Collection points placement in urban delivery: A game-theoretic analysis of public and competitive strategies

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Abstract

Collection point networks are rapidly expanding as delivery companies and public authorities promote their implementation to consolidate deliveries and reduce urban congestion. However, rather than catering to public interest by maximizing accessibility, the placement of collection points remains primarily driven by competition among delivery companies, which seek to maximize their market share. This paper thus aims to investigate how delivery companies strategically open collection points in response to competitors and how their decisions diverge from an optimal public placement. In this context, we propose a competitive location model where collection points are placed within a network of districts and a multinomial logit framework is used to model customer choices. In this competitive setting, we analyze the Nash equilibria and measure potential inefficiencies using the price of stability. This analysis includes a theoretical two-district setting, where we find a significant mismatch between competitive and optimal public placement, leading to potential efficiency losses of up to 33%. In contrast, a case study based on a Dutch city shows a narrower difference between public and strategic placements. At the same time, this case study illustrates that it is not easy for authorities to close the gap between public and strategic placements, for example, by promoting the attractiveness of all or those collection points of particular interest to a public authority.

Keywords— Game theory; Facilities planning and design; Location; Collection points; Discrete choice modelling

1 Introduction

Delivery companies, particularly in Europe, are rapidly introducing and expanding networks of urban *collection points* for their last-mile delivery (DHL, 2024; DPD, 2023; Reuters, 2025). These designated points, for example, in supermarkets or train stations, serve as temporary storage hubs where parcels are delivered and later retrieved by customers. They can be automated, such as parcel lockers, or staffed service points, and are utilized for both the collection and return of parcels. The motivation behind these investments in collection points is their potential to consolidate deliveries, significantly reduce travel distances, and decrease the number of failed deliveries. This positively impacts last-mile routing costs while supporting environmental goals, such as lower levels of emissions, noise, and air pollution, as well as traffic congestion within cities (Janinhoff et al., 2024). As such, many cities aim to promote the installation of collection points (Molin, Kosicki, and Van Duin, 2022). However, in most cases, the positioning of these points remains primarily driven by *competitive* factors, with delivery companies seeking to increase their *market share* rather than maximizing overall utilization of collection points. As a result, competing delivery companies may place their collection points near areas that are particularly busy or populated, creating overlaps in these areas while other neighborhoods remain underserved. This suboptimal distribution creates inconvenience for consumers who live in less densely populated areas, and may result in inefficient use of urban space and resources from a public authority perspective. A more coordinated effort in terms of location decisions could thus lead to a more balanced distribution of collection points across different regions, improving accessibility for all consumers. Inspired by these dynamics, this paper investigates the strategic positioning of collection points in competitive settings and how this positioning may differ from the optimal placement of a public authority (e.g., a municipality locating collection points).

For this investigation, we propose a game-theoretical model between two delivery companies, where each company can position a set of collection points within a network. Nodes in this network represent individual *districts* or neighborhoods in a city center, and are weighted according to the number of potential customers within each district. Edges represent the streets between districts, with their weights corresponding to the lengths of the roads. Collection points can, in this context, only be placed in a subset of districts due to specific prerequisites, partnerships, or regulations (e.g., they may need to be situated adjacent to a store rather than arbitrarily placed on the sidewalk, or in areas with higher accessibility for people with disabilities). In contrast, customers in districts may choose to either go to collection points or opt for home delivery. Customer choices are modeled using a *multinomial logit* (MNL) model, which considers the willingness of customers to use a particular collection point, location, or company, as well as their sensitivity to travel distances. The payoff for both companies is then determined by the expected number of customers using their collection points. Given that collection points can be relocated somewhat easily, it is plausible that companies adjust their locations in response to competitors' decisions similarly to best-response behavior. While such dynamics do not always converge, when they do, the resulting outcome corresponds to a pure Nash equilibrium of the simultaneous game (Amiet et al., 2021). This provides a practical motivation for using pure Nash equilibria as the solution concept. By studying these equilibria, we can capture the strategic decision-making of companies and compare it to the optimal decision-making of a public authority. For this comparison, we assume that the public authority centrally controls the decision-making, maximizing the capture of the interested population. Subsequently, we compare the total payoff, i.e., the total expected number of customers using the collection points, in an equilibrium solution with the total payoff under the solution with a public authority, which we refer to as the social solution. We quantify this loss using the well-known price of stability (Anshelevich et al., 2008). To gain insights, we first focus our analysis on a simplified, stylized setting before investigating the dynamics in the context of a case study for the Dutch city of Eindhoven. For the stylized setting, with two districts and one collection point per company, we find that companies prefer the same district when travel sensitivity of customers is low, while higher sensitivity incentivizes companies to spread the collection points —though this tendency is constrained by asymmetries in the population size of districts. The decision-making of the public authority, in contrast, indicates a stronger tendency toward spreading locations, especially when customer travel sensitivity is high. As a result, competitive decisions can lead to a loss of up to 33% of potential customers relative to the public authority decision. For the case study in Eindhoven, where each company has multiple collection points and 220 districts are considered, the loss of potential customers is considerably lower, lying below 10% in all experiment settings. Furthermore, in almost half of the settings, the competitive equilibrium coincides with the public authority's decision. These findings suggest that expanded locational flexibility on a given city region may reduce the gap between strategic behavior and the public authority's decision. In the context of this analysis, we also examine the location preferences of companies and public authorities in relation to customers' attitudes toward traveling and their interest in collection points. In cases where customers are less sensitive to travel, we observe that companies tend to choose the same locations in both the stylized and case study settings. Specifically, they select the city center as the primary collection point and then opt for the most densely populated areas if additional collection points are placed. On the other hand, when customers are more sensitive to travel, companies tend to differ in their location choices, focusing on densely populated areas if there is low interest in collection points, or spreading the collection points throughout the city if customer interest is higher. As a general observation, we also see that the strategic decisions of companies in a competitive setting lead to less diversification of selected locations in terms of the authority's interest, corresponding to a loss in population capture. This reduced differentiation is most evident when both interest toward collection points and travel sensitivity are high.

Given these findings, we also investigate how delivery companies can be encouraged to adopt location decisions that align with the public authority's decision, e.g., by promoting collection points through public awareness campaigns or selectively investing in infrastructure around some collection point locations. Our experiments in this regard demonstrate that these initiatives cannot always guarantee the alignment of strategic collection point locations with the public authority's decisions. However, these efforts consistently result in an overall increase in the use of collection points.

The remainder of the paper is organized as follows: In §2, we provide a literature review of the most relevant contributions to the related research disciplines. In §3 we introduce the model. §4 then provides an analysis of the two-district stylized setting, while §5 shows the case study in the city of Eindhoven. Finally, our conclusions are given in §6.

2 Literature overview

Given their practical relevance, collection points have become an increasingly prominent field of study within the operations research literature. More specifically, research in this area has focused on three key themes: facility location, vehicle routing, and location-routing models (Janinhoff et al., 2024). Facility location models aim to identify the optimal placement of collection points to maximize accessibility and minimize costs, considering factors such as customer demand and geographic distribution (Deutsch and Golany, 2018; Lin et al., 2020; Lin et al., 2022; Lyu and Teo, 2022; Mercurio et al., 2023). Vehicle routing and location-routing models, in contrast, focus on optimizing the routes taken by delivery vehicles to these collection points and compare them with home delivery relative to travel distance, timely deliveries, and estimated savings (Hong et al., 2019; Orenstein, Raviv, and Sadan, 2019; Peppel and Spinler, 2022; Zhang, Xu, and S. Wang, 2023). Our work aligns with the first theme, for which location decisions are generally highly dependent on the customers' choice regarding the selected locations.

These customer choices towards locations have been modeled in various ways within existing location models (Marianov and H.A. Eiselt, 2024, Section 3.3). Some studies consider customers to make deterministic choices, e.g., always opting for the closest collection points (see, e.g., Deutsch and Golany (2018); Lee et al. (2019); Y. Wang et al. (2022); Mercurio et al. (2023)). However, probabilistic models of customer behavior, such as discrete choice frameworks (Lin et al., 2022; Lin et al., 2020; Lyu and Teo, 2022), provide a more realistic approach to model the customer decision process, by accounting for preference heterogeneity, as argued in Janinhoff et al. (2024). Lin et al. (2020) and Lyu and Teo (2022), for example, model customer behavior using the MNL model and test its applicability through case studies in Singapore, using real data on factors such as distance sensitivity. Our paper departs from these works by moving from a single-company placement problem to a competitive two-company setting, and by including the possibility of customers opting out, which differs from Lin et al. (2020).

By modeling the strategic location of collection points, our work also closely aligns with the literature on game-theoretic competitive facility location. However, this stream of literature (Z. Drezner and HA Eiselt, 2024) is primarily based on settings where the placement of a facility has long-lasting implications and cannot be easily reversed or relocated, such as supermarkets (see, e.g., T. Drezner (2019); Z. Drezner and HA Eiselt (2024)). As such, the strategic positioning of a facility often depends on the choices made by earlier movers, leading to dynamics where players must anticipate their competitors' actions and thus make decisions in a sequential manner. In many of these sequential location games, which are modeled as bi-level optimization problems, the primary focus is then on developing methods and techniques to find Stackelberg equilibria efficiently (see, e.g., Z. Drezner and HA Eiselt (2024), Section 5.5, T. Drezner (2019); Aboolian, Berman, and Krass (2007b); Aboolian, Berman, and Krass (2007a)). This sequential structure, triggered by the difficulty in relocating open facilities, contrasts with the more flexible nature of collection points, which can be relocated more easily, and for which a simultaneous modeling approach is therefore more natural. Several papers in the competitive facility location literature consider simultaneous decision-making settings. Customer choices have been modeled both deterministically and probabilistically within this context. Deterministic variants are presented by Gur, Saban, and Stier-Moses (2018); Godinho and Dias (2010), and Fournier and Scarsini (2019), where the latter two also analyze the inefficiency of strategic location decisions, similar to our paper. Probabilistic choice modeling is considered in Sáiz, Hendrix, and Pelegrín (2011) and Saidani, Chu, and Chen (2012). However, these works differ from our setting, as they use the Huff model instead of MNL. MNL, differently to Huff model, allows to accommodate uncertainty as there can be unobserved factors in the utility which is random in the MNL, but deterministic in the Huff model. Moreover, an analysis of the price of stability is not included in these papers.

Finally, in recent years, significant developments have been made in finding Nash equilibria for competitive, simultaneous location problems. For example, Dragotto and Scatamacchia (2023) and Crönert and Minner (2024) develop methodologies for the computation of pure and mixed Nash equilibria for general integer programming games, respectively, and test them on location games, of which our model is a special case.[1] The focus of our paper is on characterizing the nature of Nash equilibria and the inefficiency of competition; however, the methods used in integer programming games can be relevant for efficiently finding equilibria in our context.

In Table 1, we frame our paper in the referenced literature according to the distinction between sequential and simultaneous decisions, deterministic or probabilistic selection from customers, and the presence of analysis of the inefficiency of strategic decisions.

Table 1: Comparison with existing work in Competitive Facility Location. *S-P* refers to the analysis of the mismatch between strategic and public optimal solutions.

	Sequential	*Simultaneous*	*Deterministic*	*Probabilistic*	*S-P*
Lin, Tian, and Zhao (2022)	x			Multiple	x
Qi, Jiang, and Shen (2024)	x			MNL (without opt-out)	
Gur, Saban, and Stier-Moses (2018)		x	x		
Godinho and Dias (2010)		x	x		x
Fournier and Scarsini (2019)		x	x		x
Sáiz, Hendrix, and Pelegrín (2011)		x	x	Huff	
Saidani, Chu, and Chen (2012)		x		Huff	
Crönert and Minner (2024)		x		Multiple (without opt-out)	
Dragotto and Scatamacchia (2023)		x		Multiple (without opt-out)	x
This paper		x		MNL (with opt-out)	x

3 Collection point location game

We consider a game $\mathscr{G}$ between two companies, named company 1 and 2, who can open collection points to serve customers in a city center. This center is divided into several districts, and we denote the set of districts by $D \subseteq \mathbb{N}_+$. These districts could, for instance, represent small neighborhoods in a city center. The number of (potential) customers in district $i \in D$ is denoted by $m_i \in \mathbb{N}_+$. We denote the number of collection points of company 1 by $B_1 \in \mathbb{N}_+$ and the number of collection points of company 2 by $B_2 \in \mathbb{N}_+$. These collection points can only be positioned in a subset $L \subseteq D$ of the districts, which we refer to as collection point locations. In practice, these locations could represent sites with high foot traffic such as supermarkets, pharmacies, or schools. The (average) time for each customer of district $i \in D$ to reach collection point location $j \in L$ is represented by $t_j^i \in \mathbb{R}_+$. We denote the (positioning) strategy of company 1 by $x = (x_j)_{j \in L} \in \{0,1\}^L$ with $x_j = 1$ if a collection point has been positioned in district $j \in L$,

[1]We refer to Carvalho et al. (2023) and Carvalho et al. (2025) for a gentle introduction into integer programming games.

and $x_j = 0$ otherwise. Similarly, we denote the strategy of company 2 by $y = (y_j)_{j\in L} \in \{0,1\}^L$. Thus, the sets of all pure strategies of company 1 and company 2, respectively, read

$$\mathscr{X} = \left\{ (x_j)_{j\in L} \in \{0,1\}^L \middle| \sum_{j\in L} x_j \leq B_1 \right\} \text{ and } \mathscr{Y} = \left\{ (y_j)_{j\in L} \in \{0,1\}^L \middle| \sum_{j\in L} y_j \leq B_2 \right\}.$$

Once collection points are positioned, the customers within the districts face a delivery choice. In practice, this choice may arise, for example, when a customer purchases a product online and is subsequently asked to select a delivery option, such as home delivery or pickup at a nearby collection point, often linked to a specific delivery company. We model this choice according to a multinomial logit model (see, e.g., Bierlaire and Lurkin (2017)). This logit model assumes that decision-makers opt for their preference from a finite, non-empty set of alternatives. For the traceability of our game, we consider customers per district as one homogeneous group of decision-makers who need to choose between the open collection points (from company 1 or 2) and the option not to select any of them, which we denote by "0". This option may represent the choice to be served alternatively (e.g., delivery at home). Thus, for any district $i \in D$ and location decisions $x \in \mathscr{X}$ and $y \in \mathscr{Y}$ the choice set is given by:

$$\mathscr{C}(x,y) = \left\{ (j,1)_{j\in L} \middle| x_j = 1 \right\} \cup \left\{ (j,2)_{j\in L} \middle| y_j = 1 \right\} \cup \{0\}.$$

A fundamental assumption behind logit models is that the decision-makers are rational utility maximizers. That means the decision makers select the option that maximizes their utility function. However, the exact specification of this utility function U is unknown and, therefore, modeled as a continuous random variable. In the case of our game $\mathscr{G}$, given district $i \in D$, the positions of collection points $x \in \mathscr{X}$ and $y \in \mathscr{Y}$, and choice $c \in \mathscr{C}(x,y)\backslash\{0\}$, the random utility function contains a deterministic function V depending on the attributes of the possible choices, such as location distance from the customer or attractiveness, as well as a continuous error term ε_c^i, following an extreme value distribution, which captures unobservable attributes. Formally, the random utility function is $U_c^i(x,y) = V_c^i(x,y) + \varepsilon_c^i$, with $V_0^i(x,y) = 0$ and $U_0^i(x,y) = V_0^i(x,y) + \varepsilon_c^i = \varepsilon_c^i$[2]. In our case, the deterministic utility function, $V_c^i(x,y)$, associated with choice $c = (j,k) \in \mathscr{C}(x,y)\backslash\{0\}$ equals:[3]

$$V_{jk}^i = \alpha_{jk}^i - \beta t_j^i \tag{1}$$

[2]We decided to normalize this utility to zero, which is not restrictive.

[3]Please, note that the dependence of V_{jk}^i in Expression (1) of (x,y) is implicit in $c = (j,k) \in \mathscr{C}(x,y)\backslash\{0\}$.

with $\alpha^i_{jk} \in \mathbb{R}_+$ being the initial interest of customers of district $i \in D$ to use a collection point at location $j \in L$, owned by company $k \in \{1, 2\}$, and $\beta \in \mathbb{R}_+$ being a time-dependent parameter describing the sensitivity of customers to travel to a collection point. Please note that the sign in front of parameter β indicates that the further away a collection point is, the less attractive it becomes for customers, which aligns with field observations discussed in Niemeijer and Buijs (2023). Moreover, note that the parameter α^i_{jk} allows for differentiation based on the type of collection point location (e.g., a supermarket versus a gas station), the company, and the number of customers per district.

Consequently, the probability that customers of district $i \in D$ decide to select the collection point of company 1 at location $j \in L$ equals

$$\mathbb{P}^{ij}_1(x, y) = \frac{x_j e^{V^i_{j1}}}{e^{V^i_0} + \sum_{l \in L} \left(x_l e^{V^i_{l1}} + y_l e^{V^i_{l2}} \right)} = \frac{x_j e^{\alpha^i_{j1} - \beta t^i_j}}{1 + \sum_{l \in L} \left(x_l e^{\alpha^i_{l1} - \beta t^i_l} + y_l e^{\alpha^i_{l2} - \beta t^i_l} \right)}.$$

By summing over all locations $j \in L$ where company 1 has placed a collection point, we obtain the probability that customers of district i are using collection points of company 1, and define $\mathbb{P}^i_1(x, y) = \sum_{j \in L} \mathbb{P}^{ij}_1(x, y)$. The expected number of customers using the collection points of company 1, i.e., the expected market share of company 1, then equals

$$I_1(x, y) = \sum_{i \in D} m_i \mathbb{P}^i_1(x, y) = \sum_{i \in D} \left(m_i \frac{\sum_{j \in L} x_j e^{\alpha^i_{j1} - \beta t^i_j}}{1 + \sum_{j \in L} \left(x_j e^{\alpha^i_{j1} - \beta t^i_j} + y_j e^{\alpha^i_{j2} - \beta t^i_j} \right)} \right)$$

and, similarly, we obtain the expected market share for company 2 given by

$$I_2(x, y) = \sum_{i \in D} m_i \mathbb{P}^i_2(x, y) = \sum_{i \in D} \left(m_i \frac{\sum_{j \in L} y_j e^{\alpha^i_{j2} - \beta t^i_j}}{1 + \sum_{j \in L} \left(x_j e^{\alpha^i_{j1} - \beta t^i_j} + y_j e^{\alpha^i_{j2} - \beta t^i_j} \right)} \right).$$

3.1 Pure Nash equilibria

As discussed in the introduction of this paper, we are interested in how companies strategically choose their collection points. To analyze this, we investigate Nash equilibria, a well-established concept in game theory that provides valuable insights into the dynamics of competitive decision-making and optimal strategy selection, in our game $\mathscr{G}$. A pure Nash equilibrium specifies players' decisions from which no player can unilaterally increase their payoff (Osborne and Rubinstein, 1994). In our game $\mathscr{G}$, this corresponds to a *strategy profile* (x^*, y^*) such that

$I_1(x^*, y^*) \geq I_1(x, y^*)\, \forall x \in \mathscr{X}$ and $I_2(x^*, y^*) \geq I_2(x^*, y)\, \forall y \in \mathscr{Y}$. We define $\Pi(\mathscr{G})$ as the set of (pure) Nash equilibria of $\mathscr{G}$. Note that it is a pure strategy for a company $k \in \{1, 2\}$ to locate fewer collection points than its maximum number B_k. However, this strategy does not lead to a Nash equilibrium since the company can unilaterally increase its payoff by opening more collection points, as reported in Proposition 1.

Proposition 1. *A Nash equilibrium* $(x^*, y^*) \in \Pi(\mathscr{G})$ *satisfies* $\sum_{j\in L} x_j^* = B_1$ *and* $\sum_{j\in L} y_j^* = B_2$.

Proof. For conciseness, let $u_{jk}^i = e^{\alpha_{jk}^i - \beta t_j^i}$ for any $i \in D$, $j \in L$, and $k \in \{1, 2\}$. We show that, if company 1 locates less than B_1 collection points, it can always unilaterally increase its payoff by locating more collection points until B_1 collection points are open. The payoff of company 1, given a location decision $y \in \mathscr{Y}$ of company 2, is

$$I_1(x, y) = \sum_{i\in D} m_i \frac{\sum_{j\in L} u_{j1}^i x_j}{\sum_{j\in L} u_{j1}^i x_j + (1 + \sum_{j\in L} u_{j2}^i y_j)}.$$

Let $z_i(x, y) = \sum_{j\in L} u_{j1}^i x_j$, and $h(z) = \frac{z}{z+(1+\sum_{j\in L} u_{j2}^i y_j)}$. Function z_i is linear in x_j for all $j \in L$, strictly increasing and non-negative since $u_{jk}^i > 0$. Function $h(z)$ is also strictly increasing. Thus, $h(z_i(x, y))$ is a strictly increasing function for x_j for all $j \in L$. Finally, we can rewrite $I_1(x, y)$ as $I_1(x, y) = \sum_{i\in D} m_i h(z_i(x, y))$. So I_1 being the sum of strictly increasing functions in x_j, is also strictly increasing in x_j for all $j \in L$. Suppose that company 1 selects a strategy $\hat{x} \in \mathscr{X}$ such that $\sum_{j\in L} \hat{x}_j < B_1$. Then, there is another strategy $\bar{x} \in \mathscr{X}$ such that $\sum_{j\in L} \bar{x}_j = B_1$ and $\{j \in L \,|\, \hat{x}_j = 1\} \subset \{j \in L \,|\, \bar{x}_j = 1\}$. Since $z_i(\hat{x}, y) < z_i(\bar{x}, y)$, $I_1(\hat{x}, y) < I_1(\bar{x}, y)$ for all actions $y \in \mathscr{Y}$ of company 2. Thus, company 1 can always unilaterally increase its payoff by locating more collection points until $\sum_{j\in L} x_j = B_1$, independently of the choice of the other company. The same holds for company 2.

□

Proposition 1 shows that companies will always locate all the available collection points according to their budget, since this policy will maximize the amount of customers using their services. Hence, we can restrict the search of Nash equilibria to such pure strategies. At the same time, the following example shows that pure Nash equilibria do not always exist.

Example 1. *Let* $D = L = \{1, 2\}$, $\alpha_{j1}^i = \ln(4)$ *for all* $i, j \in \{1, 2\}$, $\alpha_{j2}^i = \ln(2)$ *for all* $i, j \in \{1, 2\}$, $\beta = 1$, $m_1 = 1, m_2 = 1$, $t_1^1 = t_2^2 = 0$, $t_2^1 = t_1^2 = \ln(2)$, *and* $B_1 = B_2 = 1$. *The payoff matrix is given in Table 2.*

		Company 1	
		$x = (1,0)$	$x = (0,1)$
Company 2	$y = (1,0)$	$\frac{450}{420}, \frac{225}{420}$	$\frac{448}{420}, \frac{238}{420}$
	$y = (0,1)$	$\frac{448}{420}, \frac{238}{420}$	$\frac{450}{420}, \frac{225}{420}$

Table 2: Payoffs table of Example 1. The first term represents the payoff for company 1, and the second term for company 2.

From the payoffs of Table 2, we learn that there is no Nash equilibrium: company 1 prefers to locate its collection point in the same district as company 2, while company 2 prefers to deviate. $\triangle$

Example 1 illustrates that the existence of a Nash equilibrium is not guaranteed. This is particularly true when some companies are more attractive to consumers than others. This could, for instance, be the case when one of the companies is a well-established and recognized company. At the same time, the other one is new (and as such less known) in the market, or when one of the two companies can offer a superior service quality (e.g., same-day delivery versus a two-day delivery service). However, in practice, differences in carriers' attractiveness are likely to be minimal as carriers offering collection points are usually well-established firms, such as DHL and PostNL in the Netherlands, which provide similar service quality. The upcoming example illustrates that a Nash equilibrium might exist without company-dependent attractiveness.[4]

Example 2. *Let* $D = L = \{1, 2\}$, $\alpha^i_{jk} = \ln(4)$ *for all* $i, j \in \{1, 2\}$ *and all* $k \in \{1, 2\}$, $\beta = \ln(16)$, $m_1 = 2, m_2 = 1$, $t^1_1 = t^2_2 = 0$, $t^1_2 = t^2_1 = 1$, *and* $B_1 = B_2 = 1$. *The payoff matrix is given in Table 3.*

Observing the payoffs in Table 3, a unique Nash equilibrium exists where both companies locate their collection point at the most dense district, i.e., $x^* = y^* = (1, 0)$. *Moreover, note that the game is symmetric, i.e.,* $\mathscr{X} = \mathscr{Y}$ *and* $I_1(x, y) = I_2(y, x)$ *for all* $x \in \mathscr{X}$ *and all* $y \in \mathscr{Y}$. $\triangle$

In the remainder of this paper, our focus is on games $\mathscr{G}$ for which the attractiveness is the same between companies (i.e., $\alpha^i_{j1} = \alpha^i_{j2}$ for all $i \in D$ and $j \in L$), representing the case where

[4] A sufficient condition for the existence of pure Nash equilibria is that the game is supermodular, based on the definition of Topkis (1998). One might wonder whether this condition holds for our game in the absence of company-dependent attractiveness. In §A in the supplementary materials, we prove that even without company-dependent attractiveness, our game is not supermodular.

		Company 1	
		$x = (1, 0)$	$x = (0, 1)$
Company 2	$y = (1, 0)$	$\frac{19}{18}, \frac{19}{18}$	$\frac{6}{7}, \frac{11}{7}$
	$y = (0, 1)$	$\frac{11}{7}, \frac{6}{7}$	$\frac{7}{9}, \frac{7}{9}$

Table 3: Payoffs table of Example 2. The first term represents the payoff for company 1, and the second term for company 2.

two well-established companies with similar service quality (e.g., one-business-day delivery) enter or operate in the collection point market.

3.2 Price of stability

In our game $\mathscr{G}$, companies locate collection points strategically, i.e., they consider the positions of competitors' collection points to optimize the position of their collection points. This competitive behavior may lead to a suboptimal utilization when compared to a non-competitive scenario where a centralized public authority is in charge of locating the collection points to maximize their overall utilization. In this section, we introduce some new notations to investigate the impact of competitive versus non-competitive (i.e., public authority) decision-making on the overall utilization of collection points. For a given $x \in \mathscr{X}$ and $y \in \mathscr{Y}$, we refer to the total number of customers using collection points as the *social payoff*, which reads formally

$$I_s(x, y) = I_1(x, y) + I_2(x, y) \quad \forall\, x \in \mathscr{X},\, y \in \mathscr{Y}.$$

Moreover, a strategy profile (x^s, y^s) that solves $\max_{x \in \mathscr{X}, y \in \mathscr{Y}} I_s(x, y)$, is called a *social solution* and the associated social payoff the *social optimum*. Note that the social solution requires all available collection points to be located, as stated in Proposition 2, indicating that a public authority will also locate all the collection points to maximize utilization.

Proposition 2. *The strategy profile (x^s, y^s) that solves $\max\limits_{x \in \mathscr{X}, y \in \mathscr{Y}} I_s(x, y)$ satisfies $\sum\limits_{j \in L} x_j^s = B_1$ and $\sum\limits_{j \in L} y_j^s = B_2$.*

Proof. The proof follows the same structure as the proof of Proposition 1 (see §B in the supplementary materials). □

To measure the impact of strategic decision-making on the overall utilization of collection points for game $\mathscr{G}$, we then introduce the ratio between the highest possible social payoff resulting from the set of all pure Nash equilibria in $\mathscr{G}$ and the social optimum. In the game-theoretical literature, this ratio is also referred to as the *price of stability*, abbreviated as PoS (see, e.g., Roughgarden (2005)). We will therefore use this term from hereon. For our game $\mathscr{G}$, the price of stability is given by:

$$PoS(\mathscr{G}) = \frac{\max\limits_{(x^*,y^*)\in\Pi(\mathscr{G})} I_s(x^*, y^*)}{I_s(x^s, y^s)},$$

where we recall that $\Pi(\mathscr{G})$ presents the set of Nash equilibria of game $\mathscr{G}$. We now illustrate the PoS for Example 2.

Example 3. *Reconsider the game of Example 2. To identify the social optimum, we need to compare the social payoff for each strategy profile as presented in Table 4.*

		Company 1	
		$x = (1,0)$	$x = (0,1)$
Company 2	$y = (1,0)$	$\frac{19}{18} + \frac{19}{18} = \frac{19}{9}$	$\frac{6}{7} + \frac{11}{7} = \frac{17}{7}$
	$y = (0,1)$	$\frac{11}{7} + \frac{6}{7} = \frac{17}{7}$	$\frac{7}{9} + \frac{7}{9} = \frac{14}{9}$

Table 4: Social payoffs table of Example 2.

Consequently, the social optimum value is $\frac{17}{7}$ *with social solutions* $(1,0),(0,1)$ *or* $(0,1),(1,0)$. *Recall that the Nash equilibrium is* $(1,0),(1,0)$ *with associated social payoff* $\frac{19}{9}$. *And so, the price of stability reads* $\frac{19}{9}/\frac{17}{7} \approx 0.87$. *The reader can verify that, for* $m_1 > 6$, *both the Nash equilibrium and social solution are* $(1,0),(1,0)$, *leading to a PoS of 1.* $\triangle$

4 A stylized two-district setting

In this section, we study a specific game $\mathscr{G}$ with only two districts to gain insights into the structure of Nash equilibria for varying combinations of i) the initial interest of customers towards collection points (α), ii) the willingness of customers to travel (β) and iii) the distribution of customers between the two districts. For this purpose, we assume that each company can position a single collection point and customers' behavior is driven solely by travel distance (i.e., $\alpha^i_{jk} = \alpha$ for all $i \in D$, $j \in L$ and $k \in \{1,2\}$). For this specific game, we furthermore assume that both districts contain customers (i.e., $m_1, m_2 > 0$), with the first district having the highest number of customers, i.e., $m_1 \geq m_2$. The first district could, in this sense, represent a city center characterized by high population density, while the other district could represent an adjacent neighborhood with a lower population density. For notational convenience, we define a scale parameter $\theta = \frac{m_1}{m_2} \geq 1$ which describes the customer ratio between the two districts. Finally, we assume that the districts are at unit distance from each other, i.e., $t_2^1 = t_1^2 = 1$, and that travel time within a district is negligible, i.e., $t_1^1 = t_2^2 = 0$. We want to stress that these last two assumptions are made without loss of generality, as long as the travel times between the districts and within them remain the same, which holds because we can then rescale α and β. In §4.1, we describe the parameter combinations under which the Nash equilibrium leads the companies to select the same or opposite locations. In §4.2, this analysis is done for the solution of the public authority, and §4.3 reports the corresponding price of stability.

4.1 Nash equilibria: Selecting the same location, or opposite?

For this specific two-district scenario, we prove that if customers are not sensitive to traveling at all, positioning a collection point in any district leads to a Nash equilibrium. If the populations of the districts are similar and customers are not accustomed to traveling long distances, companies will position themselves in districts opposite to each other. In contrast, if customers are less sensitive to long distances, or one district is significantly more populated than the other, both companies will benefit from selecting the most populated district.

Theorem 1. *Consider the two-district scenario. Let $\beta = 0$. Positioning a collection point in any district leads to a Nash equilibrium. Let $\beta > 0$. If $\theta > \dfrac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$, both companies position their collection point at the most dense district in the unique Nash equilibrium. However, if $\theta = \dfrac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$, positioning a collection point at any district leads to a Nash equilibrium, except for the one where both position their collection point at the least dense district. Finally, if $\theta < \dfrac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$, the Nash equilibria correspond to companies positioning collection points oppositely to each other.*

Proof. For identifying Nash equilibria, we can divide $I_1(x, y)$ and $I_2(x, y)$ by m_2 for all $x \in \mathscr{X}$ and all $y \in \mathscr{Y}$. We refer to these scaled payoff functions as $I_1^\theta(x, y)$ and $I_2^\theta(x, y)$ which read as follows for every $x \in \mathscr{X}$ and every $y \in \mathscr{Y}$

$$
\begin{aligned}
I_1^\theta(x, y) &= \theta \frac{x_1 + x_2 e^{-\beta}}{e^{-\alpha} + x_1 + y_1 + (x_2 + y_2)e^{-\beta}} + \frac{x_1 e^{-\beta} + x_2}{e^{-\alpha} + (x_1 + y_1)e^{-\beta} + x_2 + y_2} \\
I_2^\theta(x, y) &= \theta \frac{y_1 + y_2 e^{-\beta}}{e^{-\alpha} + x_1 + y_1 + (x_2 + y_2)e^{-\beta}} + \frac{y_1 e^{-\beta} + y_2}{e^{-\alpha} + (x_1 + y_1)e^{-\beta} + x_2 + y_2}.
\end{aligned}
$$

Recall that, according to Proposition 1, strategy (0,0) cannot be a Nash equilibrium. We report in Table 5 the payoffs for both companies and their corresponding strategies.

		Company 1	
		$x = (1,0)$	$x = (0,1)$
Company 2	$y = (1,0)$	$\frac{\theta}{e^{-\alpha}+2} + \frac{e^{-\beta}}{e^{-\alpha}+2e^{-\beta}}, \frac{\theta}{e^{-\alpha}+2} + \frac{e^{-\beta}}{e^{-\alpha}+2e^{-\beta}}$	$\frac{\theta e^{-\beta}+1}{1+e^{-\alpha}+e^{-\beta}}, \frac{\theta + e^{-\beta}}{1+e^{-\alpha}+e^{-\beta}}$
	$y = (0,1)$	$\frac{\theta + e^{-\beta}}{1+e^{-\alpha}+e^{-\beta}}, \frac{\theta e^{-\beta}+1}{1+e^{-\alpha}+e^{-\beta}}$	$\frac{\theta e^{-\beta}}{e^{-\alpha}+2e^{-\beta}} + \frac{1}{e^{-\alpha}+2}, \frac{\theta e^{-\beta}}{e^{-\alpha}+2e^{-\beta}} + \frac{1}{e^{-\alpha}+2}$

Table 5: Payoffs table. The first term represents the payoff for company 1, and the second term for company 2.

We now show that ((0,1),(0,1)) is never a Nash equilibrium. We do so by showing that company 1 strictly prefers to position its collection point in the most densely populated district. That is,

$$\begin{aligned}
&I_1^\theta((1,0),(0,1)) - I_1^\theta((0,1),(0,1)) \\
&= \frac{\theta + e^{-\beta}}{1+e^{-\alpha}+e^{-\beta}} - \left(\frac{\theta e^{-\beta}}{e^{-\alpha}+2e^{-\beta}} + \frac{1}{e^{-\alpha}+2}\right) \\
&= \theta\left(\frac{1}{1+e^{-\alpha}+e^{-\beta}} - \frac{e^{-\beta}}{e^{-\alpha}+2e^{-\beta}}\right) + \left(\frac{e^{-\beta}}{1+e^{-\alpha}+e^{-\beta}} - \frac{1}{e^{-\alpha}+2}\right) \\
&= \frac{1-e^{-\beta}}{1+e^{-\alpha}+e^{-\beta}}\left(\frac{e^{-\alpha}+e^{-\beta}}{e^{-\alpha}+2e^{-\beta}}\theta - \frac{e^{-\alpha}+1}{e^{-\alpha}+2}\right) > 0.
\end{aligned} \tag{2}$$

The equalities hold by some rewriting. The inequality holds, because $\frac{1-e^{-\beta}}{1+e^{-\alpha}+e^{-\beta}} > 0$, and

$$\theta \geq 1 > \frac{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}$$

where the strict inequality follows from the fact that (i):

$$(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta}) - (e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta}) = e^{-\alpha}(e^{-\beta}-1) < 0.$$

and (ii) both terms of the fraction are positive. Hence, ((0,1),(0,1)) is never a Nash equilibrium.

We now investigate when ((1,0)),(1,0)), ((1,0),(0,1)), and ((0,1),(1,0)) are Nash equilibria. Given that our game $\mathscr{G}$ is symmetric, it suffices to only study ((1,0),(1,0)) and ((1,0),(0,1)). In doing so, we distinguish between three cases: (i) $\theta > \frac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$, (ii) $\theta = \frac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$, and (iii) $\theta < \frac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$.

Case (i) $\theta > \dfrac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$

We first show that $((1,0),(1,0))$ is a Nash equilibrium. In doing so, we first prove that company 2 has no incentive to deviate from $((1,0),(1,0))$. That is,

$$\begin{aligned}
&I_2^\theta((1,0),(1,0)) - I_2^\theta((1,0),(0,1)) \\
&= \left(\frac{\theta}{e^{-\alpha}+2} + \frac{e^{-\beta}}{e^{-\alpha}+2e^{-\beta}}\right) - \frac{\theta e^{-\beta}+1}{1+e^{-\alpha}+e^{-\beta}} \\
&= \left(\frac{1}{e^{-\alpha}+2} - \frac{e^{-\beta}}{1+e^{-\alpha}+e^{-\beta}}\right)\theta + \frac{e^{-\beta}}{e^{-\alpha}+2e^{-\beta}} - \frac{1}{1+e^{-\alpha}+e^{-\beta}} \\
&= \frac{1-e^{-\beta}}{1+e^{-\alpha}+e^{-\beta}}\left(\frac{e^{-\alpha}+1}{e^{-\alpha}+2}\theta - \frac{e^{-\alpha}+e^{-\beta}}{e^{-\alpha}+2e^{-\beta}}\right) > 0.
\end{aligned} \tag{3}$$

The equalities hold by some rewriting. The last inequality holds, because $\theta > \frac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$ by the case condition. By symmetry of $\mathscr{G}$ and (3), we also know that

$$0 < I_2^\theta((1,0),(1,0)) - I_2^\theta((1,0),(0,1)) = I_1^\theta((1,0),(1,0)) - I_1^\theta((0,1),(1,0)),$$

implying that there is also no incentive for company 1 to deviate from $((1,0),(1,0))$. Thus, $((1,0),(1,0))$ is a Nash equilibrium. By (3), we also know that there is an incentive for company 2 to deviate from $((1,0),(0,1))$, implying that $((1,0),(0,1))$ is not a Nash equilibrium, and making $((1,0),(1,0))$ the unique Nash equilibrium.

Case (ii) $\theta = \dfrac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$

In this case, we show that ((1,0),(1,0)) and ((1,0),(0,1)) are both Nash equilibria. We start with ((1,0),(1,0)). By using the same derivation as used in (3), we have

$$I_2^\theta((1,0),(1,0)) - I_2^\theta((1,0),(0,1)) = \frac{1-e^{-\beta}}{1+e^{-\alpha}+e^{-\beta}}\left(\frac{e^{-\alpha}+1}{e^{-\alpha}+2}\theta - \frac{e^{-\alpha}+e^{-\beta}}{e^{-\alpha}+2e^{-\beta}}\right) = 0 \tag{4}$$

because $\theta = \frac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$ by the case condition. Hence, company 2 has no incentive to deviate from ((1,0),(1,0)). Next, by symmetry of $\mathscr{G}$ and (4), we know that:

$$0 = I_2((1,0),(1,0)) - I_2((1,0),(0,1)) = I_1((1,0),(1,0)) - I_1((0,1),(1,0)) \tag{5}$$

implying that company 1 has no incentive to deviate from ((1,0),(1,0)). Hence, ((1,0),(1,0)) is a Nash equilibrium. Next, we focus on ((1,0),(0,1)). By (4), we already know that company 2 has no incentive to deviate from ((1,0),(0,1)). By (2), we also know that $I_1((1,0),(0,1)) - I_1((0,1),(0,1)) > 0$, implying that company 1 has no incentive to deviate from ((1,0),(0,1)). In conclusion, ((1,0),(0,1)) is also a Nash equilibrium.

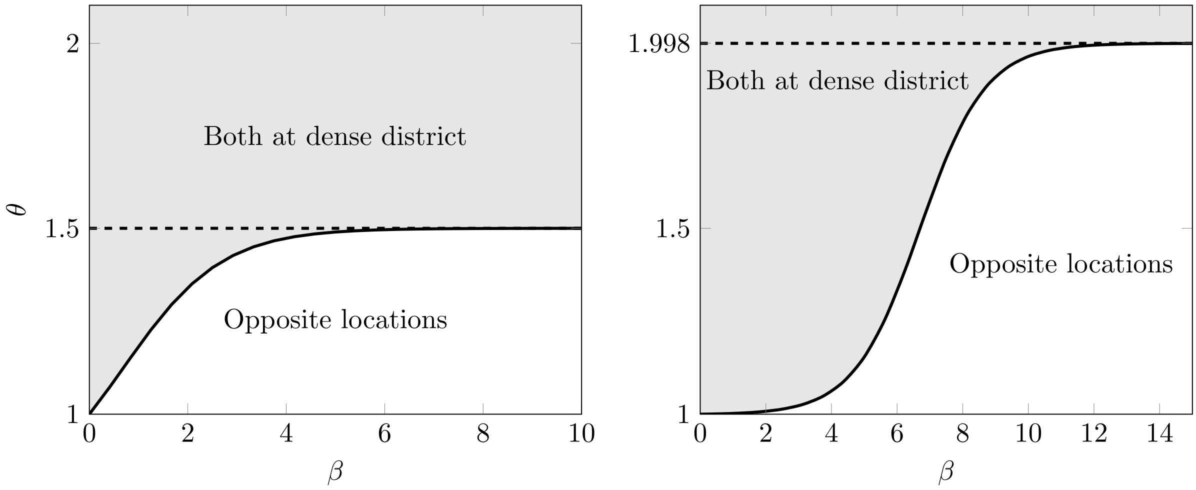
Case (iii) $\theta < \dfrac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$
In this case, we show that ((1,0),(1,0)) is not a Nash equilibrium, while ((1,0),(0,1)) is. By using the same derivation as used in (3), we have

$$I_2^{\theta}((1,0),(1,0)) - I_2^{\theta}((1,0),(0,1)) = \frac{1-e^{-\beta}}{1+e^{-\alpha}+e^{-\beta}}\left(\frac{e^{-\alpha}+1}{e^{-\alpha}+2}\theta - \frac{e^{-\alpha}+e^{-\beta}}{e^{-\alpha}+2e^{-\beta}}\right) < 0 \quad (6)$$

because $\theta < \frac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$ by the case condition. Hence, there is a reason for company 2 to deviate from ((1,0),(1,0)), and so ((1,0),(1,0)) is not a Nash equilibrium. Next, we show that ((1,0),(0,1)) is a Nash equilibrium. From (6), we know that company 2 has no incentive to deviate from ((1,0),(0,1)). At the same time, using (2), we know that company 1 has no incentive to deviate from ((1,0),(0,1)). Hence, ((1,0), (0,1)) is a Nash equilibrium, and, by symmetry, $((0,1),(1,0)$ is also a Nash equilibrium. □

Applying Theorem 1, Figure 1 illustrates for which combinations of β and θ both companies locate their collection point at the most dense district, or spread the collection points instead, given that the initial interest equals $\alpha = 0$ and $\alpha = 6$. Both figures show a non-linear threshold dividing how companies locate their collection points. For the setting with $\alpha = 0$, this threshold has a concave behavior, while for $\alpha = 6$ the threshold follows an S-shape curve. In both figures, we also included a dashed threshold, indicating from which θ onwards it is always better to locate the collection points at the most dense district. For $\alpha = 0$ this is exactly at $3/2$ and for $\alpha = 6$ this is close to, but not exactly equal to 2. For the specific setting with $\alpha = 0$ and $\theta \geq 3/2$, each company attracts 33.3% of the customers of the most dense district if both companies locate their collection points in it. Given that the number of customers in the most dense district is at least $3/2$ times as large as that in the least dense one, both companies would each serve more customers than they could individually if they decided to reposition their collection point to the least dense district. This implies that there is no reason for an individual company to switch its collection point to another district. A similar type of reasoning holds for Figure 1b.

In summary, for scenarios where customers are slightly sensitive to travel, consolidating collection points in the most dense district is the best option. However, as customers' sensitivity to travel increases, spreading the collection points becomes a better option, provided the districts do not differ significantly in customer size.



(a) Location decisions for β and θ given α=0. (b) Location decisions for β and θ given α=6.

Figure 1: Nash equilibria location decisions for β and θ for two different given α.

4.2 Solution under public authority

Studying the social optimum for this specific game, we show that a threshold, similar to that of the Nash Equilibrium, exists that determines when to switch between concentrating the collection points in the most densely populated district and spreading them.

Theorem 2. *Consider the two-district scenario. Let $\beta = 0$. Positioning a collection point in any district leads to a social solution. Let $\beta > 0$. If $\theta > \dfrac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$, both companies position their collection point at the most dense district in the social solution. If $\theta = \dfrac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$, positioning a collection point at any district leads to a social solution, except for the one where both position their collection point at the least dense district. If $\theta < \dfrac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$, the companies should position collection points opposite to each other, implying the existence of two social solutions.*

Proof. The proof follows the same structure as the proof of Theorem 1 (see §C in the supplementary materials). □

Theorem 2 tells us that there is, again, a single threshold identifying when companies should locate collection points in the same district (namely the most dense district) and when to locate them opposite to each other. This threshold, however, differs only slightly from the threshold in Theorem 1. More precisely, the threshold of Theorem 1 is the product of the threshold of Theorem 2 and constant $\frac{e^{-\alpha}+e^{-\beta}}{e^{-\alpha}+1}$. Given that this constant is between zero and one, we learn that whenever it is optimal to position collection points opposite to each other under a social solution, it is also the case for a Nash equilibrium. However, if it is optimal to locate the collection points in the same district (the most densely populated district) under a social solution, it depends on the number of customers in that district whether it is also a Nash equilibrium, as shown in Figure 2.

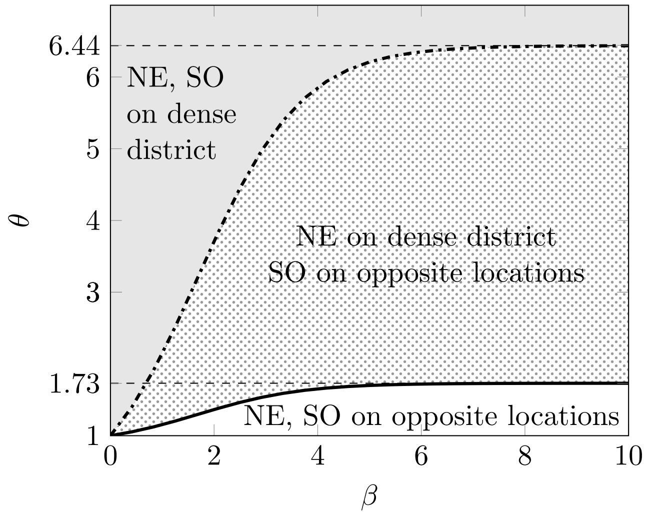


Figure 2: Nash equilibria (NE) and social optima (SO) relationships between β and θ for α=1.

More precisely, Figure 2 describes the three possible scenarios, namely (i) both the social solution and the Nash equilibrium locate the collection points opposite to each other, (ii) the social solution locates collection points opposite to each other, but the Nash equilibrium locates them in the same district and (iii) both the social solution and the Nash equilibrium locate the collection points at the same district. Please note that there does not exist a case where the social solution positions collection points in the same district while the Nash equilibrium spreads them. This coincides with the intuition that public authorities are more likely to promote diversification of collection point locations than delivery companies. Note that for scenarios (i) and (iii), the price of stability equals one, and companies position collection points the same way as the authority would. At the same time, it is strictly smaller than one in scenario (ii), representing a mismatch in the companies' and municipality's objectives. Note that Example 3 dealt with the second scenario, with $PoS(\mathscr{G}) \approx 0.87$. Our next goal is to investigate how closely the outcome of strategic decisions of companies aligns with the outcome under the public authority, using the price of stability as the comparison measure for these decisions.

4.3 Bound on the price of stability

Recall that for determining the price of stability of game $\mathscr{G}$, we need to select a Nash equilibrium from $\Pi(\mathscr{G})$ for which the social payoff is the highest amongst all Nash equilibria in $\Pi(\mathscr{G})$. For our specific game $\mathscr{G}$ with two districts, set $\Pi(\mathscr{G})$ only consists of multiple Nash equilibria if $\theta = \frac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$, but then all Nash equilibria have the same social payoff. This implies that for our specific game $\mathscr{G}$, the price of stability can be rewritten as:

$$PoS(\mathscr{G}) = \frac{I_s(x^*, y^*)}{I_s(x^s, y^s)},$$

with (x^*, y^*) a Nash equilibrium in $\Pi(\mathscr{G})$. It turns out that the price of stability is bound by 2/3, implying that at most one third of interested customers can be lost due to the strategic behaviors of companies. We will now formalize this result.

Theorem 3. *Consider the two-district scenario. Then,* $PoS(\mathscr{G}) > \frac{2}{3}$.

Proof. By Theorem 1 and Theorem 2, we know that for $\theta \leq \frac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$ and $\theta \geq \frac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$, the social solution and the Nash equilibrium coincide. Consequently, $PoS(\mathscr{G}) = 1$.

Now, let $\frac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})} < \theta < \frac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$. By Theorem 1, we know that the Nash equilibrium corresponds to ((1,0),(1,0)), while the social solution is ((1,0),(0,1)) or ((0,1),(1,0)). Given the symmetry of the game, we focus on ((1,0),(0,1)) only in this proof. Thus,

$$\begin{aligned} PoS(\mathscr{G}) &= \frac{I_s((1,0),(1,0))}{I_s((1,0),(0,1))} \\ &= \frac{\dfrac{2\theta}{e^{-\alpha}+2} + \dfrac{2e^{-\beta}}{e^{-\alpha}+2e^{-\beta}}}{\dfrac{(\theta+1)(e^{-\beta}+1)}{e^{-\alpha}+e^{-\beta}+1}} \\ &= \frac{2(e^{-\beta}+e^{-\alpha}+1)(2\theta e^{-\beta}+\theta e^{-\alpha}+e^{-\beta}e^{-\alpha}+2e^{-\beta})}{(\theta+1)(e^{-\beta}+1)(e^{-\alpha}+2)(2e^{-\beta}+e^{-\alpha})}. \end{aligned} \tag{7}$$

To identify a lower bound for the price of stability, we describe the last expression of (7) as an explicit function. That is, we define

$$PoS(\alpha', \beta', \theta') = \frac{2(e^{-\beta'}+e^{-\alpha'}+1)(2\theta' e^{-\beta'}+\theta' e^{-\alpha'}+e^{-\beta'}e^{-\alpha'}+2e^{-\beta'})}{(\theta'+1)(e^{-\beta'}+1)(e^{-\alpha'}+2)(2e^{-\beta'}+e^{-\alpha'})} \tag{8}$$

for all $\alpha', \beta', \theta' \in \mathbb{R}_+$. Now, observe that for all $\alpha', \beta', \theta' \in \mathbb{R}_+$ we have

$$
\begin{aligned}
&\frac{\partial}{\partial \theta'} PoS(\alpha', \beta', \theta') \\
=&\frac{\partial}{\partial \theta'} \left(\frac{2(e^{-\beta'} + e^{-\alpha'} + 1)(2\theta' e^{-\beta'} + \theta' e^{-\alpha'} + e^{-\beta'} e^{-\alpha'} + 2e^{-\beta'})}{(\theta' + 1)(e^{-\beta'} + 1)(e^{-\alpha'} + 2)(2e^{-\beta'} + e^{-\alpha'})} \right) \\
=&\frac{2(e^{-\beta'} + e^{-\alpha'} + 1)}{(e^{-\beta'} + 1)(e^{-\alpha'} + 2)(2e^{-\beta'} + e^{-\alpha'})} \cdot \frac{d}{d\theta'} \left(\frac{2\theta' e^{-\beta'} + \theta' e^{-\alpha'} + e^{-\beta'} e^{-\alpha'} + 2e^{-\beta'}}{\theta' + 1} \right) \\
=&\frac{2(e^{-\beta'} + e^{-\alpha'} + 1)}{(e^{-\beta'} + 1)(e^{-\alpha'} + 2)(2e^{-\beta'} + e^{-\alpha'})} \cdot \frac{e^{-\alpha'}(1 - e^{-\beta'})}{(\theta' + 1)^2} \\
=&\frac{2(e^{-\beta'} + e^{-\alpha'} + 1)e^{-\alpha'}(1 - e^{-\beta'})}{(e^{-\beta'} + 1)(e^{-\alpha'} + 2)(2e^{-\beta'} + e^{-\alpha'})(\theta' + 1)^2} > 0,
\end{aligned}
$$

where the inequality follows from the fact that all terms in the fraction are strictly positive. Because the function is strictly increasing in θ', we conclude that:

$$
PoS(\alpha, \beta, \theta) \geq \frac{2(e^{-\beta} + e^{-\alpha} + 1)(e^{-\beta}(e^{-\alpha} + 2) + e^{-\alpha})}{(e^{-\beta} + 1)(e^{-\beta}(3e^{-\alpha} + 4) + (2e^{-\alpha} + 3)e^{-\alpha})}, \tag{9}
$$

where the right-hand size of the inequality is expression (8) with $\theta = \frac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$ substituted and some rewriting. Next, observe that for any $\alpha', \beta' \in \mathbb{R}_+$ we have

$$
\begin{aligned}
&\frac{\partial}{\partial \beta'} \left(\frac{2\left(e^{-\beta'} + e^{-\alpha'} + 1\right)\left(e^{-\beta'}\left(e^{-\alpha'} + 2\right) + e^{-\alpha'}\right)}{\left(e^{-\beta'} + 1\right)\left(e^{-\beta'}\left(3e^{-\alpha'} + 4\right) + \left(2e^{-\alpha'} + 3\right)e^{-\alpha'}\right)} \right) \\
=& -\frac{2e^{-\beta'}e^{-\alpha'}(1 - e^{-\beta'})\left(e^{-\beta'}\left(e^{-2\alpha'} + 6e^{-\alpha'} + 6\right) + 3e^{-2\alpha'} + 6e^{-\alpha'} + 2\right)}{\left(e^{-\beta'} + 1\right)^2\left(e^{-\beta'}\left(3e^{-\alpha'} + 4\right) + e^{-\alpha'}\left(2e^{-\alpha'} + 3\right)\right)^2} < 0
\end{aligned}
$$

where the inequality holds because all factors are strictly positive. Hence, the lower bound in (9) is a strictly decreasing function in β. Consequently, we have

$$
\frac{2(e^{-\beta} + e^{-\alpha} + 1)(e^{-\beta}(e^{-\alpha} + 2) + e^{-\alpha})}{(e^{-\beta} + 1)(e^{-\beta}(3e^{-\alpha} + 4) + (2e^{-\alpha} + 3)e^{-\alpha})} > \frac{2e^{-\alpha} + 2}{2e^{-\alpha} + 3}. \tag{10}
$$

Next, observe that for any $\alpha' \in \mathbb{R}_+$ we have

$$
\frac{\partial}{\partial \alpha'} \left(\frac{2e^{-\alpha'} + 2}{2e^{-\alpha'} + 3} \right) = -\frac{2e^{-\alpha'}}{(2e^{-\alpha'} + 3)^2} < 0,
$$

where the inequality holds because all factors are strictly positive. Hence, the lower bound in (10) strictly decreases in α. Consequently, we have

$$
\frac{2e^{-\alpha} + 2}{2e^{-\alpha} + 3} > \frac{2}{3} \quad \forall \alpha \in \mathbb{R}_+ . \tag{11}
$$

Combining (9), (10), and (11) we conclude that $PoS(\mathscr{G}) = PoS(\alpha, \beta, \theta) > \frac{2}{3}$. $\square$

The following example demonstrates that combinations of α and β of our game $\mathscr{G}$ exist such that the price of stability converges to 2/3, making the bound tight.

Example 4. *Consider the two-district scenario. Let $\gamma \in \mathbb{R}_+$ be a constant, $\beta \in \mathbb{R}_+$ with $\beta > \ln(\gamma)$, $\alpha = \beta - \ln(\gamma)$ and $\theta = \frac{(e^{-\alpha}+2)(e^{-\alpha}+e^{-\beta})}{(e^{-\alpha}+1)(e^{-\alpha}+2e^{-\beta})}$. Then, the price of stability reads*

$$\begin{aligned} PoS(\mathscr{G}) &= \frac{2(\gamma e^{-\beta} + e^{-\beta} + 1)(\gamma + 2 + \gamma e^{-\beta})}{(1 + e^{-\beta})(2\gamma^2 e^{-\beta} + 3\gamma(1 + e^{-\beta}) + 4)} \\ &= \frac{(2\gamma^2 + 2\gamma)e^{-\beta} + (2\gamma^2 + 8\gamma + 4)e^{-\beta} + 2\gamma + 4}{(2\gamma^2 + 3\gamma)e^{-2\beta} + (2\gamma^2 + 6\gamma + 4)e^{-\beta} + 3\gamma + 4}. \end{aligned} \tag{12}$$

We observe that the second equation of (12) converges to $\frac{2\gamma+4}{3\gamma+4}$ when β goes to infinity. Since α and θ are defined for all $\beta \in \mathbb{R}_+$, $PoS(\mathscr{G})$ converges to $\frac{2\gamma+4}{3\gamma+4}$ when β goes to infinity too, with γ as given. We can set γ as large as we want, which leads $PoS(\mathscr{G})$ to converge to $\frac{2}{3}$. $\triangle$

5 Case study: locating collection points in the city of Eindhoven

In this section, we apply our collection point location game to a more complex setting that considers multiple collection locations and permits each company to operate multiple collection points. Within this real-life context, we then investigate numerically whether and how the theoretical results from §4 still apply. In doing so, we utilize publicly available data from the city of Eindhoven in the Netherlands.

5.1 Case study design

This section describes the case study design for the placement of collection points within the Dutch city of Eindhoven, using open-source map data from OpenStreetMap (OpenStreetMap contributors, 2017) based on which we constructed a weighted graph representation of the walkable streets within the three central areas of the city, i.e., *Binnenstad* (4,074 inhabitants), *Witte Dame* (2,262 inhabitants), and *Bergen* (2,906 inhabitants) (Eindhoven, 2024)[5]. In this graph, nodes represent intersections, while edges correspond to streets, and the weights of the edges relate to street lengths. The nodes are the districts D in our game $\mathscr{G}$. Due to the lack of more detailed information about the population distribution within districts (i.e., around each intersection), we uniformly divided the total population of each central area across the districts it contains, rounding it to the nearest integer. This results in approximately 10

[5]For an intuition of the size of the study area, its overall surface is 1.2 km^2.

potential customers per district for *Binnenstad*, 23 for *Witte Dame*, and 15 for *Bergen*. As such, *Witte Dame* emerges as the densest area, followed by *Bergen* and *Binnenstad*. The assumption of a uniform population distribution per district is reasonable, particularly in small, densely populated urban areas like Eindhoven. We assume that collection points may be located next to points of interest provided by OpenStreetMaps, where high foot traffic is expected, such as supermarkets, schools, or pharmacies. However, the position of these points of interest does not always coincide with the intersections on the map, which is a key assumption in our game $\mathscr{G}$ (i.e., the set L of collection points is a subset of D). To address this, L corresponds to the closest intersection for each point of interest that we have selected[6]. Using the street lengths provided by OpenStreetMaps, we then calculated for each collection point location $j \in L$ the distances t_j^i to each district $i \in D$ based on the shortest path distance using Dijkstra's algorithm (Dijkstra, 1959). In Figure 3, we represent the 220 districts as black dots and the 28 candidate collection point locations as squares.

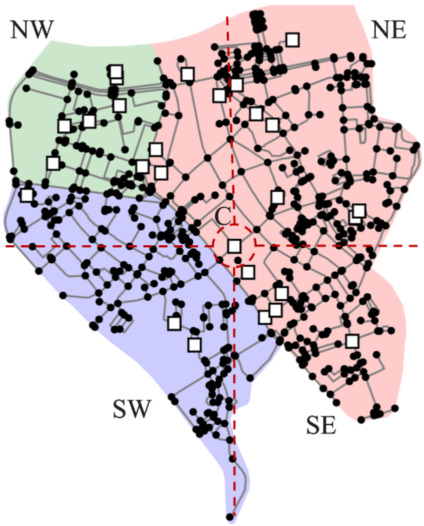


Figure 3: Visual representation of the central areas *Binnenstad* (red), *Witte dame* (green), and *Bergen* (purple) with intersections (black dots), points of interest (white squares), and streets, based on OpenStreetMaps. The five orientation areas (C, NE, NW, SE, SW) introduced in §5.3 are shown.

The parameters of our MNL model (i.e., α_{jk}^i and β) can, in theory, be estimated through a discrete choice experiment, where participants are asked to make choices between different delivery options according to their preferences. However, this approach is time-intensive and could constitute a separate study in itself, as determining the appropriate values for these parameters involves significant data collection, experimental design, and statistical analysis. Given the complexity and resources required for such an experiment, we instead chose to study a broad

[6]In most cases, these alternative locations are very close by (the maximum distance between a point of interest and the nearest intersection is around 165 m, but 95% of interest points are at most 64 m away from the closest intersection).

range of these parameter values. This strategy enables us to examine how our model performs under various parameter combinations while minimizing the additional workload associated with conducting a full-scale discrete choice experiment. Moreover, by exploring a broad range of parameter values, we account for different phases of collection point usage and a variety of urban and logistical contexts, making the results more broadly applicable. In line with recent case studies on collection points (see, e.g., Lin et al. (2022)), and to facilitate a more accurate comparison between our numerical results and the analytical outcomes presented in §3, we start with $\alpha^i_{jk} = \alpha$ for all $i \in D$, $j \in L$, and $k \in \{1, 2\}$, where α represents the willingness of customers to use lockers[7]. To ensure that we include both scenarios where customers are not interested to visit collection points, and situations where everyone is highly willing to do so, we consider $\alpha \in \{-1, 0, 1, 2, 3, 4\}$ and $\beta \in \{1 \text{ km}^{-1}, 2 \text{ km}^{-1}, \ldots, 12 \text{ km}^{-1}\}$[8]. In addition, we alter the number of collection points per company to examine the different decisions that may be made depending on the desired type of investment for each company. The scenarios we consider are "1 vs. 1", "2 vs. 1", "2 vs. 2", and "3 vs. 1", where the first number is the number of available collection points of one company, and the latter is the number of the other company (i.e., "B_1 vs B_2"). This enables us to evaluate how changes in these parameters impact the model's performance, thereby ensuring that our findings are reliable across various configurations. We propose 288 scenarios from all combinations of α, β, and number of collection points per company ($6 \times 12 \times 4$ number of collection points scenarios). For each scenario, we calculate *all* Nash equilibria via complete enumeration and *all* social optima[9]. Moreover, we calculate the PoS and *relative loss* of customers using collection points in equilibrium concerning the social optimum, i.e., we calculate $(1 - \text{PoS}) \cdot 100\% = \frac{I_s(x^s,y^s) - I_s(x^*,y^*)}{I_s(x^s,y^s)} \cdot 100\%$, with (x^s, y^s) the social optimal strategy profile, and (x^*, y^*) the Nash equilibrium with highest social payoff, which we briefly refer to as the *best Nash equilibrium*. Every time we refer to a "loss" in this section, we refer to this relative loss.

5.2 Relative loss

We begin by conducting an overarching analysis of the frequency of relative loss across all scenarios, which is summarized in Figure 4. In §E in the supplementary materials, we pro-

[7]Later on in this study (i.e., §5.5), we will also discuss cases with varying α^i_{jk}.

[8]In the remainder of this paper, we keep the unit measure of β, being km^{-1}, implicit.

[9]The experiments were performed on the Dutch supercomputer Snellius, using 16 CPUs, including AMD Genoa 9654, 2.4GHz, and 360W. Relatively to the number of collection points, we point out that our work is subject to computational challenges, since computation time increases with the number of facilities per player; for each scenario of α and β, a 1 vs. 1 instance is solved in 34 seconds, 3.8 minutes for a 2 vs. 1 instance, 40 minutes on a 3 vs. 1 instance, and 52 minutes on a 2 vs. 2 instance. Finally, although full enumeration is sufficient for finding the social solution together with the Nash equilibrium, we find the social solutions with the model described in §D in the supplementary materials, which returns solutions in less than one second.

vide detailed information for each scenario in Table 8, including the overall market share and the relative loss, plus the figures with the corresponding location outcomes of the best Nash equilibrium and social solution.

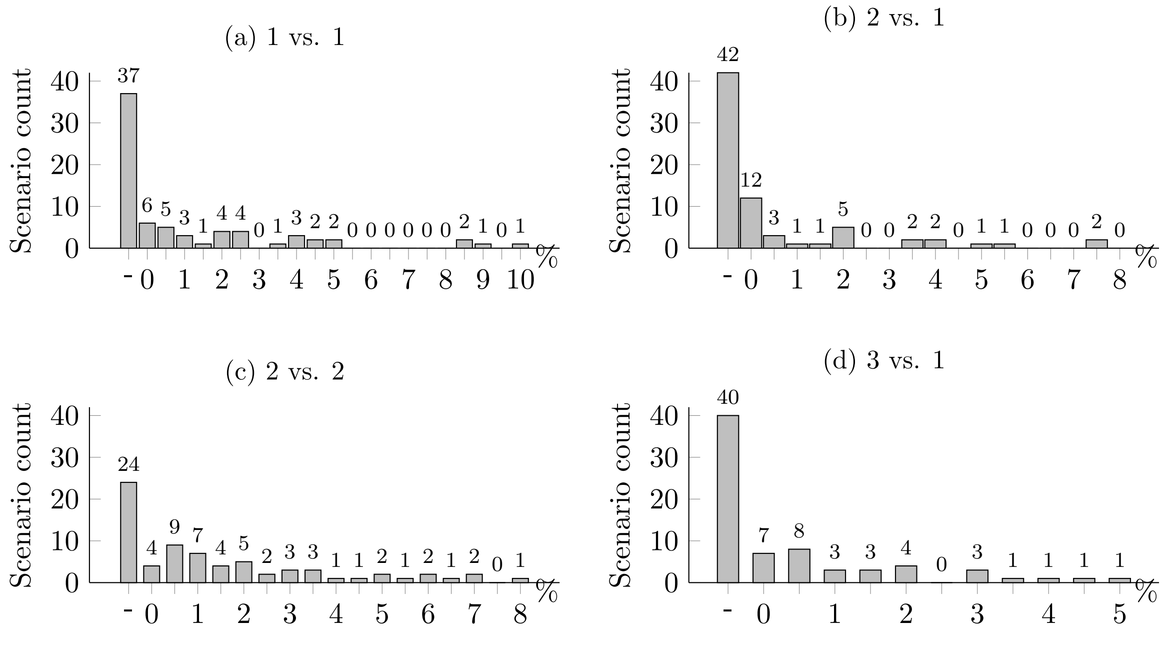


Figure 4: Histograms of the losses for each number of collection points scenario. The "-" bin represents the coincident best Nash equilibrium and the social solution. Bin x groups up the losses in the range (x-0.25%, x+0.25%].

A key observation is that the relative loss across all scenarios is generally low. More specifically, only 16 scenarios exhibit a loss greater than 5%, while 93 scenarios report a loss exceeding 1%, and 143 scenarios show no loss at all. This finding is promising for authorities who may consider accepting a slight loss in overall market share to ensure the stability of location decisions. Another observation is that the frequency of no losses is lower in scenarios where delivery companies have the same number of collection points (1 vs. 1 and 2 vs. 2) compared to those with different numbers of collection points (2 vs. 1 and 3 vs. 1). This could mean that *even* competition (in terms of the number of lockers) may lead to strategic decisions that are further away from the social optimum. Conversely, the company with more collection points in an uneven competition is more aligned with the social solution, as it may choose locations that are similar to those in the social solution, thereby attracting a higher market share.

5.3 Spatial distribution of collection points

The next step is to study the spatial distribution of the collection points. For describing the outcomes, it proves handy to split the three areas of Eindhoven into five subareas: the central location (the small circle at the center of the picture) and four subareas, identified based on the north-south and east-west opposition. Thus, we will refer to these areas as C, NW, NE, SW, and SE. A visualization of these five subareas is represented in Figure 3.

In Table 6, we provide a summary of the collection point positions for all 288 scenarios, categorized into the five areas shown in Figure 3, with aggregation by α and three β intervals (i.e., 1-4, 5-8, 9-12). Moreover, in Figure 5, we report on the frequency of coinciding locations, categorized by the number of collection points per company and travel sensitivity ranges.

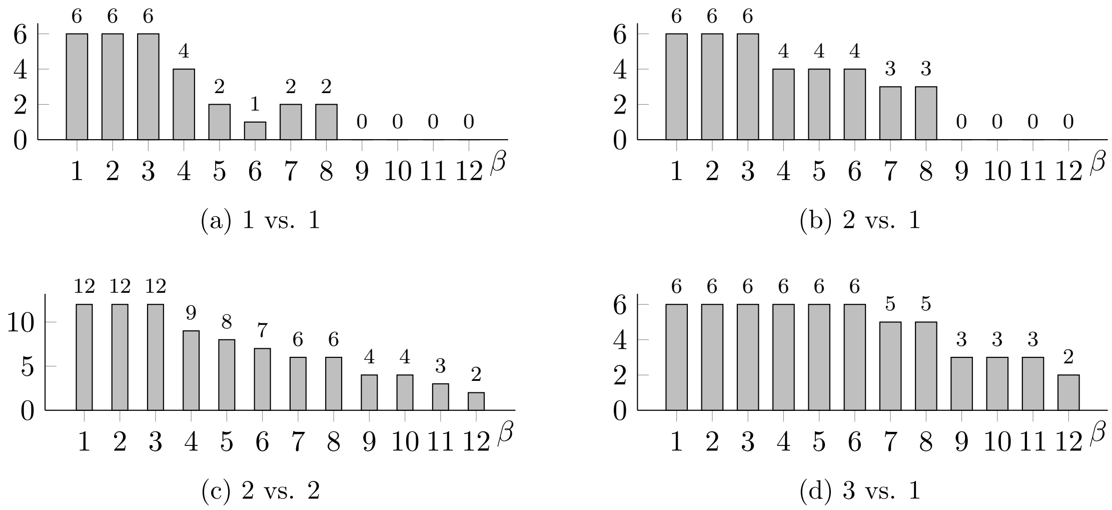


Figure 5: Histograms of the number of coinciding locations for each number of collection points scenario. Each bar may have a maximum value of 6 (i.e., the different α), except in the "2 vs. 2" scenario since two coinciding locations are possible for each α, so the maximum is 12.

The first key insight we derive from Figure 5 is that the number of coinciding location decisions decreases in β, which is in line with the theoretical results of the two-district case of §4.1, where our findings indicate that for a wide range of θ values, there is a threshold value of β beyond which it becomes advantageous to position collection points in different locations. This diversification in locations reflects that, if customers are more sensitive to travel (i.e., have larger β), companies benefit from positioning collection points in different areas of the city, rather than concentrating them in a single location. However, the precise location of the collection points, if customers are travel-sensitive, depends on the initial interest of customers towards these points. If the initial interest is relatively low, companies focus on the most densely populated area, which is Witte Dame in the NW area: the smallest area, but also the one with the highest number of potential customers per district. Note, however, that some collection points are also positioned in NE, due to the population density cluster at the top of this region.

When customers have a higher initial interest in collection points, indicating a greater willingness to travel longer distances, we observe that, in addition to being placed in the NW area, a collection point tends also to be located in the city's central area. Compare, for instance, the

Table 6: For each α and ranges of β, we report the number of open collection points at the equilibrium, and at the social optimum in brackets, in each area described in Figure 3. If SE or SW areas are not reported in the columns, the companies and the authority have selected no location in those areas. These are summed over the β's in the reported ranges, corresponding to low, medium, or high travel sensitivity.

α	β	C	NW	NE
-1	1-4	7 (7)	1 (1)	-
	5-8	0 (1)	8 (7)	-
	9-12	-	8 (8)	-
0	1-4	7 (5)	1 (3)	-
	5-8	2 (2)	6 (4)	0 (2)
	9-12	-	8 (4)	0 (4)
1	1-4	8 (5)	0 (3)	-
	5-8	3 (3)	4 (4)	1 (1)
	9-12	-	8 (4)	0 (4)
2	1-4	8 (4)	0 (4)	-
	5-8	4 (4)	4 (4)	-
	9-12	1 (1)	4 (4)	3 (3)
3	1-4	8 (4)	0 (4)	-
	5-8	4 (4)	4 (4)	-
	9-12	3 (3)	4 (4)	1 (1)
4	1-4	8 (4)	0 (4)	-
	5-8	5 (4)	3 (4)	-
	9-12	4 (4)	4 (4)	-

(a) 1 vs. 1

α	β	C	NW	NE	SW	SE
-1	1-4	8 (8)	8 (7)	0 (1)	-	-
	5-8	3 (2)	10 (10)	3 (4)	-	-
	9-12	-	12 (12)	4 (4)	-	-
0	1-4	8 (8)	7 (6)	1 (2)	-	-
	5-8	5 (5)	8 (7)	3 (4)	-	-
	9-12	-	12 (12)	4 (4)	-	-
1	1-4	8 (8)	7 (6)	1 (2)	-	-
	5-8	5 (5)	7 (5)	4 (4)	-	0 (2)
	9-12	2 (2)	10 (8)	4 (5)	-	0 (1)
2	1-4	8 (8)	8 (6)	0 (2)	-	-
	5-8	6 (4)	6 (4)	4 (4)	-	0 (4)
	9-12	4 (4)	8 (5)	4 (4)	-	0 (3)
3	1-4	8 (7)	8 (6)	0 (2)	-	-
	5-8	7 (4)	5 (4)	4 (4)	-	0 (4)
	9-12	4 (4)	8 (4)	4 (4)	-	0 (4)
4	1-4	8 (7)	8 (6)	0 (2)	-	-
	5-8	8 (1)	6 (4)	2 (4)	0 (3)	0 (4)
	9-12	5 (3)	7 (4)	4 (4)	0 (1)	4 (4)

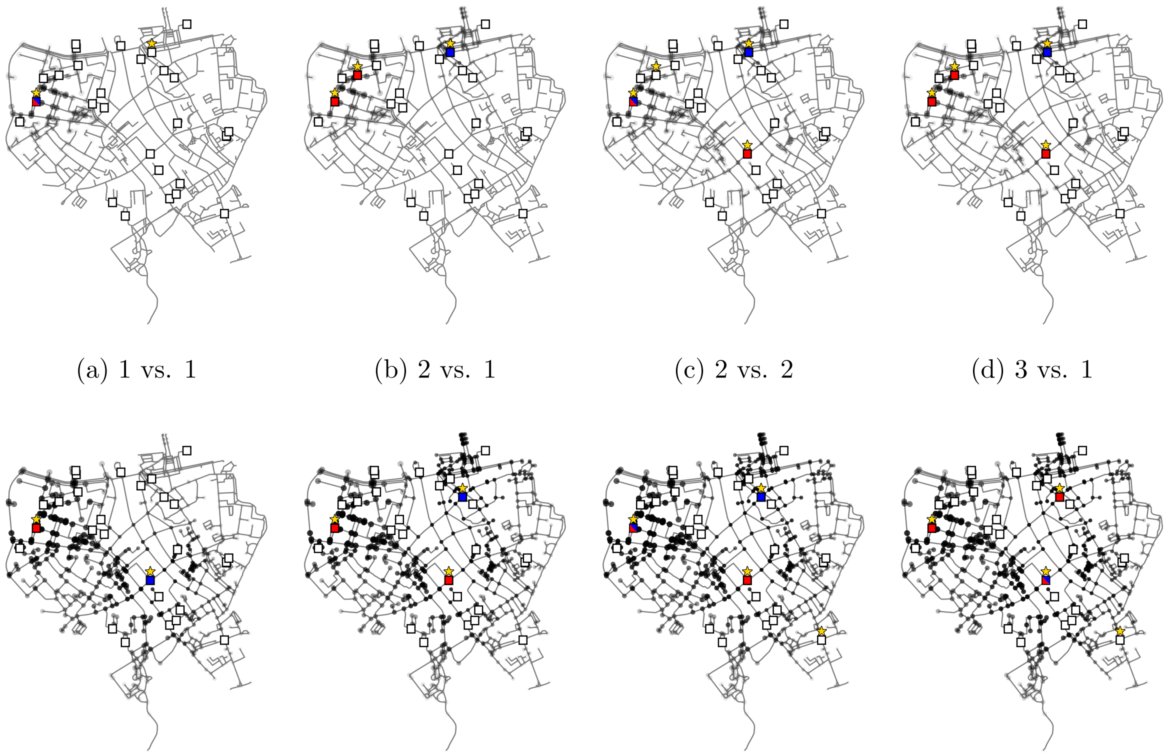
(b) 2 vs. 2

α	β	C	NW	NE	SE
-1	1-4	8 (8)	4 (4)	-	-
	5-8	2 (1)	10 (7)	0 (4)	-
	9-12	-	8 (8)	4 (4)	-
0	1-4	7 (7)	4 (4)	1 (1)	-
	5-8	3 (3)	7 (5)	2 (4)	-
	9-12	-	8 (8)	4 (4)	-
1	1-4	7 (6)	4 (4)	1 (2)	-
	5-8	3 (4)	5 (4)	4 (4)	-
	9-12	0 (1)	8 (7)	4 (4)	-
2	1-4	8 (6)	4 (4)	0 (2)	-
	5-8	4 (4)	4 (4)	4 (4)	-
	9-12	4 (4)	4 (4)	4 (4)	-
3	1-4	8 (7)	4 (4)	0 (1)	-
	5-8	6 (4)	4 (4)	2 (4)	-
	9-12	4 (4)	4 (4)	4 (4)	-
4	1-4	8 (6)	4 (4)	0 (1)	0 (1)
	5-8	8 (1)	4 (4)	0 (4)	0 (3)
	9-12	4 (4)	4 (4)	4 (4)	-

(c) 2 vs. 1

α	β	C	NW	NE	SW	SE
-1	1-4	8 (8)	6 (6)	2 (2)	-	-
	5-8	2 (2)	10 (10)	4 (4)	-	-
	9-12	-	12 (12)	4 (4)	-	-
0	1-4	8 (8)	6 (6)	2 (2)	-	-
	5-8	4 (5)	8 (7)	4 (4)	-	-
	9-12	-	12 (12)	4 (4)	-	-
1	1-4	8 (8)	6 (6)	2 (2)	-	-
	5-8	5 (5)	7 (5)	4 (4)	-	0 (2)
	9-12	2 (2)	10 (8)	4 (5)	-	0 (1)
2	1-4	8 (8)	6 (6)	2 (2)	-	-
	5-8	7 (4)	5 (4)	4 (4)	-	0 (4)
	9-12	4 (4)	5 (5)	4 (4)	-	3 (3)
3	1-4	8 (7)	6 (6)	2 (2)	-	0 (1)
	5-8	8 (4)	4 (4)	4 (4)	-	0 (4)
	9-12	4 (4)	4 (4)	4 (4)	-	4 (4)
4	1-4	8 (7)	5 (6)	3 (2)	-	0 (1)
	5-8	8 (1)	4 (4)	4 (4)	0 (3)	0 (4)
	9-12	6 (3)	4 (4)	4 (4)	0 (1)	2 (4)

(d) 3 vs. 1



(a) 1 vs. 1 (b) 2 vs. 1 (c) 2 vs. 2 (d) 3 vs. 1

(e) 1 vs. 1 (f) 2 vs. 1 (g) 2 vs. 2 (h) 3 vs. 1

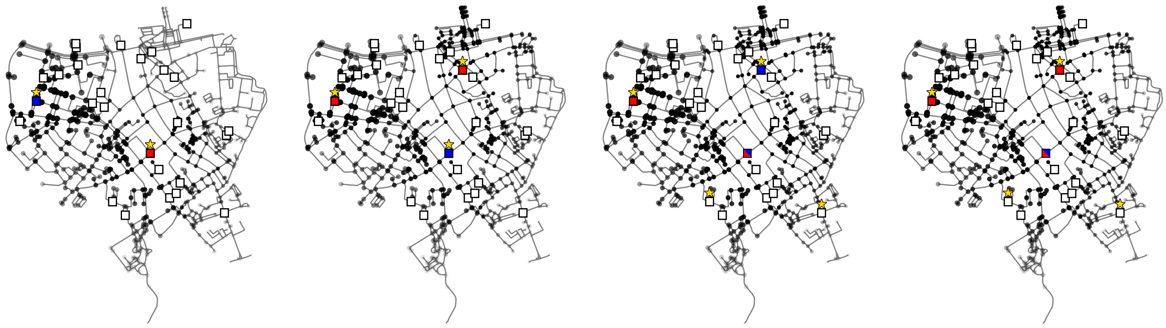
Figure 6: Comparison for 1 vs. 1, 2 vs. 1, 2 vs. 2, and 3 vs. 1, with $\beta = 8$ for $\alpha = 0$ (upper row) and $\alpha = 3$ (bottom row).

top and bottom rows of Figure 6, where there are more collection points in the center (except in the "2 vs. 2" scenario). Furthermore, note that in the bottom row of Figure 6, the number of customers using the collection point, indicated by the node density in black, has increased compared to the first row, which is a result of an increased initial interest in all collection points ($\alpha = 3$ in the bottom row, compared to $\alpha = 0$ in the top row).

A second key insight emerging from Table 6 and Figure 5 is that, when customers are less sensitive to travel distance (i.e., for low values of β), the placements tend to shift towards the central location, as this maximizes reach and accessibility to a larger share of the customer population. This is, for example, clearly illustrated in Figure 5, where, in the large majority of the cases, both companies opt for the central location (see, e.g., the cases with for $\beta \leq 3$ in the 1 vs. 1 and 2 vs. 1 scenarios). Note that this observation also aligns with the theoretical results in the two-district setting, which indicate that, when β is below a certain threshold, companies are more inclined to choose coinciding locations.

5.4 Social solution

As shown in the previous subsection and summarized in Figure 4, the relative loss across all scenarios is limited, suggesting that the location choices under strategic behavior may not deviate substantially from those in the socially optimal solution. This is indeed what we observe when investigating the spatial distribution of the social optimum compared to the Nash equilibrium. Nevertheless, in line with the theoretical results presented in §4, we find that the social solution tends to spread collection point locations slightly more than the Nash equilibria, particularly in cases of high attractiveness and travel sensitivity. This tendency becomes particularly evident in Tables 6b and 6d for $\alpha = 4$ and $\beta \geq 5$, where the social optimum proposes to locate a collection point in the SW area, a location that is consistently excluded from all equilibrium outcomes. As an example, Figure 7 shows that the social optimum places a collection point in each of the four corners. However, strategic behavior leads to both companies locating a collection point in the city center, thereby reducing the diversification of locations.



(a) 1 vs. 1 (b) 2 vs. 1 (c) 2 vs. 2 (d) 3 vs. 1

Figure 7: Comparison for 1 vs. 1, 2 vs. 1, 2 vs. 2, and 3 vs. 1 for $\alpha = 4$ and $\beta = 9$.

Finally, Table 7 summarizes the main insights from §5.2, §5.3, and §5.4, categorized by high and low levels of attractiveness and travel sensitivity.

Table 7: Location outcomes at the best Nash equilibria and social solutions for different parameter ranges.

Attractiveness	Travel sensitivity	Nash equilibrium	Social solution
Low or high	Low	Coincident locations (first at the center, then at the dense area)	Almost same as the equilibrium (slightly more spread)
Low	High	Different locations at the dense area	
High	High	Different locations over all the city	

5.5 Increasing the attractiveness of the social solution

In this final section of our analysis, we examine how a central authority can potentially influence the positioning of delivery companies to ensure their strategic decisions align with the optimal placement of collection points. A natural way for central authorities to support this alignment is by increasing public awareness of and interest in the use of collection points. In practice, this could involve promoting the use of collection points through public awareness campaigns, advertisements, or media coverage. Such campaigns could, for example, encourage the use of collection points as a convenient and sustainable alternative to home delivery. This centralized approach can, within our model, be represented by uniformly increasing the initial interest in all collection points by a constant factor. We want to stress that we have already executed such an experiment in the previous sections (see, e.g., Table 8 of §E in the supplementary materials, where we present the relative loss for increasing α and several βs). As illustrated by this experiment, increasing the initial interest parameter α does not necessarily lead delivery companies to make the same strategic choices as a central authority. For instance, in the 2 vs. 2 scenario, we observe that with $\beta = 11$ the relative loss goes to 8% for increasing α. This suggests that efforts by a central authority to promote collection point usage in general does not necessarily result in improved alignment with socially optimal outcomes. On the other hand, while the selected strategic locations may not fully align with the socially optimal solution, the overall use of collection points increases when they are actively promoted (i.e., overall market share increases). This can still be considered a positive outcome in itself.

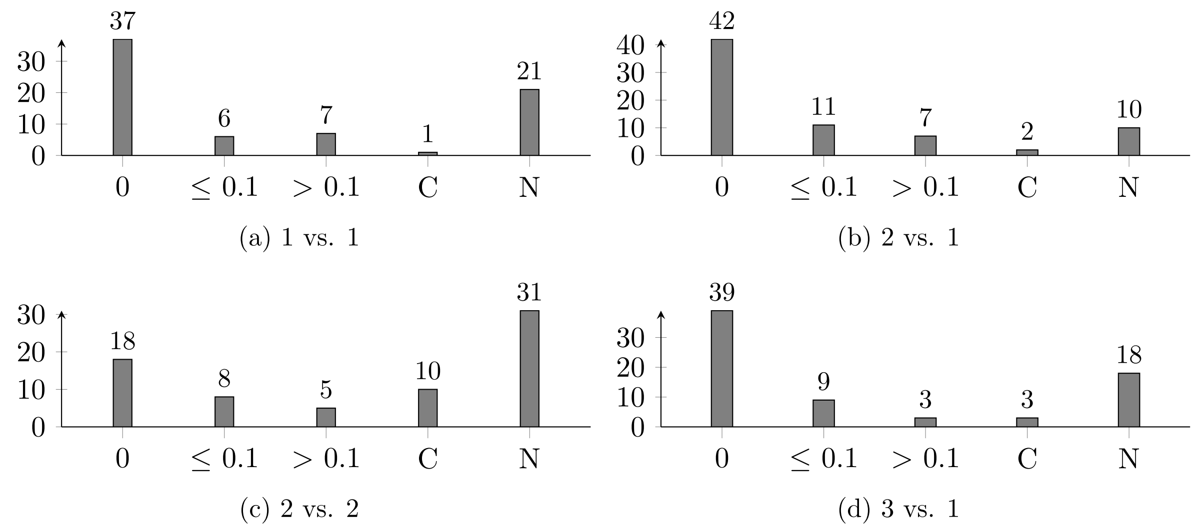
Next to a centralized approach, a central authority could also adopt a more targeted approach, e.g., by making specific collection points more attractive to potential customers. This might be achieved by improving the accessibility and convenience of the collection point, for instance, by enhancing the surrounding infrastructure through increased public transportation connections or the addition of parking facilities. This more targeted approach can, within our model, be represented by increasing the initial interest only for those collection points that are part of the centrally optimal solution. Formally, this means that we add a constant value (specifically, 0.01) to all α's that belong to the social solution, check if that solution is still the social solution, and check if any assignment of the locations of the social solution to the companies makes that assignment a Nash equilibrium[10]. If an assignment is a Nash equilibrium, we stop our procedure and give as output the total increase in α so far; otherwise, we increase the constant value by 0.01 and repeat until it reaches a Nash equilibrium or a predefined maximum

[10]In all our experiments, there was only one unique social solution, which we refer to as the social solution. In theory, however, multiple optimal social solutions may exist.

value (in our case, 5). Note that the social optimum may also change as attractiveness increases. In such a case, we also stop our procedure and report the outcome "C".

We show in Figure 8 the bar plots of the increase in α for all 288 scenarios as introduced in §5.1. Note that a value of zero implies that the social solution was already a Nash equilibrium. We use "N" if no increase made the social solution a Nash equilibrium, and report the outcome "C". We observe a relatively high frequency of "N" outcomes (27.4%), illustrating that the social solution and Nash equilibria do not often coincide, even if the locations of the social solution become very attractive. However, for almost half of all scenarios (47.6%) the social solution coincides with the Nash equilibrium directly. In 11.5% of the scenarios, the increase is below 1% and in the remaining 8% of the scenarios it is equal to or above 1%. It is worthwhile to mention that the 2 vs. 2 scenarios have a relatively high number of "N" cases and more "C" outcomes than the 1 vs. 1, 2 vs. 1, and 3 vs. 1 together. This may be in line with the observations in §5.2, where we argue that more balanced competition (i.e., each company having the same number of collection points) makes a match between the social solution and a Nash equilibrium less likely.

Figure 8: Barplots summarizing Table 9 in §F of the supplementary material on the increase of α for each scenario of collection points number. Label "N" refers to non-converging, while "C" refers to changing social optimum while finding the increase of α.



In summary, it does not seem to be straightforward for central authorities to align the strategic locations of collection points with the centrally optimal solution, whether through general promotion or a more targeted approach. However, these efforts do lead to an overall increase in market share. In practice, it is up to the central authority to strike an appropriate balance between improving usage rates and ensuring that the strategic positioning of delivery companies aligns with the optimal placement of collection points.

6 Concluding remarks

In this work, we investigated the strategic positioning of collection points in competitive settings and how this positioning differs from the optimal placement of a public authority. We did so by developing a competitive location model between two delivery companies where collection points are placed in a network of districts and customer choices are modeled using a multinomial logit framework. We began with a stylized setting comprising only two districts, where each company has a single collection point to locate, allowing us to characterize how players place their collection points within the same district or separately, according to the proposed parameters. We then compared the results of the stylized setting with those of a case study based on the Dutch city of Eindhoven, highlighting how geography as well as the population distribution in the city influence the decisions of companies and public authorities. From this analysis, we have observed that the public authority prefers to spread collection points more than companies in a competitive setting, with the customers' behavior influencing which areas collection points should be located in. Moreover, in the case study, the loss of customers due to companies' competition is limited (the highest loss is 10%), but a promotion of socially desirable collection point locations by the public authority does not necessarily lead companies to select the proposed locations.

We would now like to discuss some interesting directions for future research. Recall that in the presence of company-dependent attractiveness parameters, a pure Nash equilibrium may not exist. However, in the absence of such dependence, we found that a pure Nash equilibrium always exists in theoretical analysis, and our numerical experiments never yielded any counterexamples. It would therefore be interesting to prove whether a pure Nash equilibrium always exists in this restricted setting. However, proving this seems to be non-trivial, given that our game can be seen as a generalization of assortment games, for which it is already complex to demonstrate the existence of pure Nash equilibria (Besbes and Sauré, 2016; Nip and C. Wang, 2024). Another promising direction for future research would be to estimate the parameters of the discrete choice model in a real-world context, such as through a field experiment conducted in a city that has been adopting collection points, like Eindhoven. This would allow us to assess how well the model aligns with the actual choice behavior of customers and the location decisions of delivery companies (e.g., PostNL and DHL in Eindhoven). In doing so, it would also be logical to include various modes of travel and the sensitivity of traveling to collection points. As another future research direction, one could also consider different types of collection points, where customers may have varying preferences for each (e.g., related to services such as capacity and opening hours). Finally, our case study is primarily based on a walkable area, where the mode of transport does not influence the distance. This representation is proper in city centers and high-density residential or office areas. However, some collection points may attract customers in a different way, e.g., a collection point in a mall where people go to do their weekly big groceries by car. Our model could thus be extended to account for other modes of transportation.

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A Supermodularity

A typical technique to demonstrate that a game has a (pure) Nash equilibrium is to show that it is a supermodular game. However, as the following counter-example demonstrates, this is not the case of our game.

Example 5. *Let a game with* $D = L = \{1, 2, 3, 4\}$*,* $m_1 = m_2 = m_3 = m_4 = 1$*,* $\alpha^i_{j1} = \alpha^i_{j2} = \alpha_j$ *for each* $i \in D$ *and* $j \in L$*,* $B_1 = B_2 = 4$*, and* $\beta = 0$*, making* t^i_j *irrelevant. Moreover, let* $\alpha_1 = \ln(2)$*,* $\alpha_2 = \ln(3)$*,* $\alpha_3 = 0$*, and* $\alpha_4 = \ln(4)$*, so that* $u_1 = e^{\alpha_1} = 2$*,* $u_2 = e^{\alpha_2} = 3$*,* $u_3 = e^{\alpha_3} = 1$ *and* $u_4 = e^{\alpha_4} = 4$*. We now consider strategies* $x^+ = (1, 1, 0, 0)$*,* $x^- = (1, 0, 0, 0)$*,* $y^+ = (0, 0, 1, 1)$*, and* $y^- = (0, 0, 1, 0)$*. For our game to be supermodular (see Topkis (1998)), it should, amongst others, hold that:*

$$I_1(x^+, y^+) - I_1(x^-, y^+) \geq I_1(x^+, y^-) - I_1(x^-, y^-)$$

However, for our example we have

$$\begin{aligned} & I_1(x^+, y^+) - I_1(x^-, y^+) = 5 \cdot \frac{2+3}{1+2+3+1+4} - 5 \cdot \frac{2}{1+2+1+4} = \frac{45}{44} \\ < \quad & I_1(x^+, y^-) - I_1(x^-, y^-) = 5 \cdot \frac{2+3}{1+2+3+1} - 5 \cdot \frac{2}{1+2+1} = \frac{85}{14}, \end{aligned}$$

implying that our game is not supermodular.

B Proof of Proposition 2

Proposition 2. *The strategy profile* (x^s, y^s) *that solves* $\max\limits_{x \in \mathscr{X}, y \in \mathscr{Y}} I_s(x, y)$ *satisfies* $\sum\limits_{j \in L} x^s_j = B_1$ *and* $\sum\limits_{j \in L} y^s_j = B_2$*.*

Proof. For conciseness, we define $u^i_{jk} = e^{\alpha^i_{jk} - \beta t^i_j}$. The social payoff corresponds to

$$I_s(x, y) = I_1(x, y) + I_2(x, y) = \sum_{i \in D} m_i \frac{\sum\limits_{j \in L} (u^i_{j1} x_j + u^i_{j2} y_j)}{1 + \sum\limits_{j \in L} (u^i_{j1} x_j + u^i_{j2} y_j)}.$$

Let $z_i(x, y) = \sum_{j \in L} (u^i_{j1} x_j + u^i_{j2} y_j)$, and $h(z) = \frac{z}{1+z}$. Function z_i is linear in x_j and y_j for all $j \in L$, strictly increasing and non-negative since $u^i_{jk} > 0$. Function $h(z)$ is also strictly increasing. Thus, $h(z_i(x, y))$ is an increasing function for x_j and y_j for all $j \in L$. Finally, we can rewrite I_s as

$$I_s(x,y) = \sum_{i\in D} m_i h(z_i(x,y))$$

so $I_s(x,y)$, being the sum of strictly increasing functions in x_j and y_j, is also strictly increasing in x_j and y_j for all $j \in L$. Thus, let two strategy profiles $(\hat{x},\hat{y}),(\bar{x},\bar{y}) \in \mathscr{X} \times \mathscr{Y}$ such that $\sum_{j\in L} \hat{x}_j < B_1$ or $\sum_{j\in L} \hat{y}_j < B_2$, and $\sum_{j\in L} \bar{x}_j = B_1$ and $\sum_{j\in L} \bar{y}_j = B_2$, and $\{j \in L \,|\, \hat{x}_j = 1\} \subseteq \{j \in L \,|\, \bar{x}_j = 1\}$ and $\{j \in L \,|\, \hat{y}_j = 1\} \subseteq \{j \in L \,|\, \bar{y}_j = 1\}$, then $I_s(\bar{x},\bar{y}) > I_s(\hat{x},\hat{y})$ because $I_s(x,y)$ is a strictly increasing function in x_j and y_j. Since (x^s,y^s) is such that there is no other solution $(x,y) \in \mathscr{X} \times \mathscr{Y}$ such that $I_s(x,y) > I_s(x^s,y^s)$, then $\sum_{j\in L} x^s_j = B_1$ and $\sum_{j\in L} y^s_j = B_2$. □

C Proof of Theorem 2

Theorem 2. *Consider the two-district scenario. Let $\beta = 0$. Positioning a collection point in any district leads to a social solution. Let $\beta > 0$. If $\theta > \dfrac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$, both companies position their collection point at the most dense district in the social solution. If $\theta = \dfrac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$, positioning a collection point at any district leads to a social solution, except for the one where both position their collection point at the least dense district. If $\theta < \dfrac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$, the companies should position collection points opposite to each other, implying the existence of two social solutions.*

Proof. To identify the social solutions, we divide $I_1(x,y)$ and $I_2(x,y)$ by m_2 for all $x \in \mathscr{X}$ and all $y \in \mathscr{Y}$, similar to the proof of Theorem 1. We also refer to these scaled payoff functions as $I_1^\theta(x,y)$ and $I_2^\theta(x,y)$, which read as follows for every $x \in \mathscr{X}$ and every $y \in \mathscr{Y}$:

$$\begin{aligned}
I_1^\theta(x,y) &= \theta \frac{e^{-\beta x_2}}{e^{-\alpha}+e^{-\beta x_2}+e^{-\beta y_2}} + \frac{e^{-\beta x_1}}{e^{-\alpha}+e^{-\beta x_1}+e^{-\beta y_1}} \\
I_2^\theta(x,y) &= \theta \frac{e^{-\beta y_2}}{e^{-\alpha}+e^{-\beta x_2}+e^{-\beta y_2}} + \frac{e^{-\beta y_1}}{e^{-\alpha}+e^{-\beta x_1}+e^{-\beta y_1}}.
\end{aligned}$$

Recall that, according to Proposition 2, strategy (0,0) cannot be a social solution.

First, we show that the social payoff under ((1,0),(1,0)) is at least equal to the social payoff under ((0,1),(0,1)). That is,

$$
\begin{aligned}
&I_s^\theta((1,0),(1,0)) - I_s^\theta((0,1),(0,1)) \\
=&\Big(I_1^\theta((1,0),(1,0)) + I_2^\theta((1,0),(1,0))\Big) - \Big(I_1^\theta((0,1),(0,1)) + I_2^\theta((0,1),(0,1))\Big) \\
=&2I_1^\theta((1,0),(1,0)) - 2I_1^\theta((0,1),(0,1)) \\
=&2\left(\frac{\theta}{e^{-\alpha}+2} + \frac{e^{-\beta}}{e^{-\alpha}+2e^{-\beta}}\right) - 2\left(\frac{1}{e^{-\alpha}+2} + \frac{\theta e^{-\beta}}{e^{-\alpha}+2e^{-\beta}}\right) \\
=&2(\theta-1)\left(\frac{1}{e^{-\alpha}+2} - \frac{e^{-\beta}}{e^{-\alpha}+2e^{-\beta}}\right) \\
=&\frac{2e^{-\alpha}(\theta-1)(1-e^{-\beta})}{(e^{-\alpha}+2)(e^{-\alpha}+2e^{-\beta})} \geq 0
\end{aligned}
\tag{13}
$$

where the equalities hold by some rewriting. Note that in the second equality we use the symmetry of game $\mathscr{G}$. The inequality holds because all terms in the fraction are strictly positive, except for the term $(\theta - 1)$, which is non-negative. Hence, ((0,1),(0,1)) is never a social solution when $\theta > 1$, while it has the same social payoff of ((1,0)),(1,0)) when $\theta = 1$.

We now investigate when ((1,0)),(1,0)), ((1,0),(0,1)), ((0,1),(1,0)), and ((0,1),(0,1)) are social solutions. Given that our game $\mathscr{G}$ is symmetric, it suffices to only study ((1,0),(1,0)), ((1,0),(0,1)), and ((0,1),(0,1)). That is, whenever we make a conclusion about ((1,0),(0,1)) we do so as well for ((0,1),(1,0)). Similarly to the previous proof, we distinguish between three cases: (i) $\theta > \frac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$, (ii) $\theta = \frac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$, and (iii) $\theta < \frac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$.

Case (i) $\theta > \dfrac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$

We show that $((1,0),(1,0))$ is the only social solution in this case. In doing so, we first show that the social payoff of $((1,0),(1,0))$ is at least the social payoff of $((1,0),(0,1))$, i.e.,

$$
\begin{aligned}
&I_s^\theta((1,0),(1,0)) - I_s^\theta((1,0),(0,1)) \\
=&\Big(I_1^\theta((1,0),(1,0)) + I_2^\theta((1,0),(1,0))\Big) - \Big(I_1^\theta((1,0),(0,1)) + I_2^\theta((1,0),(0,1))\Big) \\
=&2\left(\frac{\theta}{e^{-\alpha}+2} + \frac{e^{-\beta}}{e^{-\alpha}+2e^{-\beta}}\right) - \left(\frac{\theta+e^{-\beta}}{1+e^{-\alpha}+e^{-\beta}} + \frac{\theta e^{-\beta}+1}{1+e^{-\alpha}+e^{-\beta}}\right) \\
=&\left(\frac{2}{e^{-\alpha}+2} - \frac{1+e^{-\beta}}{1+e^{-\alpha}+e^{-\beta}}\right)\theta + \frac{2e^{-\beta}}{e^{-\alpha}+2e^{-\beta}} - \frac{1+e^{-\beta}}{1+e^{-\alpha}+e^{-\beta}} \\
=&\frac{e^{-\alpha}(1-e^{-\beta})}{(e^{-\alpha}+2)(1+e^{-\alpha}+e^{-\beta})}\theta + \frac{e^{-\alpha}(e^{-\beta}-1)}{(e^{-\alpha}+2e^{-\beta})(1+e^{-\alpha}+e^{-\beta})} \\
=&\frac{e^{-\alpha}(1-e^{-\beta})}{1+e^{-\alpha}+e^{-\beta}}\left(\frac{1}{e^{-\alpha}+2}\theta - \frac{1}{e^{-\alpha}+2e^{-\beta}}\right) > 0.
\end{aligned}
\tag{14}
$$

The equalities hold by some rewriting. The inequality holds, because $\theta > \frac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$ by the case condition. Hence, $((0,1),(1,0))$ is not a social solution, neither is ((0,1),(0,1)) by (13) given that $\theta > 1$ in this case. Thus, $((1,0),(1,0))$ is the only social solution.

Case (ii) $\theta = \dfrac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$

In this case, we show that ((1,0),(1,0)) and ((1,0),(0,1)) are both social solutions. We start with ((1,0),(1,0)). By using the same derivation as used in (14), we have

$$I_s^{\theta}((1,0),(1,0)) - I_s^{\theta}((1,0),(0,1)) = \frac{e^{-\alpha}(1-e^{-\beta})}{1+e^{-\alpha}+e^{-\beta}}\left(\frac{1}{e^{-\alpha}+2}\theta - \frac{1}{e^{-\alpha}+2e^{-\beta}}\right) = 0, \tag{15}$$

because $\theta = \frac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$ by the case condition. Note that $\theta > 1$ and thus by (13) we know that ((0,1),(0,1)) has a smaller social payoff than ((1,0),(1,0)). This implies that ((1,0),(1,0)) and ((1,0),(0,1)) are the social solutions.

Case (iii) $\theta < \dfrac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$
In this case, we demonstrate that ((1,0), (0,1)) is the only social solution. By using the same derivation as used in (14), we have

$$I_s^{\theta}((1,0),(1,0)) - I_s^{\theta}((1,0),(0,1)) = \frac{e^{-\alpha}(1-e^{-\beta})}{1+e^{-\alpha}+e^{-\beta}}\left(\frac{1}{e^{-\alpha}+2}\theta - \frac{1}{e^{-\alpha}+2e^{-\beta}}\right) < 0 \tag{16}$$

because $\theta < \frac{e^{-\alpha}+2}{e^{-\alpha}+2e^{-\beta}}$ by the case condition. Hence, ((1,0),(1,0)) is not a social solution. By (13) we know that ((0,1),(0,1)) has a smaller social payoff than ((1,0),(1,0)). This implies that ((1,0),(0,1)) is the only social solution. □

D Mixed integer quadratic problem for the social solution

In this work, alongside using full enumeration of the payoffs to determine all Nash equilibria, we have also implemented a mixed integer quadratic optimization model for finding efficiently the social solution, implementable into commercial solvers such as Gurobi.

Finding the social solution corresponds to solving the following problem:

$$\max_{x,y} \quad \sum_{i \in D} \left(m_i \frac{\sum_{j \in L} (x_j e^{\alpha^i_{j1} - \beta t^i_j} + y_j e^{\alpha^i_{j2} - \beta t^i_j})}{1 + \sum_{j \in L} (x_j e^{\alpha^i_{j1} - \beta t^i_j} + y_j e^{\alpha^i_{j2} - \beta t^i_j})} \right) \tag{17a}$$

$$\text{s.t.} \quad \sum_{j \in L} x_j \leq B_1 \tag{17b}$$

$$\sum_{j \in L} y_j \leq B_2 \tag{17c}$$

$$x_j \in \{0,1\} \quad \forall j \in L \tag{17d}$$

$$y_j \in \{0,1\} \quad \forall j \in L. \tag{17e}$$

Note that the model of problem (17) is not linear nor quadratic. Since nowadays we have powerful solvers for linear and quadratic mixed-integer programs, the formulation of problem (17) prevent us to feed it directly to solvers such as Gurobi. Therefore, we rewrite problem (17) as a concave mixed-integer quadratic program (MIQP). In doing so, we first define $u^i_{jk} = e^{\alpha^i_{jk} - \beta t^i_j}$ for each $i \in D$, $j \in L$, and $k \in \{1, 2\}$.]Next, we define the auxiliary variables z_i and t_i for each $i \in D$, as well as the following constraints

$$z_i = 1 + \sum_{j \in L} x_j u^i_{j1} + \sum_{j \in L} y_j u^i_{j2} \tag{18a}$$

$$t_i = -\frac{1}{z_i} \tag{18b}$$

$$z_i \geq 0, \tag{18c}$$

$$t_i \leq 0. \tag{18d}$$

Note that z_i cannot be equal to zero, since it is the sum of 1 and non-negative terms. In this case, each term for $i \in D$ of the numerator becomes the following

$$m_i \frac{\sum_{j \in L} (x_j u^i_{j1} + y_j u^i_{j2})}{1 + \sum_{j \in L} (x_j u^i_{j1} + y_j u^i_{j2})} = m_i \frac{z_i - 1}{z_i} = m_i \left(1 - \frac{1}{z_i}\right) = m_i(1 + t_i).$$

Note that in the optimization problem we can relax constraints (18b) into

$$t_i \leq -\frac{1}{z_i} \tag{19}$$

since the maximization of the new objective function $\sum_{i \in D} m_i(1 + t_1)$ leads to have t_i as high as possible, making the inequality tight. The variables z_i and t_i have been defined in order to reformulate the model with a rotated second-order cone constraint. Second-order cone constraints have the following structure:

$$Q^n = \{x \in \mathbb{R}^n : 2x_1x_2 \geq x_3^2 + \ldots + x_n^2,\ x_1, x_2 \geq 0\} \tag{20}$$

which, for $n = 3$, corresponds to $Q^3 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : 2x_1x_2 \geq x_3^2,\ x_1, x_2 \geq 0\}$. Now, it is easy to see that constraints (19) can be captured through a rotated second-order cone:

$$\text{Constraint (19)} \Leftrightarrow (-z_i) \cdot t_i \geq 1 \Leftrightarrow \left(-t_i, z_i, \sqrt{\frac{1}{2}}\right) \in Q^3.$$

Note that the third term of the elements in the cone is a constant. Finally, our MIQP formulation for the finding the social solution is

$$\max_{x,y,z,t} \quad \sum_{i \in D} m_i(1 + t_i) \tag{21a}$$

$$s.t. \quad \sum_{j \in L} x_j \leq B_1 \tag{21b}$$

$$\sum_{j \in L} y_j \leq B_2 \tag{21c}$$

$$z_i = 1 + \sum_{l \in L}(x_j u_{j1}^1 + y_j u_{j2}^i) \qquad \forall i \in D \tag{21d}$$

$$\left(-t_i, z_i, \sqrt{\frac{1}{2}}\right) \in Q^3 \qquad \forall i \in D \tag{21e}$$

$$x_j, y_j \in \{0, 1\} \qquad \forall j \in L \tag{21f}$$

$$z_i \geq 0 \qquad \forall i \in D \tag{21g}$$

$$t_i \leq 0 \qquad \forall i \in D. \tag{21h}$$

Now, the model is in a form that can be directly inputted to popular commercial solvers.

E Results on the market share of the case study

We report in Table 8, for each combination of attractiveness and travel sensitivity, the market share at the equilibrium (left entry) and the loss with respect to the social optimum (right entry). We also report in Figures 9-32 the location outcomes, for each combination of attractiveness and travel sensitivity, according to the best Nash equilibrium (red and blue squares), and the social solution (yellow stars). For each figure, three α and four β parameters are reported in the caption, plus the number of available collection points per company. Each column corresponds to a different α (the smallest to the left, the largest to the right), each row to a different β (the smallest on the top, the largest on the bottom). As an example, in Figure 10 the picture on the second row from the top and to the left corresponds to a scenario with 1 vs. 1 collection points, $\alpha = 2$, and $\beta = 2$.

Table 8: Overall market share (first) and relative loss (second) of the best equilibrium for each combination of proposed α, β, and number of collection points per company.

		α											
		-1		0		1		2		3		4	
β	1	29.4%	-	52.84%	-	75.11%	-	89.07%	0.01%	95.67%	0.01%	98.36%	0.0%
	2	19.6%	-	39.06%	0.17%	62.6%	0.67%	81.51%	0.55%	92.17%	0.29%	96.94%	0.12%
	3	13.0%	-	27.72%	0.89%	49.02%	2.11%	70.7%	2.03%	86.1%	1.25%	94.23%	0.58%
	4	9.03%	-	20.31%	-	36.87%	4.46%	58.08%	4.31%	76.97%	3.26%	89.34%	1.8%
	5	6.35%	3.88%	15.27%	-	30.36%	-	49.18%	1.82%	69.92%	-	81.67%	4.11%
	6	5.06%	-	11.73%	-	24.08%	-	41.54%	-	59.42%	1.66%	77.35%	-
	7	4.13%	-	8.79%	5.18%	19.3%	-	34.46%	-	52.16%	-	68.22%	0.84%
	8	3.43%	0.74%	7.37%	4.78%	15.76%	-	28.69%	-	44.87%	-	58.78%	3.96%
	9	2.95%	-	6.43%	2.71%	12.03%	10.0%	24.02%	-	38.6%	-	54.19%	-
	10	2.54%	-	5.58%	2.44%	10.55%	9.2%	20.52%	-	33.25%	-	47.86%	-
	11	2.22%	-	4.9%	2.34%	9.33%	8.65%	17.94%	-	28.73%	-	42.24%	-
	12	1.96%	0.04%	4.34%	2.32%	8.31%	8.27%	16.0%	-	25.14%	-	37.31%	-

(a) 1 vs 1

		α											
		-1		0		1		2		3		4	
β	1	38.28%	-	62.61%	-	81.9%	0.01%	92.46%	0.01%	97.08%	0.0%	98.91%	0.0%
	2	26.57%	-	48.98%	-	71.81%	-	87.2%	-	94.84%	-	98.03%	-
	3	18.1%	0.1%	36.48%	0.1%	59.66%	0.0%	79.35%	0.0%	91.05%	-	96.47%	-
	4	12.83%	-	27.49%	-	49.05%	-	69.15%	2.24%	85.12%	1.01%	93.75%	0.42%
	5	9.34%	0.17%	20.66%	1.86%	39.92%	-	61.74%	-	76.9%	3.47%	89.34%	1.72%
	6	7.21%	0.07%	15.73%	3.9%	32.45%	-	53.12%	-	69.04%	4.76%	82.88%	4.11%
	7	5.75%	1.98%	12.73%	1.88%	26.48%	-	45.34%	-	65.0%	-	76.32%	5.53%
	8	4.83%	2.0%	10.81%	-	20.21%	7.27%	38.58%	-	57.66%	-	68.79%	7.68%
	9	4.22%	-	9.31%	-	17.67%	3.55%	32.85%	-	50.8%	-	68.07%	-
	10	3.66%	-	8.12%	0.48%	15.79%	-	28.05%	0.47%	44.61%	-	61.67%	-
	11	3.22%	-	7.24%	-	14.1%	-	24.45%	-	39.13%	-	55.55%	-
	12	2.85%	0.03%	6.47%	-	12.69%	-	21.35%	-	34.36%	0.21%	49.89%	-

(b) 2 vs 1

		α											
		-1		0		1		2		3		4	
β	1	45.11%	0.0%	68.94%	0.0%	85.73%	0.0%	94.22%	0.0%	97.79%	0.0%	99.18%	0.0%
	2	32.26%	-	55.82%	-	77.06%	-	90.01%	-	96.05%	-	98.51%	-
	3	22.44%	0.57%	42.87%	1.23%	65.93%	1.17%	83.48%	0.7%	93.07%	0.32%	97.31%	0.13%
	4	16.08%	0.94%	33.48%	-	56.23%	-	74.84%	2.34%	88.25%	1.4%	95.19%	0.63%
	5	11.97%	0.54%	25.06%	3.04%	46.54%	-	68.22%	0.32%	84.29%	0.51%	91.79%	1.87%
	6	9.29%	-	20.26%	0.03%	37.51%	1.94%	59.59%	1.2%	77.91%	1.27%	85.09%	5.98%
	7	7.48%	-	16.31%	0.76%	31.18%	1.67%	50.14%	4.61%	70.79%	2.2%	84.99%	1.86%
	8	6.3%	-	13.51%	1.02%	26.03%	1.69%	43.32%	4.84%	61.63%	6.01%	79.39%	2.92%
	9	5.39%	0.19%	11.55%	-	22.57%	0.71%	37.99%	3.45%	54.98%	6.6%	73.2%	3.88%
	10	4.67%	0.57%	10.13%	-	19.46%	1.23%	33.04%	3.38%	48.83%	7.07%	65.01%	7.2%
	11	4.14%	0.01%	8.98%	-	16.7%	3.7%	29.23%	2.23%	44.27%	5.32%	59.12%	7.92%
	12	3.69%	-	8.02%	-	15.04%	3.01%	25.87%	2.4%	39.49%	5.25%	58.56%	-

(c) 2 vs 2

		α											
		-1		0		1		2		3		4	
β	1	45.11%	-	68.94%	-	85.73%	-	94.22%	-	97.79%	-	99.18%	-
	2	32.23%	-	55.79%	-	77.04%	-	90.0%	-	96.05%	-	98.47%	0.03%
	3	22.57%	-	43.4%	-	66.72%	-	84.07%	-	92.87%	0.54%	97.23%	0.21%
	4	16.23%	-	33.48%	-	56.23%	-	76.64%	-	89.5%	0.0%	95.76%	0.04%
	5	12.03%	-	25.53%	1.21%	46.54%	-	68.22%	0.32%	84.29%	0.51%	93.24%	0.33%
	6	9.29%	-	20.26%	0.03%	37.51%	1.94%	59.59%	1.2%	77.91%	1.27%	89.66%	0.92%
	7	7.48%	-	16.39%	0.27%	31.18%	1.67%	51.4%	2.21%	70.79%	2.2%	84.99%	1.86%
	8	6.3%	-	13.65%	-	26.1%	1.44%	43.32%	4.84%	63.43%	3.25%	79.39%	2.92%
	9	5.39%	0.19%	11.55%	-	22.73%	-	39.35%	-	58.86%	-	73.2%	3.88%
	10	4.67%	0.57%	10.13%	-	19.7%	-	34.02%	0.5%	52.54%	-	66.8%	4.65%
	11	4.14%	0.01%	8.98%	-	16.7%	3.7%	29.9%	-	46.76%	-	64.2%	-
	12	3.69%	-	8.02%	-	15.04%	3.01%	26.5%	-	41.57%	0.26%	58.56%	-

(d) 3 vs 1

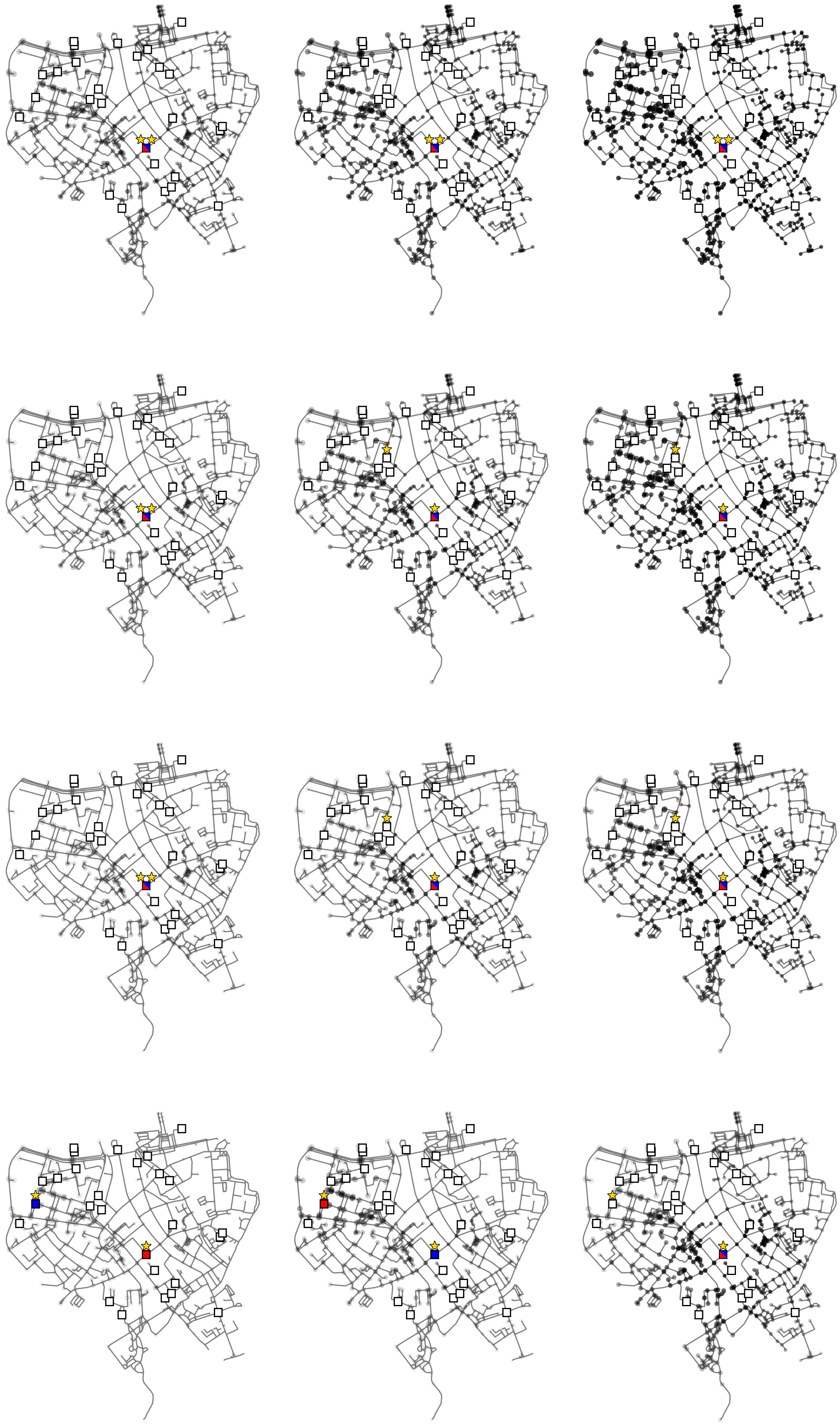

Figure 9: 1 vs 1, $\beta = 1, 2, 3, 4\,\mathrm{km}^{-1}$, $\alpha = -1, 0, 1$

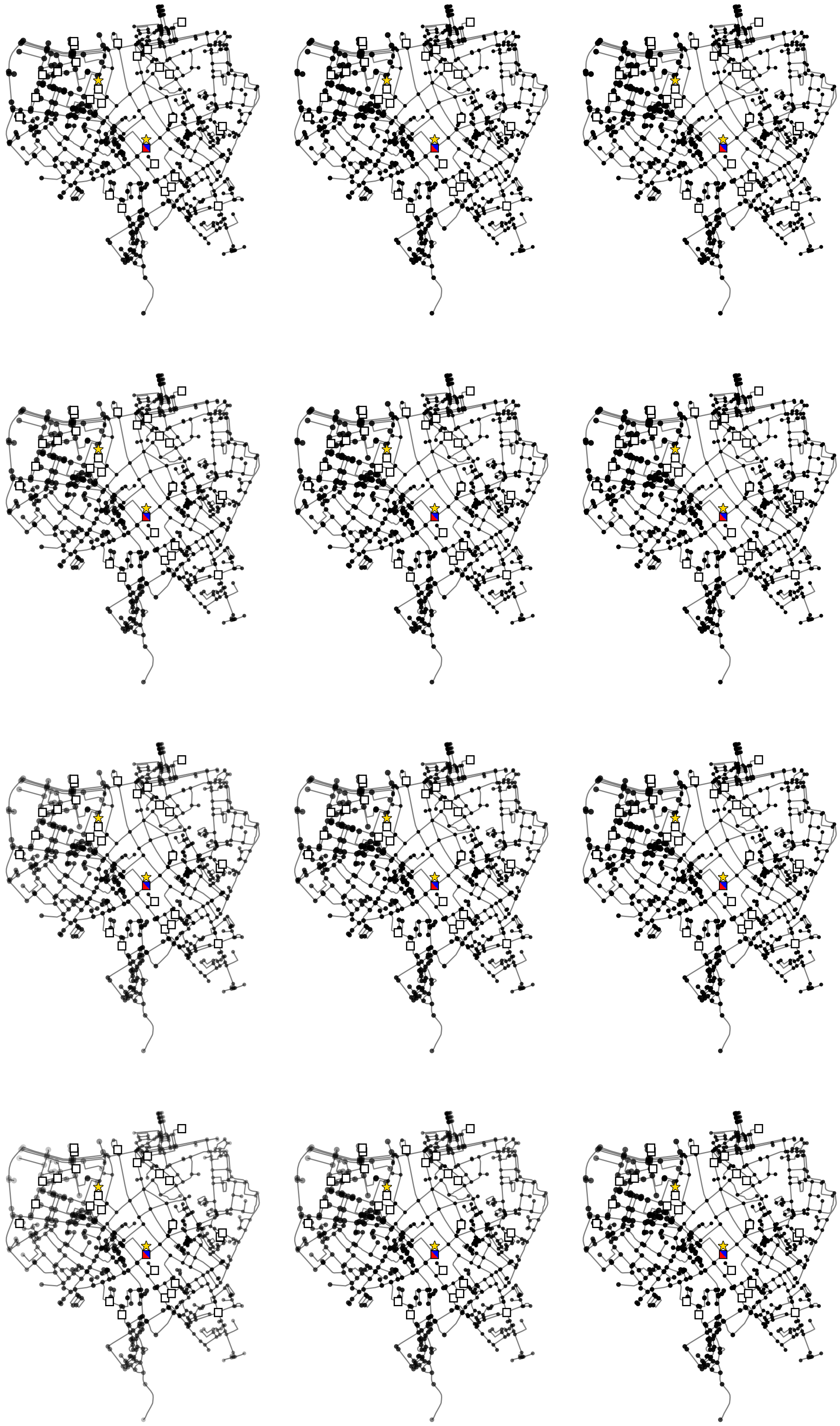

Figure 10: 1 vs. 1, $\beta = 1, 2, 3, 4\,\mathrm{km}^{-1}$, $\alpha = 2, 3, 4$

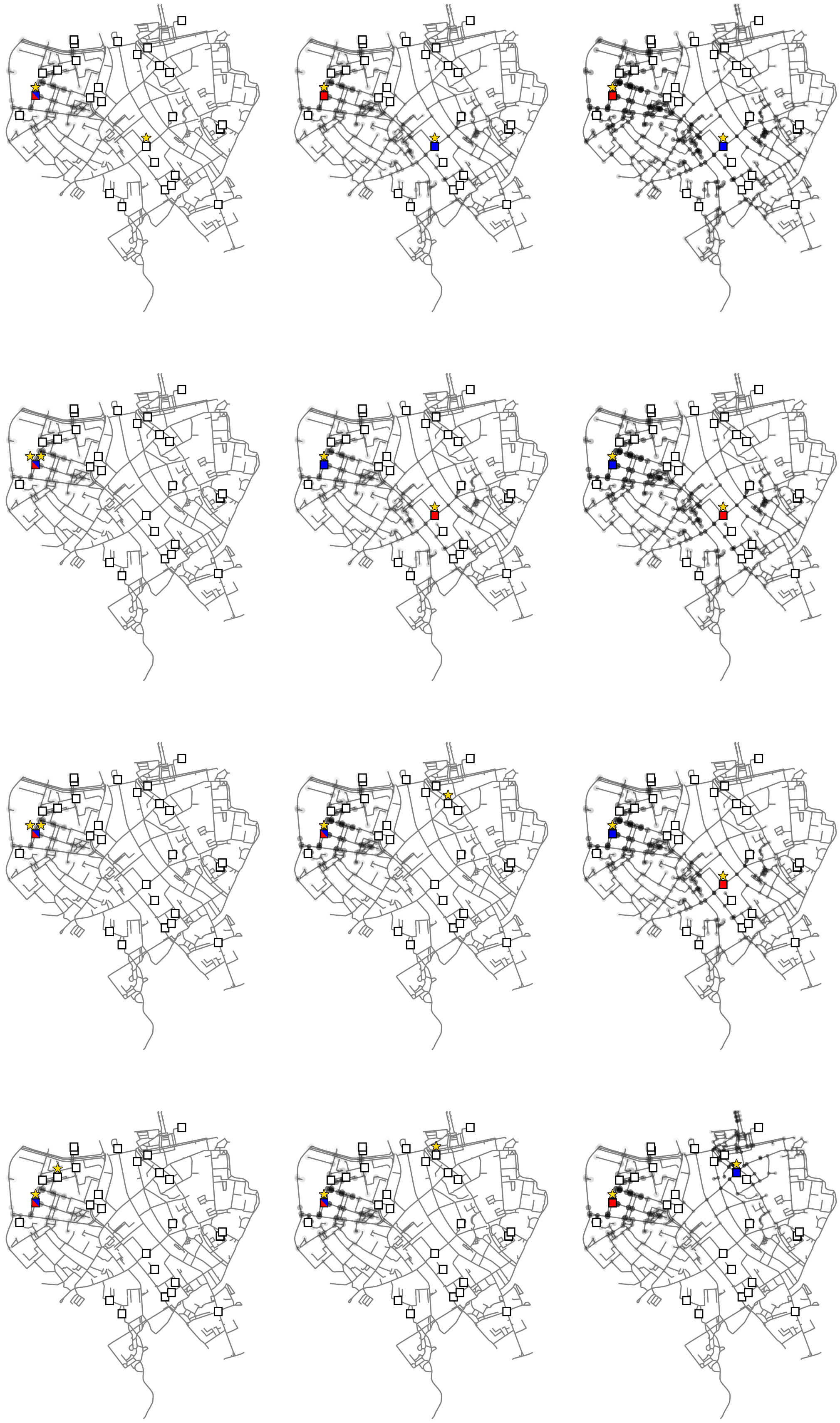

Figure 11: 1 vs. 1, $\beta = 5, 6, 7, 8\,\mathrm{km}^{-1}$, $\alpha = -1, 0, 1$

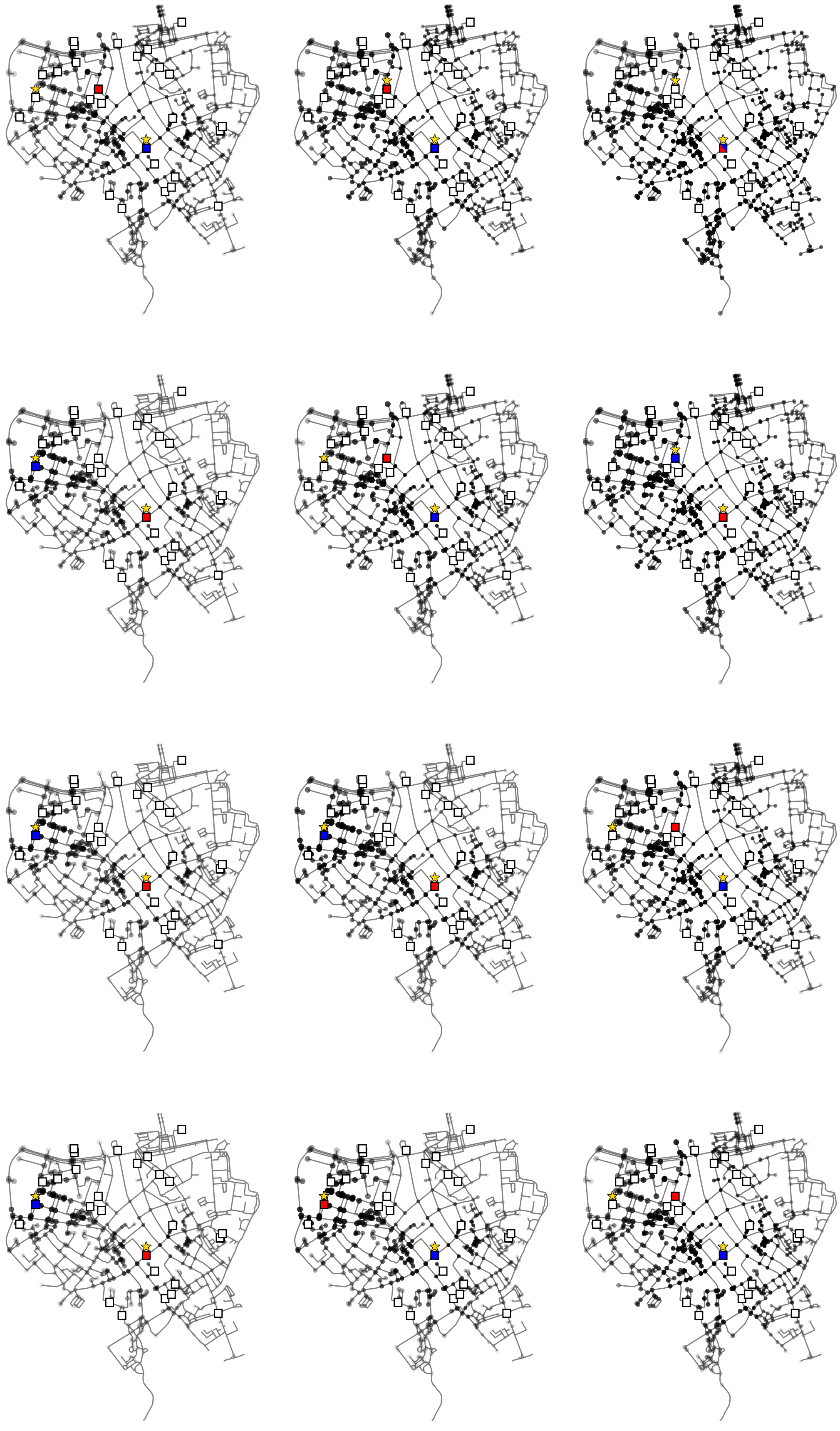

Figure 12: 1 vs. 1, $\beta = 5, 6, 7, 8\,\mathrm{km}^{-1}$, $\alpha = 2, 3, 4$

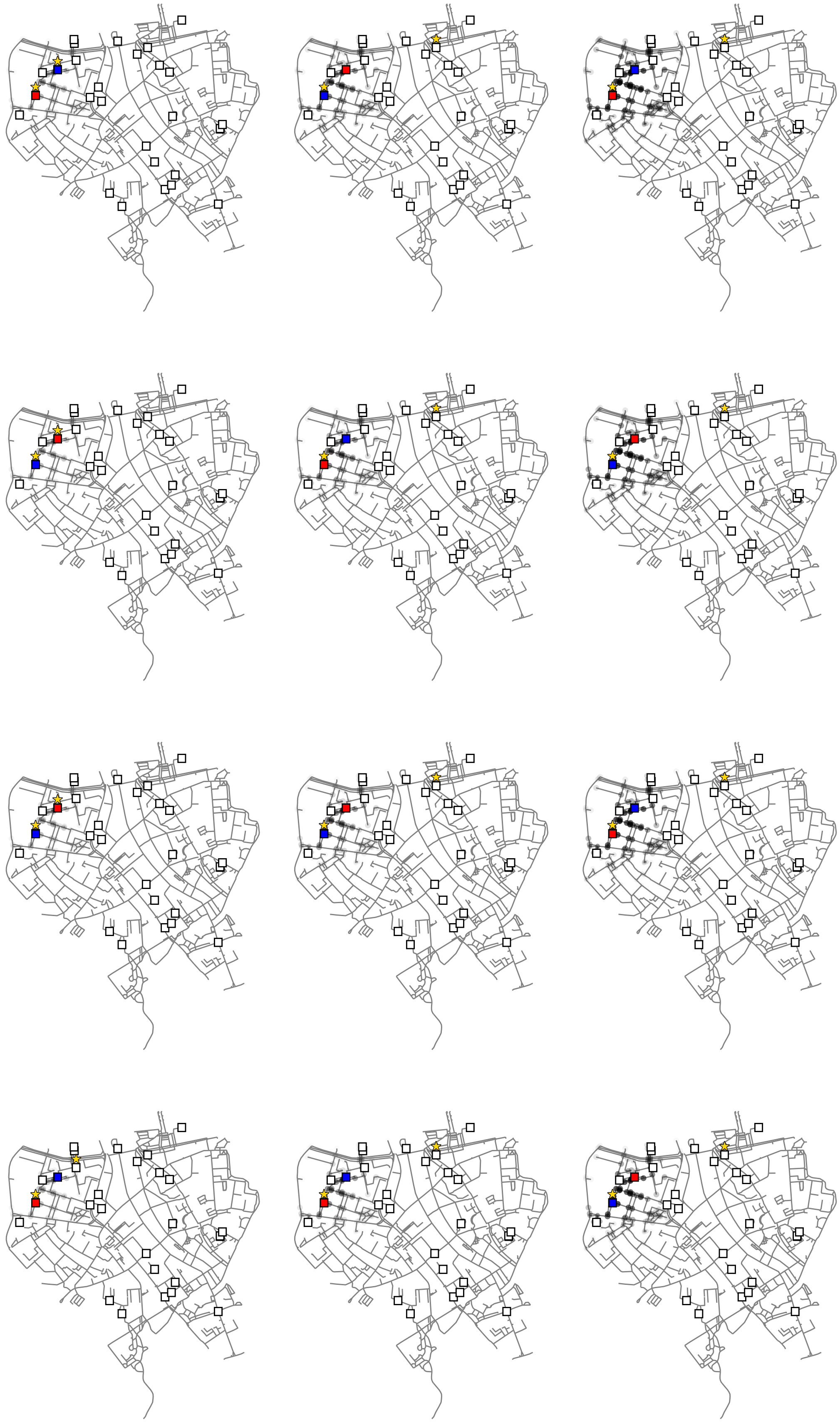

Figure 13: 1 vs. 1, $\beta = 9, 10, 11, 12\,\mathrm{km}^{-1}$, $\alpha = -1, 0, 1$

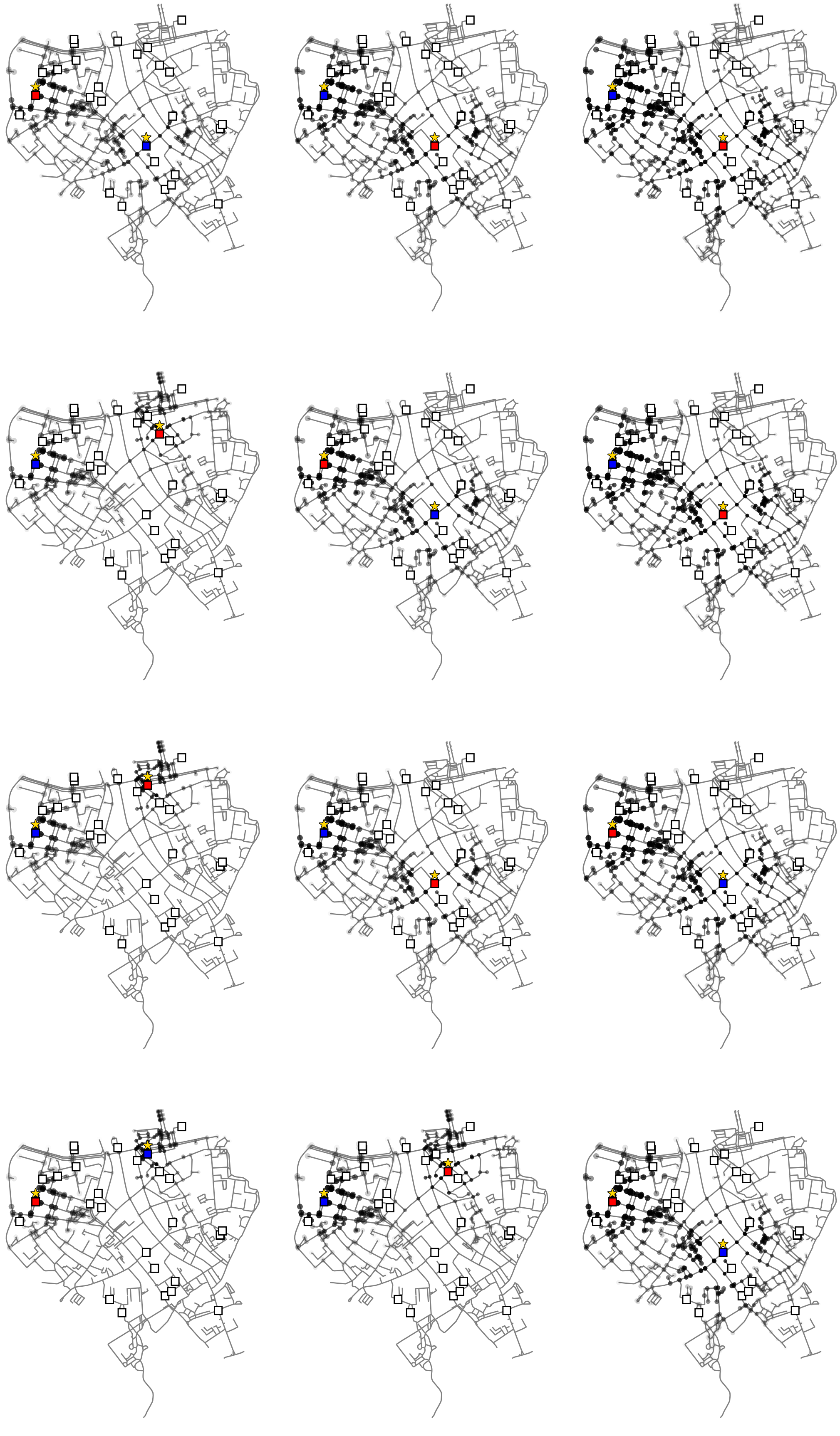

Figure 14: 1 vs. 1, $\beta = 9, 10, 11, 12\,\mathrm{km}^{-1}$, $\alpha = 2, 3, 4$

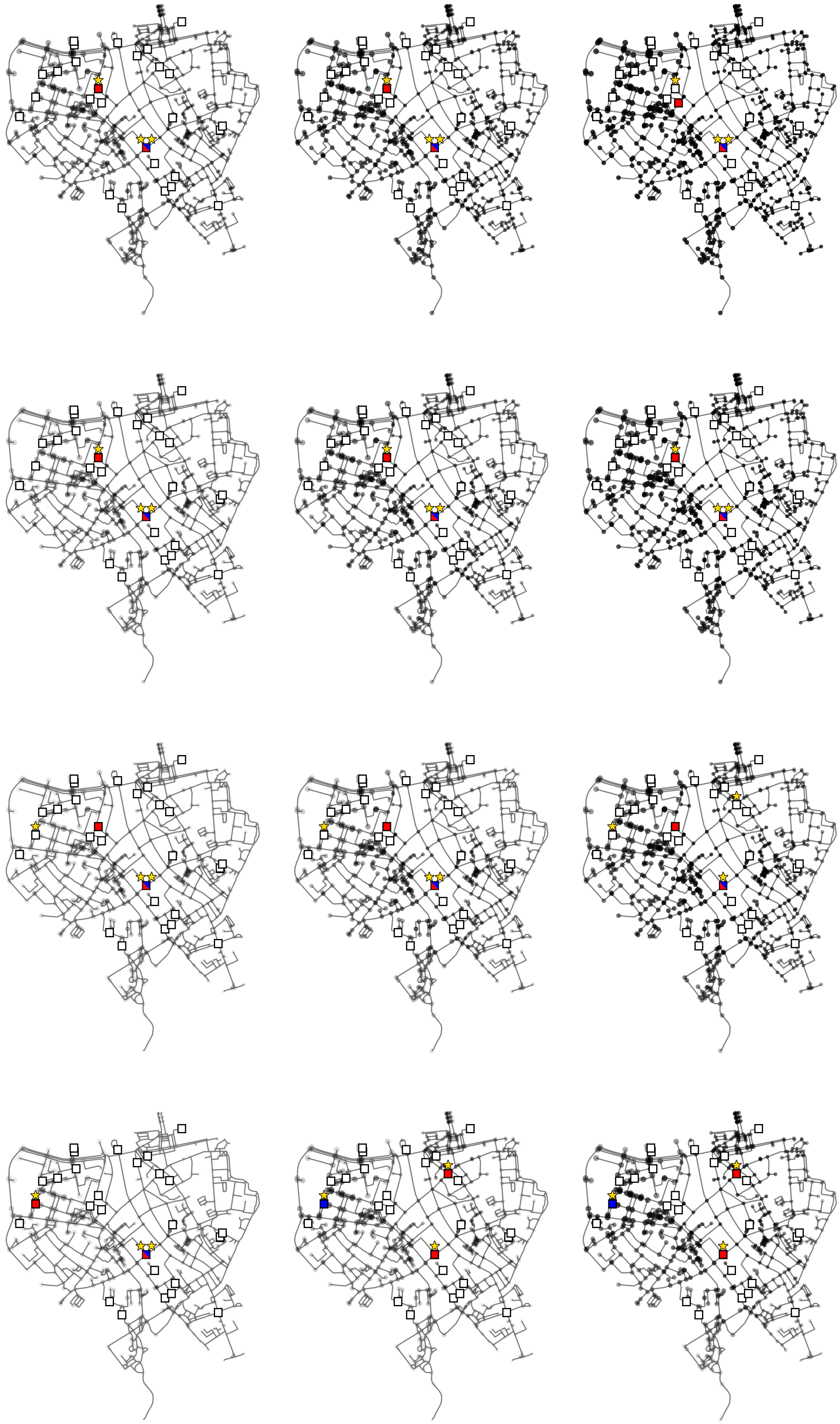

Figure 15: 2 vs. 1, $\beta = 1, 2, 3, 4\,\mathrm{km}^{-1}$, $\alpha = -1, 0, 1$

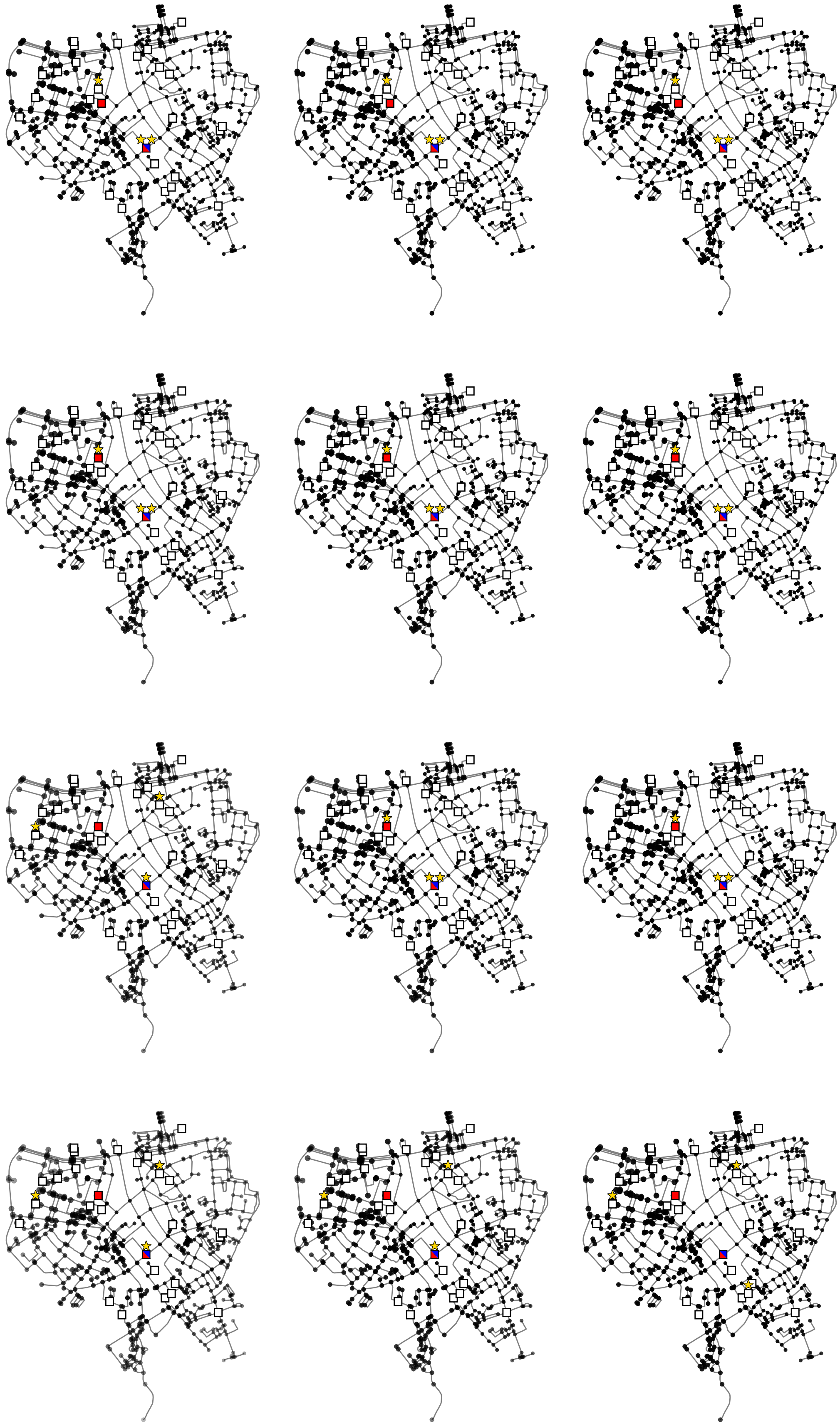

Figure 16: 2 vs. 1, $\beta = 1, 2, 3, 4\,\mathrm{km}^{-1}$, $\alpha = 2, 3, 4$

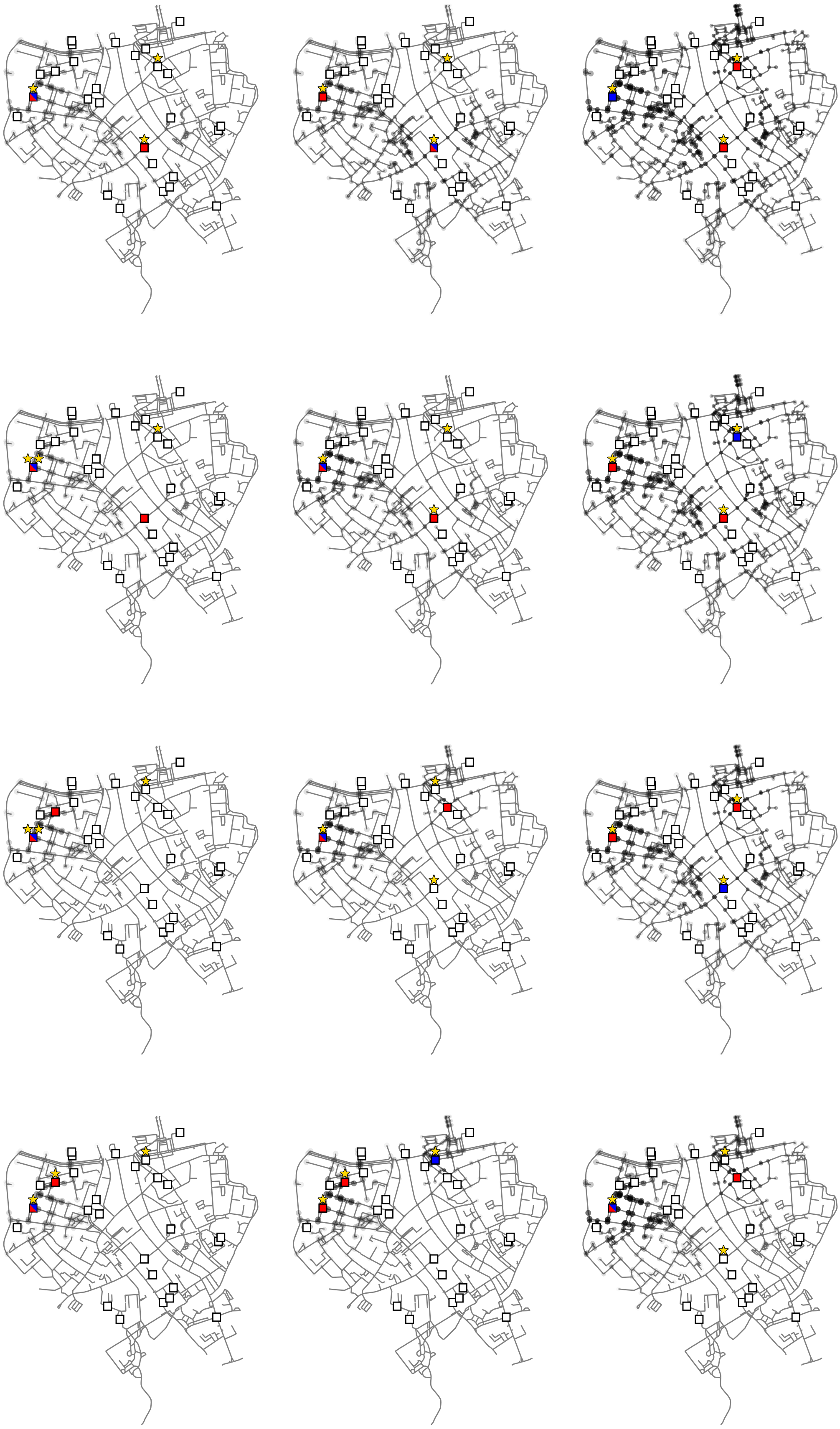

Figure 17: 2 vs. 1, $\beta = 5, 6, 7, 8\,\mathrm{km}^{-1}$, $\alpha = -1, 0, 1$

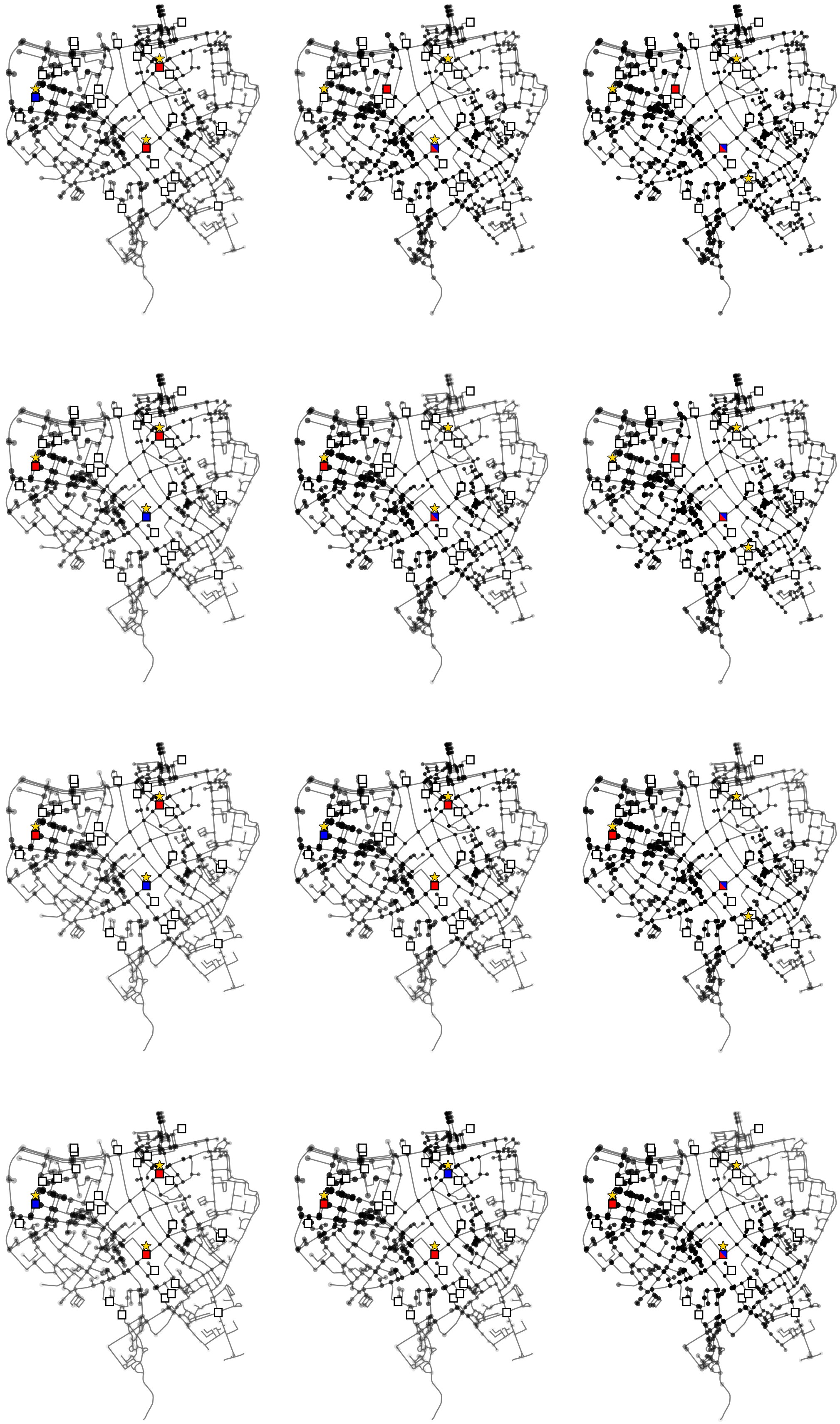

Figure 18: 2 vs. 1, $\beta = 5, 6, 7, 8\,\mathrm{km}^{-1}$, $\alpha = 2, 3, 4$

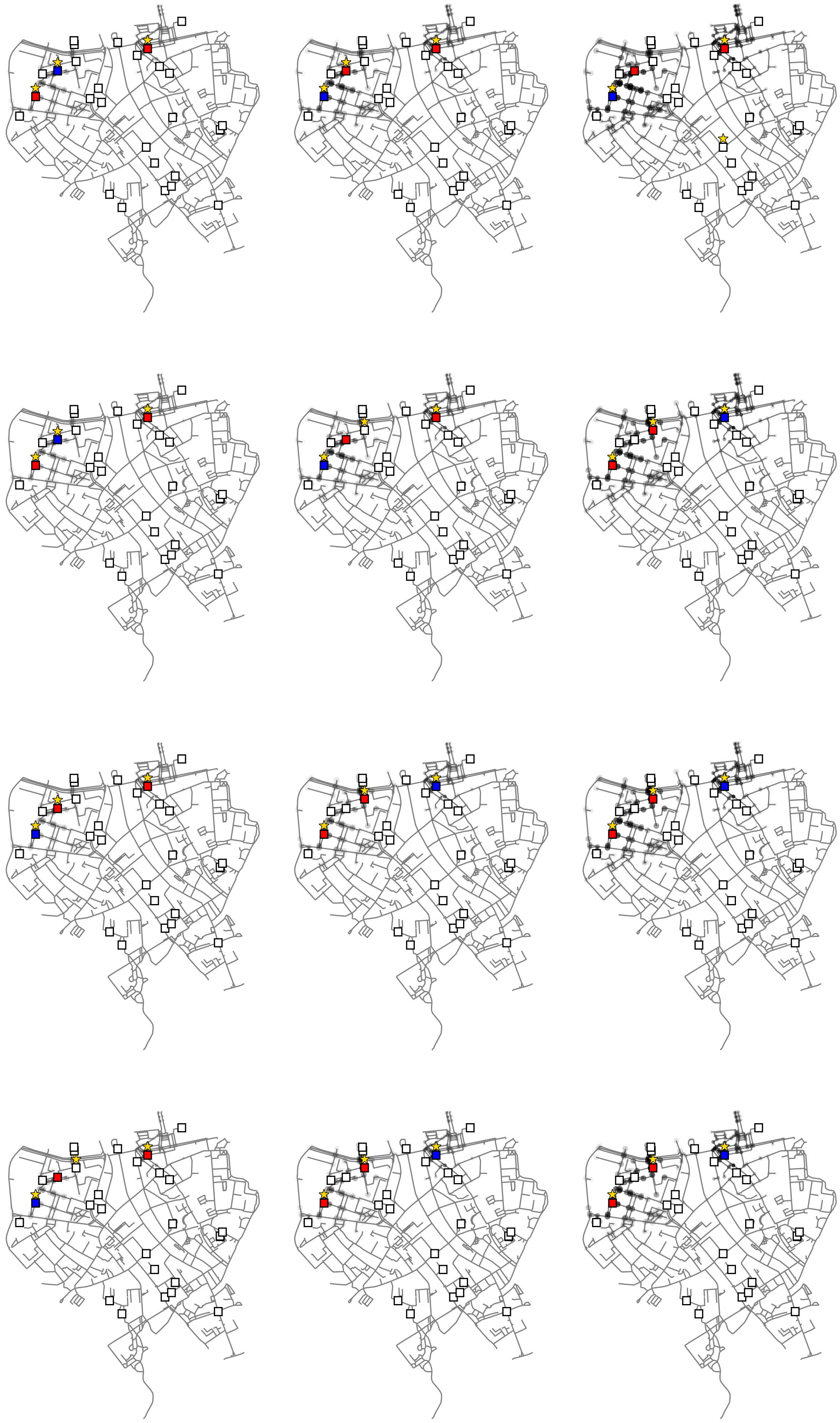

Figure 19: 2 vs. 1, $\beta = 9, 10, 11, 12\,\mathrm{km}^{-1}$, $\alpha = -1, 0, 1$

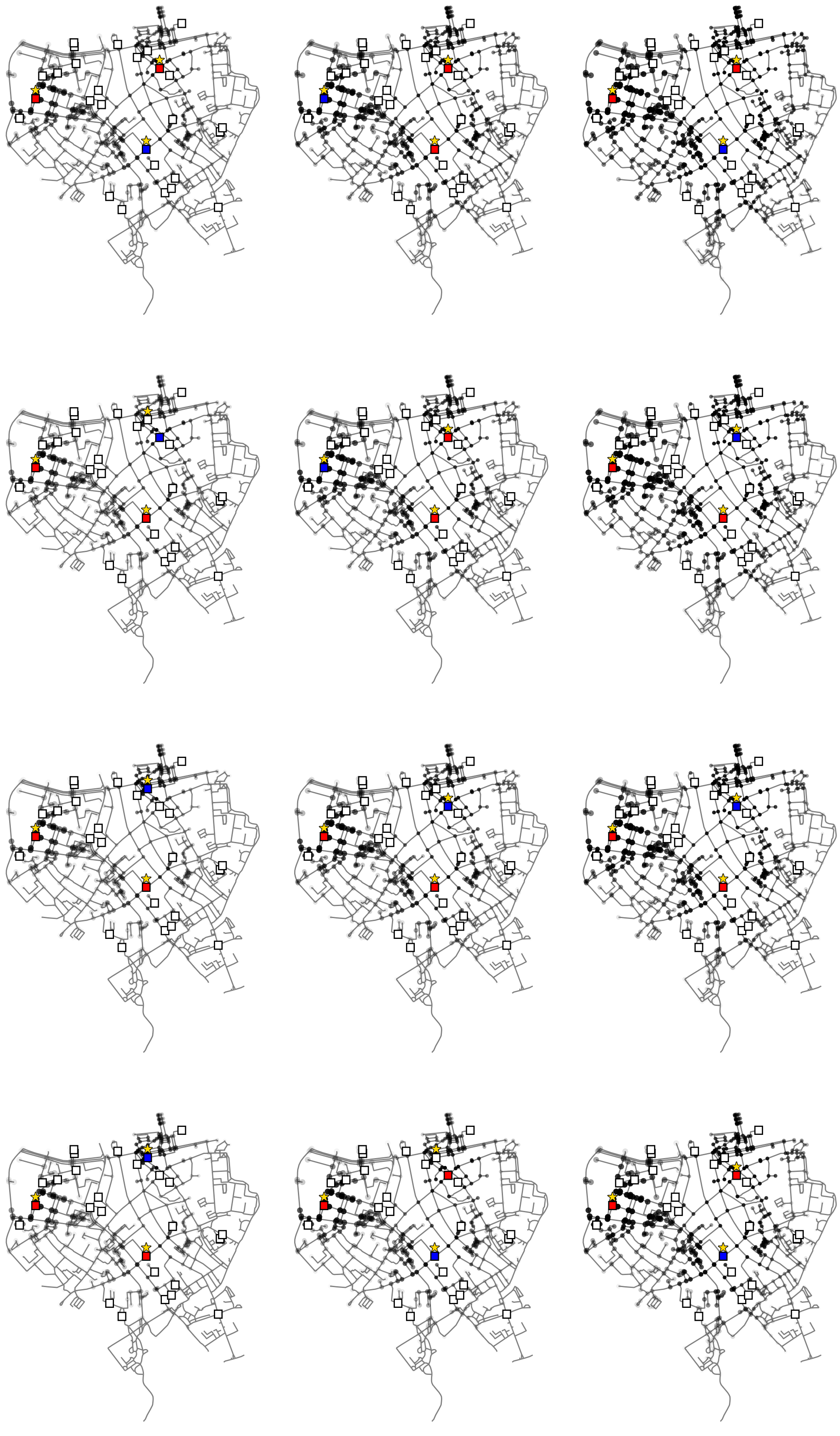

Figure 20: 2 vs. 1, $\beta = 9, 10, 11, 12\,\mathrm{km}^{-1}$, $\alpha = 2, 3, 4$

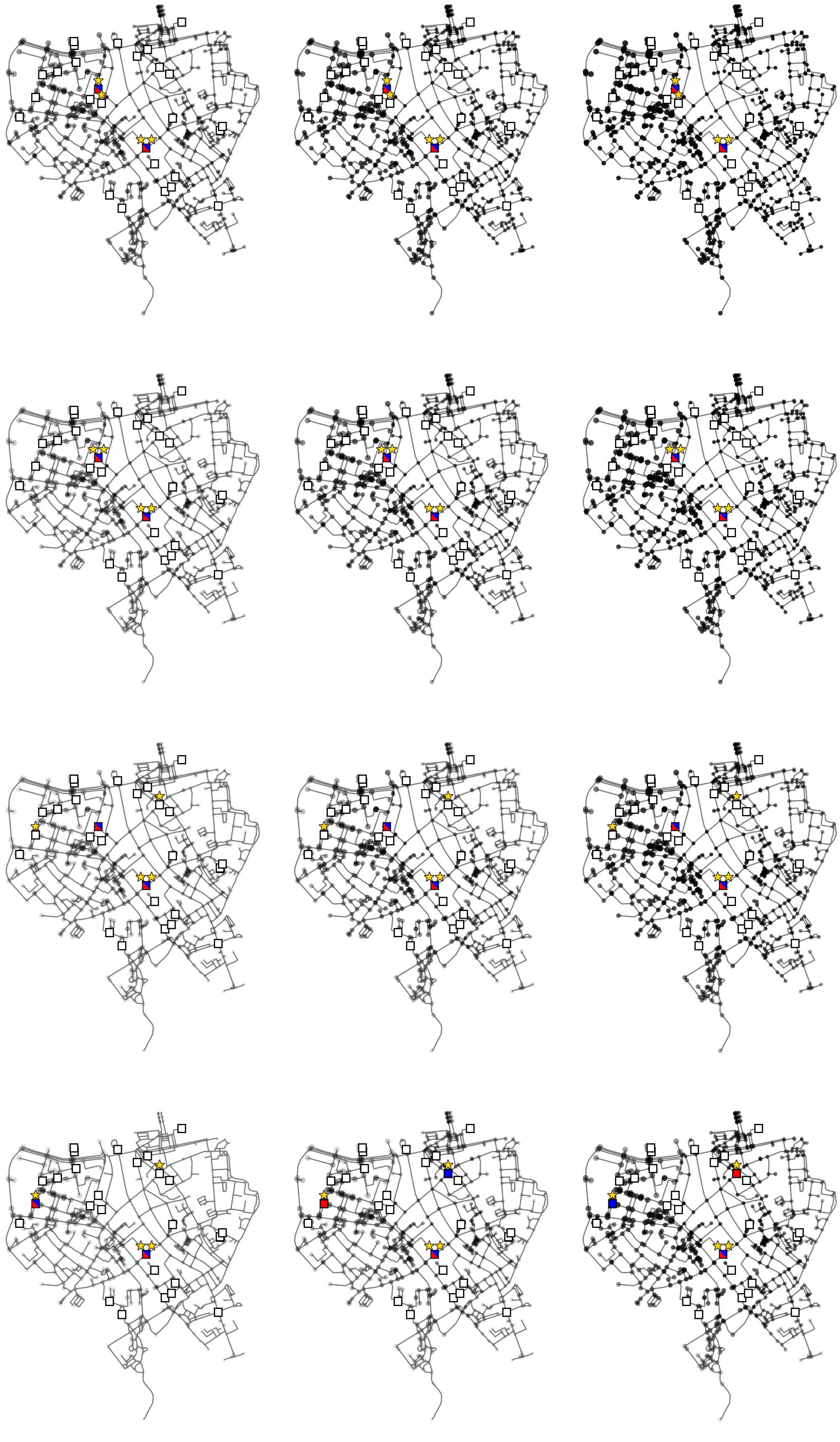

Figure 21: 2 vs. 2, $\beta = 1, 2, 3, 4\,\mathrm{km}^{-1}$, $\alpha = -1, 0, 1$

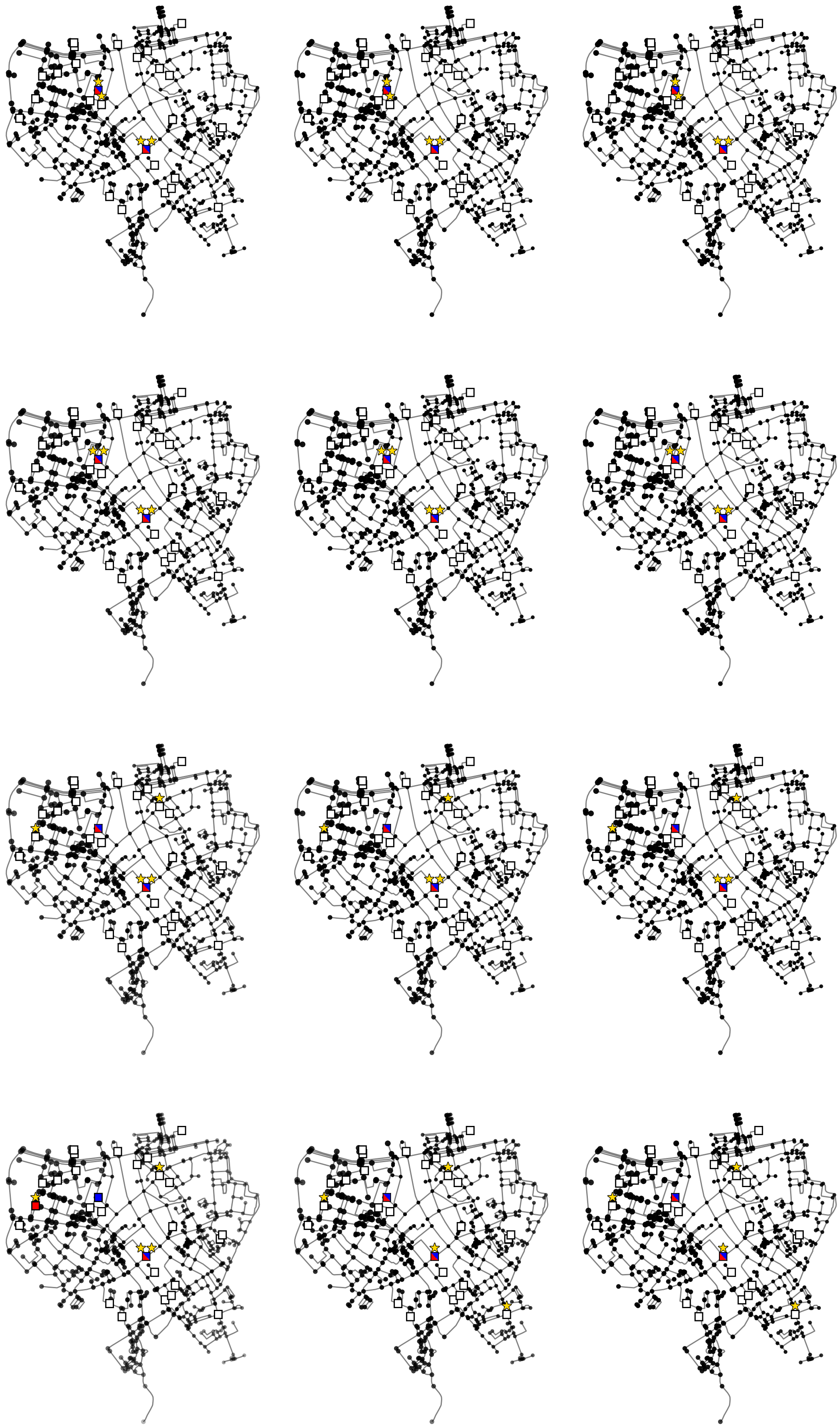

Figure 22: 2 vs. 2, $\beta = 1, 2, 3, 4\,\mathrm{km}^{-1}$, $\alpha = 2, 3, 4$

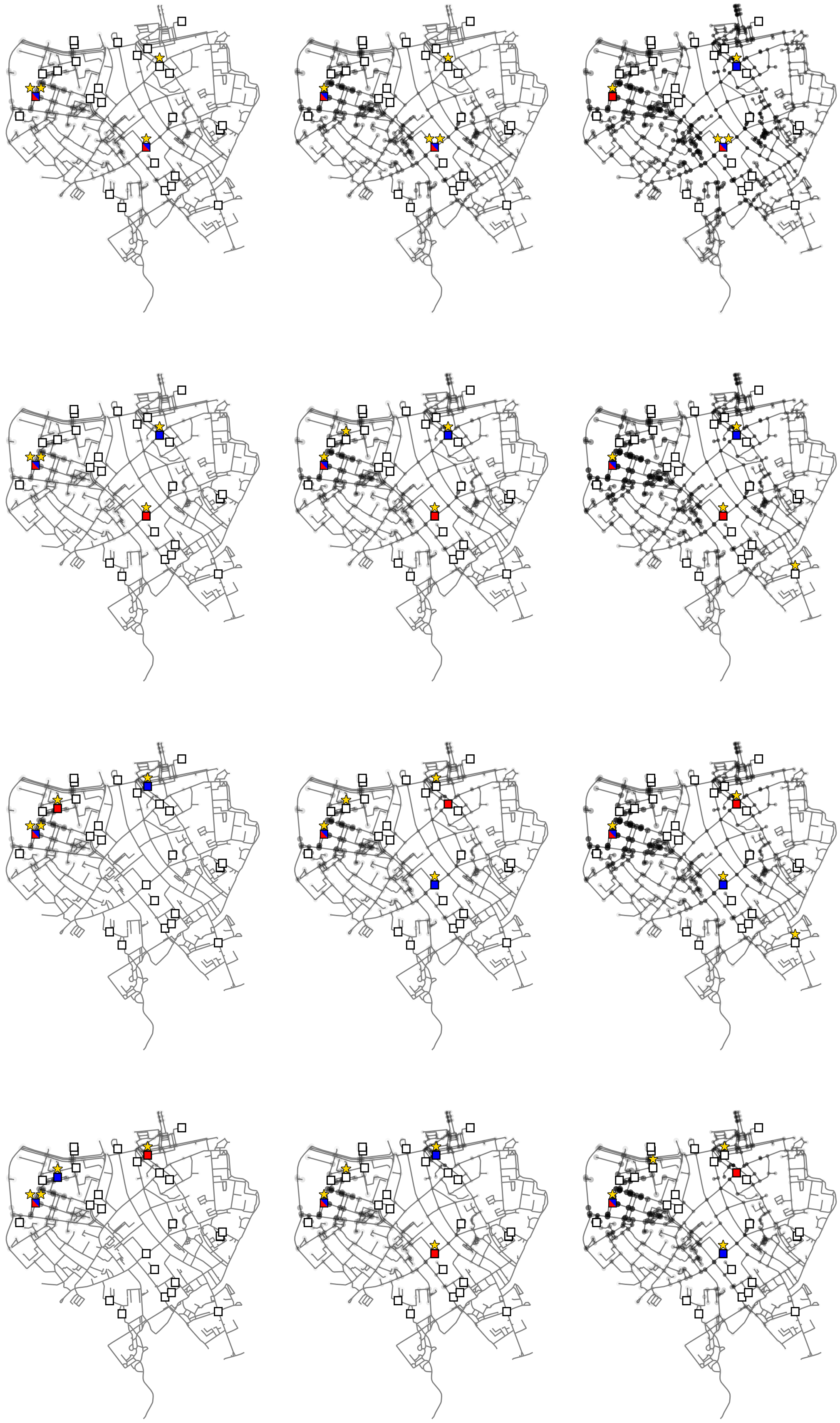

Figure 23: 2 vs. 2, $\beta = 5, 6, 7, 8\,\mathrm{km}^{-1}$, $\alpha = -1, 0, 1$

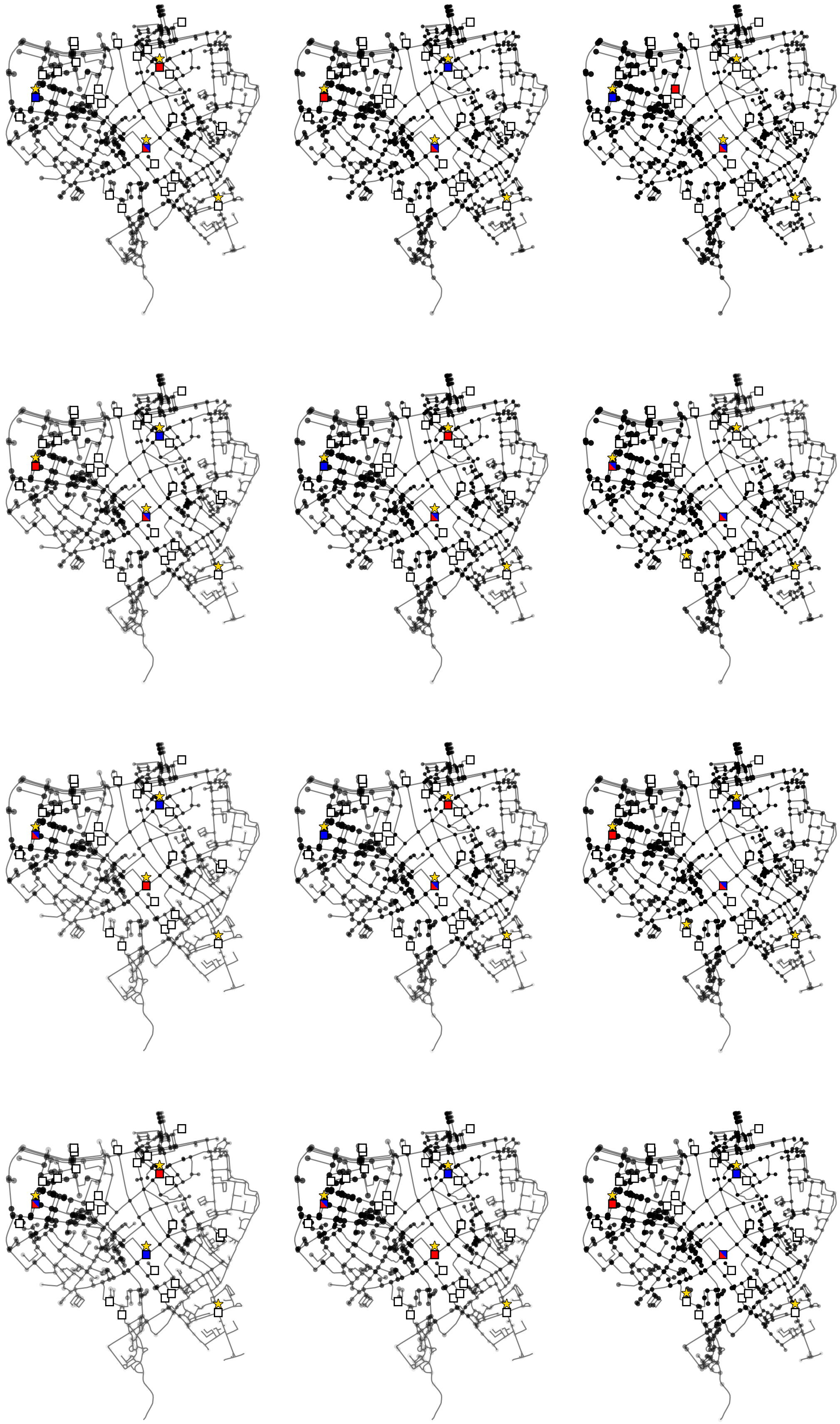

Figure 24: 2 vs. 2, $\beta = 5, 6, 7, 8\,\mathrm{km}^{-1}$, $\alpha = 2, 3, 4$

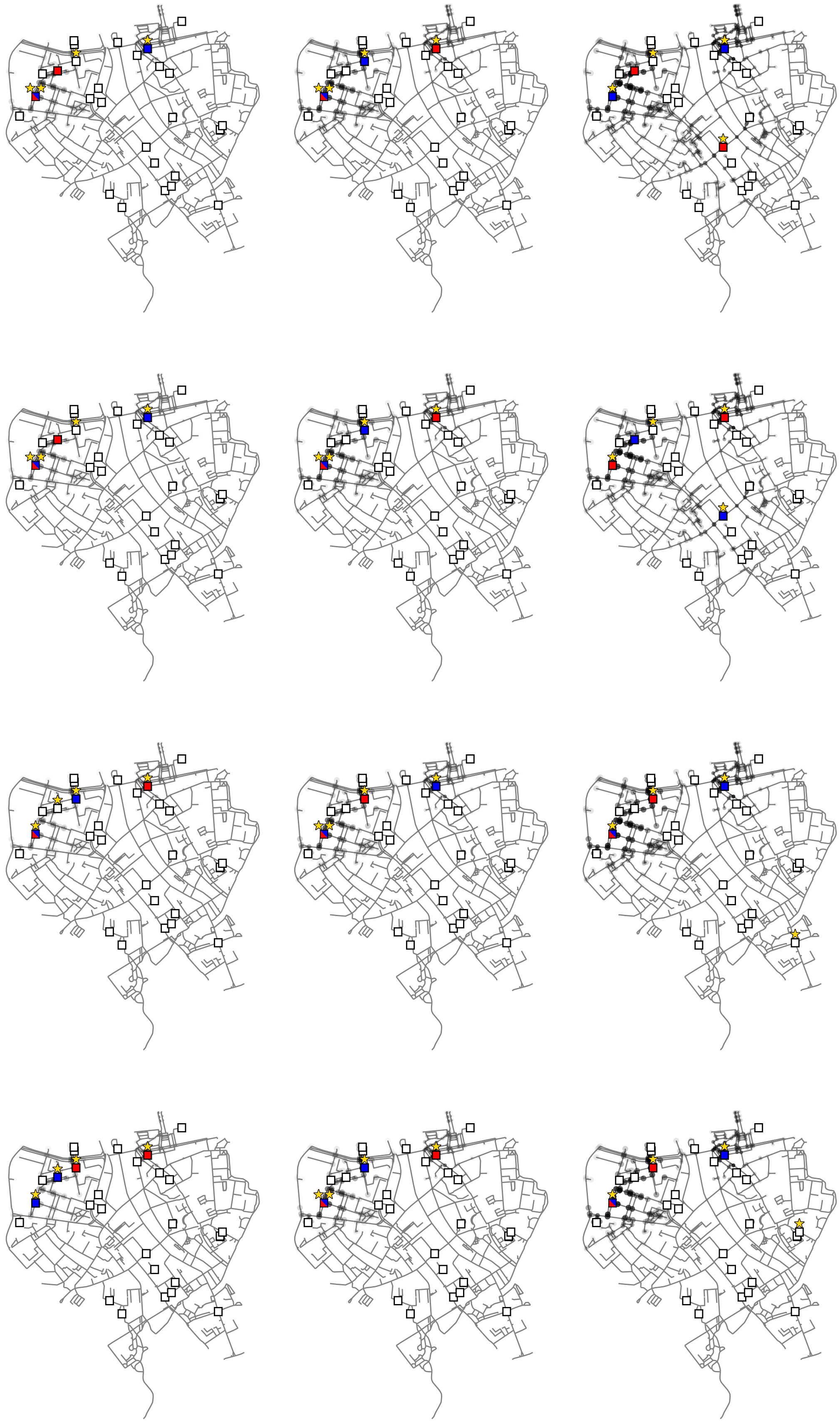

Figure 25: 2 vs. 2, $\beta = 9, 10, 11, 12\,\mathrm{km}^{-1}$, $\alpha = -1, 0, 1$

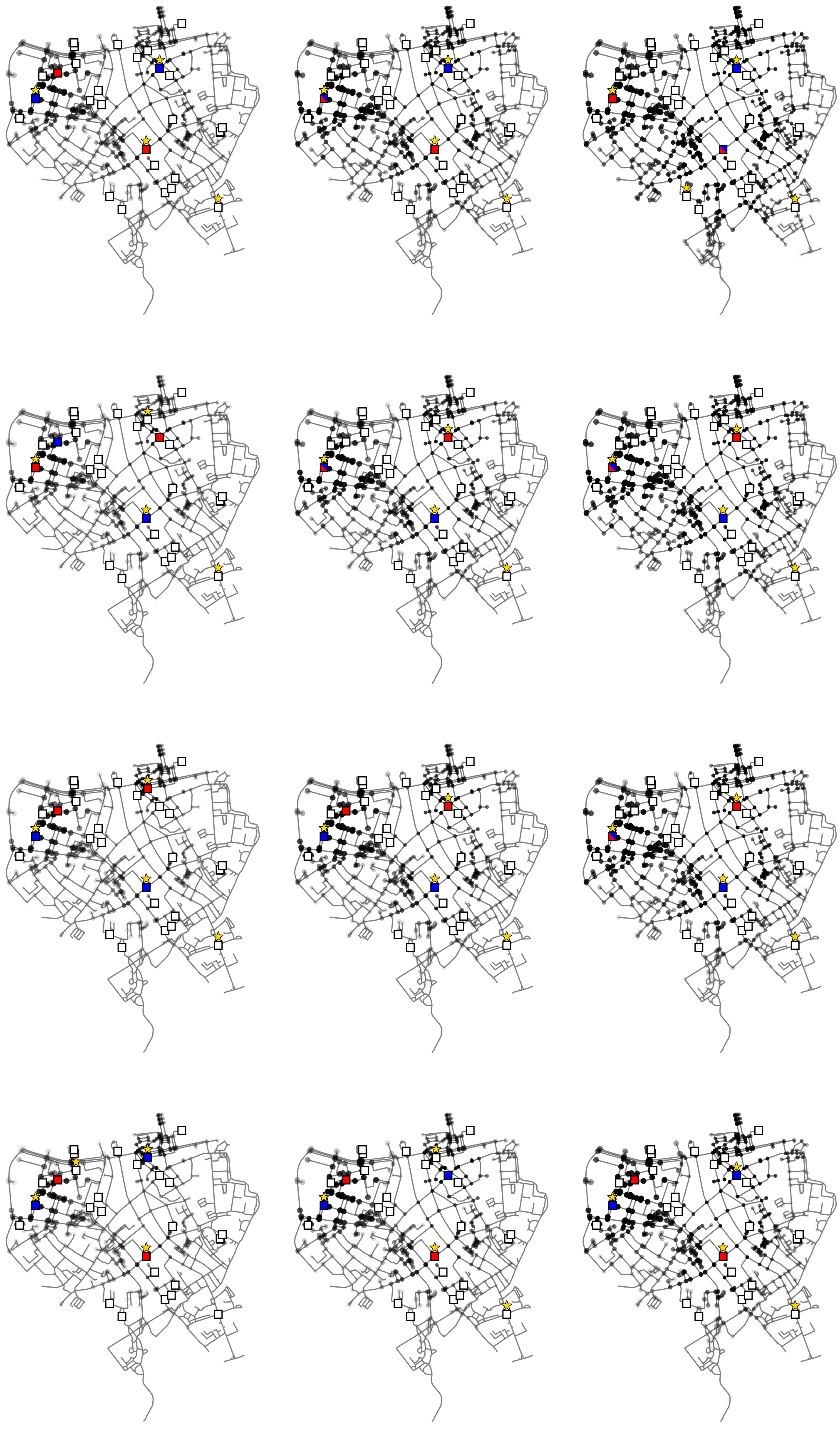

Figure 26: 2 vs. 2, $\beta = 9, 10, 11, 12\,\mathrm{km}^{-1}$, $\alpha = 2, 3, 4$

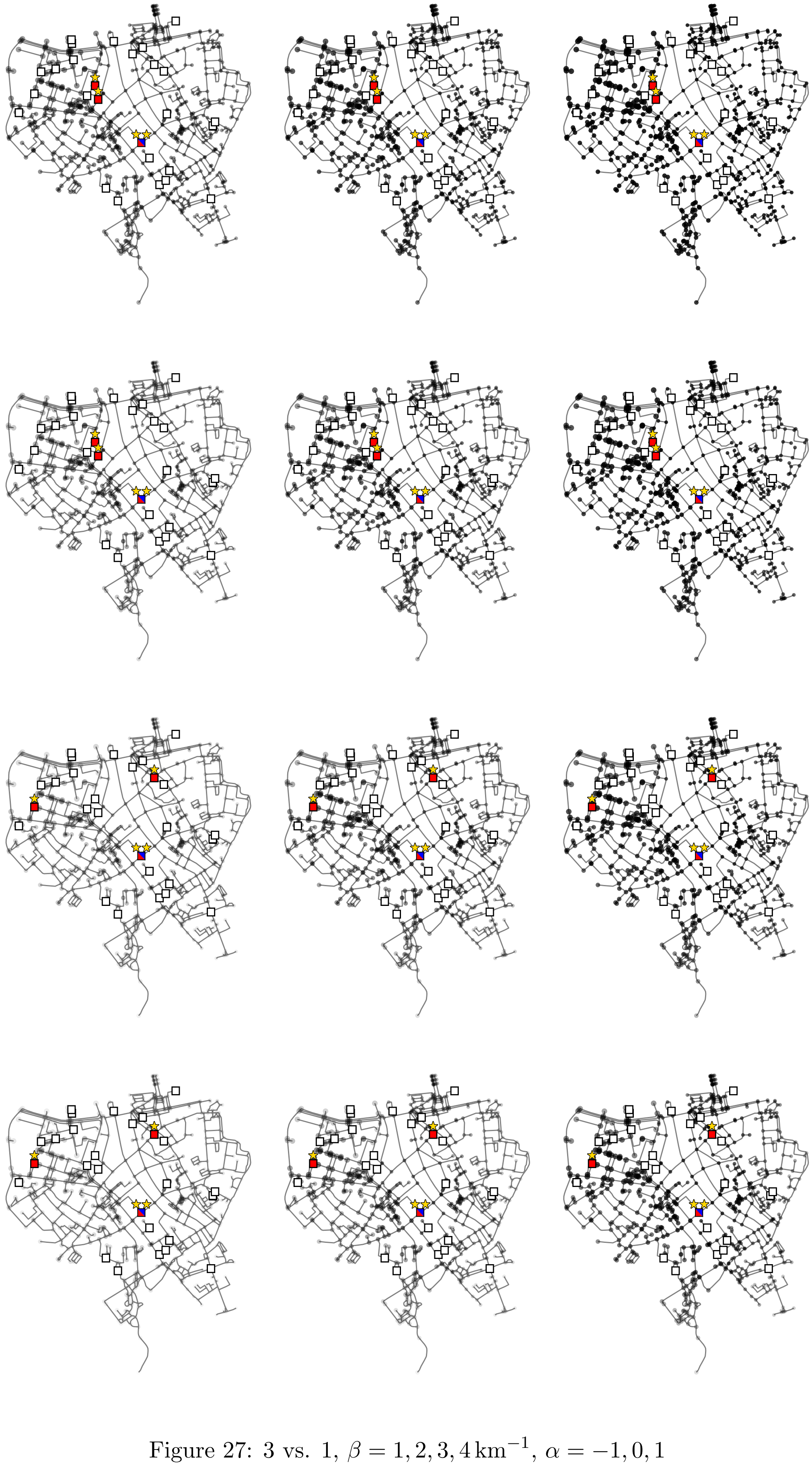

Figure 27: 3 vs. 1, $\beta = 1, 2, 3, 4\,\mathrm{km}^{-1}$, $\alpha = -1, 0, 1$

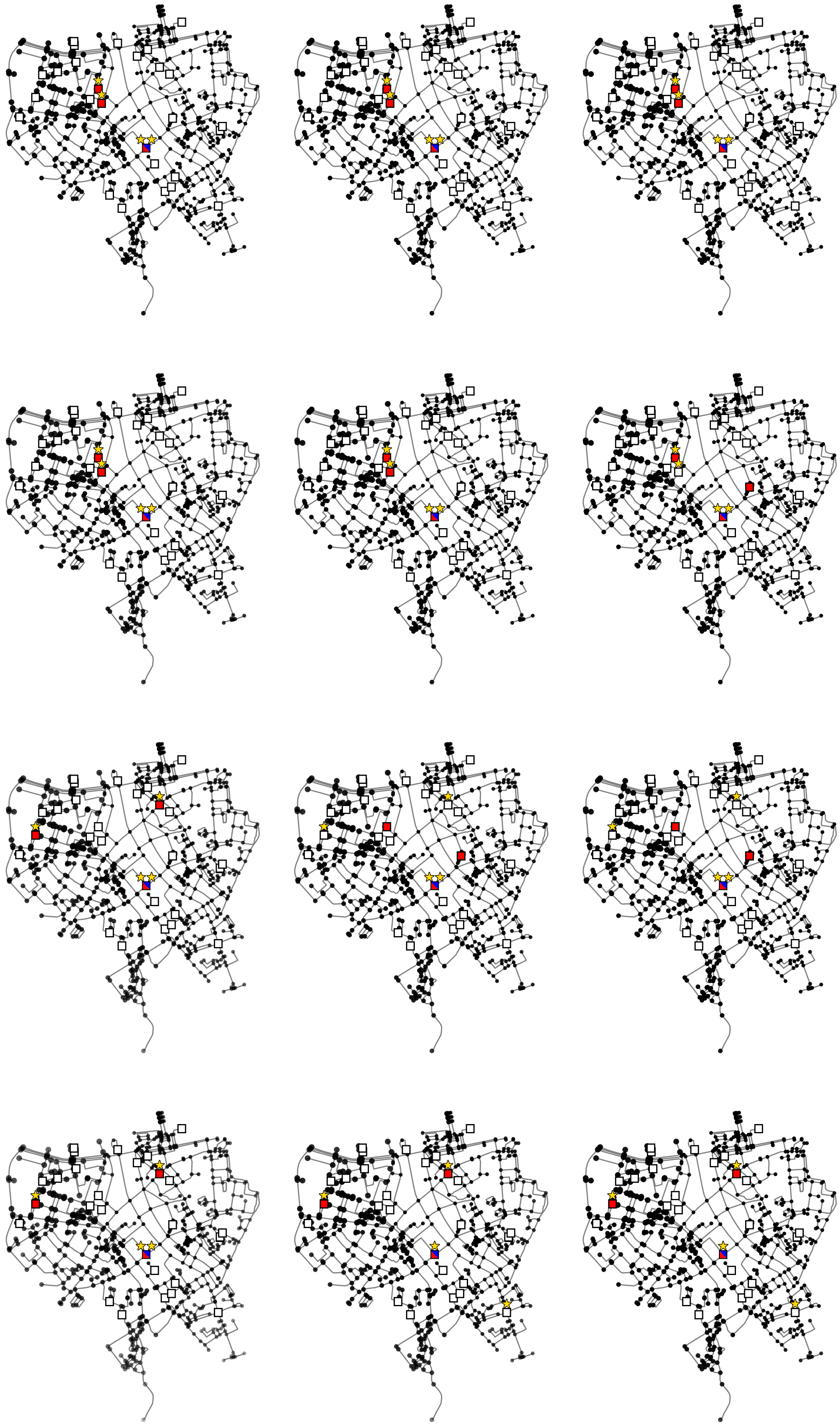

Figure 28: 3 vs. 1, $\beta = 1, 2, 3, 4\,\mathrm{km}^{-1}$, $\alpha = 2, 3, 4$

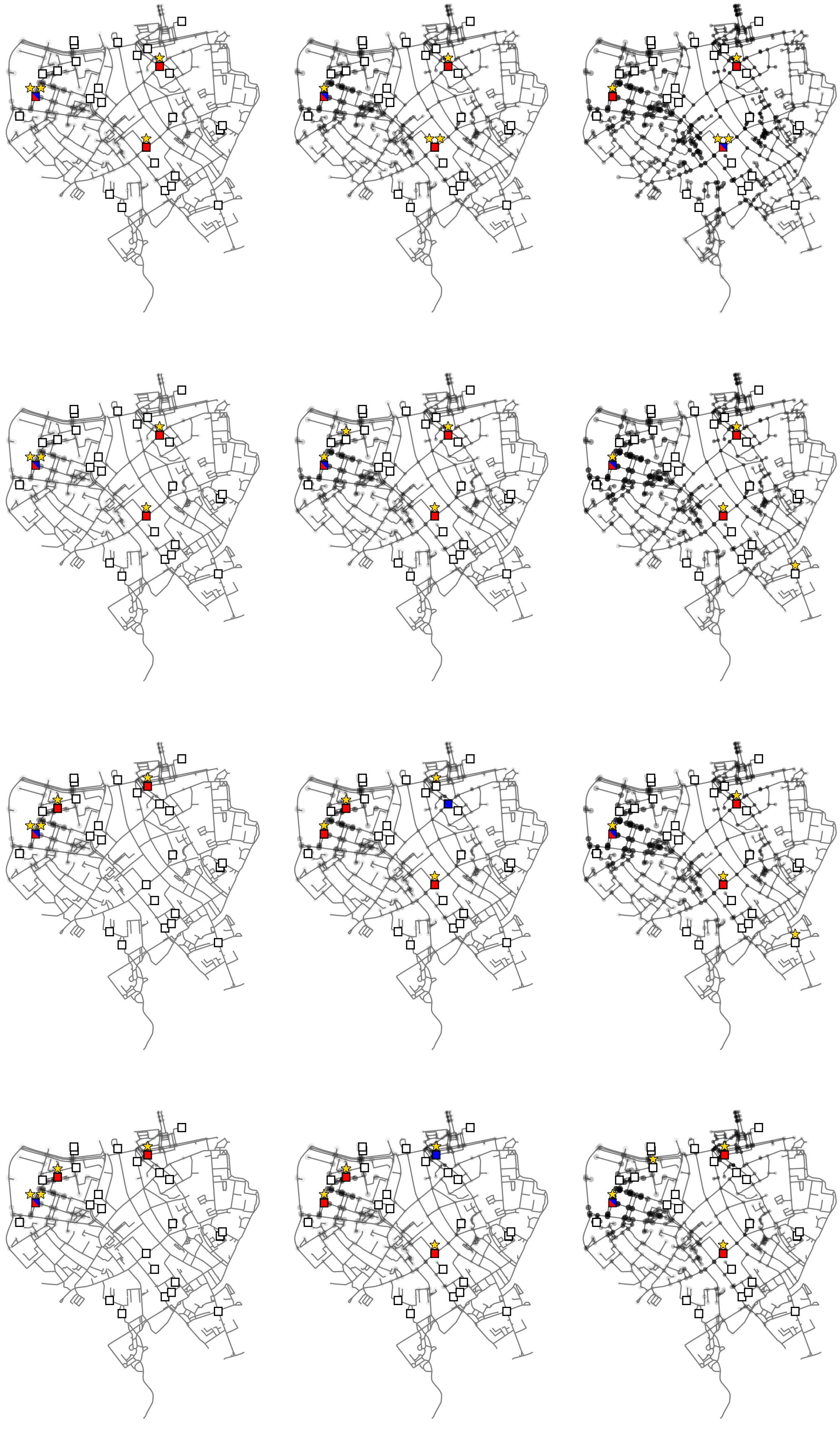

Figure 29: 3 vs. 1, $\beta = 5, 6, 7, 8\,\mathrm{km}^{-1}$, $\alpha = -1, 0, 1$

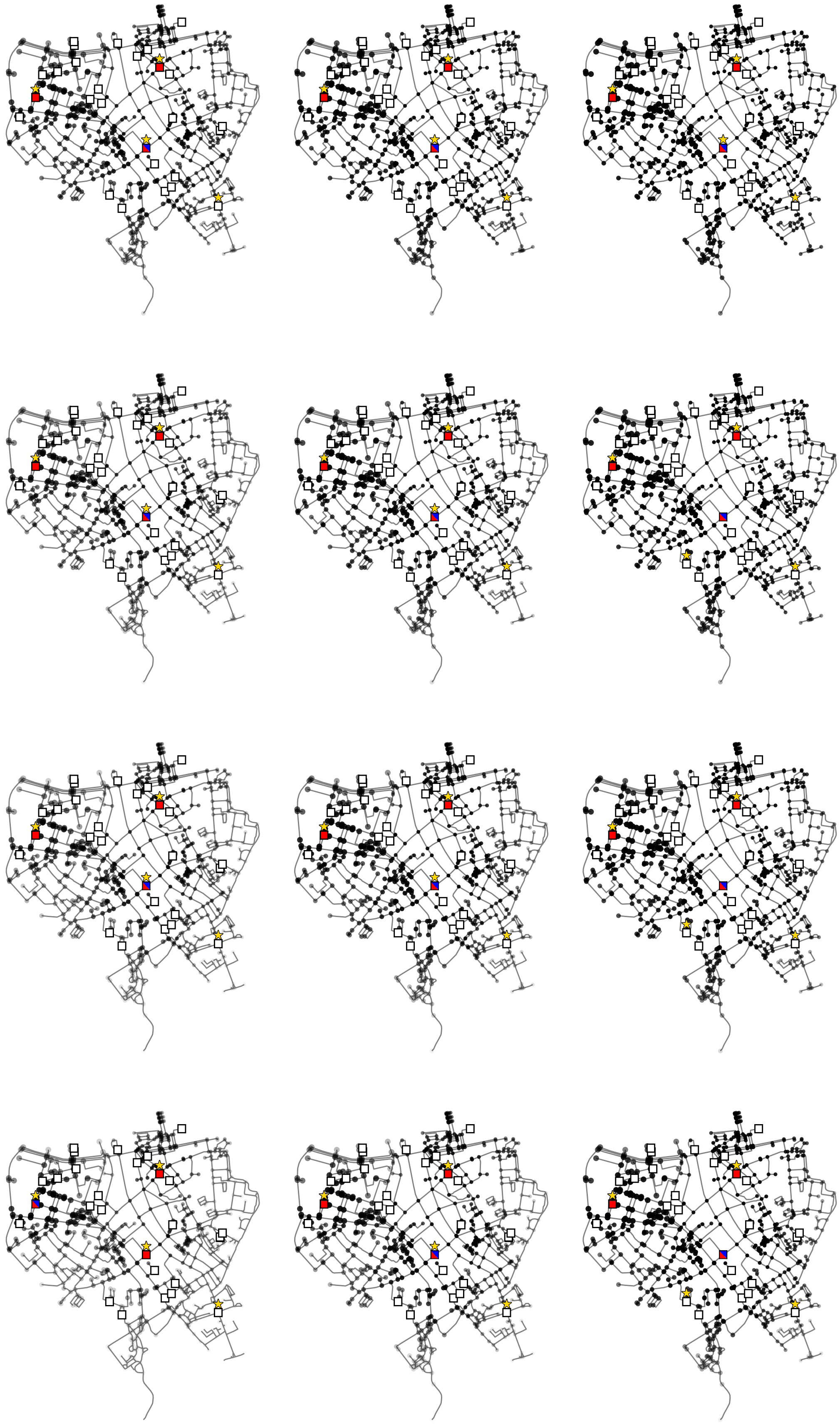

Figure 30: 3 vs. 1, $\beta = 5, 6, 7, 8\,\mathrm{km}^{-1}$, $\alpha = 2, 3, 4$

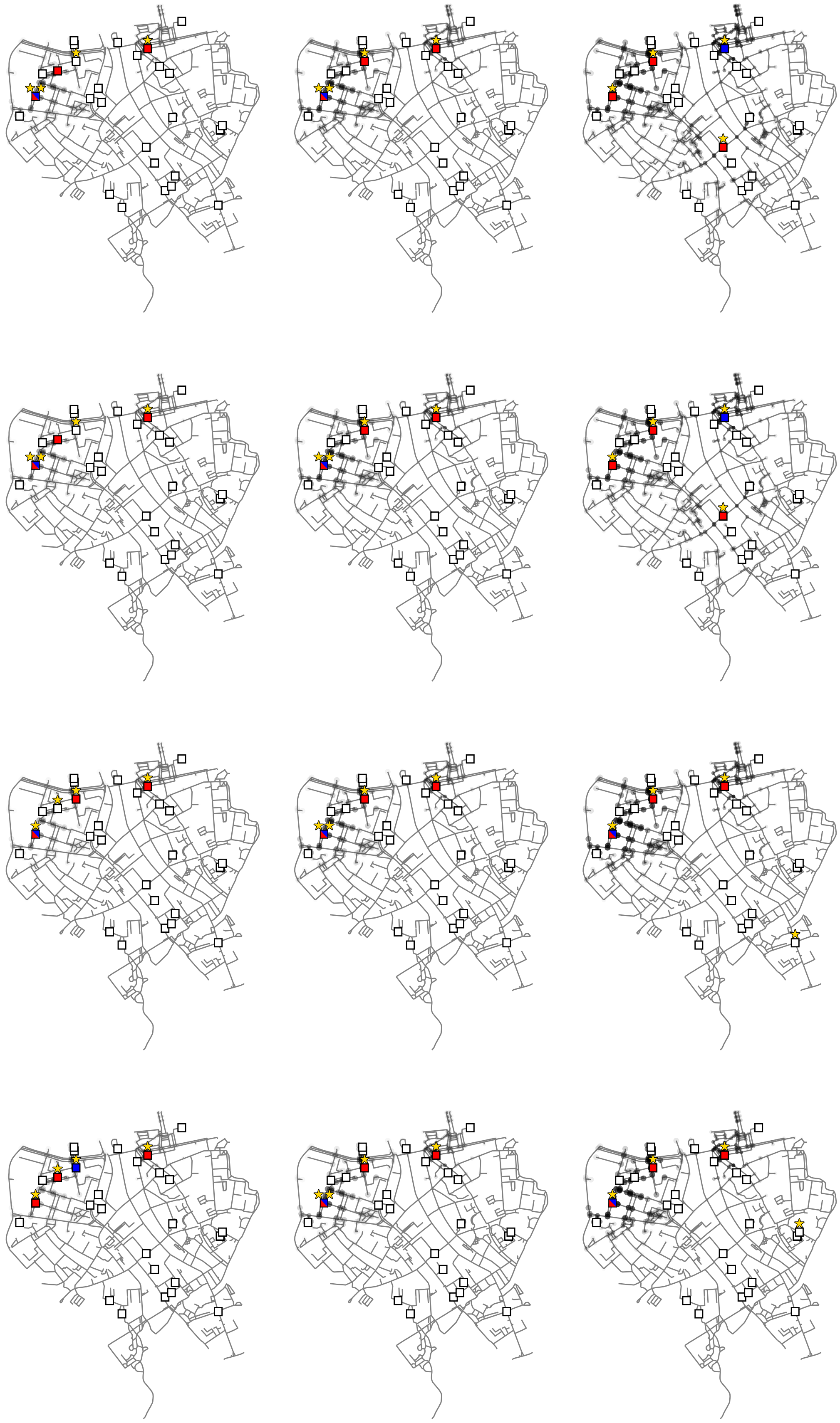

Figure 31: 3 vs. 1, $\beta = 9, 10, 11, 12\,\mathrm{km}^{-1}$, $\alpha = -1, 0, 1$

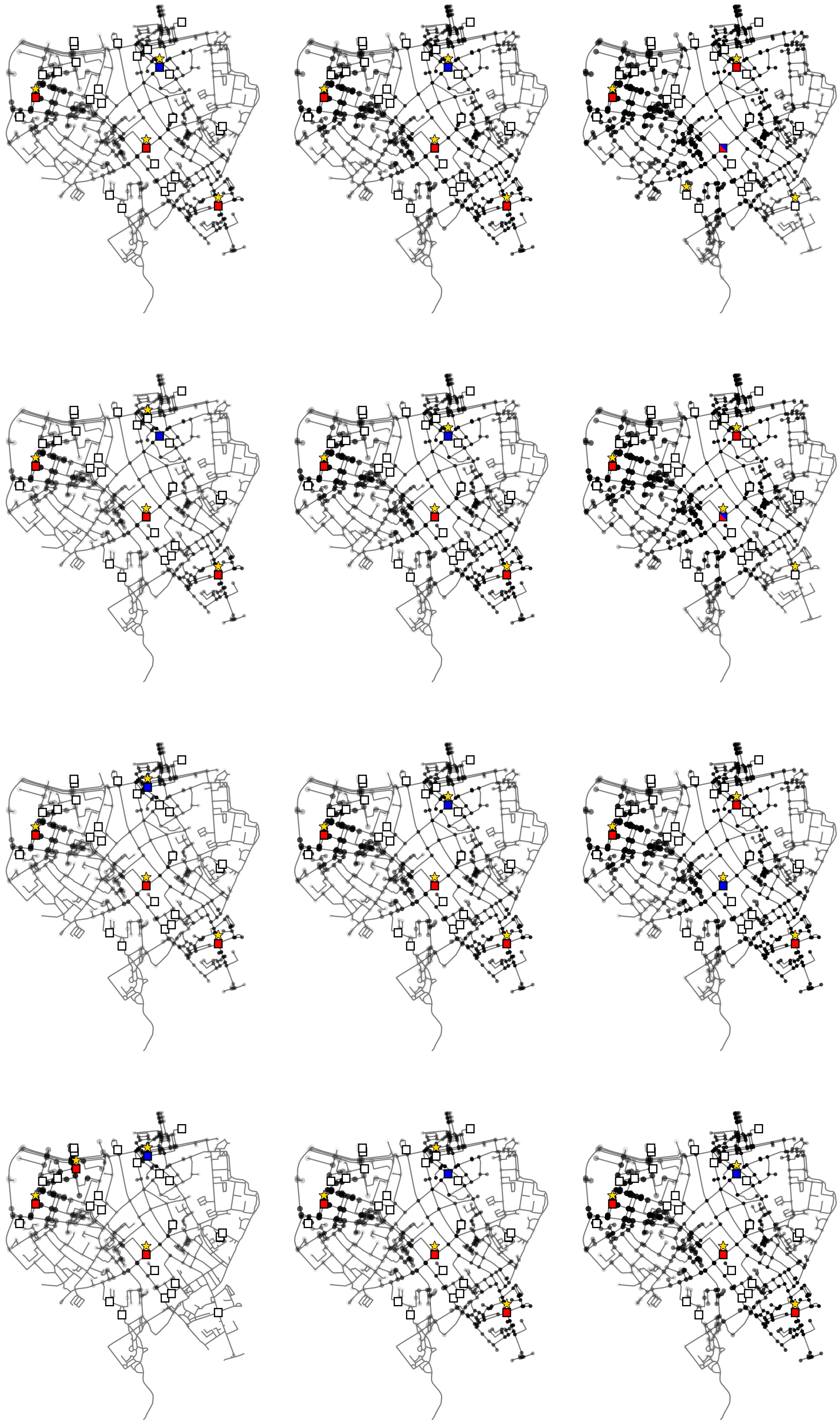

Figure 32: 3 vs. 1, $\beta = 9, 10, 11, 12\,\mathrm{km}^{-1}$, $\alpha = 2, 3, 4$

F Impact of the increase of the attractiveness

Table 9 represents, for each scenario, the increase of α for which the social solution corresponds to a Nash equilibrium. Label "N" refers to non-converging, while "C" refers to changing social optimum while finding the increase of α.

Table 9: Report on the increase of α for each combination of α, β, and number of lockers. Label "N" refers to non-converging, while "C" refers to changing social optimum while finding the increase of α.

	α											
	1 vs. 1						2 vs. 2					
β	-1	0	1	2	3	4	-1	0	1	2	3	4
1	0	0	0	N	N	N	N	N	N	N	N	N
2	0	N	N	N	N	N	0	0	0	0	0	0
3	0	N	N	N	N	N	0.03	0.02	0.03	0.07	0.11	N
4	0	0	N	N	N	N	0.76	0	0	0.01	N	N
5	0.09	0	0	N	0	N	C	0.55	0	N	N	N
6	0	0	0	0	N	0	0	C	N	N	N	N
7	0	0.65	0	0	0	N	0	C	N	N	N	N
8	C	1.61	0	0	0	0.04	0	C	C	N	N	N
9	0	1.60	0.60	0	0	0	0.03	0	C	N	N	N
10	0	1.26	0.26	0	0	0	0.01	0	C	N	N	N
11	0	0.61	0.04	0	0	0	C	0	C	N	0.28	N
12	0.04	0.07	0.03	0	0	0	0	0	C	0.22	0.10	0
	2 vs. 1						3 vs. 1					
β	-1	0	1	2	3	4	-1	0	1	2	3	4
1	0	0	0.01	0.01	0.01	0.01	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0.01
3	0.02	0.03	N	N	0	0	0	0	0	0	0.01	0.04
4	0	0	0	N	N	0.30	0	0	0	0	N	N
5	N	N	0	0	N	0.30	0	0.01	0	N	N	N
6	0.01	0.45	0	0	N	0.28	0	C	N	N	N	N
7	0.01	N	0	0	0	0.27	0	0.01	N	N	N	N
8	C	0	0.97	0	0	N	0	0	C	N	N	0.56
9	0	0	0.91	0	0	0	0.03	0	0	0	0	0.78
10	0	C	0	0.02	0	0	0.01	0	0	N	0	N
11	0	0	0	0	0	0	C	0	1.40	0	0	0
12	0.04	0	0	0	0.01	0	0	0	0.05	0	0.04	0