Multi-Leader Single-Follower Power-Market Modeling: The Impact of DC Market-Clearing on AC Feasibility

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Abstract. We study the impact of DC power flow modeling in multi-leader single-follower market models on the AC feasibility of the market outcome. To this end, we consider strategically bidding power producers that are connected to an electricity network and a market-clearing executed by an ISO. The focus is on a pay-as-bid electricity market in which each producer provides the ISO with a bid used to derive its revenues directly, and the market clearing is performed by taking into account the DC power flow constraints. This scenario naturally leads to a multi-leader single-follower game in which the upper-level solution concept is a generalized Nash equilibrium among the power producers who are jointly constrained by the market-clearing process in the lower level of this bilevel game. While the accurate modeling of AC power flows is essential for system reliability and operational feasibility, its nonconvex nature poses immense computational and conceptual challenges for the bilevel setting. To address this, we propose a two-step approach. First, the multi-leader single-follower problem is solved using a DC power flow model for the lower-level market-clearing process using a diagonalization method. Second, the operational feasibility of the obtained production plans is evaluated by computing the closest AC-feasible dispatch. This quantifies how far the DC-based market outcome is from the closest AC-feasible dispatch. The approach is tested on established benchmark instances, and the main outcome is that DC-based market-clearing results have to be considered with great care when evaluated with respect to AC-feasibility.

1. Introduction

We consider strategically bidding power producers that are connected to an electricity network and a following market-clearing process that is carried out by an independent system operator (ISO). In a nutshell, each power producer submits bids for electricity to be sold at the market. While doing so, the goal is to maximize revenues based on the difference between bid-based earnings at the market and actual production costs. After collecting the bids of all producers in the market, the ISO determines the dispatch quantities of generation so as to minimize overall production costs while satisfying the total demand and respecting the network constraints. This scenario can be modeled as a multi-leader single-follower game in which the solution concept at the upper level is the famous Nash equilibrium among the producers, who all face a bilevel problem in which the lower-level problem is the market-clearing, which is common to all leaders, i.e., producers, of this game. A Nash equilibrium in this setting occurs when each producer cannot find an alternative bid to improve revenues, assuming that the other producers do not deviate from their bids. While this upper-level problem is challenging on its own, the lower-level problem in its most accurate form includes an alternating current (AC) power flow model to obtain appropriate clearing results that are feasible w.r.t. the actual power

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flow physics in the network. Obviously, accurately capturing the physics of power flow is critical. In most of the existing literature, the mentioned market-clearing problem in the lower-level problem of the bilevel game is modeled using a direct current (DC) power flow model. This linear approximation of AC power flows is widely adopted because it enables computationally effective solution approaches—especially when the market-clearing problem is embedded in frameworks such as multi-leader single-follower models. In line with this common practice, we model the market clearing using a DC optimal power flow (DCOPF) model. This allows us to reformulate the considered multi-leader single-follower game as a generalized Nash equilibrium problem (GNEP) with shared constraints that are given by the necessary and sufficient Karush–Kuhn–Tucker (KKT) conditions of the lower-level problem. This is still a very challenging setup, and the literature on solving these problems is pretty scarce. We adopt a common approach from the literature and solve the KKT-constrained GNEPs by a diagonalization method. The use of a DCOPF model provides a convex and thus tractable lower-level problem, facilitating the analysis of strategic interactions among multiple leaders. However, DCOPF models neglect reactive power and nonlinear relationships that are covered by AC power flow models. This may lead to dispatch solutions that are suboptimal or even infeasible w.r.t. actual AC network physics. To quantify the extent of this discrepancy, once the DCOPF-based market outcome (bids and dispatch levels) is obtained, we evaluate its AC feasibility by solving a full but relaxed variant of the AC optimal power flow (ACOPF) problem. The difference between the active power production schedule resulting from the DCOPF market-clearing and the redispatch required by the ACOPF defines an "AC slack", which we use as a measure for the approximation error introduced due to DC modeling. This approach allows us to explicitly measure how far its solutions deviate from a potential AC-feasible operating point. We validate this procedure on well-established MatPower test cases, highlighting how relying on DCOPF for market clearing can affect production decisions and distort the resulting configuration of the system. This means DC-based market-clearing results have to be considered with great care when evaluated w.r.t. AC-feasibility. In particular, this leads to the clear need for novel bilevel optimization techniques that can tackle continuous and highly nonconvex lower-level problems such as the ACOPF problem.

The remainder of this paper is structured as follows. In Section 2, we review the relevant literature in the field. Then, in Section 3, AC power flow modeling as well as the DC power flow approximation are presented. With these models at hand, we describe the multi-leader single-follower model, together with its KKT reformulation, in Section 4 before we present the diagonalization approach to solve the bidding problem involving the KKT conditions of the DCOPF lower level in Section 5. In Section 6, we then discuss our novel proposal to measure the DCOPF-induced approximation error in this context. The numerical results are discussed in Section 7 and we conclude the paper in Section 8.

2. Literature Review

The literature on the OPF problem is rich and spans over half a century. In the literature, various formulations and algorithms have been proposed. We refer to [15] for an introduction to OPF and to [10, 14, 26, 27] for detailed surveys.

For the DC model, a small-angle linearization of the ACOPF problem is considered, where losses and reactive power are ignored, and a flat voltage profile is assumed. The approach is widely used in practice, since the DCOPF problem is a linear problem (LP) that is efficiently solvable even for large-sized networks; see e.g.,

[9, Section 1.2.4] or [42, Equation (5.48)]. Due to its computational tractability, DCOPF is widely used in both operational practice and electricity-market modeling.

In contrast, solving the ACOPF problem is a much more challenging task. The problem can be formulated in many ways, e.g., as a quadratically constrained quadratic problem, a polynomial programming problem, or as a general nonlinear problem (NLP). All of them involve nonconvexities [9]. While the variables (voltage, current, power) are naturally defined on continuous domains, an interesting feature of the AC model is that its variables are complex numbers. Although a separation in real and imaginary parts is always possible, matrix formulations and relaxations generally take up twice the amount of storage w.r.t. working directly in complex numbers [29].

Several algorithms, summarized in [14], tackle the nonconvex ACOPF problem using NLP techniques, which only guarantee local optimality in general. These include gradient and reduced-gradient methods [3, 22], sequential quadratic programming approaches [13], (quasi-)Newton methods [30, 47], interior-point methods [35, 43], and derivative-free optimization techniques [2, 34]. Among these, interior-point methods have shown strong empirical performance on large-scale instances, although certifying global optimality or tight optimality gaps remains difficult.

To address this issue, a substantial body of work has focused on convex relaxations of the ACOPF problem, such as LP, second-order cone programming (SOCP), and semi-definite programming (SDP) relaxations [42]. Recent works have also proposed convex relaxations in the context of the design of AC power transportation networks with line activity variables [11].

Beyond single-level models, bilevel optimization has become a prominent modeling tool in power systems, particularly for models of market design, strategic behavior, and security applications. A bilevel model with ACOPF in the lower level is proposed in [17], where the market equilibrium of an integrated heat-power distribution system with strategic providers and demand elasticity is studied. In the upper level, a provider bids offering prices of electricity and heat to the respective markets. In the lower level, the system operators solve the market-clearing problems (both for heat and power), and determine the dispatch of their generators, energy contracts with the provider in case of need, as well as locational marginal prices (LMPs). The ACOPF-based market-clearing lower level is nonlinear. To overcome such an issue, first, convex relaxation is performed on the "branch flow based OPF model", replacing a nonconvex equality constraint with a rotated second-order cone (SOC). Then, each rotated SOC is approximated via a polyhedral set, and the power market clearing finally gives rise to an LP. Bilevel programming is also used in [24] to model demand response in organized energy markets. Demand response is a modification in the power consumption of a prosumer to better match the power demand with the supply. In the proposed bilevel model, the lower level models the economic dispatch of energy and the LMPs, while the upper level minimizes the total amount of demand response subject to a net benefit requirement. In [37], the pessimistic variant of the same problem is considered, where the retailer prepares for the least favorable optimal responses from the consumers. It is indeed demonstrated that the set of optimal consumption schedules typically contains various responses that are equal for the follower, but lead to radically different profits for the leader. The main contribution of the paper is an exact procedure for solving the pessimistic variant of the problem.

Bilevel optimization has also been extensively used to formulate power network interdiction problems modeling attacker-defender Stackelberg games. The leader (the attacker) maximizes the damage to the system, and the follower (the defender) responds to the action of the leader by minimizing the attack's consequences to

the power network. In most contributions, the lower level relies on DC power flow constraints [4, 5, 45]. The AC power flow model is used in the lower level in [36], where attacks consist in increasing impedance along transmission lines, and an algorithm returning feasible solutions is proposed. The AC power flow at the lower level can also be found in [1], where an attacker-defender game is considered in which the attacker can disable multiple assets simultaneously. The ACOPF is linearized, and the whole bilevel problem is reformulated as a single-level mixed-integer linear problem using duality theory and further linearization techniques. In [19], a branch-and-bound algorithm is proposed to solve the same bilevel problem as addressed in [1]. Upper bounds are computed by solving a reformulation that avoids the relaxation of the AC power flow constraints in the lower-level problem. Lower bounds are provided using SDP relaxed solutions to the lower level.

In the context of strategic behavior at electricity markets, DCOPF-based formulations are widely used to ensure computational tractability when modeling non-cooperative interactions among producers [8, 33, 48]. Relying on DC approximations is particularly common in multi-leader game-theoretic models, where the presence of equilibrium constraints significantly increases computational complexity. A notable exception is [7], in which oligopolistic competition is modeled as a multi-leader single-follower game with a full AC network representation. In the proposed formulation, dominant producers act as Cournot leaders, while the follower (involving competitive suppliers, price-responsive demand, and the system operator) solves an ACOPF to clear the market. A key contribution of this work is the explicit modeling of strategic behavior in both active and reactive power, highlighting how reactive power can be exploited strategically even in the absence of an explicit reactive power market. To compute equilibrium outcomes, the authors embed the KKT conditions of the follower's ACOPF directly into the leaders' problems and reformulate the resulting equilibrium conditions as a single, large-scale nonlinear program. Complementarity constraints are handled through penalty-based techniques, yielding a highly nonconvex formulation for which equilibrium solutions correspond to only stationary instead of globally optimal points of the lower-level problem.

More recently, the research has focused on improving the fidelity of DC-based market models while retaining their computational efficiency. In particular, the authors of [18] propose a bilevel optimization framework that calibrates demand-dependent parameters in a DCOPF model to minimize discrepancies in dispatch and LMPs relative to ACOPF outcomes, while enforcing market properties such as cost recovery and revenue adequacy. Their approach yields a parametric and data-driven DC approximation suitable for time-sensitive market applications.

Within this landscape, our paper focuses on a strategic bidding model in a pay-as-bid electricity market formulated as a multi-leader single-follower game, where the market clearing is performed using a DCOPF formulation to preserve tractability in the presence of strategic interactions and equilibrium constraints. Rather than embedding ACOPF directly into the bilevel model, the proposed approach evaluates the AC feasibility of the resulting dispatch ex post, quantifying the distance between the DC-based equilibrium outcome and ACOPF physics. This two-step methodology enables the analysis of strategic behavior at scale while explicitly assessing the physical reliability implications of DC-based market clearing.

3. Power Flow Modeling

We start by describing the ACOPF problem and then we focus on the classic DC approximation. All the sets, parameters, as well as variables used in the models are listed in Table 1.

The power network is modeled as a directed graph $\mathcal{H} = (N, L)$, where $N = \{1, \dots, n\}$ is the set of buses and L is the set of lines. We have $|N| = n$ and suppose $|L| = m$. The set N of buses is split into buses N_+, at which power is produced and supplied to the network, and nodes N_-, at which power is withdrawn from the network and consumed. We further assume that there is a reference bus in N_-, indexed by r, which has a fixed voltage (with imaginary part set to zero) and flexible power injection for ensuring overall power balance. We assume that $N = N_+ \cup N_-$. For ease of presentation, we treat N_+ and N_- as disjoint sets; however, in practical instances a bus may both generate and consume power, i.e., $N_+ \cap N_- \neq \emptyset$. Furthermore, although multiple generators may be connected to a single bus in real-world power systems, we assume that each bus in N_+ hosts a single representative generator again for the ease of readability and notational simplicity.

3.1. **ACOPF Constraints.** The first challenge of modeling AC power flow is that, whenever the line ℓ between buses i and j has a transformer at i, the current and power that are injected on ℓ at i are different from those at j. For this reason, each variable associated with a line $\ell = (i, j)$ will also be defined by the superscript f or t indicating if it refers to a flow from bus i or to bus j, respectively.

A transformer installed at bus i is described by the "ratio" of the AC transformation, which is usually expressed in polar coordinates as $\tau_\ell e^{\mathbf{i}\nu_\ell} = \tau_\ell \cos\nu_\ell + \mathbf{i}\tau_\ell \sin\nu_\ell$, where τ_ℓ is the magnitude and ν_ℓ is the angle.

Both the generated and consumed power in the AC network are complex numbers because of the cyclic nature of the alternating current. The real part is the *active* power, while the imaginary part is the *reactive* power. The amount of active and reactive power generated at the generator bus $i \in N_+$ is modeled with the variables P_i^G and Q_i^G, respectively, i.e., $S_i^G = P_i^G + \mathbf{i}Q_i^G$. We suppose that the complex demand $S_i^D = P_i^D + \mathbf{i}Q_i^D$ is known and fixed for each consumer $i \in N_-$. With a slight abuse of notation, we extend P_i^G, Q_i^G and P_i^D, Q_i^D to all $i \in N$ by setting them equal to zero whenever the corresponding quantity is not defined (i.e., for $i \notin N_+$ or $i \notin N_-$, respectively).

For each bus $i \in N$, let V_i be its voltage, with $R_i = \Re(V_i)$ and $W_i = \Im(V_i)$ being the real and the imaginary part. $R_r > 0$ is the fixed voltage at the reference bus, where $W_r = 0$ is set. For each line $\ell = (i, j)$, let $z_\ell = r_\ell + \mathbf{i}x_\ell$ denote its impedance, with the real part r_ℓ being the resistance and the imaginary part x_ℓ being the reactance. The complex current flows along line $\ell \in L$ from bus i and to bus j are denoted as I_ℓ^{f} and I_ℓ^{t}, while the complex power flows are $s_\ell^{\mathsf{f}} = p_\ell^{\mathsf{f}} + \mathbf{i}q_\ell^{\mathsf{f}}$ and $s_\ell^{\mathsf{t}} = p_\ell^{\mathsf{t}} + \mathbf{i}q_\ell^{\mathsf{t}}$.

With this notation at hand, we can now describe the main physical relationships. First, we have the current balance and Ohm's law for all $\ell = (i, j) \in L$ given by

$$\begin{pmatrix} I_\ell^{\mathsf{f}} \\ I_\ell^{\mathsf{t}} \end{pmatrix} = \begin{bmatrix} Y_\ell^{\mathsf{ff}} & Y_\ell^{\mathsf{ft}} \\ Y_\ell^{\mathsf{tf}} & Y_\ell^{\mathsf{tt}} \end{bmatrix} \begin{pmatrix} V_i \\ V_j \end{pmatrix}, \tag{1a}$$

where the line admittance matrix Y_ℓ is defined as

$$Y_\ell = \begin{bmatrix} Y_\ell^{\mathsf{ff}} & Y_\ell^{\mathsf{ft}} \\ Y_\ell^{\mathsf{tf}} & Y_\ell^{\mathsf{tt}} \end{bmatrix} = \begin{bmatrix} (\frac{1}{z_\ell} + \mathbf{i}\frac{\beta_\ell}{2})\frac{1}{\tau_\ell^2} & -\frac{1}{z_\ell \tau_\ell e^{-\mathbf{i}\nu_\ell}} \\ -\frac{1}{z_\ell \tau_\ell e^{\mathbf{i}\nu_\ell}} & \frac{1}{z_\ell} + \mathbf{i}\frac{\beta_\ell}{2} \end{bmatrix}.$$

Furthermore, at each line, the power balance is modeled by

$$s_\ell^{\mathsf{f}} = V_i (I_\ell^{\mathsf{f}})^*, \quad s_\ell^{\mathsf{t}} = V_j (I_\ell^{\mathsf{t}})^* \quad \forall\, \ell = (i, j) \in L. \tag{1b}$$

TABLE 1. Sets, parameters, and variables of the optimal power flow model

Sets	Description
N_+	Set of generator buses
N_-	Set of demand buses
N	Set of buses of the grid; $N = \{1, \dots, n\} = N_+ \cup N_-$
L_i^{f}	Set of lines with origin at bus $i \in N$
L_i^{t}	Set of lines with destination at bus $i \in N$
L	Set of lines of the grid given by $L = \bigcup_{i \in N}(L_i^{\mathsf{f}} \cup L_i^{\mathsf{t}}) \subseteq \{(i,j) \colon i,j \in N\}$
Parameters	Description
r	Reference bus; in N_-
P_i^D	Active power demand of consumer $i \in N_-$
Q_i^D	Reactive power demand of consumer $i \in N_-$
h_i	Complex shunt admittance at bus $i \in N$
Y_ℓ	Line admittance matrix of line $\ell \in L$
$z_\ell = r_\ell + \mathbf{i}x_\ell$	Line impedance of line $\ell \in L$ (r_ℓ the resistance, x_ℓ the reactance)
β_ℓ	Line charging susceptance line $\ell \in L$
τ_ℓ	Magnitude of the turns ratio of the transformer located on line $\ell \in L$
ν_ℓ	Phase angle of the turns ratio of the transformer located on line $\ell \in L$
$\overline{P}_i^G$, $\underline{P}_i^G$	Bounds on the active generated power at bus $i \in N_+$
$\overline{Q}_i^G$, $\underline{Q}_i^G$	Bounds on the reactive generated power at bus $i \in N_+$
$\overline{s}_\ell$	Bound on the absolute value of power flow line $\ell \in L$
$\overline{V}_i$, $\underline{V}_i$	Bounds on the voltage magnitude at bus $i \in N$
$V_r = R_r$	Fixed voltage at the reference bus r
$\overline{\eta}_\ell, \underline{\eta}_\ell$	Bounds on the voltage phase difference on line $\ell \in L$
Variables	Description
P_i^G	Active power produced by generator $i \in N_+$
Q_i^G	Reactive power produced by generator $i \in N_+$
p_ℓ^{f}	Active power flow along line $\ell = (i,j) \in L$ from node $i \in N$
p_ℓ^{t}	Active power flow along line $\ell = (i,j) \in L$ to node $j \in N$
q_ℓ^{f}	Reactive power flow along line $\ell = (i,j) \in L$ from node $i \in N$
q_ℓ^{t}	Reactive power flow along line $\ell = (i,j) \in L$ to node $j \in N$
$V_i = R_i + \mathbf{i}W_i$	Complex voltage at bus $i \in N$
θ_i	Voltage phase difference at bus $i \in N$

Combining Equations (1a) and (1b), we obtain the following expressions for s_ℓ^{f} and s_ℓ^{t}:

$$s_\ell^{\mathsf{f}} = V_i V_i^* (Y_\ell^{\mathsf{ff}})^* + V_i V_j^* (Y_\ell^{\mathsf{ft}})^* \quad \forall\, \ell = (i,j) \in L_i^{\mathsf{f}}, \tag{1c}$$

$$s_\ell^{\mathsf{t}} = V_j V_i^* (Y_\ell^{\mathsf{tf}})^* + V_j V_j^* (Y_\ell^{\mathsf{tt}})^* \quad \forall\, \ell = (i,j) \in L_i^{\mathsf{t}}. \tag{1d}$$

The power flow equations read, for all $i \in N$,

$$\sum_{\ell \in L_i^{\mathsf{f}}} s_\ell^{\mathsf{f}} + \sum_{\ell \in L_i^{\mathsf{t}}} s_\ell^{\mathsf{t}} = (P_i^G + \mathbf{i}Q_i^G) - (P_i^D + \mathbf{i}Q_i^D) - h_i^* |V_i|^2 \tag{2}$$

with $h_i^* |V_i|^2$ being the shunt admittance term, related to the interaction with the ground. By splitting the real and imaginary parts of the involved quantities, we get,

for all $i \in N$,

$$\sum_{\ell \in L_i^{\mathsf{f}}} p_\ell^{\mathsf{f}} + \sum_{\ell \in L_i^{\mathsf{t}}} p_\ell^{\mathsf{t}} = P_i^G - P_i^D - \Re(h_i)(R_i^2 + W_i^2), \tag{3a}$$

$$\sum_{\ell \in L_i^{\mathsf{f}}} q_\ell^{\mathsf{f}} + \sum_{\ell \in L_i^{\mathsf{t}}} q_\ell^{\mathsf{t}} = Q_i^G - Q_i^D + \Im(h_i)(R_i^2 + W_i^2). \tag{3b}$$

Let us also split the real and imaginary parts of each component of the Y_ℓ-matrix for all $\ell \in L$:

$$\Re(Y_\ell^{\mathsf{ff}}) = \frac{r_\ell}{(r_\ell^2 + x_\ell^2)\tau_\ell^2}, \qquad \Im(Y_\ell^{\mathsf{ff}}) = \frac{\beta_\ell}{2\tau_\ell^2} - \frac{x_\ell}{(r_\ell^2 + x_\ell^2)\tau_\ell^2},$$

$$\Re(Y_\ell^{\mathsf{ft}}) = -\frac{r_\ell \cos\nu_\ell + x_\ell \sin\nu_\ell}{(r_\ell^2 + x_\ell^2)\tau_\ell}, \qquad \Im(Y_\ell^{\mathsf{ft}}) = -\frac{r_\ell \sin\nu_\ell - x_\ell \cos\nu_\ell}{(r_\ell^2 + x_\ell^2)\tau_\ell},$$

$$\Re(Y_\ell^{\mathsf{tf}}) = -\frac{r_\ell \cos\nu_\ell - x_\ell \sin\nu_\ell}{(r_\ell^2 + x_\ell^2)\tau_\ell}, \qquad \Im(Y_\ell^{\mathsf{tf}}) = \frac{r_\ell \sin\nu_\ell + x_\ell \cos\nu_\ell}{(r_\ell^2 + x_\ell^2)\tau_\ell},$$

$$\Re(Y_\ell^{\mathsf{tt}}) = \frac{r_\ell}{(r_\ell^2 + x_\ell^2)}, \qquad \Im(Y_\ell^{\mathsf{tt}}) = \frac{\beta_\ell}{2} - \frac{x_\ell}{(r_\ell^2 + x_\ell^2)}.$$

By defining $R_{ij} = R_i R_j$ and $W_{ij} = W_i W_j$, we the obtain, for all $\ell = (i,j) \in L_i^{\mathsf{f}}$,

$$\begin{aligned} p_\ell^{\mathsf{f}} = {} & (R_i^2 + W_i^2)\Re(Y_\ell^{\mathsf{ff}}) + (R_{ij} + W_{ij})\Re(Y_\ell^{\mathsf{ft}}) \\ & + (W_i R_j - R_i W_j)\Im(Y_\ell^{\mathsf{ft}}), \end{aligned} \tag{4a}$$

$$\begin{aligned} q_\ell^{\mathsf{f}} = {} & -(R_i^2 + W_i^2)\Im(Y_\ell^{\mathsf{ff}}) - (R_{ij} + W_{ij})\Im(Y_\ell^{\mathsf{ft}}) \\ & + (W_i R_j - R_i W_j)\Re(Y_\ell^{\mathsf{ft}}) \end{aligned} \tag{4b}$$

and, for all $\ell = (i,j) \in L_i^{\mathsf{t}}$,

$$\begin{aligned} p_\ell^{\mathsf{t}} = {} & (R_{ij} + W_{ij})\Re(Y_\ell^{\mathsf{tf}}) + (R_i W_j - W_i R_j)\Im(Y_\ell^{\mathsf{tf}}) \\ & + (R_j^2 + W_j^2)\Re(Y_\ell^{\mathsf{tt}}), \end{aligned} \tag{4c}$$

$$\begin{aligned} q_\ell^{\mathsf{t}} = {} & (R_i W_j - W_i R_j)\Re(Y_\ell^{\mathsf{tf}}) - (R_{ij} + W_{ij})\Im(Y_\ell^{\mathsf{tf}}) \\ & - (R_j^2 + W_j^2)\Im(Y_\ell^{\mathsf{tt}}). \end{aligned} \tag{4d}$$

In addition to Equations (3)–(4), we impose bounds on the involved quantities via

$$\underline{P}_i^G \le P_i^G \le \overline{P}_i^G \quad \forall\, i \in N_+, \tag{5a}$$

$$\underline{Q}_i^G \le Q_i^G \le \overline{Q}_i^G \quad \forall\, i \in N_+, \tag{5b}$$

$$\underline{V}_i^2 \le (R_i^2 + W_i^2) \le \overline{V}_i^2 \quad \forall\, i \in N, \tag{5c}$$

$$(p_\ell^{\mathsf{f}})^2 + (q_\ell^{\mathsf{f}})^2 \le \overline{s}_\ell^2 \quad \forall\, \ell = (i,j) \in L_i^{\mathsf{f}} \tag{5d}$$

$$(p_\ell^{\mathsf{t}})^2 + (q_\ell^{\mathsf{t}})^2 \le \overline{s}_\ell^2 \quad \forall\, \ell = (i,j) \in L_i^{\mathsf{t}}. \tag{5e}$$

The bounds $[\underline{\eta}_{ij}, \overline{\eta}_{ij}]$ on the voltage phase angle difference $\theta_i - \theta_j$ cannot be imposed directly, as θ_i is not a variable of the ACOPF problem. Instead, we impose, for all $(i,j) \in L_i^{\mathsf{f}}$, constraints

$$(W_i R_j - R_i W_j) \ge \tan(\underline{\eta}_{ij})(R_{ij} + W_{ij}), \tag{6a}$$

$$(W_i R_j - R_i W_j) \le \tan(\overline{\eta}_{ij})(R_{ij} + W_{ij}), \tag{6b}$$

$$(R_{ij} + W_{ij}) \ge 0; \tag{6c}$$

see [10] for more details. Note that imposing bounds on the tangent rather than directly on the angles requires that the interval $[\underline{\eta}_{ij}, \overline{\eta}_{ij}]$ is contained in $[-\pi/2, \pi/2]$. In practice, this limitation is not restrictive, as larger intervals typically imply that no meaningful bounds are imposed.

3.2. **DCOPF Constraints.** In practice, solving a problem involving ACOPF constraints is computationally challenging due to the nonlinear and nonconvex nature of the AC power flow equations. To obtain a tractable formulation, the AC model is often approximated using the DCOPF approach; see, e.g., [33]. The DC approximation relies on several assumptions. First, reactive power is neglected and voltage magnitudes are assumed to be constant and close to one per unit. Second, line resistances are assumed to be much smaller than line reactances, i.e., $r_{ij} \ll x_{ij}$. Third, charging susceptance, transformers, and phase shifters are ignored. Finally, voltage angle differences across lines are assumed to be small. Under these assumptions, the power flow equations can be linearized, and only active power flows and voltage phase angles are considered. In particular, for a line $\ell = (i,j) \in L$, the admittance matrix reduces to

$$Y_\ell = \frac{-\mathbf{i}}{x_\ell} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad \forall\, \ell = (i,j) \in L,$$

where $x_\ell \in \mathbb{R}$ is the line reactance with $x_\ell = x_{ij} = x_{ji}$. The simplified DCOPF power flow equations then read

$$p_{ij} = \frac{(\theta_i - \theta_j)}{x_{ij}} \quad \forall\, (i,j) \in L, \tag{7a}$$

$$\sum_{j:(j,i)\in L} p_{ji} - \sum_{j:(i,j)\in L} p_{ij} + P_i^G = P_i^D \quad \forall\, i \in N. \tag{7b}$$

For its underlying assumptions, while AC power flows are not "network flows", e.g., they do not sum up to zero over cycles, DC power flows are [9, Sections 1.2.5–1.2.6].

Let us also note that, here and in what follows, we make the simplifying assumption that there is exactly one generator per bus $i \in N_+$. We only use this assumption for better readability; a generalization of the mathematical models to multiple generators per bus is straightforward.

4. A Multi-Leader Single-Follower Model

The DCOPF model presented in Section 3.2 is used in this section to set up a multi-leader single-follower (MLSF) model of strategic power producers playing a Nash game in which all players maximize their bid-based profits. This game is subject to a DC power flow market-clearing process that is carried out by the ISO in the lower-level problem of the MLSF problem. A KKT reformulation of the lower-level problem is then presented Section 4.1.

Suppose that $A_i P_i^G + B_i (P_i^G)^2$ is the actual production cost of generator i to generate P_i^G. We consider the situation in which, at every generator bus $i \in N_+$, a producer submits a *quadratic bid (offer) function* to the market, which is parameterized by the parameters $(a_i, b_i) \in \mathbb{R}^2$. These bid parameters will usually differ from the true cost parameters $(A_i, B_i) \in \mathbb{R}^2$. Hence, for an active production level P_i^G, the resulting bid function is given by $a_i P_i^G + b_i (P_i^G)^2$. We assume that the bid's parameters are bounded from below and above by $0 < \underline{a}_i \le \overline{a}_i$ and $0 < \underline{b}_i \le \overline{b}_i$ for all $i \in N_+$. Every generator $i \in N_+$ optimizes over the two bid variables (a_i, b_i) and solves the problem

$$\max_{a_i, b_i} \quad (a_i + 2 b_i P_i^G) P_i^G - (A_i + B_i P_i^G) P_i^G \tag{8a}$$

$$\text{s.t.} \quad \underline{a}_i \le a_i \le \overline{a}_i,\ \underline{b}_i \le b_i \le \overline{b}_i, \tag{8b}$$

$$(P^G, Q^G, p, q, R, W) \in S(a,b), \tag{8c}$$

where $S(a,b)$ is the set of globally optimal solutions to the DCOPF-based market-clearing problem, which constitutes the lower-level problem of the MLSF model.

In particular, the actual power production level P_i^G is determined via the market-clearing mechanism of the lower level. The objective function to be maximized by the generator at bus i is based on the marginal price, i.e., on the derivative $a_i + 2b_i P_i^G$ of generator i's bid function. In words, the objective to be maximized is bid-based revenues minus actual production costs.

The lower-level problem is parameterized by all bids $a := (a_i)_{i \in N_+}$ and $b := (b_i)_{i \in N_+}$. The market clearing is carried out so that all demand is met, all physical restrictions are satisfied, and the overall production costs (based on the bids of the producers in the upper level) are minimized. When assuming a DC market-clearing procedure, the lower-level problem is thus given by

$$\min_{p,\theta,P^G} \quad \sum_{i \in N_+} a_i P_i^G + b_i (P_i^G)^2 \tag{9a}$$

$$\text{s.t.} \quad p_{ij} = \frac{(\theta_i - \theta_j)}{x_{ij}} \quad \forall\, (i,j) \in L, \tag{9b}$$

$$\sum_{j:(j,i) \in L} p_{ji} - \sum_{j:(i,j) \in L} p_{ij} + P_i^G = P_i^D \quad \forall\, i \in N, \tag{9c}$$

$$\underline{P}_i^G \le P_i^G \le \overline{P}_i^G \quad \forall\, i \in N_+, \tag{9d}$$

$$\underline{p} \le p_{ij} \le \overline{p} \quad \forall\, (i,j) \in L. \tag{9e}$$

The constraints in (9b)–(9c) are the DC power flow equations as presented in (7). Constraints (9d)–(9e) are the bounds on the power production and the power flow. Since p_{ij} is directly related to $\theta_i - \theta_j$ through constraints (9b), explicit bounds on the voltage phase angle differences are not required, as they are implicitly enforced by the bounds on p_{ij}.

Because Problem (9) is a convex minimization problem with a strictly convex objective, it has a unique optimal production solution P_i^G for all $i \in N_+$. This fact is of great importance in multi-level energy market models; see, e.g., [31, 32].

Note that the DC power flows p and the phase angles θ do not need to be unique, which has been discussed, e.g., in [40, 41]. These variables, however, do not enter the upper-level problems, so that we do not have any ill-posedness issues that one otherwise often encounters in multi-leader games [6].

The overall model is a multi-leader single-follower model. The leaders maximize revenues over a box-constrained feasible set. The lower-level problem consists of a quadratic objective function over a linear feasible set that is given by the DC power flow model. The suitable solution concept is thus a Nash equilibrium among the upper-level players, i.e., we consider a Nash game among Stackelberg leaders. This setup has been considered in [33]. For a recent survey of multi-leader single-follower games, we refer to [6].

4.1. **A KKT Reformulation of the Generator Bilevel Problem.** Since the lower-level problem (9) is convex, it can be equivalently replaced by its optimality conditions. In particular, several reformulation techniques can be employed in this setting, such as the KKT conditions or dualization approaches [16, 20, 21]. In this section, we propose a KKT-based reformulation of the ith bilevel problem associated with generator i. For the linear lower-level problem (9), let us define the KKT multipliers α_{ij} associated to each constraint in (9b), β_i to each constraint in (9c), γ_i to constraint $\underline{P}_i^G - P_i^G \le 0$, δ_i to constraint $P_i^G - \overline{P}_i^G \le 0$, and, finally, ζ_{ij} to constraint $\underline{p} - p_{ij} \le 0$ and η_{ij} to constraint $p_{ij} - \overline{p} \le 0$.

The corresponding Lagrangian function reads

$$\begin{aligned}\mathcal{L} = &\sum_{i\in N_+} \left(b_i (P_i^G)^2 + a_i P_i^G\right) + \sum_{(i,j)\in L} \alpha_{ij} \left(p_{ij} - \frac{(\theta_i - \theta_j)}{x_{ij}}\right) \\ &+ \sum_{i\in N} \beta_i \left(\sum_{j:(j,i)\in L} p_{ji} - \sum_{j:(i,j)\in L} p_{ij} + P_i^G - P_i^D\right) \\ &+ \sum_{i\in N_+} \gamma_i (\underline{P}_i^G - P_i^G) + \sum_{i\in N_+} \delta_i (P_i^G - \overline{P}_i^G) \\ &+ \sum_{(i,j)\in L} \zeta_{ij} (\underline{p} - p_{ij}) + \sum_{(i,j)\in L} \eta_{ij} (p_{ij} - \overline{p}).\end{aligned}$$

Thus, the KKT conditions are given by dual feasibility

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial P_i^G} &= 2b_i P_i^G + a_i + \beta_i - \gamma_i + \delta_i = 0 && \forall\, i \in N_+, \\ \frac{\partial \mathcal{L}}{\partial \theta_i} &= -\sum_{j:(i,j)\in L} \frac{\alpha_{ij}}{x_{ij}} + \sum_{j:(j,i)\in L} \frac{\alpha_{ji}}{x_{ji}} = 0 && \forall\, i \in N, \\ \frac{\partial \mathcal{L}}{\partial p_{ij}} &= \alpha_{ij} - \beta_i + \beta_j - \zeta_{ij} + \eta_{ij} = 0 && \forall\, (i,j) \in L,\end{aligned}$$

primal feasibility (9b)–(9e), non-negative of dual variables of inequality constraints

$$\begin{aligned}\gamma_i, \delta_i &\geq 0 && \forall\, i \in N_+, \\ \zeta_{ij}, \eta_{ij} &\geq 0 && \forall\, (i,j) \in L,\end{aligned}$$

as well as KKT complementarity

$$\begin{aligned}\gamma_i (\underline{P}_i^G - P_i^G) &= 0 && \forall\, i \in N_+, \\ \delta_i (P_i^G - \overline{P}_i^G) &= 0 && \forall\, i \in N_+, \\ \zeta_{ij} (\underline{p} - p_{ij}) &= 0 && \forall\, (i,j) \in L, \\ \eta_{ij} (p_{ij} - \overline{p}) &= 0 && \forall\, (i,j) \in L.\end{aligned}$$

Since the lower-level problem (9) is a convex-quadratic minimization problem over a polyhedral feasible set, its KKT conditions are necessary and sufficient, and we can thus use them to transform each producer's bilevel problem into an equivalent single-level reformulation. The latter is given by

$$\max_{a_i,b_i,p,\theta,P^G} \quad (a_i - A_i)P_i^G + (2b_i - B_i)(P_i^G)^2, \tag{10a}$$

$$\text{s.t.} \quad p_{ij} = \frac{(\theta_i - \theta_j)}{x_{ij}} \qquad \forall\,(i,j) \in L, \tag{10b}$$

$$\sum_{j:(j,i)\in L} p_{ji} - \sum_{j:(i,j)\in L} p_{ij} + P_i^G = P_i^D \qquad \forall\, i \in N, \tag{10c}$$

$$2b_i P_i^G + a_i + \beta_i - \gamma_i + \delta_i = 0 \qquad \forall\, i \in N_+, \tag{10d}$$

$$\sum_{j:(j,i)\in L} \frac{\alpha_{ji}}{x_{ji}} - \sum_{j:(i,j)\in L} \frac{\alpha_{ij}}{x_{ij}} = 0 \qquad \forall\, i \in N, \tag{10e}$$

$$\alpha_{ij} - \beta_i + \beta_j - \zeta_{ij} + \eta_{ij} = 0 \qquad \forall\,(i,j) \in L, \tag{10f}$$

$$\gamma_i(\underline{P}_i^G - P_i^G) = 0 \qquad \forall\, i \in N_+, \tag{10g}$$

$$\delta_i(P_i^G - \overline{P}_i^G) = 0 \qquad \forall\, i \in N_+, \tag{10h}$$

$$\zeta_{ij}(\underline{p} - p_{ij}) = 0 \qquad \forall\,(i,j) \in L, \tag{10i}$$

$$\eta_{ij}(p_{ij} - \overline{p}) = 0 \qquad \forall\,(i,j) \in L, \tag{10j}$$

$$\gamma_i, \delta_i \geq 0 \qquad \forall\, i \in N_+, \tag{10k}$$

$$\zeta_{ij}, \eta_{ij} \geq 0 \qquad \forall\,(i,j) \in L, \tag{10l}$$

$$\underline{P}_i^G \leq P_i^G \leq \overline{P}_i^G \qquad \forall\, i \in N_+, \tag{10m}$$

$$\underline{p} \leq p_{ij} \leq \overline{p} \qquad \forall\,(i,j) \in L, \tag{10n}$$

$$\underline{a}_i \leq a_i \leq \overline{a}_i,\ \underline{b}_i \leq b_i \leq \overline{b}_i \qquad \forall\, i \in N_+. \tag{10o}$$

The most challenging part of solving such single-level reformulations stems from the KKT complementarity constraints (10g)–(10j), which induce an inherently nonconvex and disjunctive structure. In our experiments, we model these complementarity relationships using constraints based on special ordered sets of type (SOS1). Specifically, for each complementarity pair consisting of a primal slack $\sigma \geq 0$ and its associated dual variable $\lambda \geq 0$, we impose $(\sigma, \lambda) \in \mathsf{SOS1}$, which enforces that at most one of the two variables can take a strictly positive value; see [25, 39] for the details. This encoding directly captures the mutually exclusive nature of the complementarity condition without requiring big-M, which are often used in single-level reformulations of bilevel problems and which may be problematic to compute [12, 38, 44].

5. Diagonalization Method

The MLSF game, i.e., the Nash game among the Stackelberg leaders $i \in N_+$ solving problems of the type in (8) is extremely challenging to solve. Hence, one often resorts to so-called diagonalization methods, which, in case of convergence, deliver a Nash equilibrium among the Stackelberg leaders. This convergence cannot be guaranteed in general; see, e.g., [23, 28, 46]. In what follows, we briefly discuss such a diagonalization for the MLSF game of Section 4. To this end, let $x_i = (a_i, b_i)$ denote the overall variable vector of upper-level player $i \in N_+$. We initialize the diagonalization algorithm with a starting point $x^{(0)}$, which may be the midpoint of the bid bounds.

Algorithm 1 Diagonalization method for the MLSF game of Section 4

Input: A starting point $x^{(0)} = (x_i^{(0)})_{i \in N_+}$ and a convergence tolerance $\varepsilon > 0$.

1: Set $k \leftarrow 0$.
2: **for** $i \in N_+$ **do**
3: Solve the ith leader's bilevel problem (10) fixed $x_j = x_j^{(k+1)}$ for $j \in \{1, \dots, i-1\}$ and fixed $x_j = x_j^{(k)}$ for $j \in \{i+1, \dots, n_+\}$.
4: Denote the x_i-part of the solution by $x_i^{(k+1)}$.
5: **end for**
6: **if** $\|x_i^{(k+1)} - x_i^{(k)}\| < \varepsilon$ for all $i \in N_+$ **then**
7: Return $x^{(k+1)}$ and stop.
8: **else**
9: Set $k \leftarrow k+1$ and go to Line 2.
10: **end if**

If we formally set $\varepsilon = 0$, and if Algorithm 1 terminates in Line 7, $x^{(k+1)}$ encodes a Nash equilibrium. First, from the stopping criterion in Line 6, we know that $x_i^{(k+1)} = x_i^{(k)}$ holds for all $i \in N_+$. Hence, no upper-level player's solution changed in the last iteration. Consequently, $x_i^{(k+1)}$ is the bilevel solution for player i and for given strategies $x_j^{(k+1)}$, $j \in N_+ \setminus \{i\}$. Since this holds for all i, the overall strategy profile $(x_i^{(k+1)})_{i \in N_+}$ is a Nash equilibrium among the Stackelberg leaders. Unfortunately, we cannot guarantee, in general, that the method actually converges, as it can be seen from our numerical experiments, and as it is also known in the literature [46]. In particular, the method can cycle, and even in the case of convergence, the computed Nash equilibrium may significantly depend on the used starting point or the order of the producers in the for-loop of the method.

6. ACOPF Feasibility

After solving the bilevel model with a DCOPF-based market-clearing model in the lower-level problem, the resulting generator dispatch may not be feasible under the full AC power flow model due to the DC simplifications needed to derive the DC model. To evaluate the physical feasibility of the obtained dispatch, we introduce a post-processing step in which the DCOPF market-clearing solution is used as input to an ACOPF redispatch process.

To this end, let P_i^{DC} denote the active power generation output of generator $i \in N_+$ obtained from the diagonalization approach. These values are then passed to an ACOPF model as an initial reference. By introducing the slack variables Δ_i^+ and Δ_i^- for each $i \in N_+$, the ACOPF redispatch is formulated as

$$\min \quad \sum_{i \in N_+} (\Delta_i^+ + \Delta_i^-) \tag{11a}$$

$$\text{s.t.} \quad \text{AC power flow constraints (3)–(6)}, \tag{11b}$$

$$P_i^G = P_i^{\mathrm{DC}} + \Delta_i^+ - \Delta_i^- \quad \forall i \in N_+, \tag{11c}$$

$$\Delta_i^+, \Delta_i^- \geq 0 \quad \forall i \in N_+, \tag{11d}$$

$$P^G, Q^G, \Delta^+, \Delta^- \in \mathbb{R}^{N_+}, \tag{11e}$$

$$p, q \in \mathbb{R}^{2L}, \tag{11f}$$

$$R, W \in \mathbb{R}^N. \tag{11g}$$

The slack variables quantify the adjustments needed to make the DCOPF dispatch compatible with the full nonlinear AC power flow equations. The objective

function (11a) is to minimize the total slack, thereby computing the closest (in terms of the ℓ_1-norm) AC-feasible redispatch. The resulting slack values Δ^+ and Δ^- provide a direct measure of the approximation error introduced by using the DC model in the market-clearing process: the larger the slack, the further the DC solution is from true AC feasibility. In particular, Δ^+ captures the upward deviation (increase in active power with respect to the DCOPF schedule), while Δ^- captures the downward deviation (reduction in active power relative to the DCOPF solution).

7. Numerical results

This section presents the computational analysis of the proposed approach. Our experiments are conducted on the standard "case files" from the MatPower library [49], discarding the information about parallel arcs. While the theoretical models presented above assume a single generator per bus for notational simplicity, the numerical instances considered in this section may include multiple generators connected to the same bus. In the computational implementation, each bus $i \in N_+$ is associated with the set $\mathcal{G}_i$ of generators connected to that bus. Then, $\mathcal{G} = \bigcup_{i \in N_+} \mathcal{G}_i$ denotes the set of all the generators of the power grid.

For each instance, we first apply the diagonalization scheme to solve the MLSF KKT reformulation, with a starting point

$$\left(a_i^{(0)}, b_i^{(0)}\right) = \left(\frac{\overline{a}_i + \underline{a}_i}{2}, \frac{\overline{b}_i + \underline{b}_i}{2}\right)$$

for all $i \in N_+$. Moreover, we impose an iteration limit of 100. At each iteration of the diagonalization (i.e., each for-loop), the KKT reformulation (10) of the i-th upper-level player is solved using Gurobi (Version 12.0.1), being interfaced through AMPL. To account for the potential existence of multiple Nash equilibria, the diagonalization procedure is repeated for up to 25 randomly chosen orderings of the generators. Among the convergent runs, we select and report the equilibrium associated with the highest social welfare. Given that demand is assumed to be perfectly inelastic, social welfare differs across equilibria only through the total generation cost. Specifically, for a given equilibrium (indexed with u in what follows), social welfare SW is defined as

$$\mathrm{SW}^{(u)} = \sum_{i \in N_-} U_i(P_i^D) - \sum_{i \in N_+} \left(A_i + B_i P_i^{G,(u)}\right) P_i^{G,(u)}, \tag{12}$$

where $U_i(\cdot)$ denotes some utility function for the given and fixed demand at node $i \in N_-$ and $P_i^{G,(u)}$ denotes the dispatched generation of generators at bus i in equilibrium u. Since the total demand is fixed, maximizing social welfare is equivalent to minimizing the total generation cost. Accordingly, among the equilibria obtained from different leader orderings, we select the equilibrium $u^\star$ such that

$$u^\star \in \arg\max_u \ \mathrm{SW}^{(u)}. \tag{13}$$

Once the diagonalization procedure terminates, we assess the feasibility of the resulting solution by solving model (11), using Gurobi (Version 12.0.1) interfaced via AMPL for small cases (up to 24 nodes) and Ipopt (Version 3.12.13), also interfaced via AMPL, for bigger cases. All computations were performed on a standard workstation equipped with an Apple M2 processor (8 cores) and 24 GB of RAM, running MacOS 14.7.4. The entire implementation, including model generation and solver interface, is written in AMPL.

In Table 2, we report, for each test instance, the size of the instance with $|N|$ being the total number of buses and $|\mathcal{G}|$ being the number of generators. Moreover,

TABLE 2. Numerical results for the DCOPF-based diagonalization method. Runtimes are given in seconds.

Instance			**DCOPF diagonalization**	
Name	$\|N\|$	$\|\mathcal{G}\|$	Time	#Iterations
case4_dist	4	2	0.21	3
case5	5	5	0.34	2
case6ww	6	3	0.25	2
case9	9	3	12.15 (NC)	100
case14	14	5	0.33	2
case24_ieee_rts	24	33	792.14 (NC)	100
case30	30	6	2.70	9
case39	39	10	71.63 (NC)	100
case89pegase	89	12	1.19	2
case118	118	54	4.14	2

we list the main performance data for the DCOPF diagonalization to compute the equilibrium $u^\star$. Here, "Time" is the runtime (in seconds) required by the DCOPF diagonalization method, and we also report the number of iterations performed by the diagonalization method.

If the diagonalization method does not converge within the iteration limit of 100, we report "NC" in brackets in the corresponding "Time", together with the time to perform those 100 iterations.

Then, in Table 3, we display percentage-based slack metrics for the ACOPF feasibility checks together with the respective runtime. All slack values are expressed in percentage form and computed using $\max\{P_i^{\mathrm{DC}}, 0.1\}$ in the denominator to avoid divisions by zero or by values very close to zero. In particular, the headings of the table are the following. For the negative slack values Δ_i^-, $i \in N_+$, we report

$$\min_{i\in N_+} \frac{\Delta_i^-}{\max\{P_i^{\mathrm{DC}}, 0.1\}},$$
$$\max_{i\in N_+} \frac{\Delta_i^-}{\max\{P_i^{\mathrm{DC}}, 0.1\}},$$
$$\frac{1}{|N_+|} \sum_{i\in N_+} \frac{\Delta_i^-}{\max\{P_i^{\mathrm{DC}}, 0.1\}}$$

as the minimum, maximum, and average percentage slack, respectively, and we report the same values for the positive slack values, for which we replace Δ_i^- with Δ_i^+ in the above three formulas.

If the diagonalization method does not converge, no ACOPF feasibility model can be solved, as no P^{DC} value is known, which is indicated with "—" in Table 3.

The results indicate that the diagonalization method performs well on almost all test instances, and convergence is achieved in a few iterations and short computation times. However, for larger instances, convergence issues can arise, preventing the availability of DC-based production plans. For those instances, we cannot apply any AC feasibility checks. For the remaining instances, the ACOPF feasibility checks show that all negative slack variables are zero for 6 out of 8 instances. The other two maximum values are 0.01 %, which is negligible, and 4.28 %, which seems acceptable for real-world markets and the transport of their outcomes through the respective networks.

TABLE 3. Numerical results for the ACOPF feasibility checks. Runtimes are given in seconds.

Instance	**ACOPF feasibilty**						
	negative slack (in %)			positive slack (in %)			Time
Name	min	avg	max	min	avg	max	
case4_dist	0.00 %	0.00 %	0.00 %	0.00 %	2.15 %	4.31 %	0.17
case5	0.00 %	0.00 %	0.00 %	0.00 %	0.30 %	1.51 %	0.10
case6ww	0.00 %	1.43 %	4.28 %	0.00 %	10.05 %	30.16 %	0.10
case9	—	—	—	—	—	—	—
case14	0.00 %	0.00 %	0.00 %	0.00 %	13.25 %	66.24 %	1087.47
case24_ieee_rts	—	—	—	—			—
case30	0.00 %	0.00 %	0.00 %	0.00 %	4.27 %	23.57 %	0.14
case39	—	—	—	—	—	—	—
case89pegase	0.00 %	0.00 %	0.01 %	0.00 %	1.34 %	16.05 %	0.94
case118	0.00 %	0.00 %	0.00 %	0.00 %	4.04 %	89.57 %	2.34

On the other hand, positive slack varies significantly and can be large with maximum values reaching almost 90 %. This seems to indicate potential underestimation of generation in the DC solution. Small systems, such as the case4_dist and case5 networks, exhibit small positive slack values, suggesting that DC approximations for the market-clearing process are close to AC feasibility. In larger systems, positive slack values can be substantial, clearly showing that the DC-based modeling of the market-clearing process is not a reasonable proxy for AC physics.

These findings show that, although the DC approach provides a computationally tractable starting point, it may substantially underestimate production levels in larger networks, making it necessary to develop more powerful computational techniques so that AC models can be included in lower-level problems of multi-leader single-follower power market bidding and clearing models.

8. Conclusions

In this paper, we evaluated the outcome of a DC power flow based market-clearing process, which is embedded in the lower-level problem of a multi-leader single-follower game, in terms of AC feasibility. The motivation to do so is that AC power flow models in the lower level of bilevel problems are extremely challenging to solve—even on small academic examples. However, when policy suggestions or investment decisions are based on such DC-driven market models, one cannot assume that the obtained outcome is feasible with respect to the real network physics. Our results show that the AC-infeasibility of DC-based market outcomes tends to be large and seems to increase for larger networks. This means that those results have to be considered with great care.

Moreover, these insights lead to important aspects that should be considered in research on power-market models that are connected to transport through AC networks. As one direction for future research, the framework proposed in this paper can be extended by exploring alternative convex relaxations of the lower-level problem, such as classic SOCP or SDP formulations of AC power flow. Investigating the impact of these relaxations on the computational performance and on the obtained equilibria would provide further insights regarding the balance between model accuracy and computational tractability of strategic electricity market models. Last but not least, the results of this paper make the need obvious for more powerful

bilevel optimization techniques that can also handle continuous but nonconvex lower-level problems.

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