Congressional Apportionment

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Abstract

This book chapter is a gentle introduction to the mathematics of congressional apportionment. It emphasizes the connections between mathematical optimization and the classical apportionment methods (e.g., Jefferson, Adams, Hamilton, Webster, Huntington-Hill, Dean).

1 Introduction

In April 2021, the U.S. Census Bureau announced the latest apportionment results. Apportionment answers the question: How many seats will each state get in the U.S. House of Representatives? It also plays an important role in the functioning of the Electoral College and consequently U.S. presidential elections. The latest numbers are shown in Figure 1.

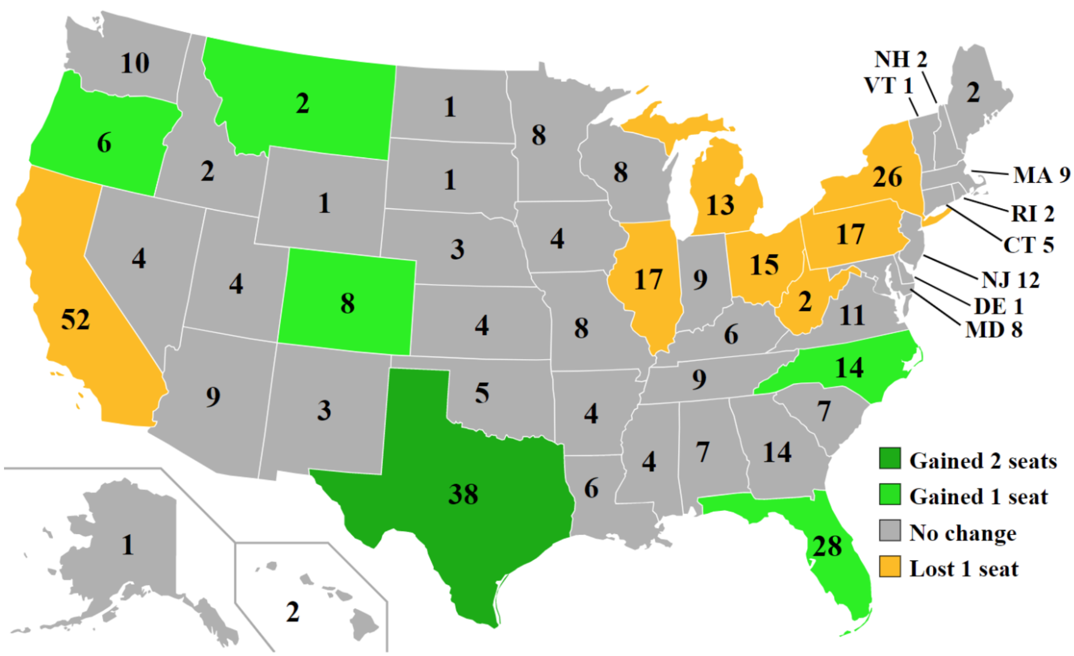

Figure 1: 2020 Census Reapportionment, from Wikimedia commons (public domain) as https://commons.wikimedia.org/wiki/File:2020_census_reapportionment.svg.

Interestingly, New York, which saw its share drop from 27 to 26 seats, would have remained at 27 seats if it had just 89 more people! Several other states, like California, Arizona, and Texas, did not receive as many seats as expected. How did the Census Bureau arrive at these numbers?

2 The Apportionment Problem

The key input to the apportionment problem is each state's population. For example, in 2020, the most populous state, California, had a population of $p_{\text{CA}} = 39,576,757$, while the least populous state, Wyoming, had a population of $p_{\text{WY}} = 577,719$. To the states, we must distribute a certain number of seats. Since 1929, this number k has been limited to $k = 435$. The total population across the 50 states was 331,108,434 in 2020, California's share or *quota* of the 435 seats was

$$q_{\text{CA}} = 435\left(\frac{39,576,757}{331,108,434}\right) \approx 51.995 \text{ seats},$$

while Wyoming's quota was

$$q_{\text{WY}} = 435\left(\frac{577,719}{331,108,434}\right) \approx 0.759 \text{ seats}.$$

If all quotas happened to take integer values, then the apportionment problem would be easy—simply allocate each state $s \in S$ its quota q_s. Since this never occurs in practice, we must essentially "round" the quotas to integers in a fair way. However, rounding each quota up would distribute too many seats, rounding down would distribute too few, and rounding to the nearest integer may not work either. The question of how best to round fractional quotas to an integer-valued apportionment has long been a conundrum, with many different techniques proposed over the years by prominent figures from U.S. history, including Thomas Jefferson, Alexander Hamilton, John Quincy Adams, and Daniel Webster, as we will see.

To formalize the discussion, we introduce some notation. Denote by S the set of states, i.e.,

$$S = \{\text{CA}, \text{TX}, \text{FL}, \text{NY}, \dots, \text{WY}\}.$$

Each state s that *belongs* to our set of states (written $s \in S$) has an associated population p_s. We denote the total population, summed across all states, by P. If there are a total of k seats to distribute, then each state s should ideally receive its quota of $q_s = k(p_s/P)$ seats.

Given this information, we must decide the *apportionment*, which we denote by x. This vector represents how many seats to give each state. For example, the entry x_{CA} indicates California's number of seats, and x_{WY} indicates Wyoming's number of seats. For the apportionment to be valid, we must distribute a total of $k = 435$ seats to the states, i.e.,

$$x_{\text{CA}} + x_{\text{TX}} + x_{\text{FL}} + x_{\text{NY}} + \cdots + x_{\text{WY}} = 435.$$

According to the U.S. Constitution, each state must receive at least one seat, i.e.,

$$x_{\text{CA}} \geq 1,\ x_{\text{TX}} \geq 1,\ x_{\text{FL}} \geq 1,\ x_{\text{NY}} \geq 1, \dots,\ x_{\text{WY}} \geq 1.$$

Finally, each state must receive an integer number of seats. Mathematically, this is expressed by requiring each entry of x to belong to the set of integers, denoted $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$, i.e.,

$$x_{\text{CA}} \in \mathbb{Z},\ x_{\text{TX}} \in \mathbb{Z},\ x_{\text{FL}} \in \mathbb{Z},\ x_{\text{NY}} \in \mathbb{Z}, \dots,\ x_{\text{WY}} \in \mathbb{Z}.$$

Together, we may call these the *constraints* of apportionment. They can be written generically and concisely as

$$\sum_{s\in S} x_s = k \tag{1a}$$

$$x_s \geq 1 \qquad \forall s \in S \tag{1b}$$

$$x_s \in \mathbb{Z} \qquad \forall s \in S. \tag{1c}$$

Here, we use sigma notation $\sum_{s\in S}$ to concisely express the summation over all states and the "for all" symbol $\forall$ (called the *universal quantifier*) to indicate that the constraints $x_s \geq 1$ and $x_s \in \mathbb{Z}$ must hold for every state $s \in S$.

Among the apportionments that satisfy these constraints, which one should be chosen? What makes one apportionment "better" than another?

A typical response is *fairness*, specifically *proportionality*; if a state has twice the population of another, then it should receive twice as many seats. This abides by the "one person, one vote" principle. In other words, we seek to have apportionments near to the quotas, i.e., $x \approx q$. Next, we see how this task has been accomplished throughout U.S. history.

3 Enter Thomas Jefferson and Alexander Hamilton

The U.S. Constitution does not specify what method should be used for apportionment. So, when results from the first census arrived in 1791, there were political disputes about how to proceed.

Alexander Hamilton proposed the following method. Choose the number of seats k to be apportioned. Compute the quotas and initially give each state s the *floor* of its quota q_s (i.e., rounded down to an integer), which is denoted by $\lfloor q_s \rfloor$. Assign any leftover seats to the states having the largest remainders $r_s = q_s - \lfloor q_s \rfloor$. For example, California in 2020 would initially receive $\lfloor q_{CA} \rfloor = \lfloor 51.995 \rfloor = 51$ seats, and Wyoming would initially receive $\lfloor q_{\mathrm{WY}} \rfloor = \lfloor 0.759 \rfloor = 0$ seats[1]. When distributing the remaining seats, California would be prioritized over Wyoming, since its remainder $r_{\mathrm{CA}} = q_{\mathrm{CA}} - \lfloor q_{\mathrm{CA}} \rfloor \approx 0.995$ exceeds Wyoming's remainder $r_{\mathrm{WY}} = q_{\mathrm{WY}} - \lfloor q_{\mathrm{WY}} \rfloor \approx 0.759$.

It turns out that Hamilton's method solves the following optimization problems. That is, from the set X of all apportionments that satisfy constraints (1a) and (1c), it returns an apportionment x that minimizes the 1-norm (or 2-norm, say) between x and the quota vector q.

$$\text{(Hamilton's Objective)} \qquad \min_{x\in X} \sum_{s\in S} |x_s - q_s| \quad \text{or equivalently} \quad \min_{x\in X} \|x-q\|_1$$

$$\min_{x\in X} \sum_{s\in S} (x_s - q_s)^2 \quad \text{or} \quad \min_{x\in X} \sqrt{\sum_{s\in S} (x_s-q_s)^2} \quad \text{or equivalently} \quad \min_{x\in X} \|x-q\|_2 .$$

Although these are seemingly natural ways to measure distances, they have notable disadvantages when applied to apportionment, and not everyone agreed on their use. For example, Thomas Jefferson proposed a different method, based on rounding. Recall that the naïve approach of rounding quotas down to integers will distribute too few seats. For example, when applied to 2020 Census data, only 410 seats would be distributed instead of 435. To remedy this, Jefferson instead

[1] Given that each state must receive at least one seat, we should arguably start by assigning $a_s = \max\{1, \lfloor q_s \rfloor\}$ seats to state s, and take the remainders to be $r_s = q_s - a_s$ rather than $r_s = q_s - \lfloor q_s \rfloor$.

proposed to choose a *divisor* d so that the ratios p_s/d, when rounded down, would distribute exactly k seats, i.e.,

$$\sum_{s \in S} \left\lfloor \frac{p_s}{d} \right\rfloor = k.$$

If we rewrite the quota definition as $q_s = p_s/(P/k)$, we see that the implicit or *standard* divisor is P/k. To distribute more seats, Jefferson would require a divisor d smaller than P/k. For the 2020 Census data, this means choosing, say, $d = 720,000$ instead of $P/k \approx 761,169$. It turns out that Jefferson's method solves the following min-max optimization problem:

$$\text{(Jefferson's Objective)} \qquad \min_{x \in X} \max_{s \in S} \left\{ \frac{x_s}{p_s} \right\}.$$

The ratio x_s/p_s records the number of seats per person in state s, and the min-max objective attempts to minimize the largest ratio across the states. This has the effect of favoring states with large populations. For example, based on 2020 Census data, California would receive 54 seats, noticeably more than its quota of 51.995 seats.

The controversy between Hamilton and Jefferson was central to the first congressional apportionment. In 1792, Congress initially approved Hamilton's apportionment for a 120-seat House [3]. President George Washington vetoed the bill on April 5, 1792, marking the first presidential veto in U.S. history. Congress then adopted Jefferson's calculation based on a ratio of one representative for every $30,000$ people. The resulting House contained 105 seats [2], and Jefferson's method was used up through the 1830 Census [3].

4 Enter John Quincy Adams and Daniel Webster

By the 1830s, Jefferson's method had been criticized for systematically favoring states with large populations. In response to this concern, John Quincy Adams and Daniel Webster proposed two alternative divisor methods in 1832 [3].

Recognizing that Jefferson's method had a bias towards large states, John Quincy Adams proposed to reverse Jefferson's rounding direction, choosing to round up rather than down. If applied to the quotas directly, this would predictably distribute too many seats. For example, when applied to 2020 Census data, 460 seats would be distributed rather than 435. Like before, a possible remedy is to choose a divisor d so that the ratios p_s/d, when rounded *up*, will distribute exactly k seats, i.e.,

$$\sum_{s \in S} \left\lceil \frac{p_s}{d} \right\rceil = k.$$

Predictably, the bias is now opposite of Jefferson's method, favoring small states rather than large states. For example, on 2020 Census data, California would receive 50 seats, noticeably less than its quota of 51.995, while a small state like South Dakota with a quota of 1.166 would receive 2 seats. It turns out that Adams' method solves a similar min-max optimization problem, but with the ratios reversed in the objective:

$$\text{(Adams' Objective)} \qquad \min_{x \in X} \max_{s \in S} \left\{ \frac{p_s}{x_s} \right\}.$$

The ratio p_s/x_s records the population per seat in state s, and the min-max objective again seeks to minimize the largest ratio.

Webster proposed a compromise. Instead of systematically rounding down as in Jefferson's method, or up as in Adams' method, Webster chose to round *to the nearest integer*. Similar to before, Webster requires a divisor d so that the ratios p_s/d, when rounded to the nearest integer, distribute exactly k seats, which we may write as

$$\sum_{s \in S} \left\lceil \frac{p_s}{d} - \frac{1}{2} \right\rceil = k.$$

When applied to the 2020 Census data, it turns out that choosing the standard divisor $d = P/k$ suffices for Webster's method. In other words, Webster would simply round each state's quota to the nearest integer. So, for example, California with quota 51.995 would receive 52 seats, and Wyoming with quota 0.759 would receive 1 seat.

It turns out that Webster's method solves the following optimization problem:

$$\text{(Webster's Objective)} \qquad \min_{x \in X} \sum_{s \in S} p_s \left(\frac{x_s}{p_s} - \frac{k}{P} \right)^2.$$

This effectively minimizes the mean squared error between the number of seats per person in the apportionment, x_s/p_s, and the ideal number of seats per person, k/P, but weighted by population.

Webster's method was used in the 1840s. Subsequently, in the 1850s, 1860s, and 1870s, Hamilton's method was instead adopted, but under the new name "Vinton's method." Its output was also altered for political reasons.

5 Paradoxes and Axioms

With the 1880 Census data came some strange results when using Hamilton's method. If $k = 299$ seats were to be distributed, then Alabama would get 8 of them. However, if the House size were increased to 300 seats, then Alabama's representation would drop to 7 seats. Even though there is more pie to go around, someone gets less! This is called the "Alabama paradox," where a state loses a seat as the House size increases.

There were other problems with Hamilton's method. In the 1900s, Virginia *grew faster* than Maine in population, but Virginia would lose a seat to Maine. This is known as the "population paradox," where a faster-growing state loses a seat to a slower-growing state.

Finally, before Oklahoma became a state in 1907, Maine and New York had 3 and 38 seats respectively, out of 386 seats total. When Oklahoma joined and got 5 seats (increasing the total to 391), Maine and New York would swap a seat under Hamilton's method, going to 4 and 37 seats respectively. This is known as the "new states paradox," where adding a new state and increasing the total number of seats accordingly can change the apportionment among the original states.

These paradoxes led mathematicians to study apportionment methods in more detail and axiomatize their study. Some prominent results include the following:

1. Divisor methods (e.g., Jefferson, Adams, Webster) are the only methods that avoid the population paradox.

2. All divisor methods avoid the Alabama paradox.

3. All divisor methods avoid the new states paradox.

These results suggest that a divisor method should be used. Intuitively, one should avoid Jefferson's and Adams' methods because they exhibit bias towards large and small states, respectively. How many other divisor methods are there? Which one should be used?

6 Enter Huntington and Hill

Joseph A. Hill began doing statistical work for the U.S. Census Bureau in 1899 and later became chief statistician in 1909. He proposed yet another apportionment method, also a divisor method. The method was also analyzed by Edward V. Huntington, a professor at Harvard [9], who called it the method of equal proportions. This method, sometimes called the Huntington-Hill method, has been used in the U.S. since the 1940s.

Before describing the Huntington-Hill method, let us describe Webster's method differently. After dividing the state's population p_s by the divisor d, Webster's method rounds the fractional number p_s/d to either the low value $l = \lfloor p_s/d \rfloor$ or the high value $h = \lceil p_s/d \rceil$ depending on whether p_s/d lies below or above the *arithmetic mean* of l and h, i.e., $(l+h)/2$. The Huntington-Hill method is similar, but the rounding threshold is instead the *geometric mean*[2] of l and h, i.e., $\sqrt{lh}$. That is, if $p_s/d < \sqrt{lh}$, then Huntington-Hill rounds down to l; otherwise, it rounds up to h.

It turns out that the Huntington-Hill method solves the following optimization problem:

$$\text{(Huntington-Hill's Objective)} \qquad \min_{x \in X} \sum_{s \in S} x_s \left(\frac{p_s}{x_s} - \frac{P}{k} \right)^2.$$

This objective function is similar to Webster's objective, minimizing the *seat-weighted* mean square error between the population per seat, p_s/x_s, and the ideal population per seat, P/k.

When applied to 2020 Census data, Huntington-Hill and Webster agree for 46 of 50 states. The only difference is that Webster's method would give New York and Ohio one more seat, at the expense of Rhode Island and Montana.

7 Pairwise Inequality Between States

In the 1920s, Huntington analyzed apportionment methods by pairwise comparisons [8, 9] and ultimately recommended the use of the now-standard Huntington-Hill method. Here we discuss this pairwise analysis between states.

To illustrate, recall New York's situation after the 2020 Census where it received $x_{\text{NY}} = 26$ seats. News reports indicated that it would have received 27 seats if its population were larger by just 89 people [6]. This hypothetical 27th seat instead went to Minnesota, which received $x_{\text{MN}} = 8$ seats. How did this 89-person figure arise?

To replicate this, let us compare New York's $x_{\text{NY}} = 26$ seats and its population $p_{\text{NY}} = 20,215,751$ with Minnesota's $x_{\text{MN}} = 8$ seats and its population $p_{\text{MN}} = 5,709,752$. Ideally, their

[2] Using the *harmonic mean* of l and h, i.e., $2/(1/l + 1/h)$, as the rounding threshold gives Dean's method.

seat-per-person ratios $x_{\mathrm{NY}}/p_{\mathrm{NY}}$ and $x_{\mathrm{MN}}/p_{\mathrm{MN}}$ would be equal. Since this is typically impossible, we may seek to minimize the inequality between them, with Huntington choosing to measure the inequality between two states i and j by

$$\frac{x_i/p_i}{x_j/p_j} - 1. \tag{2}$$

Here, the convention is that the larger ratio x_i/p_i goes in the numerator, and the smaller ratio x_j/p_j goes in the denominator, in which case the result (2) is always nonnegative, with zero inequality occurring precisely when the seat-per-person ratios are equal. Huntington suggests to transfer a seat from one state to another if doing so reduces this measure of inequality. By repeatedly performing these transfers in an inequality-reducing way, we arrive at the Huntington-Hill apportionment.

To illustrate, let us compare the actual 2020 apportionment x with an alternative apportionment x' in which New York received a 27th seat at the expense of Minnesota, i.e., with $x'_{\mathrm{NY}} = x_{\mathrm{NY}} + 1 = 27$, $x'_{\mathrm{MN}} = x_{\mathrm{MN}} - 1 = 7$, and $x'_s = x_s$ for all other states s. Then, we calculate (2) for both x and x' to see if this seat transfer would reduce the amount of inequality between them:

$$\begin{aligned} & \frac{x_{\mathrm{MN}}/p_{\mathrm{MN}}}{x_{\mathrm{NY}}/p_{\mathrm{NY}}} - 1 = \frac{8/5709752}{26/20215751} - 1 \approx 0.08940 \\ < \quad & \frac{x'_{\mathrm{NY}}/p_{\mathrm{NY}}}{x'_{\mathrm{MN}}/p_{\mathrm{MN}}} - 1 = \frac{27/20215751}{7/5709752} - 1 \approx 0.08941. \end{aligned}$$

Recall that the numerator state is chosen as the one with more seats per person. Since this transfer causes the amount of inequality to increase from 0.08940 to 0.08941, the transfer would be rejected. Moreover, all other possible transfers between pairs of states would be rejected for similar reasons, causing the apportionment to be *stable*. It is also the *unique* apportionment that lacks an inequality-reducing transfer.

However, if New York's population were larger by 89 people, i.e., $p'_{\mathrm{NY}} = p_{\mathrm{NY}} + 89 = 20,215,840$, then the MN→NY seat transfer would (just barely) have been accepted, since

$$\begin{aligned} & \frac{x_{\mathrm{MN}}/p_{\mathrm{MN}}}{x_{\mathrm{NY}}/p'_{\mathrm{NY}}} - 1 = \frac{8/5709752}{26/20215840} - 1 \approx 0.08940957 \\ > \quad & \frac{x'_{\mathrm{NY}}/p'_{\mathrm{NY}}}{x'_{\mathrm{MN}}/p_{\mathrm{MN}}} - 1 = \frac{27/20215840}{7/5709752} - 1 \approx 0.08940955. \end{aligned}$$

It turns out that similar transfer-based procedures exactly replicate the classical methods of Adams, Dean, Jefferson, and Webster, but using the following terms to measure the pairwise inequality between states. Starting with an arbitrary apportionment, seat transfers are made in an inequality-reducing way until convergence.

- Adams: $x_i - x_j(p_i/p_j)$,
- Dean: $p_j/x_j - p_i/x_i$,
- Jefferson: $x_i(p_j/p_i) - x_j$,
- Webster: $x_i/p_i - x_j/p_j$.

As with the Huntington-Hill measure of inequality (2), we assume that the ratio x_i/p_i is at least x_j/p_j, in which case the terms above are nonnegative and equal zero precisely when $x_i/p_i = x_j/p_j$.

8 Apportionments for the 2020 Census Data

Here we apply six classical apportionment methods to apportion the 435 seats of the House of Representatives to the 50 states using the 2020 Census data. Results are reported in Table 1. For the divisor methods, we apply a generic seat-priority algorithm, similar to how the U.S. Census Bureau actually computes the Huntington-Hill apportionment. We compute the Hamilton apportionment using the largest-remainder method described above. We also require each state to receive at least one seat. The Python code used to generate the apportionments is publicly available at `https://github.com/JessicaXiaocong/Apportionment-book-chapter/`.

As expected, Jefferson's method has a clear bias in favor of larger states, giving more seats to California, Texas, Florida, New York, and Pennsylvania than any other method. Contrariwise, Adams' method, which is known to favor small states, gives more seats to Delaware and South Dakota than any other method. Both methods *violate quota* in the sense that some state s receives fewer than $\lfloor q_s \rfloor$ seats or more than $\lceil q_s \rceil$ seats. It is known that all divisor methods will violate quota sometimes, but Adams and Jefferson frequently do so here.

The other divisor methods—Dean, Huntington-Hill, and Webster—largely agree, giving the same number of seats for 44 states. They disagree slightly on six states: New York (26, 26, 27), Ohio (15, 15, 16), Minnesota (7, 8, 8), Idaho (3, 2, 2), Rhode Island (2, 2, 1), and Montana (2, 2, 1). The quotas for these states are 26.559, 15.514, 7.501, 2.419, 1.443, and 1.426, respectively. These values are all far from integers, making it unsurprising that they disagree. Although these methods are generally not guaranteed to satisfy quota, they do here.

Hamilton's method is not a divisor method and is subject to the Alabama paradox. It does, however, satisfy quota here and in general. On 2020 Census data, it happens to perfectly agree with Webster's method, which is why Hamilton and Webster are placed side by side in the table.

Recall that the methods of Dean, Huntington-Hill, and Webster are closely related to the harmonic mean (HM), geometric mean (GM), and arithmetic mean (AM), respectively. Since these means are known to satisfy $HM \leq GM \leq AM$, it is not surprising that there is a pattern in their rounding tendencies. Indeed, *relative to each other*, Dean slightly favors smaller states, while Webster slightly favors larger states, with Huntington-Hill somewhere in between. This is not to say that Webster's method *objectively* favors larger states; here, it happens to round each state's quota to the nearest integer, which is hard to argue with. Nevertheless, the Huntington-Hill method is currently in use and has been for the past 85 years.

Further Reading

For more information on the mathematics of apportionment, we refer the reader to the classic book by Balinski and Young [3]. We also refer to the concise summary by Agnew [1]. To this day, apportionment remains an active research area, with recent works such as [4, 5, 7, 10].

Abbr.	State	Population	Quota	Adams	Dean	H.H.	Web.	Ham.	Jeff.
CA	California	39,576,757	51.995	50	52	52	52	52	54
TX	Texas	29,183,290	38.340	37	38	38	38	38	40
FL	Florida	21,570,527	28.339	27	28	28	28	28	29
NY	New York	20,215,751	26.559	26	26	26	27	27	28
PA	Pennsylvania	13,011,844	17.095	17	17	17	17	17	18
IL	Illinois	12,822,739	16.846	16	17	17	17	17	17
OH	Ohio	11,808,848	15.514	15	15	15	16	16	16
GA	Georgia	10,725,274	14.091	14	14	14	14	14	14
NC	North Carolina	10,453,948	13.734	14	14	14	14	14	14
MI	Michigan	10,084,442	13.249	13	13	13	13	13	13
NJ	New Jersey	9,294,493	12.211	12	12	12	12	12	12
VA	Virginia	8,654,542	11.370	11	11	11	11	11	11
WA	Washington	7,715,946	10.137	10	10	10	10	10	10
AZ	Arizona	7,158,923	9.405	9	9	9	9	9	9
MA	Massachusetts	7,033,469	9.240	9	9	9	9	9	9
TN	Tennessee	6,916,897	9.087	9	9	9	9	9	9
IN	Indiana	6,790,280	8.921	9	9	9	9	9	9
MD	Maryland	6,185,278	8.126	8	8	8	8	8	8
MO	Missouri	6,160,281	8.093	8	8	8	8	8	8
WI	Wisconsin	5,897,473	7.748	8	8	8	8	8	8
CO	Colorado	5,782,171	7.596	8	8	8	8	8	8
MN	Minnesota	5,709,752	7.501	8	7	8	8	8	7
SC	South Carolina	5,124,712	6.733	7	7	7	7	7	7
AL	Alabama	5,030,053	6.608	7	7	7	7	7	6
LA	Louisiana	4,661,468	6.124	6	6	6	6	6	6
KY	Kentucky	4,509,342	5.924	6	6	6	6	6	6
OR	Oregon	4,241,500	5.572	6	6	6	6	6	5
OK	Oklahoma	3,963,516	5.207	5	5	5	5	5	5
CT	Connecticut	3,608,298	4.740	5	5	5	5	5	5
UT	Utah	3,275,252	4.303	5	4	4	4	4	4
IA	Iowa	3,192,406	4.194	4	4	4	4	4	4
NV	Nevada	3,108,462	4.084	4	4	4	4	4	4
AR	Arkansas	3,013,756	3.959	4	4	4	4	4	4
MS	Mississippi	2,963,914	3.894	4	4	4	4	4	4
KS	Kansas	2,940,865	3.864	4	4	4	4	4	4
NM	New Mexico	2,120,220	2.785	3	3	3	3	3	2
NE	Nebraska	1,963,333	2.579	3	3	3	3	3	2
ID	Idaho	1,841,377	2.419	3	3	2	2	2	2
WV	West Virginia	1,795,045	2.358	3	2	2	2	2	2
HI	Hawaii	1,460,137	1.918	2	2	2	2	2	2
NH	New Hampshire	1,379,089	1.812	2	2	2	2	2	1
ME	Maine	1,363,582	1.791	2	2	2	2	2	1
RI	Rhode Island	1,098,163	1.443	2	2	2	1	1	1
MT	Montana	1,085,407	1.426	2	2	2	1	1	1
DE	Delaware	990,837	1.302	2	1	1	1	1	1
SD	South Dakota	887,770	1.166	2	1	1	1	1	1
ND	North Dakota	779,702	1.024	1	1	1	1	1	1
AK	Alaska	736,081	0.967	1	1	1	1	1	1
VT	Vermont	643,503	0.845	1	1	1	1	1	1
WY	Wyoming	577,719	0.759	1	1	1	1	1	1
▶	TOTAL	331,108,434	435	435	435	435	435	435	435

Table 1: Apportionments for 2020 Census data

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