Quantum Optimization: from origins to current challenges and future perspectives

Alessia Ciacco[a,*], Lucio Grandinetti[b], Francesca Guerriero[a], Alessandro Salvatore Rizzuto[a]

[a]*Department of Mechanical, Energy and Management Engineering, University of Calabria, Via Pietro Bucci, Rende, 87036, CS, Italy*
[b]*Department of Computer Engineering, Electronics, Modelling and System Sciences, University of Calabria, Via Pietro Bucci, Rende, 87036, CS, Italy*

Abstract

Quantum optimization has emerged at the intersection of quantum computing and operations research, driven by the prospect of addressing challenging combinatorial optimization problems through quantum and quantum-inspired computational approaches. This paper provides a structured review of the field from a past, present, and future perspective. We first trace the evolution of quantum optimization from the early development of quantum annealing and quantum adiabatic approaches to the emergence of gate-based quantum algorithms and quantum-inspired optimization methods. We then examine the current state of the field, focusing on the main algorithmic paradigms, their applications, and the practical challenges that currently limit their effectiveness. Particular attention is devoted to the reformulation of optimization problems, the use of quadratic unconstrained binary models and Ising formulations, embedding and connectivity requirements, noise and limited circuit depth, parameter training, and the computational overhead associated with hybrid quantum-classical workflows. Finally, we discuss the future of quantum optimization and identify research directions that may contribute to its practical development, including decomposition-based hybrid methods, cutting-plane strategies, warm-start techniques, problem-specific quantum circuits, tensor-network approaches, and quantum-inspired dynamical systems. Across these three perspectives, the review emphasizes the importance of evaluating quantum optimization through end-to-end computational workflows rather than isolated algorithmic or hardware metrics. Future progress will depend on the coordinated development of mathematical modelling, optimization algorithms, quantum hardware, and rigorous benchmarking against strong classical methods. This perspective identifies the conditions under which quantum computing may deliver measurable, practically relevant benefits for operations research.

Keywords: Quantum Optimization; Quantum Computing; Quantum Annealing; Quantum Approximate Optimization Algorithm; Quantum-Inspired Optimization; Combinatorial Optimization; Hybrid Quantum-Classical Algorithms.

1. Introduction

Quantum Computing (QC) has emerged as a computational paradigm based on the principles of quantum mechanics, exploiting qubits and phenomena like superposition, entanglement, and interference to offer fundamental alternatives to conventional digital architectures [1, 2]. While interest stems from solving complex tasks such as search, simulation, and cryptography, current processors face structural bottlenecks including noise, finite coherence times, gate errors, and restricted connectivity, defining the Noisy Intermediate-Scale Quantum (NISQ) era [3, 4]. This regime relies on shallow circuits and hybrid procedures, making overall workflow efficiency the true metric of interest rather than qubit count alone.

Optimization is a cornerstone of Operations Research (OR), seeking high-quality decisions among alternatives defined by decision variables, objective functions, and constraints. Arriving in contexts like vehicle routing, scheduling, facility location, and portfolio management, combinatorial growth frequently leads to computational hardness and NP-hardness. Over decades, OR has developed mature methodologies, such as mathematical programming, exact algorithms, branch-and-bound, heuristics, and metaheuristics, that modern solvers optimize extensively. Consequently, any alternative paradigm must be evaluated against these robust classical standards rather than simplified benchmarks.

Quantum Optimization (QO) is the use of QC techniques to solve optimization problems by exploiting quantum phenomena such as superposition, entanglement, and quantum interference. The goal is to identify high-quality or optimal solutions among a potentially large set of feasible solutions, potentially achieving computational advantages over classical optimization methods for certain problem classes. Combinatorial challenges are typically transcribed into Quadratic Unconstrained Binary Optimization (QUBO) forms or equivalent Ising spin-glass representations [5, 6].

Current QO approaches are largely characterized by hybrid quantum–classical workflows, in which quantum processors are integrated with classical optimization and post-processing routines. These workflows introduce several sources of computational overhead, including the incorporation of penalty terms and auxiliary variables, as well as hardware-dependent costs

*Corresponding author
Email addresses: alessia.ciacco@unical.it (Alessia Ciacco), lucio.grandinetti@unical.it (Lucio Grandinetti), francesca.guerriero@unical.it (Francesca Guerriero), alessandrosalvatorerizzuto@gmail.com (Alessandro Salvatore Rizzuto)

associated with circuit compilation, qubit mapping, and routing. Although numerous proof-of-concept studies have demonstrated the technical feasibility of QO across a broad range of OR problems, such demonstrations do not, by themselves, establish a computational advantage over classical approaches [7, 8]. A meaningful assessment of QO therefore requires an end-to-end evaluation of the complete computational workflow, from problem formulation and circuit compilation to quantum execution, sampling, and classical post-processing, against strong state-of-the-art classical baselines.

Motivated by these challenges, this paper provides a structured analysis of QO from an OR perspective, examining the transition from feasibility-oriented demonstrations toward rigorous performance evaluation. Specifically, the paper addresses three main research questions:

1. What are the main computational, hardware, and methodological limitations currently affecting the development of QO for OR?
2. How can quantum and quantum-inspired approaches be classified and evaluated according to their role within optimization processes, rather than solely according to their underlying computational paradigm or hardware architecture?
3. Under which conditions can quantum resources provide measurable computational value beyond state-of-the-art classical optimization methods?

To address these questions, the paper reviews theoretical foundations, computational paradigms, hardware platforms, and benchmarking challenges. It is organized around three complementary perspectives: the historical evolution and theoretical foundations, the present state and limitations of computational paradigms, and the conditions needed for future progress through rigorous, workflow-level evaluation in realistic OR settings.

The remainder of the paper is organized as follows. Section 2 explores the historical origins and theoretical foundations of QO. Section 3 examines the current QO landscape, analyzing the three main computational paradigms across application domains, identifying current hardware, modeling, and algorithmic limitations, and highlighting hybrid integration methodologies. Section 4 discusses future perspectives, detailing industrial technology roadmaps, suitable application domains, emerging professional roles, and essential solution methods required to achieve practical quantum advantage. Finally, Section 5 concludes the paper by summarizing key insights and outlining the evolving role of OR in future quantum computational ecosystems.

2. Past: the Emergence of QO

The development of QO is rooted in the broader development of QC. The first major milestones in QC demonstrated that quantum-mechanical principles could be exploited to perform computational tasks for which classical approaches face fundamental or computational limitations. Among the most influential results are Shor's algorithm for integer factorization and discrete logarithms and Grover's algorithm for unstructured search. Shor showed that factoring and discrete logarithms can be solved on a quantum computer in polynomial time [9], while Grover introduced a quantum search algorithm requiring only $O(\sqrt{N})$ queries for an unstructured search problem [10]. These results provided some of the first concrete demonstrations of the potential computational power of quantum systems and motivated the investigation of quantum algorithms for a broader range of computational problems.

As QC developed, research progressively moved from general computational tasks towards problems with a strong combinatorial and optimization structure. Optimization represented a natural application area because many combinatorial problems involve discrete decision variables and can be formulated in terms of finding low-energy configurations of interacting systems. In particular, the Ising model provided an important mathematical connection between combinatorial optimization and physical energy minimization, allowing a broad class of optimization problems to be represented through spin variables and an associated energy function [11].

From this development, two main paradigms of QC for optimization progressively emerged: Quantum Annealing (QA) and Gate-Based Quantum Optimization (Gate-based QO). Alongside these quantum-computing approaches, Quantum-Inspired Optimization (QIO) developed as a distinct paradigm that draws inspiration from quantum-mechanical principles without relying on quantum hardware. Although all three approaches aim to explore complex solution spaces efficiently, they differ substantially in their computational principles, implementation, and reliance on quantum mechanics.

The three paradigms can be distinguished as follows:

- **QA** is an optimization-oriented QC paradigm in which quantum fluctuations are exploited to explore a complex energy landscape and guide the system towards low-energy configurations corresponding to high-quality solutions. Optimization problems are typically encoded using Ising models or their equivalent QUBO formulations, where the objective function is mapped onto an energy function to be minimized. The quantum evolution then provides the mechanism for exploring the space of candidate configurations, with the annealing process gradually modifying the system so that it evolves towards favorable regions of the solution landscape. In this framework, quantum effects such as tunneling and superposition can contribute to the exploration of the energy landscape, particularly when the landscape contains local minima or narrow energy barriers [12, 13, 14].

- **Gate-based QO** follows a fundamentally different computational model, in which optimization is performed through programmable quantum circuits composed of elementary quantum gates. Rather than relying on a continuous annealing process, the algorithm encodes the optimization problem and manipulates quantum states through a sequence of discrete operations. In the NISQ regime, this paradigm is particularly associated with variational and

hybrid quantum–classical algorithms, where a parameterized quantum circuit is executed on a quantum processor and its output is evaluated according to a problem-specific objective function. A classical optimizer then updates the circuit parameters based on the measured results, and the quantum circuit is repeatedly evaluated until a suitable solution is obtained. This hybrid interaction between quantum and classical computation allows Gate-based QO to operate within the limitations of current quantum hardware while providing a flexible framework for implementing different optimization strategies [15, 16].

- **QIO** represents a third and conceptually distinct direction within the broader landscape of quantum-related optimization. Unlike QA and Gate-based QO, QIO does not require the use of quantum hardware or the execution of genuinely quantum computations. Instead, these methods translate concepts, mathematical representations, or computational mechanisms inspired by quantum mechanics into classical optimization algorithms. Such approaches may exploit quantum-inspired representations of states, amplitudes, interference-like mechanisms, or other principles derived from quantum computation while being implemented on conventional classical or specialized computational architectures. QIO methods have developed in parallel with genuine quantum approaches and can be used as independent optimization techniques for solving complex combinatorial and numerical problems. They are also particularly relevant for assessing the potential advantages of QO, since they provide classical reference methods that can reproduce or approximate certain quantum-inspired mechanisms without requiring access to quantum hardware [17, 18].

Figure 1 presents the technological taxonomy adopted in this review. It separates approaches that rely on genuine quantum computation, namely QA and Gate-based QO, from QIO methods that adopt formulations, search mechanisms, or architectural principles derived from QO while operating without quantum computational resources.

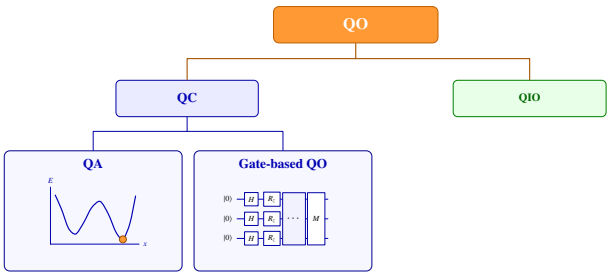


Figure 1: Taxonomy of the computational approaches considered in this review. QO includes genuine quantum-computing methods, represented by QA and Gate-based QO, as well as QIO approaches.

The historical development of these three paradigms can therefore be summarized as the transition from the general foundations of QC to optimization-oriented quantum computation and, in parallel, to quantum-inspired classical approaches.

2.1. QA

The first major approach to emerge was QA. An early formulation of QA was introduced in [19] as an extension of classical simulated annealing in which quantum effects are used to explore the energy landscape and reduce the probability of becoming trapped in local minima. Their approach was applied to the minimization of multidimensional functions, including the search for low-energy configurations of Lennard–Jones clusters. A more formal framework for QA was subsequently developed by [20] through the transverse Ising model. Their approach introduced quantum fluctuations into the annealing process and interpreted the transverse field as a mechanism for inducing transitions between configurations. The aim was to drive the system towards the ground state of the problem Hamiltonian, thereby establishing a direct connection between quantum dynamics and optimization. In parallel with the development of QA, the idea of using adiabatic quantum evolution for solving computational problems was pioneered by [21], who introduced a quantum algorithm designed to solve instances of the satisfiability problem through the adiabatic evolution of a time-dependent Hamiltonian. The central idea was to construct an initial Hamiltonian whose ground state is easy to prepare and evolve it gradually into a final Hamiltonian whose ground state encodes the solution of the computational problem. This concept was subsequently applied directly to randomly generated instances of an NP-complete problem. In their 2001 paper, the authors further investigated the performance of this quantum adiabatic evolution algorithm on small problem instances, providing numerical evidence that the approach could successfully find solutions for the cases that could be simulated. This work established an important connection between Adiabatic Quantum Computation and combinatorial optimization. The relationship between quantum adiabatic evolution and classical optimization was further investigated by [22] through a direct comparison with simulated annealing. Their analysis showed that quantum adiabatic evolution and simulated annealing can behave similarly for certain classes of cost functions, while differences can emerge for other problem structures. This comparison was an early example of the broader question that would subsequently become central to QO: whether the use of quantum resources can provide a computational benefit compared with strong classical optimization methods. The theoretical development of QA was followed by the development of programmable quantum hardware. A major experimental milestone was obtained by [23], who demonstrated QA in an artificial Ising spin system composed of eight superconducting flux qubits with programmable spin–spin couplings. This experiment represented an important transition from theoretical formulations of QA towards programmable superconducting hardware and provided an experimental basis for the subsequent development of dedicated QA processors.

2.2. Gate-based QO

A second branch of QO stems from programmable gate-based QO. Gate-based QO can be implemented using different physical hardware technologies, differing in qubit encoding, control, connectivity, gate fidelity, and scalability:

- **Superconducting qubits:** encode information in superconducting circuits, offering fast gate speeds.
- **Trapped-ion qubits:** use internal electronic states of electromagnetic-confined ions, providing high gate fidelities.
- **Topological qubits:** aim for fault tolerance through topologically protected states.
- **Silicon spin qubits:** utilize electron/nuclear spins in semiconductors, leveraging existing industrial fabrication.
- **Photonic QC:** encodes quantum information in photons and can use measurement-based approaches to perform quantum computation.

Hardware platforms are distinct from the software paradigms they run. A major milestone in this domain was the introduction of Variational Quantum Algorithms (VQAs), which optimize parameterized quantum circuits via a classical feedback loop. [24] introduced the Variational Quantum Eigensolver (VQE) in 2014 for eigenvalue estimation, establishing the hybrid quantum–classical framework central to near-term QO. In the same year, [25] introduced the Quantum Approximate Optimization Algorithm (QAOA), directly applying gate-based hybrid circuits to combinatorial optimization. [26] later explored QAOA's complexity and classical simulability in shallow circuits. The paradigm was further expanded by [27] with the Quantum Alternating Operator Ansatz, generalizing mixing operators to incorporate hard problem constraints. Techniques such as Quantum Imaginary Time Evolution [28] further expanded gate-based QO toward broader ground-state search and energy minimization strategies on noisy hardware.

2.3. QIO

A third research direction developed in parallel with the genuinely quantum approaches: QIO. Unlike QA and Gate-based QO, QIO does not require a quantum processor. Instead, it translates concepts originating from quantum mechanics, QO, or quantum computational models into classical or specialized computational procedures. One of the early examples of this direction was introduced by [29] through their work on Quantum-Inspired Genetic Algorithms. The authors proposed a classical evolutionary method in which concepts and principles inspired by quantum mechanics were used to construct an alternative genetic algorithm, including an application to the travelling salesperson problem. This work represents an early foundation of the quantum-inspired family and illustrates the fundamental idea that quantum concepts can be transferred into classical computational methods without requiring quantum hardware. The quantum-inspired direction subsequently evolved towards specialized optimization mechanisms derived from physical models. A representative example is Simulated Bifurcation and related approaches. [30] proposed a high-performance combinatorial optimization method based on classical mechanics, modifying the previously introduced simulated bifurcation approach and exploiting massively parallel classical computation. The method demonstrated that mechanisms inspired by quantum bifurcation can be reformulated into classical dynamical systems and used to solve large combinatorial optimization problems.This development is particularly relevant because it demonstrates that quantum-inspired approaches can constitute independent computational paradigms rather than merely simplified simulations of quantum hardware. They can exploit specialized classical architectures and alternative dynamical mechanisms to explore large combinatorial solution spaces, while remaining fully classical from the hardware perspective. More recently, the quantum-inspired direction has expanded beyond quantum-annealing-inspired dynamical systems towards generative approaches. [31] investigated the use of classical and quantum generative models for combinatorial optimization, including tensor-network-based approaches. This represents a broader interpretation of QIO, in which ideas originating from quantum information and quantum generative models can be transferred to classical optimization workflows. A related development in the gate-based field was the use of Quantum Imaginary Time Evolution for finite-temperature systems. [32] extended the QITE framework to the computation of finite-temperature static and dynamical properties of spin systems, demonstrating the approach on digital quantum hardware. Although this work is primarily concerned with quantum simulation rather than combinatorial optimization, it illustrates the broader development of quantum variational and imaginary-time techniques that emerged during the NISQ period.

The optimal choice among QA, Gate-based QO, and QIO is fundamentally dictated by mathematical problem structure, variable constraints, and latency requirements. Table 1 provides a structured synthesis of their domain alignments.

2.4. Past Literature Reviews

To further explore the field, several key reviews offer complementary perspectives on QO foundations, paradigms, applications, and current bottlenecks.

A broad overview is provided by [33], who analyze computational complexity, algorithmic building blocks, and conditions for quantum advantage. [34] complement this by examining quantum algorithms across practical domains such as finance, logistics, and production planning.

Regarding QA, [35] supply foundational references, while [36] examine recent hardware developments and problem formulations for combinatorial optimization.

For gate-based approaches, [37] review VQAs and their hybrid structures, while [38] provide a comprehensive analysis of the QAOA, its variants, and noise impacts. [39] further connect problem formulation with near-term algorithm implementation.

In QIO, [40] review advances focusing on QUBO and PUBO formulations, and [41] present a scientometric analysis of quantum-inspired metaheuristics.

3. Present: the current QO landscape

The current landscape of QO is characterized by the coexistence of three main computational paradigms: QA, Gate-based QO, and QIO. Although these approaches differ in their

Feature	QA	Gate-based QO	QIO
Physical Quantum System	✓	✓	✗
Universal Computation	✗	✓	✗
Native Representation	Ising / QUBO	Cost Hamiltonian / QAOA	Ising / QUBO
Search Mechanism	Analog / Fluctuations	Parameterized Interference	Stochastic / Bifurcation
Constraint Handling	Penalties	Penalties / Mixers	Penalties
Critical Resource	Connectivity / Chains	Circuit Depth	Memory / Precision
Hardware Bottleneck	Analog Noise / Embedding	Decoherence / Gate Noise	Memory Bandwidth
Documented Scale	$> 10^3$ Qubits	Small (NISQ)	$> 10^4$ Variables
Maturity Level	Commercial	Early-Stage / NISQ	Commercial
Primary OR Use	Pure Binary Combinatorics (VRP, Packing, Location)	Multi-Body Simulation and Continuous Constraints	Real-Time, Zero-Latency and Deterministic Edge
Optimality Guarantee	✗	✗	✗

Table 1: Structural comparison of the main computational paradigms. Scale and maturity reflect current hardware limits and do not imply overall performance superiority.

computational principles and in the role of quantum mechanics, current research increasingly considers them within broader optimization workflows rather than as isolated quantum solvers.

This section examines the current QO landscape from four complementary perspectives: the application of the main computational paradigms across different optimization domains, the current challenges and limitations affecting their practical use, and the methodological strategies developed to address these limitations.

3.1. Quantum Paradigms Across Application Domains

QO has been investigated across several OR domains, with different computational paradigms being adopted depending on the available hardware, algorithmic formulation, and characteristics of the application. The three QO paradigms have been investigated across a broad range of practical optimization domains, covering a wide range of applications across different problem structures and operational contexts.

- **QA:** has been explored across a wide range of combinatorial optimization problems, including scheduling [42, 43, 44], routing [45, 46, 47], facility location [48, 49], pricing [50, 51], and packing [52, 53]. Beyond general problem formulations, real-world case studies have also been investigated across diverse application domains. Notable examples include urban traffic flow optimization [54], complex scheduling problems in healthcare and educational timetabling [55, 56], large-scale operational planning [57], and facility planning in the public sector [58].

- **Gate-based QO:** approaches have been explored for combinatorial optimization problems arising in scheduling [59, 60, 61], routing [62, 63, 64], portfolio optimization [65, 66, 67], graph optimization [68, 69, 70], resource allocation [71, 72, 73], and energy systems [74, 75, 76]. These approaches generally encode the optimization problem into a parameterized quantum circuit and employ a classical optimizer to determine the circuit parameters. Beyond theoretical formulations and benchmark instances, QAOA has also been investigated in real-world and industrially relevant settings. Examples include industrial scheduling [77, 78], portfolio optimization based on real financial data [79, 80], energy resource management [81, 82], and logistics and transportation applications [83, 84, 85].

- **QIO:** has been applied to a broad range of OR applications, including scheduling [86, 87, 88], routing [89], portfolio selection [90, 91, 92], production planning [93, 94, 95], and resource allocation [96, 97, 98]. Beyond these general applications, QIO has also been investigated in several real-world and industrial case studies. Examples include industrial scheduling [99], production planning [95], logistics and delivery planning [100], energy management and microgrid optimization [101], and financial portfolio optimization [91].

3.2. Current Difficulties and Limitations

Despite the increasing number of QO algorithms and applications, current QO remains constrained by limitations arising at different levels of the computational workflow. These limitations are not restricted to quantum hardware, but also involve problem formulation, algorithmic trainability, model size, and the evaluation of computational performance.

- **At the hardware level:** current quantum processors are characterized by limited qubit connectivity, noise, gate errors, finite coherence times, and non-negligible compilation and embedding costs. These constraints directly affect the size and structure of optimization models that can be executed on quantum devices.

- **At the modelling level:** the translation of an OR problem into a quantum-compatible representation can introduce additional binary variables, auxiliary variables, dense interactions, and penalty terms. Consequently, a mathematically valid QUBO formulation may still be unsuitable for execution on current quantum hardware.

- **At the algorithmic level:** VQAs introduce further challenges related to circuit depth, parameter initialization, ansatz design, classical optimization, and trainability. These factors can substantially affect the quality and stability of the obtained solutions.

- **At the evaluation level:** demonstrating that a quantum method can solve an optimization instance is not sufficient to establish a computational advantage. Meaningful evaluation requires comparisons with strong classical methods and consideration of the complete computational workflow, including preprocessing, encoding, quantum execution, sampling, and classical post-processing.

3.3. OR methodologies for QO

OR methodologies play an increasingly important role in addressing the limitations of current quantum hardware. Classical optimization techniques such as problem reduction, decomposition, and iterative solution strategies can help adapt complex optimization problems to the restricted computational resources of quantum processors, leading to the development of increasingly integrated and problem-specific hybrid approaches. Because current quantum hardware is severely resource-constrained, bridging abstract OR problems with quantum processors requires paradigm-specific reduction and decomposition techniques. Rather than applying a single uniform workflow, optimization strategies are tailored to the architectural and mathematical nature of each computational paradigm: QA, Gate-Based optimization, and QIO.

3.3.1. QA

To fit large-scale OR problems onto limited hardware graphs with strict qubit connectivity and embedding limits, rigorous classical reduction and decomposition are mandatory:

- **Preprocessing and Model Reduction:** several studies have proposed preprocessing and model reduction techniques aimed at reducing the size and complexity of optimization models before their execution on quantum hardware. These approaches include variable fixing and persistency detection, probing, decomposition, and other reduction techniques that can simplify QUBO formulations and reduce the number of variables required for QO [102, 103, 104].

- **Column Generation and Decomposition:** for problems with an exponentially large number of variables column generation can be integrated with QA. Classical master problems generate promising columns or variables iteratively, while a quantum annealer can be used to solve the resulting pricing or separation subproblems [105, 106]. Similarly, decomposition methods such as Benders decomposition can separate a discrete master problem, which can be formulated as a QUBO and solved using QA, from continuous subproblems that are handled classically [107, 108, 109].

- **Cutting-Plane / Iterative Methods:** rather than encoding a massive or complex system of constraints all at once into a single quantum model, which may quickly exceed current hardware capacity, cutting-plane approaches operate iteratively. Classical algorithms solve a relaxation and identify violated constraints, which are then added dynamically to generate updated subproblems for the quantum annealer [110, 111]. This iterative strategy can reduce the size of the quantum models while progressively tightening the feasible region and improving the quality of the resulting solutions.

3.3.2. Gate-based QO

Gate-based optimization relies on programmable circuits and variational algorithms. Because hardware noise, limited qubit counts, and barren plateaus severely limit scalability and circuit depth, optimization methods focus on reducing circuit complexity and improving trainability:

- **Warm-Start Strategies:** classical relaxations (e.g., linear programming or semidefinite programming relaxations) are used to initialize quantum parameters or states, effectively providing a more informed starting point for the variational search. Warm-start strategies have been investigated for QAOA in problems such as Max-Cut and portfolio optimization, where solutions obtained from classical relaxations are incorporated into the initial quantum state and, in some cases, into the mixer Hamiltonian [112, 113, 114].

- **Problem-Specific Ansätze and Custom Mixers:** rather than using generic circuits, problem structure and constraints can be exploited to design custom mixing operators and problem-specific ansätze that restrict the quantum evolution to the feasible subspace and can reduce implementation overhead. Such constructions have been developed for a variety of constrained optimization problems and can incorporate problem-specific symmetries or feasibility-preserving operations into the QAOA circuit [27, 115, 116].

- **Hybrid Iterative Refinement:** objective functions or expectation values are evaluated on the quantum circuit, while classical optimizers iteratively update the variational parameters, balancing quantum state exploration with classical optimization. This hybrid quantum-classical structure is a defining feature of VQAs and has been employed to iteratively refine circuit parameters based on measurements obtained from the quantum processor [24, 25, 116].

3.3.3. QIO

QIO bypasses physical quantum hardware limitations by utilizing classical algorithms and specialized architectures inspired by quantum mechanics (such as tensor networks, simulated bifurcation, or coherent Ising machine simulations):

- **Tensor-Network Methods and Scalability:** provide a classical framework for representing and optimizing structured combinatorial problems, particularly when the underlying interactions or correlations can be captured by low-dimensional tensor representations. These approaches avoid the physical-qubit connectivity and hardware-error constraints of quantum processors while enabling the treatment of relatively large optimization instances [117, 118, 119].
- **Large-Scale Classical Decomposition Integration:** can be integrated with classical decomposition and local-search strategies to address large combinatorial optimization problems by dividing the original problem into smaller subproblems that can be solved and recombined iteratively. Such approaches have been investigated for large-scale QUBO formulations using decomposition schemes, parallel subproblem solving, local search, and population-based mechanisms [120, 121, 99].
- **Benchmarking and Algorithmic Validation:** provide useful reference points for evaluating whether quantum-inspired search mechanisms and algorithmic modifications, such as decomposition, variable reduction, or alternative problem encodings, provide measurable benefits relative to established classical optimization methods. Several studies systematically benchmark quantum-inspired solvers against classical algorithms in terms of solution quality, computational time, and scalability [121, 122, 118, 123].

4. The future: future perspectives in QO

The future development of QO is transitioning from proof-of-concept demonstrations to the systematic establishment of measurable computational advantage over classical baselines. As highlighted in recent literature, quantum advantage must be evaluated at a system-wide level, taking into account end-to-end workflows including mathematical formulation, variable embedding, execution overhead, and classical post-processing.

This section explores the evolutionary outlook of the three main QO paradigms, QA, Gate-based QO, and QIO, by examining industry roadmaps, promising application domains, and critical future methodologies.

4.1. Industrial Roadmaps and Technology Trajectories

The future development of QO is closely connected to the technology roadmaps pursued by the main industrial and research actors in the QC ecosystem. These roadmaps differ substantially across computational paradigms, reflecting different hardware architectures, scalability strategies, error-correction approaches, and integration models. In particular, current trajectories range from the continued scaling of dedicated quantum annealing systems, to the development of increasingly reliable gate-based processors and fault-tolerant architectures, and to the advancement of quantum-inspired computational systems capable of addressing large-scale optimization problems on classical hardware. The following discussion summarizes the main announced technology trajectories of the main actors associated with QA, Gate-based QO, and QIO.

4.1.1. QA

QA is currently supported by a relatively mature industrial ecosystem, with **D-Wave** representing the main commercial actor pursuing dedicated annealing hardware. The future trajectory of QA is primarily concerned with increasing the scale and connectivity of quantum annealing processors, improving coherence and analog control, reducing embedding overhead, and strengthening the integration between quantum processors and classical hybrid solvers.

D-Wave's current roadmap extends the Advantage2 architecture while simultaneously expanding the company's activities towards gate-model QC. The Advantage2 system is designed around a higher-connectivity architecture and is intended to reduce the overhead associated with embedding large and densely connected optimization problems. In January 2026, D-Wave reported a 314% year-over-year increase in usage of Advantage2 systems and announced additional developments in hybrid quantum-classical optimization, including the integration of machine-learning models into optimization workflows [124]. These developments indicate a transition from quantum annealing as a standalone computational resource towards increasingly integrated optimization platforms.

An additional direction concerns the scaling of quantum annealing hardware itself. D-Wave has announced plans for a scalable input/output architecture targeting systems with up to approximately 100000 qubits, while attempting to limit the increase in the number of required I/O connections relative to current systems. Such developments are particularly relevant for optimization because increasing qubit count alone does not guarantee that larger optimization models can be efficiently embedded. Connectivity, chain length, embedding overhead, and classical control therefore remain central components of the future QA roadmap [125].

D-Wave is also expanding beyond annealing through a parallel gate-model roadmap. The company has announced intermediate milestones including a 17-physical-qubit system in 2026, a 49-qubit system in 2027, and a 181-qubit system in 2028, with the latter intended to provide a scalable architecture for fault-tolerant development [126]. This dual-platform strategy is particularly relevant to QO because it illustrates the increasing convergence between specialized annealing architectures and more general-purpose gate-based QC.

4.1.2. Gate-based QO

The future trajectory of Gate-based QO involves a broad ecosystem of companies pursuing different hardware technologies, including superconducting qubits, trapped ions, topological qubits, silicon spin qubits, and other emerging architectures. Despite these differences, the main industrial roadmaps increasingly converge towards higher-fidelity processors, logical qubits, error correction, and fault-tolerant QC. The main trajectories can be summarized as follows:

- **IBM.** IBM is pursuing a roadmap based on superconducting processors, modular quantum systems, and quantum-centric supercomputing. Its Nighthawk processor provides 120 programmable qubits and has demonstrated the execution of circuits exceeding 7500 gates. Looking ahead, IBM targets the deployment of IBM Quantum Starling in 2029, with approximately 200 logical qubits and the capability to execute around 100 million quantum gates. The subsequent Blue Jay architecture is planned to scale towards approximately 2000 qubits and one billion quantum operations [127, 128]. In parallel, IBM is developing a quantum-centric supercomputing architecture in which quantum processors are integrated with CPUs and GPUs, supporting heterogeneous workflows in which quantum resources operate as specialized computational components [129].

- **Google.** Google Quantum AI is focusing on superconducting quantum processors and, in particular, on quantum error correction as the main route towards fault-tolerant QC. The Willow processor demonstrated below-threshold error correction, providing an important step towards the development of reliable logical qubits [130]. The longer-term roadmap focuses on progressively increasing the number and lifetime of logical qubits and ultimately developing a large-scale error-corrected quantum computer capable of executing computationally demanding algorithms [131]. For QO, this trajectory could enable deeper and more reliable optimization circuits than those currently feasible in the NISQ regime.

- **Microsoft.** Microsoft is pursuing a topological-qubit strategy based on Majorana states, with the objective of developing inherently more robust qubits and reducing the overhead associated with quantum error correction. Its roadmap progresses from foundational physical-qubit systems towards resilient logical qubits and, ultimately, scalable quantum supercomputers. The development of Majorana 2 represents a further step in this direction, while the company's roadmap targets a practical quantum computer by 2029 [132, 133]. For Gate-based QO, the relevance of this trajectory lies in the possibility of supporting deeper variational circuits and more computationally demanding optimization algorithms once fault-tolerant operation becomes available.

- **IonQ.** IonQ is developing trapped-ion quantum computers, with a long-term objective of achieving scalable, fault-tolerant QC. The company's technology roadmap focuses on improving system scalability, modularity, and quantum error correction, with the aim of progressively overcoming the limitations of current quantum processors [134]. In September 2026, IonQ announced its Superion 256 platform as part of this longer-term development trajectory [135]. The company is also exploring the integration of quantum processors with high-performance computing infrastructure through its collaboration with NVIDIA, reflecting the broader interest in hybrid quantum-classical computing architectures [136]. For Gate-based QO, these developments may facilitate the implementation of increasingly complex variational circuits and hybrid optimization workflows. However, their practical benefits must be established through end-to-end benchmarking against competitive classical optimization methods.

- **Quantinuum.** Quantinuum is pursuing trapped-ion QC with a strong focus on high-fidelity operations and fault-tolerant architectures. Its current hardware roadmap includes the 98-qubit Helios system, while longer-term objectives include scaling towards thousands of physical qubits and hundreds of logical qubits, with universal fault-tolerant QC targeted for 2030 [137]. Future developments also emphasize the integration of quantum hardware, software, and error-correction technologies, creating an environment in which increasingly complex variational and hybrid optimization algorithms can be executed with improved reliability [138].

- **Rigetti.** Rigetti is developing superconducting processors through a modular, chiplet-oriented architecture. Its 108-qubit Cepheus-1-108Q system represents a step towards scaling superconducting processors while maintaining high gate fidelity and fast operation [139]. The company's future activities focus on increasing processor scale, improving manufacturing capabilities, and expanding customer access to quantum systems. These developments are relevant to QO because modular architectures may provide a route towards larger processors while maintaining the control and fabrication requirements needed for increasingly complex gate-based optimization workflows [140].

- **Amazon Web Services.** AWS is pursuing a cloud-centered strategy that combines quantum hardware access with classical cloud and high-performance computing resources. Through Amazon Braket, different quantum technologies can be incorporated into hybrid computational workflows. A major future direction is the integration of fault-tolerant QC into the cloud: AWS has announced collaboration with QuEra to provide access to the Libra fault-tolerant system through Amazon Braket, with initial applications targeted for 2028 and a planned scale of hundreds of logical qubits and approximately one million quantum operations [141]. In parallel, AWS is developing its own superconducting cat-qubit architecture through the Ocelot program, with particular emphasis on scalable error correction [142]. This ecosystem-oriented strategy is particularly relevant to QO because it facilitates the integration of quantum processors with classical optimization and HPC resources.

- **Fujitsu.** Fujitsu is pursuing a multi-stage superconducting quantum-computing roadmap in collaboration with RIKEN and other Japanese research institutions. Following the development of a 256-qubit system, the company is targeting a 1000-qubit processor and, in the longer

term, a superconducting system exceeding 10000 qubits by fiscal year 2030 [143]. Fujitsu is also investigating diamond-spin QC, with a prototype multi-module system targeted for 2027 and longer-term objectives involving hundreds of logical qubits [144]. These activities indicate a broader strategy combining hardware scaling, alternative qubit technologies, and system-level integration.

- **Intel and Hitachi.** Intel and Hitachi are investigating silicon-based QC as a potential route towards scalable and manufacturable quantum processors. Intel's Tunnel Falls platform is based on silicon spin qubits and leverages semiconductor manufacturing technologies [145]. In parallel, Hitachi and Intel have established a joint R&D program with AIST targeting the development of technologies for 100-qubit silicon quantum processors and subsequent three-dimensional integration towards the 1000-qubit scale. Hitachi also targets cloud-accessible experimental infrastructure and error-corrected silicon quantum systems at the 100- and 1000-qubit levels by 2028 and 2030, respectively [146]. These roadmaps emphasize the importance of semiconductor fabrication, packaging, interconnects, and cryogenic integration in the future scaling of Gate-based QO.

- **PsiQuantum.** PsiQuantum is pursuing a photonic approach to fault-tolerant QC, encoding quantum information in single photons and exploiting integrated silicon photonics and semiconductor manufacturing processes to support large-scale system integration [147]. In 2025, the company introduced Omega, a photonic chipset integrating key components for generating, manipulating, detecting, and networking photonic qubits. Experimental results demonstrated high-fidelity quantum operations and chip-to-chip interconnects, although the reported performance metrics do not by themselves establish the feasibility of a complete, utility-scale quantum computer [148]. The company's long-term roadmap focuses on integrating large numbers of photonic chips into modular, fault-tolerant QC systems. For Gate-based QO, photonic QC represents an alternative hardware approach that may eventually support complex quantum algorithms and hybrid optimization workflows. However, the realization of practically useful, large-scale systems remains a substantial engineering challenge, and any potential advantage for optimization over state-of-the-art classical methods will require rigorous end-to-end benchmarking.

4.1.3. QIO

The future trajectory of QIO differs fundamentally from that of physical QC because it does not depend on scaling quantum hardware or achieving fault tolerance. Instead, the main development directions concern the scalability of classical architectures, specialized accelerators, algorithmic improvements, and the integration of quantum-inspired optimization with artificial intelligence and real-time decision systems.

Toshiba represents one of the most prominent industrial actors in this area through its Simulated Bifurcation Machine (SBM) and the associated SQBM+ optimization platform. In 2026, Toshiba introduced a third-generation simulated bifurcation algorithm that modifies the control of the bifurcation parameters to improve the ability of the optimization process to escape local optima. On a fully connected 2000-spin Ising problem, the company reported a success probability close to 100% and performance improvements of approximately 100× relative to previous generations [149].

More recent developments have extended this direction towards dynamic optimization. Toshiba has developed a quantum-inspired framework that combines SBM with optimization-control AI and Ising-model compression. The framework is designed to adapt algorithm parameters and computational resources according to the characteristics of dynamically changing optimization problems, with applications including wireless communication and automotive environments. This represents an important evolution of QIO from static combinatorial optimization towards continuous and real-time decision-making.

Toshiba has also announced further scaling of its SQBM+ platform, with support for optimization problems involving up to approximately 1 billion variables. The company identifies applications in finance, logistics, drug discovery, and other large-scale optimization settings. These developments illustrate a different form of scalability from that pursued by quantum hardware: instead of increasing the number of physical qubits, QIO aims to increase the size of the optimization problems that can be processed using classical or specialized architectures.

Overall, the roadmaps of the main actors reveal three complementary technological trajectories. QA is primarily evolving through increased connectivity, system scale, and hybrid solver integration; Gate-based QO is moving towards higher-fidelity processors, logical qubits, error correction, and fault-tolerant architectures; while QIO is progressing through specialized classical hardware, algorithmic scaling, and increasingly adaptive optimization systems. These trajectories suggest that future QO systems are likely to remain heterogeneous, with the appropriate computational approach depending on problem structure, hardware availability, latency requirements, and the degree of quantum resources required by the target application.

4.2. Methodological Strategies for Future QO Development

To bridge abstract OR models with resource-constrained quantum hardware, algorithm design must focus on systematic decomposition and hybrid integration.

- **Preprocessing and Model Reduction:** preprocessing techniques aim to reduce the size and complexity of optimization models before their execution on quantum or quantum-inspired architectures. These approaches include variable fixing, persistency detection, probing, presolving, and other reduction procedures that eliminate redundant variables or constraints and simplify the resulting problem representation. Such techniques are particularly important for resource-constrained quantum hardware, where reducing the number of variables and interactions can directly alleviate qubit, connectivity, and embedding requirements.

- **Hybrid Quantum–Classical Decomposition:** for large-scale OR problems exceeding available quantum resources, classical decomposition methods can separate the original problem into smaller components that can be handled through complementary computational resources. In Column Generation, for example, a classical master problem iteratively generates promising variables, while a quantum processor can be employed to solve selected pricing subproblems. Similarly, Benders Decomposition can separate discrete binary master problems, potentially formulated as QUBO and solved through QA, from continuous subproblems handled classically.
- **Iterative Cutting-Plane Frameworks:** rather than embedding dense constraint systems into a single QUBO formulation through large penalty terms, cutting-plane methodologies introduce constraints iteratively. A classical algorithm first solves a relaxed problem and identifies violated constraints, which are then added as cuts to progressively refine the feasible region. This approach can reduce the size and complexity of the quantum subproblems while avoiding the need to encode all constraints simultaneously.
- **Warm-Starting and Problem-Specific Custom Mixers:** warm-start strategies exploit information obtained from classical relaxations to initialize QO procedures from more informed states or parameter configurations. In variational algorithms such as QAOA, classical solutions or relaxations can therefore provide a starting point for the QO process. In parallel, problem-specific mixers can replace generic mixing operators and restrict the quantum evolution to feasible or structurally relevant regions of the solution space, potentially reducing circuit complexity and improving trainability.
- **Tensor Networks and Dynamical Bifurcation Systems:** within QIO, tensor-network methods provide classical representations capable of compressing structured high-dimensional solution spaces, while avoiding the physical noise and connectivity constraints of quantum processors. In parallel, dynamical approaches such as Simulated Bifurcation exploit nonlinear classical dynamics inspired by quantum or physical systems to explore large combinatorial solution spaces. These approaches represent complementary directions for improving the scalability of quantum-inspired optimization without requiring physical quantum resources.

5. Conclusions

QO has rapidly evolved from a predominantly theoretical research area into a dynamic, interdisciplinary field at the intersection of OR and QC. In its foundational stages, research focused heavily on proving computational feasibility, demonstrating that classical combinatorial optimization problems could be mapped onto Ising or QUBO models and executed on early quantum hardware. As both hardware technologies and optimization methodologies have matured, the fundamental paradigm has shifted. The central question is no longer whether a problem can be solved on a quantum device, but whether quantum resources can deliver a measurable, reproducible, and practically relevant computational advantage over competitive classical baselines.

This review demonstrates that QO cannot be analyzed as a monolithic computing model. Instead, it represents a multifaceted ecosystem encompassing QA, Gate-Based VQAs, and an increasingly class of QIO methods. These paradigms differ substantially across computational mechanics, scalability, noise sensitivity, and hardware readiness. Consequently, evaluating performance through isolated metrics, such as raw QPU execution time or qubit count, is fundamentally insufficient. The primary bottlenecks to practical QO extend far beyond physical quantum processors, as significant overheads and failure modes emerge throughout the full optimization pipeline, including mathematical reformulation, variable encoding overheads, constraint penalty calibration, embedding chain breaks, barren plateaus, and classical-quantum communication latency. Practical quantum advantage must therefore be assessed as an emergent property of the overall computational system, integrating mathematical formulation, algorithmic design, physical architecture, classical co-processing, and experimental protocols.

A major contribution of this analysis is detailing the evolution of the relationship between OR and QC. Initially relegated to a source of benchmark application instances, OR is now positioned at the methodological core of QO research. Techniques from Mathematical Programming, such as decomposition frameworks like Benders Decomposition and Column Generation, cutting-plane cuts, primal-dual bounds, and systematic relaxations, are indispensable for bridging abstract OR models with resource-constrained quantum hardware. Rather than replacing classical algorithms, quantum processors are most effectively integrated as specialized subroutines, pricing workers, or heuristics within master classical OR frameworks.

Looking ahead, progress in QO will not depend on hardware scaling alone. True technological readiness requires a holistic hardware-software-algorithm co-design. Mathematical formulations must naturally reflect hardware connectivity topologies to minimize graph-embedding distortions, custom mixers and warm-start protocols must be developed to restrict state evolution strictly to feasible subspaces, and standardized benchmarking suites must evaluate total system runtime, time-to-solution, energy consumption, and solution quality against state-of-the-art classical solvers. Ultimately, the future of QO will be defined not by whether quantum hardware can operate independently, but by how seamlessly quantum resources can be embedded within heterogeneous computing ecosystems. In this emerging paradigm, advances in mathematical modeling, decomposition techniques, and rigorous evaluation methodologies will be fully as decisive as breakthroughs in physical quantum hardware.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] S. Gill, A. Kumar, H. Singh, M. Singh, K. Kaur, M. Usman, R. Buyya, Quantum computing: A taxonomy, systematic review and future directions, Software: Practice and Experience 52 (2020) 114 – 66. doi:10.1002/spe.3039.

[2] L. Gyongyosi, S. Imre, A survey on quantum computing technology, Comput. Sci. Rev. 31 (2019) 51–71. doi:10.1016/j.cosrev.2018.11.002.

[3] J. Preskill, Quantum computing in the nisq era and beyond, Quantum 2 (2018) 79.

[4] P. Murali, J. M. Baker, A. Javadi-Abhari, F. Chong, M. Martonosi, Noise-adaptive compiler mappings for noisy intermediate-scale quantum computers, Proceedings of the Twenty-Fourth International Conference on Architectural Support for Programming Languages and Operating Systems (2019). doi:10.1145/3297858.3304075.

[5] F. Glover, G. Kochenberger, R. Hennig, Y. Du, Quantum bridge analytics i: A tutorial on formulating and using qubo models, Annals of Operations Research 314 (2022) 141–183. doi:10.1007/s10479-022-04634-2.

[6] F. Glover, G. Kochenberger, M. Ma, Y. Du, Quantum bridge analytics ii: Qubo-plus, network optimization and combinatorial chaining for asset exchange, 4OR 18 (2020) 387–417.

[7] T. Koch, D. E. B. Neira, Y. Chen, G. Cortiana, D. J. Egger, R. Heese, N. N. Hegade, A. G. Cadavid, R. Huang, T. Itoko, T. Kleinert, P. M. Xavier, N. Mohseni, J. A. Montañez-Barrera, K. Nakano, G. Nannicini, C. O'Meara, J. Pauckert, M. Proissl, A. Ramesh, M. Schicker, N. Shimada, M. Takeori, V. Valls, D. V. Bulck, S. Woerner, C. Zoufal, The quantum optimization benchmarking library, Nature Computational Science 6 (2025) 653 – 671. doi:10.1038/s43588-026-00991-1.

[8] A. Abbas, A. Ambainis, B. Augustino, A. Bartschi, H. Buhrman, C. Coffrin, G. Cortiana, V. Dunjko, D. J. Egger, B. Elmegreen, N. Franco, F. Fratini, B. Fuller, J. Gacon, C. Gonciulea, S. Gribling, S. Gupta, S. Hadfield, R. Heese, G. Kircher, T. Kleinert, T. Koch, G. Korpas, S. Lenk, J. Mareček, V. Markov, G. Mazzola, S. Mensa, N. Mohseni, G. Nannicini, C. O'Meara, E. P. Tapia, S. Pokutta, M. Proissl, P. Rebentrost, E. Sahin, B. C. B. Symons, S. Tornow, V. Valls, S. Woerner, M. Wolf-Bauwens, J. Yard, S. Yarkoni, D. Zechiel, S. Zhuk, C. Zoufal, Challenges and opportunities in quantum optimization, Nature Reviews Physics 6 (2023) 718 – 735. doi:10.1038/s42254-024-00770-9.

[9] P. W. Shor, Algorithms for quantum computation: discrete logarithms and factoring, in: Proceedings 35th annual symposium on foundations of computer science, Ieee, 1994, pp. 124–134.

[10] L. K. Grover, A fast quantum mechanical algorithm for database search, in: Proceedings of the twenty-eighth annual ACM symposium on Theory of computing, 1996, pp. 212–219.

[11] A. Lucas, Ising formulations of many np problems, Frontiers in Physics 2 (2014) 5. doi:10.3389/fphy.2014.00005.

[12] P. Hauke, H. Katzgraber, W. Lechner, H. Nishimori, W. Oliver, Perspectives of quantum annealing: methods and implementations, Reports on Progress in Physics 83 (2019). doi:10.1088/1361-6633/ab85b8.

[13] A. Rajak, S. Suzuki, A. Dutta, B. Chakrabarti, Quantum annealing: an overview, Philosophical Transactions of the Royal Society A 381 (2022). doi:10.1098/rsta.2021.0417.

[14] S. Kim, S.-W. Ahn, I.-S. Suh, A. W. Dowling, E. Lee, T. Luo, Quantum annealing for combinatorial optimization: a benchmarking study, npj Quantum Information 11 (2025). doi:10.1038/s41534-025-01020-1.

[15] M. Cerezo, A. Arrasmith, R. Babbush, S. Benjamin, S. Endo, K. Fujii, J. McClean, K. Mitarai, X. Yuan, L. Cincio, P. J. Coles, Variational quantum algorithms, Nature Reviews Physics 3 (2020) 625 – 644. doi:10.1038/s42254-021-00348-9.

[16] K. Blekos, D. Brand, A. Ceschini, C.-H. Chou, R.-H. Li, K. Pandya, A. Summer, A review on quantum approximate optimization algorithm and its variants, Physics Reports (2023). doi:10.1016/j.physrep.2024.03.002.

[17] F. S. Gharehchopogh, Quantum-inspired metaheuristic algorithms: comprehensive survey and classification, Artificial Intelligence Review 56 (2022) 5479–5543. doi:10.1007/s10462-022-10280-8.

[18] Y. Du, H. Wang, R. Hennig, A. Hulandageri, G. Kochenberger, F. W. Glover, New advances for quantum-inspired optimization, Int. Trans. Oper. Res. 32 (2023) 6–17. doi:10.1111/itor.13420.

[19] A. Finnila, M. A. Gomez, C. Sebenik, C. Stenson, J. D. Doll, Quantum annealing: A new method for minimizing multidimensional functions, Chemical Physics Letters 219 (1994) 343–348. doi:10.1016/0009-2614(94)00117-0.

[20] T. Kadowaki, H. Nishimori, Quantum annealing in the transverse ising model, Physical Review E 58 (1998) 5355–5363. doi:10.1103/PhysRevE.58.5355.

[21] E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, D. Preda, A quantum adiabatic evolution algorithm applied to random instances of an np-complete problem, Science 292 (2001) 472–475. doi:10.1126/science.1057726.

[22] E. Farhi, J. Goldstone, S. Gutmann, Quantum adiabatic evolution algorithms versus simulated annealing, arXiv preprint quant-ph/0201031 (2002).

[23] M. W. Johnson, M. H. S. Amin, S. Gildert, T. Lanting, F. Hamze, N. Dickson, R. Harris, A. J. Berkley, J. Johansson, P. Bunyk, E. M. Chapple, C. Enderud, J. P. Hilton, K. Karimi, E. Ladizinsky, N. Ladizinsky, T. Oh, I. Perminov, C. Rich, M. C. Thom, E. Tolkacheva, C. J. S. Truncik, S. Uchaikin, J. Wang, B. Wilson, G. Rose, Quantum annealing with manufactured spins, Nature 473 (2011) 194–198. doi:10.1038/nature10012.

[24] A. Peruzzo, J. McClean, P. Shadbolt, M.-H. Yung, X.-Q. Zhou, P. J. Love, A. Aspuru-Guzik, J. L. O'brien, A variational eigenvalue solver on a photonic quantum processor, Nature communications 5 (2014) 4213.

[25] E. Farhi, J. Goldstone, S. Gutmann, A quantum approximate optimization algorithm, 2014. arXiv:1411.4028.

[26] E. Farhi, A. W. Harrow, Quantum supremacy through the quantum approximate optimization algorithm, arXiv preprint arXiv:1602.07674 (2016).

[27] S. Hadfield, Z. Wang, B. O'gorman, E. G. Rieffel, D. Venturelli, R. Biswas, From the quantum approximate optimization algorithm to a quantum alternating operator ansatz, Algorithms 12 (2019) 34.

[28] M. Motta, C. Sun, A. T. Tan, M. J. O'Rourke, E. Ye, A. J. Minnich, F. G. Brandao, G. K.-L. Chan, Determining eigenstates and thermal states on a quantum computer using quantum imaginary time evolution, Nature Physics 16 (2020) 205–210.

[29] A. Narayanan, M. Moore, Quantum-inspired genetic algorithms, in: Proceedings of IEEE international conference on evolutionary computation, IEEE, 1996, pp. 61–66.

[30] H. Goto, K. Endo, M. Suzuki, Y. Sakai, T. Kanao, Y. Hamakawa, R. Hidaka, M. Yamasaki, K. Tatsumura, High-performance combinatorial optimization based on classical mechanics, Science Advances 7 (2021) eabe7953.

[31] J. Alcazar, M. Ghazi Vakili, C. B. Kalayci, A. Perdomo-Ortiz, Enhancing combinatorial optimization with classical and quantum generative models, Nature Communications 15 (2024) 2761.

[32] S.-N. Sun, M. Motta, R. N. Tazhigulov, A. T. Tan, G. K.-L. Chan, A. J. Minnich, Quantum computation of finite-temperature static and dynamical properties of spin systems using quantum imaginary time evolution, PRX Quantum 2 (2021) 010317.

[33] A. Abbas, A. Ambainis, C. Augustino, A. Bärtschi, H. Buhrman, C. Coffrin, et al., Challenges and opportunities in quantum optimization, Nature Reviews Physics 6 (2024) 718–735. doi:10.1038/s42254-024-00770-9.

[34] A. Ciacco, F. Guerriero, G. Macrina, Review of quantum algorithms for medicine, finance and logistics, Soft Computing 29 (2025) 2129–2170.

[35] A. Das, B. K. Chakrabarti (Eds.), Quantum Annealing and Related Optimization Methods, volume 679 of *Lecture Notes in Physics*, Springer, Berlin, Heidelberg, 2005. doi:10.1007/11526216.

[36] B. Tasseff, T. Albash, Z. Morrell, M. Vuffray, A. Y. Lokhov, S. Misra, C. Coffrin, On the emerging potential of quantum annealing hardware for combinatorial optimization, Journal of Heuristics 30 (2024) 325–358. doi:10.1007/s10732-024-09530-5.

[37] M. Cerezo, A. Arrasmith, R. Babbush, S. C. Benjamin, S. Endo, K. Fujii, J. R. McClean, K. Mitarai, X. Yuan, L. Cincio, P. J. Coles, Variational quantum algorithms, Nature Reviews Physics 3 (2021) 625–644. doi:10.1038/s42254-021-00348-9.

[38] K. Blekos, D. Brand, A. Ceschini, C.-H. Chou, R.-H. Li, K. Pandya, A. Summer, A review on quantum approximate optimization algorithm and its variants, Physics Reports 1068 (2024) 1–66. doi:10.1016/j.physrep.2024.03.002.

[39] B. C. B. Symons, D. Galvin, E. Sahin, V. Alexandrov, S. Mensa, A practitioner's guide to quantum algorithms for optimisation problems, Journal of Physics A: Mathematical and Theoretical 56 (2023) 453001. doi:10.1088/1751-8121/ad00f0.

[40] Y. Du, H. Wang, R. Hennig, A. Hulandageri, G. Kochenberger, F. Glover, New advances for quantum-inspired optimization, International Transactions in Operational Research 32 (2025) 6–17. doi:10.1111/itor.13420.

[41] Pooja, S. K. Sood, Scientometric analysis of quantum-inspired metaheuristic algorithms, Artificial Intelligence Review 57 (2024). doi:10.1007/s10462-023-10659-1.

[42] D. Venturelli, D. Marchand, G. Rojo, Job shop scheduling solver based on quantum annealing, in: Proc. of ICAPS-16 Workshop on Constraint Satisfaction Techniques for Planning and Scheduling (COPLAS), 2016, pp. 25–34.

[43] L. F. Pérez Armas, S. Creemers, S. Deleplanque, Solving the resource constrained project scheduling problem with quantum annealing, Scientific Reports 14 (2024) 16784. doi:10.1038/s41598-024-67168-6.

[44] C. Li, Z. Liu, Y. Song, H. Liu, H. Liu, X. Liu, Quantum computing approaches to optimize employee scheduling in multi-task call centers, in: The International Conference on Smart Manufacturing, Industrial & Logistics Engineering (SMILE), Springer, 2023, pp. 3–9.

[45] T. D. Tambunan, A. B. Suksmono, I. J. M. Edward, R. Mulyawan, Quantum annealing for vehicle routing problem with weighted segment, 2022. arXiv:2203.13469.

[46] T. V. Le, M. V. Nguyen, S. Khandavilli, T. N. Dinh, T. N. Nguyen, Quantum annealing approach for selective traveling salesman problem, in: ICC 2023-IEEE International Conference on Communications, IEEE, 2023, pp. 2686–2691.

[47] E. Osaba, E. Garrote, P. Miranda-Rodriguez, A. Ciacco, I. Cabanes, A. Mancisidor, Quantum-assisted automatic path-planning for robotic quality inspection in industry 4.0, in: 2025 IEEE International Conference on Quantum Computing and Engineering (QCE), volume 2, IEEE, 2025, pp. 388–389.

[48] A. Ciacco, F. Guerriero, F. P. Saccomanno, Quantum annealing for the two-level facility location problem, Future Generation Computer Systems 174 (2026) 107961.

[49] G. Malviya, B. AkashNarayanan, J. Seshadri, Logistics network optimization using quantum annealing, in: International Conference on Emerging Trends and Technologies on Intelligent Systems, Springer, 2023, pp. 401–413.

[50] R. Pérez-Castillo, F. Lorente, Hybrid quantum annealing for price optimization under cross-elastic demand, Computers & Operations Research (2026) 107472.

[51] P. Kumari, et al., Pricing optimization for e-commerce using quantum computing, Antony raj, Pricing Optimization for E-Commerce Using Quantum Computing (2024).

[52] M. G. de Andoin, E. Osaba, I. Oregi, E. Villar-Rodriguez, M. Sanz, Hybrid quantum-classical heuristic for the bin packing problem, in: Proceedings of the Genetic and Evolutionary Computation Conference Companion, 2022, pp. 2214–2222.

[53] M. Garcia-de Andoin, I. Oregi, E. Villar-Rodriguez, E. Osaba, M. Sanz, Comparative benchmark of a quantum algorithm for the bin packing problem, arXiv preprint arXiv:2207.07460 (2022).

[54] F. Neukart, G. Compostella, C. Seidel, D. V. Dollen, S. Yarkoni, B. Parney, Traffic flow optimization using a quantum annealer, ArXiv abs/1708.01625 (2017). doi:10.3389/fict.2017.00029.

[55] K. Ikeda, Y. Nakamura, T. Humble, Application of quantum annealing to nurse scheduling problem, Scientific Reports 9 (2019). doi:10.1038/s41598-019-49172-3.

[56] A. Ciacco, F. Guerriero, E. Osaba, Quantum annealing for staff scheduling in educational environments, in: 2026 International Conference on Quantum Communications, Networking, and Computing (QCNC), IEEE, 2026, pp. 630–637.

[57] E. Rieffel, D. Venturelli, B. O'Gorman, M. Do, E. Prystay, V. Smelyanskiy, A case study in programming a quantum annealer for hard operational planning problems, Quantum Information Processing 14 (2014) 1 – 36. doi:10.1007/s11128-014-0892-x.

[58] A. Ciacco, L. D. P. Pugliese, F. Guerriero, School network reorganization under educational and spatial constraints using classical and quantum optimization, arXiv preprint arXiv:2608.05427 (2026).

[59] K. Kurowski, T. Pecyna, M. Slysz, R. Różycki, G. Waligóra, J. Węglarz, Application of quantum approximate optimization algorithm to job shop scheduling problem, Eur. J. Oper. Res. 310 (2023) 518–528. doi:10.1016/j.ejor.2023.03.013.

[60] P. Vikstål, M. Svensson, M. Andersson, G. Johansson, G. Ferrini, Applying the quantum approximate optimization algorithm to the tail-assignment problem, Physical Review Applied (2019). doi:10.1103/physrevapplied.14.034009.

[61] M. Baioletti, F. Fagiolo, A. Oddi, R. Rasconi, Applying quantum approximate optimization algorithm to the oversubscribed satellite scheduling problem, J. Aerosp. Inf. Syst. 23 (2026) 382–392. doi:10.2514/1.i011645.

[62] W. Qian, R. A. M. Basili, M. M. Eshaghian-Wilner, A. Khokhar, G. Luecke, J. P. Vary, Comparative study of variations in quantum approximate optimization algorithms for the traveling salesman problem, Entropy 25 (2023) 1238. doi:10.3390/e25081238.

[63] K. Bell, A. Lowe, E. Coles, N. Ridgway, G. Marshall, A comparison of quadratic and higher-order representations for qaoa, arXiv preprint arXiv:2509.20127 (2025). doi:10.48550/arXiv.2509.20127.

[64] U. Azad, B. K. Behera, E. A. Ahmed, P. Panigrahi, A. Farouk, Solving vehicle routing problem using quantum approximate optimization algorithm, IEEE Transactions on Intelligent Transportation Systems 24 (2020) 7564–7573. doi:10.1109/tits.2022.3172241.

[65] S. Brandhofer, V. Dehn, G. Hellstern, M. Huls, Y. Ji, I. Polian, A. Bhatia, T. Wellens, Benchmarking the performance of portfolio optimization with qaoa, Quantum Information Processing 22 (2022). doi:10.1007/s11128-022-03766-5.

[66] C. Huot, K. Kea, T.-K. Kim, Y. Han, Enhancing knapsack-based financial portfolio optimization using quantum approximate optimization algorithm, IEEE Access 12 (2024) 183779–183791. doi:10.1109/access.2024.3506981.

[67] N. Slate, E. Matwiejew, S. Marsh, J. B. Wang, Quantum walk-based portfolio optimisation, Quantum 5 (2020) 513. doi:10.22331/q-2021-07-28-513.

[68] R. Herrman, P. C. Lotshaw, J. Ostrowski, T. Humble, G. Siopsis, Multi-angle quantum approximate optimization algorithm, Scientific Reports 12 (2021). doi:10.1038/s41598-022-10555-8.

[69] F. Arute, K. Arya, R. Babbush, D. Bacon, J. Bardin, R. Barends, S. Boixo, M. Broughton, B. Buckley, D. Buell, B. Burkett, N. Bushnell, Y. Chen, Z.-J. Chen, B. Chiaro, R. Collins, W. Courtney, S. Demura, A. Dunsworth, E. Farhi, A. Fowler, B. Foxen, C. Gidney, M. Giustina, R. Graff, S. Habegger, M. Harrigan, A. Ho, S. Hong, T. Huang, L. Ioffe, S. Isakov, E. Jeffrey, Z. Jiang, C. Jones, D. Kafri, K. Kechedzhi, J. Kelly, S. Kim, P. Klimov, A. Korotkov, F. Kostritsa, D. Landhuis, P. Laptev, M. Lindmark, M. Leib, E. Lucero, O. Martin, J. Martinis, J. McClean, M. McEwen, A. Megrant, X. Mi, M. Mohseni, W. Mruczkiewicz, J. Mutus, O. Naaman, M. Neeley, C. Neill, F. Neukart, H. Neven, M. Niu, T. O'Brien, B. O'Gorman, E. Ostby, A. Petukhov, H. Putterman, C. Quintana, P. Roushan, N. Rubin, D. Sank, K. Satzinger, A. Skolik, V. Smelyanskiy, D. Strain, M. Streif, K. J. Sung, M. Szalay, A. Vainsencher, T. White, Z. Yao, P. Yeh, A. Zalcman, L. Zhou, Quantum approximate optimization of non-planar graph problems on a planar superconducting processor, Nature Physics 17 (2020) 332 – 336. doi:10.1038/s41567-020-01105-y.

[70] S. Bravyi, A. Kliesch, R. Koenig, E. Tang, Hybrid quantum-classical algorithms for approximate graph coloring, Quantum 6 (2020) 678. doi:10.22331/q-2022-03-30-678.

[71] W. Van Dam, K. Eldefrawy, N. Genise, N. Parham, Quantum optimization heuristics with an application to knapsack problems, in: 2021 IEEE International Conference on Quantum Computing and Engineering (QCE), IEEE, 2021, pp. 160–170.

[72] L. Cellini, A. Macaluso, M. Lombardi, Qal-bp: an augmented lagrangian quantum approach for bin packing, Scientific Reports 14 (2023). doi:10.1038/s41598-023-50540-3.

[73] P. J. Christiansen, L. Binkowski, D. Ramacciotti, S. Wilkening, Quantum tree generator improves qaoa state-of-the-art for the knapsack problem, in: 2025 IEEE International Conference on Quantum Computing and Engineering (QCE), volume 1, IEEE, 2025, pp. 1–10.

[74] A. Ajagekar, F. You, Quantum computing for energy systems optimization: Challenges and opportunities, Energy (2019). doi:10.1016/j.energy.2019.04.186.

[75] H. Jing, Y. Wang, Y. Li, Data-driven quantum approximate optimization algorithm for power systems, Communications Engineering 2 (2023). doi:10.1038/s44172-023-00061-8.

[76] C. Boy, D. J. Wales, Energy landscapes for the quantum approximate optimization algorithm, Physical Review A (2024). doi:10.1103/physreva.109.062602.

[77] D. Amaro, M. Rosenkranz, N. Fitzpatrick, K. Hirano, M. Fiorentini, A case study of variational quantum algorithms for a job shop scheduling problem, EPJ Quantum Technology 9 (2022) 5.

[78] P. Vikstål, M. Grönkvist, M. Svensson, M. Andersson, G. Johansson, G. Ferrini, Applying the quantum approximate optimization algorithm to the tail-assignment problem, Physical Review Applied 14 (2020) 034009. doi:10.1103/PhysRevApplied.14.034009.

[79] S. Boulebnane, et al., Benchmarking the performance of portfolio optimization with qaoa, Quantum Information Processing (2022). doi:10.1007/s11128-022-03766-5.

[80] R. Yalovetzky, M. J. A. Schuetz, Z. He, J. Shen, Y. Sun, R. Raymond, S. Sahay, K. Perla, R. S. Andrist, G. Salton, H. G. Katzgraber, R. Bongiovanni, N. Kumar, R. Otter, Quantum-informed portfolio selection: An end-to-end pipeline validated on trapped-ion hardware with real market data, arXiv preprint arXiv:2607.01037 (2026). doi:10.48550/arXiv.2607.01037.

[81] S. Koretsky, P. Gokhale, J. M. Baker, J. Viszlai, H. Zheng, N. Gurung, R. Burg, E. A. Paaso, A. Khodaei, R. Eskandarpour, et al., Adapting quantum approximation optimization algorithm (qaoa) for unit commitment, in: 2021 IEEE International Conference on Quantum Computing and Engineering (QCE), IEEE, 2021, pp. 181–187.

[82] J. E. Avila-Jaimes, M. A. Gutiérrez-Limón, A. Flores-Tlacuahuac, A hybrid quantum–classical qaoa approach to the unit commitment/economic dispatch problem: A mexican power-grid case study, Chemical Engineering Research and Design 231 (2026) 23–38. doi:10.1016/j.cherd.2026.05.034.

[83] X. Zhao, J. Chen, L. Liu, L. Kang, Study on quantum approximation optimization algorithm in airport

cargo transportation problem, Journal of Advanced Transportation 2026 (2026) 4046578. doi:10.1155/atr/4046578.

[84] L. A. Moncayo-Martínez, Quantum optimisation for supply chain: Qubo formulations and qaoa solutions for facility location and load balancing, Results in Engineering 29 (2026) 108373. doi:10.1016/j.rineng.2025.108373.

[85] M. Federer, D. Müssig, S. Lenk, J. Lässig, Real-world application benchmark for qaoa algorithm for an electromobility use case, in: INFORMATIK 2022, Gesellschaft für Informatik, Bonn, 2022, pp. 1145–1151.

[86] B. Liu, Y. Zhou, Q. Luo, H.-J. Huang, Quantum-inspired african vultures optimization algorithm with elite mutation strategy for production scheduling problems, J. Comput. Des. Eng. 10 (2023) 1767–1789. doi:10.1093/jcde/qwad078.

[87] P. C. Su, S.-Y. Tan, Z.-Y. Liu, W. Yeh, A mixed-heuristic quantum-inspired simplified swarm optimization algorithm for scheduling of real-time tasks in the multiprocessor system, Appl. Soft Comput. 131 (2022) 109807. doi:10.1016/j.asoc.2022.109807.

[88] T. Werner, F. Ullinger, Quantum algorithms for scheduling problems: a survey, EPJ Quantum Technology 13 (2026) 31. doi:10.1140/epjqt/s40507-026-00494-y.

[89] P. Liu, S. Parkinson, K. Best, Quantum and quantum-inspired optimisation in transport and logistics: A systematic review, Smart Cities 8 (2025) 206. doi:10.3390/smartcities8060206.

[90] N. Slate, E. Matwiejew, S. Marsh, J. Wang, Quantum walk-based portfolio optimisation, Quantum 5 (2021) 513.

[91] M. Mullings, et al., Dynamic portfolio optimization with real datasets using quantum processors and quantum-inspired tensor networks, Physical Review Research 4 (2022) 013006. doi:10.1103/PhysRevResearch.4.013006.

[92] D. Lim, P. Rebentrost, A quantum online portfolio optimization algorithm, 2023. arXiv:2208.14749.

[93] H. Song, P. Liu, A study on the optimal flexible job-shop scheduling with sequence-dependent setup time based on a hybrid algorithm of improved quantum cat swarm optimization, Sustainability 14 (2022) 9547.

[94] Y. Xu, M. Zhang, M. Yang, D. Wang, Hybrid quantum particle swarm optimization and variable neighborhood search for flexible job-shop scheduling problem, Journal of Manufacturing Systems 73 (2024) 334–348.

[95] H. Chishima, Case study of production planning optimization using quantum-inspired technology, JSAI Technical Report 2024 (2024) 34–38.

[96] R. K. Singh, A. R. Singh, R. K. Yadav, A balanced-quantum inspired evolutionary algorithm for solving disassembly line balancing problem, Applied Soft Computing 132 (2023) 109840.

[97] S. Chandra Das, A. Zidan, A. K. Manna, A. A. Shaikh, A. K. Bhunia, An application of preservation technology in inventory control system with price dependent demand and partial backlogging, Alexandria Engineering Journal 59 (2020) 1359–1369. URL: https://www.sciencedirect.com/science/article/pii/S1110016820301058. doi:https://doi.org/10.1016/j.aej.2020.03.006, a Special Section on: Computational Methods in Engineering and Artificial Intelligence.

[98] N. Kumar, A. K. Manna, A. A. Shaikh, A. K. Bhunia, Application of hybrid binary tournament-based quantum-behaved particle swarm optimization on an imperfect production inventory problem, Soft Computing 25 (2021) 11245–11267.

[99] D. Leib, T. Seidel, S. Jäger, R. Heese, C. I. Jones, A. Awasthi, A. E. Niederle, M. Bortz, An optimization case study for solving a transport robot scheduling problem on quantum-hybrid and quantum-inspired hardware, Scientific Reports 13 (2023) 18743. doi:10.1038/s41598-023-45668-1.

[100] F. Takano, M. Suzuki, Y. Kobayashi, T. Araki, Optimization of logistics using quantum-inspired technology, NEC Technical Journal 18 (2025). URL: https://www.nec.com/en/global/techrep/journal/g24/n01/240111.html.

[101] Y. Bai, Y. Sui, X. Deng, X. Wang, Quantum-inspired robust optimization for coordinated scheduling of pv-hydrogen microgrids under multi-dimensional uncertainties, Scientific Reports 15 (2025) 29589. doi:10.1038/s41598-025-12280-4.

[102] A. Ciacco, F. Guerriero, E. Osaba, Steiner traveling salesman problem with quantum annealing, in: Proceedings of the Genetic and Evolutionary Computation Conference Companion, Association for Computing Machinery, New York, NY, USA, 2025, pp. 2412–2418. URL: https://doi.org/10.1145/3712255.3734313.

[103] G. Orlandi, D. Volpe, M. Graziano, G. Turvani, Qoolchain: A qubo preprocessing toolchain for enhancing quantum optimization, Advanced Quantum Technologies 8 (2025) 2400384. doi:10.1002/qute.202400384.

[104] A. Ciacco, L. Di Puglia Pugliese, F. Guerriero, Traveling salesman problem with a preprocessing method for classical and quantum optimization, in: 2026 IEEE International Conference on Smart Computing Workshops and Other Affiliated events (SmartComp Companion), 2026, pp. 114–119. doi:10.1109/SmartComp-Companion70724.2026.00036.

[105] I. Castillo, et al., Optimization of a refinery scheduling process with column generation and a quantum annealer, Optimization and Engineering (2021). doi:10.1007/s11081-021-09662-8.

[106] J. Ossorio-Castillo, F. Pena-Brage, Optimization of a refinery scheduling process with column generation and a quantum annealer, Optimization and Engineering 23 (2022) 1471–1488.

[107] G. Bass, M. P. Henderson, J. Heath, J. Dulny, Optimizing the optimizer: decomposition techniques for quantum annealing, Quantum Machine Intelligence 3 (2020). doi:10.1007/s42484-021-00039-9.

[108] J. Ling, Q. Zhang, G.-C. Geng, Q.-Y. Jiang, Hybrid quantum annealing decomposition framework for unit commitment, Electric Power Systems Research (2025). doi:10.1016/j.epsr.2024.111121.

[109] F. Wagner, F. Liers, Quantum subroutines in branch-price-and-cut for vehicle routing, IEEE Transactions on Quantum Engineering (2026). doi:10.1109/TQE.2026.3688001.

[110] A. Ciacco, L. D. P. Pugliese, F. Guerriero, Cutting-plane methodology via quantum optimization for solving the traveling salesman problem, 2026. URL: https://arxiv.org/abs/2604.20321. arXiv:2604.20321.

[111] T. Heister, et al., A quantum constraint generation framework for binary linear programs, EPJ Quantum Technology (2025). doi:10.1140/epjqt/s40507-025-00364-z.

[112] D. J. Egger, J. Marecek, S. Woerner, Warm-starting quantum optimization, Quantum 5 (2021) 479. doi:10.22331/q-2021-06-17-479.

[113] R. Tate, M. Farhadi, C. Herold, G. Mohler, S. Gupta, Bridging classical and quantum with sdp initialized warm-starts for qaoa, ACM Transactions on Quantum Computing 4 (2023) 9. doi:10.1145/3549554.

[114] Y. Yu, X.-B. Wang, N. Shannon, R. Joynt, Warm-start adaptive-bias quantum approximate optimization algorithm, Physical Review A 112 (2025) 012422. doi:10.1103/nt3w-j4mj.

[115] R. Tate, J. Moondra, B. Gard, G. Mohler, S. Gupta, Warm-started qaoa with custom mixers provably converges and computationally beats goemans-williamson's max-cut at low circuit depths, Quantum 7 (2023) 1121. doi:10.22331/q-2023-09-26-1121.

[116] J. Sud, S. Hadfield, E. Rieffel, N. Tubman, T. Hogg, Parameter-setting heuristic for the quantum alternating operator ansatz, Physical Review Research 6 (2024) 023171. doi:10.1103/PhysRevResearch.6.023171.

[117] T. Hao, X. Huang, C. Jia, C. Peng, A quantum-inspired tensor network algorithm for constrained combinatorial optimization problems, Frontiers in Physics 10 (2022) 906590. doi:10.3389/fphy.2022.906590.

[118] M. Gili, et al., Enhancing combinatorial optimization with classical and quantum generative models, Nature Communications 15 (2024). doi:10.1038/s41467-024-46959-5.

[119] S. Patra, S. Singh, S. Jahromi, R. Orus, Quantum-inspired tensor network optimizer and method associated therewith, U.S. Patent Application No. 18/955,107, 2026.

[120] J.-R. Jiang, Q.-Y. Lin, Decomposition of large-scale quadratic unconstrained binary optimization problems for quantum annealers and quantum-inspired annealers, in: The 7th Eurasia Conference on IoT, Communication and Engineering, volume 134, 2026, p. 29. doi:10.3390/engproc2026134029.

[121] H. Oshiyama, M. Ohzeki, Benchmark of quantum-inspired heuristic solvers for quadratic unconstrained binary optimization, Scientific Reports 12 (2022) 2146. doi:10.1038/s41598-022-06070-5.

[122] J. R. Jiang, Y. C. Shu, Q. Y. Lin, Benchmarks and recommendations for quantum, digital, and gpu annealers in combinatorial optimization, IEEE Access 12 (2024) 125014–125031. doi:10.1109/ACCESS.2024.3455436.

[123] M. Ragone, et al., The quantum optimization benchmarking library, Nature Computational Science 6 (2026) 653–671. doi:10.1038/s43588-026-00991-1.

[124] D-Wave Quantum Inc., D-wave announces commercial milestones and advantage2 architecture roadmap updates, 2026. Official Press Release.

[125] D-Wave Quantum Inc., D-wave reports second quarter 2026 results, 2026. Accessed September 2026.

[126] D-Wave Quantum Inc., D-wave charts a new course to fault-tolerant quantum computing with gate-model roadmap, 2026. Accessed September 2026.

[127] IBM, Ibm quantum roadmap, 2026. Accessed September 2026.

[128] IBM Quantum, Ibm quantum roadmap: From nisq utility to fault-tolerant quantum supercomputing, 2026. Corporate Announcement.

[129] IBM, Ibm releases a new blueprint for quantum-centric supercomputing, 2026. Accessed September 2026.

[130] Google Quantum AI, Meet willow, our state-of-the-art quantum chip, 2024. Accessed September 2026.

[131] Google Quantum AI, Google quantum ai: The road to fault tolerance, 2026. Accessed September 2026.

[132] Microsoft, Microsoft quantum hardware, 2026. Accessed September 2026.

[133] Microsoft, Majorana 2: Microsoft's scalable quantum processor with reliable, long-lasting qubits, 2026. Accessed September 2026.

[134] IonQ, Fault-Tolerant Quantum Computing with Trapped Ions: The Walking Cat Architecture, 2026. Technical report.

[135] IonQ, IonQ Launches Superion Product Line: Industry-Leading, Upgradeable Platform Designed to Scale Manufacturable Fault-Tolerant Quantum Computing, 2026. Accessed: 2026-10-19.

[136] IonQ, IonQ to Advance Quantum Supercomputing by Bringing First QPU to NVIDIA Accelerated Quantum Research Center, 2026. Press release.

[137] Quantinuum, Quantinuum unveils accelerated roadmap to achieve universal, fully fault-tolerant quantum computing by 2030, 2024. Accessed September 2026.

[138] Quantinuum, Quantinuum overcomes last major hurdle to deliver scalable universal fault-tolerant quantum computers by 2029, 2025. Accessed September 2026.

[139] Rigetti Computing, Rigetti announces general availability of 108-qubit system, 2026. Accessed September 2026.

[140] Rigetti Computing, Rigetti computing provides update on 108-qubit system, 2026. Accessed September 2026.

[141] Amazon Web Services, Aws deepens strategic collaboration with quera to bring fault-tolerant quantum computing to amazon braket, 2026. Accessed September 2026.

[142] Amazon Web Services, Ocelot: A new architecture for scalable quantum error correction, 2025. Accessed September 2026.

[143] Fujitsu, Technology strategy to support customer business growth, 2024. Accessed September 2026.

[144] Fujitsu, Quantum computing explainer, 2025. Accessed September 2026.

[145] Intel, Quantum computing systems achieving quantum practicality, 2026. Accessed September 2026.

[146] Hitachi, Ltd., Hitachi launches r&d on silicon quantum computers in collaboration with intel k.k. and aist to accelerate the shift from academic demonstration to industrial application, 2026. Accessed September 2026.

[147] PsiQuantum, PsiQuantum Announces Omega, a Manufacturable Chipset for Photonic Quantum Computing, 2025. Company announcement.

[148] B. Bartlett, et al., A manufacturable platform for photonic quantum computing, Nature 638 (2025) 651–657. doi:10.1038/s41586-025-08820-7.

[149] Toshiba Corporation, Toshiba releases 3rd generation simulated bifurcation algorithm for high-speed combinatorial optimization, 2026. Technology News Release.