x_h) \quad \forall c \in \mathcal{C}, h \in \mathcal{H}, \tag{11}$$

$$\implies p_{ch} \geq \frac{\alpha_{ch}}{\alpha_c^0} p_c^0 = \frac{\alpha_{ch}}{\sum_{h' \in \mathcal{H}} \alpha_{ch'} x_{h'} + \alpha_c^0}. \tag{12}$$

Thus, constraints (9) and (11) enforce that p_{ch} is correctly evaluated. This allows us to modify Haase (2009) for the MCE-FLP (5) as a MILP:

$$\max_{x,\, p} \quad \sum_{c \in \mathcal{C}} b_c \sum_{h \in \mathcal{H}} z_{ch} p_{ch} \tag{13a}$$

$$\text{s.t.} \quad \sum_{h \in \mathcal{H}} x_h = r, \tag{13b}$$

$$\sum_{h \in \mathcal{H}} p_{ch} + p_c^0 = 1, \qquad \forall c \in \mathcal{C}, \tag{13c}$$

$$p_c^0 \leq \frac{\alpha_c^0}{\alpha_{ch}} p_{ch} + (1 - x_h), \quad \forall c \in \mathcal{C}, h \in \mathcal{H}, \tag{13d}$$

$$p_{ch} \leq \frac{\alpha_{ch}}{\alpha_c^0} p_c^0, \qquad \forall c \in \mathcal{C}, h \in \mathcal{H}, \tag{13e}$$

$$p_{ch} \leq \frac{\alpha_{ch}}{\alpha_{ch} + \alpha_c^0} x_h, \qquad \forall c \in \mathcal{C}, h \in \mathcal{H}, \tag{13f}$$

$$p_{ch} \geq 0, \qquad \forall c \in \mathcal{C}, h \in \mathcal{H}, \tag{13g}$$

$$p_c^0 \geq 0, \qquad \forall c \in \mathcal{C}, \tag{13h}$$

$$x_h \in \{0, 1\}, \qquad \forall h \in \mathcal{H}. \tag{13i}$$

where constraint (13c) requires the choice probabilities for each community sum to one, constraints (13d)-(13e) enforce that the choice probabilities take on the correct values when a obstetric unit is open, and constraint (13f) enforces that the choice probability of seeking a closed unit is zero.

4.2.2. Linear Fractional Linear Reformulation Because solution techniques that rely on concavity no longer apply, we explore other properties of the MCE-FLP. Let us again consider the

MCE-FLP under the MNL model formulated as a MINLP (6). The objective function (6a) is a sum of linear fractional functions, which are of the form: $f(x) = \frac{a^T x+b}{c^T x+d}$ where $a, c \in \mathbb{R}^{\mathcal{H}}$ are vectors of known coefficients and $b, d \in \mathbb{R}$ are known coefficients. Bazaraa *et al.* (2006) show that linear fractional functions are quasiconvex and quasiconcave, thus quasilinear, and that every local optimum of a linear fractional program is also its global solution.

We can linearly reformulate linear fractional programs via the Charnes-Cooper transformation (Charnes and Cooper 1962), which Yue *et al.* (2013) extended to mixed-integer linear fractional programs. We now apply this linear fractional reformulation to the MCE-FLP. First, similar to the Charnes-Cooper transformation (Charnes and Cooper 1962), we introduce new variables $t_c = (\sum_{h' \in \mathcal{H}} \alpha_{ch'} x_{h'} + \alpha_c^0)^{-1}$. The MCE-FLP (5) can then be transformed into an equivalent bilinear program with the introduction of these new variables (Charnes and Cooper 1962, Yue *et al.* 2013),

$$\begin{aligned}
\max_{x,t} \quad & \sum_{c \in C} b_c \sum_{h \in \mathcal{H}} z_{ch} \alpha_{ch} x_h t_c \\
\text{s.t.} \quad & \sum_{h \in \mathcal{H}} x_h = r, \\
& \sum_{h \in \mathcal{H}} \alpha_{ch} x_h t_c + \alpha_c^0 t_c = 1, && \forall c \in C, \\
& t_c \geq 0, && \forall c \in C, \\
& x_h \in \{0,1\}, && \forall h \in \mathcal{H}.
\end{aligned} \tag{14}$$

To linearize the bilinear term, $x_h t_c$, we introduce a new variable $w_{ch} = x_h t_c$, which we can linearize with Glover's linearization scheme (Glover 1975, Yue *et al.* 2013). Thus, (14) can be linearized into an equivalent MILP problem,

$$
\begin{aligned}
\max_{x,t,w} \quad & \sum_{c \in \mathcal{C}} b_c \sum_{h \in \mathcal{H}} z_{ch} \alpha_{ch} w_{ch} \\
\text{s.t.} \quad & \sum_{h \in \mathcal{H}} x_h = r, \\
& \sum_{h \in \mathcal{H}} \alpha_{ch} w_{ch} + \alpha_c^0 t_c = 1, \quad \forall c \in \mathcal{C}, \\
& w_{ch} \geq t_c - (1 - x_h) T_c, \quad \forall c \in \mathcal{C}, h \in \mathcal{H}, \\
& w_{ch} \leq t_c, \quad \forall c \in \mathcal{C}, h \in \mathcal{H}, \\
& w_{ch} \leq x_h T_c, \quad \forall c \in \mathcal{C}, h \in \mathcal{H}, \\
& w_{ch} \geq 0, \quad \forall c \in \mathcal{C}, h \in \mathcal{H}, \\
& t_c \geq 0, \quad \forall c \in \mathcal{C}, \\
& x_h \in \{0, 1\} \quad \forall h \in \mathcal{H}.
\end{aligned}
\tag{15}
$$

Here, T_c is an upper bound on t_{ch}, serving as a big-M parameter in Glover's linearization scheme.

5 Solution Methods

In the previous section, we provided two MILP reformulations of the MCE-FLP. We can directly solve each of these formulations using a commercial optimization solver, such as Gurobi. In this section, we demonstrate how these reformulations are amenable to branch-and-cut (B&C) decomposition algorithms. A summary of the model formulations and their corresponding solution methods is provided in Table 1.

Our B&C schemes for solving the MCE-FLP formulations are similar to Benders decomposition for stochastic programming. Benders decomposition breaks the extensive form of a stochastic program into a *main problem* and *subproblems*. The main problem typically only considers "complicating variables" while the subproblems will consider the other variables assuming fixed values of the complicating variables. In the context of stochastic programming, typically a "relaxed main problem" is solved, which involves only the first-stage decisions and a subset of the constraints required to specify the complete optimization problem. Then, the first-stage solutions and sub-

Formulation	Solution Method	Acronym	Description	Details
Probability-Based (13)	Solved directly	Prob MILP	MILP formulation (13) solved directly	§4.2.1
	B&C	Prob B&C	B&C algorithm where an optimality cut is generating by solving the $DSP_c^P(\hat{x})$, (19)	§5.1
	B&C (Analytical)	Prob B&C-A	B&C algorithm where an optimality cut is generated by computing the $DSP_c^P(\hat{x})$ analytical solution	Proposition 2
Linear Fractional-Based (15)	Solved directly	LF MILP	MILP formulation (15) solved directly	§4.2.2
	B&C	LF B&C	B&C algorithm where an optimality cut is generating by solving the $DSP_c^{LF}(\hat{x})$, (23)	§5.2
	B&C (Analytical)	LF B&C-A	B&C algorithm where an optimality cut is generated by computing the $DSP_c^{LF}(\hat{x})$ analytical solution	Proposition 3

B&C: branch-and-cut; MILP: mixed-integer linear program; DSP: dual subproblem

Table 1: Summary of solution methods for two linear reformulations.

problems' duals are used to subsequently add constraints, or *cuts*, to the relaxed main problem to enforce the feasibility and/or optimality of the first-stage solutions from the relaxed main problem.

In our setting, we consider the binary obstetric unit location variables to be our "complicating variables" in both the probability-based formulation (13) and the linear fractional-based formulation (15). Thus, in each B&C scheme, our completely relaxed main problem initially only includes the binary variables representing which hospitals have obstetric units. Then, we enforce the constraints via cuts that are generated from the respective subproblems corresponding to (13) or (15). For both (13) and (15), the relaxed main problem starts as:

$$\begin{aligned} \max_{\theta, x} \quad & \sum_{c \in \mathcal{C}} \theta_c \\ \text{s.t.} \quad & \sum_{h \in \mathcal{H}} x_h = r, \\ & \theta_c \leq U_c, \qquad \forall c \in \mathcal{C}, \\ & x_h \in \{0,1\}, \quad \forall h \in \mathcal{H}, \end{aligned} \tag{16}$$

where U_c is an initial upper bound on $g_c(x)$, the objective function of the subproblem for community $c \in C$. The θ_c variables take on an upper bound of $g_c(x^*)$ at the optimal value x^*. Then, when the kth integer feasible solution is found, a cut $(\Theta^{k,c}(x^k))$ is generated which provides an upper bound on θ_c for each community $c \in C$ and a constraint of the form:

$$\theta_c \leq \Theta^{k,c}(x^k) \tag{17}$$

is added for each community $c \in C$. These upper bounds get progressively lower as more cuts are generated at subsequent integer feasible solutions. In our setting, we will demonstrate that the subproblems can be decomposed by community $c \in C$. We now describe the subproblems used to generate optimality cuts for both the probability-based and linear fractional-based formulations.

5.1. Branch-and-cut (B&C) for the Probability-Based Linear Reformulation

For probability-based formulation MILP (13), the subproblem for community c at an integer feasible solution $\hat{x} \in \{0, 1\}^{|\mathcal{H}|}$ is denoted as $SP_c^P(\hat{x})$ and formulated as follows:

$$\begin{aligned}
g_c^P(\hat{x}) = \max_{p} \quad & b_c \sum_{h \in \mathcal{H}} z_{ch} p_{ch} && \text{(18a)}\\
\text{s.t.} \quad & \sum_{h \in \mathcal{H}} p_{ch} + p_c^0 = 1, && \text{(18b)}\\
& p_c^0 \leq \frac{\alpha_c^0}{\alpha_{ch}} p_{ch} + (1 - \hat{x}_h), \ \forall h \in \mathcal{H}, && \text{(18c)}\\
& p_{ch} \leq \frac{\alpha_{ch}}{\alpha_c^0} p_c^0, \qquad \forall h \in \mathcal{H}, && \text{(18d)}\\
& p_{ch} \leq \frac{\alpha_{ch}}{\alpha_{ch} + \alpha_c^0} \hat{x}_h, \qquad \forall h \in \mathcal{H}, && \text{(18e)}\\
& p_{ch}, p_c^0 \geq 0, \qquad \forall h \in \mathcal{H}. && \text{(18f)}
\end{aligned}$$

The dual of the subproblem, which we will refer to as $DSP_c^P(\hat{x})$, is then:

$$\begin{aligned}
\min_{\gamma, \varepsilon, \vartheta, \lambda} \quad & \gamma + \sum_{h \in \mathcal{H}} \varepsilon_h (1 - \hat{x}_h) && \text{(19a)} \\
& + \sum_{h \in \mathcal{H}} \lambda_h \left(\frac{\alpha_{ch}}{\alpha_{ch} + \alpha_c^0} \right) \hat{x}_h \\
\text{s.t.} \quad & \gamma - \frac{\alpha_c^0}{\alpha_{ch}} \varepsilon_h + \vartheta_h + \lambda_h \geq b_c z_{ch}, \ \forall h \in \mathcal{H}, && \text{(19b)} \\
& \gamma + \sum_{h \in \mathcal{H}} \varepsilon_h - \sum_{h \in \mathcal{H}} \frac{\alpha_{ch}}{\alpha_c^0} \theta_h \geq 0, && \text{(19c)} \\
& \varepsilon_h, \vartheta_h, \lambda_h, \geq 0, \qquad \forall h \in \mathcal{H}, && \text{(19d)}
\end{aligned}$$

where γ corresponds with (18b), ε with (18c), ϑ with (18d), and λ with (18e).

Solving the dual subproblem for each community, $DSP_c^P(\hat{x})$, we can use the optimal solutions $(\gamma^*, \varepsilon^*, \lambda^*, \vartheta^*)$ to add an optimality cut to the relaxed main problem (16) for each community $c \in C$:

$$\theta_c \leq \gamma^* + \sum_{h \in \mathcal{H}} \varepsilon_h^* (1 - x_h) + \sum_{h \in \mathcal{H}} \lambda_h^* \left(\frac{\alpha_{ch}}{\alpha_{ch} + \alpha_c^0} \right) x_h. \tag{20}$$

Because $\hat{x}$ is a fixed integer feasible solution, $DSP_c^P(\hat{x})$ is an linear program (LP) and can be solved directly using a commercial solver such as Gurobi. However, we demonstrate that the dual subproblems, $DSP_c^P(\hat{x})$, can be solved analytically, which avoids the need to build and solve an LP.

Proposition 2. *Given $\hat{x} \in \{0, 1\}^{|\mathcal{H}|}$ and assuming $r > 1$, the following is an optimal solution of the $DSP_c^P(\hat{x})$:*

$$\begin{aligned}
\gamma^* &= b_c \sum_{h \in \mathcal{H}} \frac{z_{ch} \alpha_{ch} \hat{x}_{h'}}{\sum_{h' \in \mathcal{H}} \alpha_{ch'} \hat{x}_{h'} + \alpha_c^0} \\
\vartheta_h^* &= \max\{(b_c z_{ch} - \gamma) \hat{x}_h, 0\} \\
\varepsilon_h^* &= \max\{-(b_c z_{ch} - \gamma) (\frac{\alpha_{ch}}{\alpha_c^0}) \hat{x}_h, 0\} \\
\lambda_h^* &= \max\{(b_c z_{ch} - \gamma)(1 - \hat{x}_h), 0\}
\end{aligned}$$

For the case when $r = 1$, we can solve the MCE-FLP in polynomial time by simply evaluating the impact of opening each single hospital.

Using the solution from Proposition 2, we can analytically compute the dual subproblem's optimal solution and add the optimality cuts (20) to the main problem (16) for each community.

5.2. Branch-and-cut (B&C) for the Linear Fractional Linear Reformulation

We now discuss the subproblems for the MCE-FLP using the linear fractional linear reformulation (15). The initial relaxed main problem (16) is the same as in the probability-based case. However, the cuts differ due to the different structure of the subproblems.

Given an integer feasible solution to (16), $\hat{x} \in \{0,1\}^{|\mathcal{H}|}$, the resulting subproblem for community $c \in C$ based on the linear fractional formulation, which we refer to as $SP_c^{LF}(\hat{x})$, is as follows:

$$
\begin{aligned}
g_c^{LF}(\hat{x}) = \max_{t,\,w} \quad & b_c \sum_{h\in\mathcal{H}} z_{ch}\alpha_{ch} w_{ch} && \text{(22a)}\\
\text{s.t.} \quad & \sum_{h\in\mathcal{H}} \alpha_{ch} w_{ch} + \alpha_c^0 t_c = 1, && \text{(22b)}\\
& w_{ch} \geq t_c - (1-\hat{x}_h)T_c, \quad \forall h \in \mathcal{H}, && \text{(22c)}\\
& w_{ch} \leq t_c, \quad \forall h \in \mathcal{H}, && \text{(22d)}\\
& w_{ch} \leq \hat{x}_h T_c, \quad \forall h \in \mathcal{H}, && \text{(22e)}\\
& w_{ch} \geq 0, \quad \forall h \in \mathcal{H}, && \text{(22f)}\\
& t_c \geq 0. && \text{(22g)}
\end{aligned}
$$

The dual of the subproblem, which we will refer to as $DSP_c^{LF}(\hat{x})$, is then:

$$
\begin{aligned}
\min_{\gamma,\varepsilon,\theta,\lambda} \quad & \gamma + \sum_{h\in\mathcal{H}} \varepsilon_h (1-\hat{x}_h) T_c + \sum_{h\in\mathcal{H}} \lambda_h \hat{x}_h T_c && \text{(23a)}\\
\text{s.t.} \quad & && \\
& \alpha_{ch}\gamma - \varepsilon_h + \vartheta_h + \lambda_h \geq b_c z_{ch}\alpha_{ch}, \quad \forall h \in \mathcal{H}, && \text{(23b)}\\
& \alpha_c^0 \gamma + \sum_{h\in\mathcal{H}} \varepsilon_h - \sum_{h\in\mathcal{H}} \vartheta_h \geq 0, && \text{(23c)}\\
& \varepsilon_h, \vartheta_h, \lambda_h \geq 0, \quad \forall h \in \mathcal{H}. && \text{(23d)}
\end{aligned}
$$

where γ corresponds with (22b), ε with (22c), ϑ with (22d), and λ with (22e).

Solving the dual subproblem for each community, $DSP_c^{LF}(\hat{x})$, we can use the optimal solutions $(\gamma^*, \varepsilon^*, \lambda^*, \vartheta^*)$ to add an optimality cut to the relaxed main problem (16) for each community $c \in C$:

$$\theta_c \leq \gamma^* + \sum_{h \in \mathcal{H}} \varepsilon_h^* (1 - x_h) T_c + \sum_{h \in \mathcal{H}} \lambda_h^* x_h T_c. \tag{24}$$

Similar to the probability-based formulation, when we are given $\hat{x}$, $DSP_c^{LF}(\hat{x})$ becomes an LP and can be solved using a solver, but we show that $DSP_c^{LF}(\hat{x})$ can also be solved analytically.

Proposition 3. *Given $\hat{x} \in \{0,1\}^{|\mathcal{H}|}$ and assuming $t_c < T_c, \forall c \in C$, the following is an optimal solution of the $DSP_c^{LF}(\hat{x})$:*

$$\gamma^* = b_c \sum_{h \in \mathcal{H}} \frac{z_{ch} \alpha_{ch} \hat{x}_{h'}}{\sum_{h' \in \mathcal{H}} \alpha_{ch'} \hat{x}_{h'} + \alpha_c^0}$$

$$\vartheta_h^* = \max\{\alpha_{ch}(b_c z_{ch} - \gamma)\hat{x}_h, 0\}$$

$$\varepsilon_h^* = \max\{-\alpha_{ch}(b_c z_{ch} - \gamma)\hat{x}_h, 0\}$$

$$\lambda_h^* = \max\{\alpha_{ch}\,(b_c z_{ch} - \gamma)\,(1 - \hat{x}_h), 0\}.$$

We can ensure $t_c < T_c$ by definition of T_c. In the remainder of this paper, we set $T_c = (\alpha_c^0)^{-1}$.

Using the solution from Proposition 3, we can analytically compute the dual subproblem's optimal solution and add the optimality cuts (24) to the main problem (16) for each community.

5.3. The complete B&C algorithm for MCE-FLP

We now formalize the entire B&C procedure, as shown in Algorithm 1. The algorithm is similar to the Integer L-Shaped method (Laporte and Louveaux 1993), but is customized to solve the dual subproblems analytically to obtain the dual variables and generate optimality cuts. In the next section, we will compare solution approaches that solve each formulation directly, use B&C with an LP solver to generate cuts, and Algorithm 1 that uses analytically-derived cuts. A summary of the six different solution methods is provided in Table 1.

Algorithm 1 Branch-and-Cut with Analytically Derived Cuts for the MCE-FLP

1: $LB \leftarrow -\infty$ and $UB \leftarrow \infty$

2: Solve the master problem (16). Let $(\hat{\theta}, \hat{x})$ be a candidate optimal solution. Set $UB = \sum_{c \in C} \hat{\theta}_c$.

3: Use $\hat{x}$ to obtain optimal dual variables $(\hat{\gamma}_c, \hat{\varepsilon}_{ch}, \hat{\vartheta}_{ch}, \hat{\lambda}_{ch})$ for each $(c, h) \in C \times \mathcal{H}$ using Proposition 2 (for probability-based) or Proposition 3 (for linear fractional-based)

4: Let $\hat{z}_c$ be the corresponding objective function value in (18) (for probability-based) or (22) (for linear fractional-based). Set $LB = \max\{LB, \sum_{c \in C} \hat{z}_c\}$

5: **if** $LB = UB$ **then**

6:     Current solution is optimal, STOP.

7: **else**

8:     For each community $c \in C$ such that $\hat{\theta}_c > \hat{z}_c$, use the current optimal dual solution $(\hat{\gamma}_c, \hat{\varepsilon}_{ch}, \hat{\vartheta}_{ch}, \hat{\lambda}_{ch})$ to add an optimality cut of the form (20) (for probability-based) or (24) (for linear fractional-based) to the master problem (16). Go to Step 2.

9: **end if**

6 Computational Experiments

In this section, we evaluate the performance of our solution methods on a dataset that has been used to compare formulations of the maximum capture problem.

6.1. Experimental Setting

We now describe the dataset used to compare the solution methods. We adapt the experimental setting described in Haase and Müller (2014) and in Freire *et al.* (2016) to consider the maximum travel distance problem described by (6) in which $z_{ch} = d_{ch}$. In our setting, we randomly locate $|C|$ communities of patients and $|\mathcal{H}|$hospital locations over a 30x30 plane. We consider $|C| \in \{50, 100, 200, 400, 800\}$, $|\mathcal{H}| = \{25, 50, 100\}$, and $b_c = 1$. We set $\alpha_c^0 = \frac{1}{999} \sum_{h \in \mathcal{H}} \alpha_{ch}$ so that $p_c^0 = 0.001$ ("no-choice" probability) when all hospitals have obstetric units.

For each instance, we have solved the problem for $r \in \{2, ..., 5\}$ and for ten random seeds. The distance between communities and hospitals, d_{ch}, is computed as Euclidean distance. The deterministic part of the utility function is given by $v_{ch} = -\beta \cdot d_{ch}$ for each community-hospital pair. Parameter β represents the sensitivity of patients to travel distance. We consider $\beta = 0.2$.

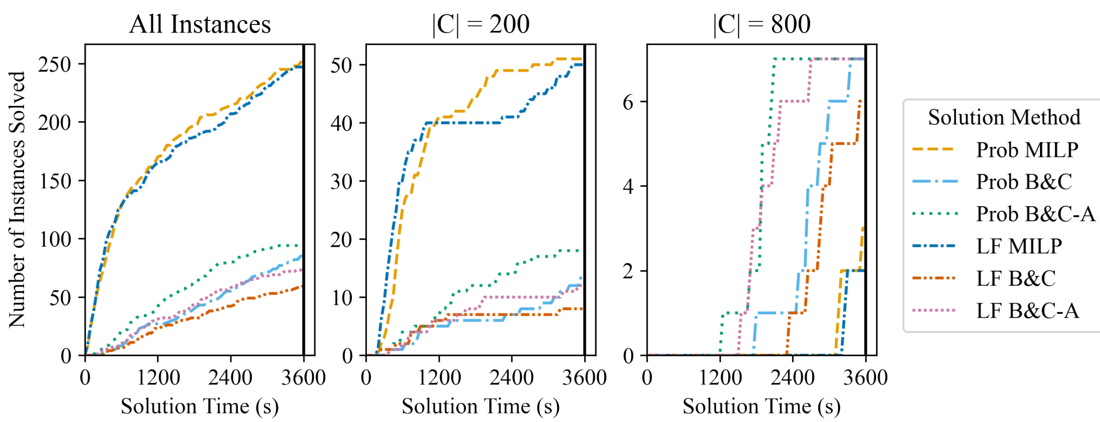


Figure 1: Number of randomly-generated instances solved within the specified time by each solution method. The maximum allotted time was 1 hour (3600 seconds). See Table 1 for a description of the solution methods.

This results in a total of 600 instances. We have given a time limit of one hour per instance.

The experiments are run on a Linux-based computational cluster with a total of 2,340 cores of x86-64 processing with over 28.9TB of memory. Our algorithms are coded in Python 3.8 and solved using Gurobi 11.0 with default parameters.

6.2. Results

Table 2 reports the computational comparison of the six approaches (see Table 1) to solve the maximum distance MCE-FLP, and Figure 1 shows the number of solved instances of each solution method for different instance sizes. We report the number of instances solved within the time limit of one hour and the average solution time in seconds as reported by Gurobi. Note that each row of the table corresponds to 40 instances.

Table 2 indicates that the probability-based and linear fractional-based MILPs can solve the majority of the smaller instances in the fastest solution time. On average, the probability-based solution methods perform slightly faster than the linear fractional-based solution methods. However, in instances where $|H| = 25$, the linear fractional-based MILP performs faster than the probability-based MILP on average. In all instance sizes, the B&C-A algorithms perform faster than the B&C algorithms on average. In larger instance sizes, the B&C-A algorithms emerge as the fastest solution methods, and, in the largest solved instances ($|C| = 800$, $|H| = 25$), the B&C-A algorithms are fastest on average.

		# Solved instances						Solution Time (CPU seconds)*					
		Probability-Based			Linear Fractional-Based			Probability-Based			Linear Fractional-Based		
$\|\mathcal{C}\|$	$\|\mathcal{H}\|$	MILP	B&C	B&C-A	MILP	B&C	B&C-A	MILP	B&C	B&C-A	MILP	B&C	B&C-A
50	25	40	22	23	40	16	20	115.53	1370.31	813.09	100.57	1256.39	1153.6
50	50	39	7	10	39	4	7	1050.74	2241.55	1901.69	1059.24	2239.99	2246.82
50	100	10	0	0	8	1	0	1533.88	-	-	2227.86	2472.39	
100	25	40	19	20	40	15	17	269.07	1530.75	1181.01	225.37	1801.88	1614.62
100	50	32	6	6	29	0	1	1393.93	2951.1	2719.78	1590.47	-	3597.58
100	100	9	0	0	0	0	0	2426.17	-	-	-	-	-
200	25	40	14	18	40	7	12	607.48	2092.68	1505.39	467.91	1209.58	1504.87
200	50	11	0	0	10	1	0	1956.30	-	-	2875.21	221.39	-
200	100	0	0	0	0	0	0	-	-	-	-	-	-
400	25	30	10	10	39	10	10	2414.56	1861.97	1441.79	2220.66	2010.77	1672.35
400	50	0	0	0	0	0	0	-	-	-	-	-	-
400	100	0	0	0	0	0	0	-	-	-	-	-	-
800	25	3	7	7	2	6	7	3274.63	2654.32	1804.43	3253.50	2862.82	1961.45
800	50	0	0	0	0	0	0	-	-	-	-	-	-
800	100	0	0	0	0	0	0	-	-	-	-	-	-
Average		16.93	5.67	6.27	16.47	4.00	4.93	1504.23	2100.38	1623.88	1557.87	1759.40	1964.47

*Average among solved instances.

Table 2: Computational experiments results for HM14 dataset, grouped by $|\mathcal{C}|$ & $|\mathcal{H}|$ (40 instances per row).

7 Case study: Determining Worst-Case Obstetric Unit Closures in Georgia

We now demonstrate the use of our modeling approach to analyze which obstetric unit closures in the state of Georgia would have the worst-case impacts on geographic access to care.

7.1. Context

According to the Georgia Obstetrical and Gynecological Society, 34% (38/112) of Georgia's obstetric units closed between 1994-2020 (Georgia Obstetrical and Gynecological Society 2020). Between 2010 and 2014, Georgia led the country in rural hospital closures in general (Ham *et al.* 2019). Although a large percentage of Georgia's obstetric units have closed in the past thirty years, it is unclear how obstetric closures have affected access to care and the quality of care among units that remain open. In response to these closures, the Rural Hospital Stabilization Committee was formed under the Georgia Department of Community Health to prevent closures by increasing utilization, strengthening financial status, and improving the health outcomes of rural residents. The committee provides support to rural hospitals by supporting staff retention and development, infrastructure improvement, and debt reduction (Ham *et al.* 2019).

In this case study, we apply the MCE-FLP to analyze the current state of obstetric access and identify which potential obstetric unit closures would most increase travel distances to care across the state when considering patient bypassing behavior. This analysis provides insights into which obstetric hospitals should receive closure prevention resources through state and nation-wide programs.

7.2. Problem Setting

In this problem, we assume that there is a set of existing, open obstetric hospitals, $\mathcal{H}$, that serve communities ("consumer zones") of pregnant patients, C. Rather than locating new obstetric units, we are interested in the context where an adversary can close a set number of obstetric units, B, to achieve the worst-case impact on access to care. Our decision variable is as follows $\forall h \in \mathcal{H}$,

$$x_h = \begin{cases} 1, & \text{if } h\text{'s obstetric unit remains open,} \\ 0, & \text{if } h\text{'s obstetric unit is closed.} \end{cases}$$

We adapt (6) so that $z_{ch} = d_{ch}$ where d_{ch} is the distance between community $c \in C$ and hospital $h \in \mathcal{H}$. This objective function reflects that we want to identify the worst-case closures that maximize expected travel distance. Further, we modify constraint (6b) to be $\sum_{h \in \mathcal{H}} (1 - x_h) = B$ to reflect that the decision maker closes B existing obstetric units (as opposed to opening r new units).

We compare the solutions of the MCE-FLP with a model assuming that patients seek care at their closest obstetric hospital. We model this "closest hospital" problem as a MILP,

$$\max_{x} \quad \sum_{c \in C} b_c \sum_{h \in \mathcal{H}_c} d_{ch} y_{ch} \tag{25a}$$

$$\text{s.t.} \quad \sum_{h \in \mathcal{H}} (1 - x_h) = B, \tag{25b}$$

$$d_{ch} y_{ch} \leq d_{ch'} y_{ch} + D_c (1 - x_{h'}), \quad \forall c \in C, h, h' \in \mathcal{H}_c, \tag{25c}$$

$$y_{ch} \leq x_h, \quad \forall c \in C, h \in \mathcal{H}_c, \tag{25d}$$

$$x_h \in \{0, 1\}, \quad \forall h \in \mathcal{H} \tag{25e}$$

where constraints (25c)-(25d) ensure that $y_{ch} = 1$ if obstetric hospital $h \in \mathcal{H}_c$ is the closest open hospital to community $c \in C$, $\mathcal{H}_c$ is a set of the ten closest obstetric hospitals to community

c, and D_c is an upper bound on travel distance of a communities closest obstetric hospital, i.e., $D_c = \max(d_{ch})$ for $h \in \mathcal{H}_c$.

7.3. Data

We parameterize our model using data from the state of Georgia. First, we collected data to infer the spatial distribution of communities that demand obstetric services and the spatial distribution of obstetric hospitals. To estimate the demand for obstetric care services, we used data from the US Census Bureau and electronic birth certificate data obtained from the state Vital Records Office in Atlanta, Georgia. We defined communities of patients as census block groups, of which Georgia has 7,432 (i.e., $|C| = 7,432$). To estimate the location of these communities, we used the latitude and longitude of the population centroid of each census block group (U.S. Census Bureau 2021). To estimate the demand of each community (b_c), we used the number of patients who gave birth and lived in the census block group $c \in C$ in 2019 as reported in the Georgia birth certificate data.

We included the 102 (i.e., $|\mathcal{H}| = 102$) obstetric hospitals in Georgia where more than 10 births occurred in 2019, as reported by Georgia birth certificate data. We obtained each hospital's obstetric level of care (ℓ_h) from Georgia's Department of Public Health public records from 2017 (Barrentine *et al.* 2017). We include the following levels of care: $L = \{\text{BC}, 1, 2, 3, \text{PRC}\}$, where BC indicates Birth Center (the lowest level of care), higher levels of care indicate more specialized obstetric care, and PRC indicates Perinatal Regional Center (the highest level of care). The address, latitude, and longitude of each obstetric hospital were located using Python's geopy package (GeoPy Development Team 2024) and cross-referenced by the study team using Google Maps.

We calculated the distance between obstetric hospitals and communities of patients (d_{ch}) as the road distance in miles between each hospital and each census block group's population centroid coordinates using Open Source Routing Machine (OSRM) in Python (Luxen and Vetter 2011).

7.4. Discrete Choice Model

Using Georgia's birth certificate data on where patients lived and where they delivered, we fit a discrete choice model to describe patients' labor & delivery (L&D) care-seeking behavior. In these models, the obstetric hospitals are considered the alternatives, and the road distance and obstetric level of care are features of these alternatives. Thus, the deterministic part of the utility of a patient

Parameter	Estimate	CI lower	CI upper
β_d	-0.111	-0.111	-0.112
β_{BC}	-2.161	-2.232	-2.090
β_1 (ref)	0	NA	NA
β_2	0.744	0.727	0.760
β_3	1.473	1.456	1.489
β_{PRC}	1.087	1.070	1.105

Table 3: Discrete choice model parameters used in Georgia case study. β_d describes the coefficient related to travel distance to care. β_ℓ describes the coefficient associated with level ℓ care for $\ell \in \{\text{BC}, 1, 2, 3, \text{PRC}\}$ where BC indicates Birth Center and PRC indicates Perinatal Regional Center.

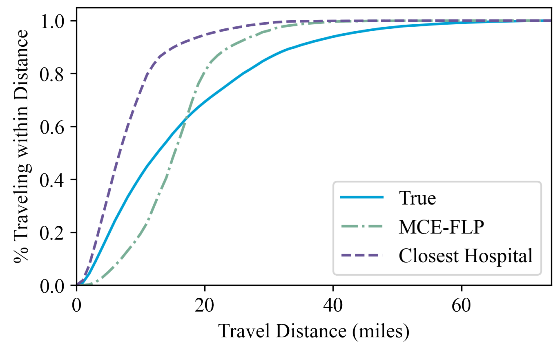


Figure 2: Percentage of births that travel to an obstetric unit within a distance under different patient behavior assumptions. "True" describes the observed distance traveled from Georgia birth records. "MCE-FLP" describes estimated distance traveled according the MCE-FLP when all units remain open. "Closest hospital" describes the estimated distance traveled assuming patients seek care at their closest unit.

from community $c \in C$ seeking care at obstetric hospital $h \in \mathcal{H}$ can be described as,

$$v_{ch} = \beta_d d_{ch} + \sum_{\ell \in L} \beta_\ell a_{\ell h}$$

where $a_{\ell h}$ is equal to 1 if obstetric hospital h is of level of care $\ell \in L = \{\text{BC}, 1, 2, 3, \text{PRC}\}$ and 0 otherwise, and β_d and β_ℓ are calibrated parameters of the MNL model. Using Georgia's birth certificate data, we calibrated the MNL model to obtain the parameters reported in Table 3.

We assume that the "no-choice" alternative is choosing a home birth. In the state of Georgia in 2017, the rate of home births was 0.65% (MacDorman and Declercq 2019). We round this to 0.5% and set $\alpha_c^0 = \frac{1}{199} \sum_{h \in \mathcal{H}} \alpha_{ch}$ so that $p_c^0 = 0.005, \forall c \in C$ when all obstetric units are open.

7.5. Results

We now present the results of our models exploring the impacts of worst-case obstetric unit closures on travel distance to care. *First,* we compare the distance that patients travel to obstetric hospitals for L&D care as observed in birth certificate records with the projected distance that patients would travel according to our MCE-FLP model and the "closest-hospital" model.

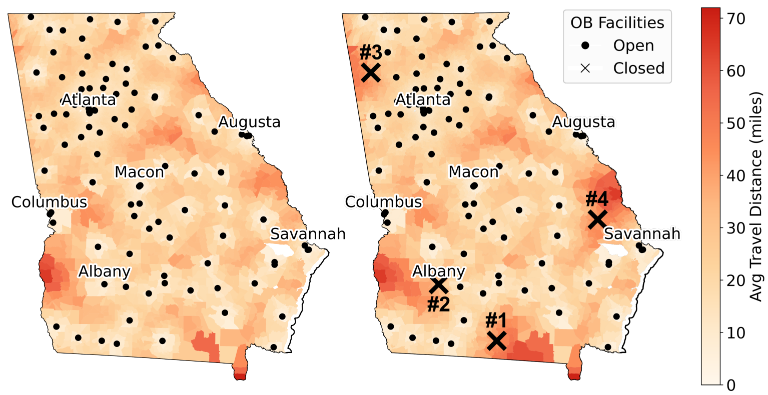
Observation 1. The MCE-FLP under the MNL model better approximates true travel distance to obstetric care, compared with the "closest-hospital" problem.

Figure 2 shows the percentage of deliveries that occur at hospitals within a set travel distance in Georgia in 2019. The estimated average travel distance according to the "closest-hospital" model is 8.1 miles and according to the MCE-FLP is 15.6 miles ($B = 0$), compared to the true average travel distance of 17.0 miles.

Observation 2. The "closest hospital" problem's optimal worst-case closures underestimate the number of births affected and overestimate the impact on travel distance compared with the MCE-FLP under the MNL model.

We solved the MCE-FLP and the "closest hospital" worst-case closure problems for zero to four closures, $B \in \{0, 1, 2, 3, 4\}$. We solved the MCE-FLP worst-case closure model with the probability-based B&C analytical algorithm because it performed the fastest on the largest instances in the computational experiments. To reduce the number of very small choice probabilities, we assumed that patients from each community would not seek obstetric hospitals where the MNL model estimated choice probability was less than their probability of "no-choice".

Both problems' single ($B = 1$) worst-case obstetric unit closure is at a level 2 hospital in south central Georgia (closure #1) (Figure 3). The "closest hospital" solutions are presented in Appendix B. According to the MCE-FLP under the MNL model, this single closure would increase the travel distance to obstetric care of 2,316 (1.9%) patients from an average of 13.3 miles to 41.6 miles (Table 4). We visualize this increase in Figure 4. According to the "closest hospital" model, this closure would increase the travel distance to obstetric care of 2,170 (1.8%) patients from an average of 9.8 miles to 42.3 miles (Table 4, Figure 8). For more than one worst-case closure ($B > 1$), the MCE-FLP and "closest hospital" problems' optimal solutions differ. For any number of closures, $B \in \{1, 2, 3, 4\}$, the "closest hospital" problem's solutions project that closures will affect a smaller population but increase their travel distance more compared with the MCE-FLP solutions (Figure 5). This is because the MCE-FLP under the MNL model assumes that an obstetric unit can capture patients from more communities than just those for which it is the closest hospital. Thus, some patients who would have chosen an obstetric unit before its closure may have an even closer alternative that they were previously choosing to bypass.



(a) Current State (b) Worst-Case Closures

Figure 3: Impacts of one (#1), two (#1 & #2), three (#1 − #3), and four (#1 − #4) worst-case closures on average travel distance in miles by census block group according to the MCE-FLP.

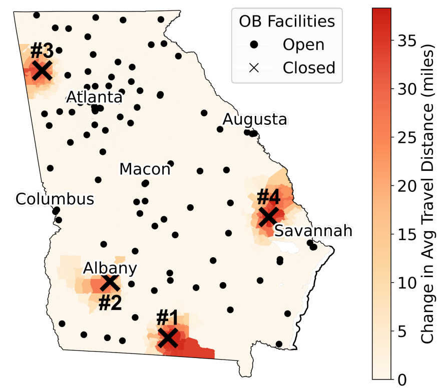


Figure 4: Change in average travel distance after one (#1), two (#1 & #2), three (#1 − #3), and four (#1 − #4) worst-case closures according to the MCE-FLP.

	Current State	**Worst-Case Scenarios**			
	No Closures	One Closure	Two Closures	Three Closures	Four Closures
		Proportion of Populations			
	Georgia Overall	*Projected Population Affected by MCE-FLP Closures*			
Total	122,004 (100%)	2,316 (1.9%)	4,106 (3.4%)	6,389 (5.2%)	8,019 (6.6%)
Rural	24.9%	940 (40.6%)	1430 (34.8%)	2406 (37.7%)	3326 (41.5%)
Below Poverty Level	10.3%	466 (20.1%)	882 (21.5%)	1250 (19.6%)	1493 (18.6%)
Race*					
White	55.9%	1261 (54.5%)	1872 (45.6%)	3653 (57.2%)	4677 (58.3%)
Black	31.5%	863 (37.3%)	1961 (47.8%)	2278 (35.6%)	2756 (34.4%)
AIAN	0.3%	14 (0.6%)	15 (0.4%)	25 (0.4%)	32 (0.4%)
Asian	4.2%	29 (1.3%)	53 (1.3%)	70 (1.1%)	84 (1.1%)
Other	3.2%	45 (1.9%)	65 (1.6%)	132 (2.1%)	166 (2.1%)
Two+	4.9%	101 (4.4%)	136 (3.3%)	227 (3.6%)	298 (3.7%)
		Travel Distance, miles (change from current state)			
Georgia Overall					
Average Travel Distance	15.6	16.1 (+0.5)	16.4 (+0.8)	16.7 (+1.1)	16.9 (+1.4)
To Closest Hospital	8.0	8.6 (+0.6)	8.9 (+0.8)	9.1 (+1.0)	9.4 (+1.3)
To Closest CCO Hospital	21.6	21.6 (+0.0)	26.2 (+4.7)	26.8 (+5.2)	26.8 (+5.2)
Population Affected by Closures					
Average Travel Distance	-	41.6 (+28.2)	37.9 (+25.0)	37.4 (+22.3)	38.3 (+22.3)
To Closest Hospital	-	41.3 (+30.5)	34.7 (+24.3)	30.9 (+19.7)	32.1 (+19.9)
To Closest CCO Hospital	-	79.7 (+0.0)	120.7 (+71.9)	92.7 (+56.1)	84.8 (+44.7)

AIAN: American Indian and Alaska Native

*Native Hawaiian and Other Pacific Islander race statistic not reported due to insufficient sample size.

Table 4: Demographics and travel distances of the projected current state Georgia population and worst-case obstetric unit closures according to the MCE-FLP solutions.

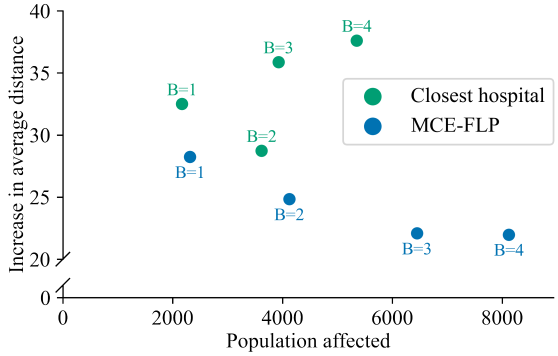


Figure 5: Population affected vs. increase in travel distance among those affected for 1-4 simultaneous obstetric closures in the state of Georgia under MCE-FLP and "closest hospital" solutions.

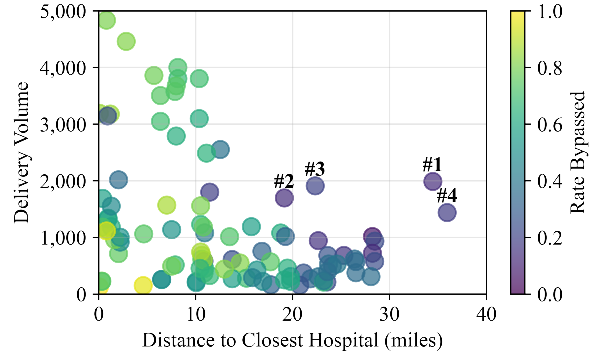


Figure 6: Distance from other obstetric hospitals vs. true delivery volume, color-coded by rate at which each obstetric hospital is bypassed. #1 – #4 represent MCE-FLP worst-case closure solutions.

Next, we explore the demographics of those affected by the MCE-FLP worst-case closures.

Observation 3. The MCE-FLP worst-case closure solutions affect above-average rates of people who live in rural areas, live below the poverty level, are Black or African American, and are American Indian or Alaskan Native.

The MCE-FLP single worst-case obstetric unit closure solution would affect 2,316 (1.9%) of births. However, among those affected, an estimated 40.6% live in rural areas, 20.1% live below the poverty level, 37.3% are Black or African American, and 0.6% are American Indian or Alaskan Native, all of which are higher than Georgia's average rates (Table 4). All of the MCE-FLP worst-case closure solutions would disproportionately affect these groups, and each of these groups experiences increased rates of adverse pregnancy outcomes compared with their urban, above the poverty level, and White counterparts, respectively (Hoyert 2025, Nicholls-Dempsey *et al.* 2023, Merkt *et al.* 2021, Hansen *et al.* 2022). Thus, preventing these worst-case obstetric unit closures is important for mitigating worsening pregnancy outcome disparities.

Finally, we explore the features of the MCE-FLP worst-case obstetric unit closures.

Observation 4. The MCE-FLP worst-case obstetric unit closures are isolated from other obstetric hospitals and have relatively high delivery volume for their isolation. Also, the MCE-FLP worst-case obstetric unit closures are not frequently bypassed.

Figure 6 shows each obstetric hospital's distance to its next closest obstetric hospital and delivery volume in 2019 derived from Georgia's birth certificate data. This figure shows that the

MCE-FLP worst-case obstetric unit closure solutions (#1 – #4) have the highest delivery volume among obstetric hospitals that are more than 15 miles from their next closest obstetric hospital. Also, these obstetric unit closure solutions (#1 – #4) are within the bottom 15% of obstetric units when ranked in terms of the rates at which they are bypassed, meaning that among patients for whom these obstetric units are their closest, these patients rarely bypass these obstetric units to seek L&D care further away. This indicates that the obstetric units at these hospitals provide vital obstetric care in their regions, which concurs with our result that these obstetric unit closures would have the largest population-level impact on travel distance.

8 Conclusion

A widespread trend of obstetric unit closures is decreasing access to care and is associated with worsening pregnancy outcomes, especially in rural regions. Most existing work on obstetric unit closures has studied the impacts of these closures retrospectively (Carroll *et al.* 2022, Fischer *et al.* 2024, Lorch *et al.* 2013, Sullivan *et al.* 2021). However, there has been little work forecasting the potential impacts of future closures to inform efforts to strategically prevent closures in the first place. In this article, we have introduced the Maximum Choice-based Expectation Facility Location Problem (MCE-FLP), which we use to determine the worst-case obstetric unit closures in the state of Georgia. Our framework incorporates patients' obstetric care-seeking behavior and captures the tendency of a substantial proportion of patients to bypass their nearest obstetric units (Thorsen *et al.* 2023, Carrel *et al.* 2023). We find that the MCE-FLP under the multinomial logit (MNL) model better estimates true travel distance to obstetric care, compared with assuming patients choose their closest obstetric hospital. We also find that the worst-case closures affect above-average populations of already marginalized groups, and that these closures are isolated from other obstetric units and are not frequently bypassed. This indicates that the worst-case obstetric unit closures provide critical care to regions of Georgia, and their closures should be prevented.

The MCE-FLP is a generalized facility location problem under random utilities, which extends the maximum capture problem. We show that the MCE-FLP under the MNL model does not have a convex or concave objective function. Thus, the state-of-the-art solution methods for the maximum capture problem (Ljubić and Moreno 2018, Mai and Lodi 2020), which exploit concavity, cannot be directly applied to solve the MCE-FLP. However, we show that the MCE-FLP can be represented as a mixed-integer nonlinear program (MINLP) wherein the objective function is a sum of linear fractional functions. We then proposed two linear reformulations of the MCE-FLP: one probability-based and one linear fractional-based. We also proposed a branch-and-cut (B&C) algorithm for these linear reformulations, where the problem is decomposed into a main problem and subproblems, which evaluate the expected value of the objective for each community. We show that the duals of the respective subproblems can be solved analytically and used to generate optimality cuts. We compare directly solving the linear reformulations, the B&C algorithms, and the B&C algorithms with analytically derived cuts. For smaller instances, we find that directly solving the probability-based formulation has the fastest solution time. However, the B&C analytical solution algorithm had the fastest average solution time on the largest instances tested.

Our article is not without limitations. We spend the majority of this article characterizing the MCE-FLP under the MNL model. While the MNL is popular due to its simple properties, it assumes the independence of irrelevant alternatives (IIA) property, which has been found to violated to some degree in many contexts. Several alternative choice models relax this property, such as the mixed MNL model, where choice probabilities are randomly sampled. Future work may characterize the MCE-FLP under other choice models. Our case study also has limitations. First, we do not consider demand or facilities outside the state of Georgia due to data availability, so our models may not fully capture the obstetric care system in and around the state. Second, we use a very simple MNL model that assumes patients seek their delivery hospital based only on travel distance and level of care. Level of care is a proxy for services offered and quality of care, but future work may incorporate hospital features more aligned with how patients perceive obstetric hospitals' services. We also do not incorporate patient-level characteristics, like pregnancy risk, which could be incorporated to better approximate patient care-seeking behavior. Third, we only consider the travel distance objective as a proxy for access. Future work may incorporate other aspects of access to high-quality care, such as delivery volume, risk-appropriateness or affordability of the obstetric care facilites. Fourth, we do not consider the risk of an obstetric unit's closure in identifying worst-case closures. When identifying which closures should be strategically prevented, it may be worth considering both the risk of the closure occurring and the impact of the closure. Fifth, while we examine the impact on marginalized communities in a post-hoc analysis, we do not explicitly incorporate equity constraints in our mathematical model. Any healthcare system's policy recommendations should explicitly consider the impact on marginalized populations, ensuring that health outcome disparities will not be exacerbated. Future work may involve further investigation into who the potential closures would impact. Finally, we do not explicitly model hospital closure prevention programs' decisions. Future work may assume that a program has a set budget of closure prevention resources and determine where to optimally distribute these resources.

In summary, this article demonstrates a new modeling approach and solution methods for incorporating patient care-seeking behavior into healthcare facility location modeling. We use these approaches to identify the worst-case obstetric unit closures in the state of Georgia, which could inform which hospitals and their obstetric units should be protected to maintain access to obstetric care. These approaches may be applied more widely to facility location modeling problems in which the objective function is an expectation based on consumer choice probabilities.

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A Proofs

Proof. Proof of Proposition 1. To prove that the continuous relaxation of the MCE-FLP is not convex or concave in general, we will use the fact that a function defined on a convex set, S, is concave if and only if its Hessian matrix, $H(x)$, is negative semidefinite $\forall x \in S$ and is convex if and only if its Hessian matrix is positive semidefinite $\forall x \in S$. We will show that there exist cases in which the Hessian of the objective function of the MCE-FLP is positive semidefinite for some feasible decision variables x' and negative semidefinite for other feasible decision variables x''. Denote the objective function as:

$$G(x) = \sum_{c \in C} b_c \frac{\sum_{h \in \mathcal{H}} z_{ch} \alpha_{ch} x_h}{\sum_{h' \in \mathcal{H}} \alpha_{ch'} x_{h'} + \alpha_c^0}.$$

The partial derivative with respect to x_k is then:

$$\frac{\partial G(x)}{\partial x_k} = \sum_{c \in C} b_c \frac{\alpha_{ck}(z_{ck}\gamma - \sum_{h \in \mathcal{H}} z_{ch} \alpha_{ch} x_h)}{\gamma^2},$$

where $\gamma = \sum_{h \in \mathcal{H}} \alpha_{ch} x_h + \alpha_c^0$.

The Hessian matrix, $H(x)$, is then defined by the second partial derivatives, h_{kw}, where

$$
\begin{aligned}
h_{kw} &= \frac{\partial G(x)}{\partial x_k \partial x_w} \\
&= \sum_{c\in C} b_c \Bigg(\frac{\alpha_{ck}\alpha_{cw}(z_{ck} - z_{cw})\gamma}{\gamma^3} \\
&\qquad - \frac{2\alpha_{ck}\alpha_{cw}(z_{ck}\gamma - \sum_{h\in\mathcal{H}} z_{ch}\alpha_{ch}x_h)}{\gamma^3} \Bigg).
\end{aligned}
$$

The Hessian matrix, $H(x)$, is positive semidefinite if and only if all of its principal minors are nonnegative, i.e., $\det(H_i) \geq 0$ for $1 \leq i \leq |\mathcal{H}|$. The Hessian is negative semidefinite if and only if all ith order principal minors are nonpositive for i odd and nonnegative for i even, i.e., $(-1)^i \det(H_i) \geq 0$ for $1 \leq i \leq |\mathcal{H}|$.

Consider a case where $C = [a]$ ($|C| = 1$) and $\mathcal{H} = [1, 2]$ ($|\mathcal{H}| = 2$). In this case, we examine one principal minor,

$$
\begin{aligned}
\det(H_1) &= h_{11} \\
&= b_a \Bigg(-\frac{2\alpha_{a1}^2 (z_{a1}\gamma - (z_{a1}\alpha_{a1}x_1 + z_{a2}\alpha_{a2}x_2)}{\gamma^3} \Bigg) \\
&= -\frac{2b_a\alpha_{a1}^2}{\gamma^3} \Bigg(z_{a1}(\alpha_{a1}x_1 + \alpha_{a2}x_2 + \alpha_a^0) \\
&\qquad - z_{a1}\alpha_{a1}x_1 - z_{a2}\alpha_{a2}x_2 \Bigg) \\
&= -\frac{2b_a\alpha_{a1}^2}{\gamma^3} \left(z_{a1}\alpha_{a2}x_2 + z_{a1}\alpha_a^0 - z_{a2}\alpha_{a2}x_2 \right) \\
&= -\frac{2b_a\alpha_{a1}^2}{\gamma^3} \left(\alpha_{a2}x_2(z_{a1} - z_{a2}) + z_{a1}\alpha_a^0 \right)
\end{aligned}
$$

If $x_2' = 0$, then $\det(H_1) < 0$. However, if $x_2'' = 1$ and $z_{a2} > (1 + \frac{\alpha_a^0}{\alpha_{a2}})z_{a1}$, then $\det(A_1) > 0$ showing that this matrix is positive semidefinite for some feasible values of x and negative semidefinite for others.

□

Proof. Proof of Proposition 2. We can take advantage of complementary slackness to derive an analytical solution to the dual subproblem, $DSP_c^P(x)$.

We have two cases for each facility $h \in \mathcal{H}$:

Case 1: The facility is closed ($\hat{x}_h = p_{ch} = 0$). Let $\mathcal{H}_0$ be the set of facilities for which this is true.

In this case, we have that constraint (18c) reduces to $p_c^0 \leq 1$, which is loose in this case assuming there is at least one open facility $h \in \mathcal{H}$ $(r > 0)$. By complementary slackness, this constraint's corresponding dual variable $\varepsilon_h = 0, \forall h \in \mathcal{H}_0$. In this case, constraint (18d) reduces to $0 \leq \frac{\alpha_{ch}}{\alpha_c^0} p_c^0$, which again is loose in this case. By complementary slackness, this constraint's corresponding dual variable $\vartheta_h = 0, \forall h \in \mathcal{H}_0$.

We can also reduce the terms in the objective for this case. Term $\varepsilon_h(1 - \hat{x}_h) = 0$ because $\varepsilon_h = 0$, $\forall h \in \mathcal{H}_0$. Term $\lambda_h \left(\frac{\alpha_{ch}}{\alpha_{ch}+\alpha_c^0}\right) \hat{x}_h = 0$ because $\hat{x}_h = 0, \forall h \in \mathcal{H}_0$.

Case 2: The facility is open ($\hat{x}_h = 1, p_{ch} > 0$). Let $\mathcal{H}_1$ be the set of facilities for which this is true.

In this case, we have that constraint (18e) reduces to $p_{ch} \leq \frac{\alpha_{ch}}{\alpha_{ch}+\alpha_c^0}$, which is loose in this case assuming there are more than one open facilities $h \in \mathcal{H}$ $(r > 1)$. By complementary slackness, this constraint's corresponding dual variable $\lambda_h = 0, \forall h \in \mathcal{H}_1$. Because $p_{ch} > 0 \ \forall h \in \mathcal{H}_1$, by complementary slackness, the corresponding dual constraints (19b) hold at equality.

We can also reduce the terms in the objective for this case. Term $\varepsilon_h(1 - \hat{x}_h) = 0$ because $\hat{x}_h = 1$, $\forall h \in \mathcal{H}_1$. Term $\lambda_h \left(\frac{\alpha_{ch}}{\alpha_{ch}+\alpha_c^0}\right) \hat{x}_h = 0$ because $\lambda_h = 0, \forall h \in \mathcal{H}_1$.

In all cases, $p_c^0 > 0$, and by complementary slackness, the corresponding dual constraint (19c) holds at equality.

Thus, we can rewrite the dual subproblem, $DSP_c^{LF}(x)$ as:

$$\min_{\gamma,\varepsilon,\vartheta,\lambda} \quad \gamma \tag{26a}$$

$$\text{s.t.} \quad \gamma + \lambda_h \geq b_c z_{ch}, \qquad \forall h \in \mathcal{H}_0, \tag{26b}$$

$$\gamma - \frac{\alpha_c^0}{\alpha_{ch}}\varepsilon_h + \vartheta_h = b_c z_{ch}, \qquad \forall h \in \mathcal{H}_1, \tag{26c}$$

$$\gamma + \sum_{h\in\mathcal{H}} \varepsilon_h - \sum_{h\in\mathcal{H}} \frac{\alpha_{ch}}{\alpha_c^0}\vartheta_h = 0, \tag{26d}$$

$$\varepsilon_h, \vartheta_h = 0, \qquad \forall h \in \mathcal{H}_0, \tag{26e}$$

$$\lambda_h = 0, \qquad \forall h \in \mathcal{H}_1, \tag{26f}$$

$$\varepsilon_h, \vartheta_h, \lambda_h \geq 0, \qquad \forall h \in \mathcal{H} \tag{26g}$$

By strong duality,

$$\gamma^* = b_c \frac{\sum_{h\in\mathcal{H}} z_{ch}\alpha_{ch}\hat{x}_{h'}}{\sum_{h'\in\mathcal{H}} \alpha_{ch'}\hat{x}_{h'} + \alpha_c^0}$$

And we can derive,

$$\vartheta_h - \frac{\alpha_c^0}{\alpha_{ch}}\varepsilon_h = (b_c z_{ch} - \gamma)\hat{x}_h \qquad \forall h \in \mathcal{H} \tag{27}$$

$$\lambda_h \geq (b_c z_{ch} - \gamma)(1 - \hat{x}_h) \qquad \forall h \in \mathcal{H} \tag{28}$$

Thus, the solutions in Proposition 2 satisfy the constraints, and by strong duality, this is an optimal solution.

□

Proof. Proof of Proposition 3. Assuming that the bound on t_c is never tight (i.e., $t_c < T_c, \forall c \in C$), we can take advantage of complementary slackness to derive an analytical solution to the dual subproblem, $DSP_c^{LF}(x)$.

Under this assumption, we have two cases for each facility $h \in \mathcal{H}$:

Case 1: The facility is closed ($\hat{x}_h = w_{ch} = 0, 0 < t_c < T_c$). Let $\mathcal{H}_0$ be the set of facilities for which this is true.

In this case, we have that constraint (22c) reduces to $t_c \leq T_c$, which is loose by assumption in this case. By complementary slackness, this constraint's corresponding dual variable $\varepsilon_h = 0, \forall h \in \mathcal{H}_0$. Constraint (22d) reduces to $0 \leq t_c$, which is also loose in this case. Again, by complementary slackness, this constraint's corresponding dual variable $\vartheta_h = 0, \forall h \in \mathcal{H}_0$.

We can also reduce the terms in the objective for this case. Term $\varepsilon_h(1 - \hat{x}_h)T_c = 0$ because $\varepsilon_h = 0, \forall h \in \mathcal{H}_0$. Term $\lambda_h \hat{x}_h T_c = 0$ because $\hat{x}_h = 0, \forall h \in \mathcal{H}_0$.

Case 2: The facility is open ($\hat{x}_h = 1, 0 < w_{ch} = t_c < T_c$). Let $\mathcal{H}_1$ be the set of facilities for which this is true.

In this case, we have that constraint (22e) reduces to $w_{ch} \leq T_c$, which is always loose by assumption in this case. By complementary slackness, this constraint's corresponding dual variable $\lambda_h = 0, \forall h \in \mathcal{H}_1$. Because $t_c > 0 \; \forall h \in \mathcal{H}_1$, by complementary slackness, the corresponding dual constraint (23b) holds at equality.

We can reduce the terms in the objective for this case. Term $\varepsilon_h(1 - \hat{x}_h)T_c = 0$ because $\hat{x}_h = 1$, $\forall h \in \mathcal{H}_1$. Term $\lambda_h \hat{x}_h T_c = 0$ because $\lambda_h = 0, \forall h \in \mathcal{H}_1$.

Thus, we can rewrite the dual subproblem, $DSP_c^{LF}(x)$ as:

$$\min_{\gamma, \varepsilon, \vartheta, \lambda} \quad \gamma \tag{29a}$$

$$\text{s.t.} \quad \alpha_{ch}\gamma + \lambda_h \geq b_c z_{ch} \alpha_{ch}, \qquad \forall h \in \mathcal{H}_0, \tag{29b}$$

$$\alpha_{ch}\gamma - \varepsilon_h + \vartheta_h = b_c z_{ch} \alpha_{ch}, \qquad \forall h \in \mathcal{H}_1, \tag{29c}$$

$$\alpha_c^0 \gamma + \sum_{h \in \mathcal{H}} \varepsilon_h - \sum_{h \in \mathcal{H}} \vartheta_h \geq 0, \tag{29d}$$

$$\varepsilon_h, \vartheta_h = 0, \qquad \forall h \in \mathcal{H}_0, \tag{29e}$$

$$\lambda_h = 0, \qquad \forall h \in \mathcal{H}_1, \tag{29f}$$

$$\varepsilon_h, \vartheta_h, \lambda_h \geq 0, \qquad \forall h \in \mathcal{H} \tag{29g}$$

By strong duality,

$$\gamma^* = b_c \frac{\sum_{h \in \mathcal{H}} z_{ch} \alpha_{ch} \hat{x}_{h'}}{\sum_{h' \in \mathcal{H}} \alpha_{ch'} \hat{x}_{h'} + \alpha_c^0}$$

And we can derive,

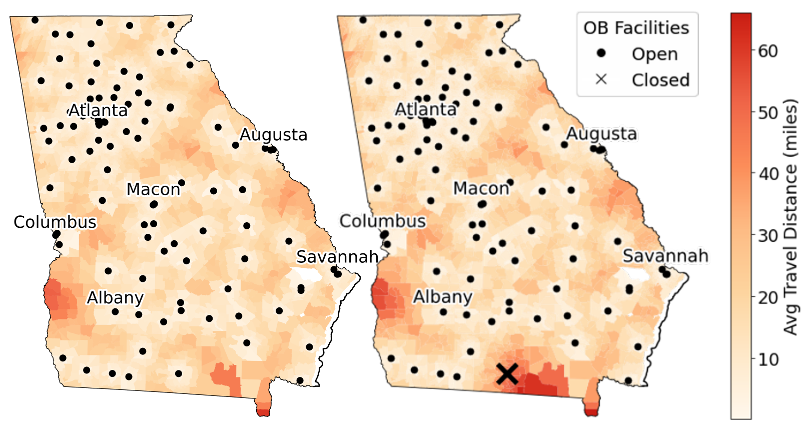
$$\vartheta_h - \varepsilon_h = (b_c z_{ch} \alpha_{ch} - \alpha_{ch}\gamma)\hat{x}_h \qquad \forall h \in \mathcal{H} \tag{30}$$

$$\lambda_h \geq (b_c z_{ch} \alpha_{ch} - \alpha_{ch}\gamma)(1 - \hat{x}_h) \qquad \forall h \in \mathcal{H} \tag{31}$$

Thus, the solutions in Proposition 3 satisfy the constraints, and by strong duality, this is an optimal solution.

□

B "Closest hospital" solutions



(a) Current State (b) One Worst-Case Closure

Figure 7: Impacts of one worst-case closure on average travel distance in miles by census block group according to the closest hospital problem.

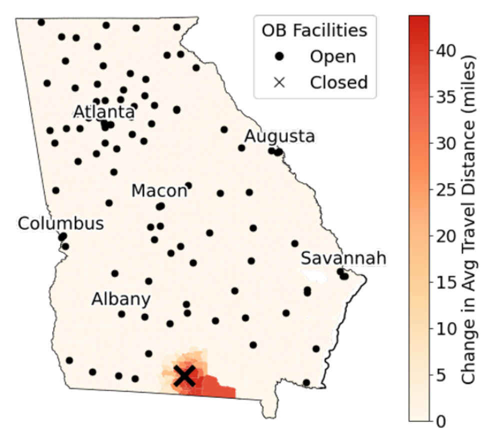
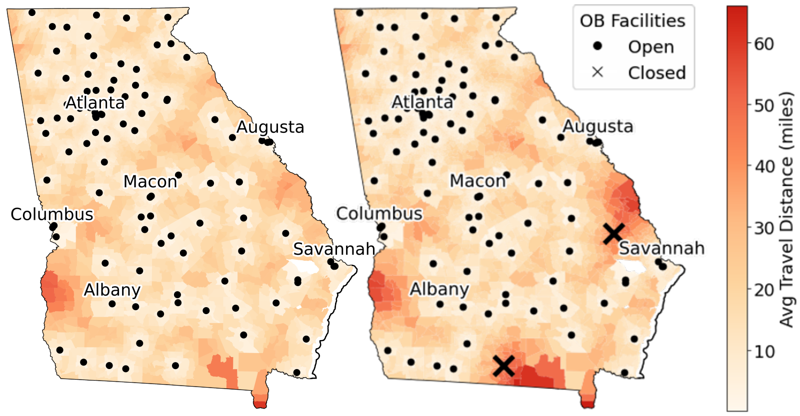


Figure 8: Change in average travel distance after one worst-case closure according to the closest hospital problem.



(a) Current State (b) Two Worst-Case Closures

Figure 9: Impacts of two worst-case closures on average travel distance in miles by census block group according to the closest hospital problem.

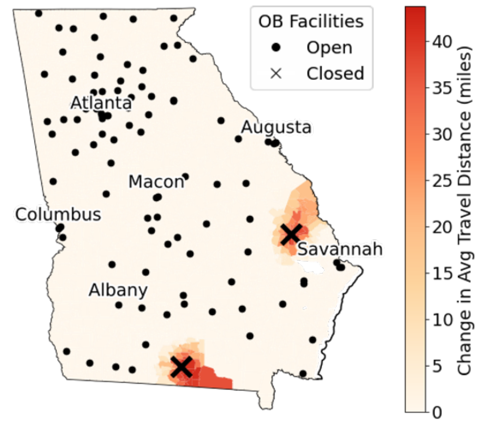
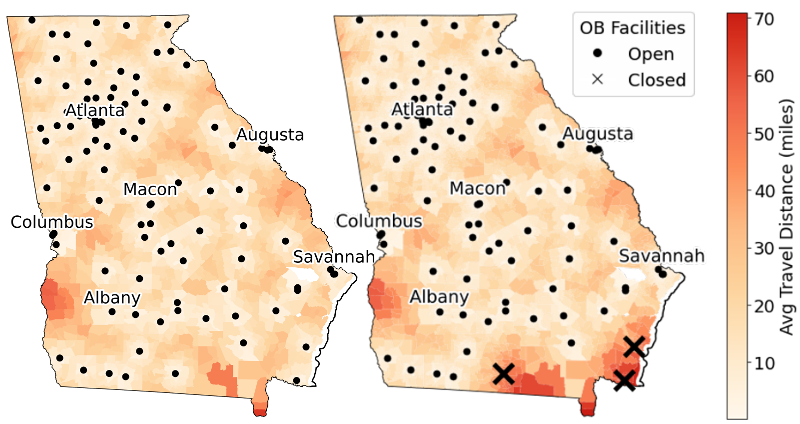


Figure 10: Change in average travel distance after two worst-case closures according to the closest hospital problem.



(a) Current State (b) Three Worst-Case Closures

Figure 11: Impacts of three worst-case courses on average travel distance in miles by census block group according to the closest hospital problem.

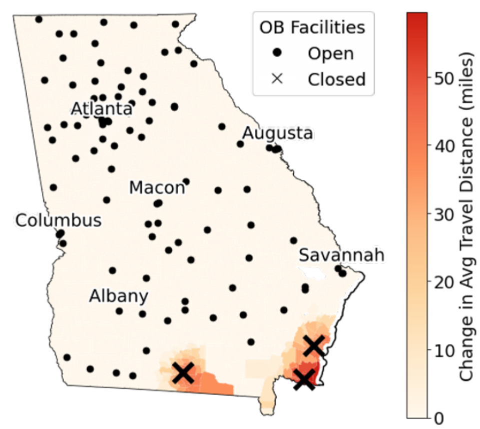
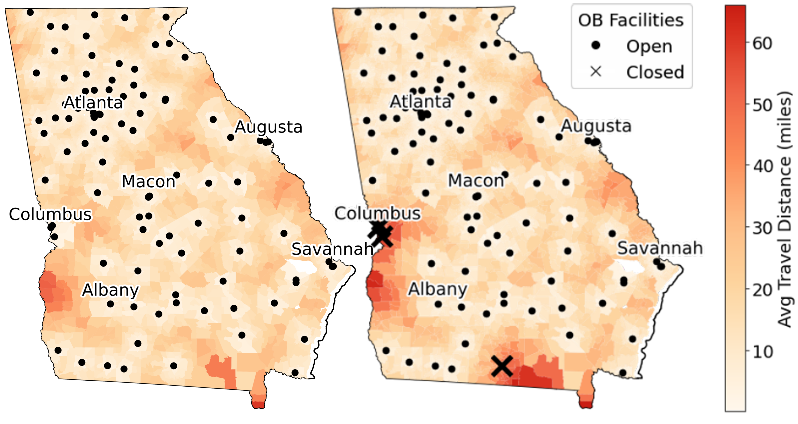


Figure 12: Change in average travel distance after three worst-case closures according to the closest hospital problem.



(a) Current State

(b) Four Worst-Case Closures

Figure 13: Impacts of four worst-case closures on average travel distance in miles by census block group according to the closest hospital problem.

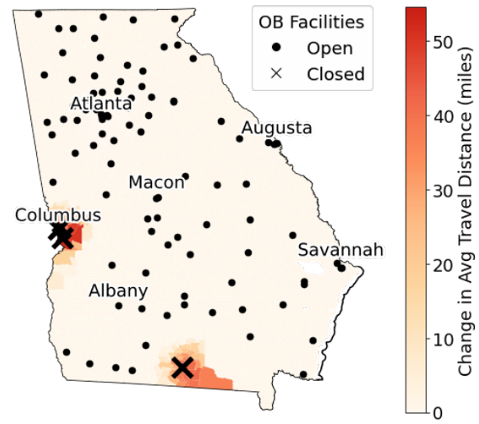


Figure 14: Change in average travel distance after four worst-case closures according to the closest hospital problem.